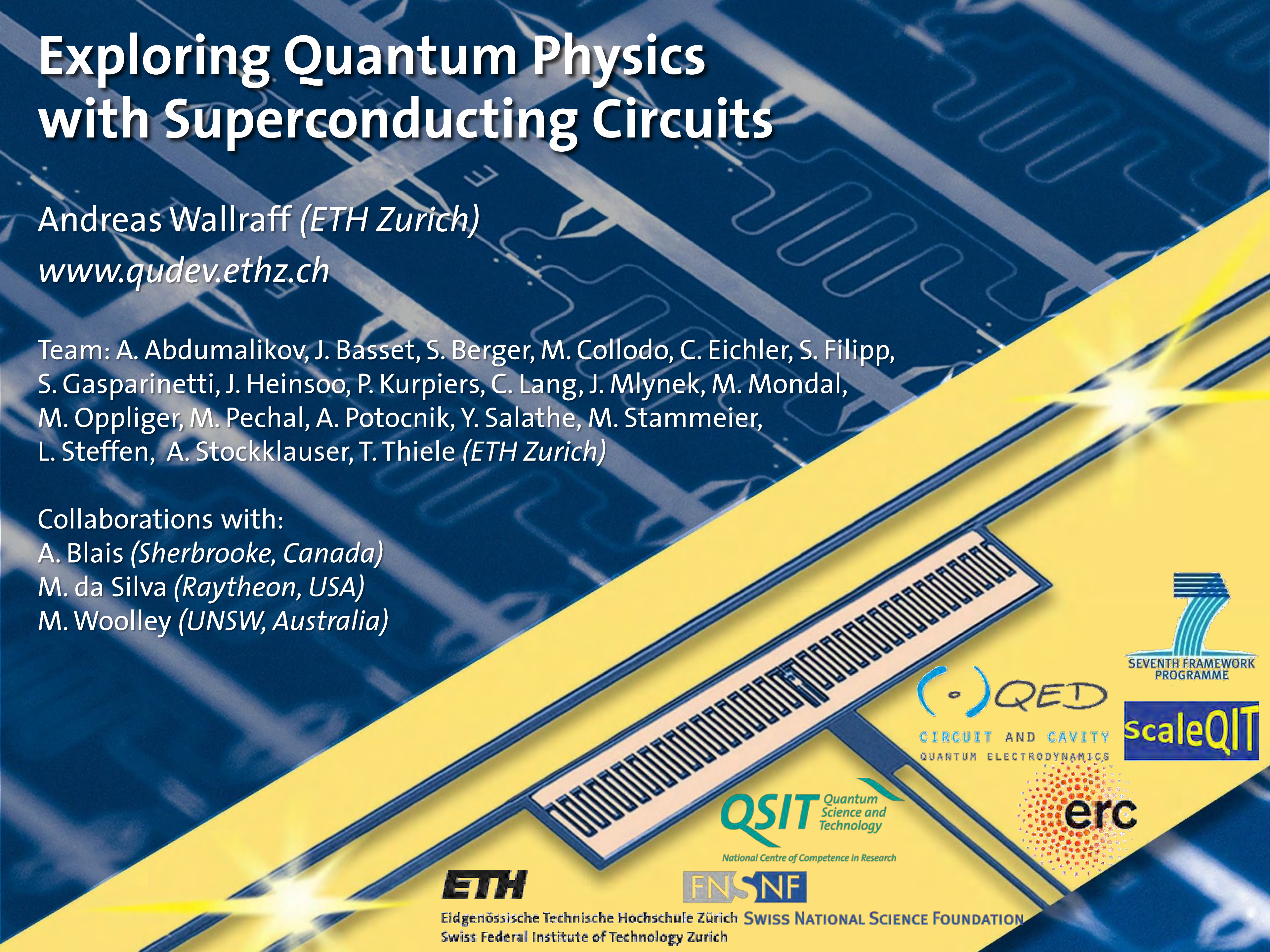


Exploring Quantum Physics with Superconducting Circuits

Andreas Wallraff *(ETH Zurich)*
www.qudev.ethz.ch

Team: A. Abdumalikov, J. Basset, S. Berger, M. Collodo, C. Eichler, S. Filipp, S. Gasparinetti, J. Heinsoo, P. Kurpiers, C. Lang, J. Mlynek, M. Mondal, M. Oppliger, M. Pechal, A. Potocnik, Y. Salathe, M. Stammeier, L. Steffen, A. Stockklauser, T. Thiele *(ETH Zurich)*

Collaborations with:
A. Blais *(Sherbrooke, Canada)*
M. da Silva *(Raytheon, USA)*
M. Woolley *(UNSW, Australia)*

SEVENTH FRAMEWORK PROGRAMME
(•)QED
CIRCUIT AND CAVITY QUANTUM ELECTRODYNAMICS
scaleQIT
QSIT Quantum Science and Technology
National Centre of Competence in Research
erc
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

Acknowledgements

www.qudev.ethz.ch

Former group members now
Faculty/PostDoc/PhD/Industry
M. Baur (ABB)
J. Basset (U. Paris Sud)
R. Bianchetti (ABB)
D. Bozyigit (ETH Zurich)
C. Eichler (Princeton)
A. Fedorov (UQ Brisbane)
A. Fragner (Yale)
S. Filipp (IBM)
J. Fink (Caltech, IST Austria)
T. Frey (Bosch)
M. Goppl (Sensirion)
J. Govenius (Aalto)
L. Huthmacher (Cambridge)
D.-D. Jarausch (Cambridge)
K. Juliusson (CEA Saclay)
C. Lang (Radionor)
P. Leek (Oxford)
P. Maurer (Stanford)
J. Mlynek (Siemens)
G. Puebla (IBM)
L. Steffen (AWK Group)
A. van Loo (Oxford)
S. Zeytinoğlu (ETH Zurich)

Collaborations with (groups of):
A. Blais (Sherbrooke)
C. Bruder (Basel)
M. da Silva (Raytheon)
L. DiCarlo (TU Delft)
K. Ensslin (ETH Zurich)
J. Faist (ETH Zurich)
J. Gambetta (IBM)
T. Ihn (ETH Zurich)
F. Merkt (ETH Zurich)
L. Novotny (ETH Zurich)
B. Sanders (Calgary)
S. Schmidt (ETH Zurich)
R. Schoelkopf (Yale)
C. Schoenenberger (Basel)
E. Solano (UPV/EHU)
W. Wegscheider (ETH Zurich)

FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

QSIT Quantum Science and Technology
National Centre of Competence in Research

(·)QED
CIRCUIT AND CAVITY
QUANTUM ELECTRODYNAMICS

SOLID

erc

SEVENTH FRAMEWORK PROGRAMME

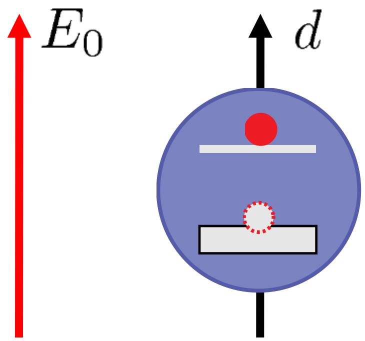
GEOMDISS

aqute

scaleQIT

Investigating the Interaction of Light and Matter

challenging on the level of single (artificial) atoms and single photons



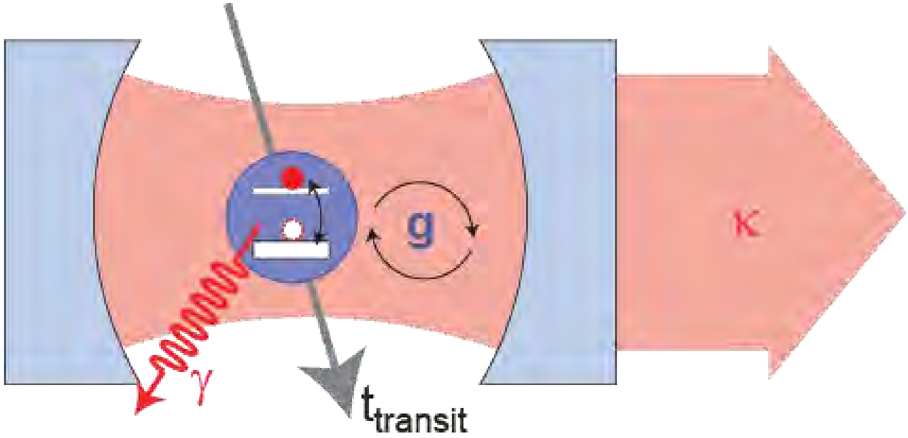
- single photon fields E_o (small in 3D)
- dipole moment d (usually small ~ ea_o)
- photon/dipole interaction $\hbar g \sim dE_0$ (usually small)
- mode-matching (controlling the absorption probability)

What to do?

- confine atom and photon in a cavity or a waveguide (QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity QED with Superconducting Circuits



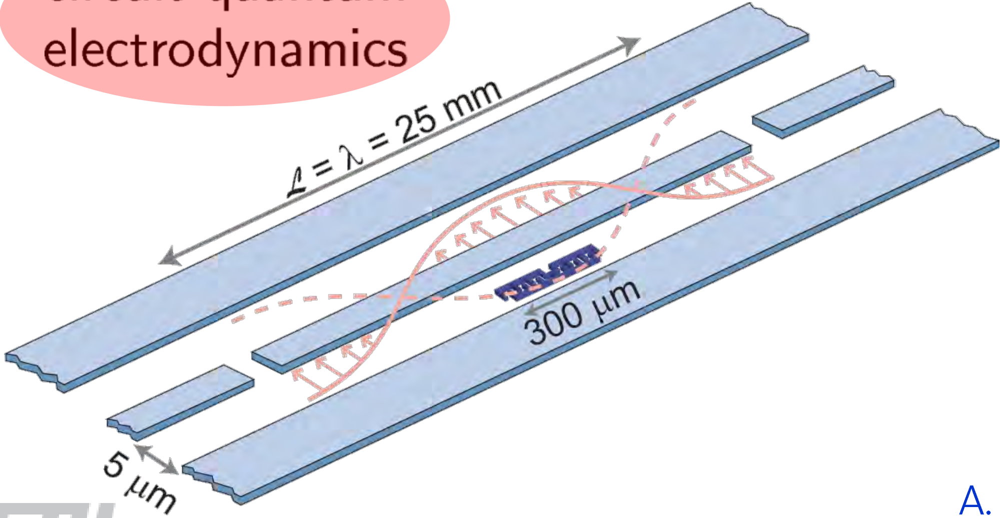
coherent interaction of photons with quantum two-level systems ...

J. M. Raimond *et al.*, *Rev. Mod. Phys.* **73**, 565 (2001)
S. Haroche & J. Raimond, *OUP Oxford* (2006)
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

Properties:

- strong coupling in solid state sys.
- ‘easy’ to fabricate and integrate

circuit quantum electrodynamics



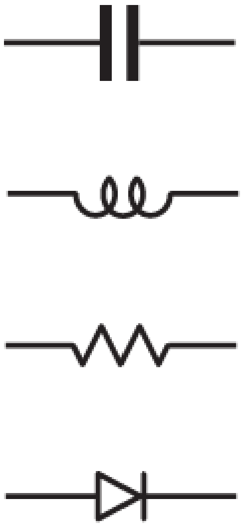
Research directions:

- quantum optics
- hybrid quantum systems
- quantum information

A. Blais, *et al.*, *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)
R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Conventional Electronic Circuits

basic circuit elements:

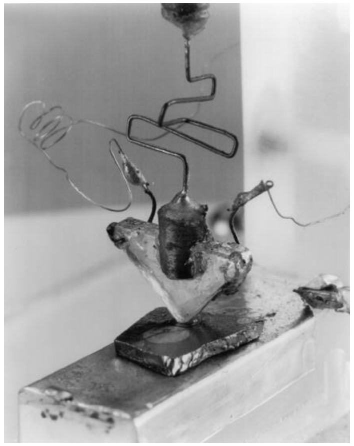


basis of modern information and communication technology

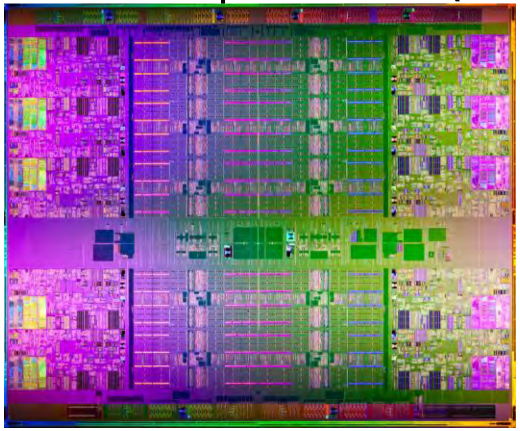
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



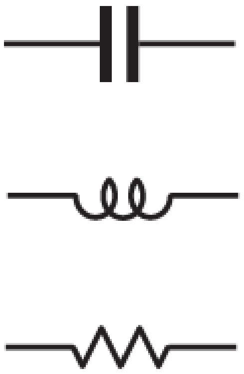
intel xeon processors (2011)



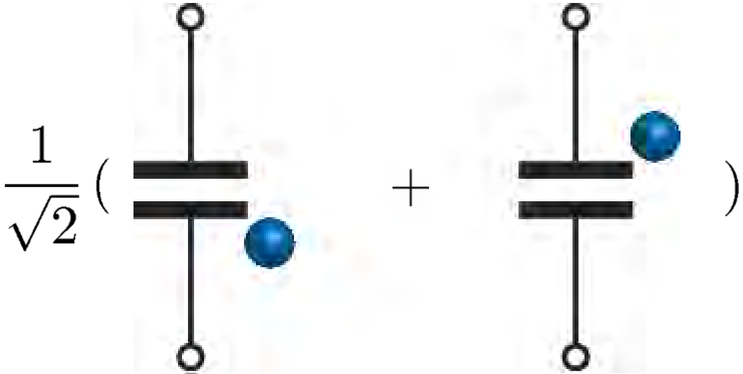
3.000.000.000 transistors
smallest feature size 32 nm
clock speed ~ 3 GHz
power consumption 10 W

Classical and Quantum Electronic Circuit Elements

basic circuit elements:



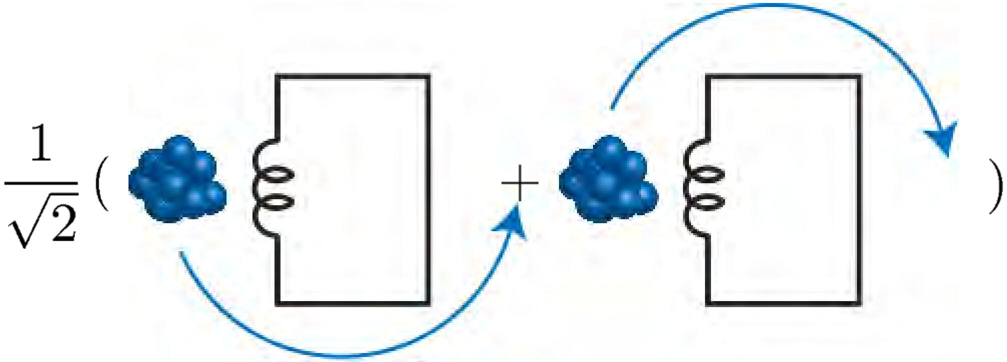
charge on a capacitor:



quantum superposition states of:

- charge q
- flux ϕ

current or magnetic flux in an inductor:

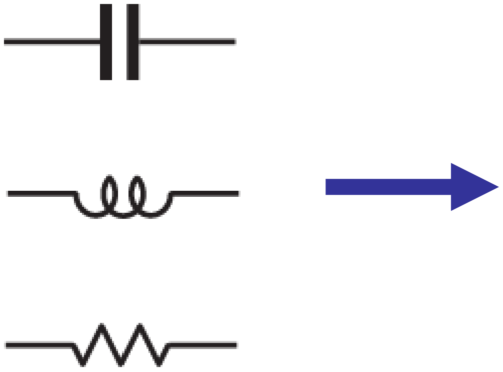


commutation relation (c.f. *x*, *p*):

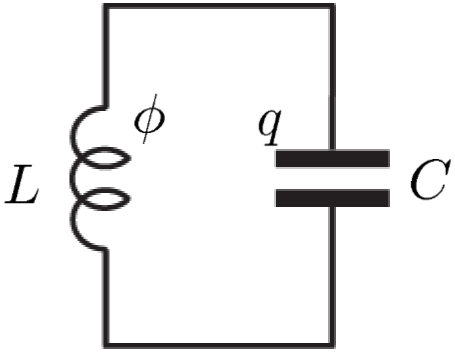
$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$

Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

classical physics:

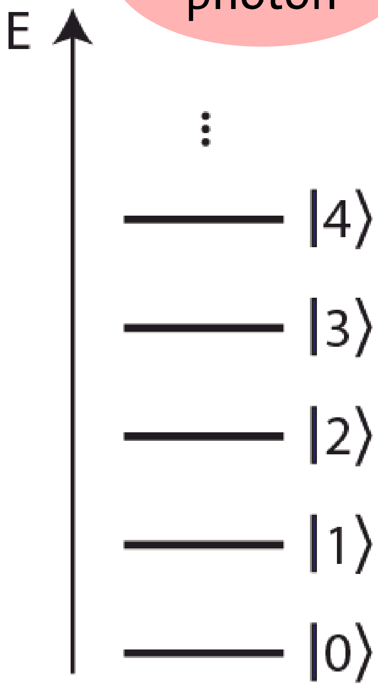
$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \qquad \left[\hat{\phi}, \hat{q}\right] = i\hbar$$

energy:

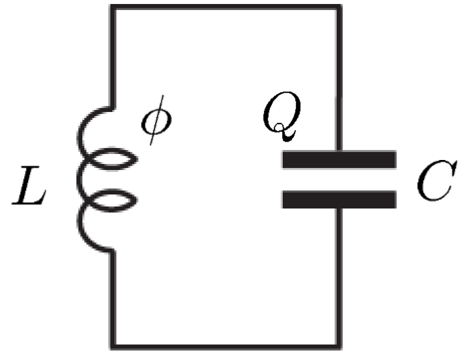
electronic photon



Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Quantization of an Electronic Harmonic Oscillator

Harmonic LC oscillator:



$$\begin{aligned} Q &= CV \\ \phi &= LI \\ V &= -L\dot{I} = -\dot{\phi} \end{aligned}$$

Charge on capacitor

Flux in inductor

Voltage across inductor

Classical Hamiltonian:

$$H = \frac{CV^2}{2} + \frac{LI^2}{2} = \frac{Q^2}{2C} + \frac{\phi^2}{2L}$$

Conjugate variables:

$$\begin{aligned} \frac{\partial H}{\partial \phi} &= \frac{\phi}{L} = I = \dot{Q} \\ \frac{\partial H}{\partial Q} &= \frac{Q}{C} = V = -L\dot{I} = -\dot{\phi} \end{aligned}$$

Hamilton operator:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C}$$

Flux and charge operator:

$$\begin{aligned} \hat{\phi} &= \phi \\ \hat{Q} &= -i\hbar\frac{\partial}{\partial \phi} \end{aligned}$$

Commutation relation:

$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$

Creation and Annihilation Operators for Circuits

Hamilton operator of harmonic oscillator in second quantization:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2)$$

$$\hat{a}^\dagger \left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$ Creation operator

$$\hat{a}\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle$$ Annihilation operator

$$\hat{a}^\dagger\hat{a}\left|n\right\rangle = n\left|n\right\rangle$$ Number operator

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_C}}(\hat{a}^\dagger + \hat{a})$$ Charge/voltage operator $$\hat{V} = \frac{\hat{Q}}{C}$$

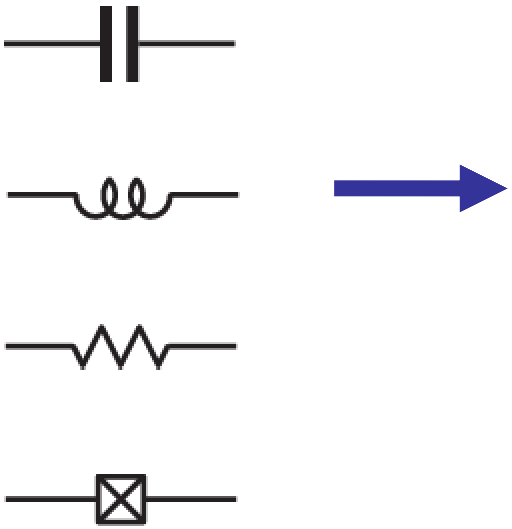
$$\hat{\phi} = i\sqrt{\frac{\hbar Z_C}{2}}(\hat{a}^\dagger - \hat{a})$$ Flux/current operator $$\hat{I} = \frac{\hat{\phi}}{L}$$

With characteristic impedance:

$$Z_C = \sqrt{\frac{L}{C}}$$

Constructing Non-Linear Quantum Electronic Circuits

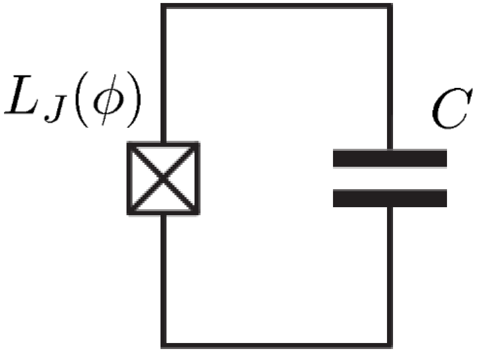
circuit elements:



Josephson junction:

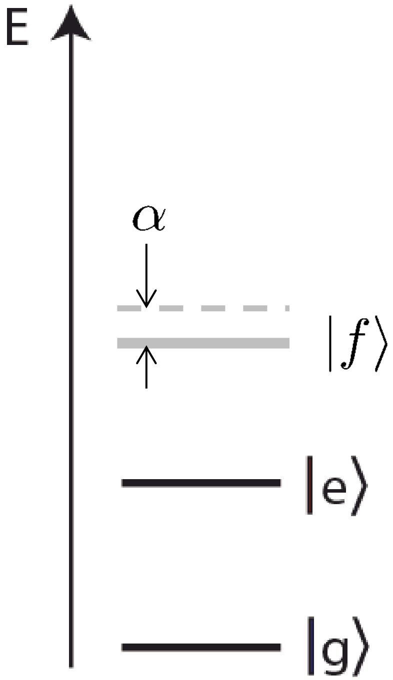
a non-dissipative nonlinear element (inductor)

anharmonic oscillator:



$$H \approx \hbar\left(\omega_{ge}\,\hat{b}^\dagger\hat{b} - \frac{\alpha}{2}\,\hat{b}^{\dagger 2}\hat{b}^2\right)$$

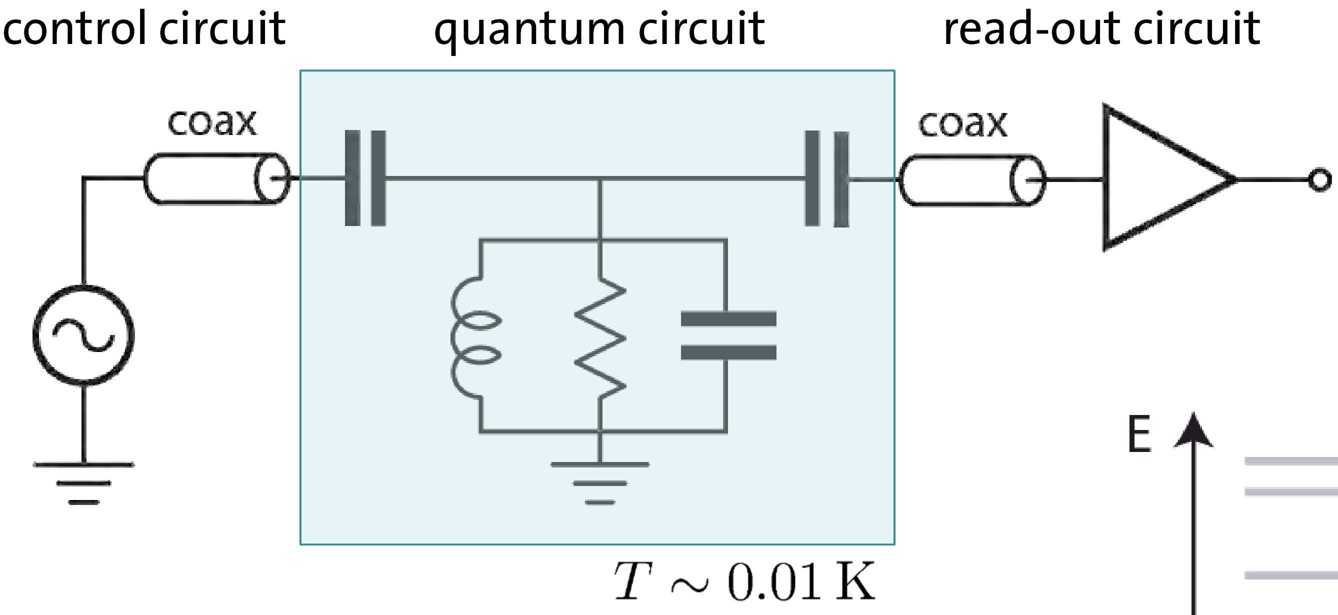
non-linear energy level spectrum:

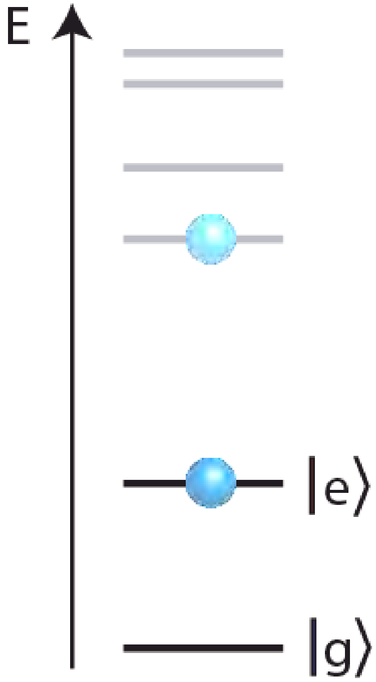


electronic artificial atom

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

How to Operate Circuits in the Quantum Regime?



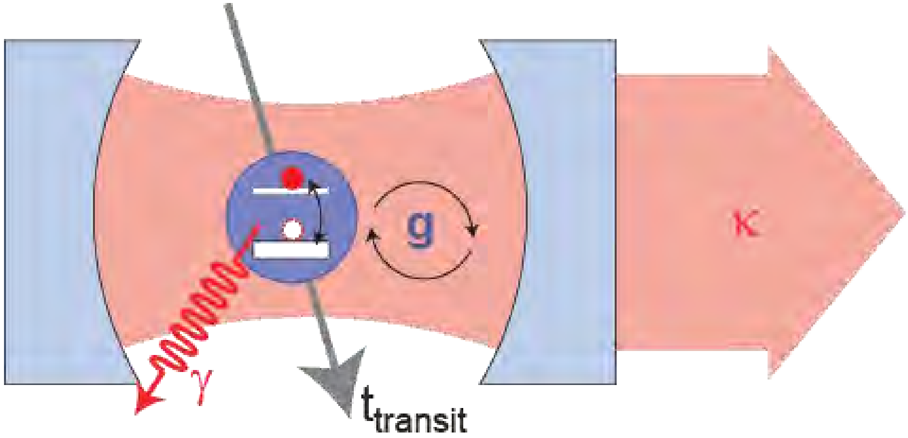


recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

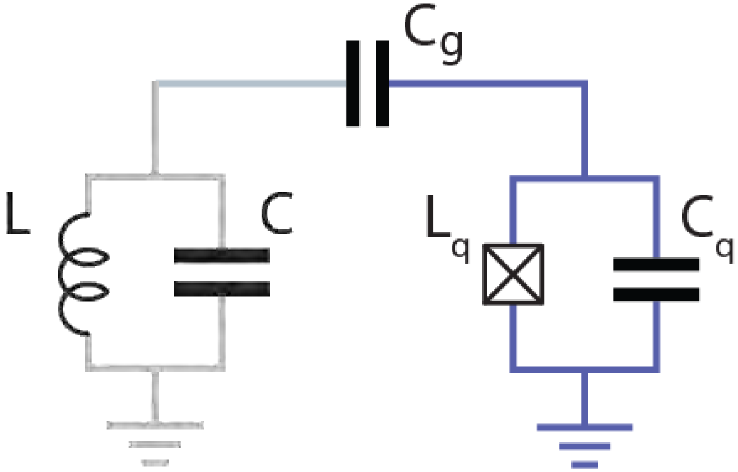
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411174* (2004)

Cavity QED with Superconducting Circuits

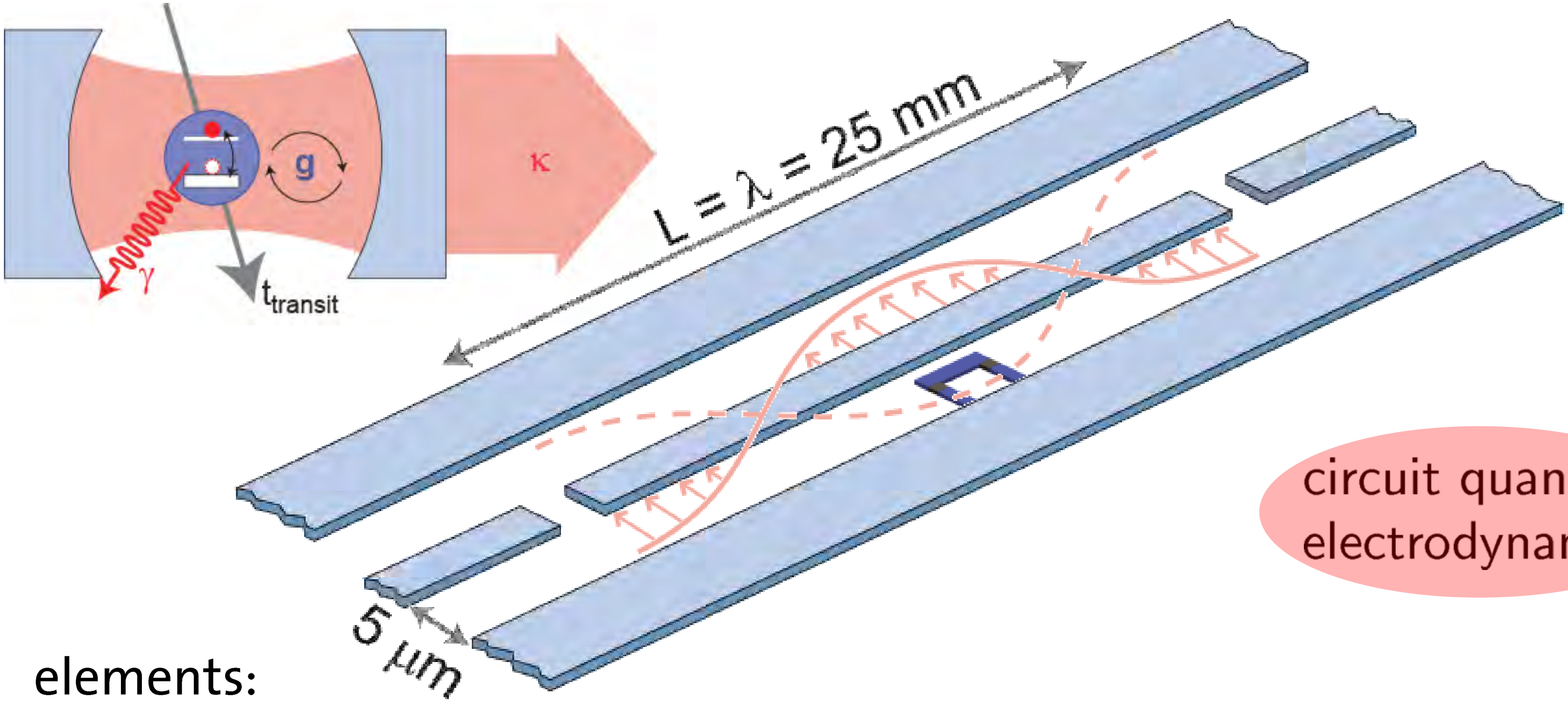


coherent quantum mechanics
with individual photons and qubits ...

... basic approach:



Cavity QED with Superconducting Circuits



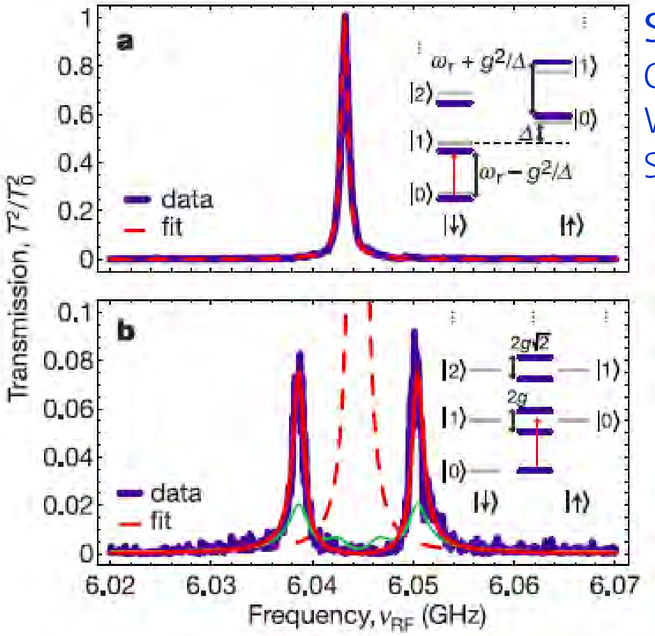
circuit quantum electrodynamics

elements:

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the atom: a superconducting qubit with large dipole moment *d* and long coherence time $1/\gamma$ and fixed position ...
- ... or any microscopic/macroscopic quantum element or ensemble thereof with an appreciable dipole moment

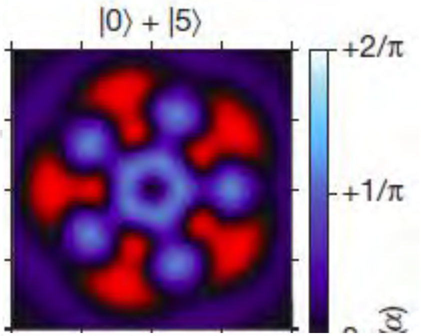
A. Blais, *et al.*, *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)
R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

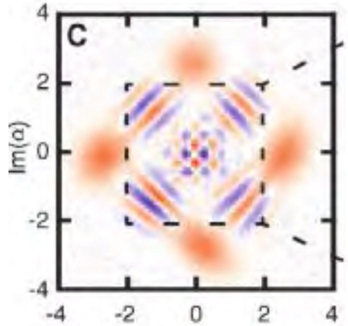
Quantum Optics with Supercond. Circuits



Strong Coherent Coupling

Chiorescu *et al.*, *Nature* **431**, 159 (2004)
Wallraff *et al.*, *Nature* **431**, 162 (2004)
Schuster *et al.*, *Nature* **445**, 515 (2007)



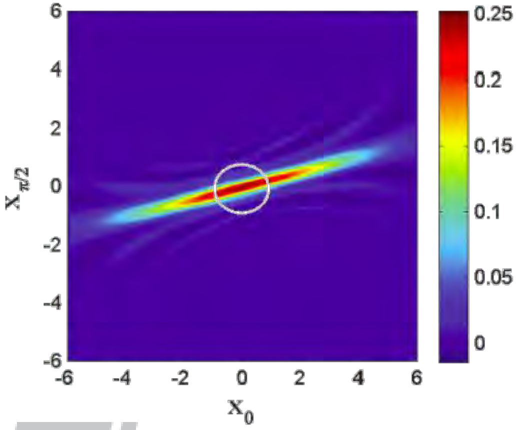


Root n Nonlinearities

Fink *et al.*, *Nature* **454**, 315 (2008)
Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)
Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

Microwave Fock and Cat States

Hofheinz *et al.*, *Nature* **454**, 310 (2008)
Hofheinz *et al.*, *Nature* **459**, 546 (2009)
Kirchmair *et al.*, *Nature* **495**, 205 (2013)
Vlastakis *et al.*, *Science* **342**, 607 (2013)

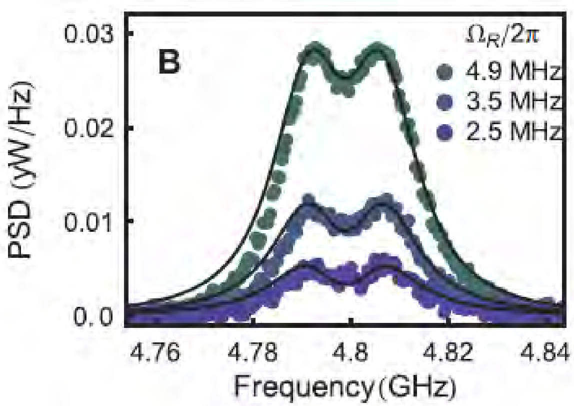


Parametric Amplification & Squeezing

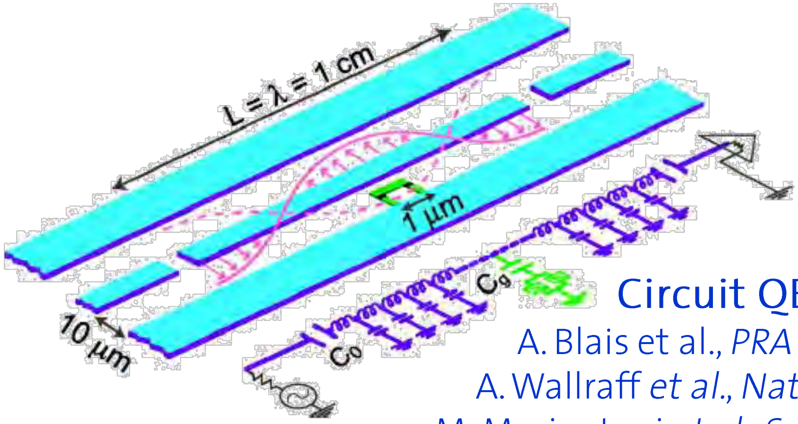
Castellanos-Beltran *et al.*,
Nat. Phys. **4**, 928 (2008)
Abdo *et al.*, *PRX* **3**, 031001 (2013)

Waveguide QED – Qubit Interactions in Free Space

Astafiev *et al.*, *Science* **327**, 840 (2010)
van Loo *et al.*, *Science* **342**, 1494 (2013)

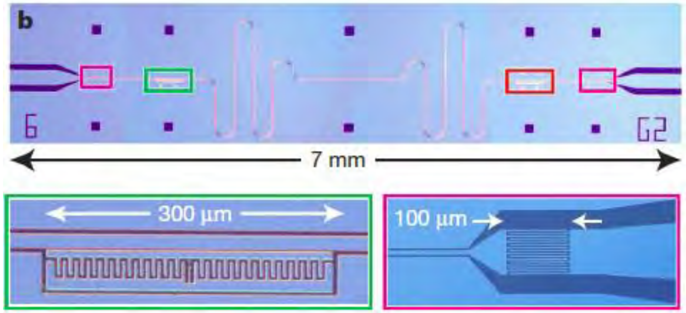


Quantum Computing with Superconducting Circuits



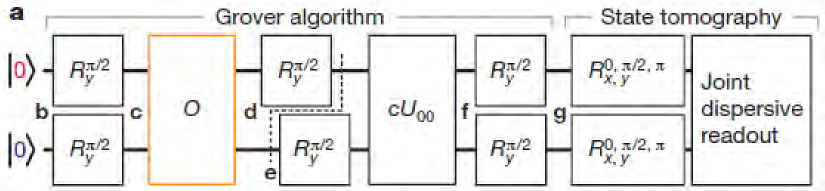
Circuit QED Architecture

A. Blais et al., *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)
R. Barends *et al.*, *Nature* **508**, 500 (2014)



Resonator as a Coupling Bus

M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)
H. Majer *et al.*, *Nature* **449**, 443 (2007)

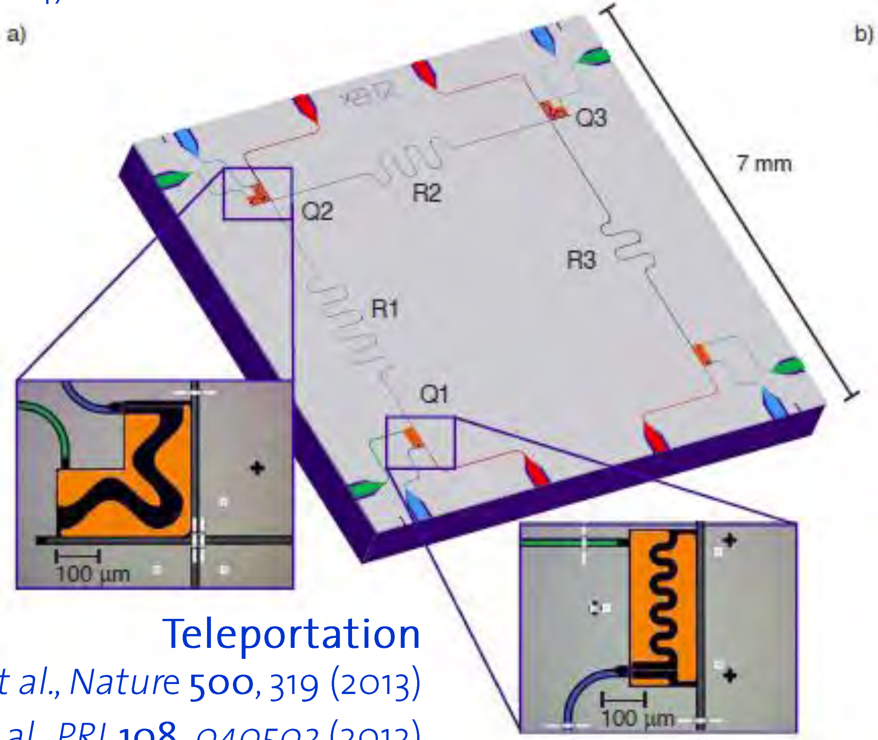


Deutsch, Grover Algorithms

L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)
L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)

Toffoli Gates & Error Correction

A. Fedorov *et al.*, *Nature* **481**, 170 (2012)
M. Reed *et al.*, *Nature* **481**, 382 (2012)



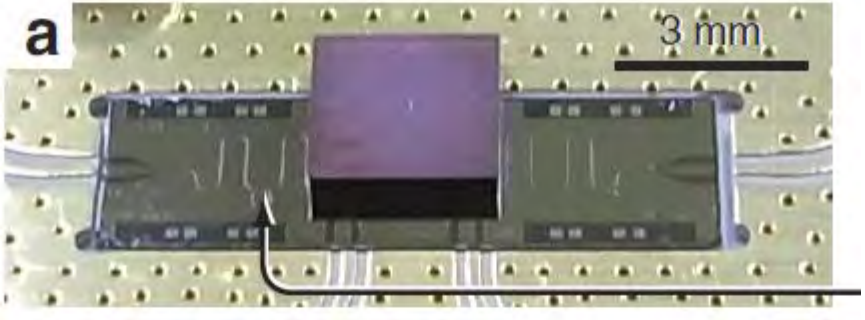
Teleportation

L. Steffen *et al.*, *Nature* **500**, 319 (2013)
M.. Baur *et al.*, *PRL* **108**, *040502* (2012)

Hybrid Systems with Superconducting Circuits

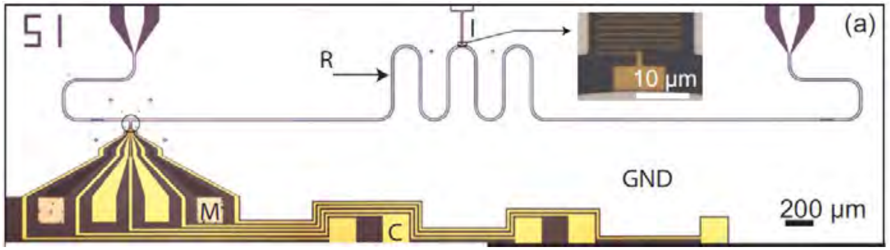
Spin Ensembles: e.g. NV centers

D. Schuster *et al.*, *PRL* **105**, 140501 (2010)
Y. Kubo *et al.*, *PRL* **105**, 140502 (2010)



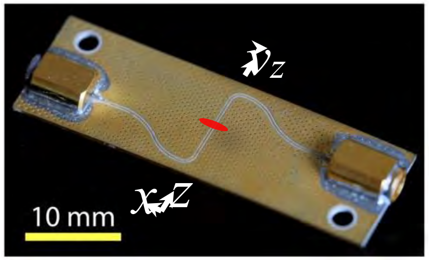
CNT, Gate Defined 2DEG, or nanowire Quantum Dots

M. Delbecq *et al.*, *PRL* **107**, 256804 (2011)
T. Frey *et al.*, *PRL* **108**, 046807 (2012)
K. Petersson *et al.*, *Nature* **490**, 380 (2013)



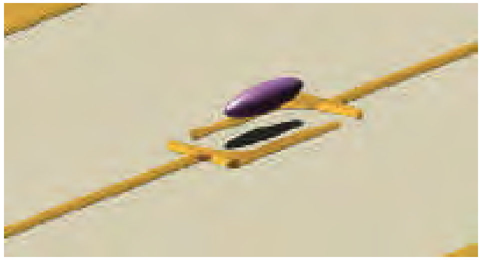
Rydberg Atoms

S. Hogan*et al.*, *PRL* **108**, 063004 (2012)



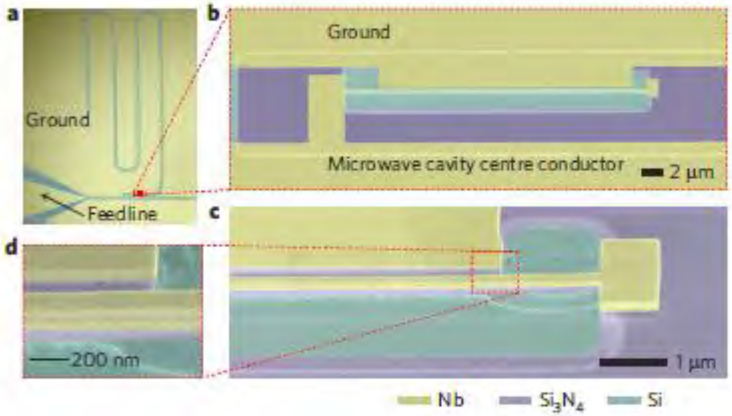
Polar Molecules, Rydberg, BEC

P. Rabl *et al*, *PRL* **97**, 033003 (2006)
A. Andre *et al*, *Nat. Phys.* **2**, 636 (2006)
D. Petrosyan *et al*, *PRL* **100**, 170501 (2008)
J. Verdu *et al*, *PRL* **103**, 043603 (2009)



Nano-Mechanics

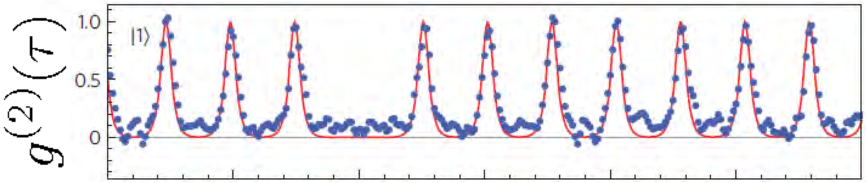
J. Teufel *et al.*, *Nature* **475**, 359 (2011)
X. Zhou *et al.*, *Nat. Phys.* **9**, 179(2013)



... and many more

Experiments with Propagating Quantum Microwaves

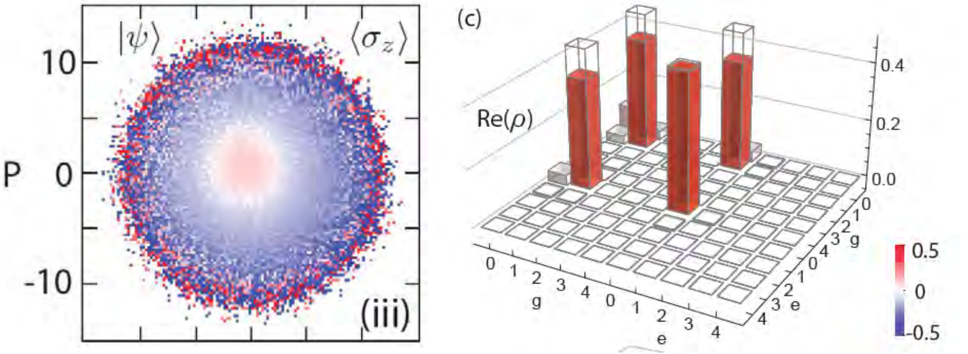
Single photon sources and their anti-bunching



Bozyigit *et al.*, Nat. Phys **7**, 154 (2011)

Lang *et al.*, *PRL* **107**, 073601 (2011)

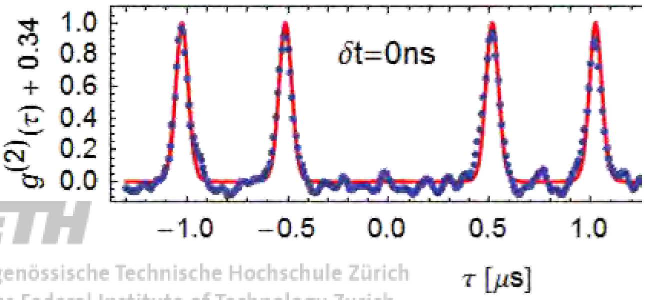
Preparation and characterization of qubit-**propagating** photon entanglement

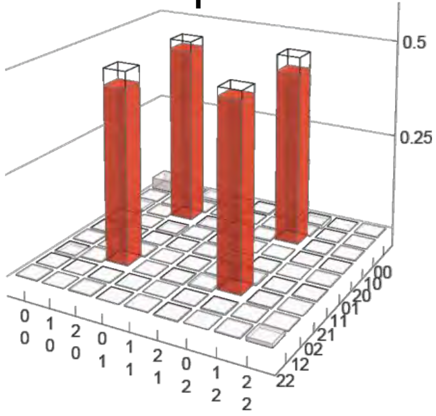


Eichler *et al.*, *PRL* **109**, *240501* (2012)
Eichler *et al.*, *PRA* **86**, 032106 (2012)

Hong-Ou-Mandel: Two-photon interference incl. msrmnt of coherences at microwave freq.

