Estimating the Average Fidelity on a NMR Quantum Computer

Based on "Experimental Estimation of Average Fidelity of a Clifford Gate on a 7-Qubit Quantum Processor" [1]

John McCann       Marlon Azinović

ETH Zürich

QSIT Lecture, 2016
Outline

- Why?
  - Clifford Gates
- How (Theory)?
  - Standard Way
  - Better Way
    - Twirling protocol
- Reminder NMR
- How (Experiment)?
- Results
Why do we want to know the fidelity?

- In the construction of any gate, imperfections and noise are inevitable.
- We might hence consider a physical gate as \( \tilde{U} = \Lambda \circ U \)
- The fidelity characterises the quality of a gate

\[
F = \langle \psi | \Lambda (|\psi\rangle \langle \psi|) |\psi\rangle
\]
What are Clifford Gates?

The Clifford Group of operators on n qubits is defined as

\[ C_n = \{ U \in U(2^n) | \sigma \in \pm P_n \Rightarrow U \sigma U^\dagger \in \pm P_n \} / U(1) \]

where \( P_n \) are the (non-identity) n-qubit Pauli Matrices.

For one qubit, this just corresponds to permuting the axes on the Bloch sphere.

This includes common gates such as Hadamard and Phase gates:

\[
\begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}, \quad \begin{pmatrix}
1 & 0 \\
0 & i
\end{pmatrix}
\]
Why are we interested in Clifford Gates?

- For $n$ qubits, the Clifford Group is generated by Hadamard, Phase and CNOT gates.
- Many common gates are Clifford Gates, e.g. Error Correction.
- Universal Quantum Computation can be achieved using only Clifford Gates and the ability to create certain states.
- We thus might be interested in characterising the Fidelity of such gates.
Why can we not just make quantum process tomography?
Why can we not just make quantum process tomography?

→ because it needs too many measurements

How many measurements does quantum process tomography need?
Remember: quantum state tomography

- Let $|\psi\rangle \in \mathcal{H}$ (Hilbert space of n-qubits)
- What’s the dimension of $\mathcal{H}$?
Remember: quantum state tomography

- Let $|\psi\rangle \in \mathcal{H}$ (Hilbert space of n-qubits)
- What’s the dimension of $\mathcal{H}$?
  \[ d := \dim(\mathcal{H}) = 2^n \]
- Performing quantum state tomography on state $|\psi\rangle$ means estimating the density matrix $\rho = |\psi\rangle \langle \psi|$. Remember $\rho$ lives in space of positive operators with trace one.
- Basis: $\{1, \sigma_x, \sigma_y, \sigma_z\}$ (dimension $d^2$ TRUE?)
How can we get $\rho$?

One qubit case

We expand (one qubit case): 

$$\rho = \frac{1}{2} \left( \text{Tr}(\rho) 1 + \text{Tr}(\sigma_x \rho) \sigma_x + \text{Tr}(\sigma_y \rho) \sigma_y + \text{Tr}(\sigma_z \rho) \sigma_z \right)$$

where:

- $\text{Tr}(\rho) = 1$
- $\text{Tr}(\sigma_i \rho), i \in \{x, y, z\}$ is experimentally obtainable. Remember: $\text{Tr}(\sigma_i \rho) =$ average value when performing measurement $\sigma_i$

How many measurements do we hence need?
How can we get $\rho$?

One qubit case

We expand (one qubit case):

$$\rho = \frac{1}{2}(\text{Tr}(\rho)1 + \text{Tr}(\sigma_x \rho)\sigma_x + \text{Tr}(\sigma_y \rho)\sigma_y + \text{Tr}(\sigma_z \rho)\sigma_z)$$

where:

- $\text{Tr}(\rho) = 1$
- $\text{Tr}(\sigma_i \rho), i \in \{x, y, z\}$ is experimentally obtainable.
  Remember: $\text{Tr}(\sigma_i \rho) =$ average value when performing measurement $\sigma_i$

How many measurements do we hence need?

