

## Nuclear Composition

- the forces binding protons and neutrons in the nucleus are much stronger (binding energy of MeV) than the forces binding electrons to the atom (binding energy of eV)
- the constituents of a nucleus are called **nucleons**
- a nucleus is characterized by its **atomic number  $Z$**  (the number of protons) and the **nucleon number  $A$**  (the total number of protons and neutrons in the nucleus)
- nuclei with the same  $Z$  but different  $A$  are called **isotopes**

- the naming convention for **nuclides** is  $\begin{matrix} A \\ Z \\ X \end{matrix}$

- the **atomic mass** includes the mass of the electrons and is measured in atomic mass unit  $u$  that is

$$1 u = \frac{1}{12} m({}^{12}_6\text{C}) = 1.66 \cdot 10^{-27} \text{ kg}$$

- energy equivalent

$$1 u c^2 = 931.49 \text{ MeV}$$

## Masses in Atomic Units

- proton	1.007 u	938.28 MeV
- neutron	1.008 u	939.57 MeV
- electron	0.00054 u	0.511 MeV

## Size of Nucleus

- the size of nuclei can be determined from scattering experiments (Rutherford), the size resolution is best for particles with small deBroglie wavelength (highly energetic electrons) and/or small charge (neutrons)

- volume of nucleus  $V = \frac{4\pi}{3} R^3 \propto A$

- nuclear radius  $R = R_0 A^{1/3}$  with  $R_0 \approx 1.2 \text{ fm}$

## Nuclear Spin and Magnetic Moment

- similar to electrons, protons and neutrons are fermions with spin  $s = 1/2$

$$S = \sqrt{s(s+1)} \hbar = \frac{\sqrt{3}}{2} \hbar \quad s = \frac{1}{2}$$

$$S_z = m_s \hbar \quad m_s = \pm \frac{1}{2}$$

- the magnetic moment of a nucleus is expressed in nuclear magnetons

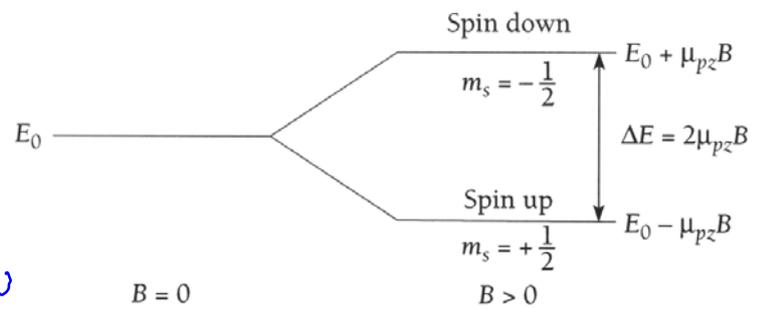
$$\mu_N = \frac{e\hbar}{2m_p} = 3.152 \cdot 10^{-8} \text{ eV/T}$$

$$\frac{\mu_B}{\mu_N} = \frac{m_p}{m_e} = 1836$$

- magnetic energy

$$E = -\vec{\mu} \cdot \vec{B} = -\mu_z B$$

$$\mu_{pz} = \pm 2.8 \mu_N \quad \mu_{nz} = \pm 1.9 \mu_N$$



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## Stability of Nuclei and Binding Energy

- stability diagram of nuclei

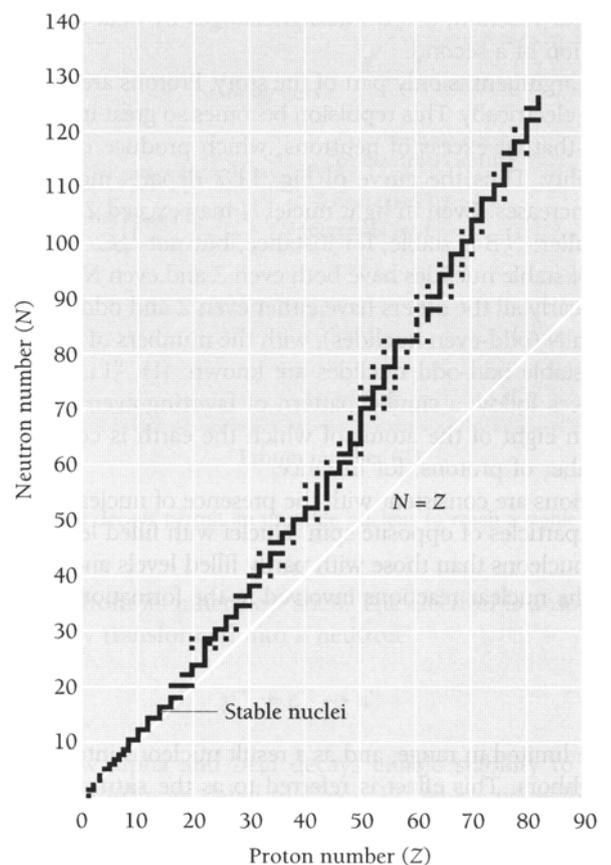
- no stable nuclei with  $Z > 83$ ,  $N > 126$ ,  $A > 209$