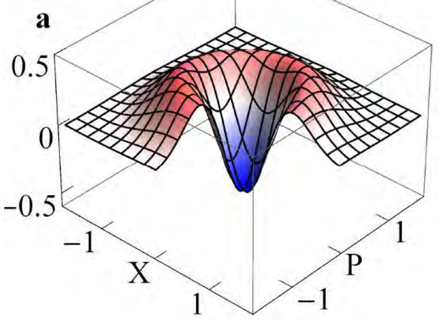
Lang *et al.*, *Nat. Phys.* **9**, 345 (2013)





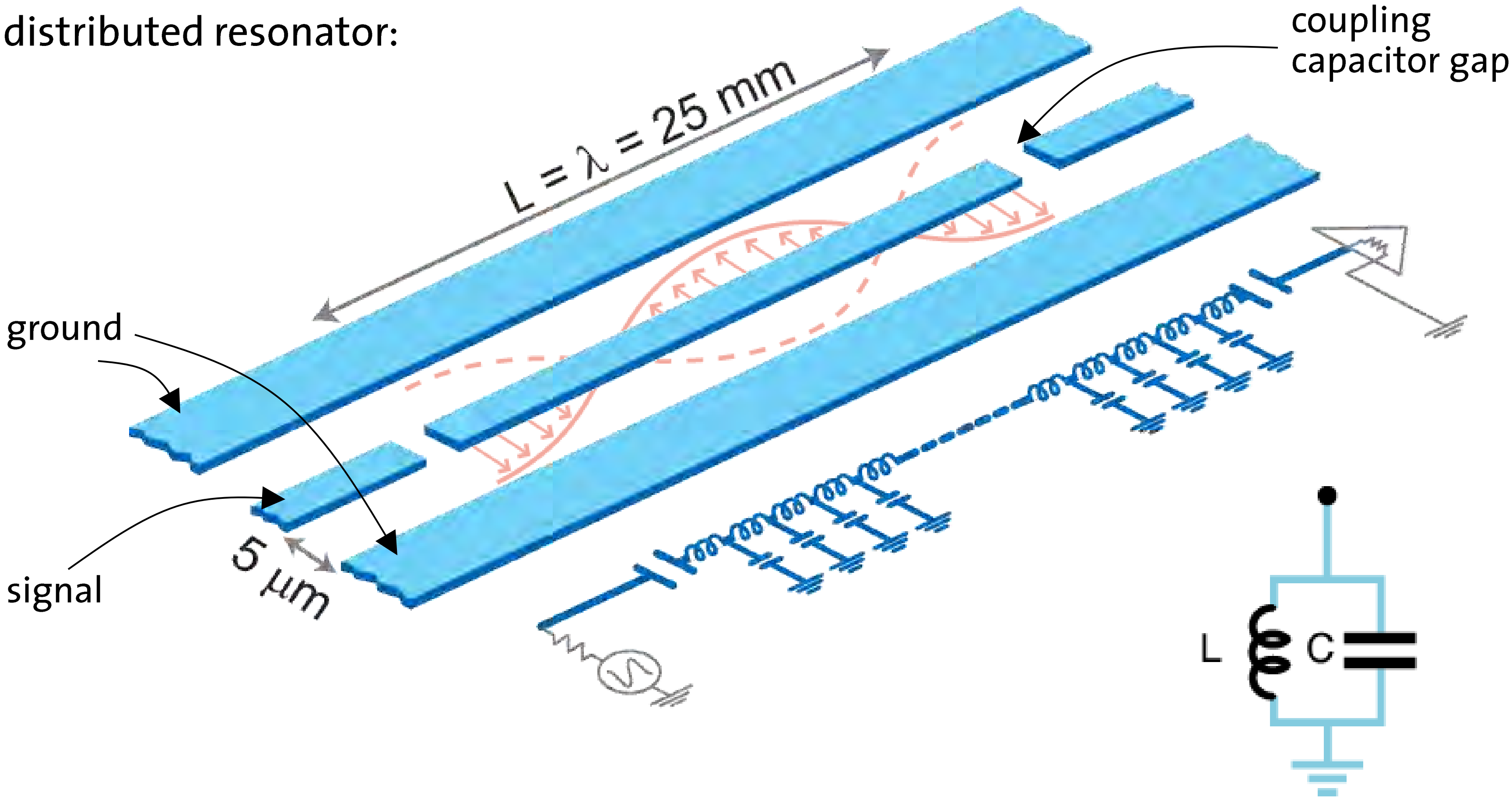
Full state tomography and Wigner functions of **propagating** photons

Eichler *et al.*, *PRL* **106**, 220503 (2011)



Realization of H.O.: Transmission Line Resonator

distributed resonator:

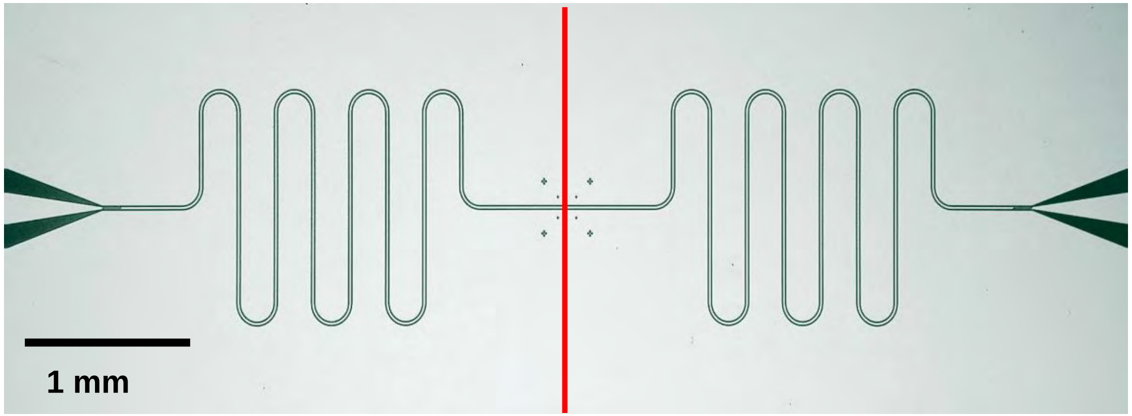


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

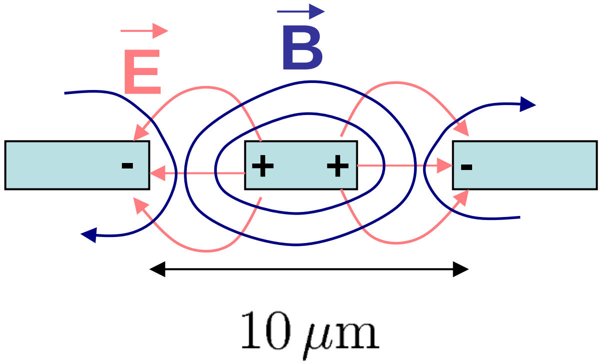
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

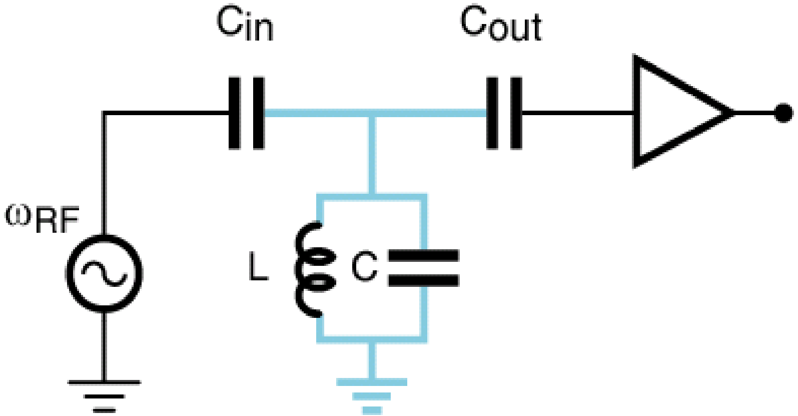
coplanar waveguide:



cross-section of transm. line (TEM mode):

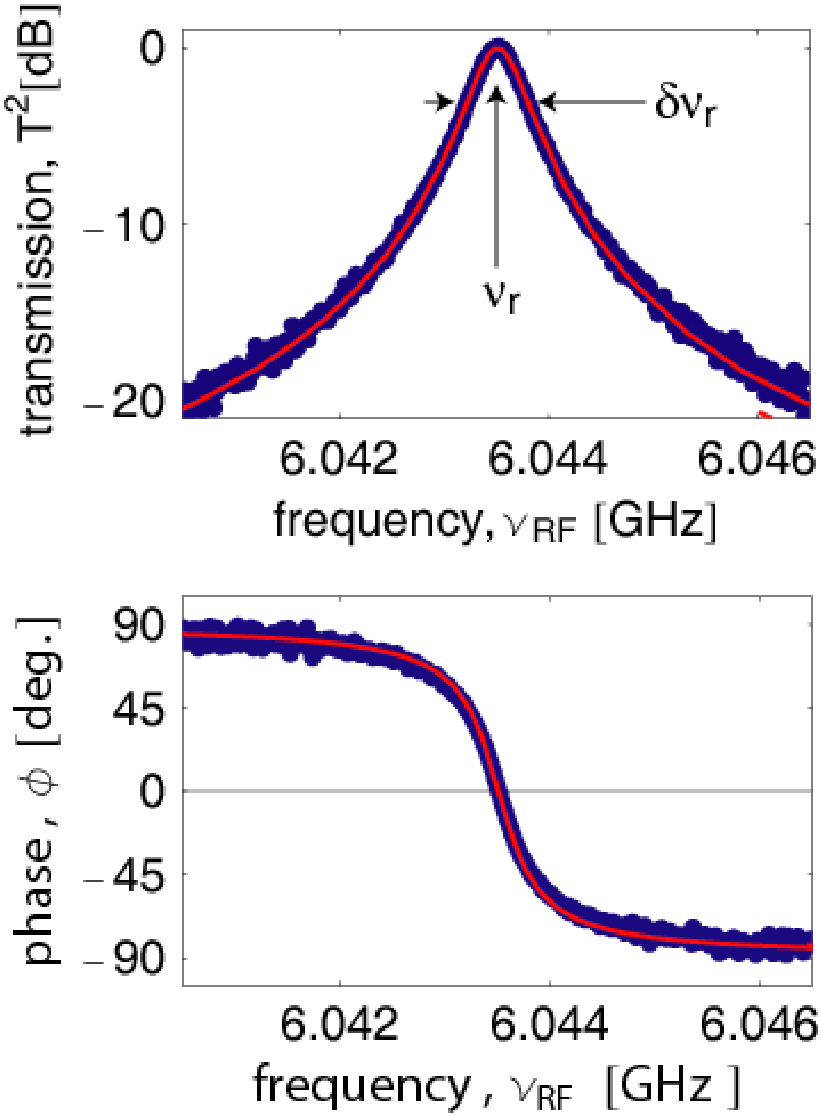


measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

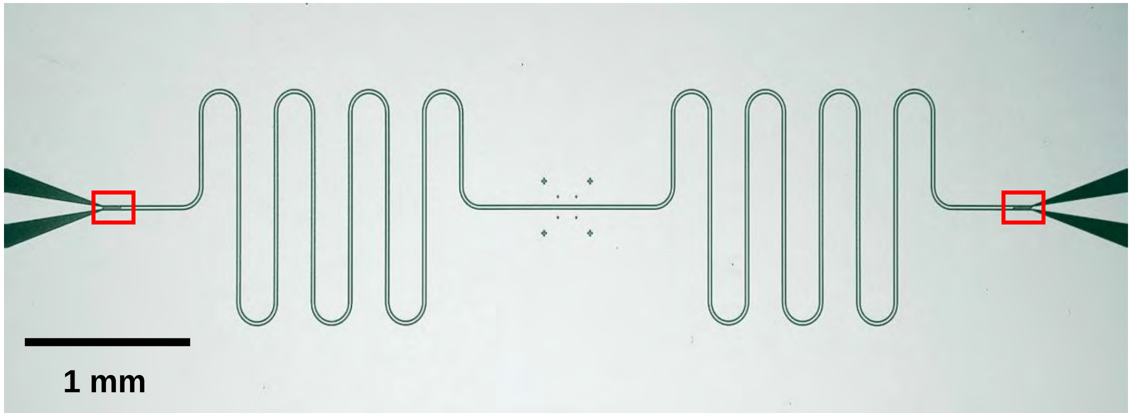
photon decay rate:

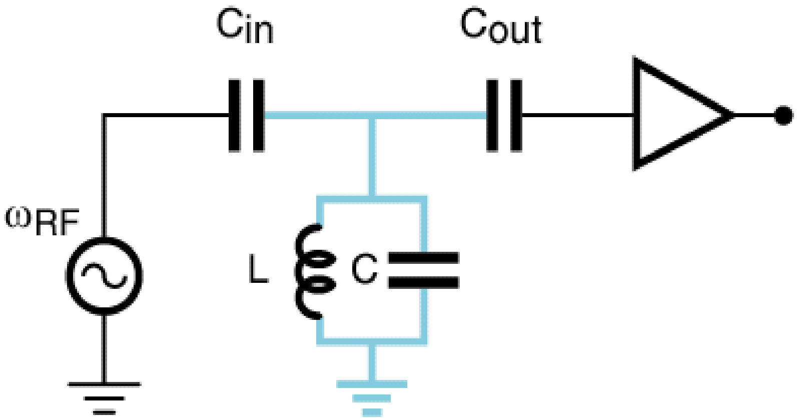
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

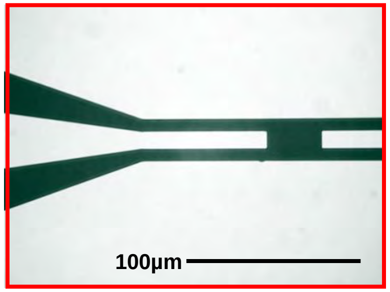
$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

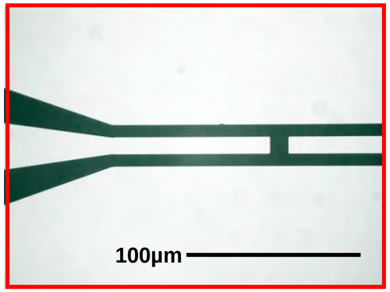
Controlling the Photon Life Time

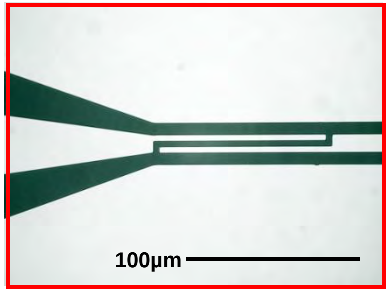


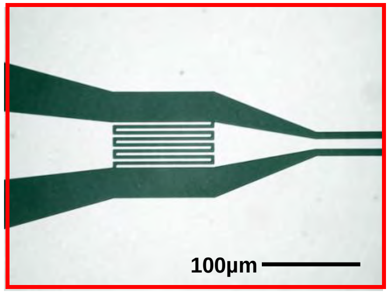


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

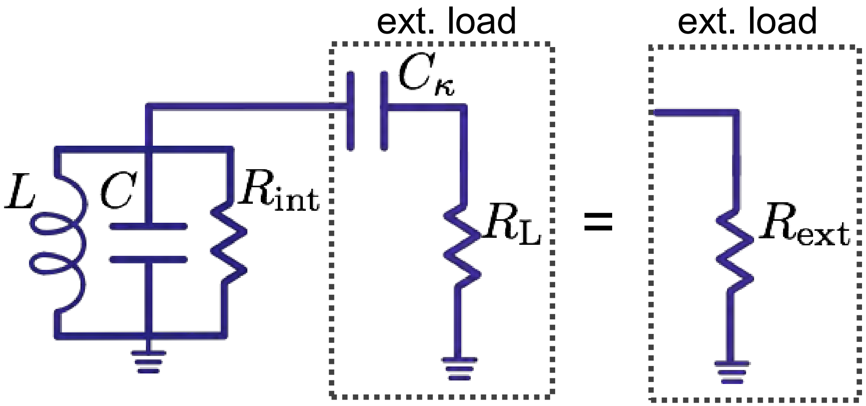


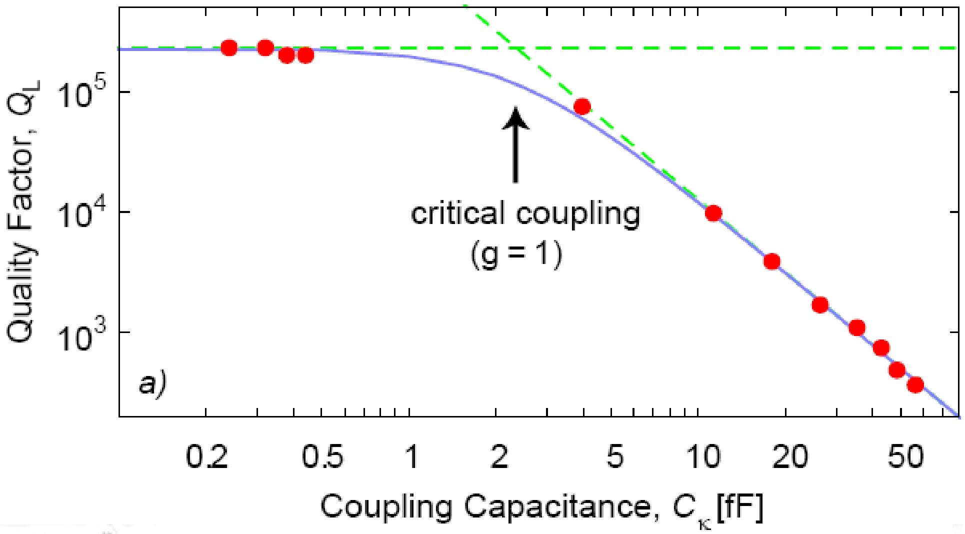


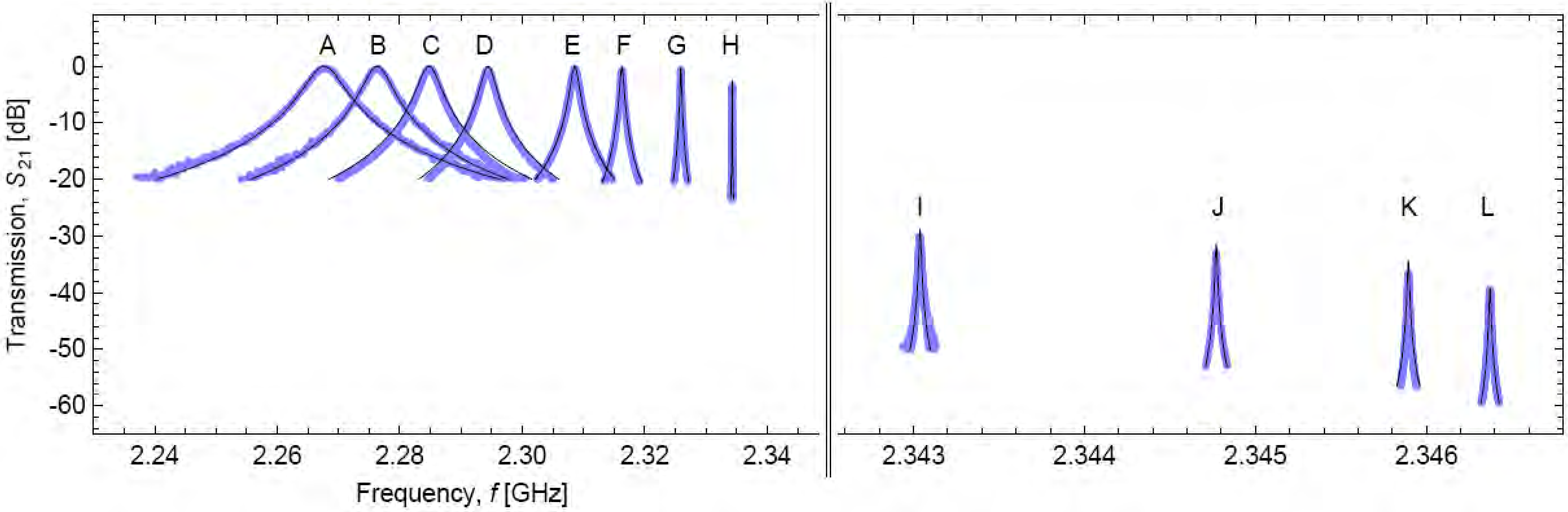




Quality Factor Measurement

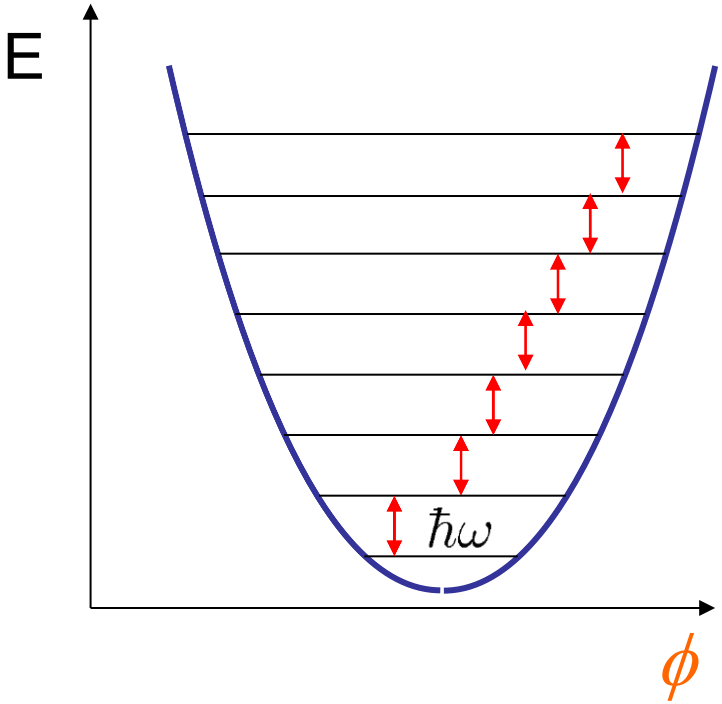






M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\mathrm{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

low temperature required:

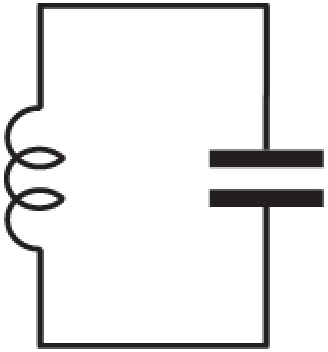
$$\hbar\omega \gg k_B T$$

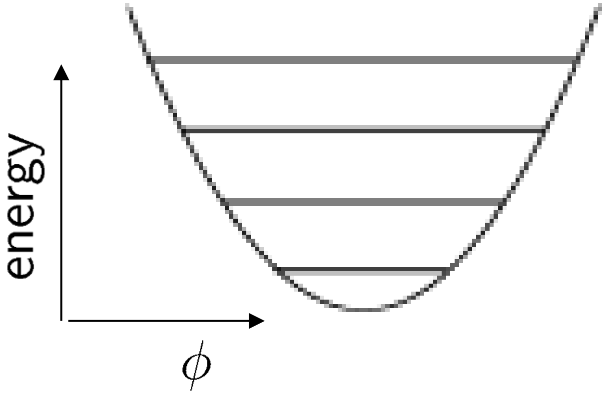
10 GHz ~ 500 mK ↗ (for $\hbar\omega$) 20 mK ↖ (for $k_B T$)

$$\langle n_{\mathrm{th}} \rangle \sim 10^{-11}$$

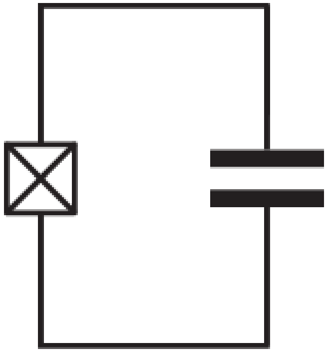
Linear vs. Nonlinear Superconducting Oscillators

LC resonator:

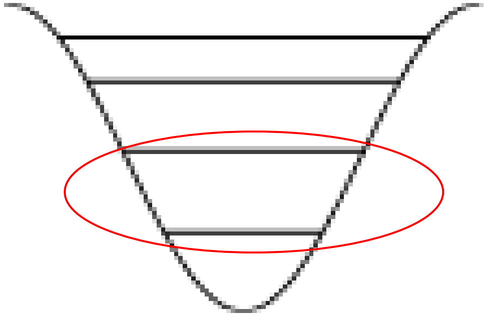




Josephson junction resonator:
Josephson junction = nonlinear inductor

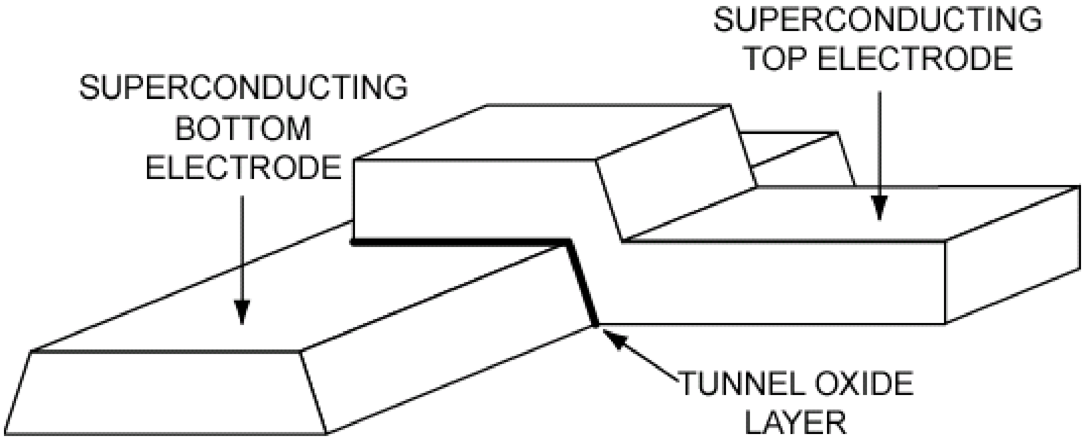


anharmonicity defines effective two-level system



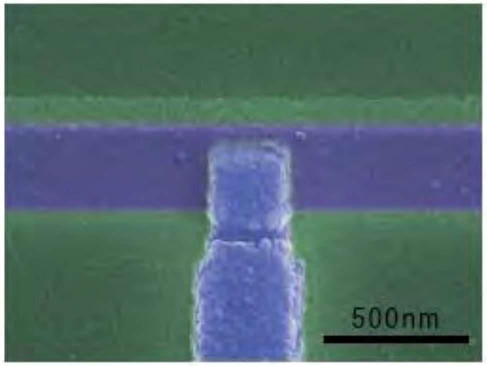
A Low-Loss Nonlinear Element

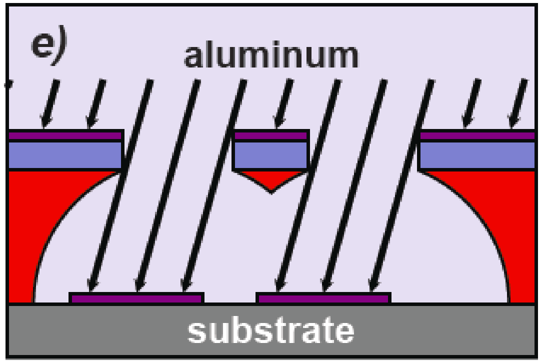
a (superconducting) Josephson junction:

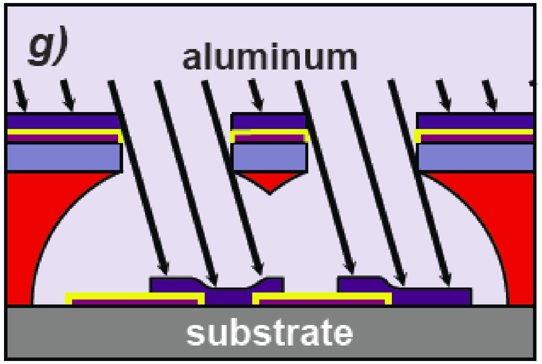


- superconductors: Nb, Al
- tunnel barrier: AlO_x

Josephson junction fabricated by shadow evaporation:



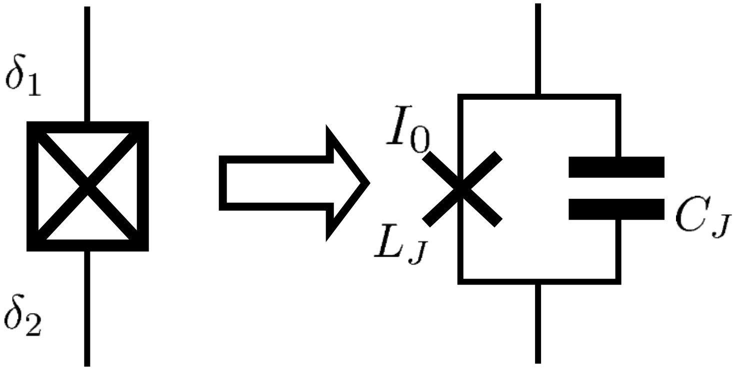




M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

The Josephson Junction as an ideal Non-Linear Inductor

a nonlinear inductor without dissipation



Josephson relations:

$$I = I_0 \sin\delta = I_0 \sin\left[2\pi\phi(t)/\phi_0\right]$$

nonlinear current/phase relation

$$V = \frac{\phi_0}{2\pi}\dot{\delta} = \dot{\phi}$$

gauge inv. phase difference:

$$\delta = \delta_2 - \delta_1 = 2\pi\phi(t)/\phi_0$$

specific Josephson inductance L_{J0}

Josephson inductance:

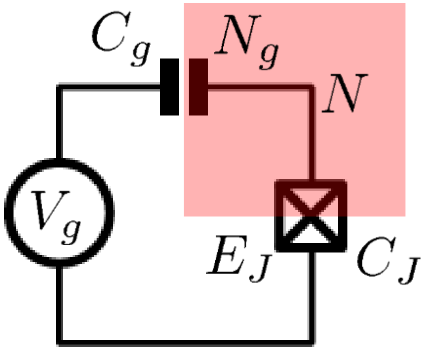
$$V = -L_J\dot{I} = \frac{\phi_0}{2\pi I_0}\frac{1}{\cos\delta}\dot{I}$$

specific Josephson energy E_{J0}

Josephson energy:

$$E_J = \int VI dt = \frac{I_0\phi_0}{2\pi}\cos\delta$$

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \dfrac{Q}{2e}$

continuous gate charge: $N_g = \dfrac{C_g V_g}{2e}$

total box capacitance $C_\Sigma = C_g + C_J$

Hamiltonian: $H = H_{\mathrm{el}} + H_{\mathrm{mag}}$

electrostatic part:

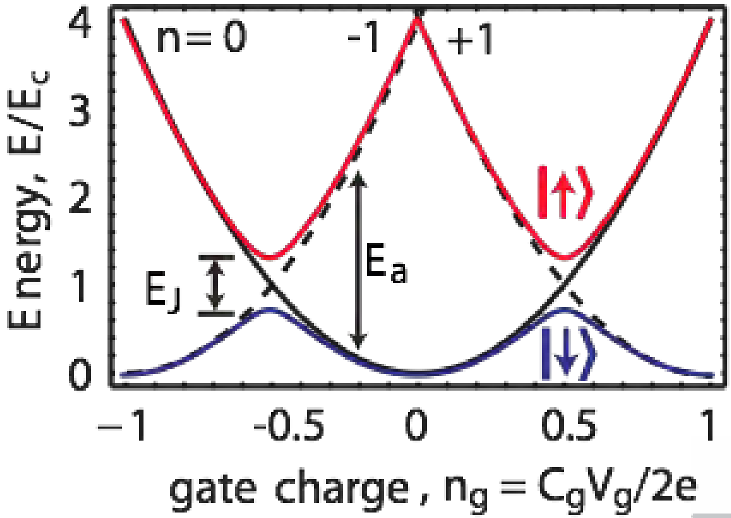
$$H_{\mathrm{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\mathrm{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator: $$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis: $$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** ***N***:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2}(|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 + E_J \cos\hat{\delta}$$

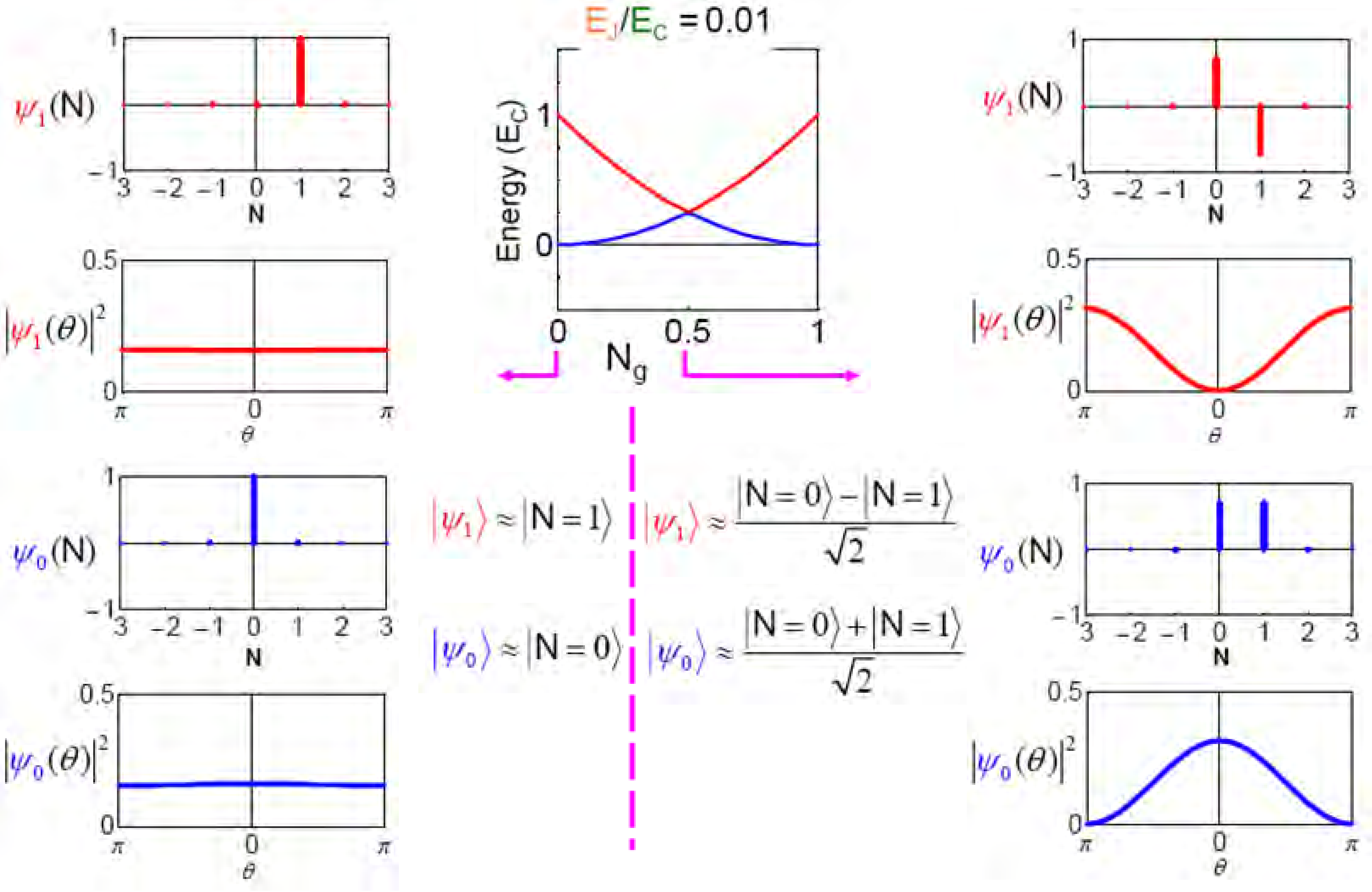
transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Charge and Phase Wave Functions ($E_J << E_C$)



$|\psi_1\rangle \approx |N=1\rangle$

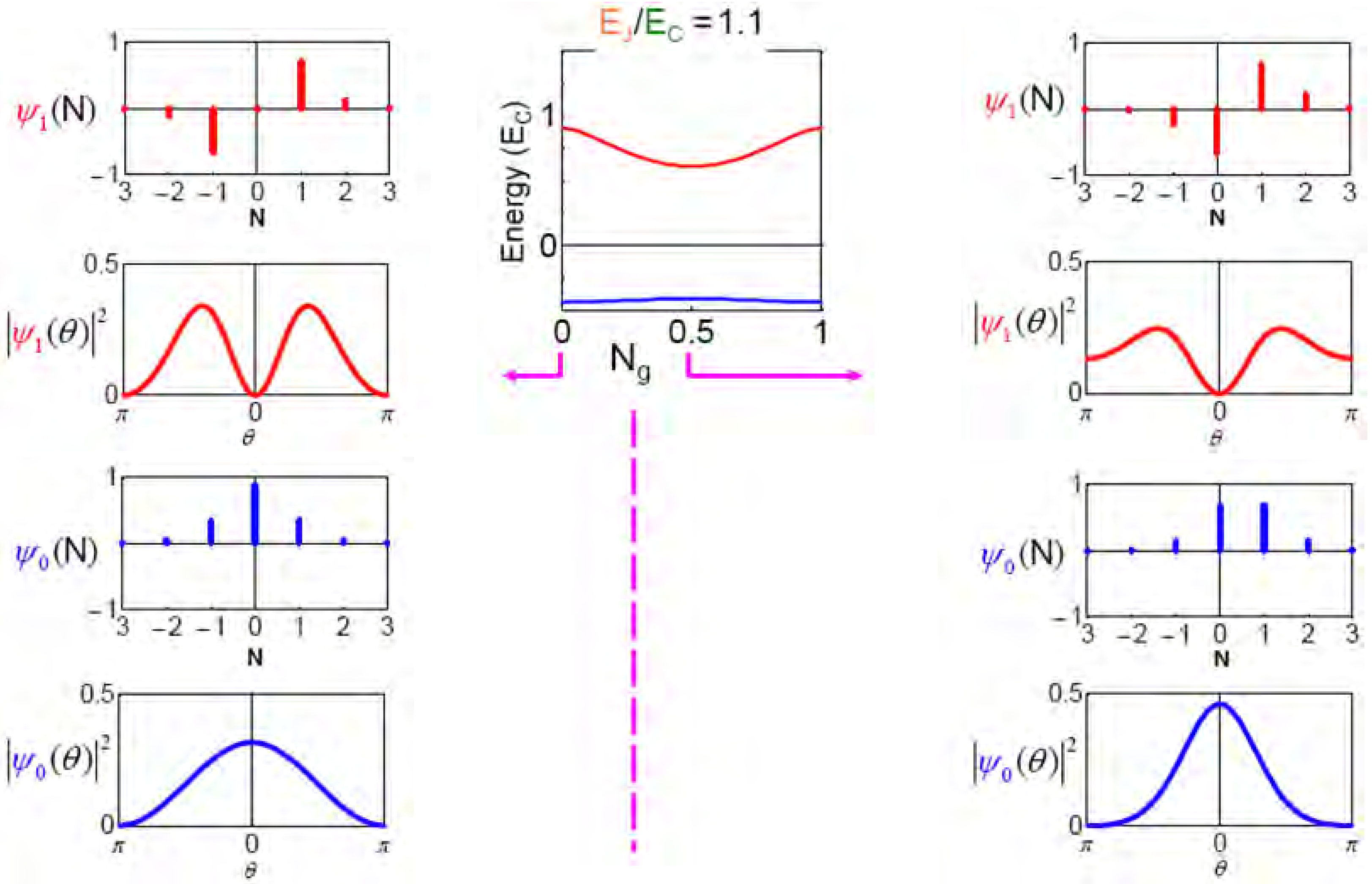
$|\psi_0\rangle \approx |N=0\rangle$

$$|\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$

courtesy CEA Saclay

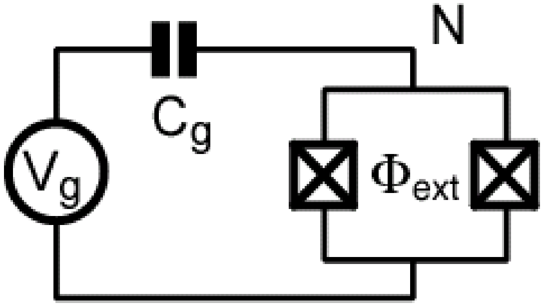
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

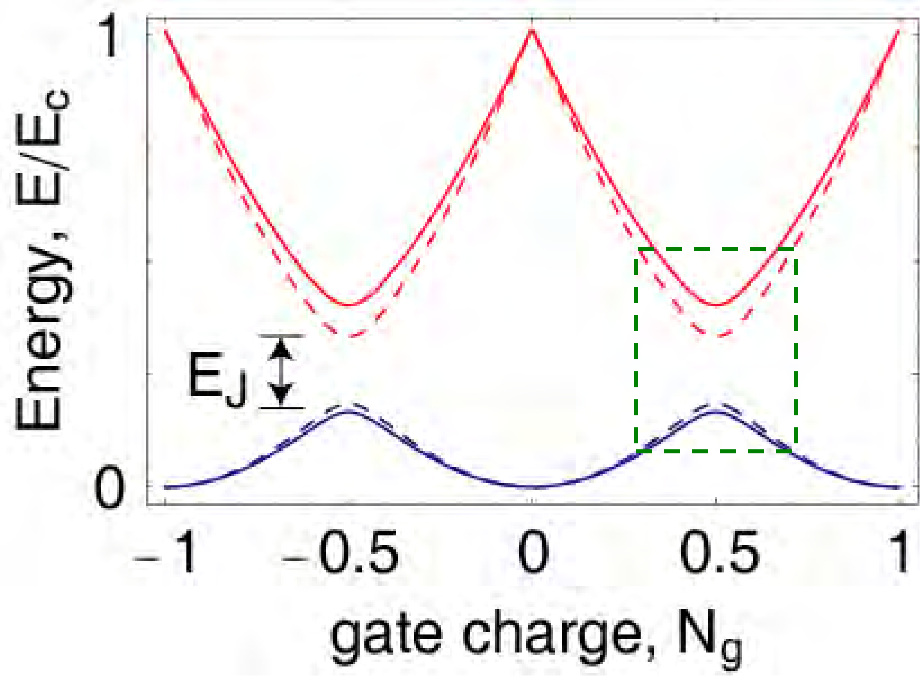
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\,\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\,\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

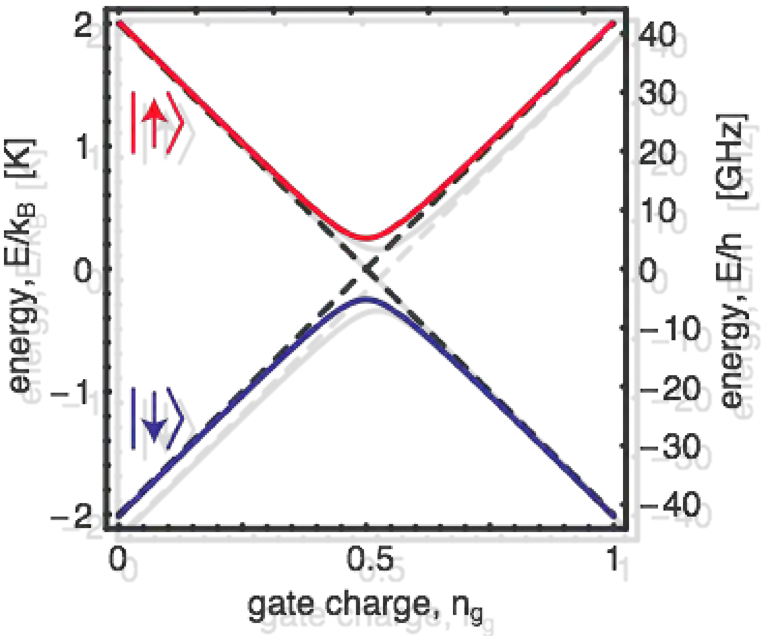
$$\hat{H}_{\mathrm{CPB}} = \sum_N \left[E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\mathrm{J}}}{2} \left(\left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\mathrm{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

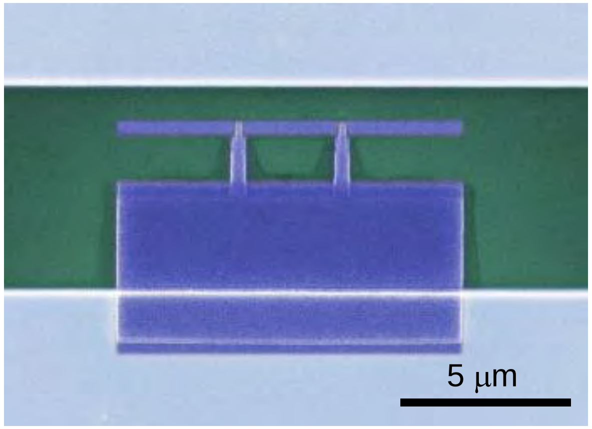


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

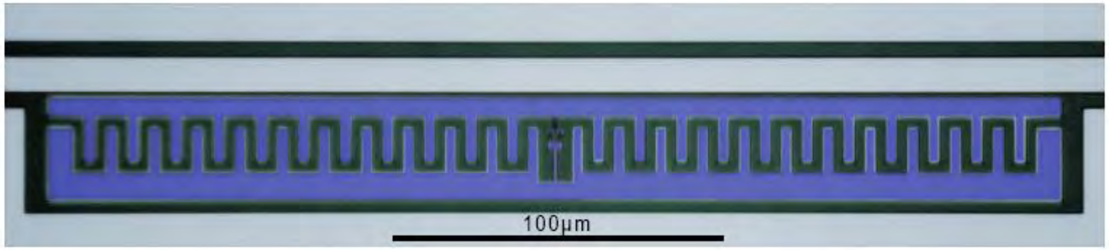
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

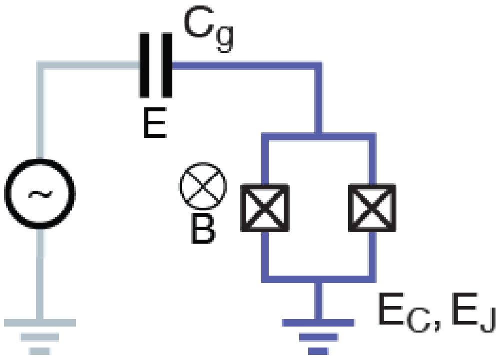
standard CPB:

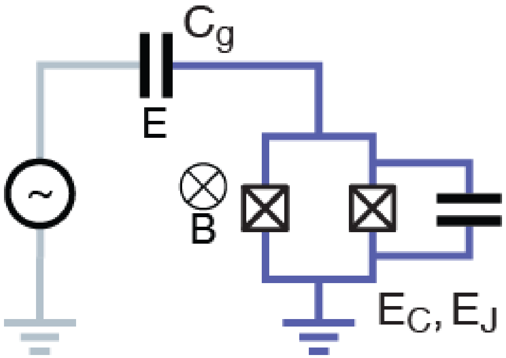


Transmon qubit:



circuit diagram:

