

# Physics IV - Final Exam - SS 2007

9:00 – 11:00, Thursday, 30th August 2007

Please note:

- There are a total of 7 questions printed on **FIVE PAGES**.
- There are a total of 60 points. Points for each part of each question are shown in square brackets in the right margin.
- A table of values of physical constants and useful formulae is printed on the back of this cover sheet.
- You are allowed 10 sides of handwritten notes, and a non-programmable calculator in the exam.
- Please write **CLEARLY**, as if we cannot read your handwriting we cannot award you marks.
- Please **WRITE YOUR NAME BELOW**. This sheet will be stapled to your answers at the end of the exam.
- Please put your name on all of your answer sheets.
- Throughout the exam the exam overseers are available to answer your questions, do not hesitate to ask for clarification if needed.

NAME:	FIRST NAME:

Question:	1	2	3	4	5	6	7	TOTAL
Marks:								
Max:	8	12	10	7	11	6	6	60

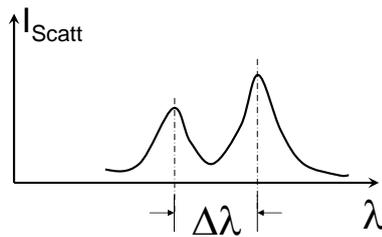
## Table of physical constants

speed of light, $c$	$3.00 \times 10^8 \text{ ms}^{-1}$
Planck constant, $h$	$6.63 \times 10^{-34} \text{ Js}$
$h/(2\pi)$	$1.05 \times 10^{-34} \text{ Js}$
electron charge, $e$	$1.60 \times 10^{-19} \text{ C}$
electron volt, eV	$1.60 \times 10^{-19} \text{ J}$
electron mass, $m_e$	$9.11 \times 10^{-31} \text{ kg}$
atomic mass unit, $m_u$ ,	$1.66 \times 10^{-27} \text{ kg}$
Boltzmann constant, $k_B$	$1.38 \times 10^{-23} \text{ JK}^{-1}$
Bohr magneton, $\mu_B$	$9.27 \times 10^{-24} \text{ JT}^{-1}$
Stefan-Boltzmann constant, $\sigma$	$5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

1. In a Compton effect experiment, X-ray photons are incident on a piece of metal and are scattered by the free electrons.

(a) The scattered radiation spectrum,  $I_{Scatt}(\lambda)$ , measured at a fixed scattering angle  $\theta$  with respect to the direction of incident photons, is shown in the graph below. What does each peak correspond to?

[1]



(b) Draw a diagram of the scattering particles indicating the momentum vectors. Write down the energy and momentum of the photons before and after the scattering in terms of incident X-ray wavelength  $\lambda$  and the wavelength shift  $\Delta\lambda$ . Assuming that the free electron was stationary before the scattering, write down an expression for the kinetic energy of the scattered electron as a function of  $\lambda$  and  $\Delta\lambda$ .

[4]

(c) The Compton shift  $\Delta\lambda$  is given by:

$$|\Delta\lambda| = \frac{h}{mc}(1 - \cos\theta)$$

For X-rays with a wavelength of  $\lambda = 1 \text{ \AA}$ , how large is the Compton shift observed perpendicular to the direction of the incident X-rays? What fraction of its original energy does the photon lose? How large would this fraction be if the X-rays had a wavelength of  $\lambda = 0.1 \text{ \AA}$ ?

[3]

2. A particle moving in 1 dimension is bound to a square well potential  $V(x)$  given by:

$$\begin{array}{lll} V(x) = 0 & x > \epsilon & \text{I} \\ V(x) = -\beta & -\epsilon \leq x \leq \epsilon & \text{II} \\ V(x) = 0 & x < -\epsilon & \text{III} \end{array}$$

- (a) What is the sign of the energy  $E$  of this bound state? (A bound state is a state with a wavefunction localized at the potential.) How does the absolute value of energy  $|E|$  compare to  $\beta$ ? ( $\beta > 0$ ) [1]
- (b) Write down the time-independent Schrödinger equation for the regions I, II, and III shown above. Find the general form of the wavefunction of the bound state in each region. [3]
- (c) Write the boundary conditions at  $x = \epsilon$  and  $x = -\epsilon$ . (Do not attempt to solve the boundary condition equations.) [2]
- (d) A special case of a square well potential is a delta function potential  $V(x) = -\beta\delta(x)$ , where  $\delta(x)$  is the Dirac delta function which satisfies the identities:

$$\int_{-\infty}^{+\infty} \delta(x) dx = 1$$

$$\int_{-\infty}^{+\infty} \delta(x)f(x) dx = f(0)$$

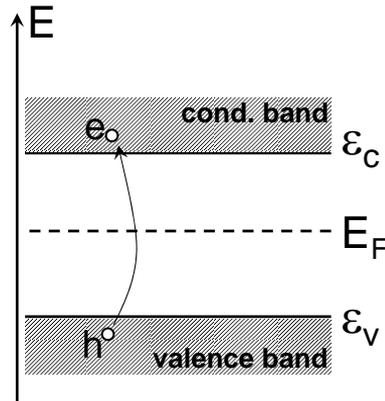
Write down the time-independent Schrödinger equation for this potential. What is the energy  $E$  of the bound state?