Error of mean goes with $\mathcal{O}\left(\frac{1}{\sqrt{\# \text{ measurements}}}\right) \rightarrow$ if $m$

measurements enough for precision:

$$3m = (2^2 - 1)m = \mathcal{O}(d^2 - 1)$$
How can we get $\rho$?

n qubit case

- We expand (n qubit case):
  $\rho = \frac{1}{d} \sum_{\{i_1, \ldots, i_n\}} \text{Tr} (\sigma_{i_1} \otimes \ldots \otimes \sigma_{i_n}) \sigma_{i_1} \otimes \ldots \otimes \sigma_{i_n}$
  where $i_j \in \{1, \sigma_x, \sigma_y, \sigma_z\}$ and:
    - $\text{Tr}(\rho) = 1$
    - $\text{Tr}(\sigma_i \rho), i \in \{x, y, z\}$ is experimentally obtainable.
      Remember: $\text{Tr}(\sigma_i \rho) = \text{average value when performing measurement } \sigma_i$

- How many measurements do we hence need?
How can we get $\rho$?

n qubit case

- We expand (n qubit case):
  \[
  \rho = \frac{1}{d} \sum_{\{i_1, \ldots, i_n\}} \text{Tr}(\sigma_{i_1} \otimes \cdots \otimes \sigma_{i_n}) \sigma_{i_1} \otimes \cdots \otimes \sigma_{i_n}
  \]
  where $i_j \in \{1, \sigma_x, \sigma_y, \sigma_z\}$ and:
  - $\text{Tr}(\rho) = 1$
  - $\text{Tr}(\sigma_i \rho), \ i \in \{x, y, z\}$ is experimentally obtainable.
  Remember: $\text{Tr}(\sigma_i \rho)$ = average value when performing measurement $\sigma_i$

- How many measurements do we hence need?
  \[
  4^n - 1 = (2^n)^2 - 1 = d^2 - 1
  \]
  averages need to be estimated.
How can we get $\rho$?

n qubit case

We expand (n qubit case):

$$\rho = \frac{1}{d} \sum_{\{i_1, \ldots, i_n\}} \text{Tr}(\sigma_{i_1} \otimes \cdots \otimes \sigma_{i_n})\sigma_{i_1} \otimes \cdots \otimes \sigma_{i_n}$$

where $i_j \in \{1, \sigma_x, \sigma_y, \sigma_z\}$ and:

- $\text{Tr}(\rho) = 1$
- $\text{Tr}(\sigma_i \rho), i \in \{x, y, z\}$ is experimentally obtainable. Remember: $\text{Tr}(\sigma_i \rho) =$ average value when performing measurement $\sigma_i$

How many measurements do we hence need?

$$4^n - 1 = (2^n)^2 - 1 = d^2 - 1$$

averages need to be estimated.

Measurements needed for quantum state tomography:

$$\rightarrow \mathcal{O}(d^2)$$
Question: does our process do what we want him to do?

Situation:

- We have a process: \( \rho \rightarrow \mathcal{E}^{\text{have}}(\rho) \)
- We want to be able to put the action of the process into numbers (to compare how far it is from \( \rho \rightarrow \mathcal{E}^{\text{ideal}}(\rho) \)).
- A map \( \rho \rightarrow \mathcal{E}(\rho) \) is determined by its action on the \( d^2 \) elements of a basis set of the set of matrices.
Number of measurements required for quantum process tomography

- We generate \( d^2 \) pure input states \( |\psi_1\rangle, \ldots, |\psi_{d^2}\rangle \), whose density matrices \( \rho_1 := |\psi_1\rangle \langle \psi_1|, \ldots, \rho_{d^2} := |\psi_{d^2}\rangle \langle \psi_{d^2}| \) form a basis for the matrix space of \( \rho \).
- For all \( \rho_i \), we determine the output of \( \mathcal{E}^{\text{have}}(\rho_i) \) with quantum state tomography (\( \mathcal{O}(d^2) \) measurements).
Number of measurements required for quantum process tomography

- We generate $d^2$ pure input states $|\psi_1\rangle, \ldots, |\psi_{d^2}\rangle$, whose density matrices $\rho_1 := |\psi_1\rangle \langle \psi_1|, \ldots, \rho_{d^2} := |\psi_{d^2}\rangle \langle \psi_{d^2}|$ form a basis for the matrix space of $\rho$.
- For all $\rho_i$, we determine the output of $\mathcal{E}^{\text{have}}(\rho_i)$ with quantum state tomography ($\mathcal{O}(d^2)$ measurements).