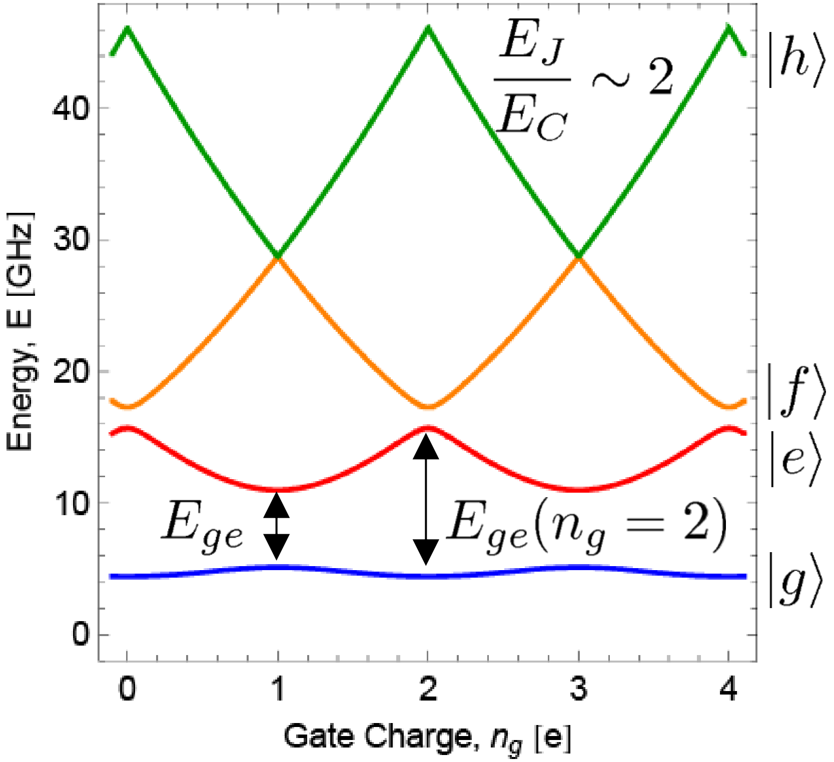




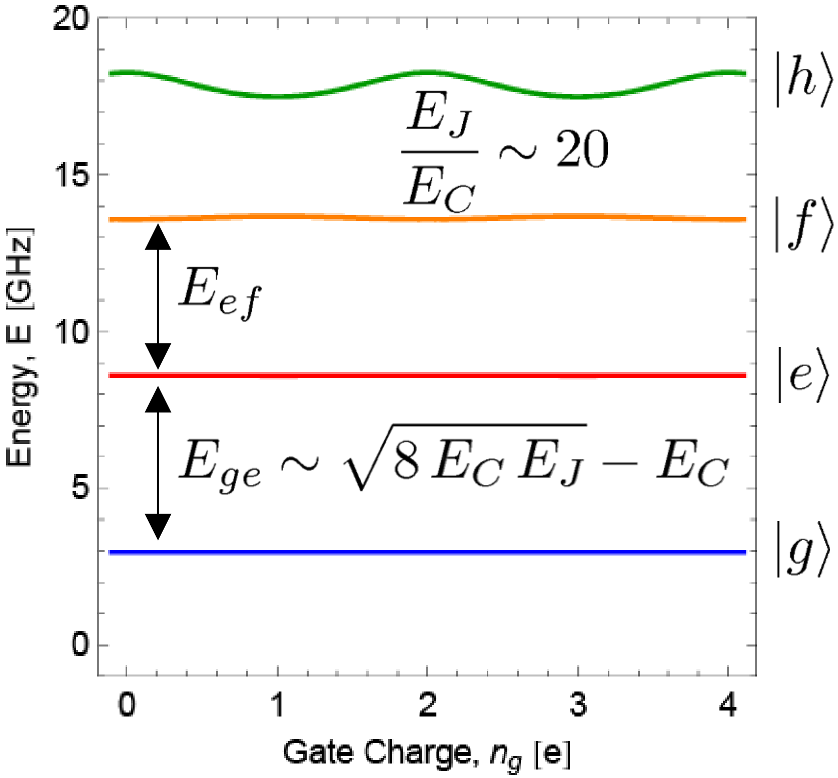
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

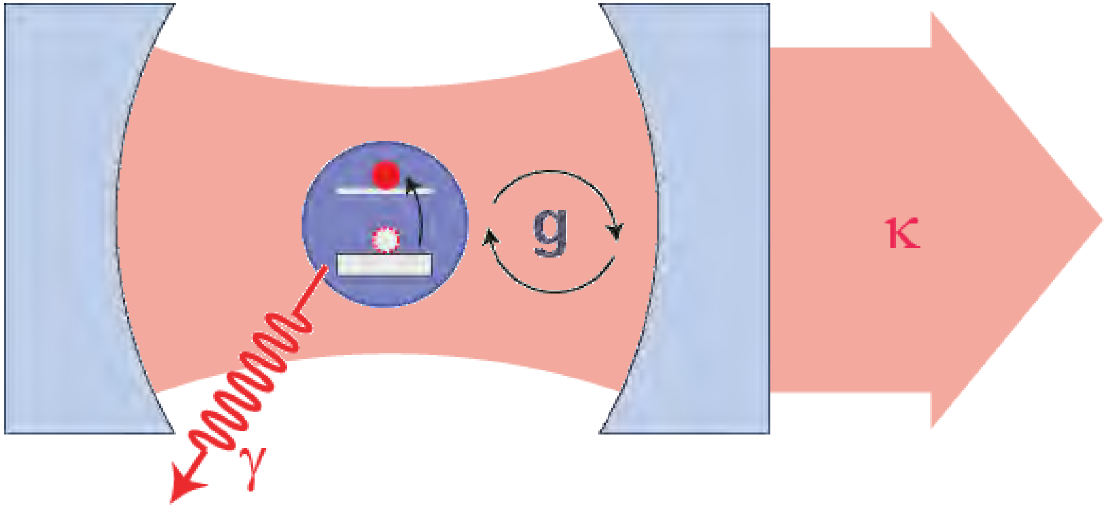
relative anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa\ ,1/t_{\text{transit}}$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Dressed States Energy Level Diagram

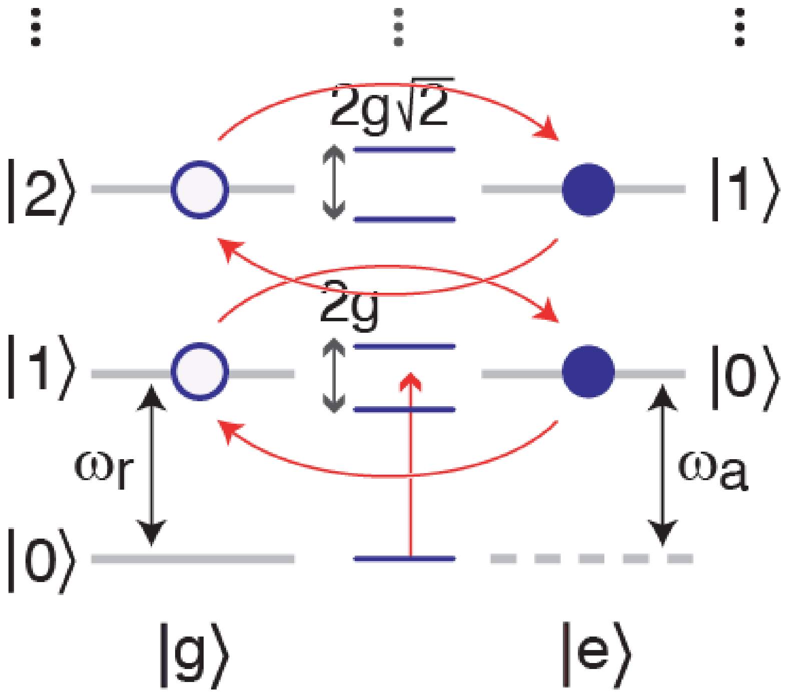
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \, \kappa$$



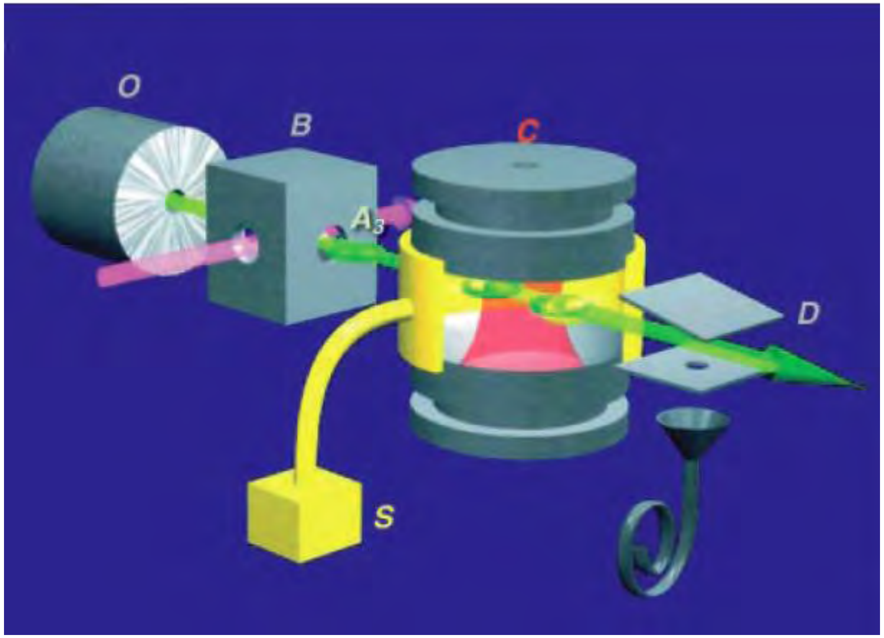
Jaynes-Cummings Ladder

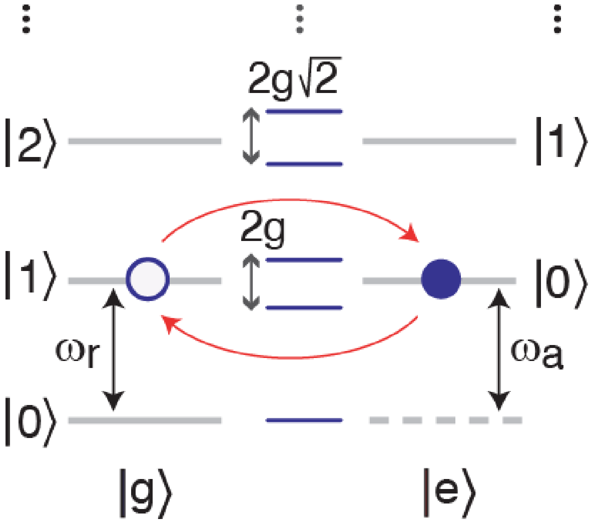
atomic cavity QED reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734-1738 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

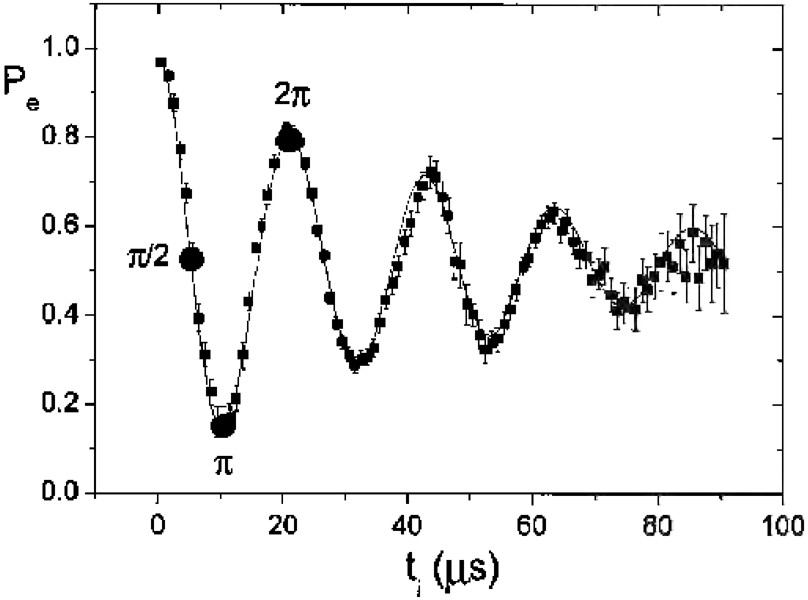
Vacuum Rabi Oscillations with Rydberg Atoms





with Rydberg atoms in microwave domain:

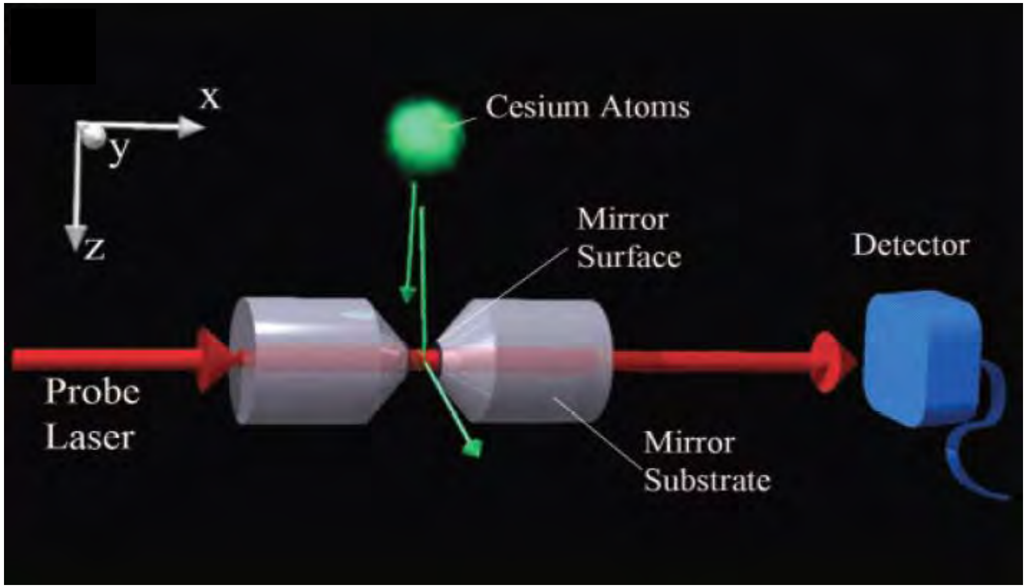
- large d

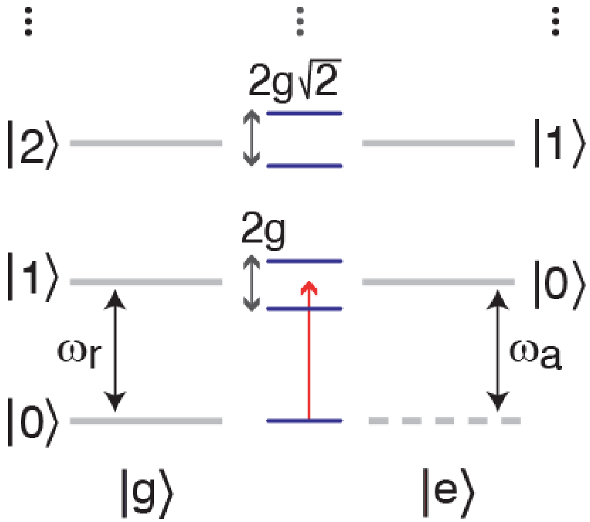


reviews:

Haroche, Raimond, *OUP Oxford* (2006)
Raimond, Brune, Haroche *RMP* **73**, 565 (2001)
this data: Brune *et al*, *PRL* **76**, 1800 (1996)

Vacuum Rabi Mode Splitting with Alkali Atoms



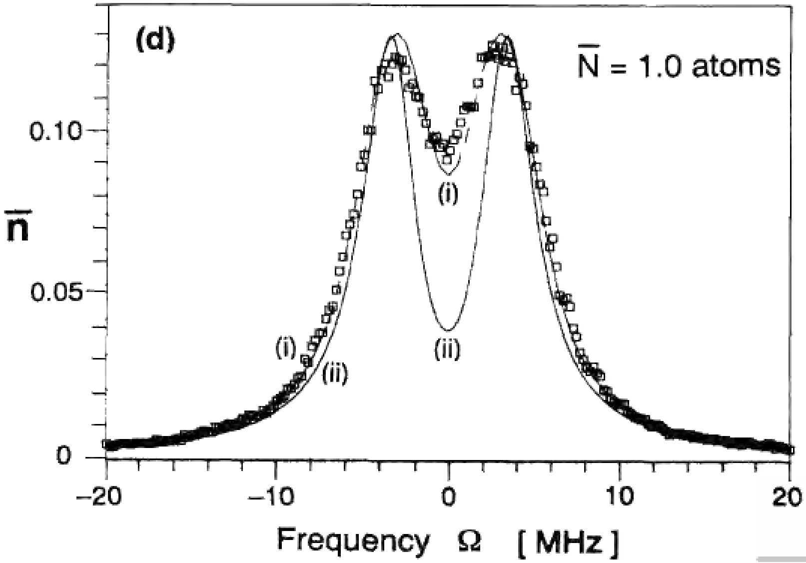


with alkali atoms in optical domain:

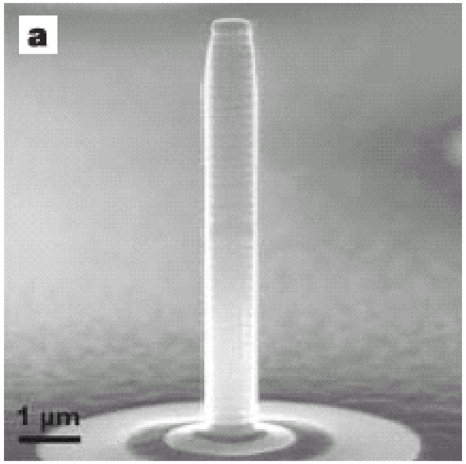
- large E_0

reviews:

Ye, Kimble, Katori, *Science* **320**, 1734 (2008)
Mabuchi, Doherty, *Science* **298**, 1372 (2002)
this data: Thompson, Rempe, Kimble *PRL* **68**, 1132 (1992)

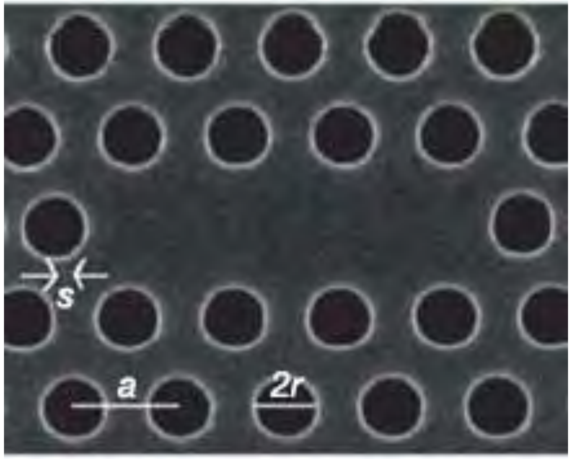


Cavity QED in Solid State with Semiconductors



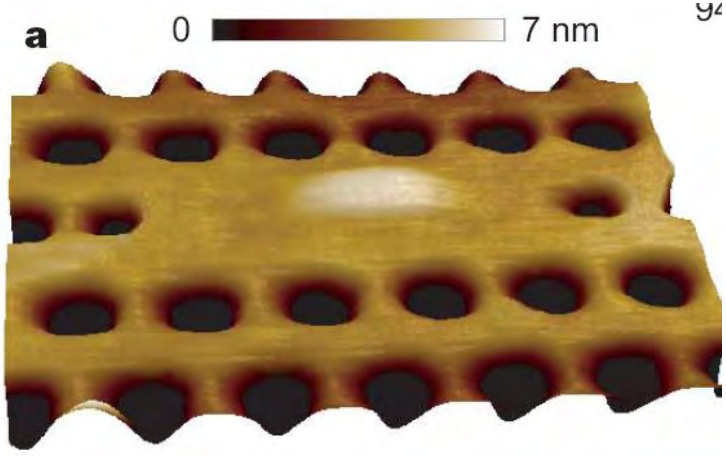
Wurzburg

*Nature **432**, 197 (2004)*



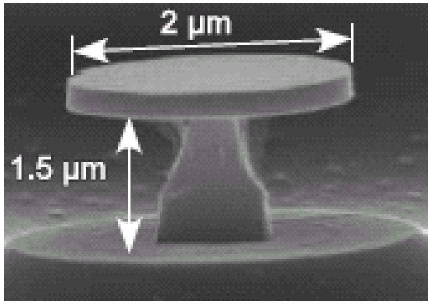
Arizona

*Nature **432**, 200 (2004)*



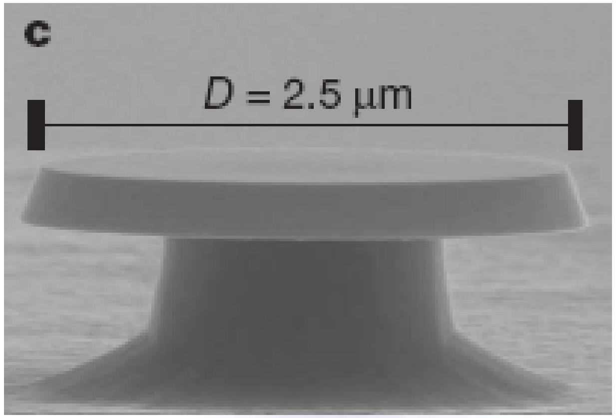
ETH Zurich

*Nature **445**, 896 (2007)*



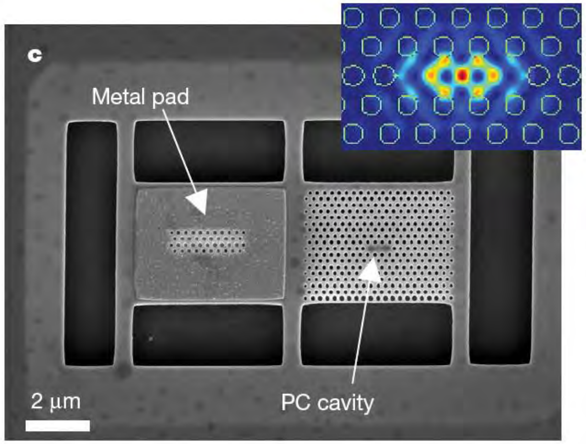
Paris

PRL (2004)



Caltech

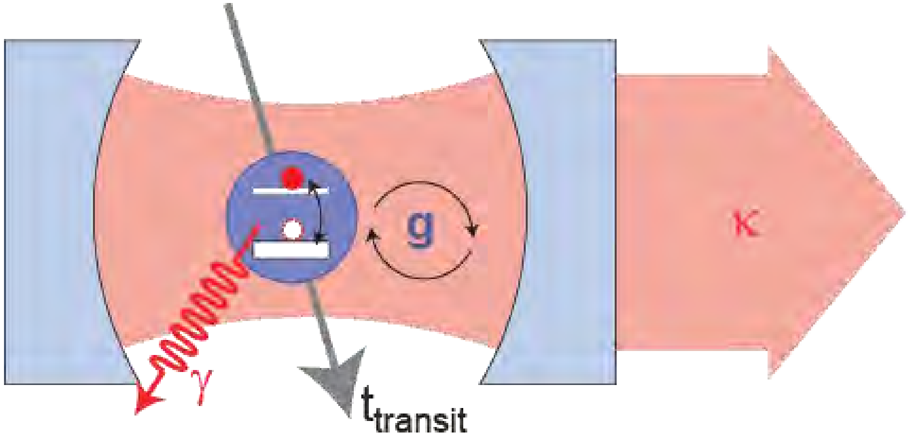
*Nature **450**, 862 (2007)*



Stanford

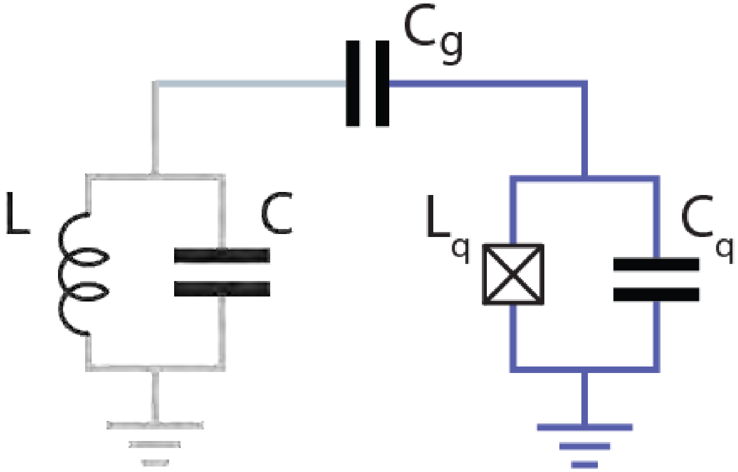
*Nature **450**, 857 (2007)*

Cavity QED with Superconducting Circuits

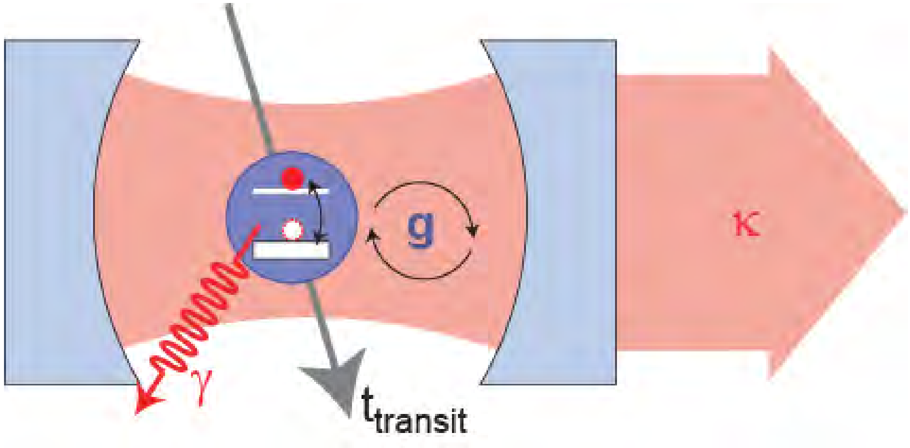


coherent quantum mechanics
with individual photons and qubits ...

... basic approach:



Proposals for Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

a number of approaches suggested at the time:

discrete LC circuits:

- Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
- O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).

large Josephson junctions:

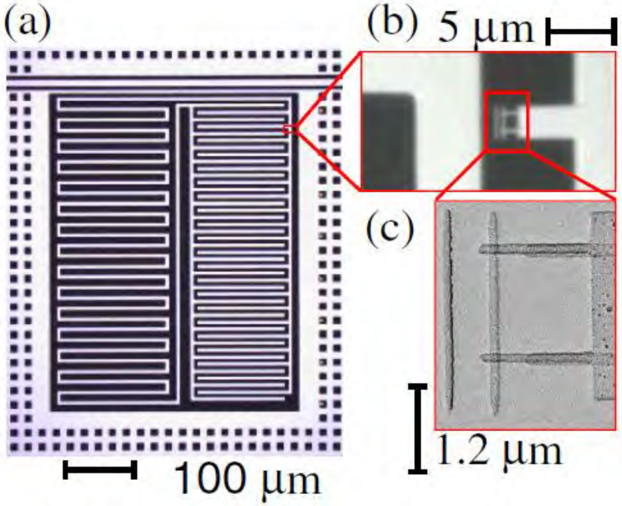
- F. Marquardt and C. Bruder, Phys. Rev. B **63**, 054514 (2001).
- F. Plastina and G. Falci, Phys. Rev. B **67**, 224514 (2003).
- A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. **90**, 127901 (2003).

3D cavities:

- W. Al-Saidi and D. Stroud, Phys. Rev. B **65**, 014512 (2001).
- C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).
- J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).

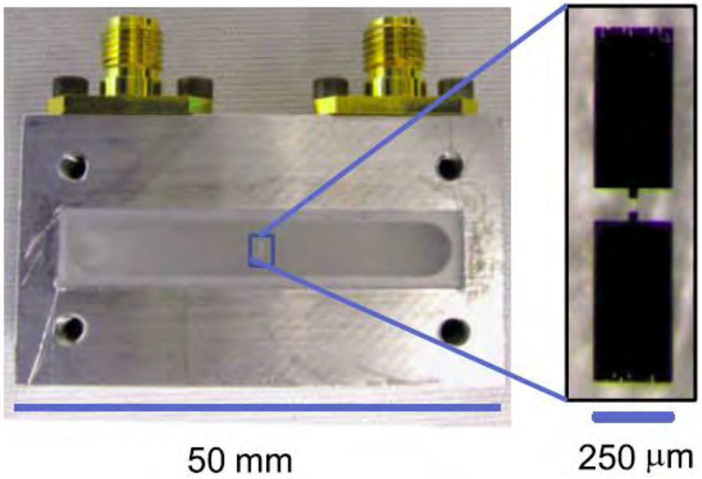
Circuit QED and its Different Realizations

lumped element resonator:



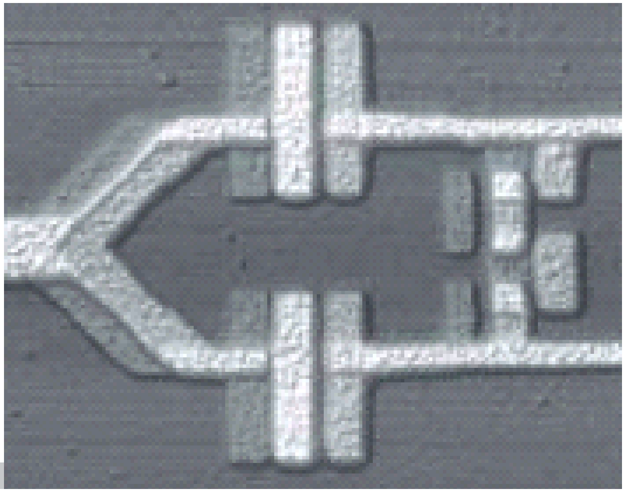
Z. Kim *et al.*, *PRL* **106**, 120501 (2011)

3D cavity:



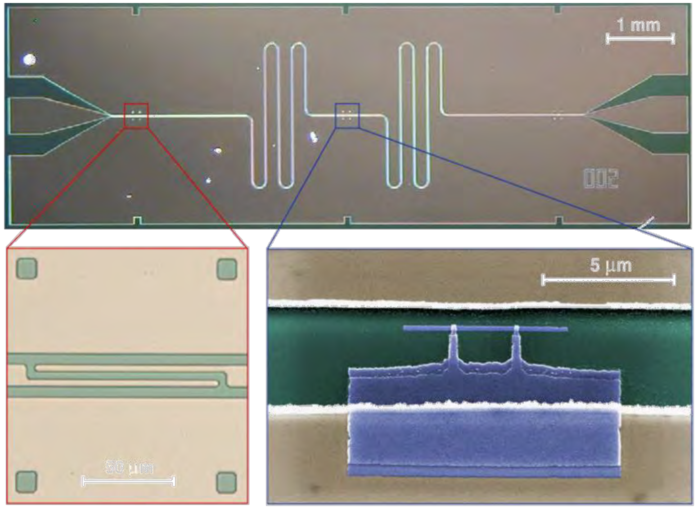
H. Paik *et al.*, *PRL* **107**, 240501 (2011)

weakly nonlinear junction:



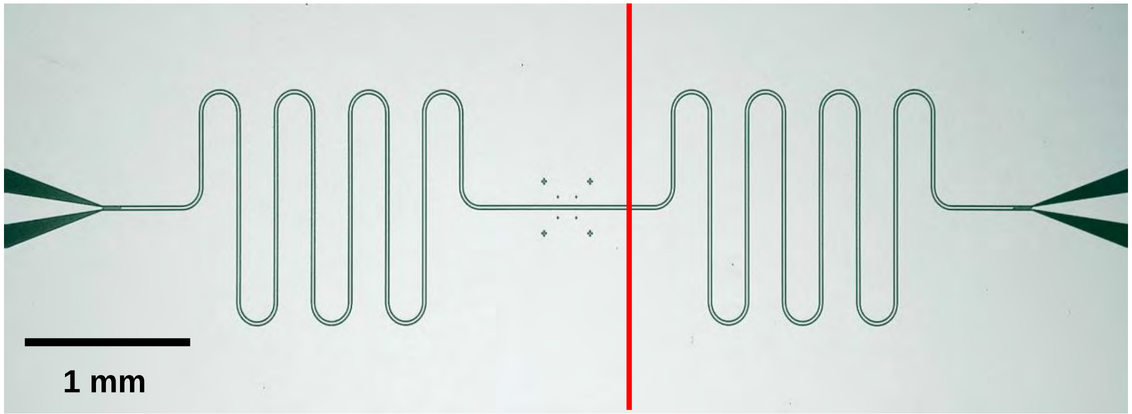
I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

planar transmission line resonator:

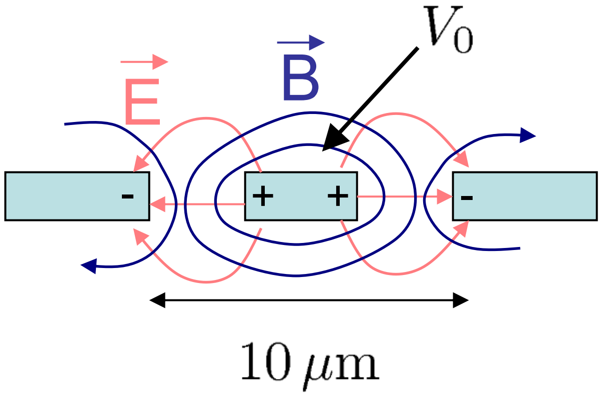


A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Large Vacuum Field in 1D Cavity



optical microscope image of strip line resonator



electric field across resonator in vacuum state (*n*=0):

$$\int \epsilon_0 E_{0,\mathrm{rms}}^2\, dV_{\mathrm{mod}} = \frac{\hbar\omega_r}{2}$$

harmonic oscillator

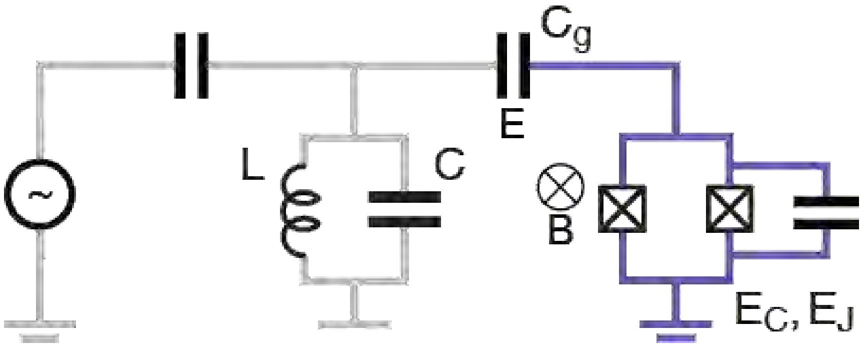
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right)$$

$$E_{0,\mathrm{rms}} \approx 0.2\,\mathrm{V/m}$$

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$

$\times 10^6$ larger than E_0 in 3D microwave cavity

Qubit/Photon Coupling



Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy $N_g = 1/2$ and change of qubit basis (*z* to *x*, *x* to -*z*)

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

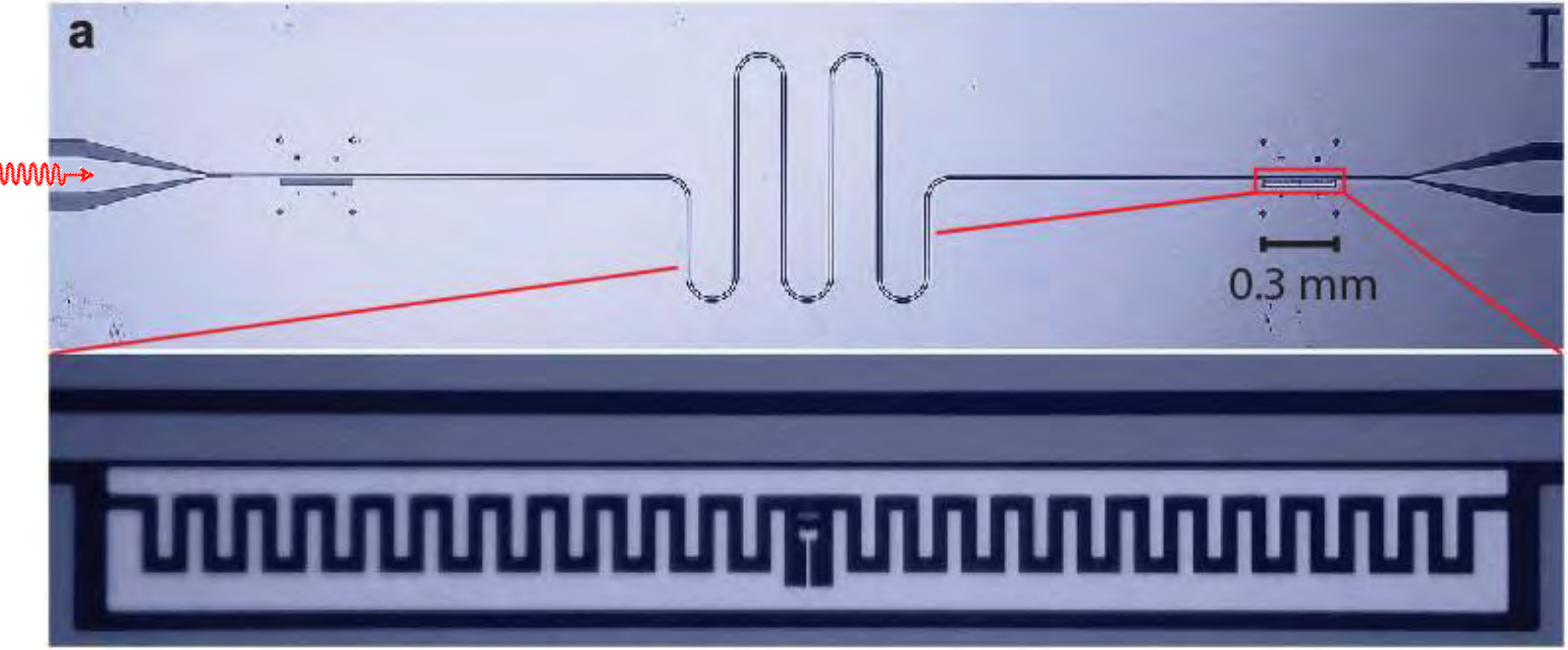
$$\hat{H}_g = \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \cancel{\hat{a}\hat{\sigma}}^- + \cancel{\hat{a}^\dagger\hat{\sigma}}^+ + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma} 2e \sqrt{\frac{\hbar\omega_r}{2C}}$

Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1\ldots 300\,\mathrm{MHz}$

$$g \gg [\kappa, \gamma] \text{ possible!}$$

Realization



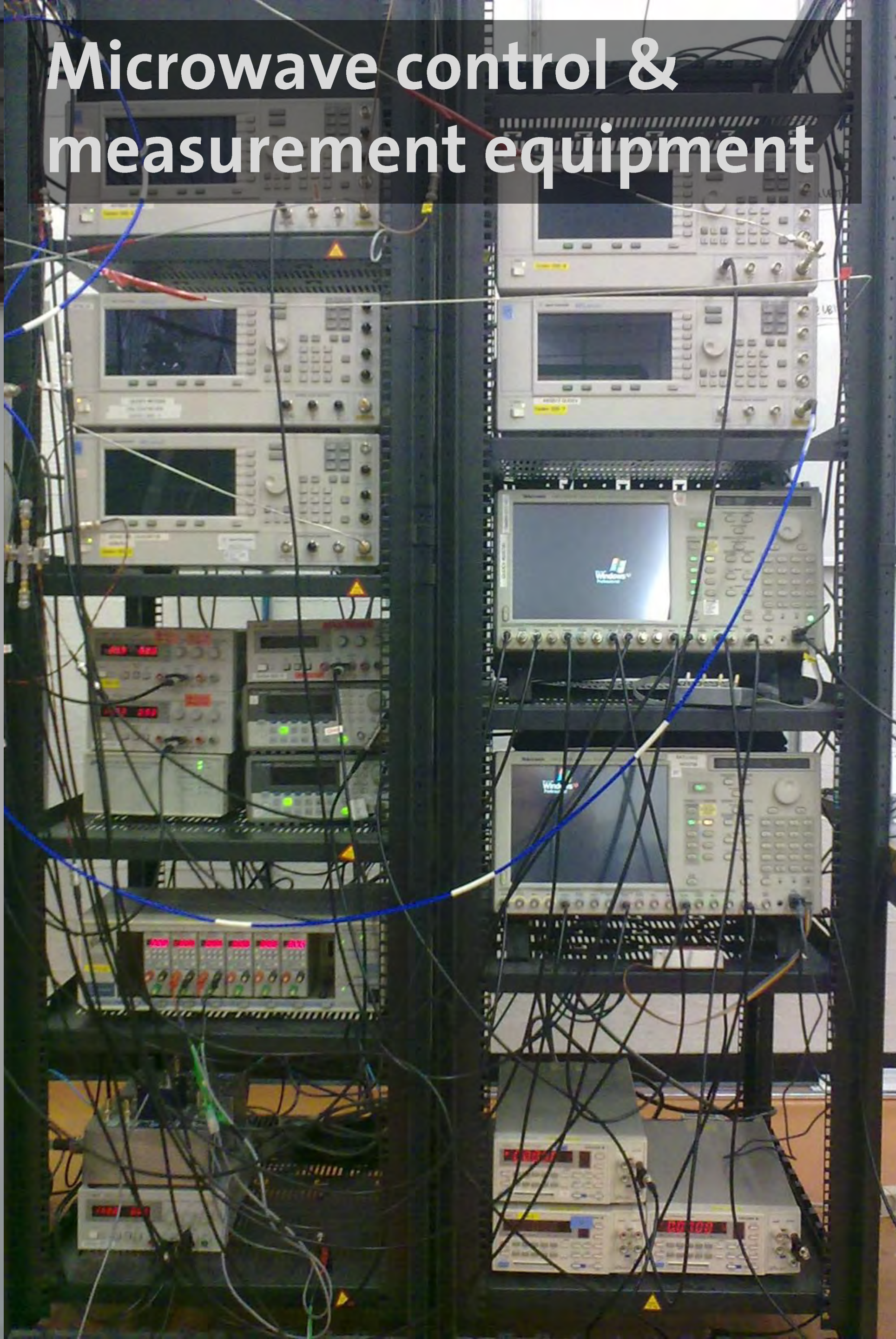
device fabrication: L. Frunzio et al., IEEE Trans. on Appl. Supercon. **15**, 860 (2005)

Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

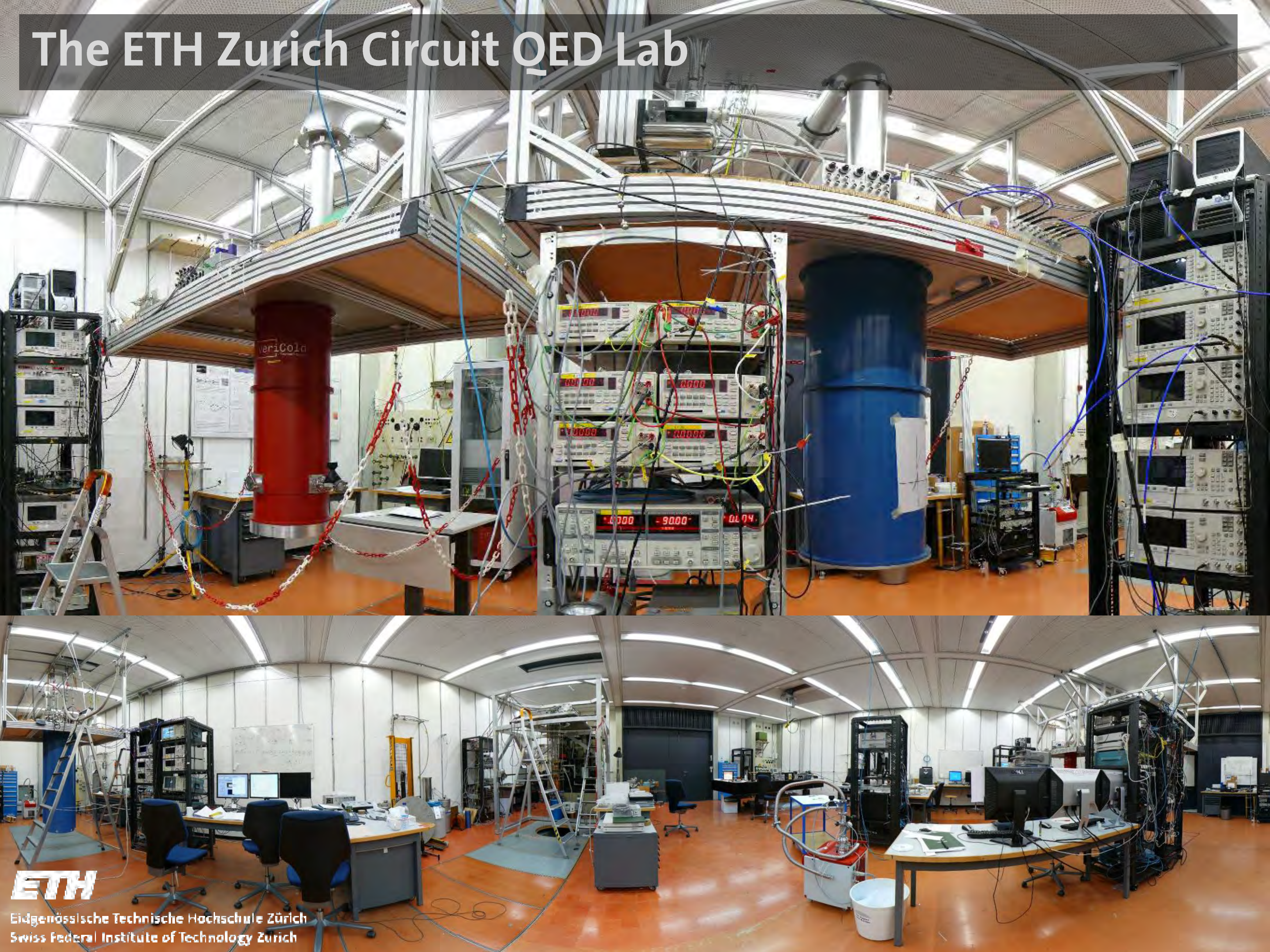
Cryostate for temperatures down to 0.02 K

Microwave control & measurement equipment



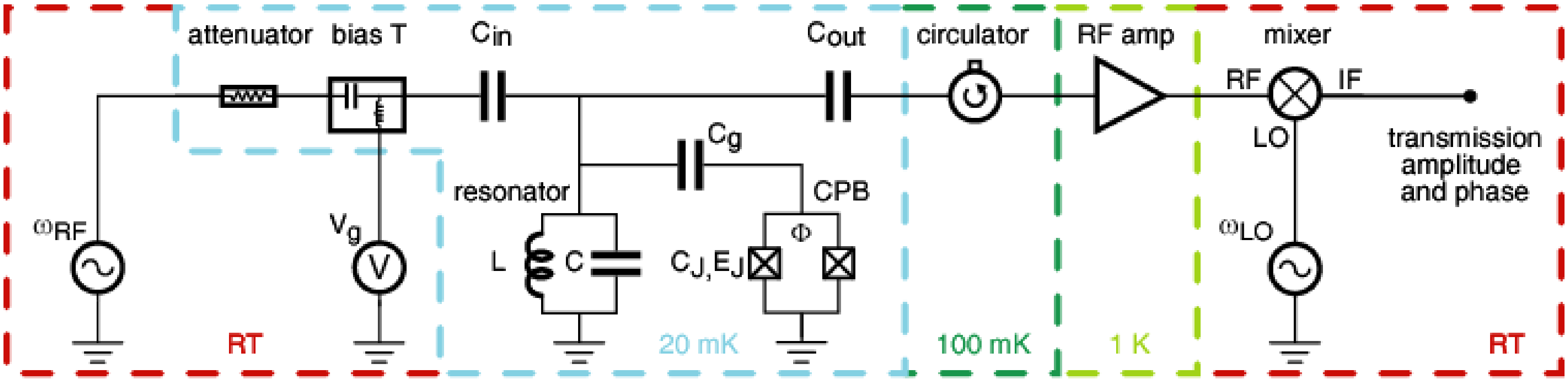
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

The ETH Zurich Circuit QED Lab



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

How to do the Measurement



- prevent leakage of thermal photons (cold attenuators and circulators)
- average power to be detected ($\omega_r/2\pi$ = 6 GHz, $\kappa/2\pi$ = 1 MHz)

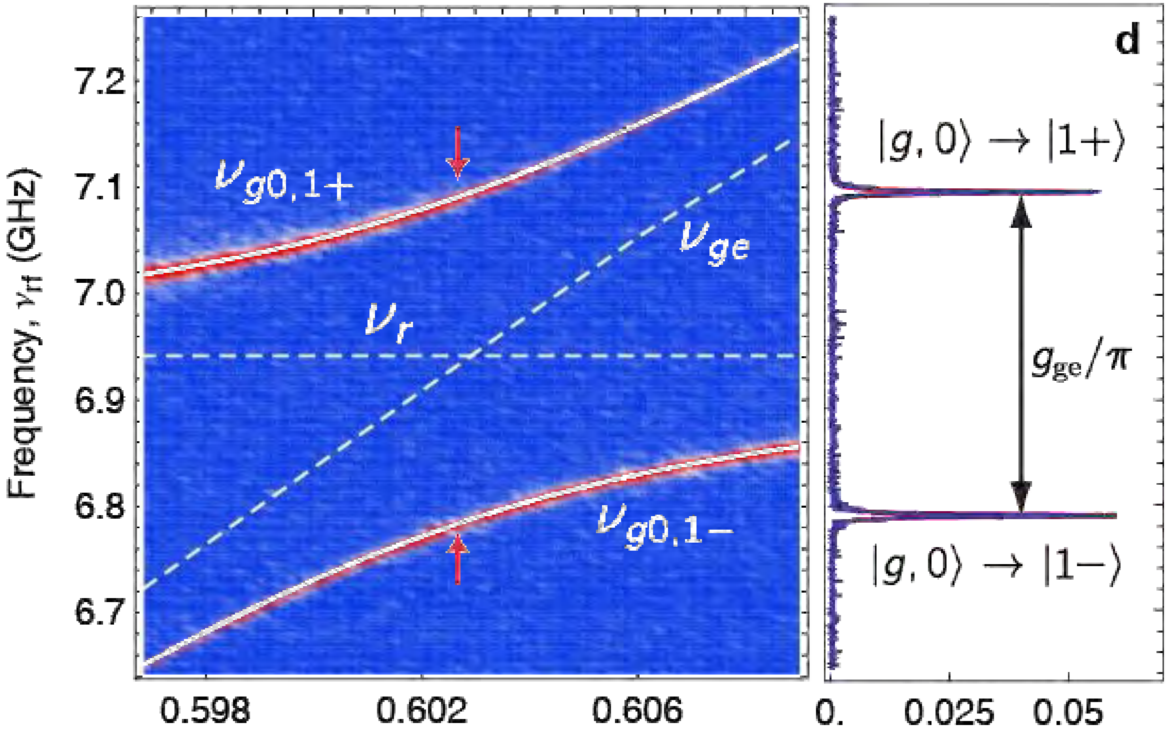
$$\langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

- efficient with cryogenic low noise HEMT amplifier T_N = 6 K

Resonant Vacuum Rabi Mode Splitting ...

