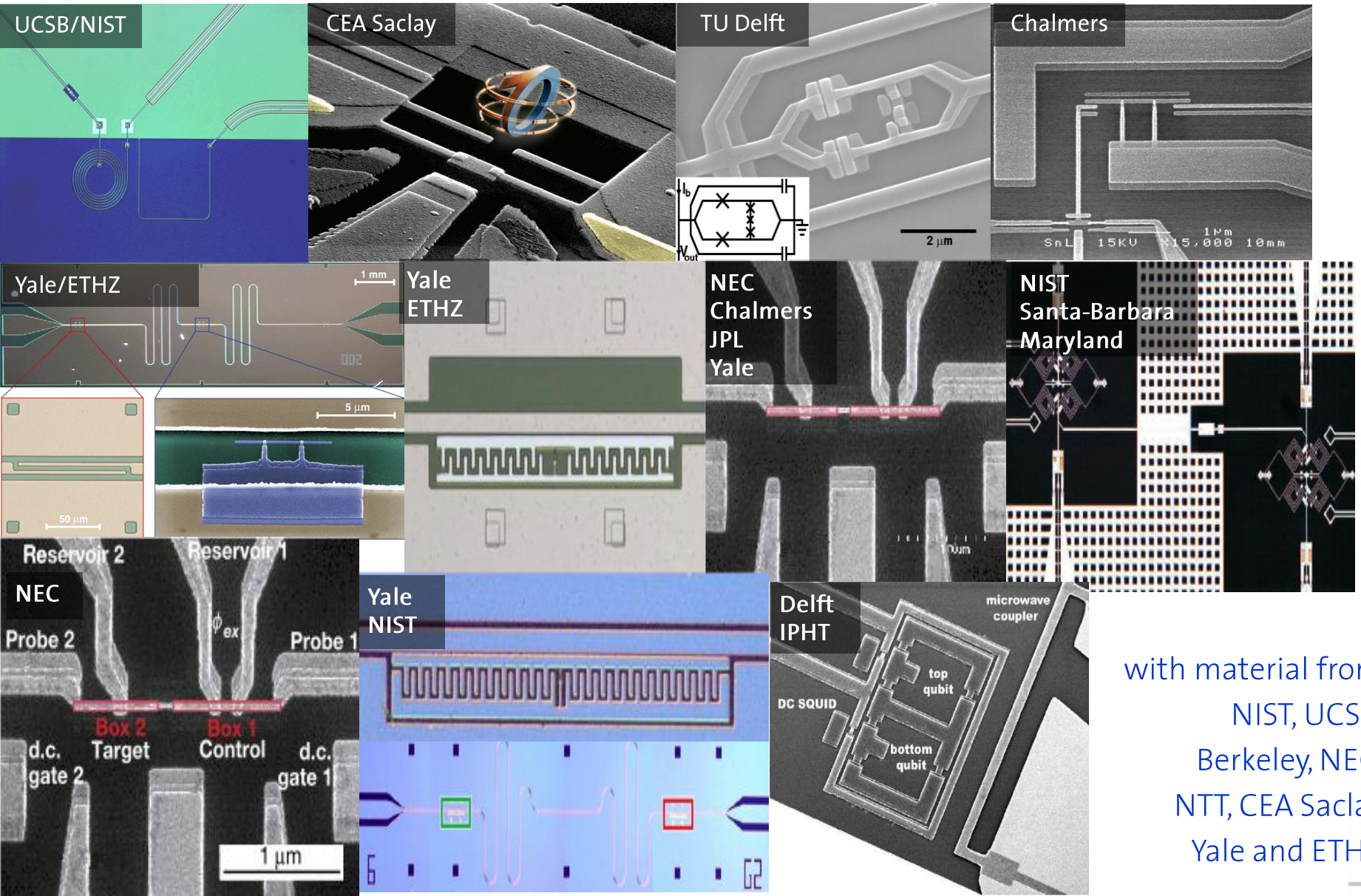


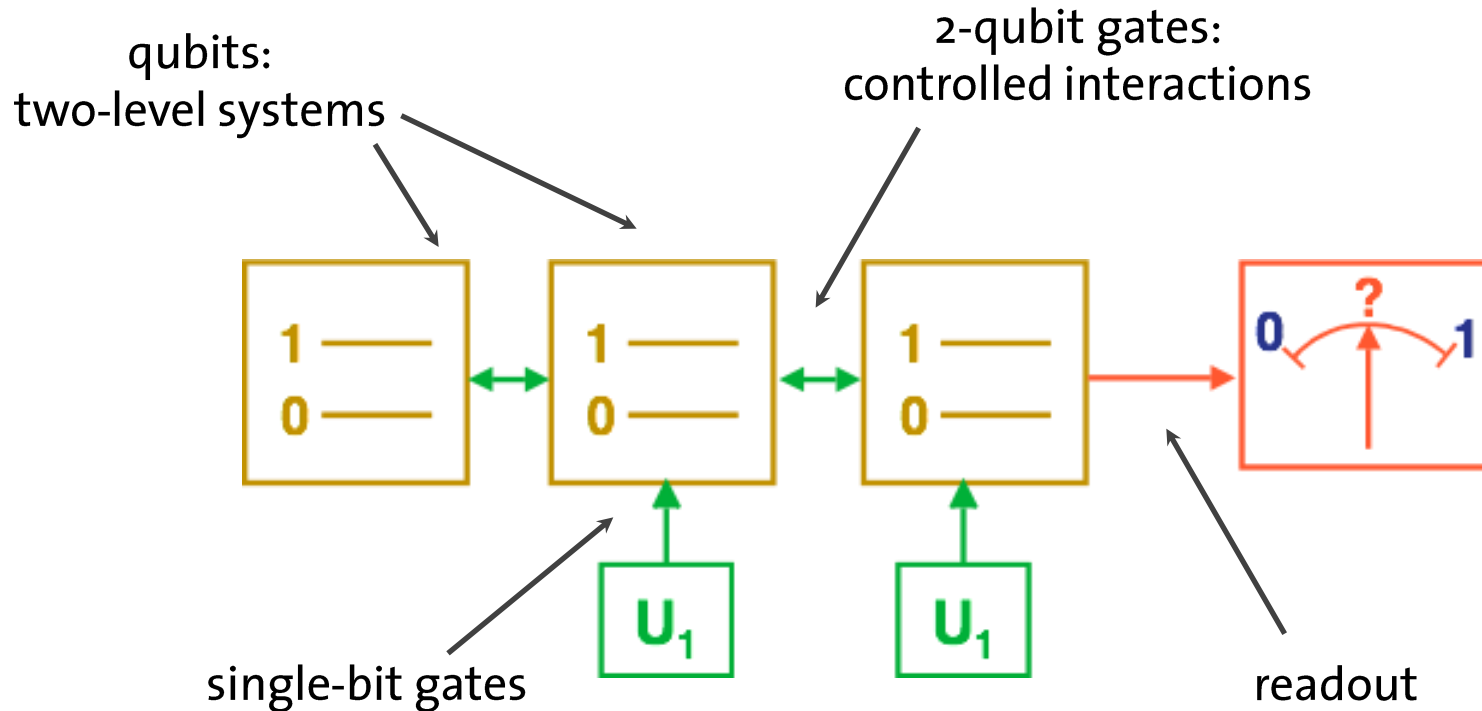
# Building a Quantum Information Processor using Superconducting Circuits



with material from  
 NIST, UCSB,  
 Berkeley, NEC,  
 NTT, CEA Saclay,  
 Yale and ETHZ

# Generic Quantum Information Processor

The challenge:



- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

# The DiVincenzo Criteria

for Implementing a quantum computer in the standard (circuit approach) to quantum information processing (QIP):

- #1. A scalable physical system with well-characterized qubits.
- #2. The ability to initialize the state of the qubits.
- #3. Long (relative) decoherence times, much longer than the gate-operation time.
- #4. A universal set of quantum gates.
- #5. A qubit-specific measurement capability.

plus two criteria requiring the possibility to transmit information:

- #6. The ability to interconvert stationary and mobile (or flying) qubits.
- #7. The ability to faithfully transmit flying qubits between specified locations.