Measurements needed for quantum process tomography:

$\mathcal{O}(d^4) = \mathcal{O}(2^{4n})$
We generate \( d^2 \) pure input states \(|\psi_1\rangle, \ldots, |\psi_{d^2}\rangle\), whose density matrices \( \rho_1 := |\psi_1\rangle \langle \psi_1 |, \ldots, \rho_{d^2} := |\psi_{d^2}\rangle \langle \psi_{d^2} | \) form a basis for the matrix space of \( \rho \).

For all \( \rho_i \), we determine the output of \( E^{\text{have}}(\rho_i) \) with quantum state tomography (\( \mathcal{O}(d^2) \) measurements).

Measurements needed for quantum process tomography:

\[ \mathcal{O}(d^4) = \mathcal{O}(2^{4n}) \]

for \( n = 7 \rightarrow 2^{4n} = 268435456 \approx 2.7 \times 10^8 \)

has been done for at most 3 qubits (date 10th of April 2015)
What can we do then?

- We would like to benchmark a gate acting on 7 qubits
- The average fidelity of our gate is enough for us
- We don’t mind if it has to consist from Clifford gates
What can we do then?

- We would like to benchmark a gate acting on 7 qubits.
- The average fidelity of our gate is enough for us.
- We don’t mind if it has to consist from Clifford gates.
  → we can use the twirling protocol.
How does the twirling protocol work?

Some things to know:

- It works to estimate the average fidelity of clifford gates.
- A superoperator $\mathcal{U}$ is an linear operator acting on the space of linear operators.
- In our case: superoperator $\mathcal{U}$ acts on the density matrix $\rho$.
- $\mathcal{U}(\rho)$ is the resulting density matrix after applying gate $\mathcal{U}$ to the state $\rho$. Correct??
- Superoperator we want: $\mathcal{U}$
  
  Superoperator we have: $\tilde{\mathcal{U}} = \Lambda \circ \mathcal{U}$, where $\Lambda$ is the noise superoperator.
- In the end we want: a good enough estimate of the average fidelity $\bar{F}(\mathcal{U}, \tilde{\mathcal{U}})$ without making too many measurements.
The average Fidelity between $\tilde{U}$ and $U$:

$$F(\tilde{U}, U) = \int d\mu(\psi) \langle \psi | U^\dagger \tilde{U} (|\psi\rangle \langle \psi |) |\psi\rangle$$

$$= \int d\mu(\psi) \langle \psi | U^\dagger \circ \Lambda \circ U (|\psi\rangle \langle \psi |) |\psi\rangle$$

$$= \int d\mu(\psi) \langle \psi | \Lambda (|\psi\rangle \langle \psi |) |\psi\rangle$$

$$= F(\Lambda)$$

Equivalently: $d\mu(\psi)$ (sums over states) $\leftrightarrow d\mu(\mathcal{V})$ (sums over random unitaries)[2].

$$\tilde{F}(\Lambda) = \int d\mu(\mathcal{V}) \langle \psi | \mathcal{V}^\dagger \circ \Lambda \circ \mathcal{V}(|\psi\rangle \langle \psi |) |\psi\rangle = \langle \psi | \int d\mu(\mathcal{V}) \mathcal{V}^\dagger \circ \Lambda \circ \mathcal{V}(|\psi\rangle \langle \psi |) |\psi\rangle$$

$$= \tilde{F}(\tilde{\Lambda}). \text{ where } \tilde{\Lambda} := \int d\mu(\mathcal{V}) \mathcal{V}^\dagger \circ \Lambda \circ \mathcal{V}$$
Twirling mathematics 2