... with one photon ($n=1$):

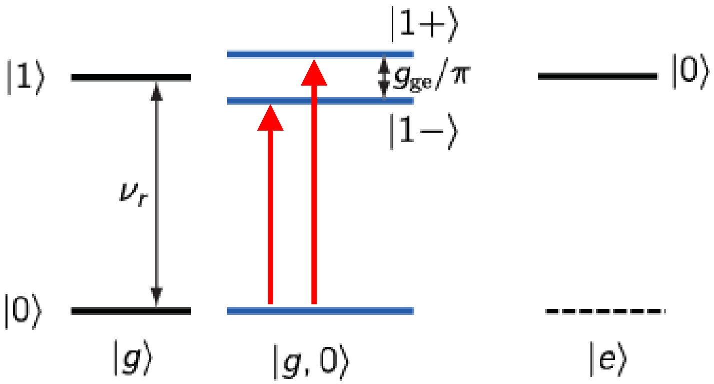
very strong coupling:



$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{\mathrm{ge}} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

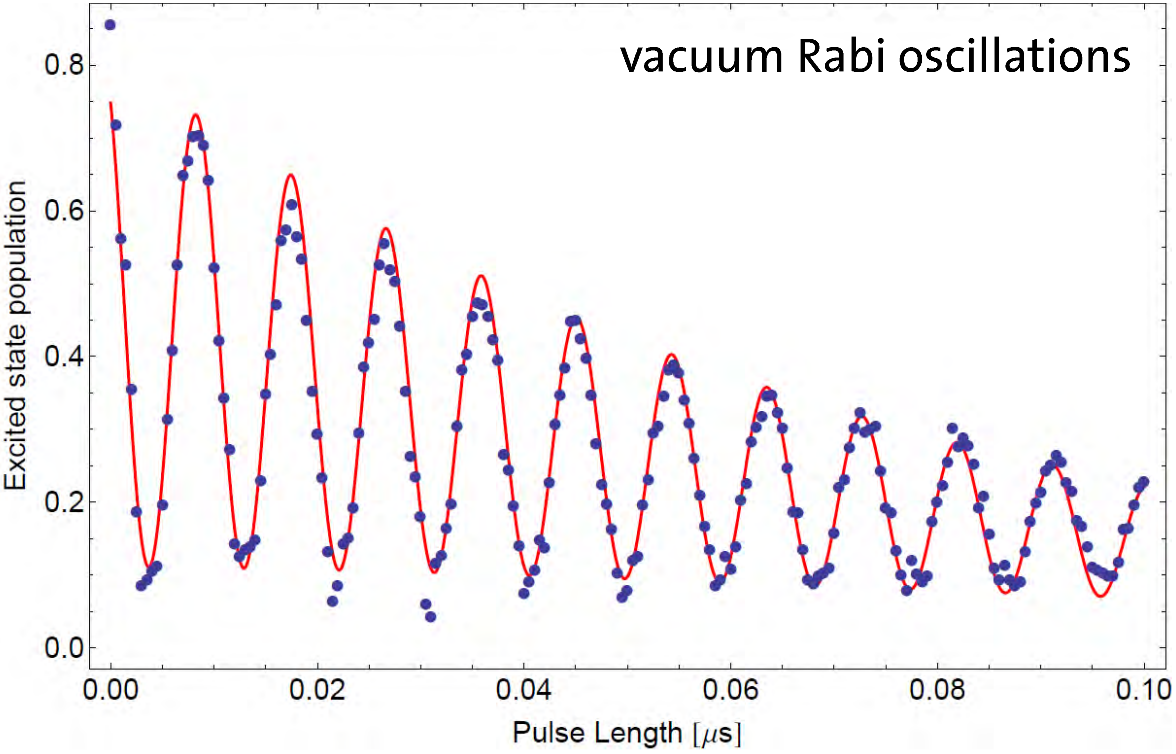
first demonstration in a solid: A. Wallraff et *al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

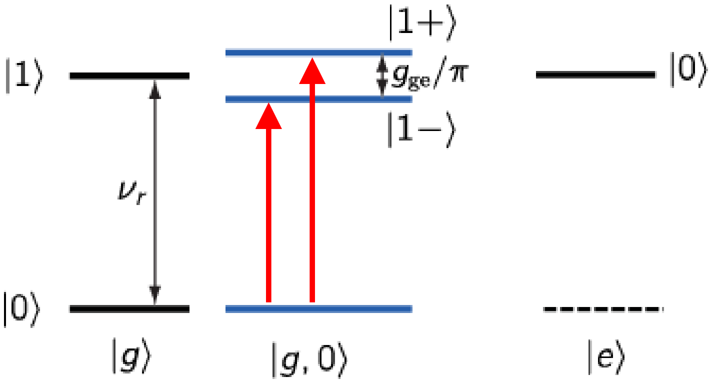


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{\mathrm{ge}} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

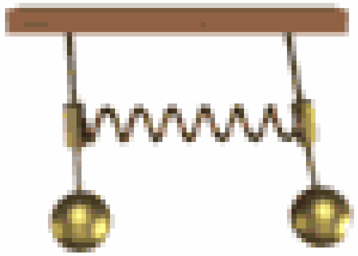
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Probing Field Quantization on a Chip ...

... by measuring the quantum nonlinearity of the J-C ladder

What can be learned from a measurement of the vacuum Rabi mode splitting?

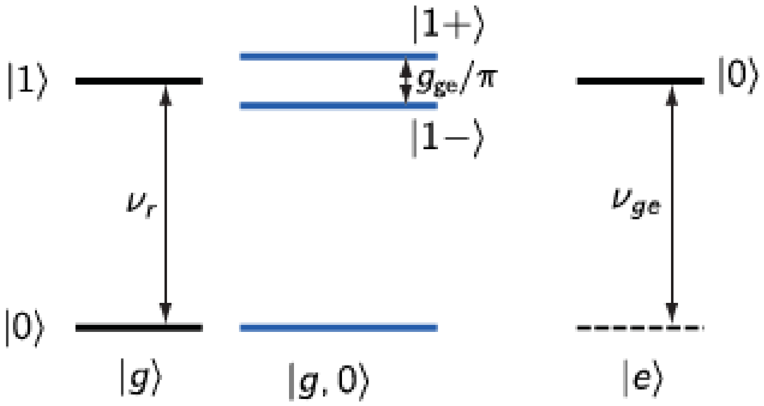


classical interpretation:

- coupled harmonic oscillators
- normal mode coupling

quantum effects:

- scaling of coupling g_{eff} with square root of the photon number n
- direct proof of field quantization



time-resolved data in atomic physics exps. (Haroche, Walther, ...) [but no spectroscopic data until recently]

$$|n\pm\rangle = (|g,n\rangle \pm |e,n-1\rangle)/\sqrt{2}$$

Climbing the Jaynes-Cummings Ladder

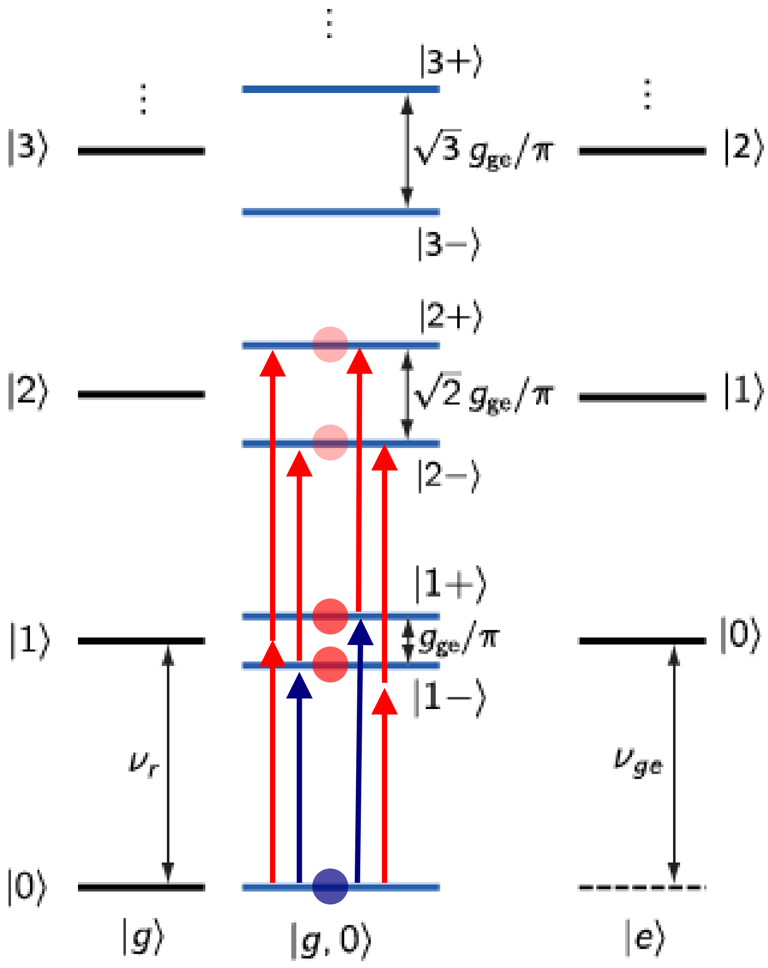
How to climb the ladder?

start on the lowest rung:

- cool to the ground state |g,0>

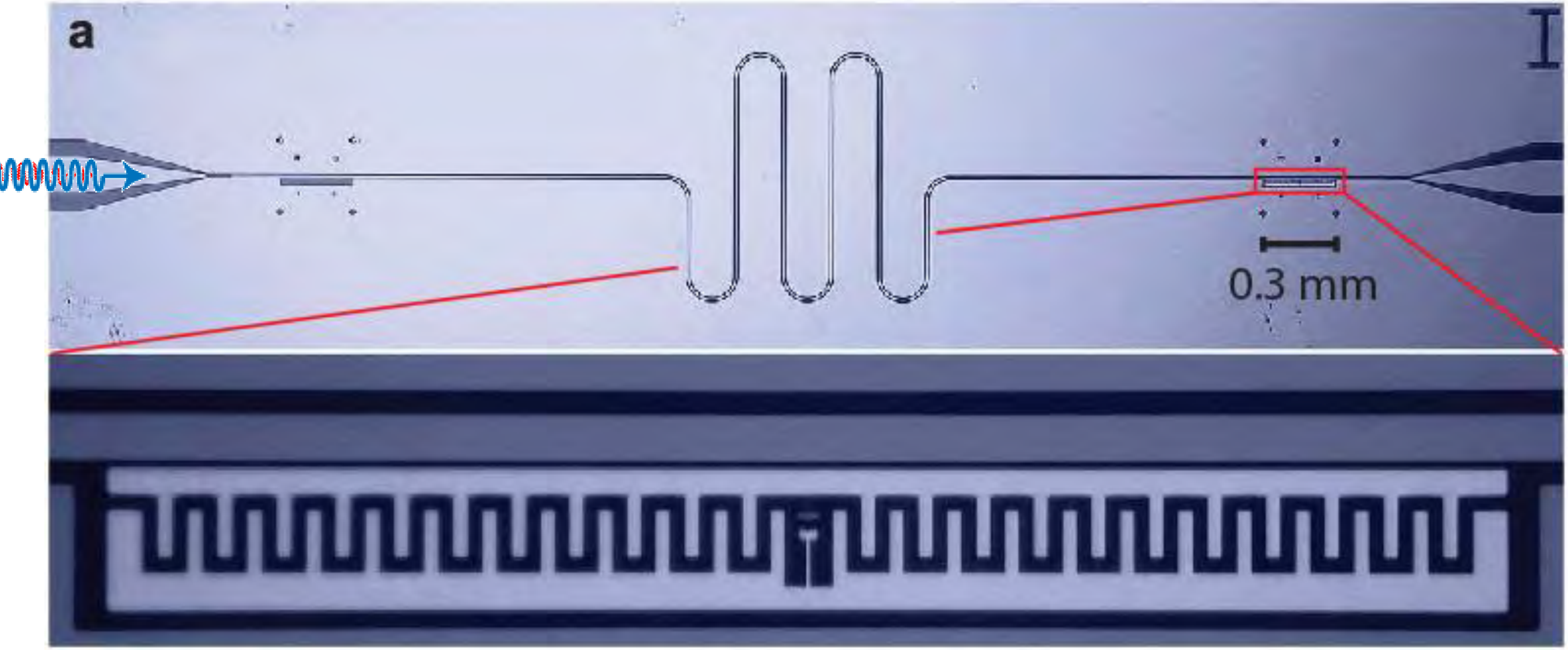
climb towards higher rungs:

- step by step:
  ‚pump & probe‘ excitation
  J. Fink *et al., Nature* **454**, 315 (2008)
  I. Schuster *et al., Nat. Phys.* **4**, 382 (2008)
  M. Hofheinz *et al., Nature* **454**, 310 (2008)
- many rungs at the same time:
  multi-photon excitation
  L. S. Bishop *et al., Nature Phys.* **5**, 105 (2009)
- thermal excitation
  J. Fink *et al., PRL* **105**, 163601 (2010)
  J. Fink *et al., Physica Scripta* **T137**, 014013 (2009)
  with full control over phase:
  M. Hofheinz *et al., Nature* **459**, 546 (2009)



$$|n\pm\rangle = \left(|g,n\rangle \pm |e,n-1\rangle\right)/\sqrt{2}$$

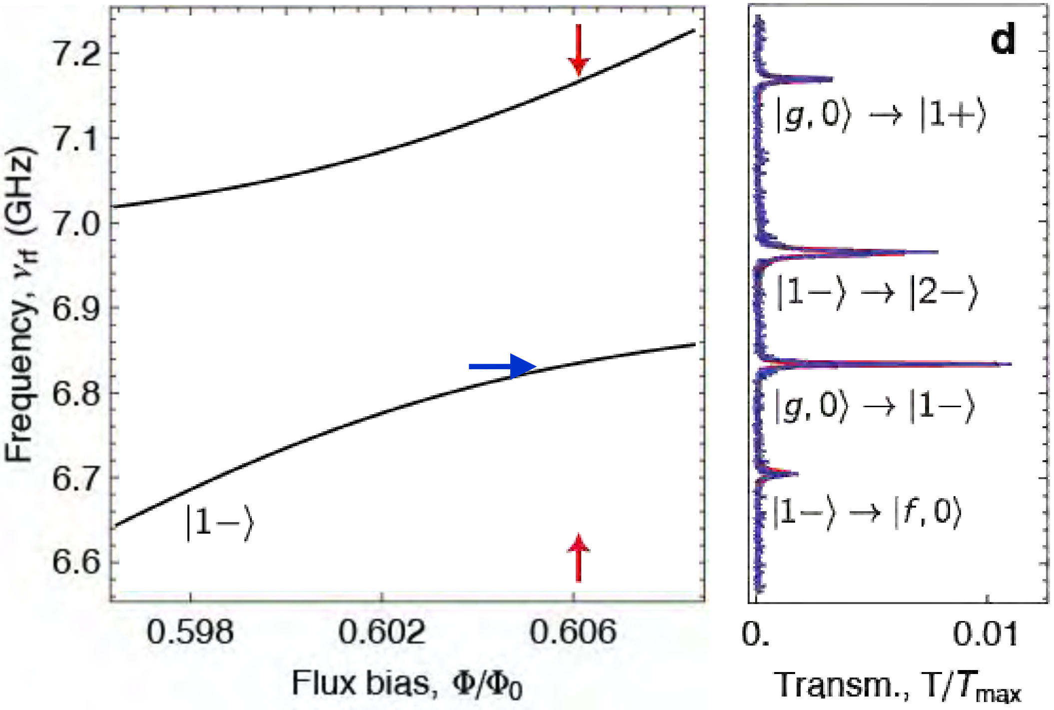
Two-Photon Pump and Probe Spectroscopy



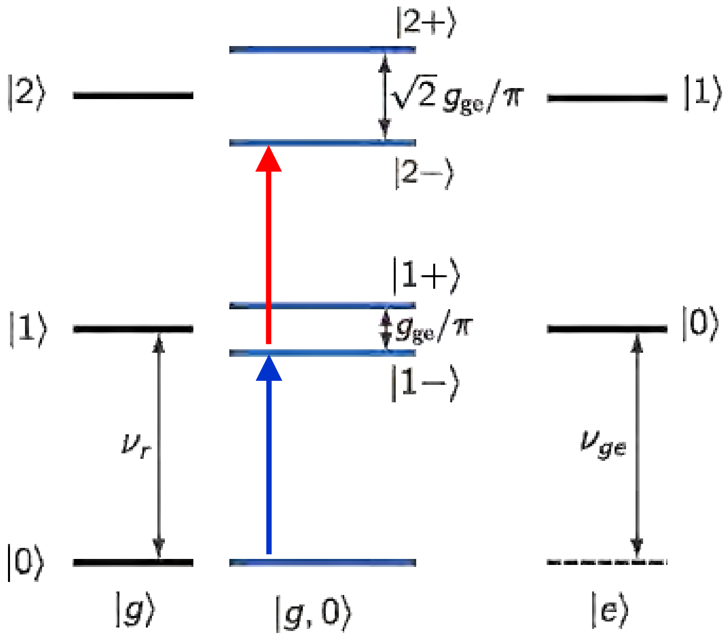
J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting …

… with two photons ($n = 2$):



pump and probe: $|n-\rangle$

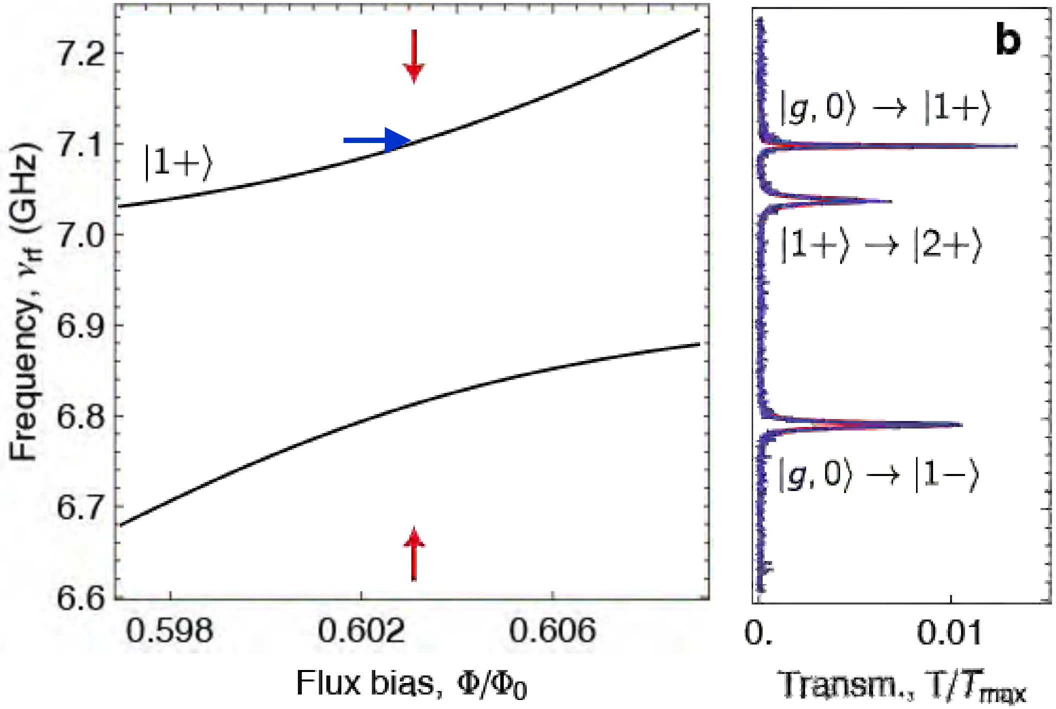


- $|n-\rangle \rightarrow |n+\rangle$ is weak

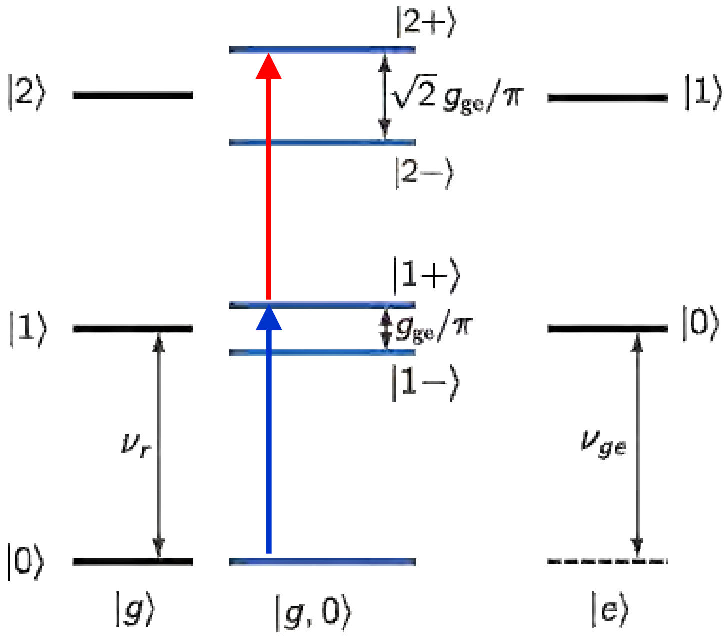
J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with two photons ($(n = 2)$):



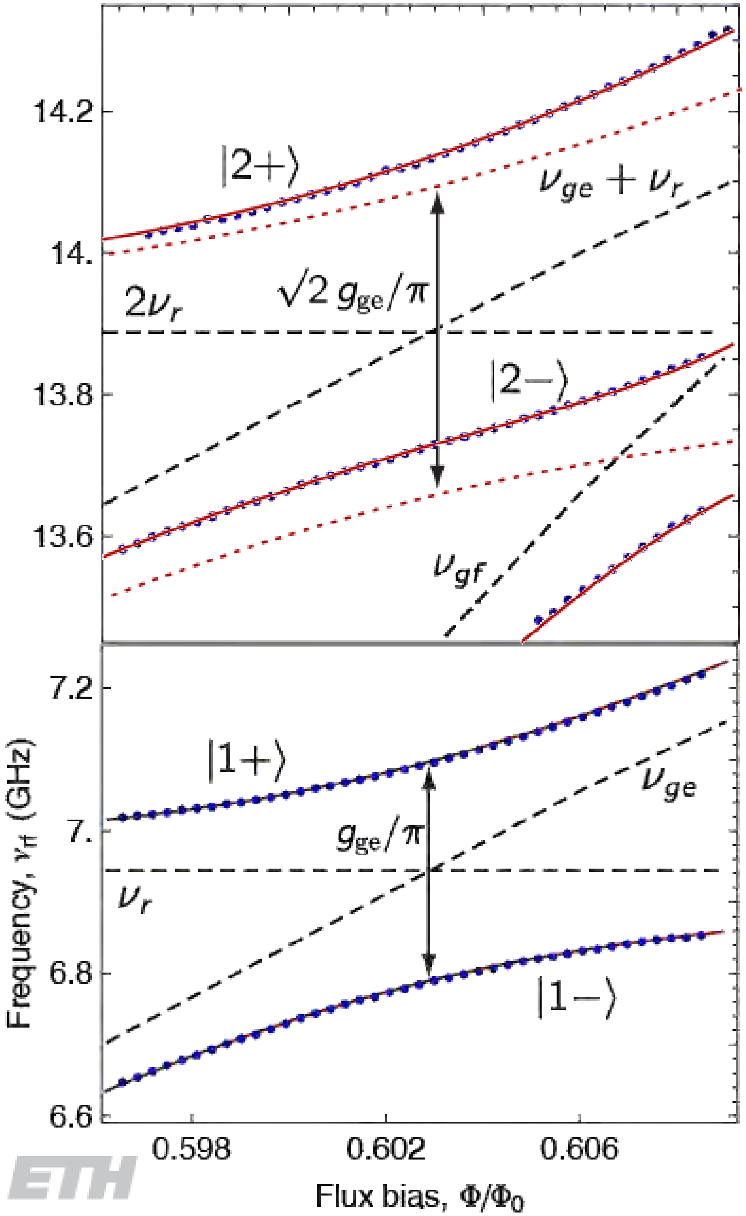
pump and probe: $|n+\rangle$



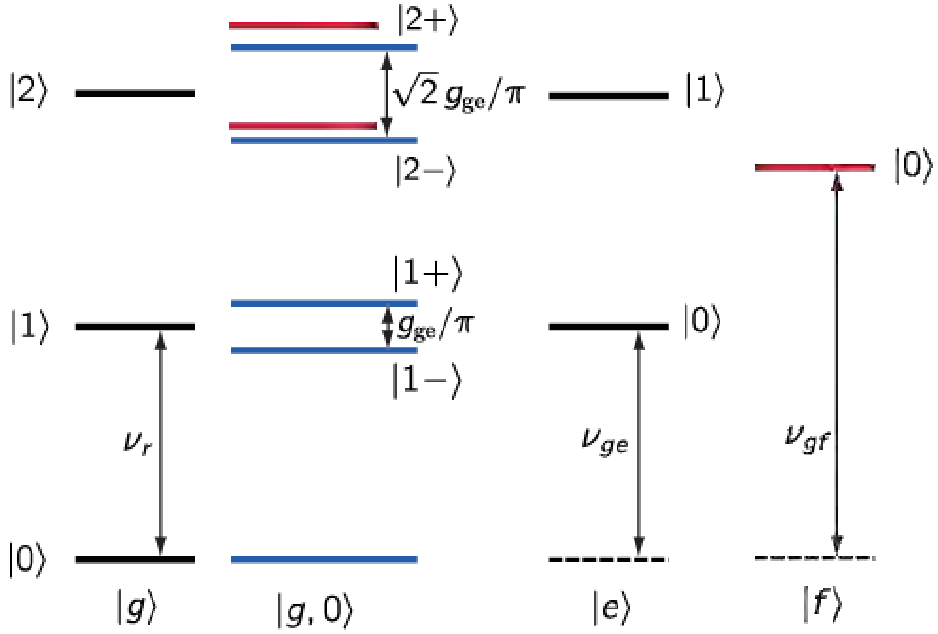
- $|n+\rangle \rightarrow |n-\rangle$ is weak

J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Sqrt(n) Quantum Nonlinearity



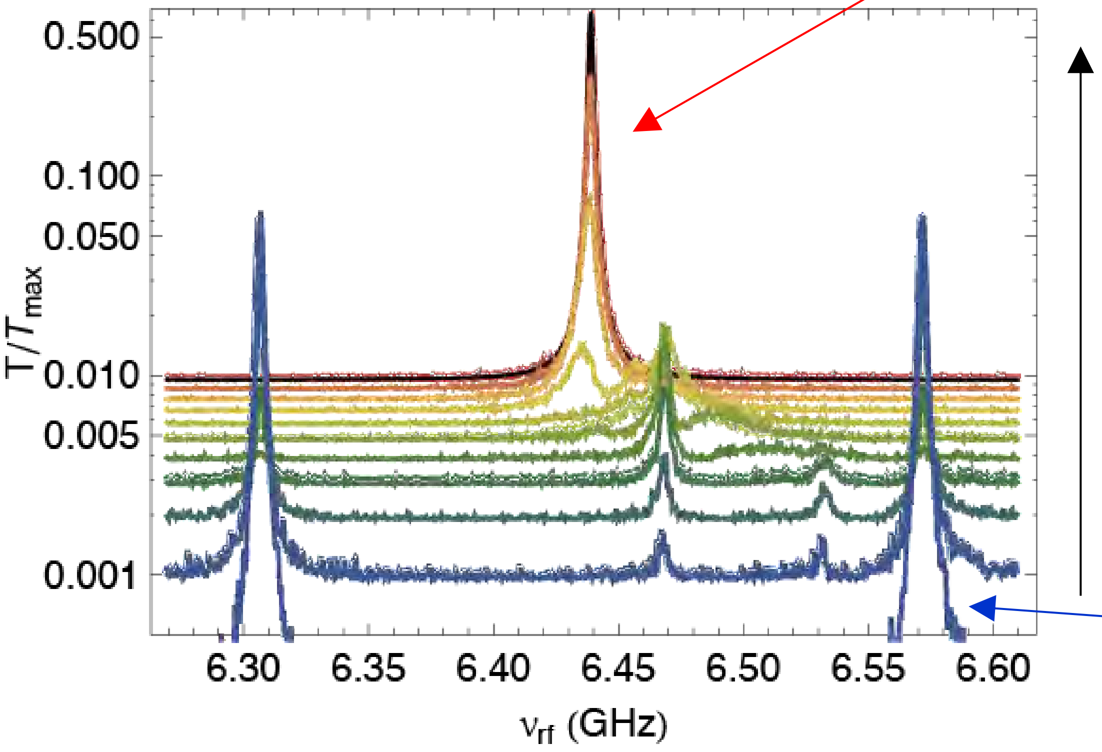
- energies reconstructed from pump + probe
- shifts due to 3rd qubit level $|f\rangle$
- full Hamiltonian yields good agreement
- clear spectroscopic demonstration of field quantization in cavity QED



J. Fink, … , A. Wallraff, *Nature (London)* **454**, 315 (2008)

Vacuum Rabi Mode Splitting at High Photon Numbers

linear response spectra:



classical

increasing temperature

quantum

- vacuum Rabi split spectrum at lowest temperatures
- single Lorentzian line at highest field temperatures
- cross-over from quantum to classical spectrum
- perspective: investigate entanglement at elevated temperatures

J. Fink *et al.*, *PRL* **105**, 163601 (2010)

Cavity QED
with one, two and three artificial atoms …
… probing the collective interaction of a number of atoms with a single photon

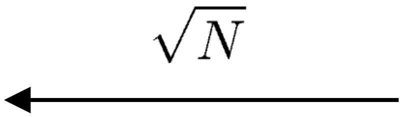
Cavity QED with Multiple ~~Photons~~ Atoms

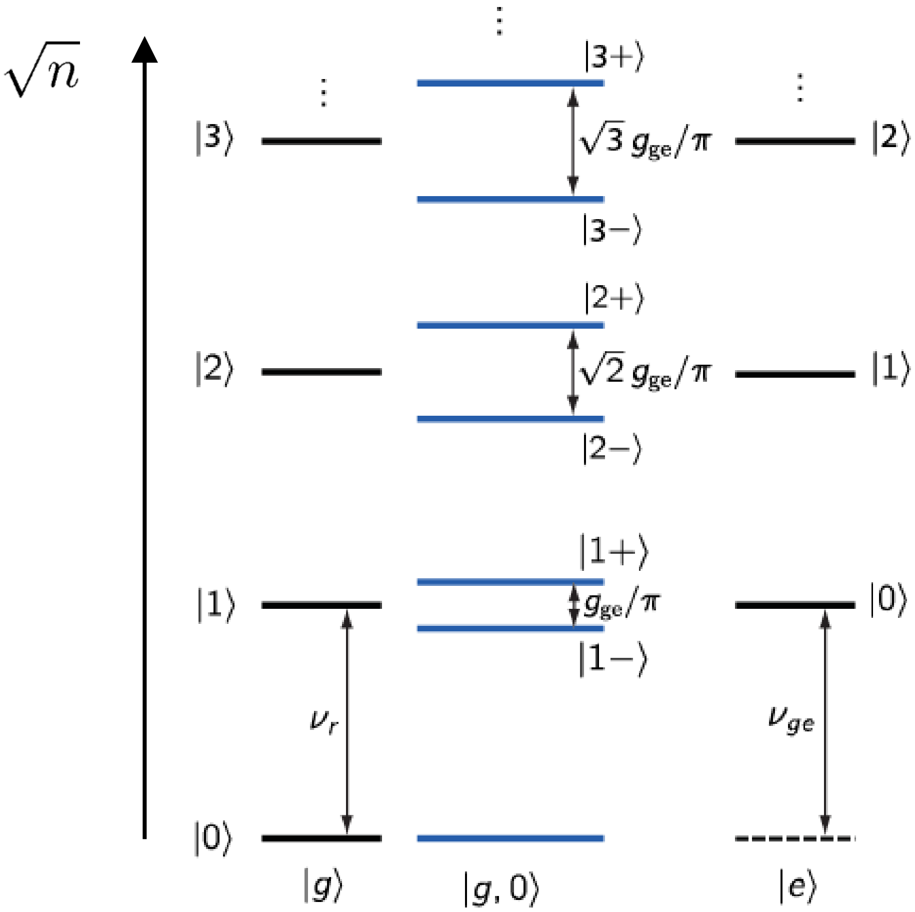
coupling *n* photons to single atom

J. Fink et al., *Nature (London)* **454**, 315 (2008)

coupling *N* atoms to a single photon

J. Fink et al., *Phys. Rev. Lett.* **103**, 083601 (2009)



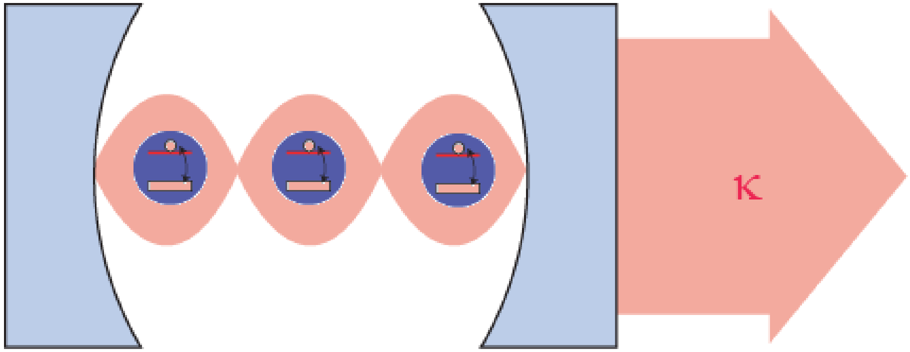


Tavis-Cummings Model

Jaynes-Cummings Model

Multi-Atom Cavity QED

- early on: $\bar{N}$ falling through cavity

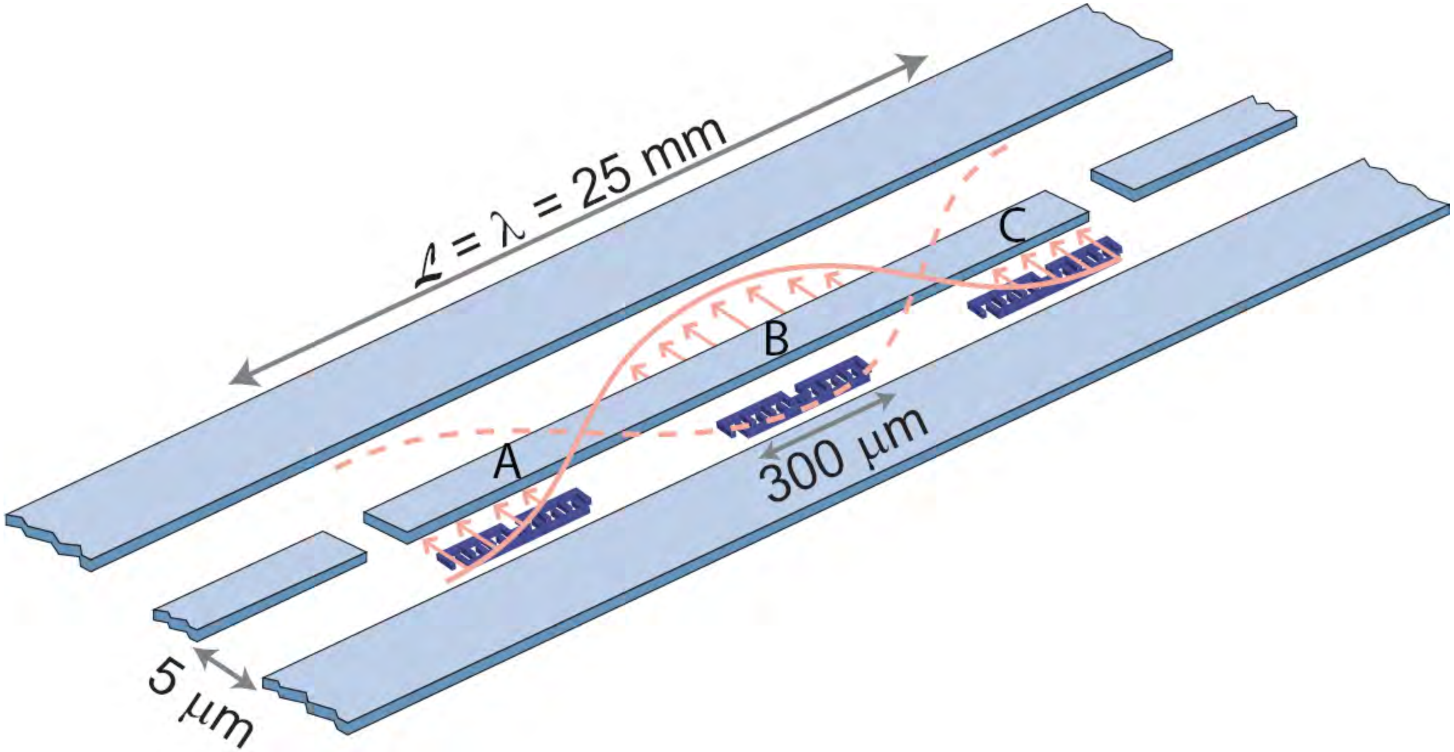


- atom number δN and coupling δg fluctuations
- Tavis-Cummings model

$$\hat{\mathcal{H}}_{\mathrm{TC}} = \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\left(\frac{\hbar}{2}\omega_{\mathrm{j}}\hat{\sigma}_{\mathrm{j}}^{z} + \hbar g_{\mathrm{j}}(\hat{a}^{\dagger}\hat{\sigma}_{\mathrm{j}}^{-} + \hat{\sigma}_{\mathrm{j}}^{+}\hat{a})\right)$$

- difficult to trap atoms

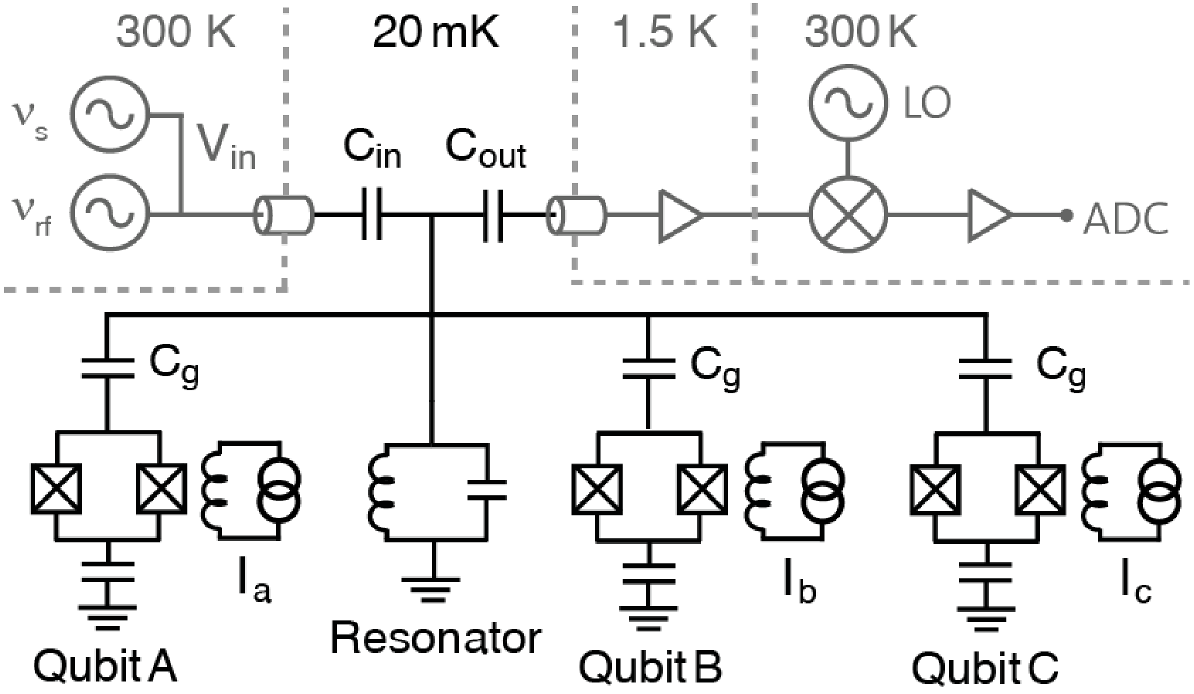
Multi-Qubit Circuit QED Schematic



in circuit QED:

- small well defined number of qubits
- no qubit number fluctuations
- fixed coupling strength
- full single qubit control

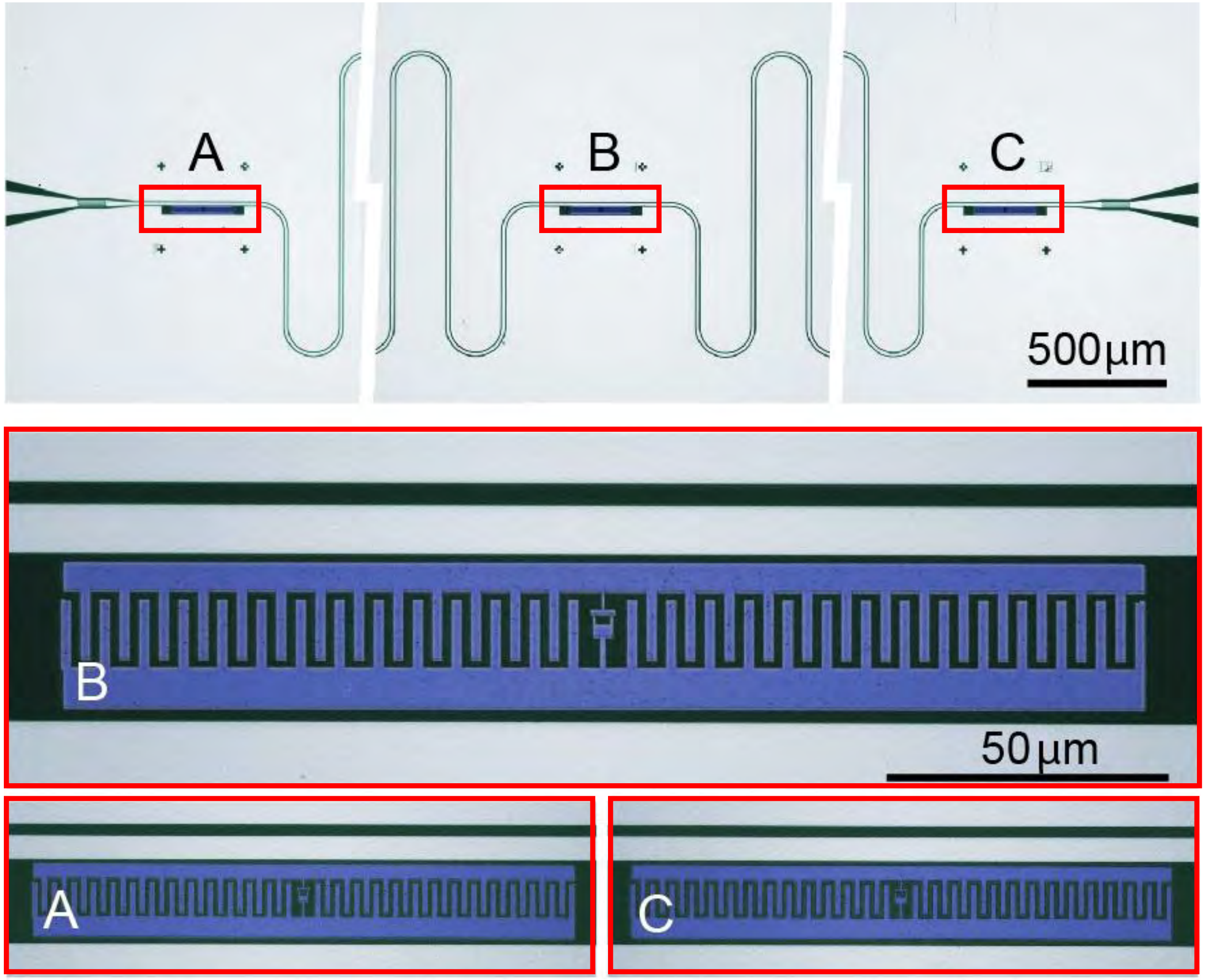
Three Qubit Circuit QED Setup



- one cavity
- three qubits
- local flux control $\Phi_{\mathrm{A,B,C}}$

Three Qubit Circuit QED Sample

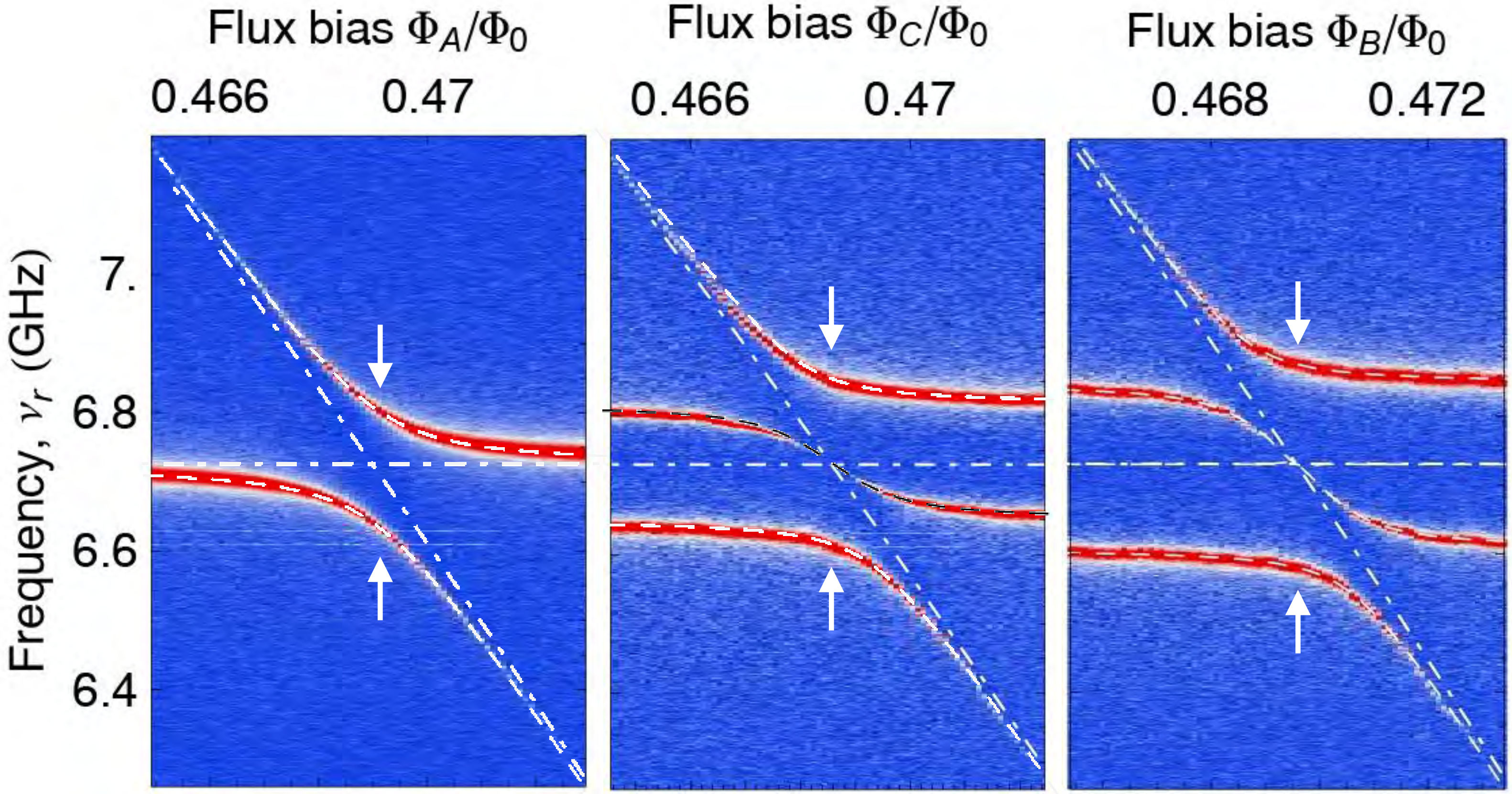
- three qubits integrated into one cavity