- larger nuclei radioactively decay into smaller fragments

- for large  $Z$  there is an excess of neutrons contributing to nuclear binding counteracting the Coulomb repulsion of the protons

- the binding energy is given by the difference between the mass of the nucleus and the sum of the masses of the individual neutrons and protons making up the nucleus

$$E_B = c^2 (M_{A,Z} - (A-Z)m_n - Zm_p - Zm_e)$$



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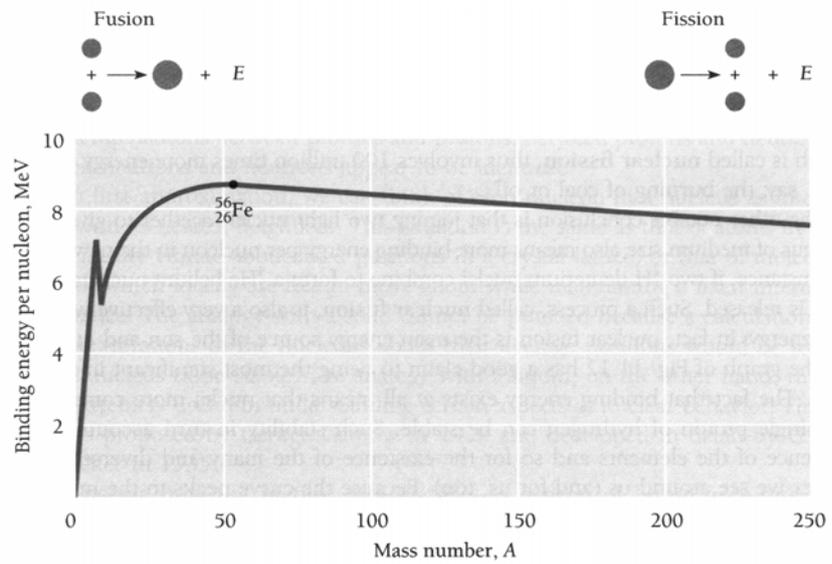
## Binding Energy per Nucleon

- small nuclei can lower their energy by forming larger nuclei in a process called **fusion**

- large nuclei can lower their energy by breaking up into smaller nuclei in a process called **fission**

- the most stable nucleus is an isotope of iron (Fe)

- the binding energy of nucleons is extremely large compared to other atomic energy scales



## Liquid Drop Model for Nuclear Binding Energy

- consider densely packed nuclei, then each nucleus has 12 nearest neighbors to each of which it is bound with an binding energy  $1/2 u$  resulting in a volume energy

$$E_v = 6uA = a_1 A$$

- nucleons on the surface of the nucleus do not bind to neighbors resulting in a negative surface energy