- Trick: replace integral by sum over a finite set of unitaries \( C_i \) in the finite n-qubit Clifford group \( C_n \) [3]:
  \[
  \bar{\Lambda} := \int d\mu(\mathcal{V}) \mathcal{V}^\dagger \circ \Lambda \circ \mathcal{V} \to \bar{\Lambda}_{C_n} := \frac{1}{|C_n|} \sum_{C_i \in C_n} C_i^\dagger \circ \Lambda \circ C_i
  \]
  \[
  \Rightarrow \bar{F}(\Lambda) = F(\bar{\Lambda}) = F(\bar{\Lambda}_{C_n}) = \langle \psi | \bar{\Lambda}_{C_n} (|\psi\rangle \langle \psi|) |\psi\rangle
  \]
Twirling mathematics 2

- Trick: replace integral by sum over a finite set of unitaries $C_i$ in the finite n-qubit Clifford group $C_n$ [3]:
  \[ \tilde{\Lambda} := \int d\mu(V) V^\dagger \circ \Lambda \circ V \rightarrow \tilde{\Lambda} C_n := \frac{1}{|C_n|} \sum_{C_i \in C_n} C_i^\dagger \circ \Lambda \circ C_i \]

- $\tilde{\Lambda} C_n$ is called the $C_n$ twirl of $\Lambda$. The twirl $C_n$ twirl of any superoperator $\Lambda$ is a *depolarizing channel*.

- Thus, can be written as ($P_0 :=$ probability of no error) [4]:
  \[ \tilde{\Lambda} C_n (\rho) = P_0 \rho + (1 - P_0) \frac{1}{2^n} \]
Twirling mathematics 2

- Trick: replace integral by sum over a finite set of unitaries $C_i$ in the finite n-qubit Clifford group $C_n$ [3]:
  \[ \overline{\Lambda} := \int d\mu(V)V^\dagger \circ \Lambda \circ V \rightarrow \overline{\Lambda}_{C_n} := \frac{1}{|C_n|} \sum_{C_i \in C_n} C_i^\dagger \circ \Lambda \circ C_i \]

- $\overline{F}(\Lambda) = F(\overline{\Lambda}) = F(\overline{\Lambda}_{C_n}) = \langle \psi | \overline{\Lambda}_{C_n}(|\psi \rangle \langle \psi |) |\psi \rangle$

- $\overline{\Lambda}_{C_n}$ is called the $C_n$ twirl of $\Lambda$. The twirl $C_n$ twirl of any superoperator $\Lambda$ is a depolarizing channel.

- Thus, can be written as ($P_0 :=$ probability of no error) [4]:
  \[ \overline{\Lambda}_{C_n}(\rho) = P_0 \rho + (1 - P_0) \frac{1^\otimes n}{2^n} \]

- Hence:

**Average Fidelity**

\[ \overline{F}(\Lambda) = F(\overline{\Lambda}_{C_n}) = \frac{2^n P_0 + 1}{2^n + 1} \]
Twirling mathematics 2

- Trick: replace integral by sum over a finite set of unitaries $C_i$ in the finite n-qubit Clifford group $C_n$ [3]:
  \[ \bar{\Lambda} := \int d\mu(V)V^\dagger \circ \Lambda \circ V \rightarrow \bar{\Lambda}_{C_n} := \frac{1}{|C_n|} \sum_{C_i \in C_n} C_i^\dagger \circ \Lambda \circ C_i \]

- $\bar{\Lambda}_{C_n}$ is called the $C_n$ twirl of $\Lambda$. The twirl $C_n$ twirl of any superoperator $\Lambda$ is a depolarizing channel.

- Thus, can be written as ($P_0 :=$ probability of no error) [4]:
  \[ \bar{\Lambda}_{C_n}(\rho) = P_0 \rho + (1 - P_0) \frac{1^\otimes 2^n}{2^n} \]

- Hence:

  **Average Fidelity**

  \[ \bar{F}(\Lambda) = F(\bar{\Lambda}_{C_n}) = \frac{2^n P_0 + 1}{2^n + 1} \]