- realization of superconducting quantum electronic circuits
  - harmonic oscillators (photons)
  - non-harmonic oscillators (qubits)
- controlled qubit/photon interactions
  - cavity quantum electrodynamics with circuits
- qubit read-out
- single qubit control
- decoherence
- two-qubit interactions
  - generation of entanglement
  - realization of quantum algorithms

# Conventional Electronic Circuits

basic circuit elements:

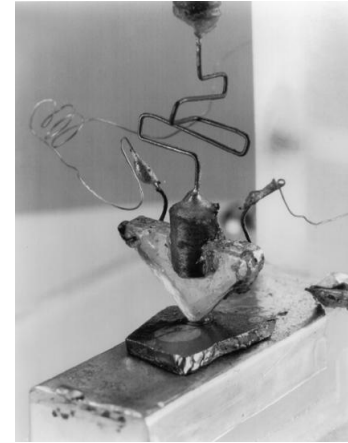


basis of modern  
information and  
communication  
technology

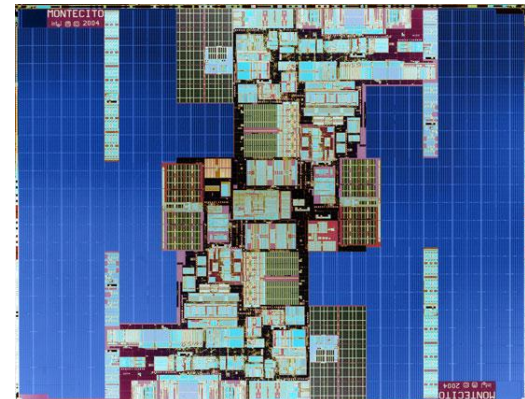
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



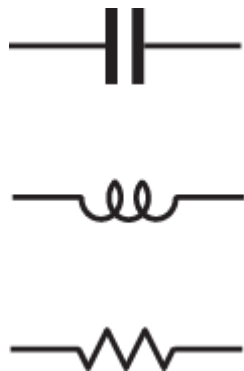
intel dual core processor (2006)



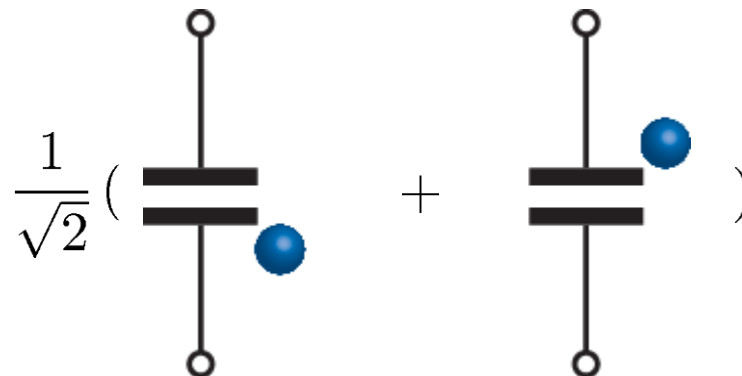
2.000.000.000 transistors  
smallest feature size 65 nm  
clock speed ~ 2 GHz  
power consumption 10 W

# Classical and Quantum Electronic Circuit Elements

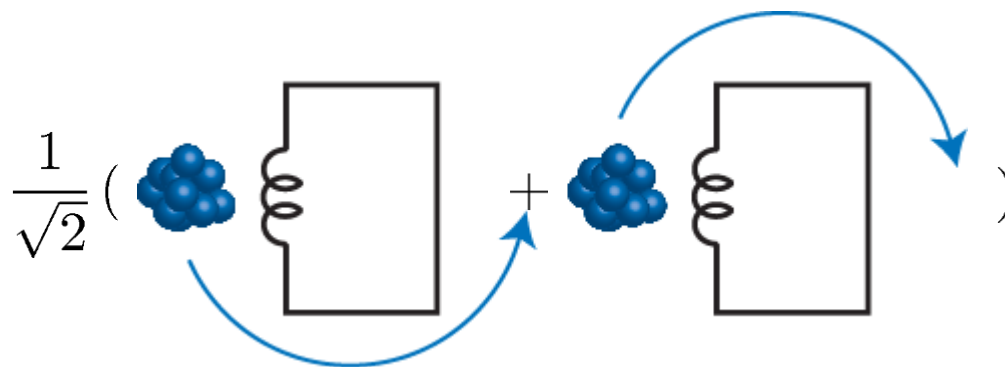
basic circuit elements:



charge on a capacitor:



current or magnetic flux in an inductor:

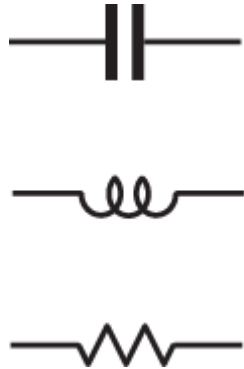


quantum superposition states:

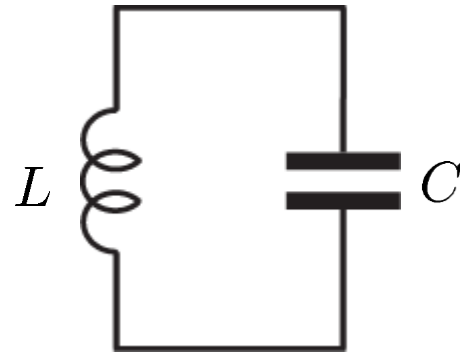
- charge  $q$
- flux  $\phi$

# Constructing Linear Quantum Electronic Circuits

basic circuit elements:

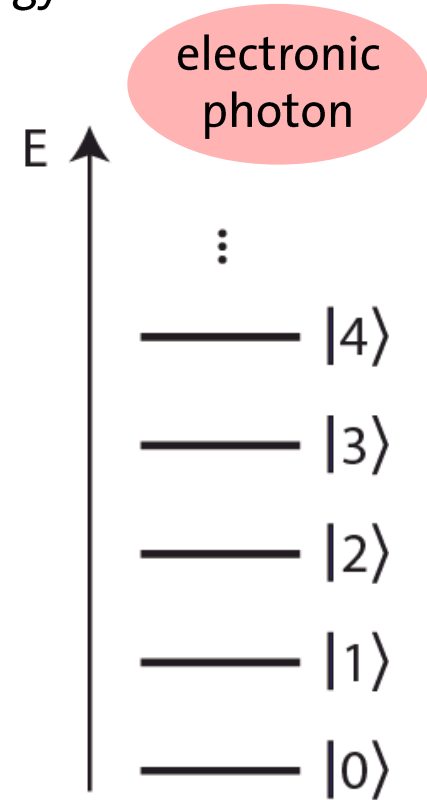


harmonic LC oscillator:



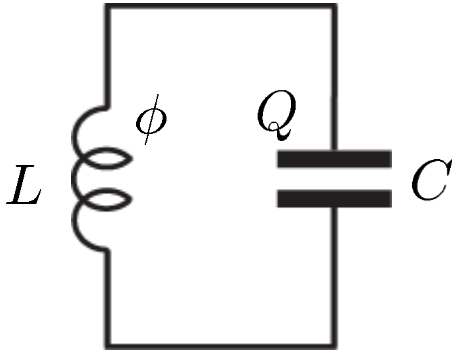
$$\omega = \frac{1}{\sqrt{LC}} \sim 5 \text{ GHz}$$

energy:



# Quantization of an Electronic Harmonic Oscillator

Harmonic LC oscillator:



$$Q = CV$$

Charge on capacitor

$$\phi = LI$$

Flux in inductor

$$V = -L\dot{I} = -\dot{\phi}$$

Voltage across inductor

Classical Hamiltonian:

$$H = \frac{CV^2}{2} + \frac{LI^2}{2} = \frac{Q^2}{2C} + \frac{\phi^2}{2L}$$

Conjugate variables:

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = V = -L\dot{I} = -\dot{\phi}$$

Hamilton operator:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C}$$

Flux and charge operator:

$$\hat{\phi} = \phi$$
$$\hat{Q} = -i\hbar \frac{\partial}{\partial \phi}$$

Commutation relation:

$$[\hat{\phi}, \hat{q}] = i\hbar$$



# Creation and Annihilation Operators for Circuits

Hamilton operator of harmonic oscillator in second quantization:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2)$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle \quad \text{Creation operator}$$