$$E_s = -a_2 A^{2/3}$$

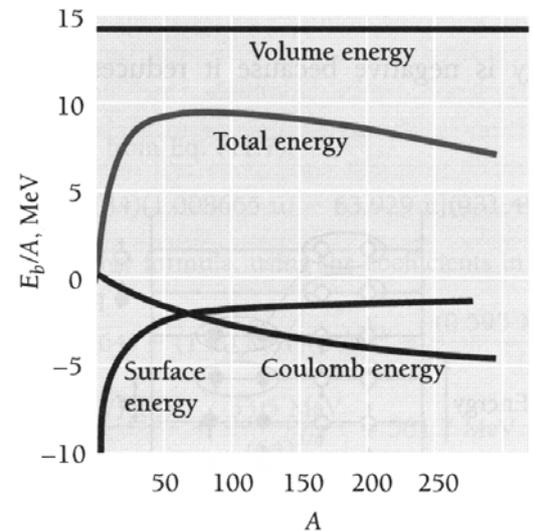
$$S = 4\pi R^2$$

$$R \propto A^{1/3}$$

- the Coulomb energy between all of the Z protons in the nucleus is

$$E_c = - \underbrace{\frac{Z(Z-1)}{2}}_{\substack{\# \text{ pairs of} \\ \text{protons}}} \frac{1}{4\pi\epsilon_0} \frac{1}{r} = -a_3 \frac{Z(Z-1)}{A^{1/3}}$$

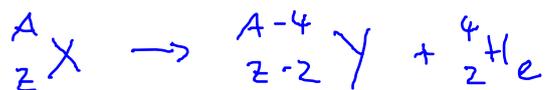
- sum of energies approximates the observed binding energy per nucleon



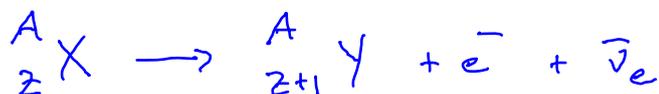
## Radioactive Decay Processes

atomic nuclei can undergo a variety of radioactive decays

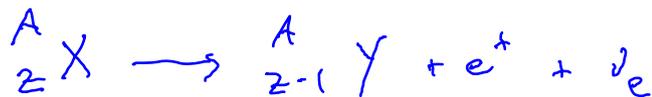
alpha decay



beta decay



positron emission



electron capture



gamma decay



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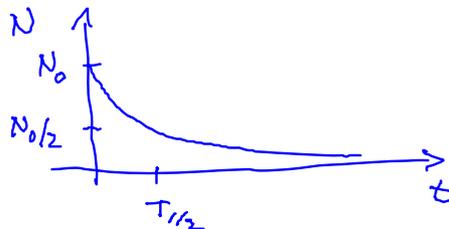
## Radioactive Decay Law

the number of nuclei  $dN$  decaying per unit time  $dt$  a radioactive material containing  $N$  nuclei is proportional to a decay constant  $\lambda$

$$dN = -N \lambda dt \quad \Leftrightarrow \quad \int_{N_0}^N \frac{dN'}{N'} = \int_0^t -\lambda dt'$$

therefore the number of nuclei  $N$  remaining after a time  $t$  when the initial number of nuclei is  $N_0$  is given by

$$N = N_0 e^{-\lambda t}$$



which is known as the radioactive decay law

the half-life ( $T_{1/2}$ ) of a radioactive material is known as the time at which half of the material ( $N = N_0/2$ ) has undergone nuclear decay

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \Leftrightarrow \quad T_{1/2} = \frac{\ln 2}{\lambda}$$

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## Activity Law

the activity  $R$  of a radioactive material is given by the number of decays occurring per unit time

$$R = \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

in contrast to the half-life  $T_{1/2}$  the mean life-time  $\langle T \rangle$  of a nucleus is given by

$$\langle T \rangle = \frac{1}{\lambda}$$

therefore

$$\langle T \rangle = \frac{T_{1/2}}{\ln 2} = 1.44 T_{1/2}$$

The time dependence of activity of radioactive materials can be used for **dating** purposes in cases that one can assume to know the initial concentration of a radioactive material in the sample the age of which is to be determined. This works for organic materials by looking at the activity of  $^{13}\text{C}$  that is naturally incorporated at a certain concentration into plants and bodies of living objects.