- qubits are almost identical
- almost identical coupling constants $g_{\mathrm{A,B,C}}$

Qubit j	E_{C_j}/h (MHz)	$E_{J\max_j}/h$ (GHz)	$g_j/2\pi$ (MHz)
A	283	224	83.7
B	287	226	-85.7
C	294	214	85.1

N = 1, 2, 3 Qubit – Cavity Anti Crossing

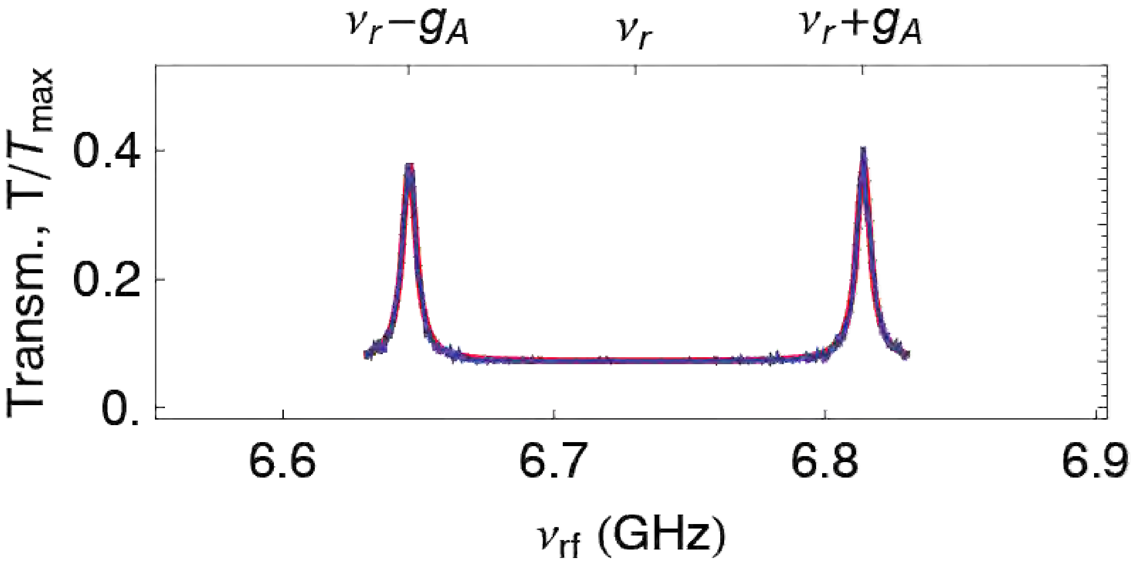


- in agreement with Jaynes-Cummings model (Δ-dependence)
- in agreement with Tavis-Cummings model (Δ-dependence)
- 2 bright and $N-1$ dark states on resonance

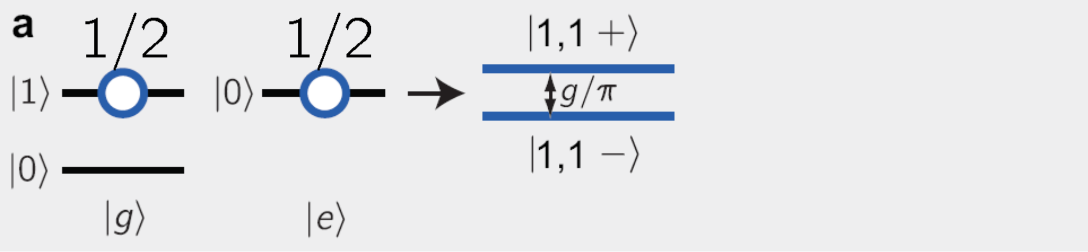
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



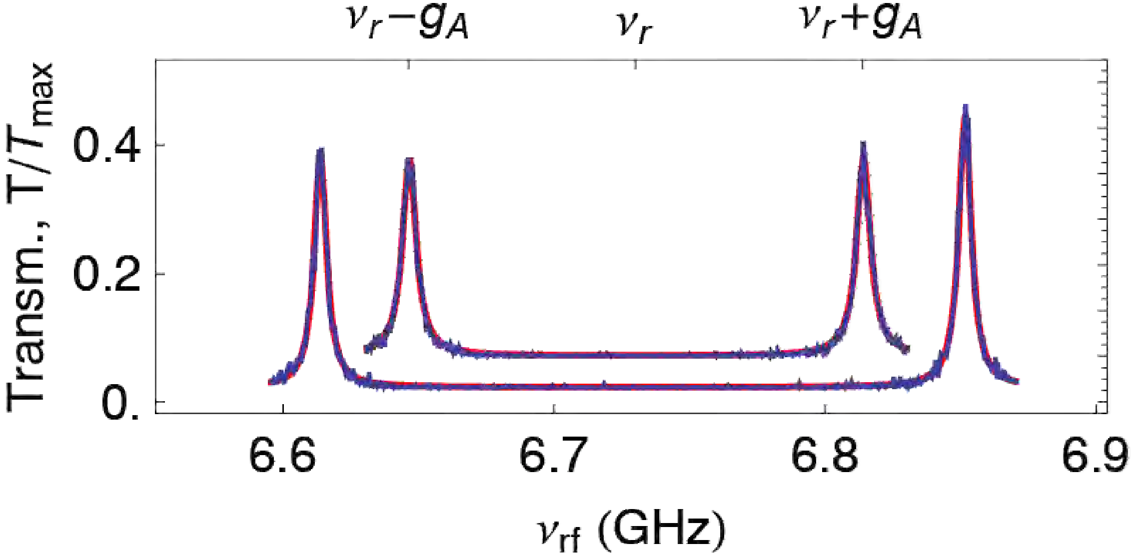
$$|1,1\pm\rangle = 1/\sqrt{2}(|g,1\rangle \pm |e,0\rangle)$$

states equally shared between photon and qubit

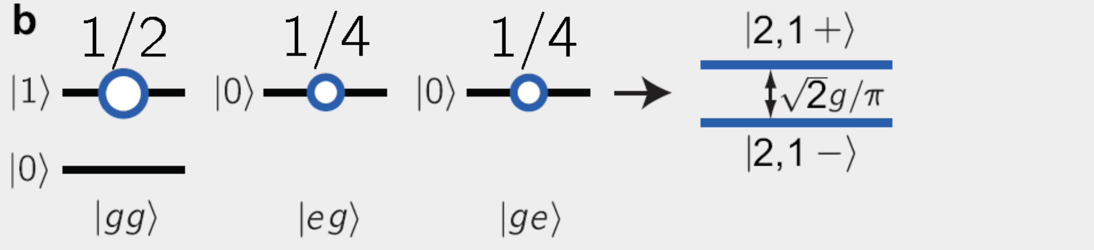
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



$$|2,1\pm\rangle = 1/\sqrt{2}\,|g,g\rangle \otimes |1\rangle \pm 1/2\,(|e,g\rangle + |g,e\rangle) \otimes |0\rangle$$

bright states: superposition of a photon and a Bell state

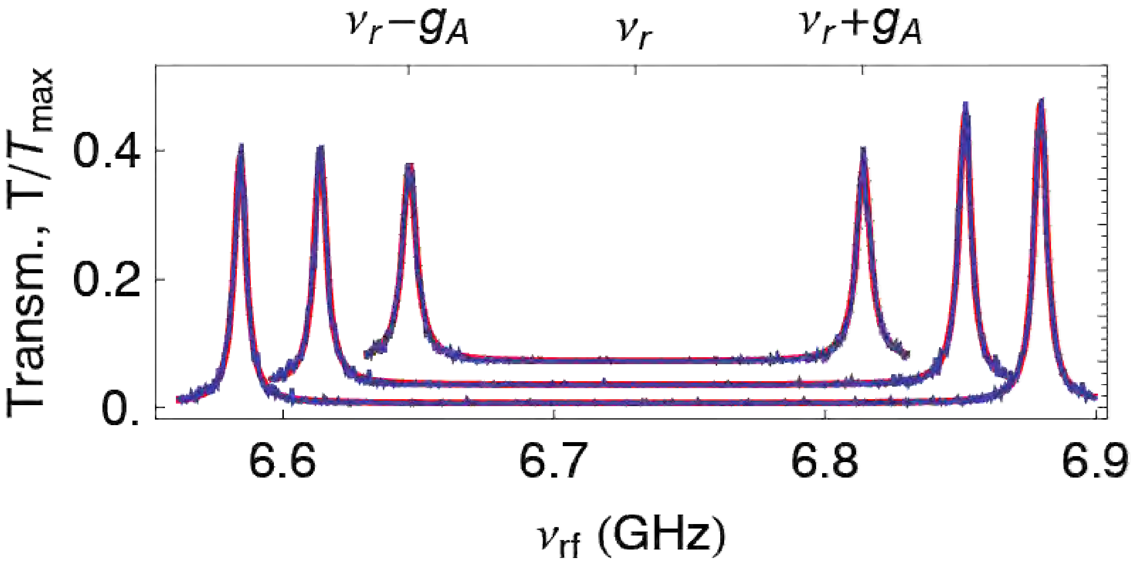
$$|2,1d\rangle = -1/\sqrt{2}(|g,e\rangle + |e,g\rangle) \otimes |0\rangle$$

dark state

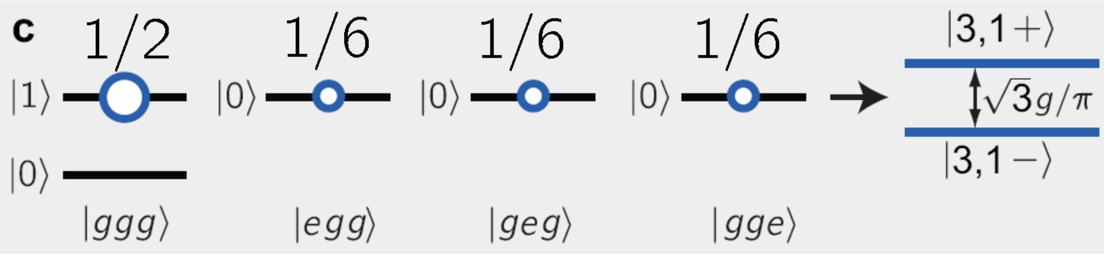
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Multi-Qubit Vacuum Rabi Mode Splitting

- the spectrum:



- the states:



$$|3,1\pm\rangle = 1/\sqrt{2}\,|g,g,g\rangle \otimes |1\rangle \pm 1/\sqrt{6}\,(|e,g,g\rangle - |g,e,g\rangle + |g,g,e\rangle) \otimes |0\rangle$$

one photon plus three qubit entagled W-state

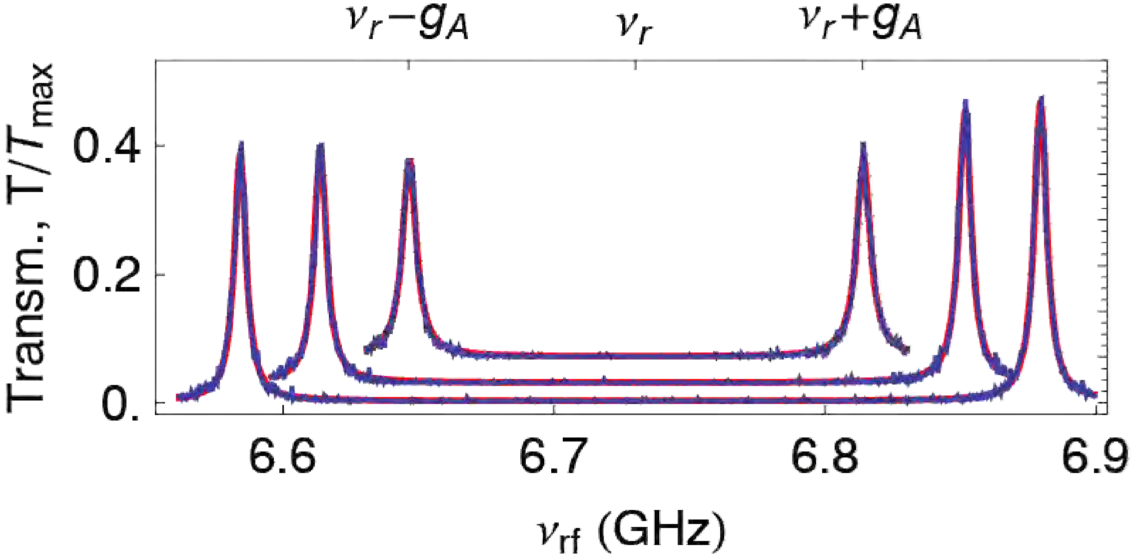
$$|3,1d_1\rangle = 1/\sqrt{2}\,(|e,g,g\rangle - |g,g,e\rangle) \otimes |0\rangle$$
$$|3,1d_2\rangle = 1/\sqrt{2}\,(|g,g,e\rangle + |g,e,g\rangle) \otimes |0\rangle$$

two dark states

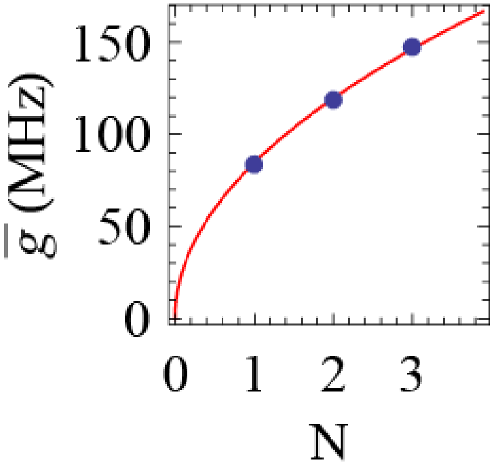
J. Fink *et al.*, Phys. Rev. Lett. **103**, 083601 (2009)

Atom Number Scaling of Coupling Strength

- the spectrum:



- scaling of collective coupling with $\sqrt{N}$



J. Fink *et al*., Phys. Rev. Lett. **103**, 083601 (2009)

Non-Resonant Interactions in Circuit QED …

… strong coupling in the dispersive limit

The Lamb and AC-Stark Shifts

signatures of the dispersive interaction with a quantum field

for $\Delta = \omega_a - \omega_r \gg g$

$$H \approx \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \overset{\substack{\text{Lamb shift}\\ /\!/}}{\frac{g^2}{\Delta}} + \overset{\substack{\text{AC Stark shift}\\ /\!/}}{\frac{2g^2}{\Delta}a^\dagger a}\right)\sigma_z$$

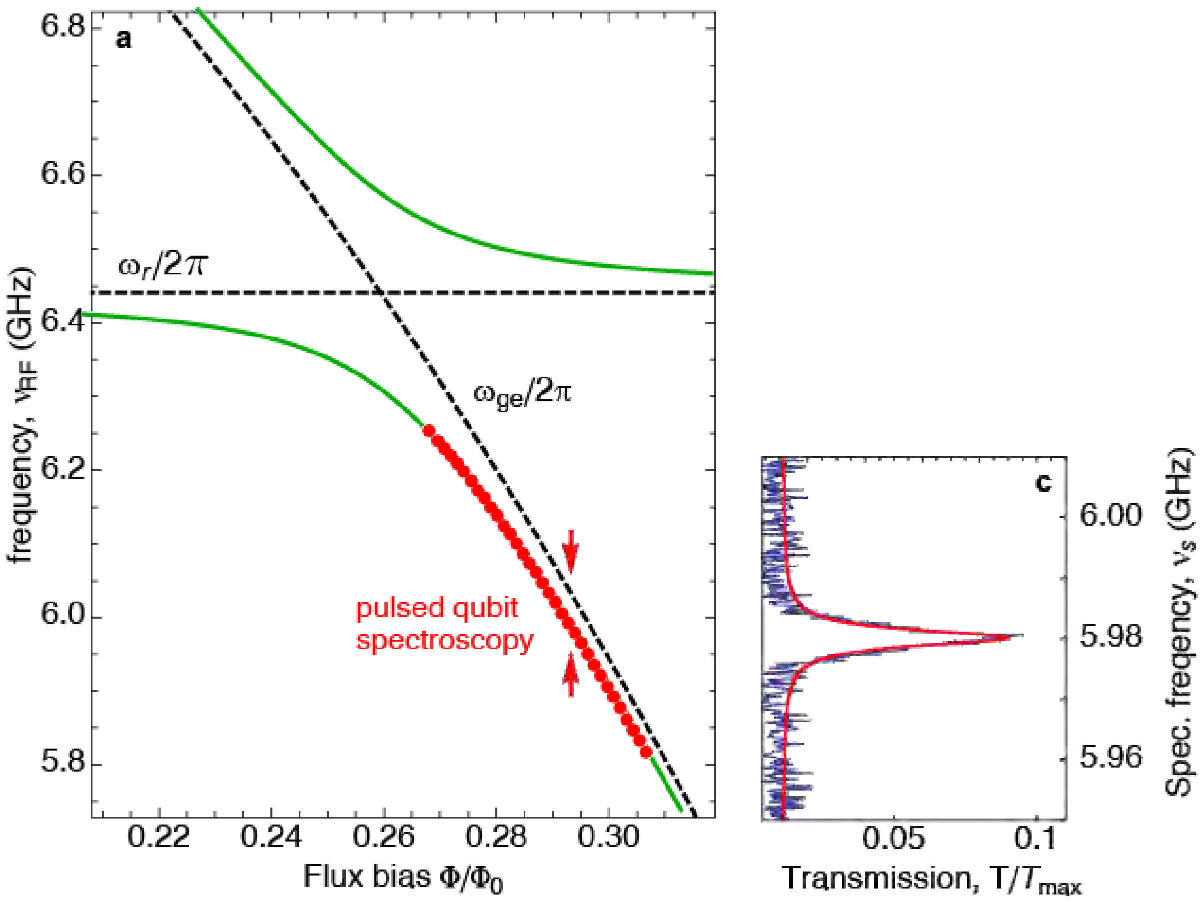
$$\tilde{\omega}_a \approx \omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta}n$$

challenges measuring the Lamb shift:

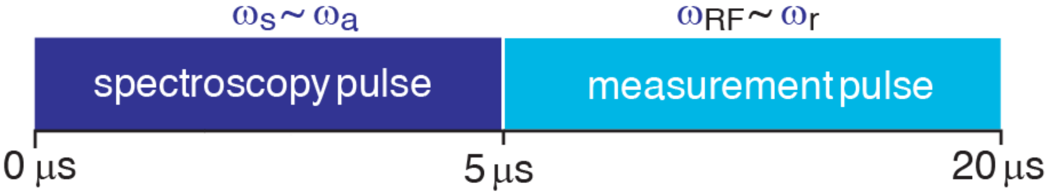
- small frequency shift
- relative to the 'bare' qubit transition frequency
- presence of (thermal) photon induced Stark shifts
- use Δ as control parameter

M. Brune *et al., Phys. Rev. Lett.* **72**, 3339 (1994)
A. Blais *et al., PRA* **69,** 062320 (2004)

Measurement of the Lamb Shift



pulsed dispersive
qubit spectroscopy



$\omega_s \sim \omega_a$

$\omega_{RF} \sim \omega_r$

spectroscopy pulse

measurement pulse

0 µs

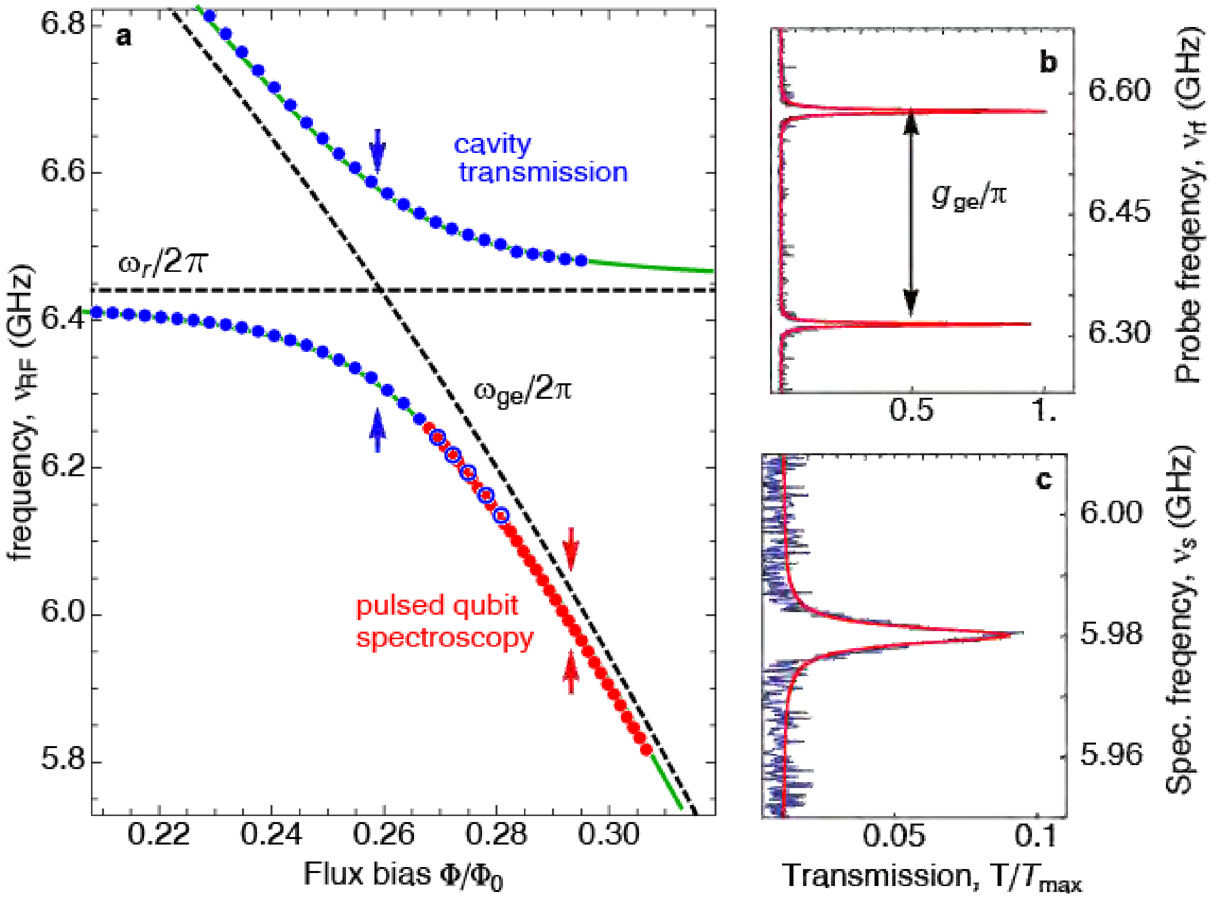
5 µs

20 µs

pulsed spectroscopy scheme

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

Measurement of the Lamb Shift



resonator transmission spectrum

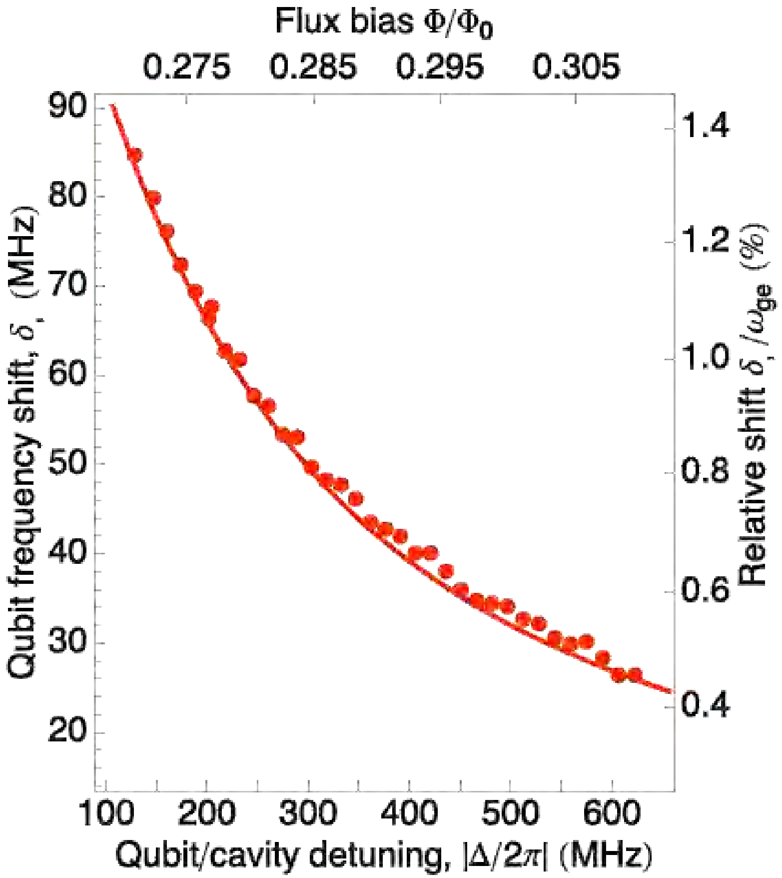
pulsed dispersive qubit spectroscopy

- qubit and photon component of joint state are measured
- accurate knowledge of qubit parameters possible

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

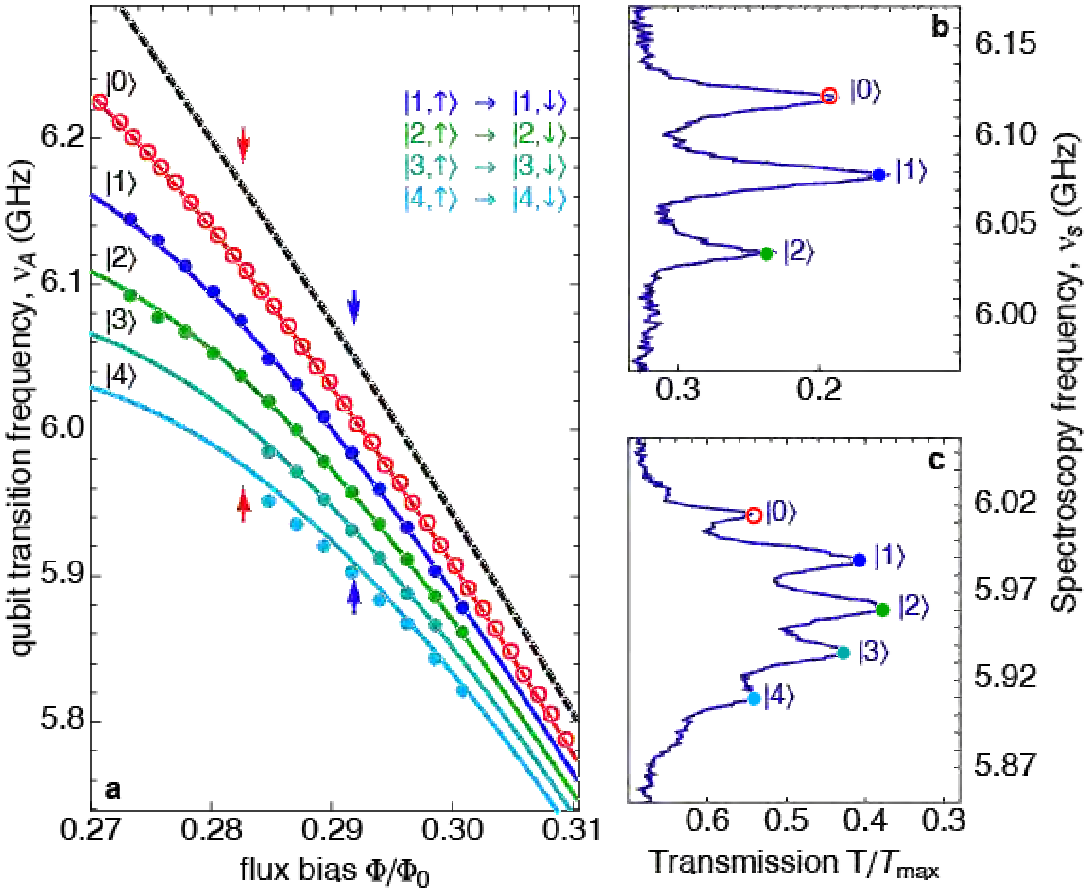
The Lamb Shift



- measurement in a solid state system
- large cavity enhanced relative shift: $\delta_L/\omega_{\mathrm{ge}} \approx 1.4\%$ (c.f. Hydrogen in vac. $\delta_L/\omega_{\mathrm{ge}} \approx 0.0001\%$)
- closely related to physical effects such as:
  - spontaneous emission
  - Purcell effect
  - vacuum Rabi effects
  - vacuum noise and squeezing

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

Quantum AC-Stark Shift and Lamb Shift



cw spectroscopy:

- populate resonator with coherent field

note:

- $\delta_{AC} < 2\delta_L$
- good agreement with multi-level theory
- clear identification of Lamb-Shift

use quantum a.c. Stark shift to measure photon number statistics

D. Schuster *et al.*, *Nature* **445**, 515 (2007)
A. Fragner *et al.*, *Science* **322**, 1357 (2008)

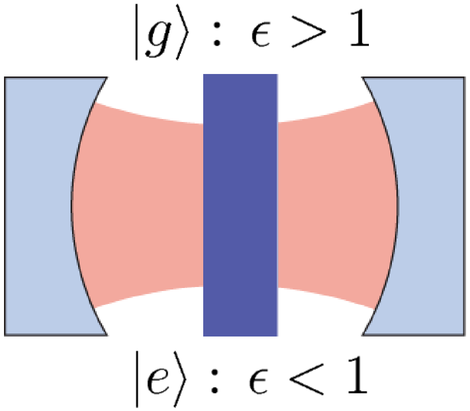
Dispersive Qubit/Photon Interaction for Qubit Read-Out

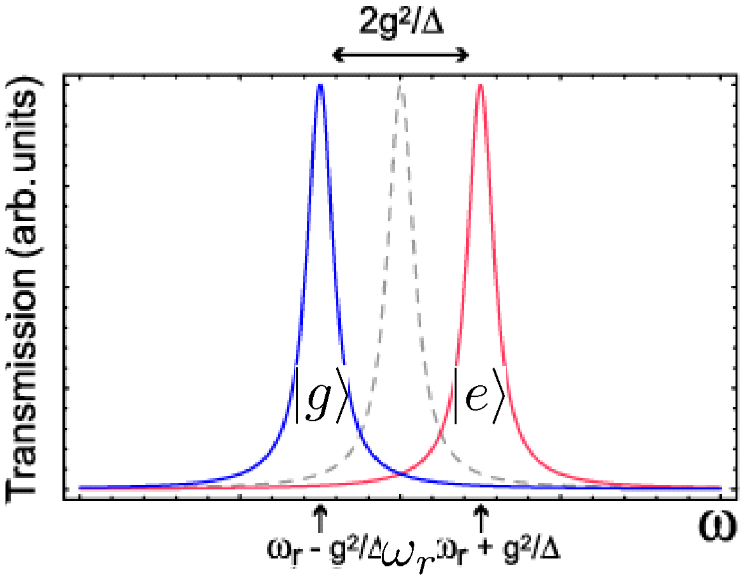
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

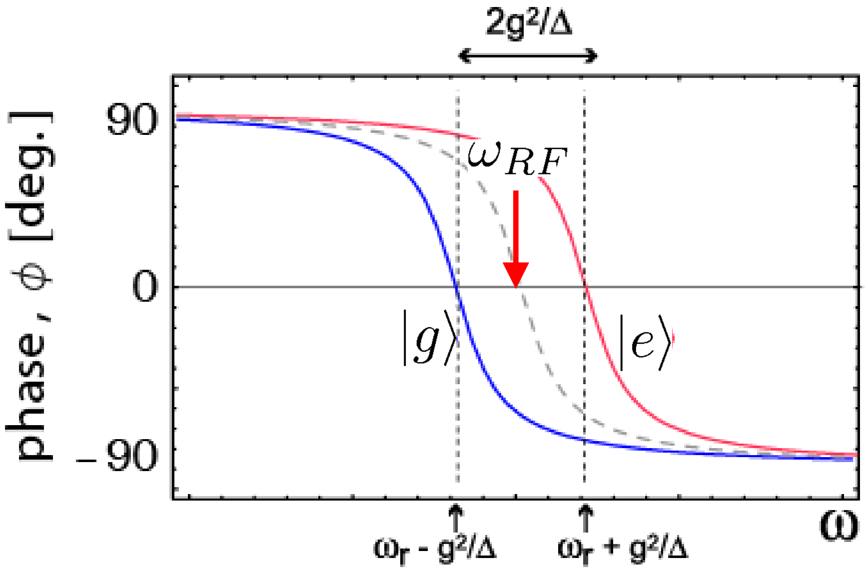
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//

cavity frequency shift
and qubit ac-Stark shift

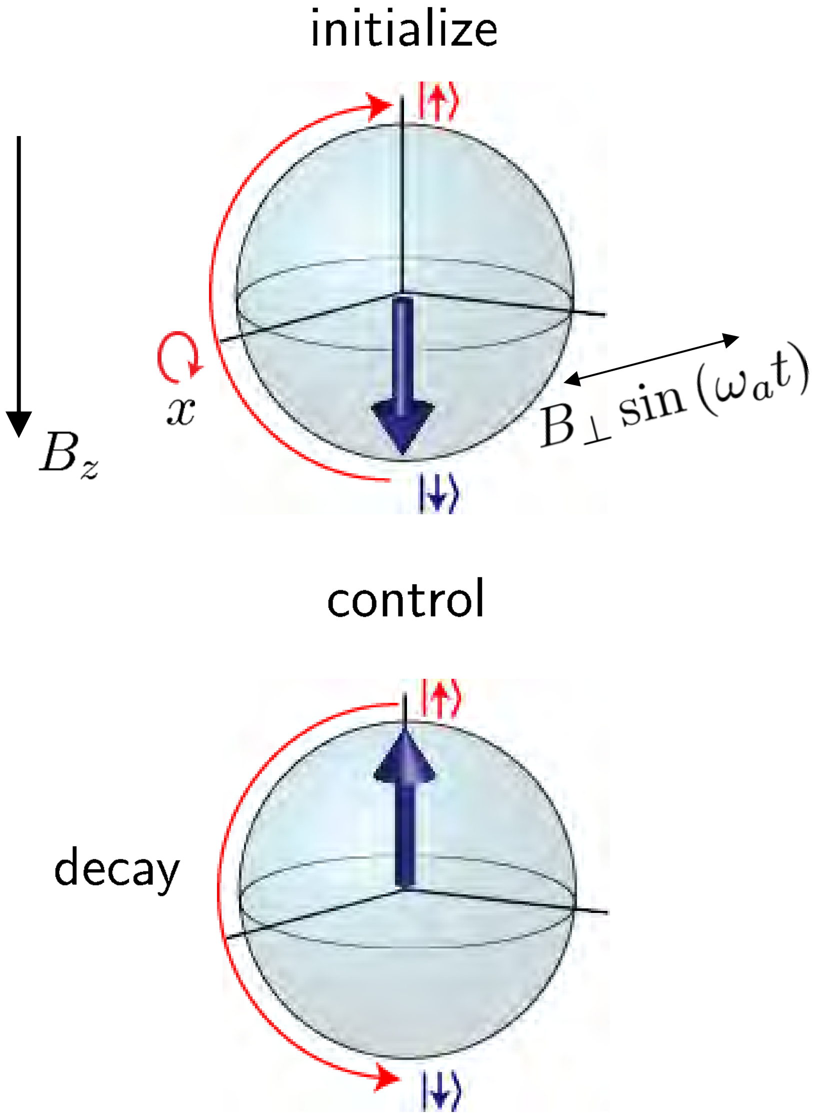


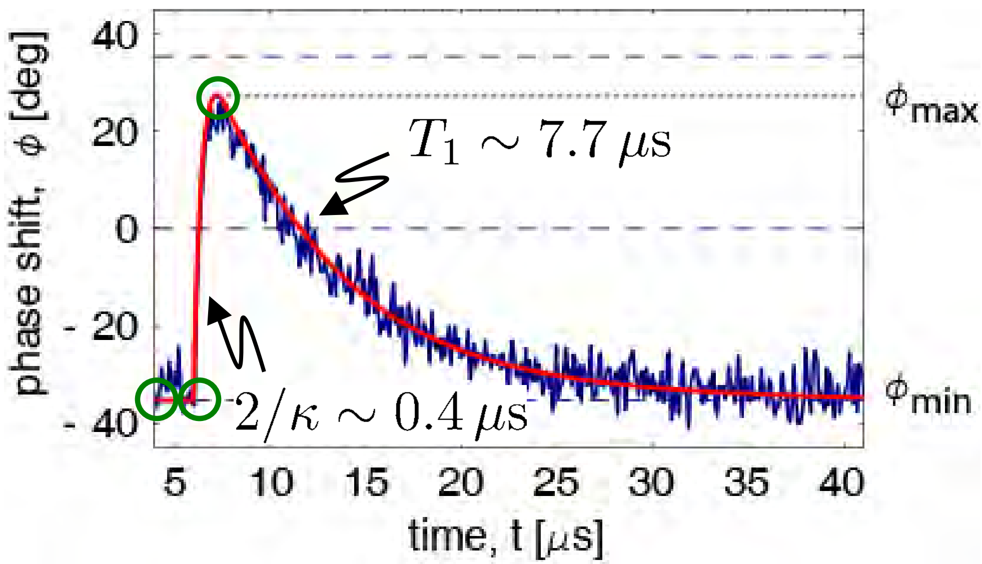




A. Blais *et al.*, *PRA* **69**, 062320 (2004)

Qubit Control and Readout



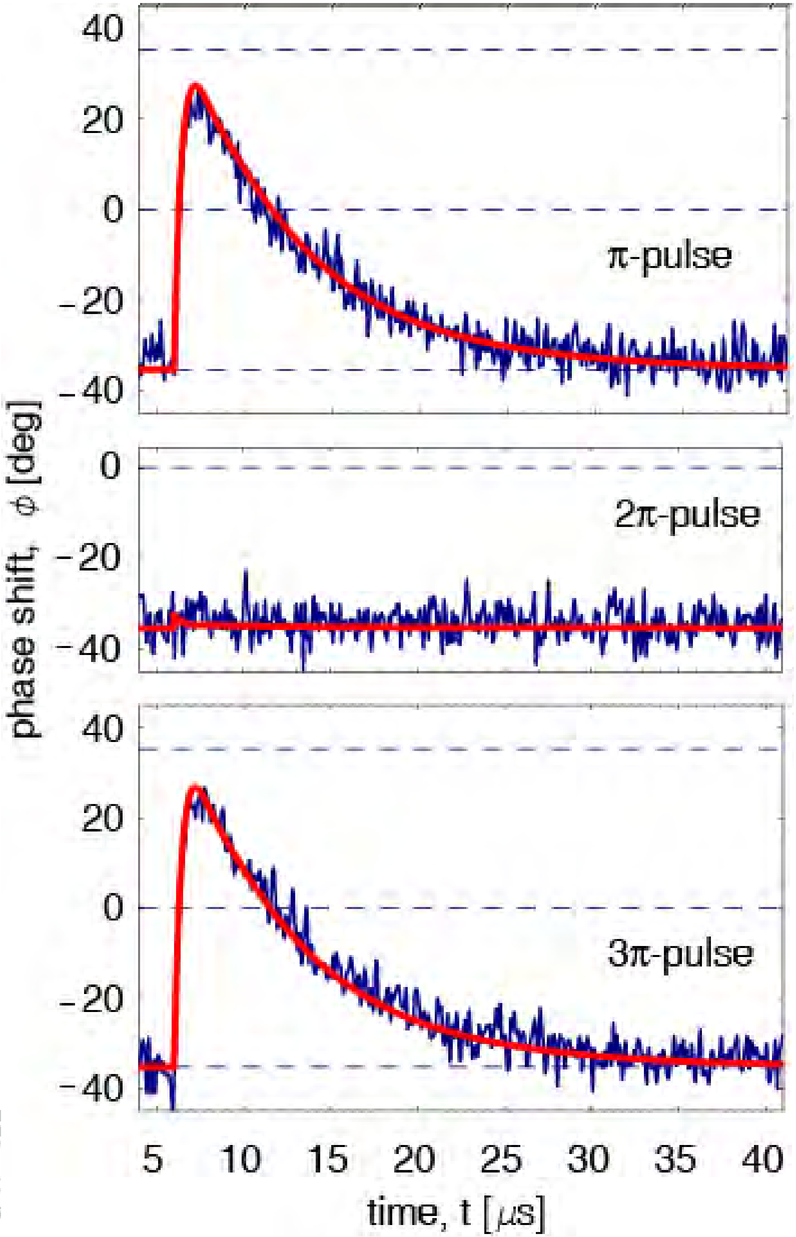


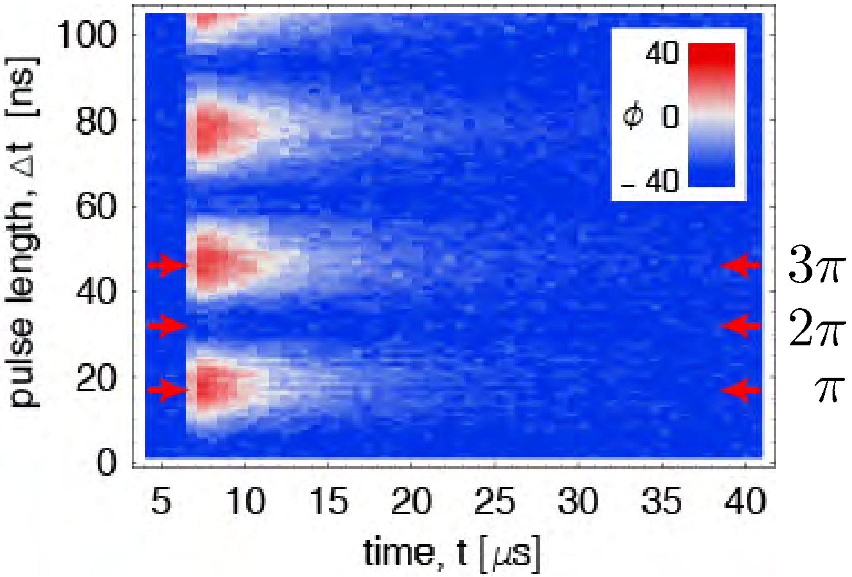
measurement properties:

- continuous
- dispersive
- quantum non-demolition
- in good agreement with predictions

Wallraff, Schuster, Blais, ... Girvin, and Schoelkopf, *Phys. Rev. Lett.* **95**, 060501 (2005)

Rabi Oscillations (weak cont. measurement)

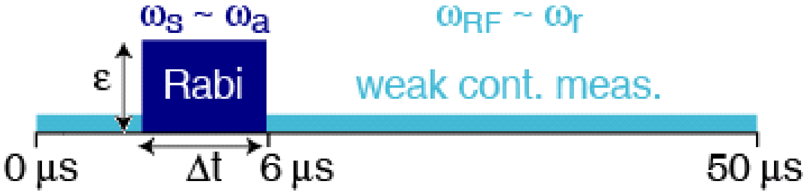




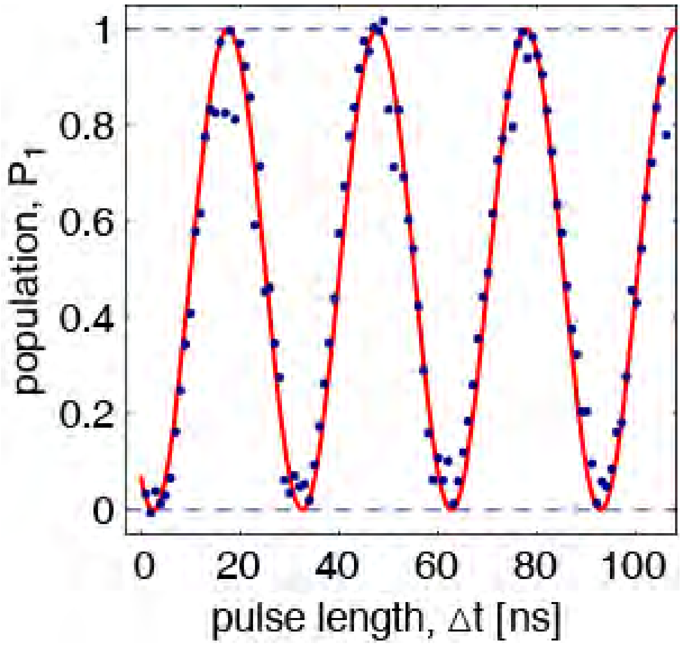
- high control fidelity
- high read-out fidelity
- good understanding of field-qubit interaction

High Fidelity Control & Read Out

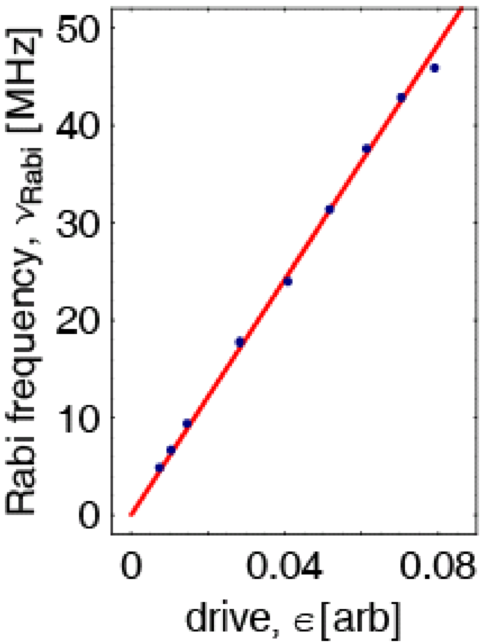
pulse scheme:



Rabi oscillations:



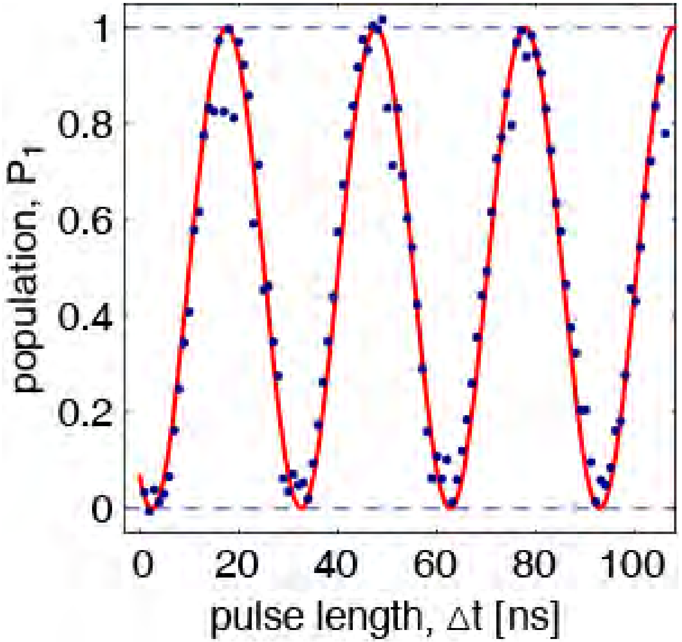
Rabi frequency:



- high visibility $95 \pm 5\%$
- detailed understanding of qubit/read-out interaction

A. Wallraff *et al.*, *Phys. Rev. Lett.* **95**, 060501 (2005)

High Fidelity Control and Read-Out



close to unit contrast

combined preparation and read-out fidelity

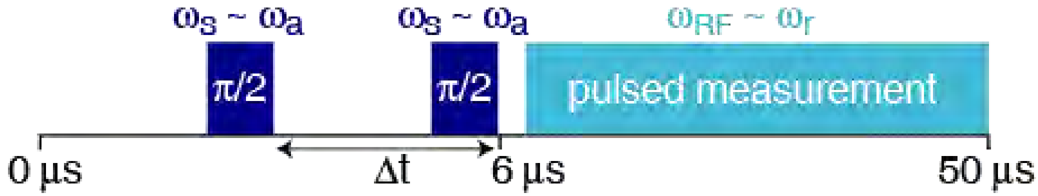
$$95 \pm 5\,\%$$

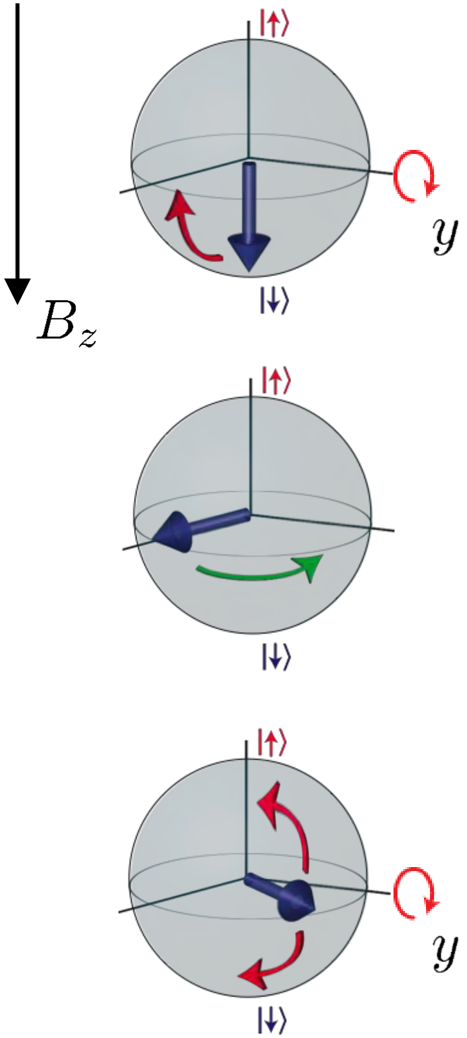
- continuous dispersive time-resolved read-out
- detailed understanding of qubit/read-out interaction
- high visibility

A. Wallraff *et al., Phys. Rev. Lett.* **95**, 060501 (2005)

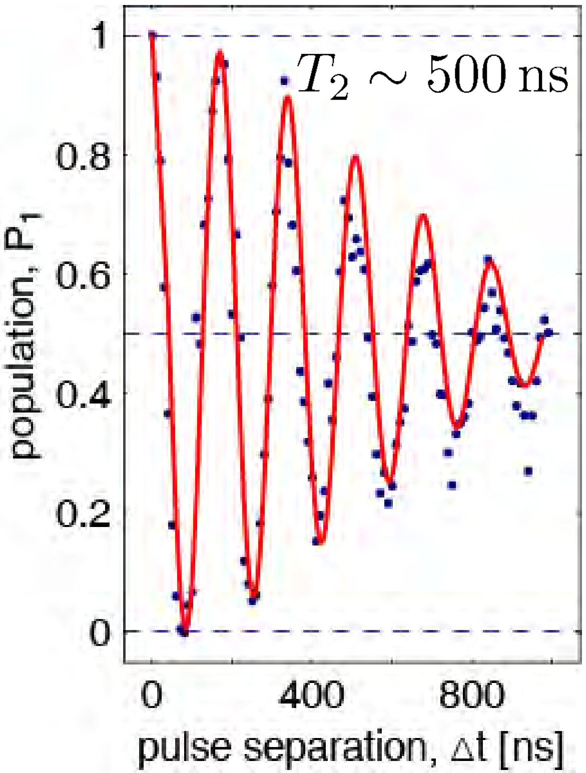
Single Qubit Coherence: Ramsey Fringes

pulse scheme:



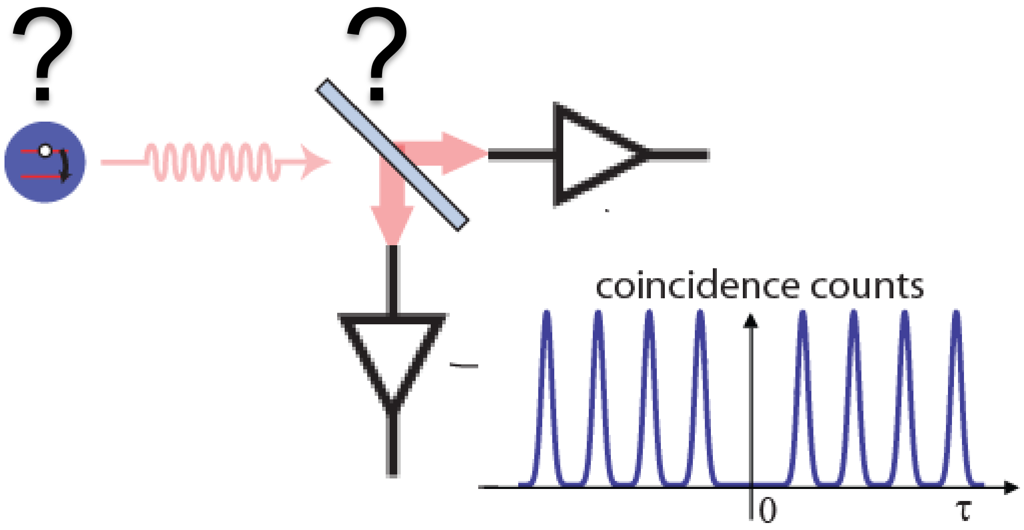


Ramsey fringes:



A. Wallraff *et al.*, *Phys. Rev. Lett.* **95**, 060501 (2005)

Exploring the Properties of Propagating Photons



quantum optics in the **visible**:

- single photon sources
- beam splitters
- photon counters

o.k. at **optical frequencies**

But in the **microwave domain**?