Hint: To find  $E$  take the integral of both sides of the Schrödinger equation from  $-\epsilon$  to  $\epsilon$ . Use the relation  $\int \frac{\partial^2 \psi}{\partial x^2} dx = \frac{\partial \psi}{\partial x}$ , and calculate  $\frac{\partial \psi}{\partial x}$  from the results of part (b). Then take the limit of  $\epsilon \rightarrow 0$ . [5]

- (e) Draw the wavefunction of the bound state of part (d) as a function of position  $x$ . [1]

3. The energy levels in the hydrogen atom spectrum, including fine structure, can be labeled with quantum numbers  $n$ ,  $l$  and  $j$ .
- What is the meaning of each of these quantum numbers? How is  $j$  related to  $l$ ? For a particular value of  $n$  what values can  $l$  and  $j$  take? [3]
  - Sketch the energy levels of the hydrogen atom, including the fine-structure, up to  $n = 3$ . Label the levels using the  $nl_j$  format, e.g.  $2s_{1/2}$ . Hint: For simplicity draw the levels with different  $l$  quantum numbers in different columns. Draw the levels in the correct order of energies in each column. [3.5]
  - What are the selection rules for electric dipole transitions in this system? List the possible transitions for levels up to  $n = 3$ . How many lines can be seen in the spectrum of hydrogen corresponding to these transitions? [3.5]
4. The vibrational motion of a linear diatomic molecule can be approximated as simple harmonic motion.
- Sketch the form of the potential energy of a diatomic molecule, as a function of distance of separation between the atoms. Use this to explain briefly when the above approximation is valid. [2]
  - A CO molecule has a bond with force constant  $k = 1900 \text{ Nm}^{-1}$ . What frequency of radiation would excite transitions between the different vibrational energy levels? [3]  
(C and O have masses of  $12 m_u$  and  $16 m_u$  respectively, where  $m_u$  is the atomic mass unit.)
  - Explain why electromagnetic radiation can not excite vibrational transitions in an  $O_2$  or an  $N_2$  molecule. How is this important for the heating of the earth's atmosphere by the sun (the greenhouse effect)? [2]

5. The figure below shows the electronic band structure of a semiconductor. The Fermi energy  $E_F$  lies in the band gap, in between the top of the valence band  $\varepsilon_v$  and the bottom of the conduction band  $\varepsilon_c$ .



- (a) Which distribution function,  $f(\varepsilon)$ , describes the electron statistics in semiconductors? Write down the expression for  $f(\varepsilon)$ . [1.5]
- (b) If the density of electronic states in the conduction band is given by  $g_c(\varepsilon)$ , what is the probability,  $\rho(\varepsilon)$ , of finding an electron in the conduction band at energy  $\varepsilon$ ? [1]
- (c) Write down the expression for the total number of electrons in the conduction band  $n_c(T)$ , as a function of temperature  $T$ .  
In an intrinsic semiconductor, i.e. a semiconductor with no impurities, how does the number of electrons in the conduction band  $n_c(T)$  compare to the number of holes in the valence band  $p_v(T)$ ? [2]
- (d) Show that in the limit where  $\varepsilon_c - E_F \gg k_B T$  the expression for  $n_c(T)$  reduces to:

$$n_c(T) = N_c(T)e^{-(\varepsilon_c - E_F)/k_B T} \quad (1)$$

where  $N_c(T)$  is a function of temperature only and does not depend on  $E_F$ . [3]

- (e) A similar relation is valid for holes in the valence band in the limit where  $E_F - \varepsilon_v \gg k_B T$ :

$$p_v(T) = P_v(T)e^{-(E_F - \varepsilon_v)/k_B T} \quad (2)$$

Using equations (1) and (2) above find the product of the two populations  $n_c(T)$  and  $p_v(T)$ ? What is the significance of this result? [1.5]

- (f) GaAs has a band gap of 1.4 eV at 300°K. Using the following expressions for  $N_c(T)$  and  $P_v(T)$  find the total number of carriers in the conduction band of intrinsic GaAs at room temperature. [2]

$$N_c(T) = \left( \frac{T}{300K} \right)^{3/2} \times 4.33 \times 10^{17}/\text{cm}^3$$

$$P_v(T) = \left( \frac{T}{300K} \right)^{3/2} \times 30.17 \times 10^{17}/\text{cm}^3$$

6. You are designing a spherical planet to be placed at the orbit of Mercury. The planet will have an atmosphere that mixes heat rapidly enough that the entire planetary surface is at the same temperature. What reflection coefficient  $\alpha$  should the atmosphere of the planet have in order for its surface temperature to be a comfortable 300°K? The radius of Mercury and Earth's orbits around the Sun are  $R_M = 57.9 \times 10^6$  Km and  $R_E = 150 \times 10^6$  Km respectively. The Sun radiates  $L_E = 1366$  Watts of power per square meter at the Earth's surface. [6]

7. In computing, a classical 'bit' can take one of only two different values, 0 and 1. A quantum version, the 'qubit', can be made from a quantum mechanical system with two different eigenstates, labeled  $|0\rangle$  and  $|1\rangle$ .
- (a) What is the main difference between a classical bit and a qubit? [2]
- (b) In vector form, we could write the states of a classical bit as '0' =  $(1, 0)$  and '1' =  $(0, 1)$ . Write down the equivalent vector representation of a general qubit state. What constraints are there on the components of the vector, and why? [2]
- (c) An operation on a bit or qubit can now be represented as  $v_{out} = Mv_{in}$  where  $v_{in/out}$  are the vectors representing the input and output states of the bit, and  $M$  is a matrix representing the operation. Write down an example of a matrix which represents a valid operation on a qubit, but not on a classical bit. [2]