QSIT 2013 - Questions 6

26. April 2013, HIT F 13

1. **Resonator as a quantum bus: entanglement of two qubits**

   Consider a transmission line resonator which is coupled to two well separated transmon qubits. The total Hamiltonian of this system is written as

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) - \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_{i,z} + \hbar \sum_{i=1,2} g_i(a^\dagger \sigma_i^- + a\sigma_i^+). \quad (1)$$

   In this Hamiltonian, there is no direct qubit-qubit interaction term. However, resonator can mediate a coupling between them. To illustrate this we write the relevant hamiltonian in the dispersive regime:

$$H = \hbar\left(\omega_\mathrm{r} + \chi_1\sigma_{1,z} + \chi_2\sigma_{2,z}\right) a^\dagger a + \sum_{i=1,2} \hbar\frac{\chi_i - \omega_i}{2}\sigma_{i,z} + \hbar J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+). \quad (2)$$

   Here, $\chi_i = \frac{g_i^2}{\omega_i - \omega_\mathrm{r}}$ and $J = \frac{1}{2}g_1 g_2 \left(\frac{1}{\omega_1 - \omega_\mathrm{r}} + \frac{1}{\omega_2 - \omega_\mathrm{r}}\right)$.

   What kind of two qubit gates can be implemented with this Hamiltonian?