- smaller photon energy ...

$$\frac{\nu_{\mathrm{opt}}}{\nu_{\mu\mathrm{w}}} = \frac{500\,\mathrm{THz}}{5\,\mathrm{GHz}} = 10^5$$

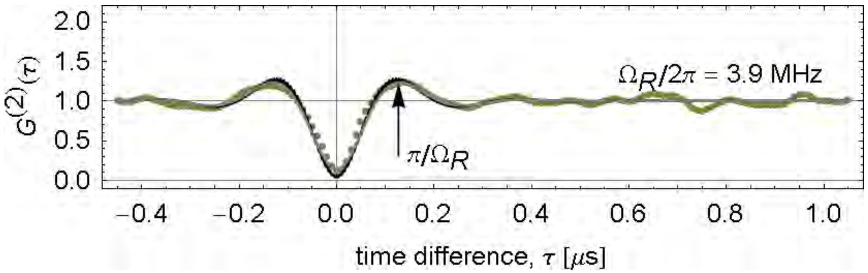
instead:

- linear amplifiers
- signal processing

J. Gabelli et al., *Phys. Rev. Lett.* **93**, 056801 (2004)
E. P. Menzel et al., *Phys. Rev. Lett.* **105**, 100401 (2010)
M. P. da Silva et al.. Phys. *Rev. A* **82**, 043804 (2010)
C. Eichler et al., *Phys. Rev. A* **86**, 032106 (2012)

Experiments with Propagating Quantum Microwaves

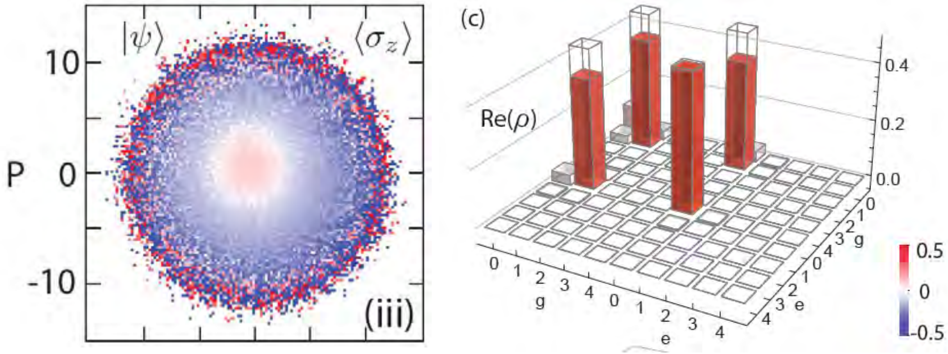
Single photon sources and their anti-bunching



Lang *et al.*, *PRL* **107**, 073601 (2011)

Bozyigit *et al.*, Nat. Phys **7**, 154 (2011)

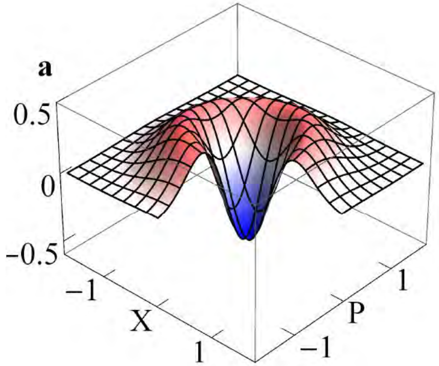
Preparation and characterization of qubit-**propagating** photon entanglement



Eichler *et al.*, *PRL* **109**, *240501* (2012)
Eichler *et al.*, *PRA* **86**, 032106 (2012)

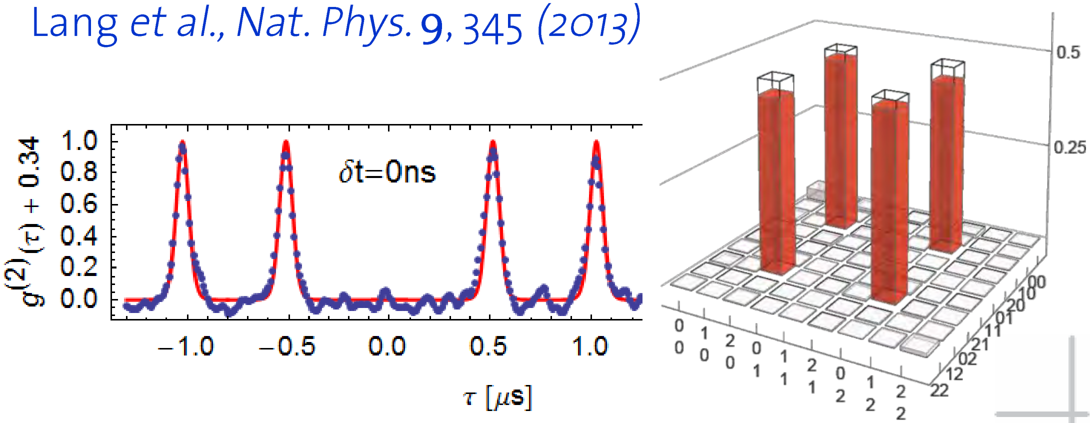
Wigner functions and full state tomography of propagating photons:

Eichler *et al.*, *PRL* **106**, 220503 (2011)

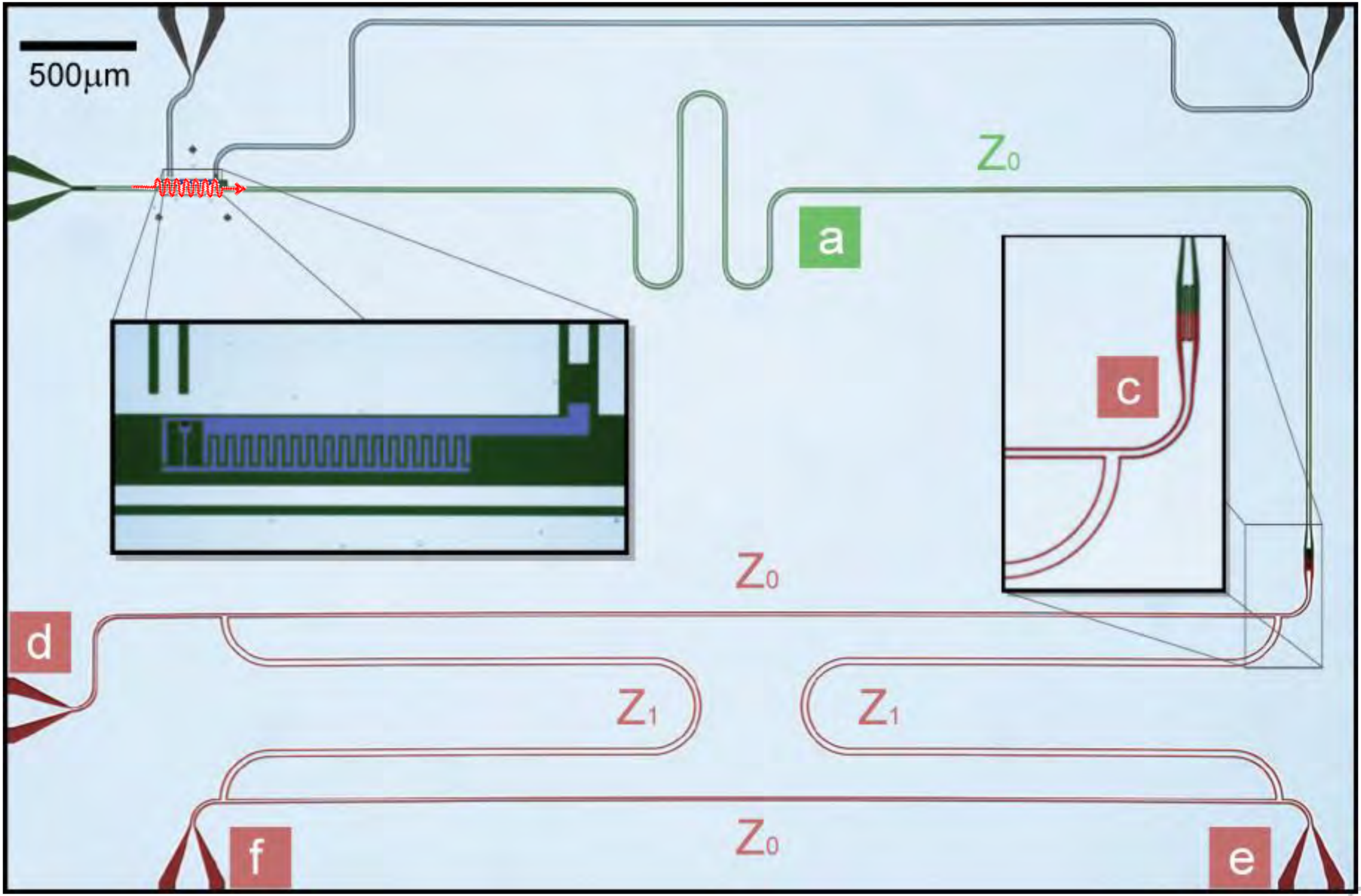


Hong-Ou-Mandel: Two-photon interference with coherences at microwave frequencies

Lang *et al.*, *Nat. Phys.* **9**, 345 *(2013)*

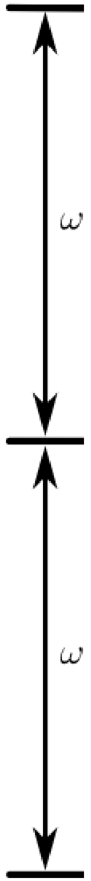


Single Sided Cavity and Beam Splitter

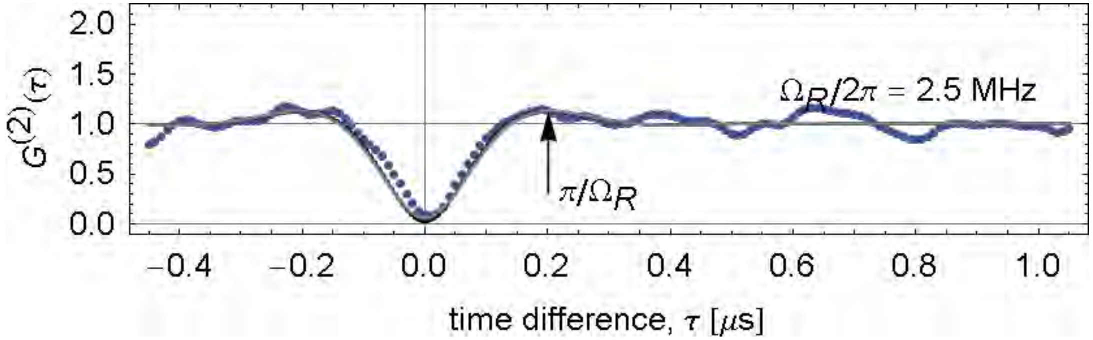


$G^{(2)}$ of Continuously Pumped Single Photon Source

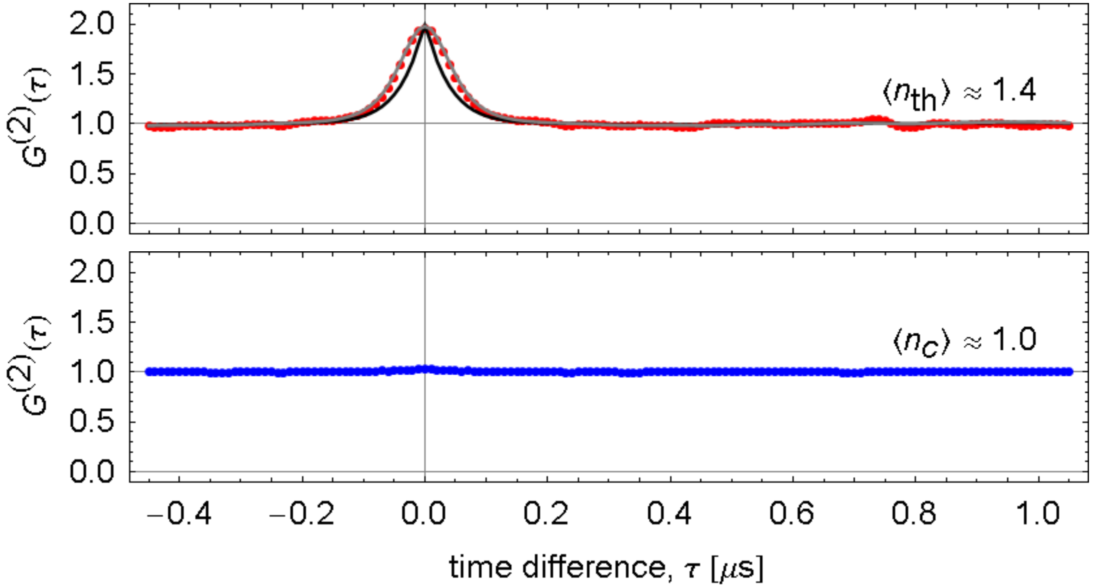
resonant photon blockade:



anti-bunching and sub-poissonian photon statistics



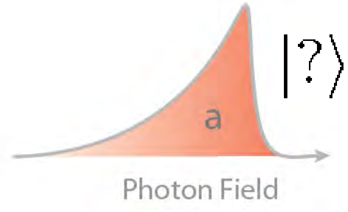
thermal (bunching) & coherent (poissonian) fields



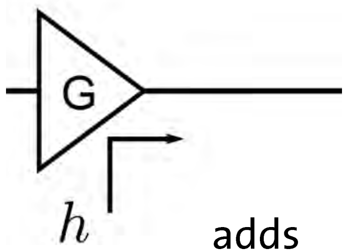
C. Lang *et al.*, *PRL* **106**, 243601 (2011)
M. P. da Silva et al., *PRA* **82**, 043804 (2010)
C. Eichler et al., *PRA* **86**, 032106 (2012)

Microwave Photon Field Detection

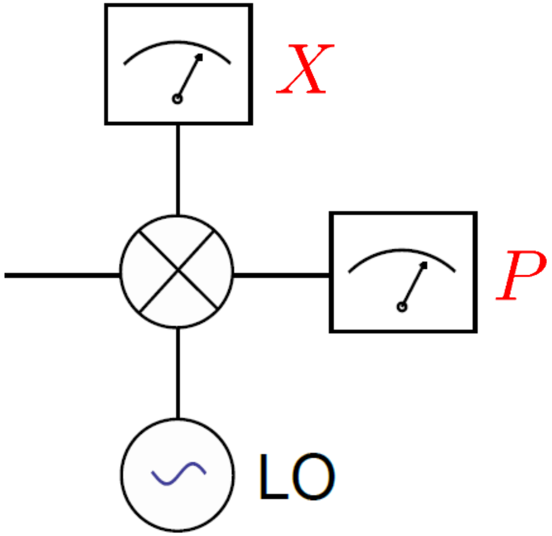
amplitude detector = ADC & digital filtering



linear amplifier



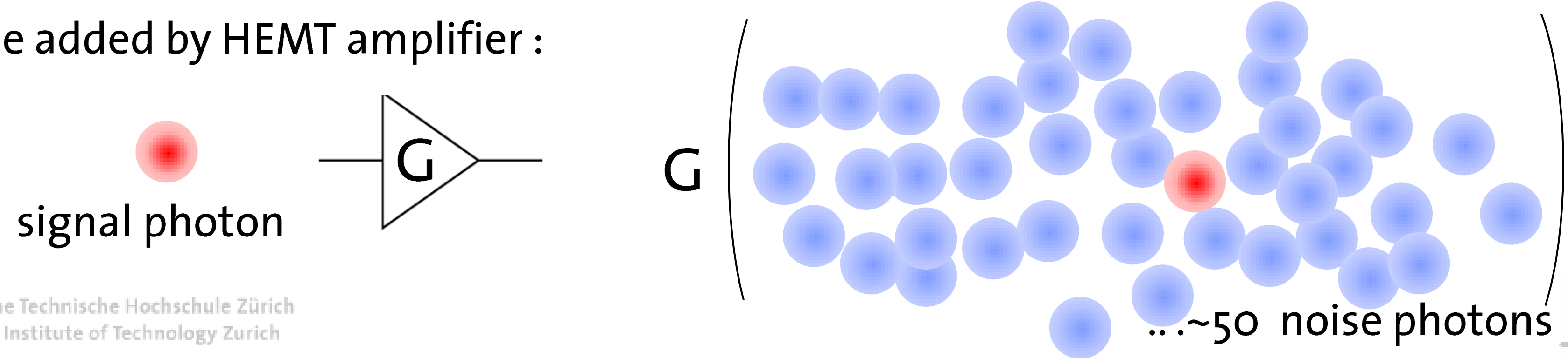
adds vacuum/thermal noise



complex amplitude: $$\boxed{S = X + iP = \underset{\text{“signal”}}{a} + \underset{\text{“noise”}}{h^\dagger}}$$

Eichler *et al.*, *PRA* **86**, 032106 (2012)
M. P. da Silva et al., *PRA* **82**, 043804 (2010)
C. M. Caves, *PRD* **26**, 1817 (1982)

noise added by HEMT amplifier :

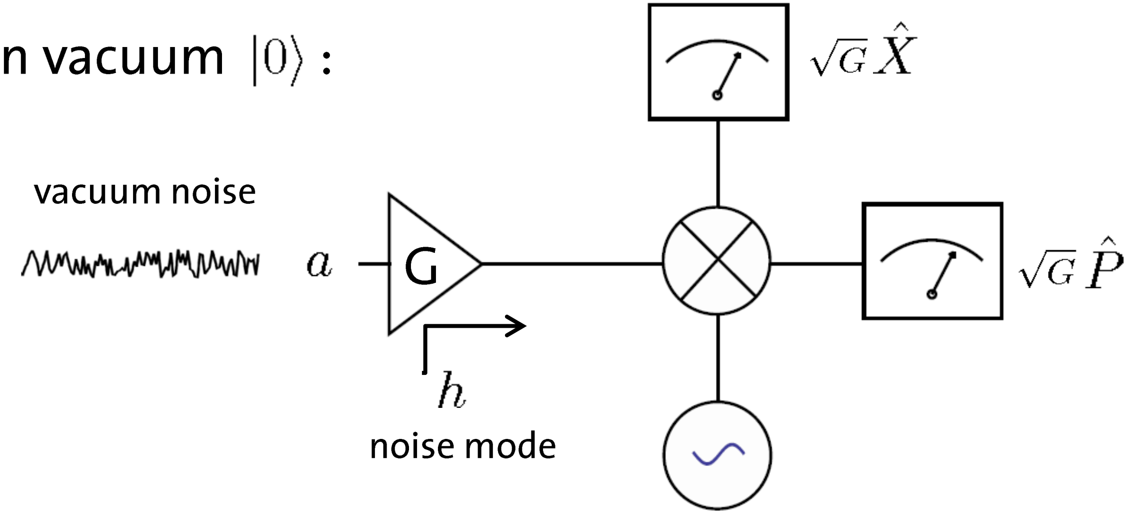


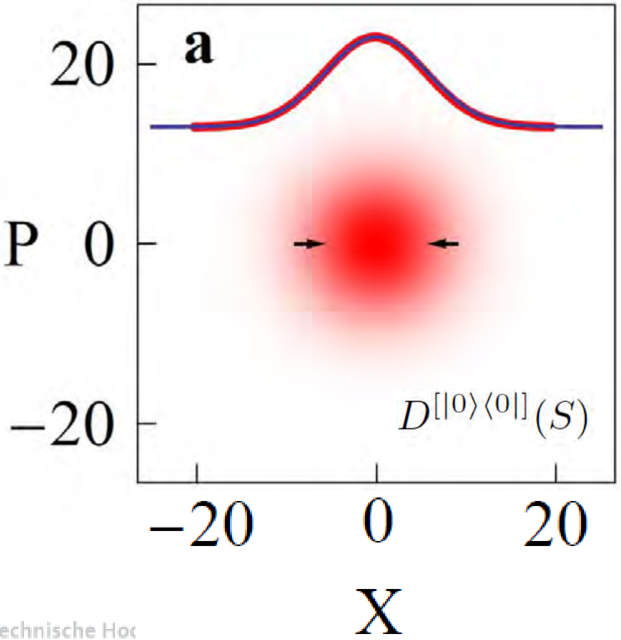
signal photon

…~50 noise photons

Full Tomography of a Single Propagating Mode

1) prepare a in vacuum $|0\rangle$:





record histogram $D^{[|0\rangle\langle 0|]}(S)$

of measurement results $S/\sqrt{G} = X + iP$

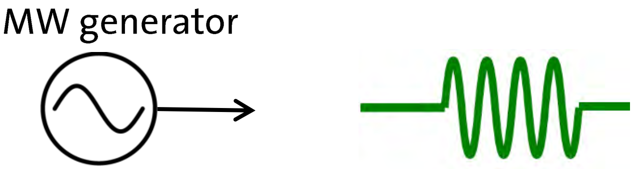
normal distribution with variance

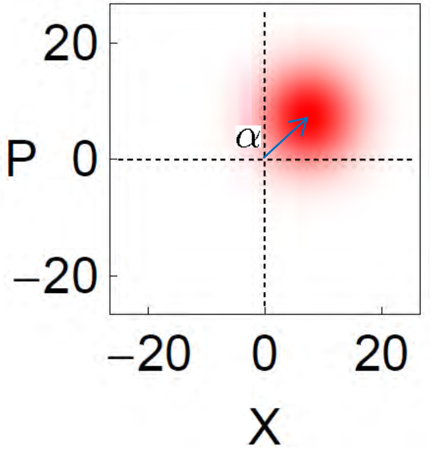
$$2\sigma^2 = \langle \hat{S}^\dagger \hat{S}\rangle / G = \frac{1}{G}\int \mathrm{d}^2 S\, D^{[|0\rangle\langle 0|]}(S)\, S^* S = 67$$

h introduces thermal noise
with mean photon number N_{noise}

Coherent State Histograms

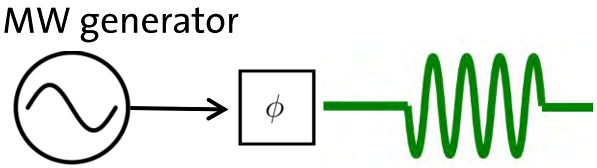
2) prepare a in coherent state $|\alpha\rangle$:

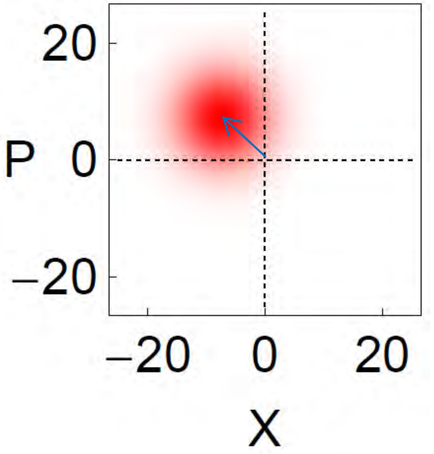




$$|\alpha| \approx 6.3$$
$$\Leftrightarrow$$
$$\langle a^\dagger a \rangle \approx 41 \sim N_{\text{noise}}$$

3) rotate phase $|e^{i\phi}\alpha\rangle$:





Question: What can we learn about state when $\langle a^\dagger a \rangle \leq 1$?

Single Photon Source Histogram

store 2D histogram $D^{[\rho]}(S)$ from $S/\sqrt{G} = X + iP$ measurement results:

corresponding
phase space
distribution

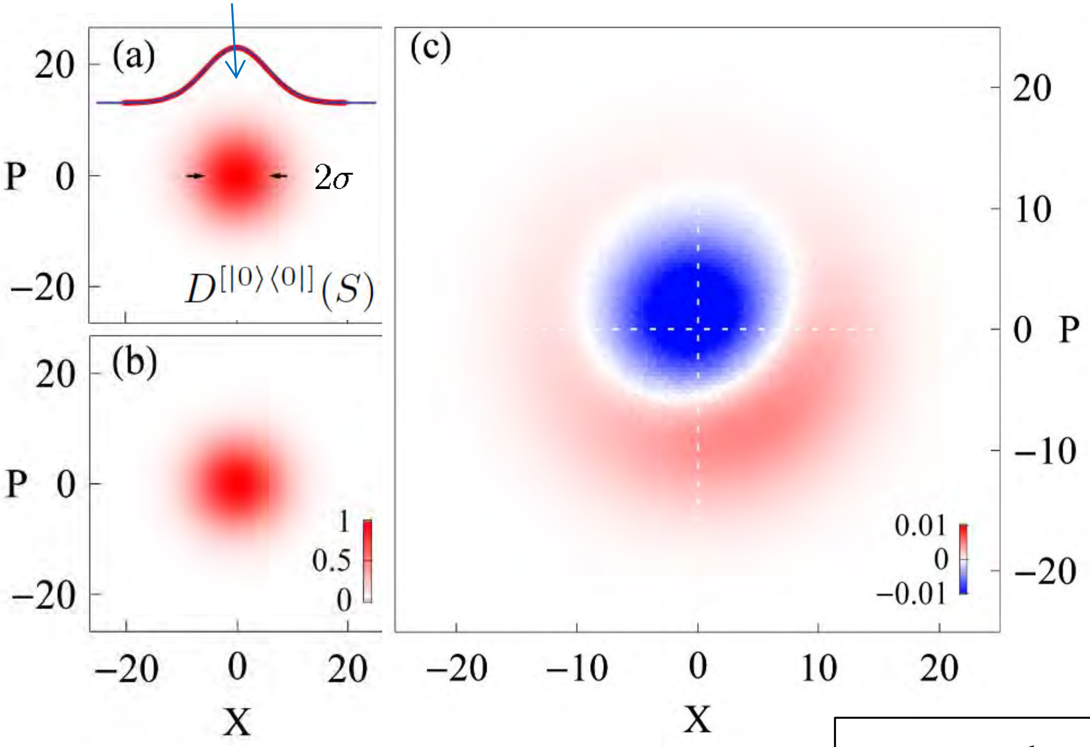
Q - function
of noise mode :

Q_h

convolution
with **P** – function
of signal

$Q_h * P_a$

signal mode a
in **vacuum**



subtracted
histograms
to visualize
difference

signal mode a
in **single photon**
Fock state

separate noise h from
signal a systematically!

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

Statistical Analysis of Histograms

systematic mode separation:

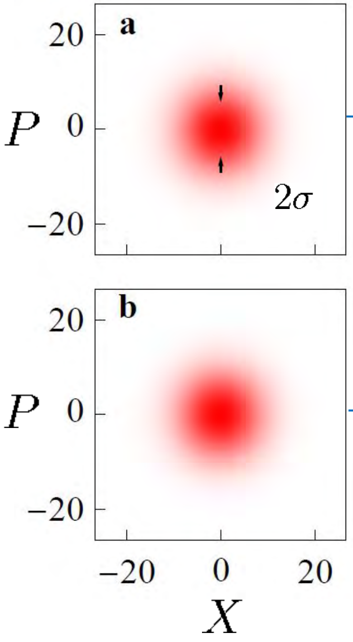
histogram moments: $\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_\rho = \int \mathrm{d}^2 S\ (S^*)^n S^m\ D^{[\rho]}(S)$

1. calculate histogram moments

2. algebraic inversion

vacuum $|0\rangle$

Fock state $|1\rangle$



$$\xrightarrow{\quad} \frac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|0\rangle\langle 0|}}{G^{(n+m)/2}} = \langle h^n (h^\dagger)^m \rangle$$

$$\xrightarrow{\quad} \frac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|1\rangle\langle 1|}}{G^{(n+m)/2}} = \langle (h + a^\dagger)^n (h^\dagger + a)^m \rangle$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \langle (a^\dagger)^n a^m \rangle$$

reminder: $X + iP = S/\sqrt{G} = (a + h^\dagger)$

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

State Dependent Moments of Probability Distribution

moments $|\langle (a^\dagger)^n a^m \rangle|$ for different prepared states:

Fock state

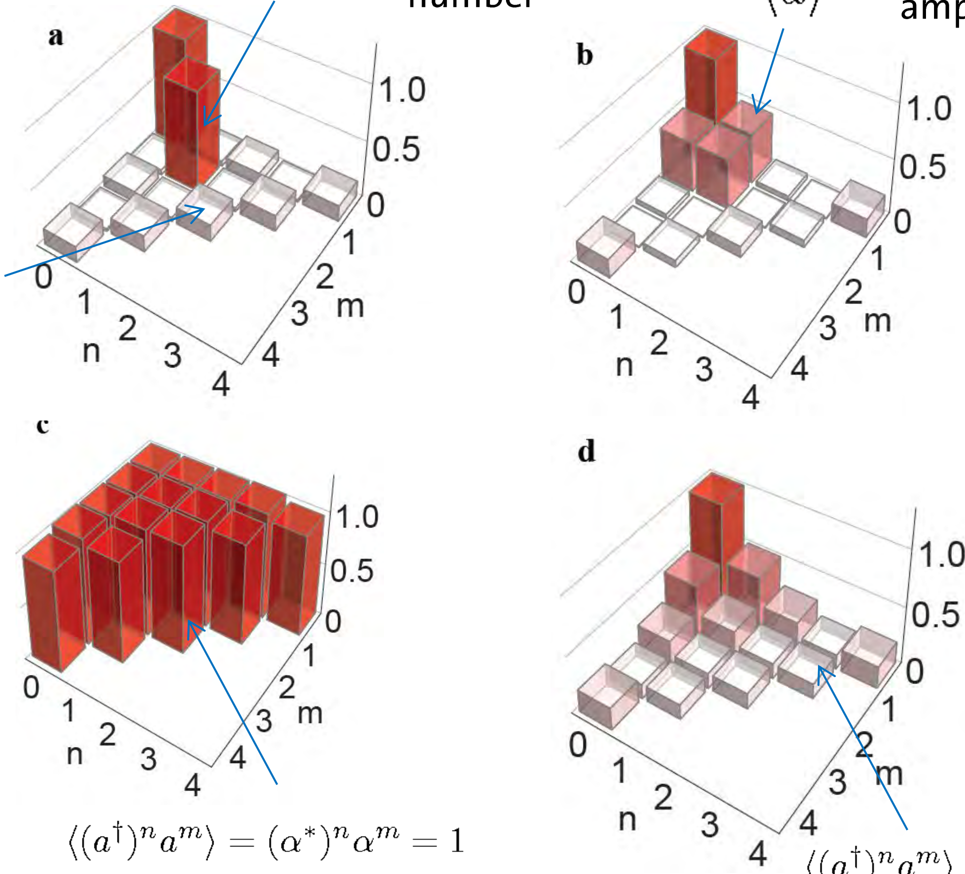
$|1\rangle$

$\langle a^\dagger a \rangle =$ "mean photon number"

$\langle a \rangle =$ "mean amplitude"

$\langle (a^\dagger)^2 a^2 \rangle \approx 0$

"anti bunching"



superposition

$$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$$

coherent state

$|\alpha = 1\rangle$

$\langle (a^\dagger)^n a^m \rangle = (\alpha^*)^n \alpha^m = 1$

$\langle (a^\dagger)^n a^m \rangle = (1/2)^{n+m}$

coherent state

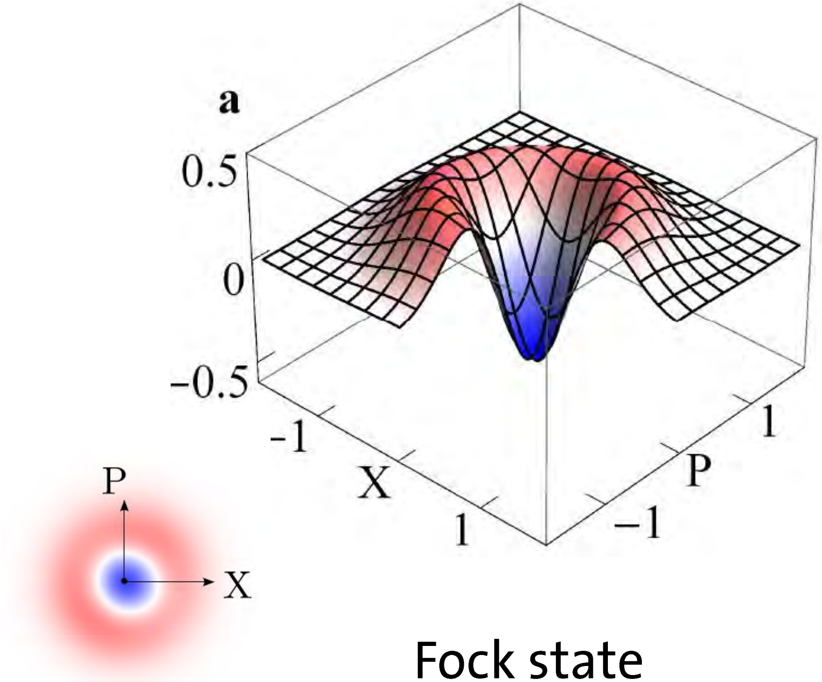
$|\alpha = 0.5\rangle$

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

Reconstructed Wigner Function of Itinerant Photon

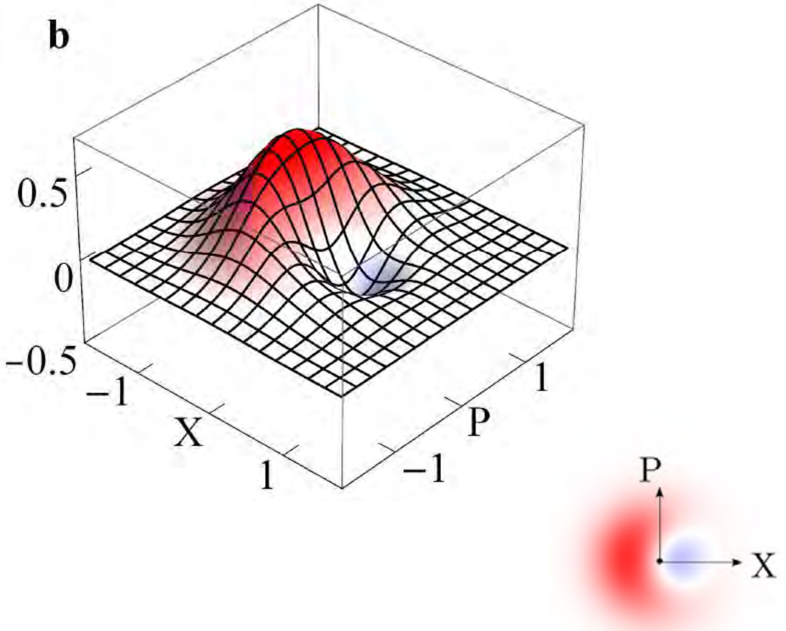
Wigner function reconstructed from measured moments:

$$W(\alpha) = \sum_{n,m} \int d^2\lambda \frac{\boxed{\langle (a^\dagger)^n a^m \rangle} (-\lambda^*)^m \lambda^n}{\pi^2 n! m!} e^{(-1/2)|\lambda|^2 + \alpha\lambda^* - \alpha^*\lambda} \quad \text{with} \quad n + m < 4$$



Fock state

$|1\rangle$



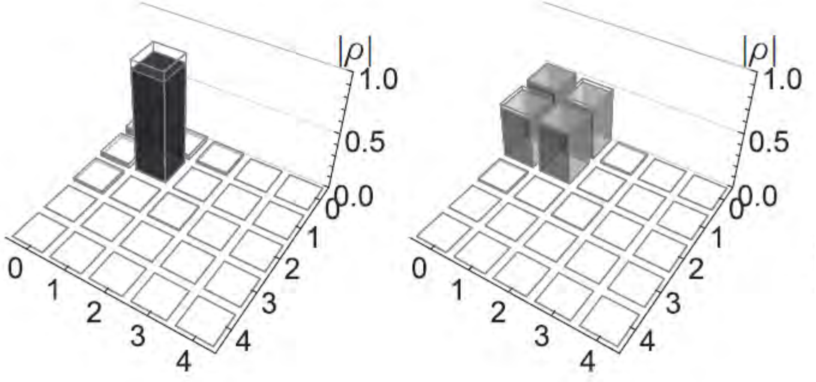
superposition

$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

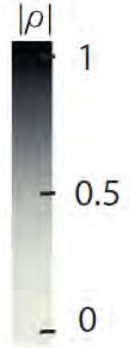
C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

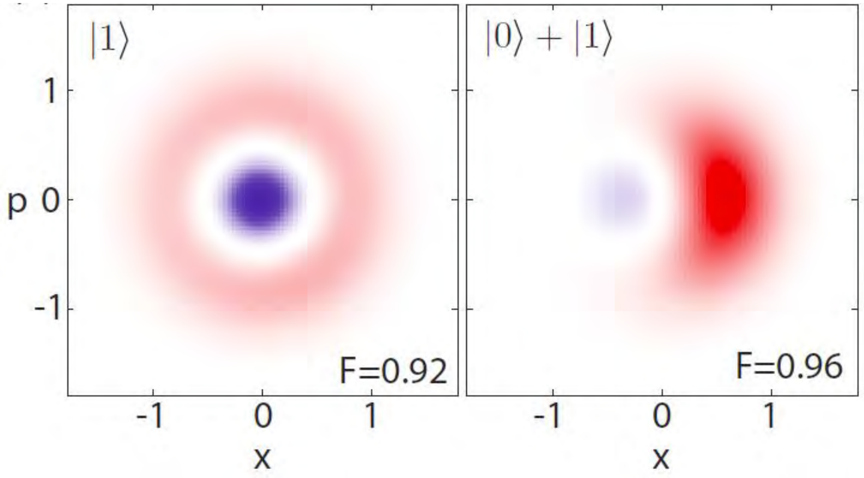
Reconstruct Density Matrices and Wigner functions…

… for **propagating multi-photon** Fock states and their superpositions:



Density matrices





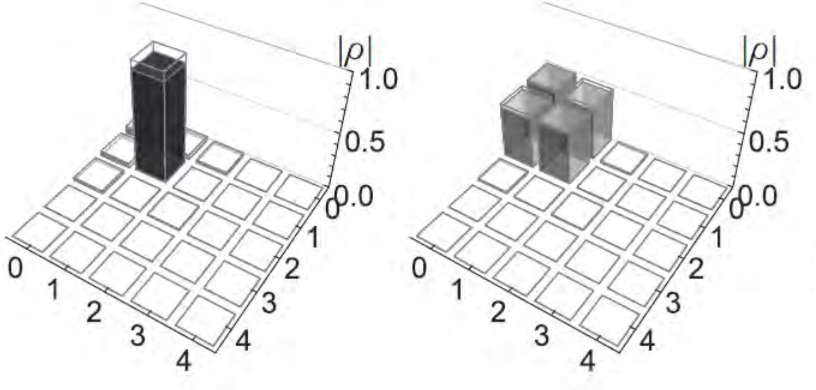
Wigner functions

measured using near-quantum-limited parametric amplifier

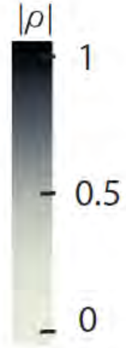
C. Eichler, et al., *PRL* **106**, 220503 (2011)
C. Eichler et al., *PRA* **86**, 032106 (2012)

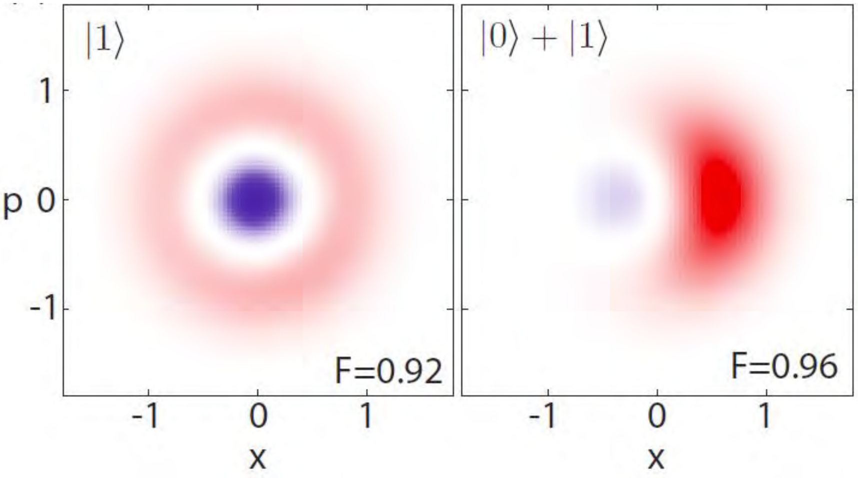
Reconstruct Density Matrices and Wigner Functions...