## Alpha Decay

- the strong force mediates the binding of nucleons in the nucleus
- for nuclei with  $A > 210$  the binding energy can hardly overcome the Coulomb repulsion between the protons, in this case the stability can be increased by decreasing the size of the nucleus
- this reduction in size is realized by an alpha decay



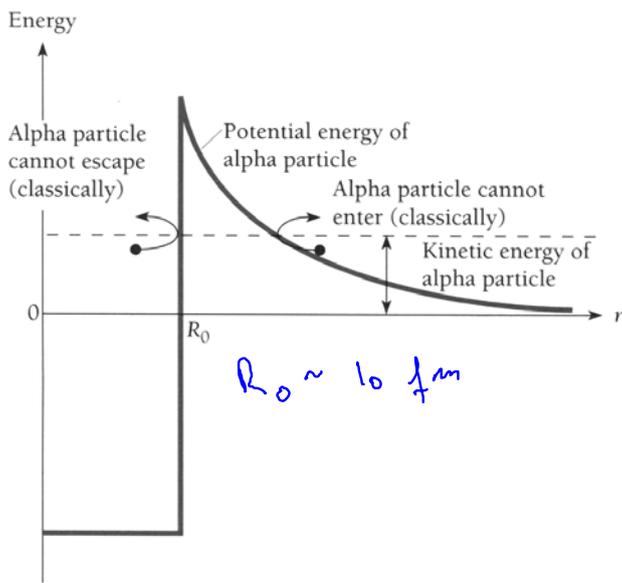
- the energy  $Q$  released in an alpha decay is

$$Q = (m_X - m_Y - m_{{}^4_2\text{He}}) c^2$$

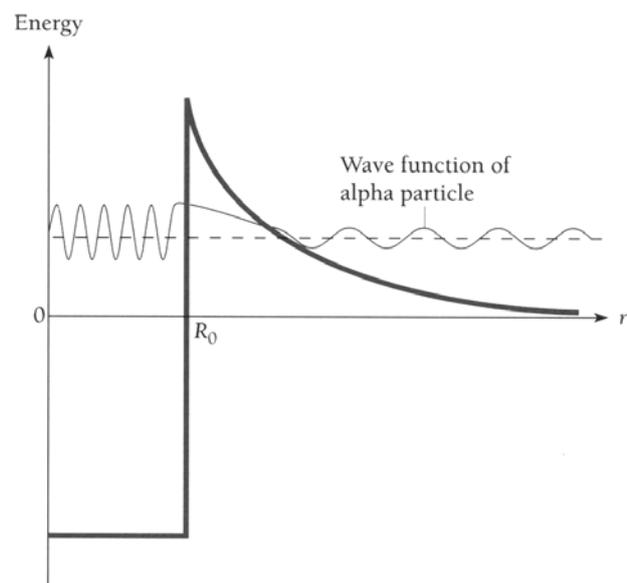
- the released energy is stored in the kinetic energy of the  $\alpha$  particle and the final nucleus  $Y$
- all nucleons with  $A > 210$  decay in processes generating alpha particles

## Tunneling of an $\alpha$ Particle out of the Nucleus

- the alpha particle has to overcome a barrier due to the Coulomb repulsions of about 25 MeV



(a)

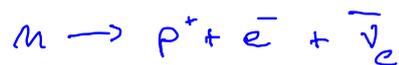


(b)

- the alpha particles tunnel through Coulomb barrier

## Beta<sup>-</sup> Decay

- decay process



- in the beta decay an **electron antineutrino  $\bar{\nu}_e$**  is generated along with the electron, it has a mass smaller than a few eV/c<sup>2</sup> and interacts extremely weakly with anything

- without it energy, momentum and angular momentum conservation would be violated

- the half life of a free neutron is **616 s**

## Positron emission (Beta<sup>+</sup> decay)

- decay process  $p \rightarrow n + e^+ + \nu_e$

- positron emission can only occur in a nucleus, a free proton is stable and does not show positron emission

## Electron Capture

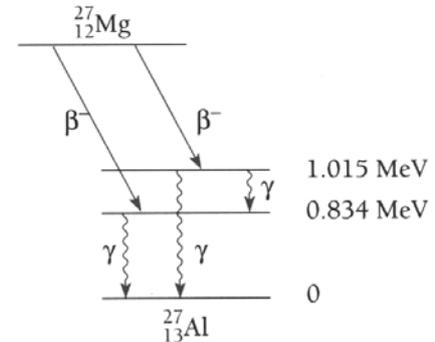
- decay process:  $p^+ + e^- \rightarrow n + \nu_e$

- a K shell electron gets captured in the nucleus and transforms a proton to a neutron, this process is accompanied by the emission of an X-ray photon

## Gamma Decay

- relaxation of an excited state nucleus (indicated by a \*) to a lower energy nucleus by emission of a high energy photon