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle \quad \text{Annihilation operator}$$

$$\hat{a}^\dagger\hat{a} |n\rangle = n |n\rangle \quad \text{Number operator}$$

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_C}}(\hat{a}^\dagger + \hat{a}) \quad \text{Charge/voltage operator} \quad \hat{V} = \frac{\hat{Q}}{C}$$

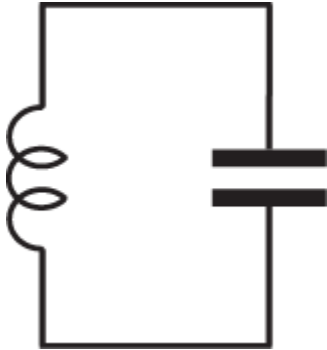
$$\hat{\phi} = i\sqrt{\frac{\hbar Z_C}{2}}(\hat{a}^\dagger - \hat{a}) \quad \text{Flux/current operator} \quad \hat{I} = \frac{\hat{\phi}}{L}$$

With characteristic impedance:

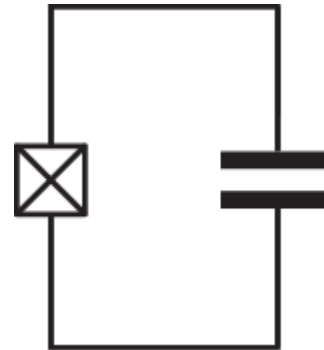
$$Z_C = \sqrt{\frac{L}{C}}$$

# Linear vs. Nonlinear Superconducting Oscillators

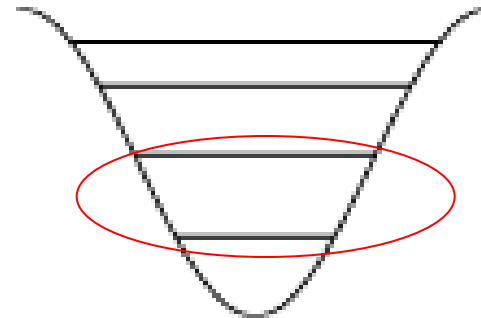
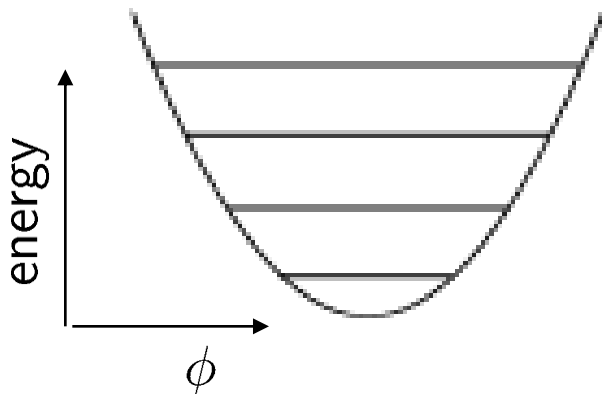
LC resonator:



Josephson junction resonator:  
Josephson junction = nonlinear inductor

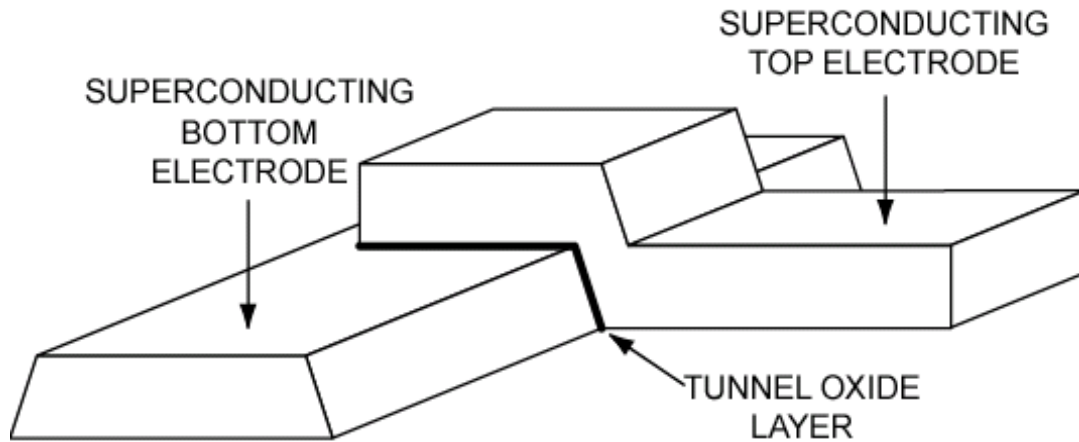


anharmonicity defines effective two-level system



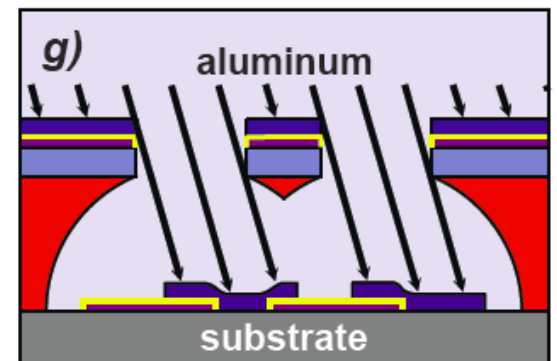
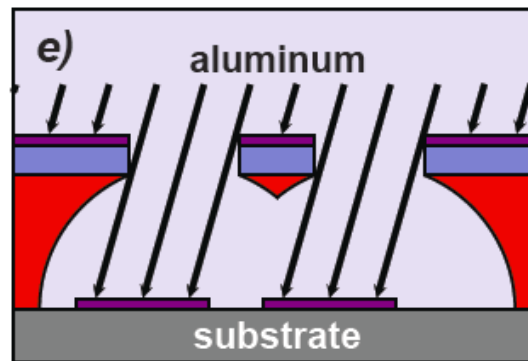
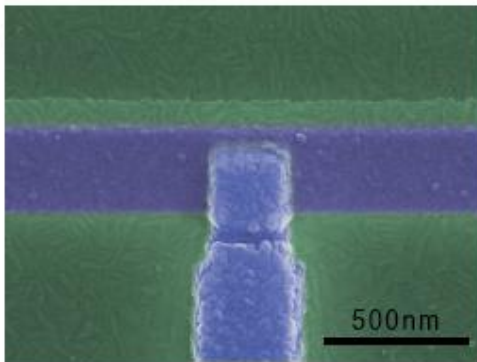
# A Low-Loss Nonlinear Element

a (superconducting) Josephson junction:



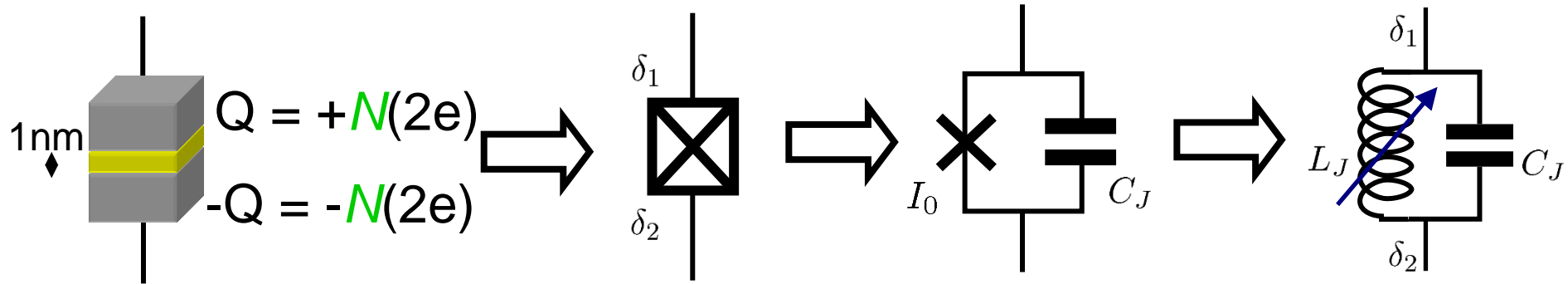
- superconductors: Nb, Al
- tunnel barrier:  $\text{AlO}_x$

Josephson junction fabricated by shadow evaporation:



# Josephson Tunnel Junction

The only non-linear resonator with no dissipation (BCS,  $k_B T < \Delta$ )