... for **propagating multi-photon** Fock states and their superpositions:



Density matrices



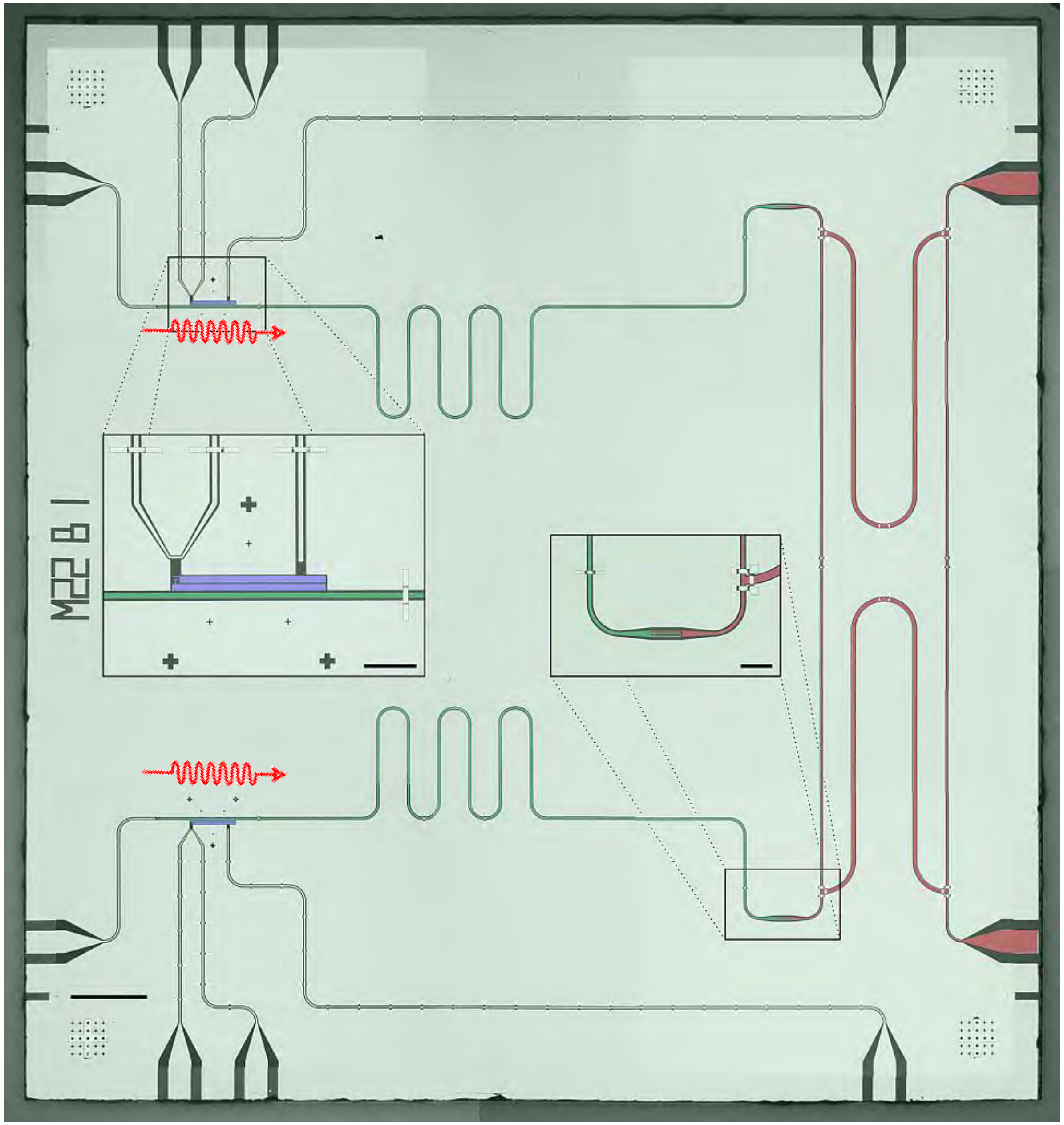


Wigner functions

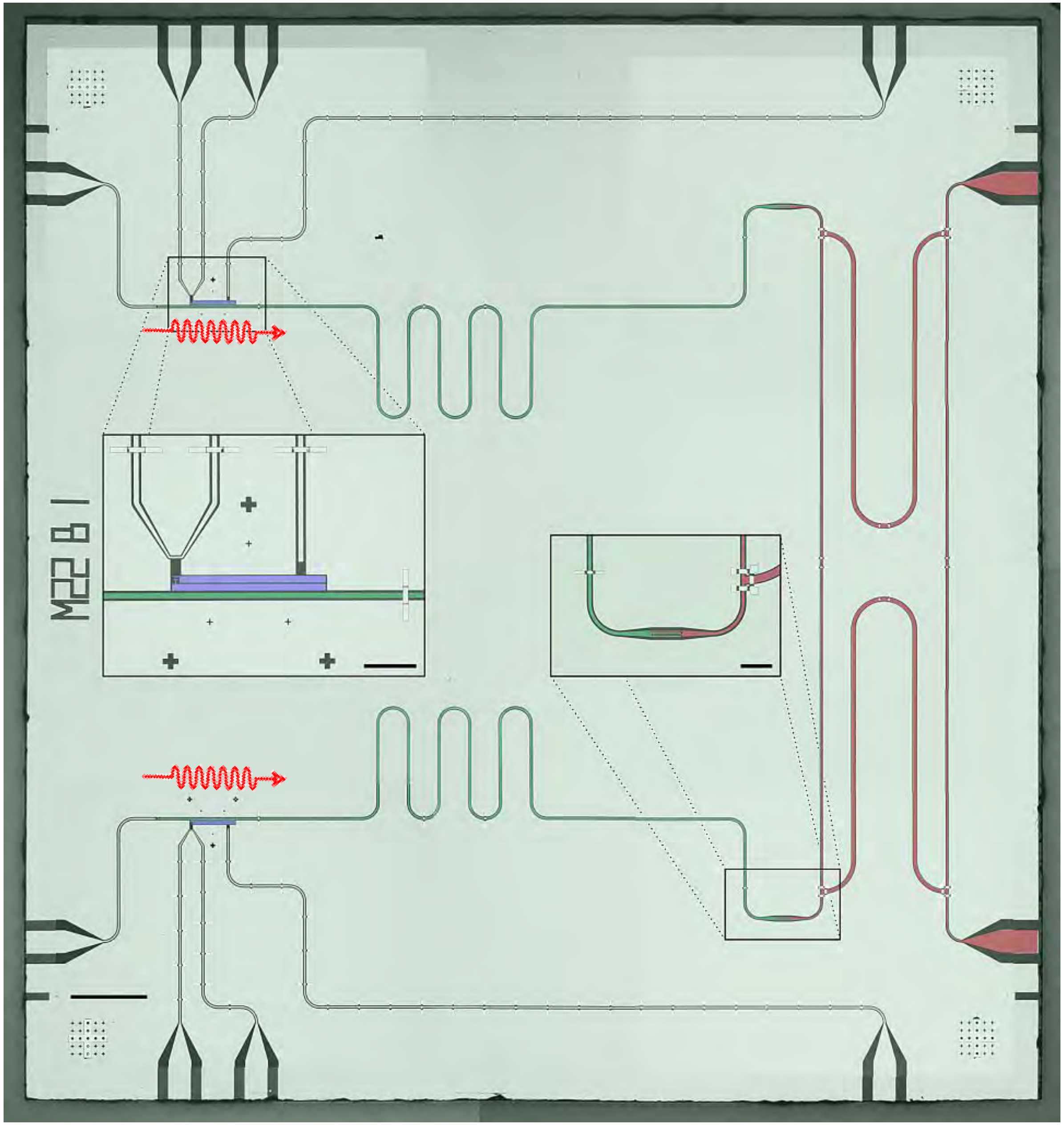
measured using near-quantum-limited parametric amplifier

C. Eichler, et al., *PRL* **106**, 220503 (2011)
C. Eichler et al., *PRA* **86**, 032106 (2012)

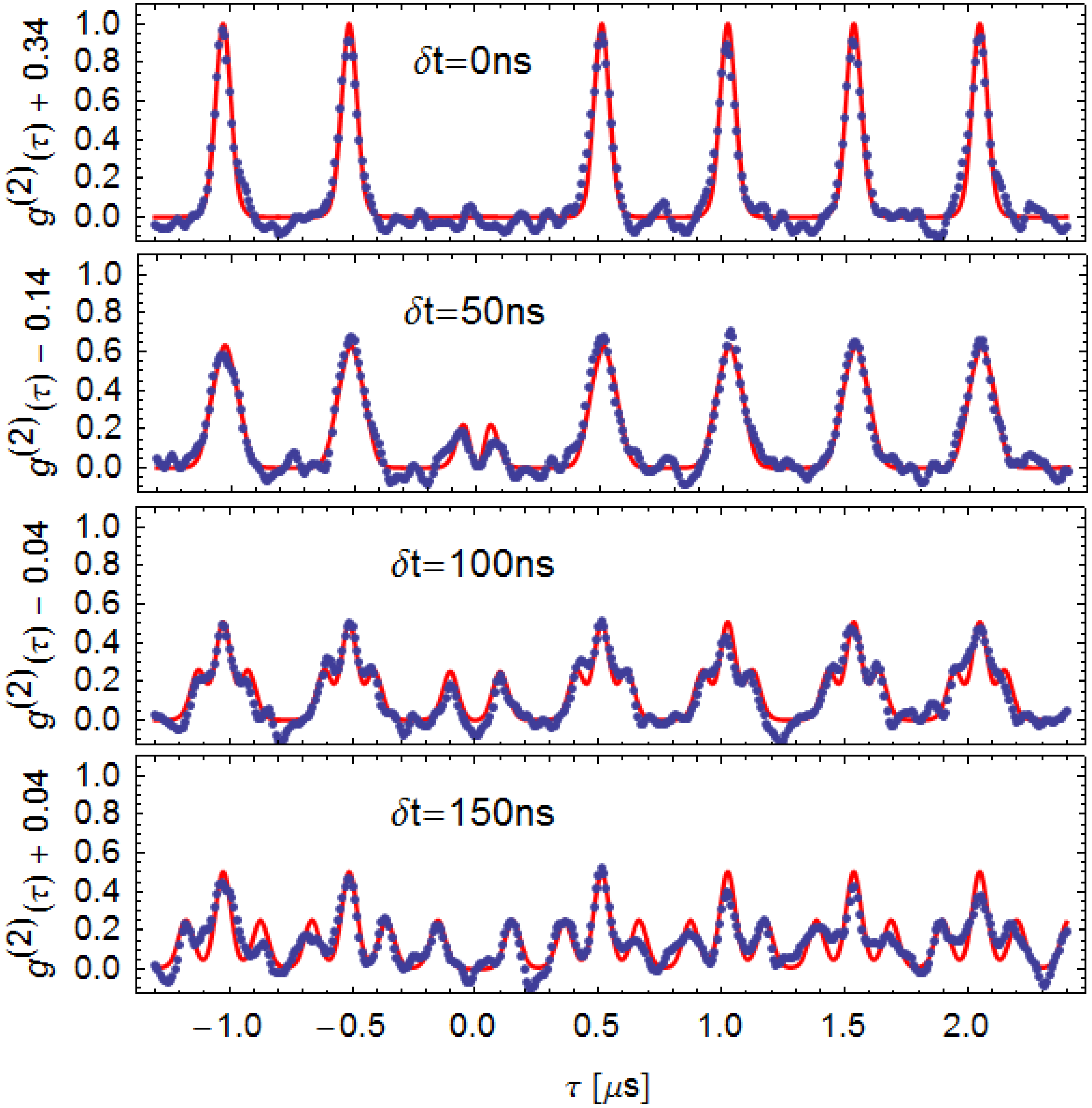
Two Single Photon Sources and Beam Splitter



Two Single Photon Sources and Beam Splitter



Hong-Ou-Mandel $g^{(2)}(\tau)$ for Microwave Photons



Observations:

- Photon-Pair anti-bunching

For $\tau > 0$:

- Broadening of satellite peaks
- Triple-peak structure of satellite peaks
- Full recovery of double-peak at $\tau \approx 0$

Exp.: C. Lang, C Eichler *et al.*,
Nat. Phys. **9**, 345 (2013)

Theo.: M. Woolley *et al.*,
New J. Phys. **15**, 105025–19 (2013)

Learning More: Two-Channel Tomography

Idea: Measure 4D histogram and evaluate relevant photon statistics

$$D_{\mathrm{ON}}(X_a, P_a, X_b, P_b)$$

$$D_{\mathrm{OFF}}(X_a, P_a, X_b, P_b)$$

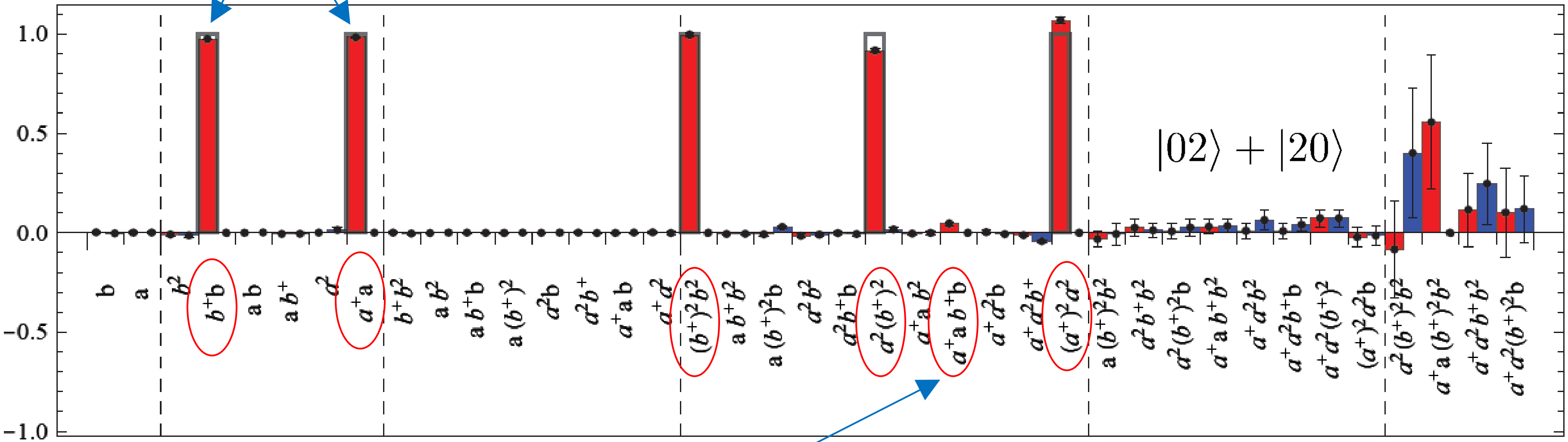
analogous to $\longrightarrow$ 1-channel case

$$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$$

1 average photon
in each channel

2-photon
bunching

quantum
superposition!



no simultaneous “click”
in both channels

C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

Density Matrix Displaying Two-Mode Entanglement

Density matrix reconstruction:

$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$

moments

linear map $\xrightarrow{}$

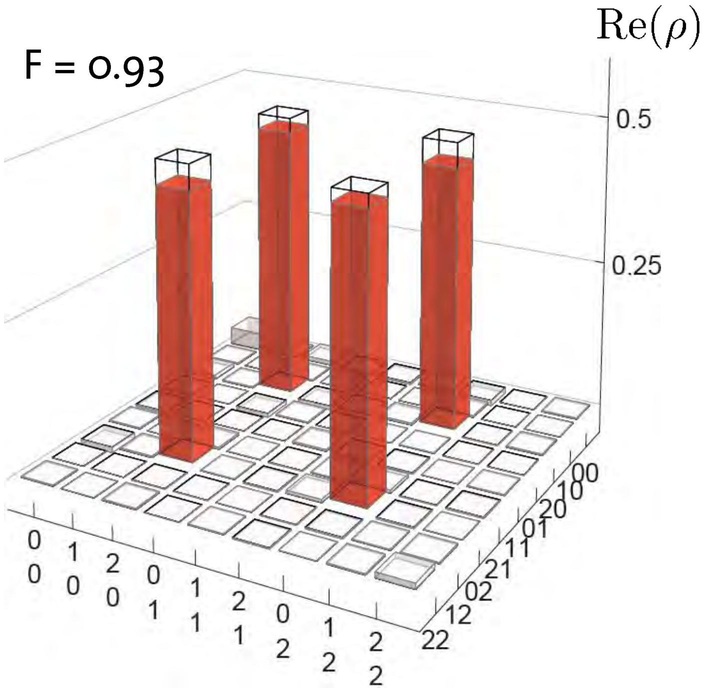
$\langle nm|\rho|kl\rangle$

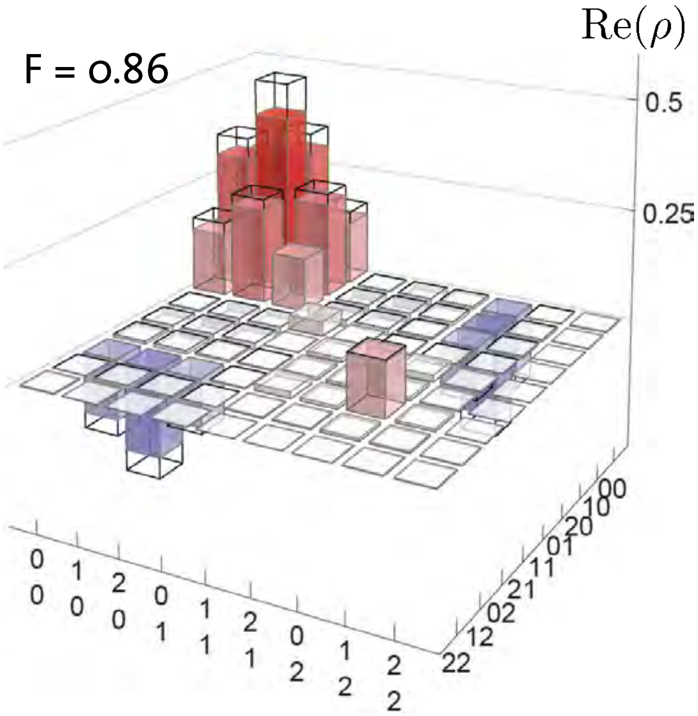
Fock space density matrix

use maximum likelihood procedure

final state: $|02\rangle + |20\rangle$

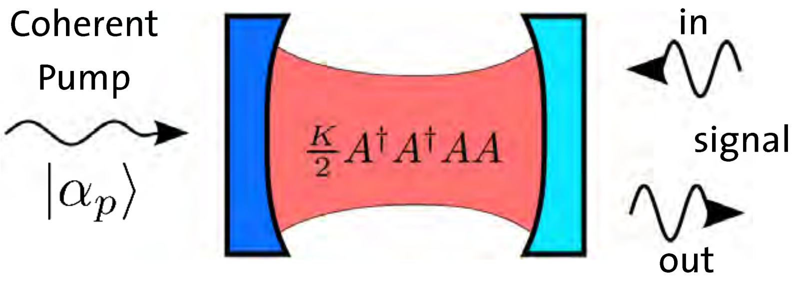
$|00\rangle + \sqrt{2}|10\rangle + (|20\rangle - |02\rangle)\sqrt{2}$

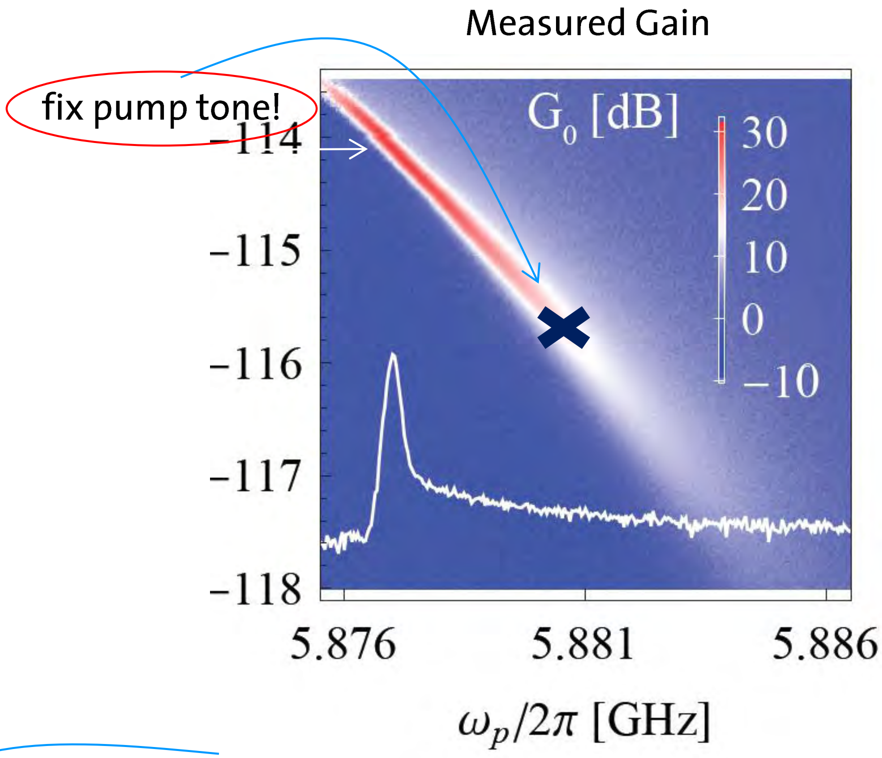




C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

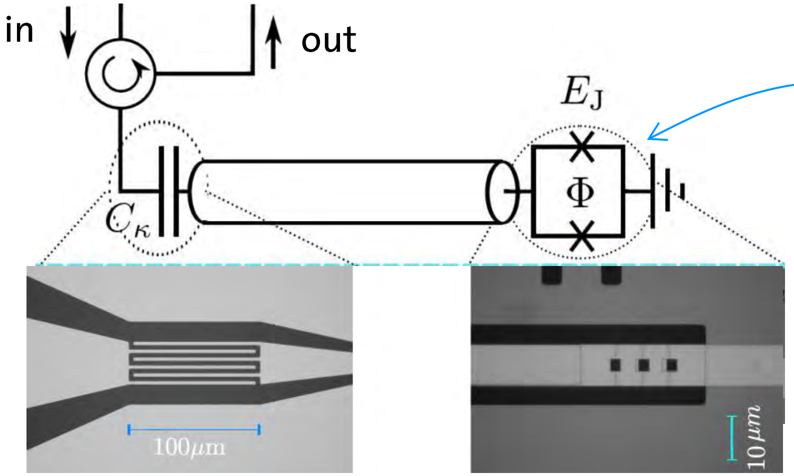
Parametric Amplifier




Measured Gain

fix pump tone!

Circuit QED implementation:



SQUID provides nonlinearity!

Caves, *Phys. Rev. D* **26**, 1817 (1982)

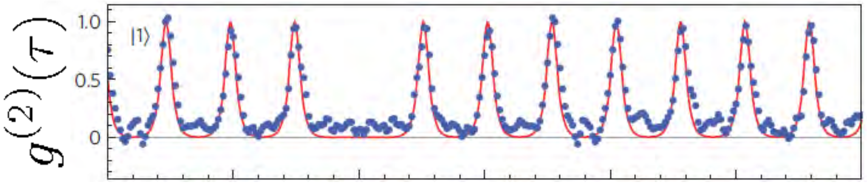
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)

Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)

Eichler et al., *Phys. Rev. Lett.* **107**, 113601 (2011)

Experiments with Propagating Quantum Microwaves

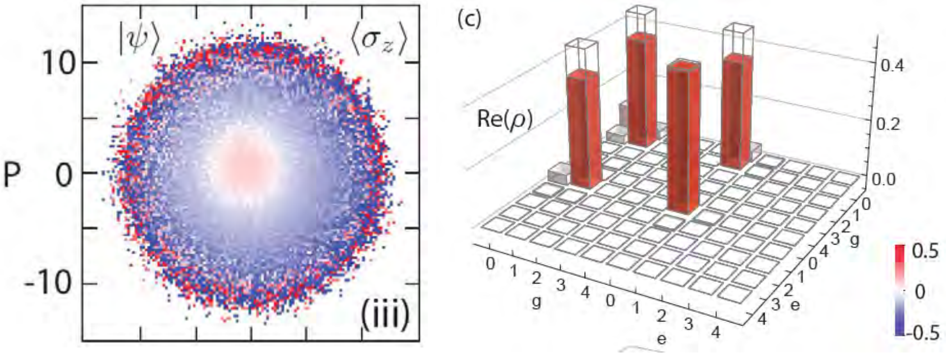
Single photon sources and their anti-bunching



Bozyigit *et al.*, Nat. Phys **7**, 154 (2011)

Lang *et al.*, *PRL* **107**, 073601 (2011)

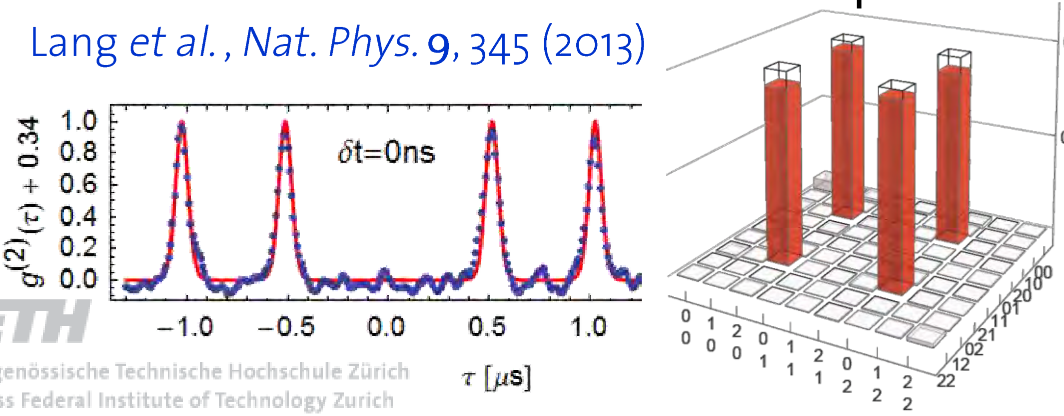
Preparation and characterization of qubit-**propagating** photon entanglement



Eichler *et al., PRL* **109**, *240501* (2012)
Eichler *et al., PRA* **86**, 032106 (2012)

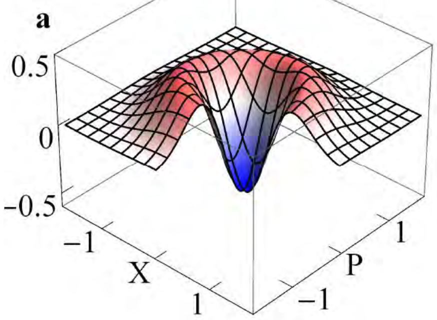
Hong-Ou-Mandel: Two-photon interference incl. msrmnt of coherences at microwave freq.

Lang *et al.*, *Nat. Phys.* **9**, 345 (2013)



Full state tomography and Wigner functions of **propagating** photons

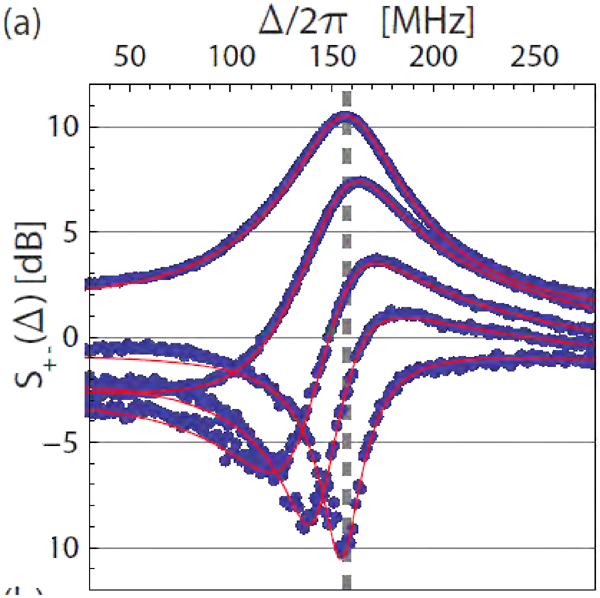
Eichler *et al., PRL* **106**, 220503 (2011)



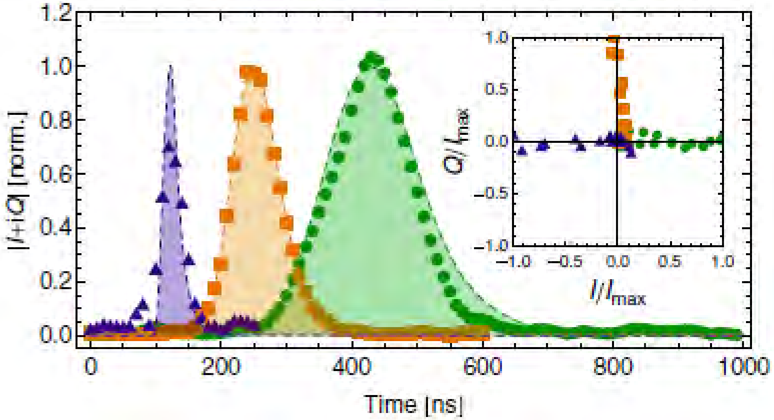
Recent Experiments with Propagating Microwaves

Squeezing in a Josephson parametric dimer

Eichler *et al.*, *PRL* **113**, *110502* (2014)



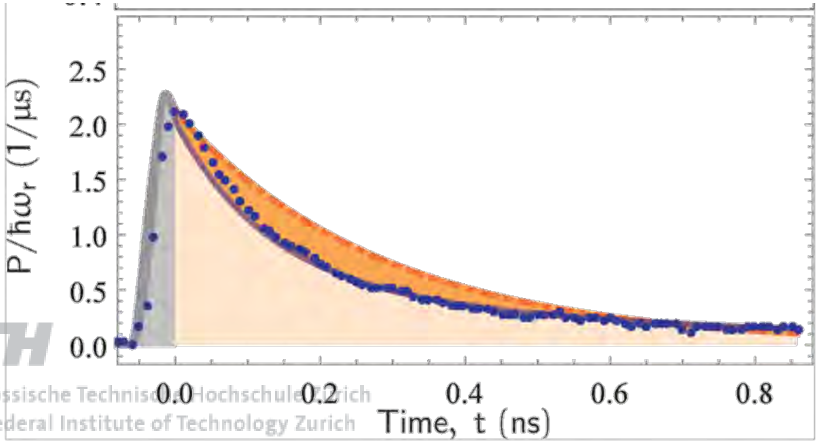
Creation of **time-reversal symmetric photons** for networking



Pechal *et al.*, *PRX* **4**, *041010* (2014)

Single and two-atom **Dicke superradiance**

Mlynek *et al.*, *Nat. Comm.* **5**, *5186* (2014)



Prospects:

- explore linear & non-linear microwave quantum optics
- shaped photons for communication across different length scales
- characterize microwave radiation emission from novel sources (e.g. nano-structures)

Teleportation ... what one may wish for !?

Teleportation in the Quantum World

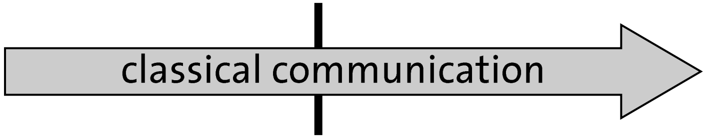
Objective:

- transfer information stored in an quantum bit from a sender to receiver

Resources:

- a pair of entangled qubits shared between the sender and receiver
- a small quantum computer at the sender and at the receiver
- a classical communication channel

Alice



Bob

Features:

- exploits non-local quantum correlations
- uses all essential ingredients required for realizing a universal quantum computer
- full protocol demonstrates use of real-time feed-forward

Applications:

- universal quantum computation
- simplification of quantum circuits
- repeaters for quantum comm.

Has been demonstrated for photons, ions and recently also in solid state systems.

proposal: Bennett *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993)

Teleportation Protocol

Task:

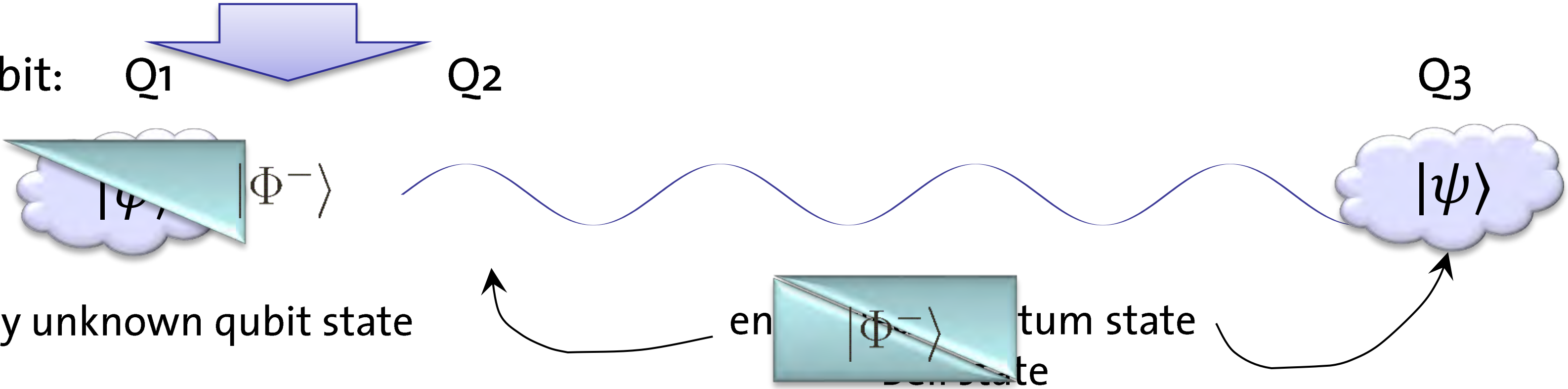
- transfer unknown quantum state from Alice to Bob

Resources:

- a pair of entangled qubits (Q1 + Q2)

Alice

Bob



Bell measurement

Qubit: Q1 Q2 Q3

$|\psi$ $|\Phi^-\rangle$

$|\psi\rangle$

arbitrary unknown qubit state

en $|\Phi^-\rangle$ tum state

te

proposal: Bennett *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993)

Teleportation Protocol

Task:

- transfer unknown quantum state from Alice to Bob

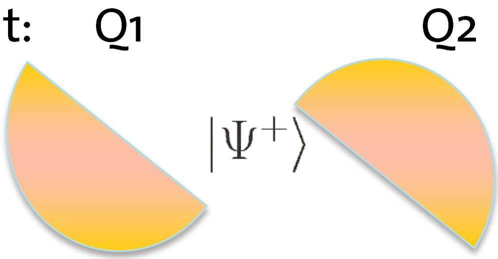
Resources:

- a pair of entangled qubits (Q1 + Q2)
- classical communication

Alice

Bob

Qubit:



proposal: Bennett *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993)

Teleportation in other Systems

Single photons

D. Bouwmeester, *et al.*, *Nature*, **390**, 575–579 (1997)
I. Marcikic, *et al.*, *Nature*, **421**, 509–513 (2003)
J. Yin, *et al.*, *Nature*, **488**, 185–188 (2012)
X.-S. Ma, *et al.*, *Nature*, **489**, 269–273 (2012)

Ion traps

M. Riebe *et al.*, *Nature*, **429**, 734–737 (2004)
M. Barrett, *et al.*, *Nature*, **429**, 737–739 (2004)
S. Olmschenk, *et al.*, *Science*, **323**, 486–489 (2009)

Atomic ensembles

X.-H. Bao, *et al.*, *PNAS*, **109**, 20347 (2012)

Single atoms

C. Nölleke, *et al.*, *Phys. Rev. Lett.*, **110**, 140403 (2013)

NMR

M. A. Nielsen, *et al.*, *Nature*, **396**, 52–55 (1998)

Continuous variables

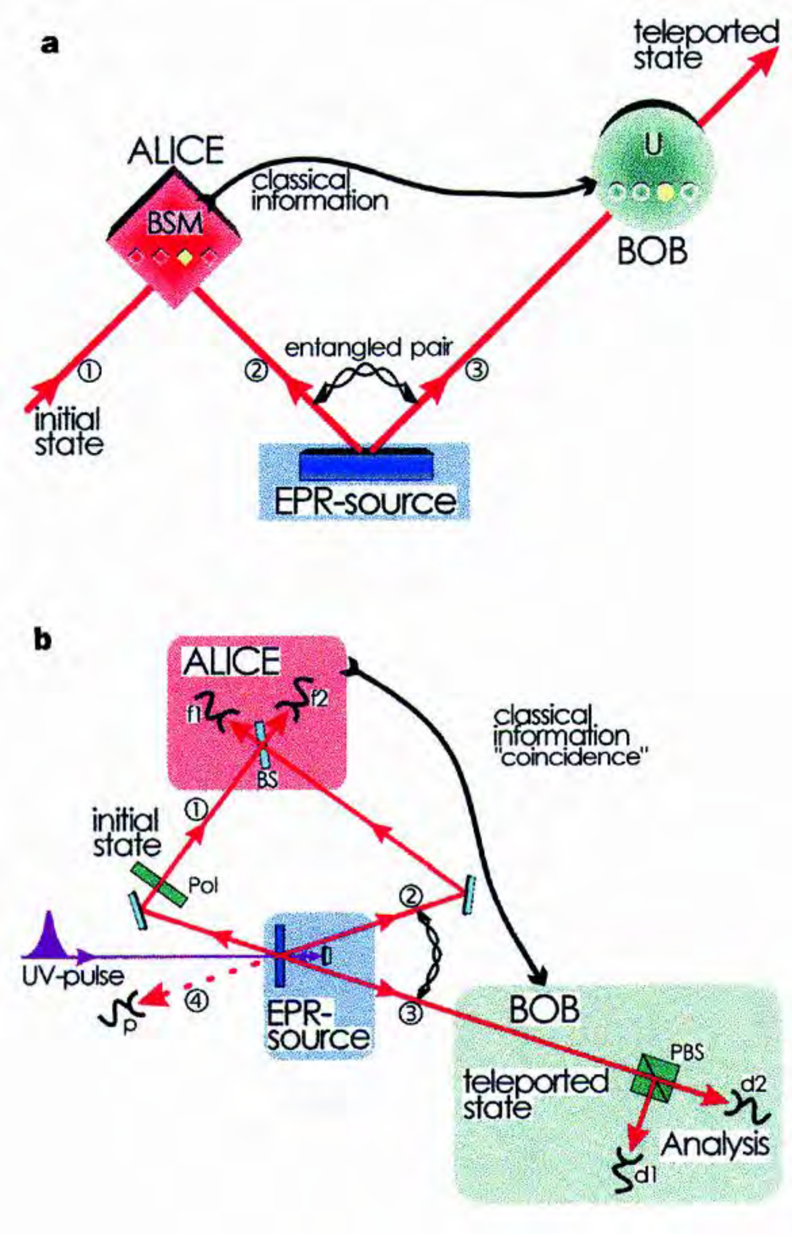
A. Furusawa, *et al.*, *Science*, **282**, 706–709 (1998)
N. Lee, *et al.*, *Science*, **332**, 330–333 (2011)
S. Takeda, *et al.*, *Nature*, **500**, 315–318 (2013)

Semiconductor Quantum Dots

W.B. Gao, et al., *Nat. Comm* **4**, 2744 (2013)

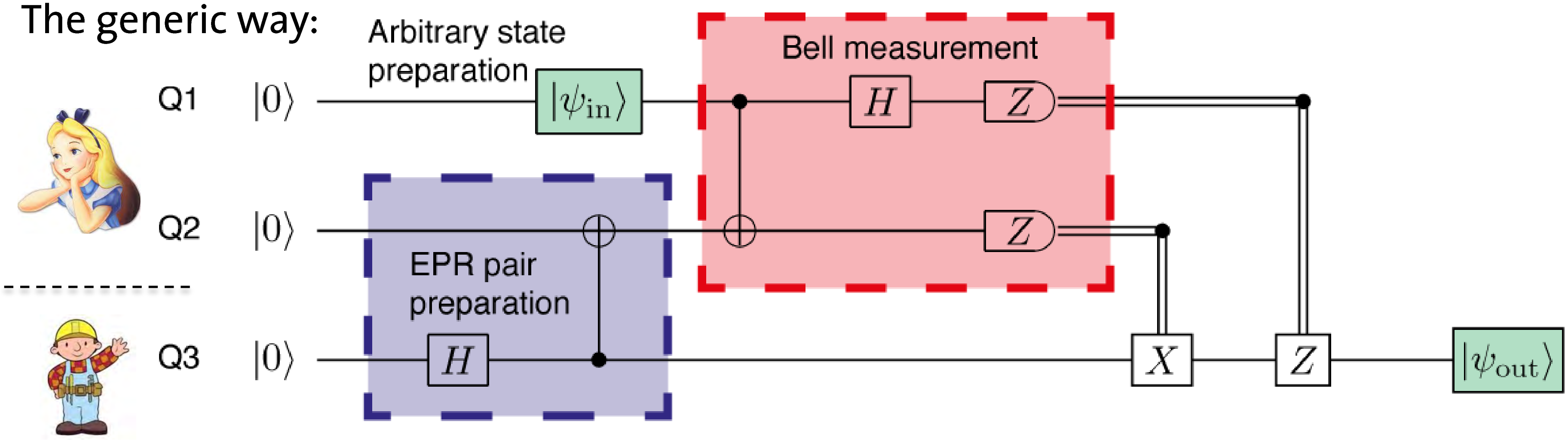
NV Centers

W. Pfaff, et al., *Science* **345**, 532 (2014)



Implementation of the Teleportation Protocol

The generic way:



Hadamard

Rotation around Y-axis

Controlled NOT

Controlled phase gate

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

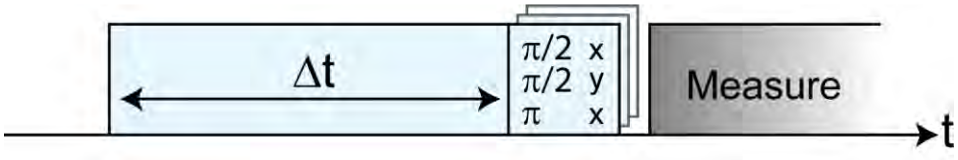
Measurement along Z-axis

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).

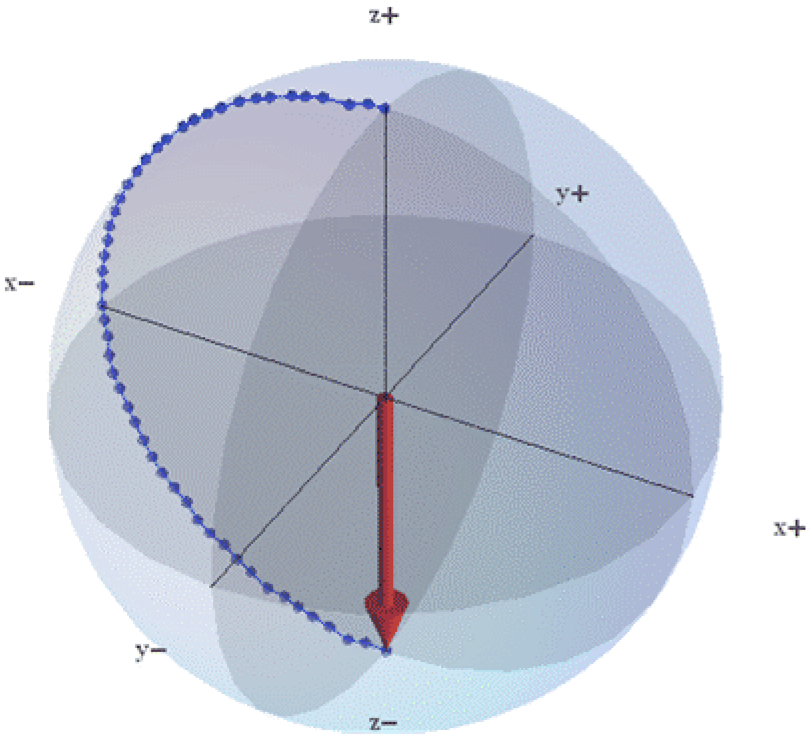
implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Single Qubit Gates

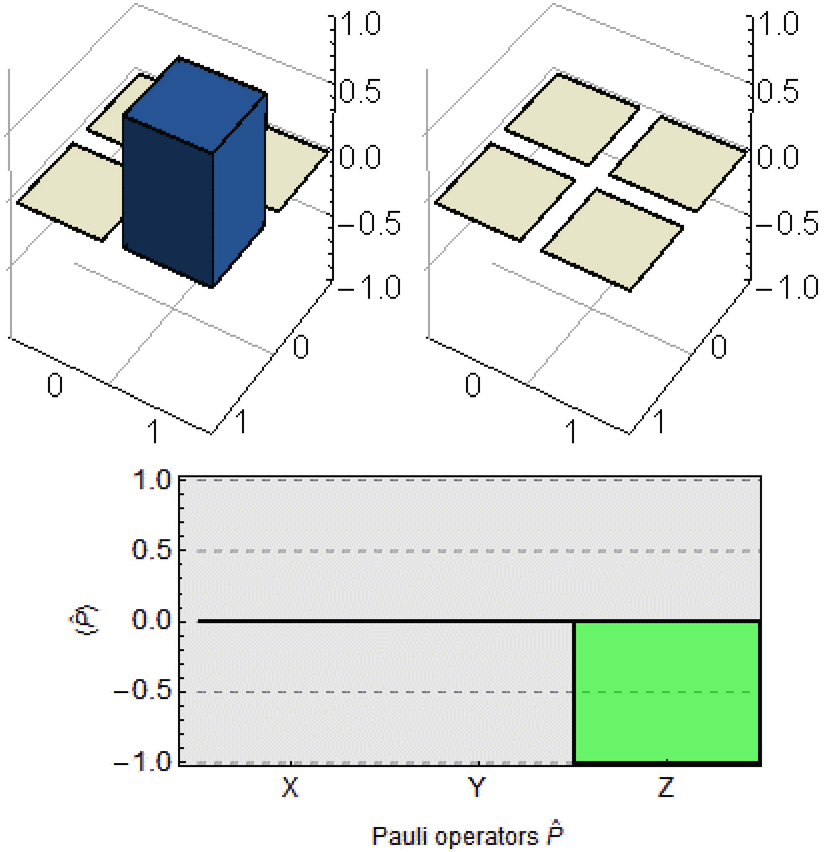
Pulse sequence for qubit rotation and readout:



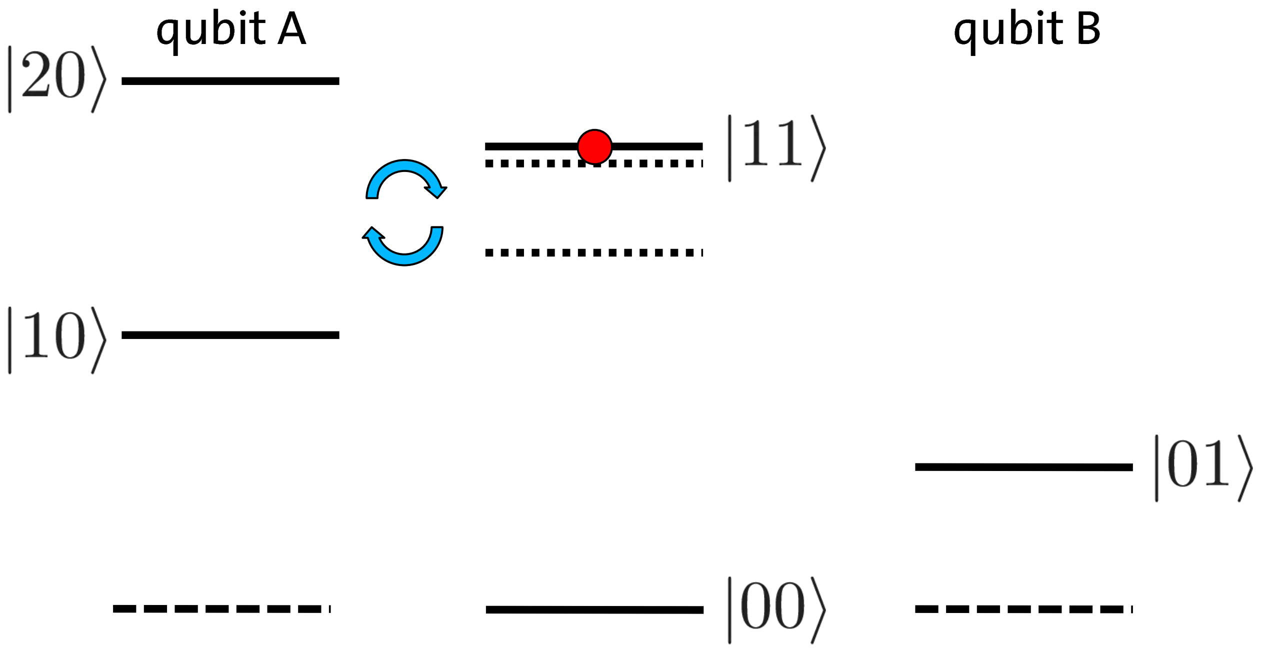
experimental Bloch vector:



experimental density matrix and Pauli set:



Controlled phase gate



Tune levels into resonance using magnetic field

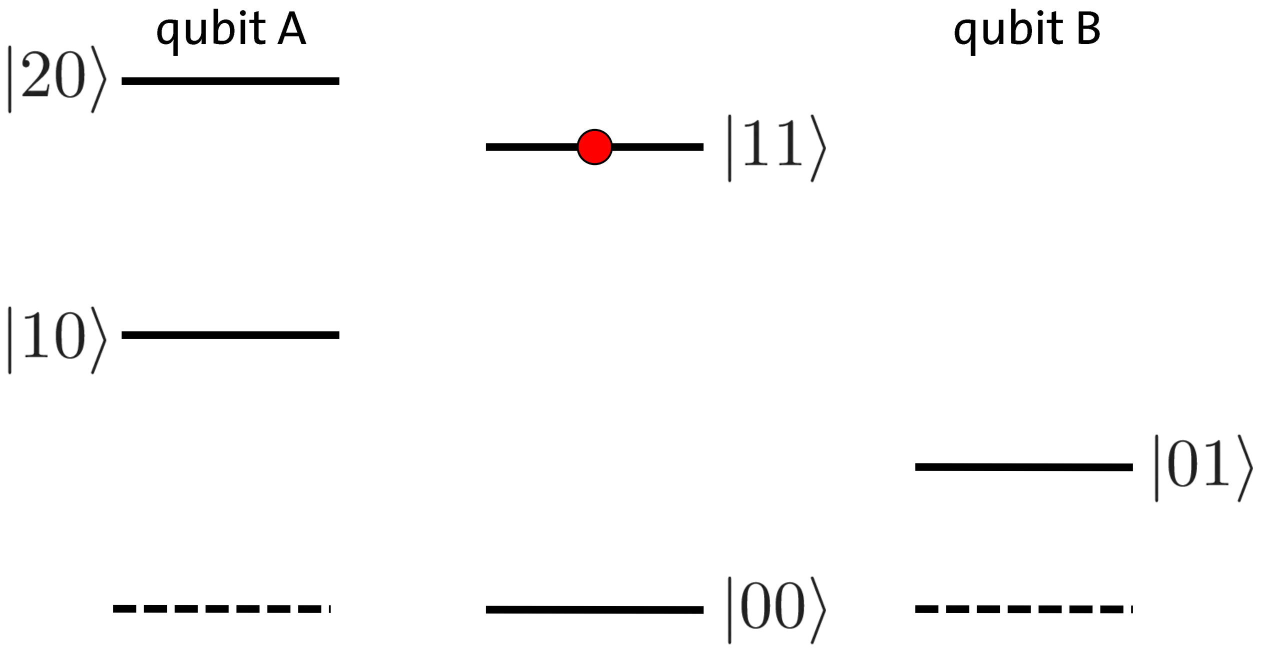
Interaction mediated by virtual photon exchange through resonator

$$|11\rangle \longrightarrow i|02\rangle \longrightarrow -|11\rangle$$

proposal: F. W. Strauch *et al*, *PRL* **91**, 167005 (2003)

first implementation: L. DiCarlo *et al.*, *Nature* **467**, 467 (2010)

Controlled phase gate



Tune levels into resonance using magnetic field

Interaction mediated by virtual photon exchange through resonator

This experiment:
Universal two-qubit gates: F ~ 90%.
Single-qubit gates: F ~ 98%.
.. to realize needed quantum operations.

