Physics IV - Mock Exam - SS 2007

13:45 – 15:45, 20th June 2007

Please note:

- There are a total of 8 questions printed on **THREE PAGES**.
- There are a total of 60 points. Points for each part of each question are shown in square brackets in the right margin.
- A table of values of physical constants and useful formulae is printed on the back of this cover sheet.
- You are allowed 10 sides of handwritten notes, and a non-programmable calculator in the exam.
- Please write CLEARLY, as if we cannot read your handwriting we cannot award you marks.
- Please WRITE YOUR NAME BELOW. This sheet will be stapled to your answers at the end of the exam.

NAME:	FIRST NAME:

Question:	1	2	3	4	5	6	7	8	TOTAL
Marks:									
Max:	6	6	4	14	9	11	5	5	60

Table of physical constants

speed of light, c	3.00×10^{8} ms^{-1}
Planck constant, h	6.63×10^{-34} Js
$h/(2\pi)$	1.05×10^{-34} Js
electron charge, e	1.60×10^{-19} C
electron volt, eV	1.60×10^{-19} J
electron mass, m_e	9.11×10^{-31} kg
atomic mass unit, m_u,	1.66×10^{-27} kg
Boltzmann constant, k_B	1.38×10^{-23} JK^{-1}
Rydberg constant, R_∞	1.10×10^{7} m^{-1}
Bohr magneton, μ_B	9.27×10^{-24} JT^{-1}
Stefan-Boltzmann constant, σ	5.67×10^{-8} Wm^{-2}K^{-4}

Useful formulae

$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

1. The photoelectric effect occurs when photons incident on a metal surface cause the emission of electrons.

   (a) Sketch a graph of the maximum kinetic energy E_{max} of the ejected electrons as a function of the frequency of the incident light ν, for two metals with different work functions ϕ_1 and $\phi_2 = 2\phi_1$, explaining the main features. [3]

   (b) The work function of a tungsten metal surface is $\phi = 5.4$ eV. When the surface is illuminated by light of wavelength $\lambda = 175$ nm, the maximum electron energy is $E_{max} = 1.7$ eV. Use this data to find Planck's constant h. [3]

2. Electrons of kinetic energy 50 eV are diffracted by a simple cubic crystal of lattice constant 0.22 nm.

   (a) Estimate the angle θ to the first diffraction maximum (measured from the normal to the surface). [4]

   (b) What would be the energy of X-rays diffracted at the same angle? [2]

3. What property does the combined wavefunction of two fermions have? Prove that two identical fermions cannot occupy the same single particle state. [4]

4. A spin-1/2 particle is in a cubic box, with side a. The potential energy is given by:

$$\begin{array}{ll} V(x,y,z) = 0 & 0 < x < a,\ 0 < y < a,\ 0 < z < a \\ V(x,y,z) = \infty & \text{elsewhere.} \end{array}$$

   The possible wavefunctions of the particle are given by

$$\psi = A \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right) \sin\left(\frac{n_z \pi z}{a}\right).$$

   (a) Find the normalisation constant A. [4]

   (b) Use the time independent Schrödinger equation in three dimensions to find the possible energies of the particle. [3]

   (c) Taking spin into account, what is the degeneracy of the first excited state of the particle? [3]

   (d) The particle is in its ground state. What is the probability of finding it inside the region defined by $0 \le x \le a/4$, $0 \le y \le a/4$, $0 \le z \le a/4$? How does this compare to the classical probability (assuming that the classical particle has non-zero energy)? [4]

5. The time dependent Schrödinger equation is

$$i\hbar \frac{d}{dt}\psi = H\psi.$$

Consider a quantum mechanical system with two energy eigenstates ψ_1 and ψ_2, with energies E_1 and E_2 respectively (i.e. $H\psi_1 = E_1\psi_1$ and $H\psi_2 = E_2\psi_2$).

(a) At time $t = 0$, the system is in state ψ_1. Solve the Schrödinger equation to find the wavefunction at later times $\psi(t)$. [3]

(b) An observable operator A has eigenvalues a_1 and a_2, with corresponding eigenstates:

$$\phi_1 = (\psi_1 + i\psi_2)/\sqrt{2}, \qquad \phi_2 = (\psi_1 - i\psi_2)/\sqrt{2},$$

$$A\phi_1 = a_1\phi_1, \qquad A\phi_2 = a_2\phi_2.$$

Consider now that at time $t = 0$, the system is in state ψ_1, and A is measured. What are the possible results and their probabilities? [2]

(c) Assuming a_1 is the result, how does the probability $p_1(t)$ of measuring a_1, now vary as a function of time? *[Hint: you can factor out a global phase.]* [4]

6. In a high magnetic field, atomic energy levels are shifted according to the Zeeman effect, by an energy

$$\Delta E = \frac{eB}{2m_e}(L_z + 2S_z)$$

(a) Work out the possible values of ΔE for the $n = 1$ and $n = 2$ energy levels in Hydrogen. [3]

(b) Draw a diagram of only the $1s$ and $2p$ energy levels without Zeeman splitting (you may ignore fine structure), and a second diagram showing the Zeeman split levels. Label the new levels by their quantum numbers m_l and m_s. [3]

(c) Remembering the selection rules $\Delta m_l = 0, \pm 1$, and $\Delta m_s = 0$, draw the allowed transitions between the split $2p$ and $1s$ levels on your previous diagram. How many different spectral lines does this give rise to? [3]

(d) In a magnetic field of $B = 10$ T, what is the frequency splitting of the spectral lines? [2]

7. The lowest frequency in the rotational absorption spectrum of the molecule HF is 1.25×10^{12} Hz. Estimate the bond length in the molecule, given that the mass of F is 19 m_u. [5]

8. A solar panel on the earth's surface of area 10 m^2 transforms solar radiation into electrical power, at an efficiency of 30%. The earth is 150×10^6 km from the sun, the sun's temperature is 6000 K, and its radius is 7×10^5 km. Neglecting absorption of radiation by the atmosphere, what is the solar panel's maximum power output? [5]