Tunnel junction parameters:

- Critical current  $I_0$
- Junction capacitance  $C_J$
- Internal resistance  $R_J$

Josephson relations:  $I = I_0 \sin \delta$

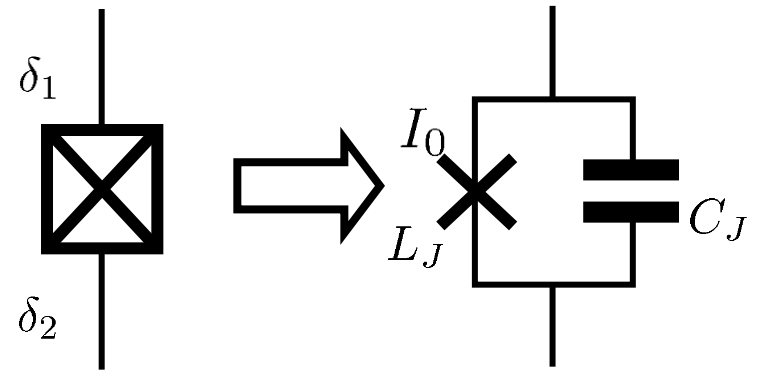
$$V = \frac{\phi_0}{2\pi} \dot{\delta}$$

Flux quantum:  $\phi_0 = \frac{h}{2e}$

Phase difference:  $\delta = \delta_2 - \delta_1$

# The Josephson Junction as an ideal Non-Linear Inductor

a nonlinear inductor without dissipation



Josephson relations:

$$I = I_0 \sin \delta = I_0 \sin [2\pi\phi(t)/\phi_0]$$

nonlinear  
current/phase  
relation

$$V = \frac{\phi_0}{2\pi} \dot{\delta} = \dot{\phi}$$

gauge inv. phase difference:

$$\delta = \delta_2 - \delta_1 = 2\pi\phi(t)/\phi_0$$

Josephson inductance:

$$V = -L_J \dot{I} = \frac{\phi_0}{2\pi I_0} \frac{1}{\cos \delta} \dot{I}$$

specific Josephson  
inductance  $L_{J0}$

Josephson energy:

$$E_J = \int V I dt = \frac{I_0 \phi_0}{2\pi} \cos \delta$$

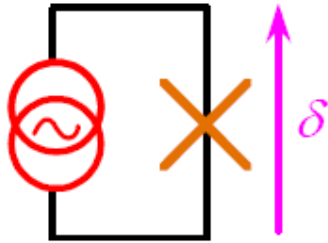
specific Josephson  
energy  $E_{J0}$

# A Classification of Josephson Junction Based Qubits

How to make use in of Jospelson junctions in a qubit?

Common options of bias (control) circuits:

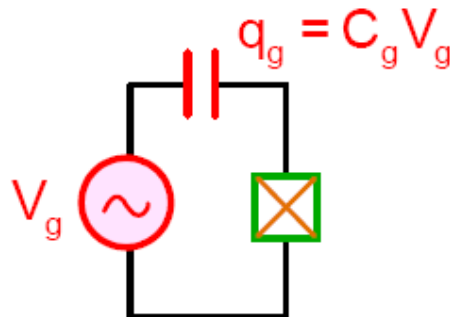
phase qubit



current bias

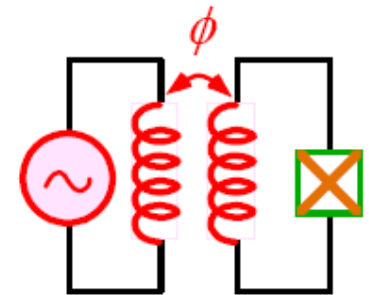
charge qubit

(Cooper Pair Box, Transmon)



charge bias

flux qubit



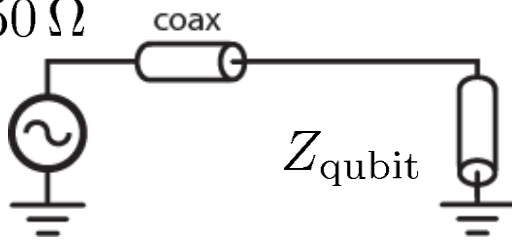
flux bias

How is the control circuit important?