- Now we need a technique to measure $P_0$
Twirling mathematics 3

- \( Pr(w) \) denotes the probability that a Pauli error of weight \( w \) occurs.
- \( \tilde{U}_C = \Lambda \circ U_C \) denotes an arbitrary faulty Clifford gate.
- It can be shown that [4, 5]:

\[
P_0 = Pr(0) = \frac{1}{4^n} \left( 1 + \frac{1}{2^n} \sum_{i=1}^{4^n-1} \text{Tr}(\Lambda(\rho_i)\rho_i) \right)
\]

where \( \rho_i \in \mathcal{P}_n \) (\( \mathcal{P}_n \) denotes the Pauli group and contains \( 4^n - 1 \) elements.)
Twirling mathematics 3

- $Pr(w)$ denotes the probability that a Pauli error of weight $w$ occurs.
- $\tilde{U}_C = \Lambda \circ U_C$ denotes an arbitrary faulty Clifford gate.
- It can be shown that [4, 5]:

$$P_0 = Pr(0) = \frac{1}{4^n} \left( 1 + \frac{1}{2^n} \sum_{i=1}^{4^n-1} \text{Tr}(\Lambda(\rho_i)\rho_i) \right)$$

where $\rho_i \in \mathcal{P}_n$ ($\mathcal{P}_n$ denotes the Pauli group and contains $4^n - 1$ elements.)

Hence:

$$P_0 = Pr(0) = \frac{1}{4^n} \left( 1 + \frac{1}{2^n} \sum_{i=1}^{4^n-1} \text{Tr}(U_C(\rho_i)\tilde{U}_C(\rho_i)) \right)$$
So how many measurements do we need?

- At most $4^n - 1 = \mathcal{O}(d^2)$, for $n = 7$, $4^7 - 1 = 16384$
- Denote:
  - $\text{Prob}(\epsilon)$: probability of event $\epsilon : |\bar{x} - \mu| > \delta$
  - $m$: number of measurements
- Hoeffding’s inequality:
  \[
  \text{Prob}(\epsilon) \leq 2e^{-2\delta^2 m}
  \]
- $\Rightarrow$ number of needed measurements $m$:
  \[
  m \leq \frac{\ln(2/\text{Prob}(\epsilon))}{2\delta^2}
  \]
So how many measurements do we need?

- At most $4^n - 1 = \mathcal{O}(d^2)$, for $n = 7$, $4^7 - 1 = 16384$
- Denote:
  - $\text{Prob}(\epsilon)$: probability of event $\epsilon : |\bar{x} - \mu| > \delta$
  - $m$: number of measurements
- Hoeffding’s inequality:

  \[
  \text{Prob}(\epsilon) \leq 2e^{-2\delta^2 m}
  \]

- $\Rightarrow$ number of needed measurements $m$:

  \[
  m \leq \frac{\ln(2/\text{Prob}(\epsilon))}{2\delta^2}
  \]

For given $\text{Prob}(\epsilon)$ and $\delta$:

Number of measurements needed is independent of number of qubits!

For $\epsilon = 1\%$ and $\delta = 0.04 \Rightarrow m \leq 1656$.
Remember: full quantum process tomography: $m \approx 2.7 \times 10^6$
How do we perform the experiments?

The gate $\mathcal{U}$ of interest we choose evolves $\rho_1 = ZI^{\otimes 6}$ to $\rho_2 = Z^{\otimes 7}$

$$\rho_3 = |0\rangle \langle 0|^{\otimes 7} + |1\rangle \langle 1|^{\otimes 7} \quad \text{and} \quad \rho_4 = |0\rangle \langle 0|^{\otimes 6} \otimes Z_7$$
## Molecule we use