- photons with energies up to MeV are called **gamma rays**

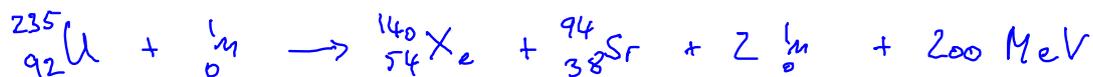


- excited state nuclei have usually a short life time against gamma decay, typically much less than a few hours

## Fission

- the process of a large nucleus decaying into two smaller nuclei while releasing energy is called fission

- a well known process is the neutron induced fission of  $^{235}\text{U}$



- most of the released energy is converted to kinetic energy of the fragments

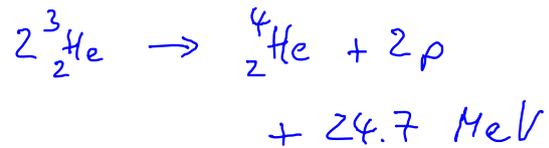
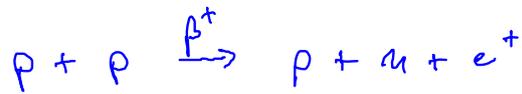
- the additional neutrons released upon the fission process of one U nucleus can induce fission of other nuclei in a **chain reaction** process

- the controlled chain reaction can be used for generation of energy in **nuclear reactors**

- one gram of U generated 1 MW of power for a day, equivalently 2.6 tons of coal would be needed to generate that amount of energy conventionally

## Fusion

- when lighter nuclei are combined into heavier ones energy can be released in a process called fusion
- the fusion of hydrogen to helium powers the sun
- the proton-proton cycle



- power of the sun is  $4 \cdot 10^{26}$  W, thus  $10^{38}$  fusion processes per second take place in the sun
- there is enough hydrogen in the sun to keep the fusion process going for a few billion years

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## Interactions and Particles

there are four fundamental interactions governing the physical world around us

Interaction	particles affected	strength	exchange particle
strong	quarks	1	gluons/mesons
electromagnetic	charges	$10^{-2}$	photons
weak	quarks/leptons	$10^{-5}$	intermediate bosons
gravitational	all	$10^{-39}$	gravitons

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## Leptons

lepton	antiparticle		mass, MeV/c <sup>2</sup>	lifetime, s	spin
electron	$e^-$	$e^+$	0.511	stable	1/2
e-neutrino	$\nu_e$	$\bar{\nu}_e$	small	stable	1/2
muon	$\mu^-$	$\mu^+$	106	$2.2 \cdot 10^{-6}$	1/2
$\mu$ -neutrino	$\nu_\mu$	$\bar{\nu}_\mu$	small	stable	1/2
tau	$\tau^-$	$\tau^+$	1777	$2.9 \cdot 10^{-23}$	1/2
$\tau$ -neutrino	$\nu_\tau$	$\bar{\nu}_\tau$	small	stable	1/2

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## Quarks

the constituents of hadrons

3 generations of quarks

quark/symbol	mass, GeV/c <sup>2</sup>	charge	strangeness
up u	0.3	+2/3	0
down d	0.3	-1/3	0
strange s	0.5	-1/3	-1
charm c	1.5	+2/3	0
top t	174	+2/3	0
bottom b	4.3	-1/3	0

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# Protons, Neutrons and other Hadrons

composition of the proton:

quarks  
charge  
spin

$$uud$$
$$+2/3 + 2/3 - 1/3 = +1$$
$$+1/2 + 1/2 - 1/2 = +1/2$$

composition of the neutron:

quarks  
charge  
spin

$$ddu$$
$$-1/3 - 1/3 + 2/3 = 0$$
$$-1/2 - 1/2 + 1/2 = -1/2$$

Hadrons composed of 3 quarks are called **baryons**

Hadrons composed of 2 quarks are called **mesons**

composition of the  $\pi^+$ -meson:

quarks  
charge  
spin

$$u\bar{d}$$
$$+2/3 + 1/3 = +1$$
$$+1/2 - 1/2 = 0$$

composition of the  $K^+$ -meson:

quarks  
charge  
spin

$$u\bar{s}$$
$$+2/3 + 1/3 = +1$$
$$+1/2 - 1/2 = 0$$