# Control of Coupling to Electromagnetic Environment

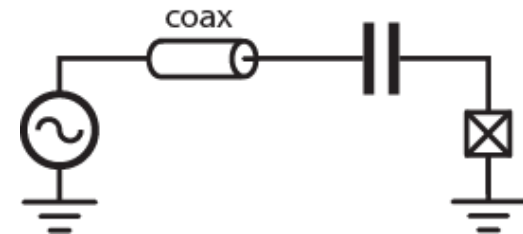
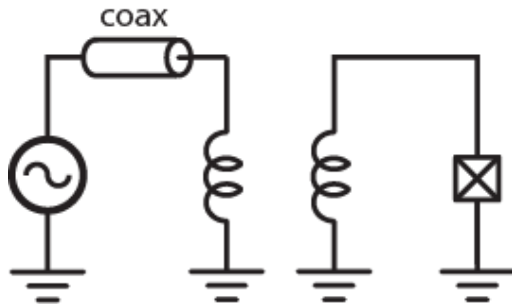
coupling to environment (bias wires):

$$Z_{\text{line}} \sim 50 \Omega$$

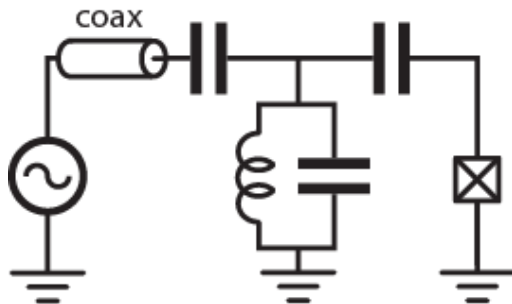


decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...

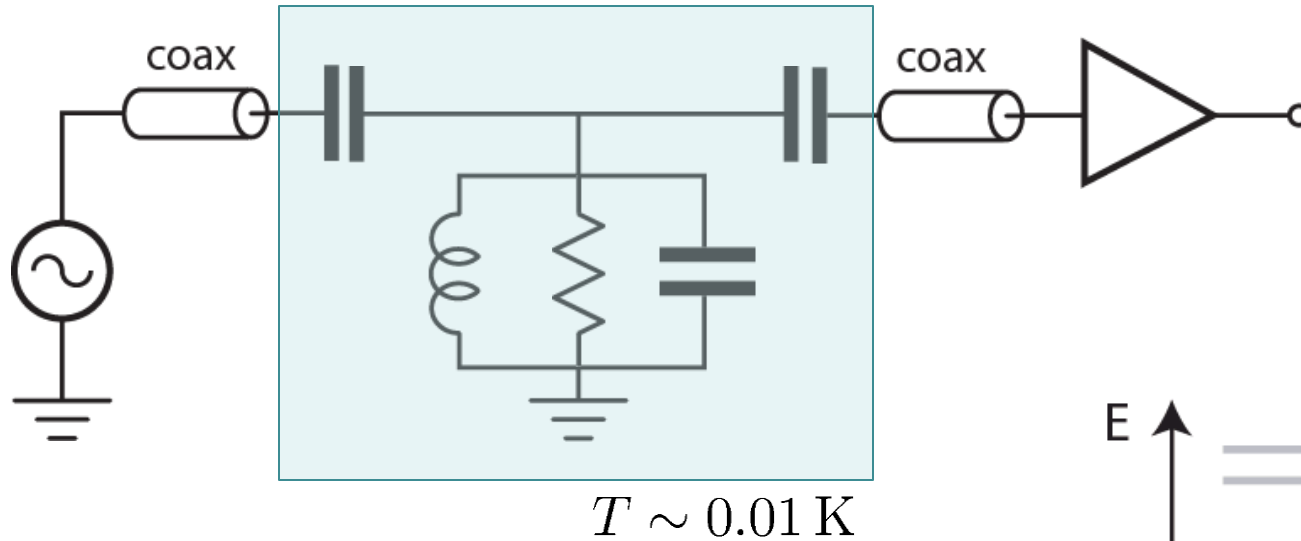


... or resonant impedance transformers



control spontaneous emission by circuit design

# How to Operate Circuits Quantum Mechanically?



recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