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>30020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>57.58</td>
<td>8779</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>-2.00</td>
<td>32.70</td>
<td>6245</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>0</td>
<td>0.30</td>
<td>0</td>
<td>10333</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>1.25</td>
<td>2.62</td>
<td>-1.11</td>
<td>33.16</td>
<td>15745</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>5.54</td>
<td>-1.66</td>
<td>0</td>
<td>-3.53</td>
<td>33.16</td>
<td>34381</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>-1.25</td>
<td>37.48</td>
<td>0.94</td>
<td>29.02</td>
<td>21.75</td>
<td>34.57</td>
<td>11928</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1</td>
<td>0</td>
<td>0</td>
<td>2.36</td>
<td>166.6</td>
<td>4.06</td>
<td>5.39</td>
<td>8.61</td>
<td>3310</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>4.41</td>
<td>1.86</td>
<td>146.6</td>
<td>2.37</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2468</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>1.81</td>
<td>3.71</td>
<td>146.6</td>
<td>2.37</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>-12.41</td>
<td>2158</td>
<td></td>
</tr>
<tr>
<td>H4</td>
<td>-13.19</td>
<td>133.6</td>
<td>-6.97</td>
<td>6.23</td>
<td>0</td>
<td>5.39</td>
<td>3.78</td>
<td>-0.68</td>
<td>1.28</td>
<td>6.00</td>
<td>2692</td>
<td></td>
</tr>
<tr>
<td>H5</td>
<td>7.87</td>
<td>-8.35</td>
<td>3.35</td>
<td>8.13</td>
<td>2.36</td>
<td>8.52</td>
<td>148.5</td>
<td>8.46</td>
<td>-1.06</td>
<td>-0.36</td>
<td>1.30</td>
<td>3649</td>
</tr>
<tr>
<td>T2</td>
<td>1.611</td>
<td>0.877</td>
<td>1.122</td>
<td>0.792</td>
<td>1.143</td>
<td>1.912</td>
<td>0.531</td>
<td>0.337</td>
<td>N/A</td>
<td>N/A</td>
<td>0.318</td>
<td>0.276</td>
</tr>
</tbody>
</table>

**Dichloro-cyclobutanone**

C-13 labeled 12-qubit system
How do we perform the experiments?

We are interested in isolating just the error caused by the gate

- After creating the state, we calibrate it by comparing its nmr signal to that of the equilibrium state
- Error due to decoherence was calculated theoretically and removed from the measured result
## Results

<table>
<thead>
<tr>
<th>Weight</th>
<th>Number of Expt.</th>
<th>Calibration</th>
<th>Average Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k</td>
<td>k_w</td>
<td>t (ms)</td>
</tr>
<tr>
<td>w=1</td>
<td>21</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>w=2</td>
<td>189</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>w=3</td>
<td>945</td>
<td>101</td>
<td>34</td>
</tr>
<tr>
<td>w=4</td>
<td>2835</td>
<td>272</td>
<td>49</td>
</tr>
<tr>
<td>w=5</td>
<td>5103</td>
<td>505</td>
<td>53</td>
</tr>
<tr>
<td>w=6</td>
<td>5103</td>
<td>524</td>
<td>55</td>
</tr>
<tr>
<td>w=7</td>
<td>2187</td>
<td>229</td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>16383</td>
<td>1656</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Results

Estimating the Average Fidelity on a NMR Quantum Computer
Results

\[ \bar{F}(\Lambda) = \frac{2^n P_0 + 1}{2^n + 1} \]

Measured value of \( P_0 = 87.4\% \) gives fidelity \( \bar{F} = 87.5\% \).

- Decomposing the gate into 18 1- and 2-qubit gates gives average fidelity of 99\% or 96\% before accounting for decoherence
- The best previous nmr result on 3-qubit gates was 99\% (86\%)
- Xmon 2-qubit gates have shown 99.0-99.4\% fidelities
- This is the only such gate characterisation for more than 3-qubits
Summary

- Clifford gates were introduced and their importance was highlighted.
- We showed how to estimate the average fidelity of Clifford gates using a number of measurements which is constant in the number of qubits by employing the twirling protocol.
- We described the measurement setup for measuring the average fidelity of a 7-qubit NMR system.
- Finally we analysed the results and found that they compared favourably to other experiments.
Thank you for your attention!
Thank you for your attention!

Questions?

or

Comments?
Experimental estimation of average fidelity of a clifford gate on a 7-qubit quantum processor.

Scalable noise estimation with random unitary operators.

Exact and approximate unitary 2-designs and their application to fidelity estimation.

*Suppression and characterization of decoherence in practical quantum information processing devices.*

Liquid crystal state nmr quantum computing - characterization, control and certification.