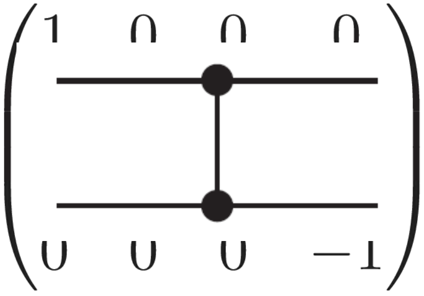
$$|11\rangle \longrightarrow i|02\rangle \longrightarrow -|11\rangle$$

$$|01\rangle \longrightarrow |01\rangle$$

$$|10\rangle \longrightarrow |10\rangle$$

$$|00\rangle \longrightarrow |00\rangle$$

C-Phase gate:



proposal: F. W. Strauch *et al*, *PRL* **91**, 167005 (2003)

first implementation: L. DiCarlo *et al.*, *Nature* **467**, 467 (2010)

Process Tomography: C-Phase Gate

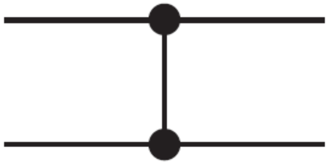
arbitrary quantum process $$\boxed{\rho' = \mathcal{E}(\rho)}$$

decomposed into $$\boxed{\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}}$$

$\{\tilde{E}_k\}$ is an operator basis

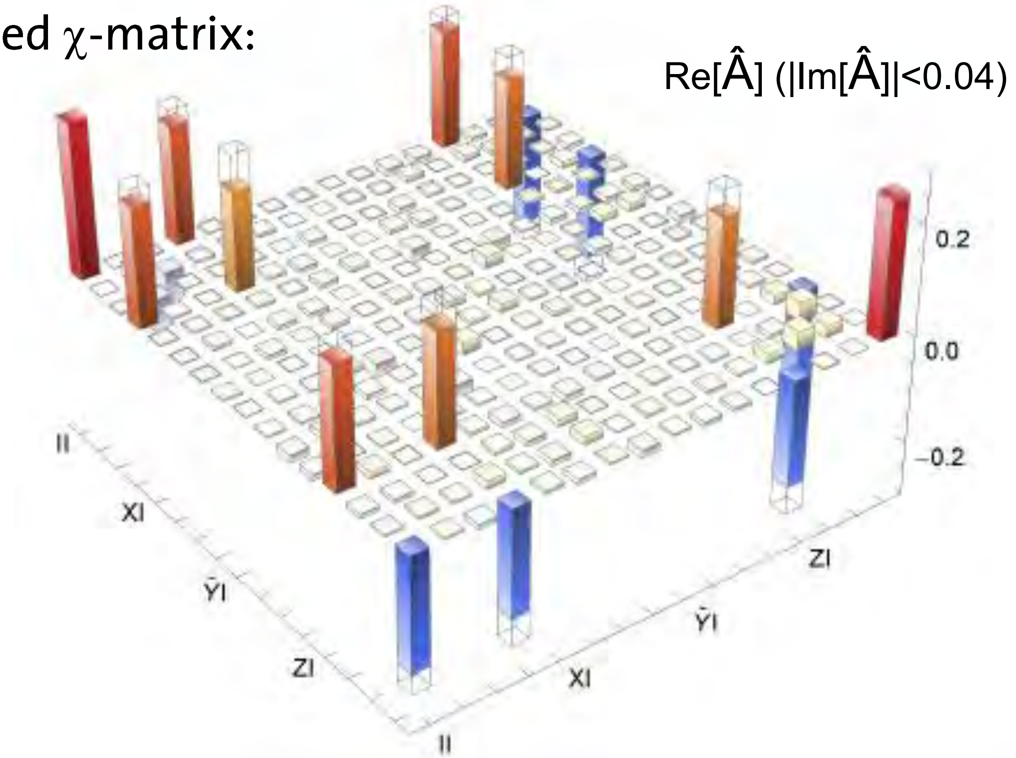
χ is a positive semi definite Hermitian matrix characteristic for the process

Controlled phase gate



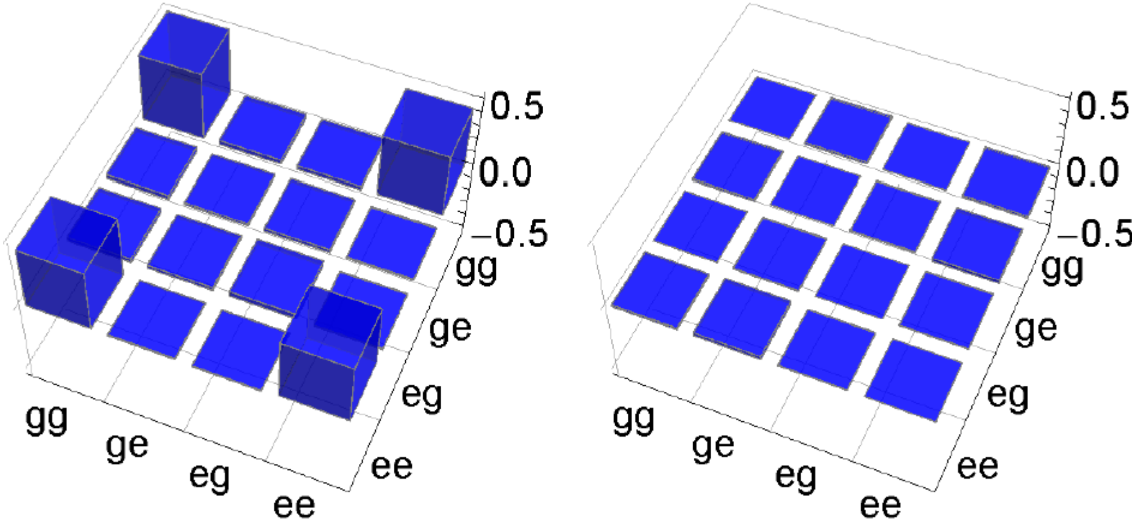
$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured χ-matrix:

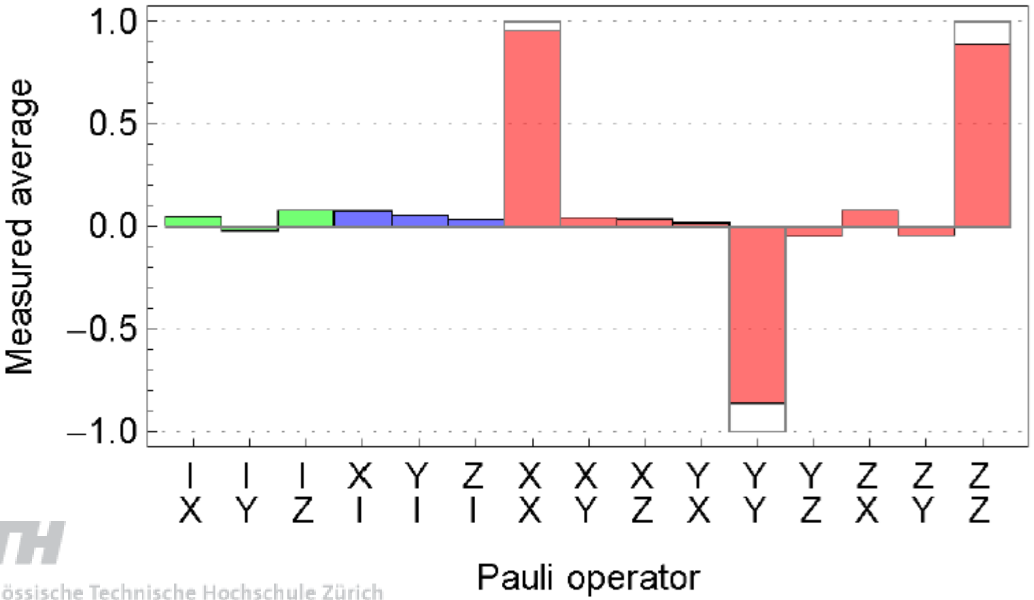


$$F = \mathrm{Tr}[\chi_{\mathrm{meas}}\chi_{\mathrm{ideal}}] > 0.90$$

Generation of Bell States



F = 0.93



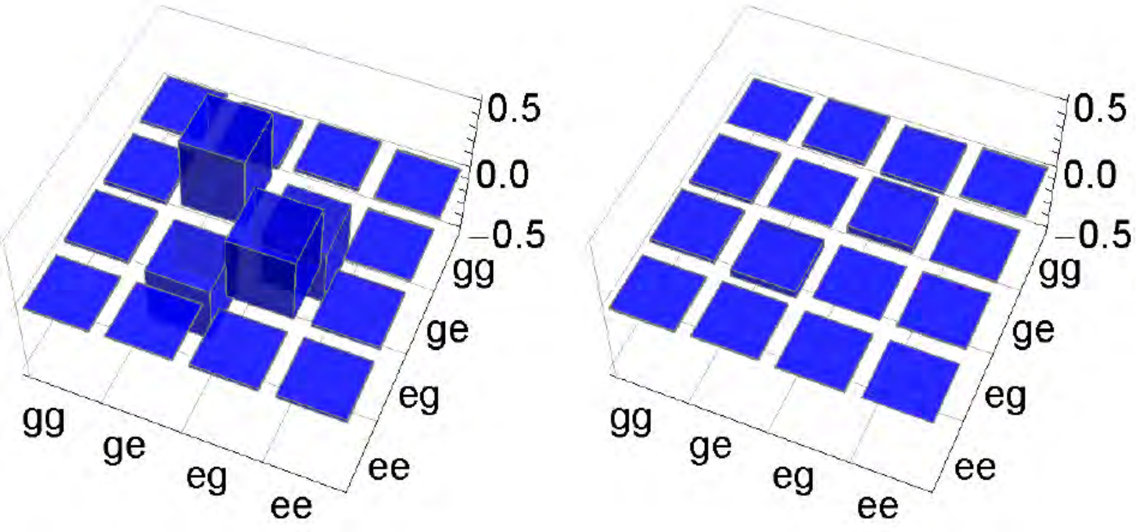
using different processes:

- controlled phase gate via 11 - 02 interaction
- sideband transitions
- iSWAP gate via 10 - 10 interaction
- resonant coupling to the cavity

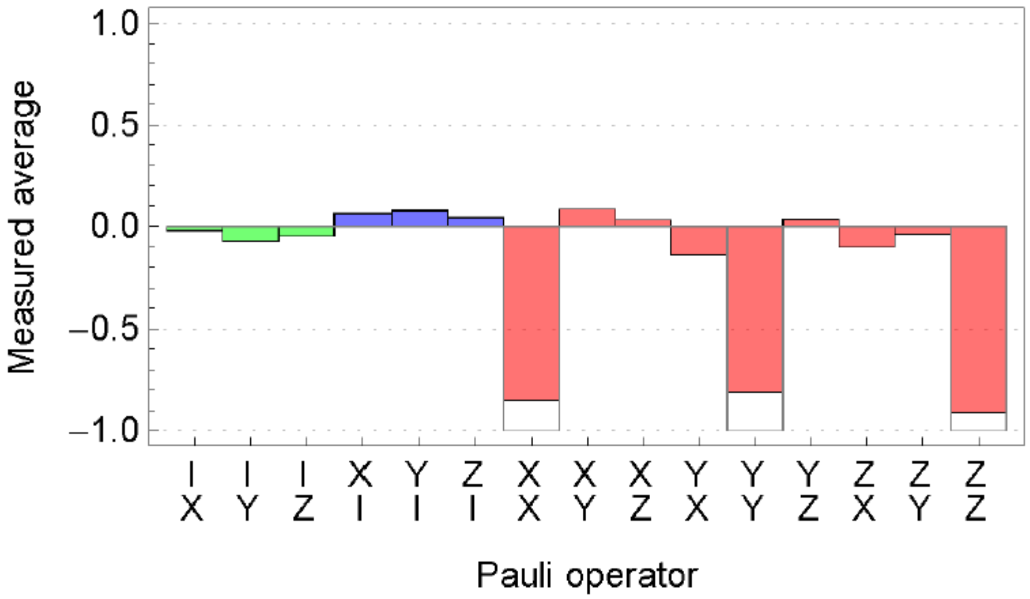
Fidelity measure:

$$F = \mathrm{Tr}[\rho_{\mathrm{meas}}\rho_{\mathrm{ideal}}]$$

Generation of Bell States



F = 0.89



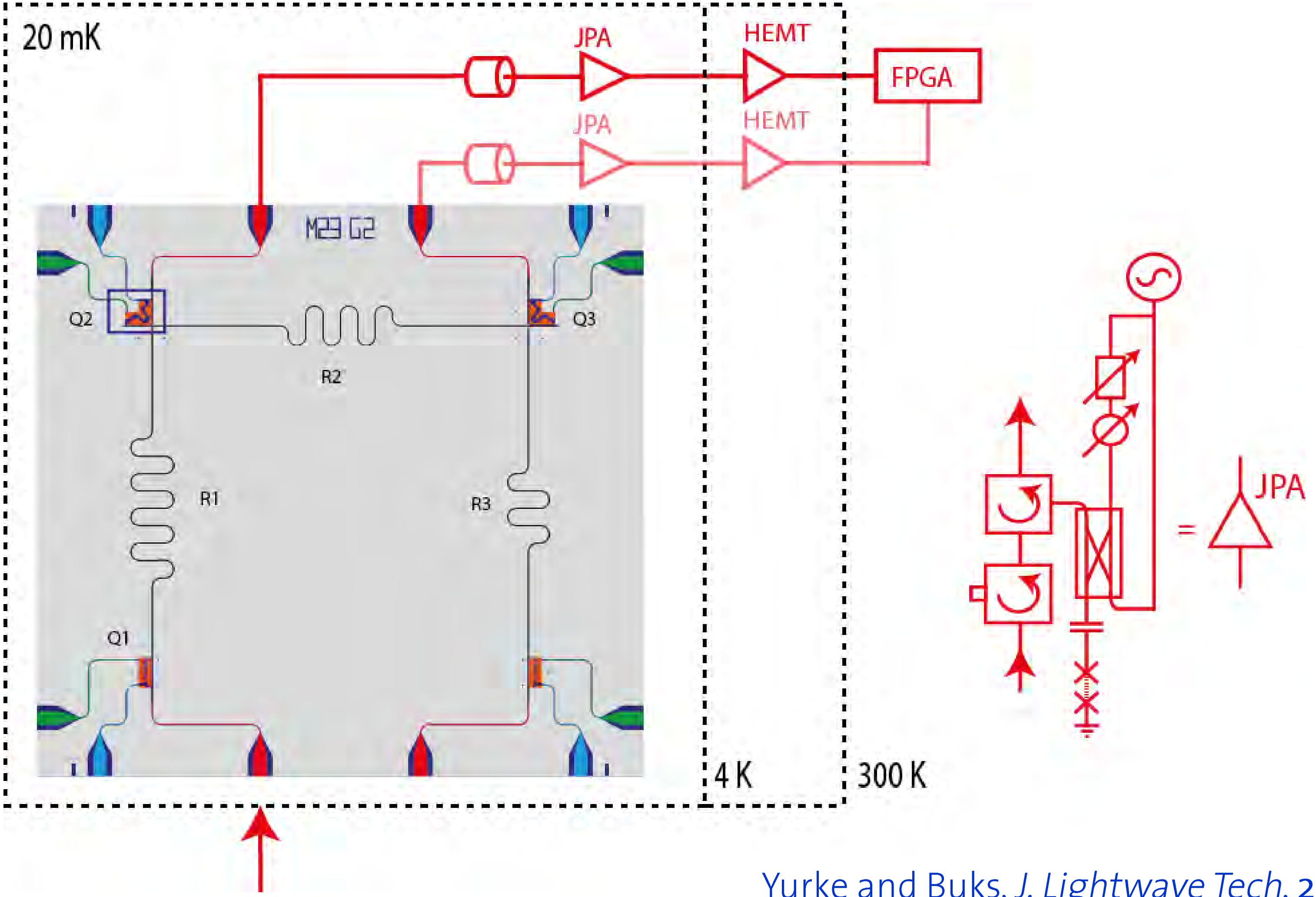
using different processes:

- controlled phase gate via 11 - 02 interaction
- sideband transitions
- iSWAP gate via 10 - 10 interaction
- resonant coupling to the cavity

Fidelity measure:

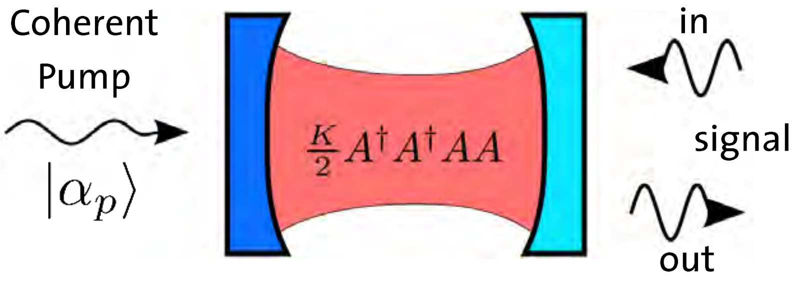
$$F = \mathrm{Tr}[\rho_{\mathrm{meas}}\rho_{\mathrm{ideal}}]$$

Dispersive Qubit Readout with Parametric Amplifiers

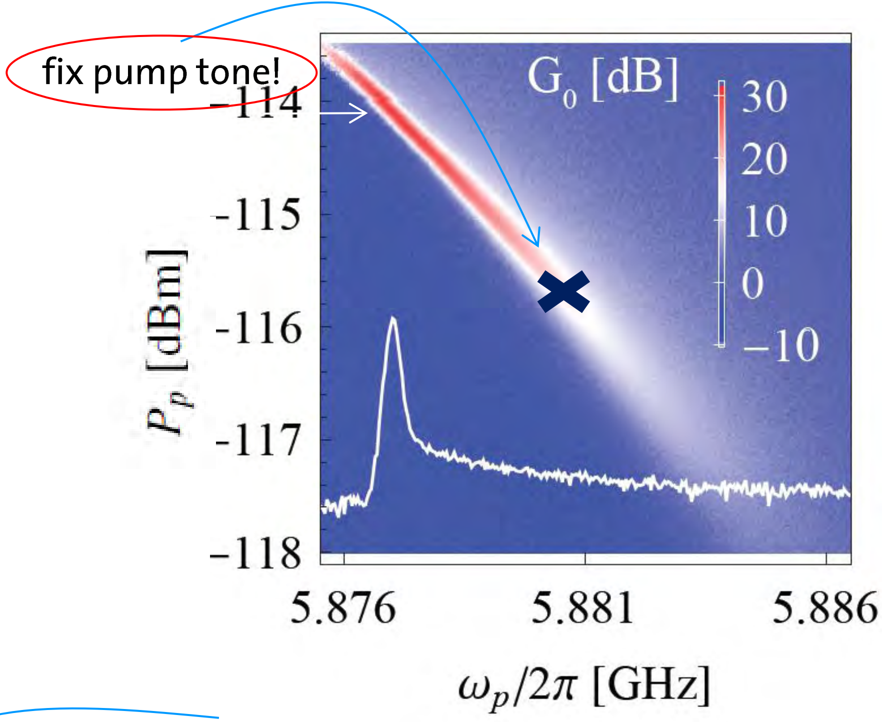


Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)
Eichler *et al.*, *PRL* **107**, 113601 (2011)
R. Vijay *et al*, *PRL* **106**, 110502 (2011)

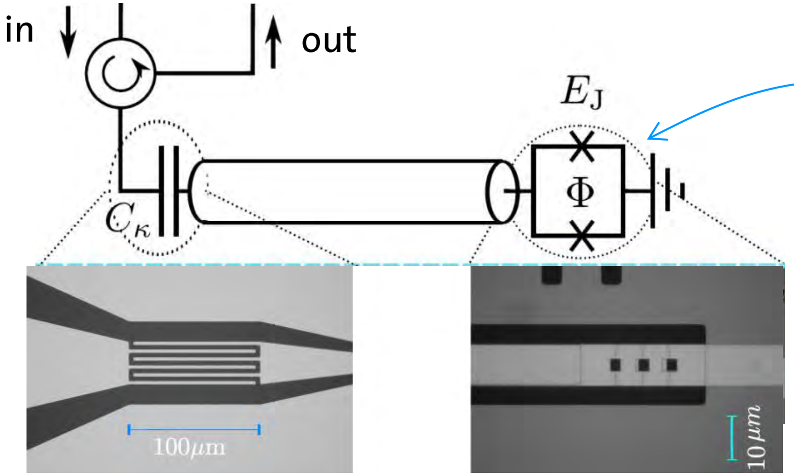
Near Quantum-Limited Parametric Amplifier



Measured Gain



Circuit QED implementation:



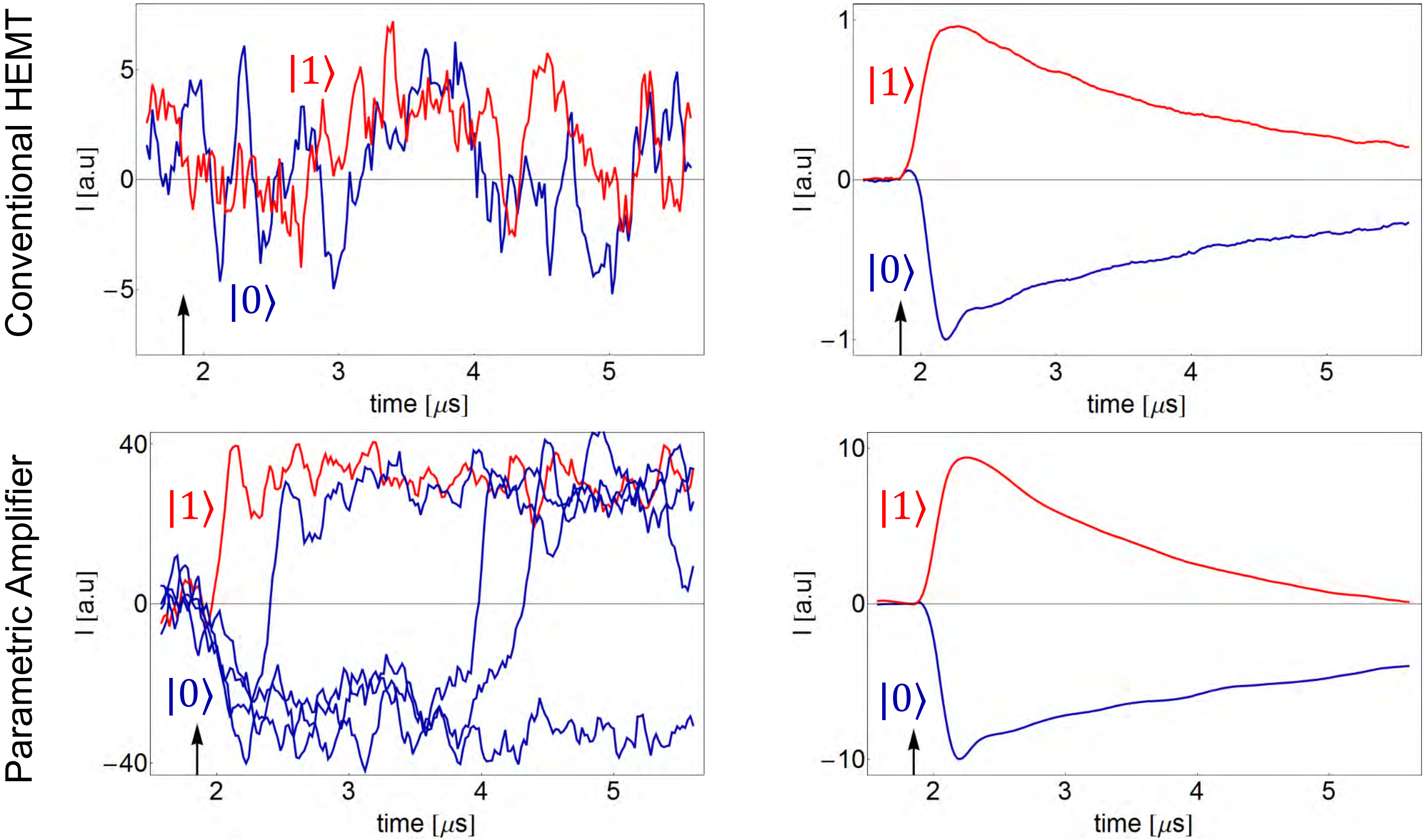
SQUID(-array) provides
required nonlinearity

Eichler *et al.*, EPJ Quantum Technology **1**, 2 (2014)
Eichler *et al.*, *Phys. Rev. Lett.* **107**, 113601 (2011)
Eichler et al., *Phys. Rev. Lett.* **113**, 110502 (2014)
Caves, *Phys. Rev. D* **26**, 1817 (1982)
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 929 (2008)

Single-Shot Single-Qubit Readout

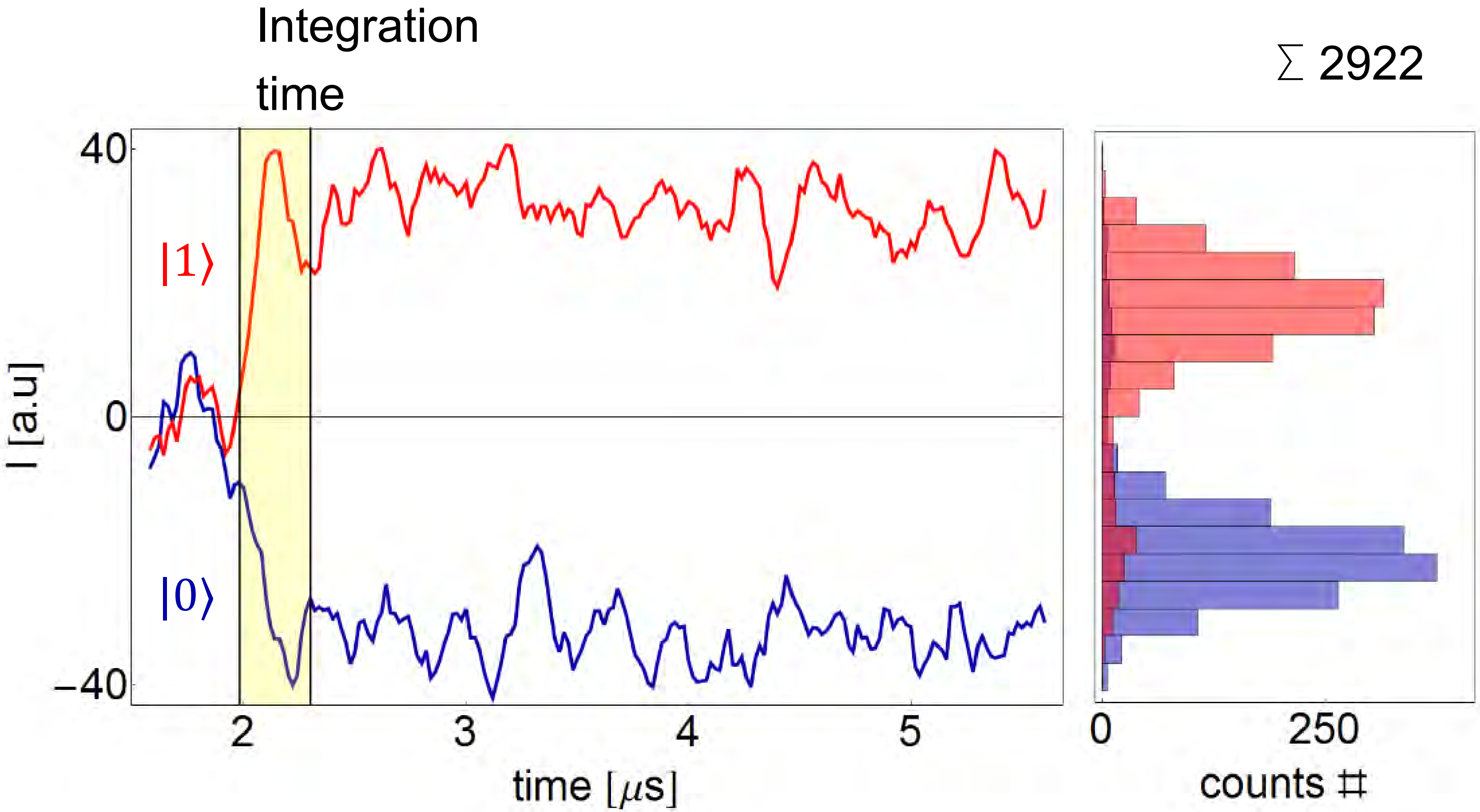
single-shot measurements:

averaged measurements ($8\ 10^4$):



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* **106**, 110502 (2011)

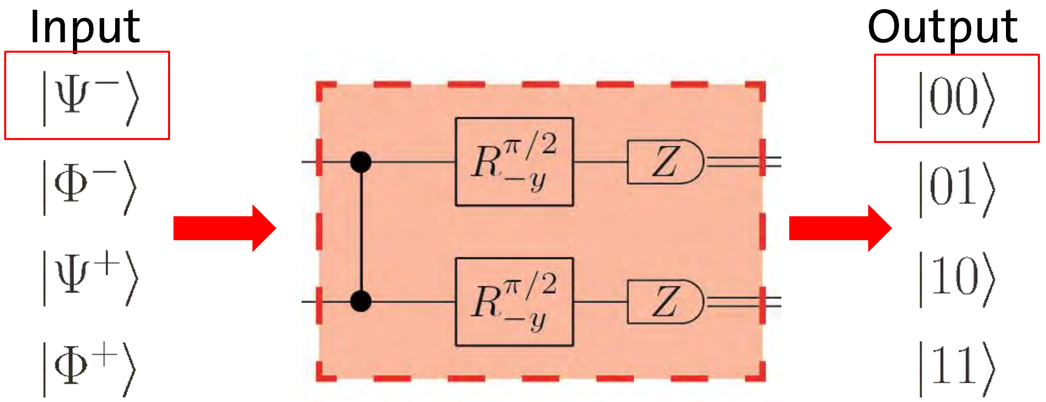
Statistics of Integrated Single-Shot Readout



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)


Quantum Teleportation with Superconducting Circuits
1 µeV ⟺ 240 MHz
10 µeV ⟺ 2.4 GHz
100 µeV ⟺ 24 GHz
1 meV ⟺ 240 GHz
10 meV ⟺ 2.4 THz
QUDEV Phonelist
DELL

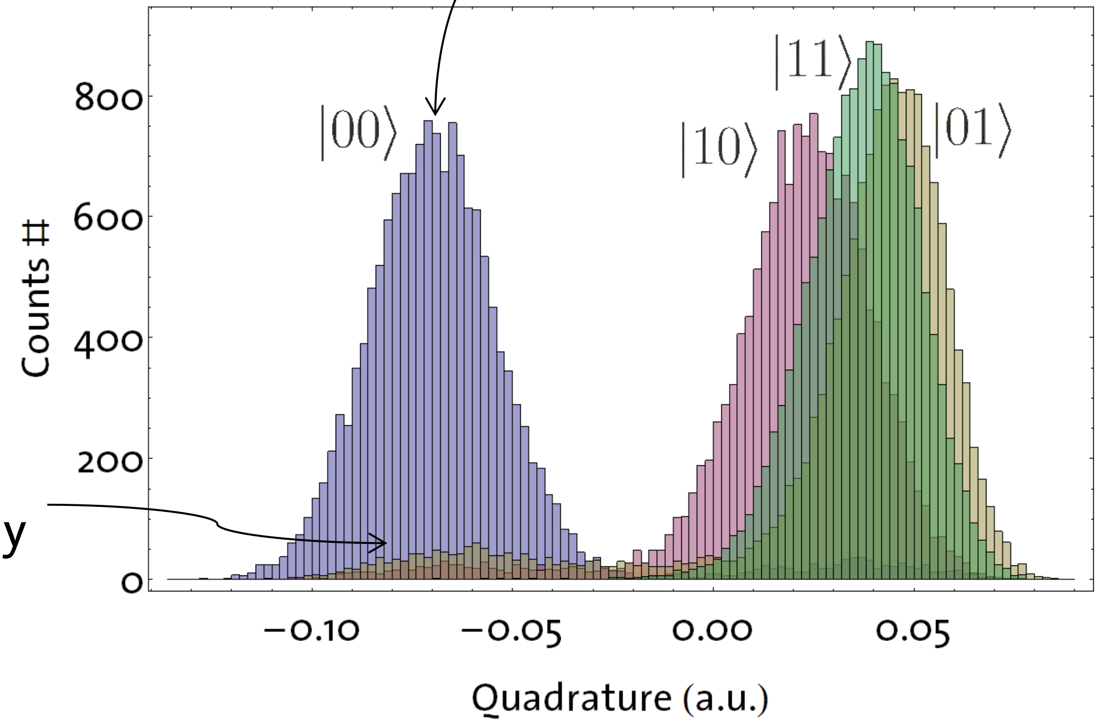
Post-Selected Teleportation: Bell Measurement



Operate parametric amplifier in phase sensitive mode

Maximize contrast of $|00\rangle$ to other states

detection fidelity of 94 %



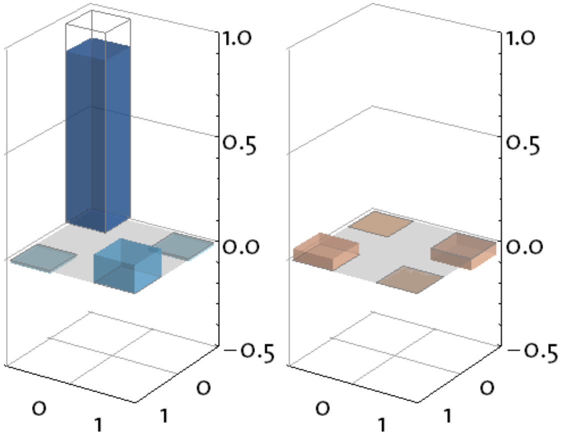
Limited by decay

Steffen *et al.*, *Nature* **500**, 319 (2013)

Tomography of Teleported States with Post-Selection

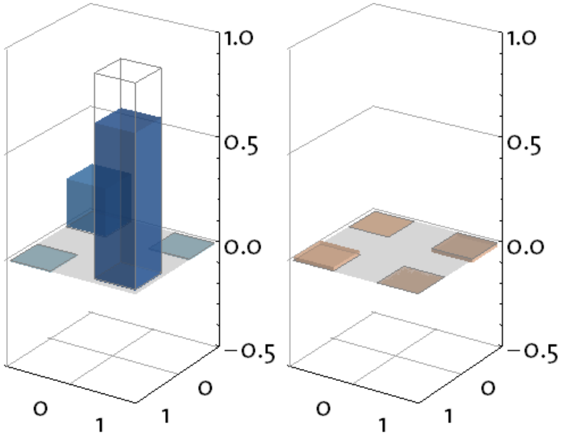
$\psi_{in} = |0\rangle$

82.2 %



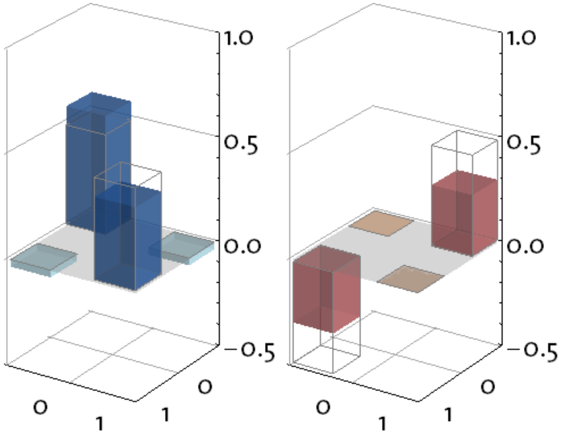
$\psi_{in} = |1\rangle$

80.5 %



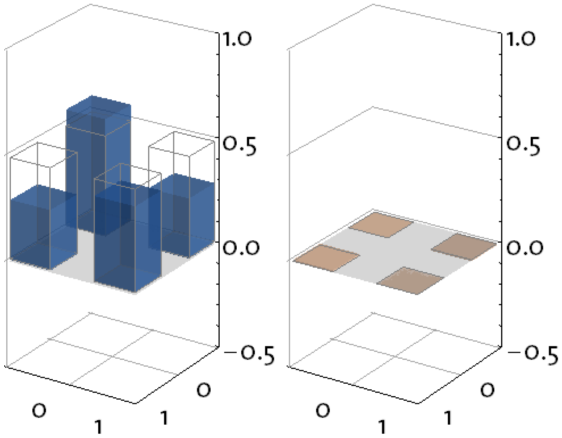
$\psi_{in} = |0\rangle - i\,|1\rangle$

79.4 %



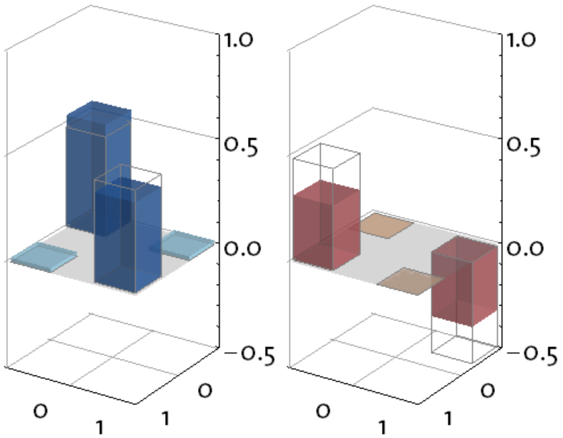
$\psi_{in} = |0\rangle + |1\rangle$

84.2 %



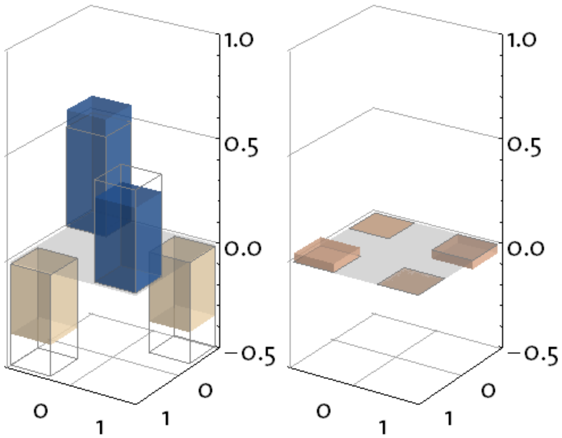
$\psi_{in} = |0\rangle + i\,|1\rangle$

79.5 %



$\psi_{in} = |0\rangle - |1\rangle$

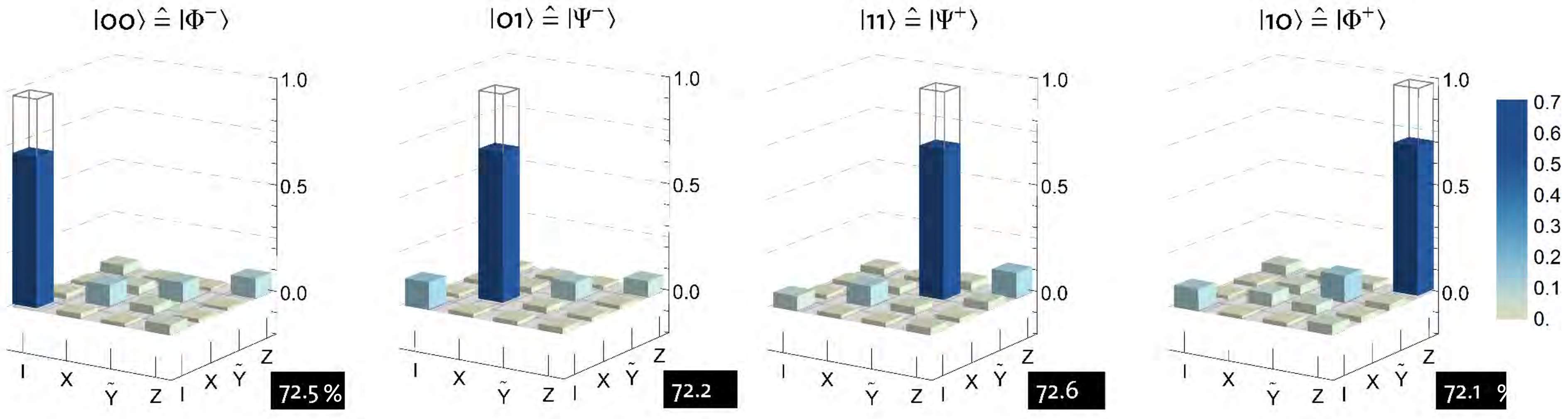
83.6 %



Steffen *et al.*, *Nature* **500**, 319 (2013)

Process Tomography: Teleportation with Post-Selection

absolute value of process matrices $|\chi|$ for state transfer from qubit 1 to qubit 3:



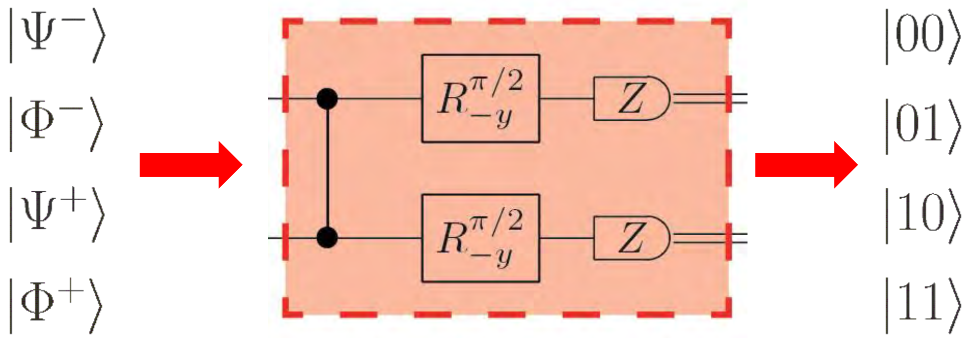
$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = X\,|\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = \tilde{Y}\,|\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = Z\,|\psi_{\text{in}}\rangle$$

$$X = \hat{\sigma}_x, \tilde{Y} = i\hat{\sigma}_y, Z = \hat{\sigma}_z$$

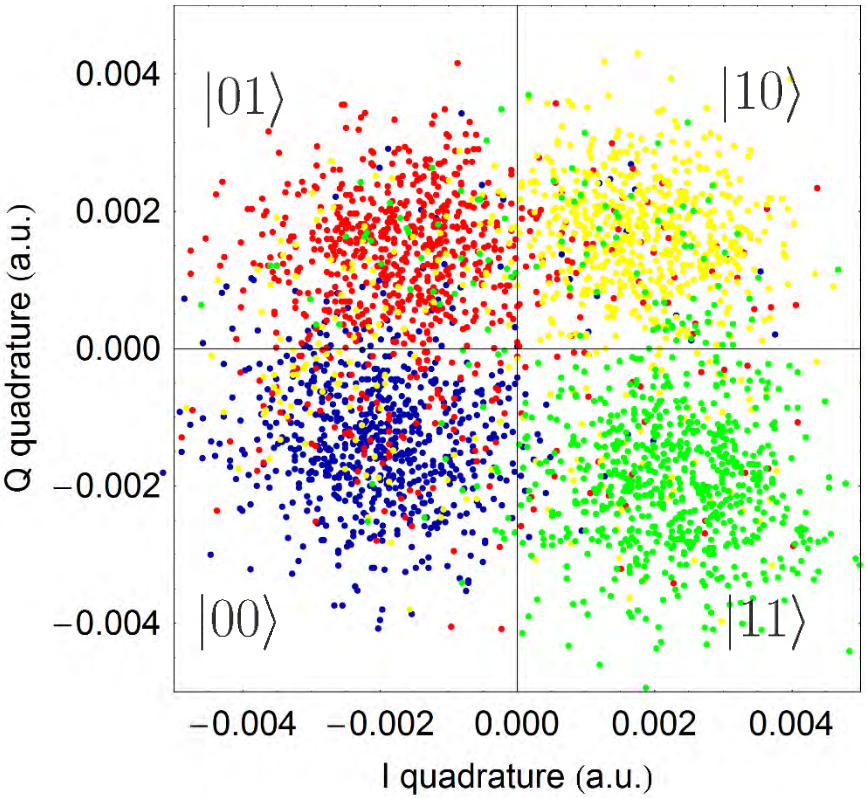
Average process fidelity **72.3 ± 0.7 %**

Steffen *et al.*, *Nature* **500**, 319 (2013)

Deterministic Bell-Measurement of all 4 States



- map Bell states on basis states
- perform joint two-qubit read-out
- paramp operated in the phase preserving mode to amplify both quadratures



States are identified correctly with ~80% probability

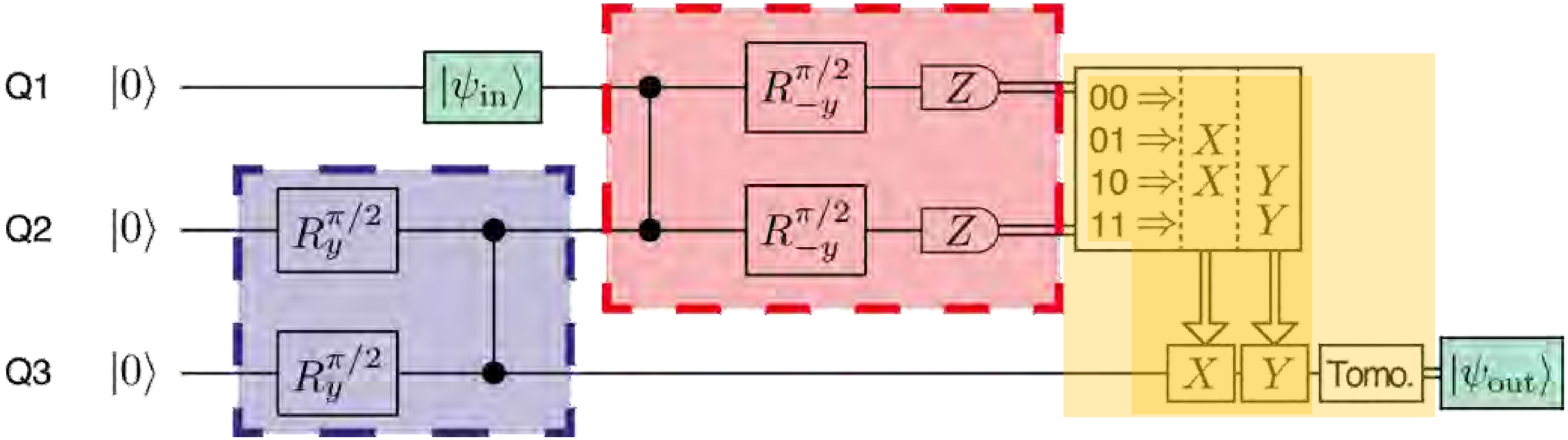
identified as

$$\begin{array}{c|cccc} & |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline |00\rangle & 0.86 & 0.09 & 0.02 & 0.02 \\ |01\rangle & 0.14 & 0.73 & 0.04 & 0.09 \\ |10\rangle & 0.03 & 0.05 & 0.84 & 0.09 \\ |11\rangle & 0.08 & 0.10 & 0.09 & 0.73 \end{array}$$

Prepared as

Steffen *et al.*, *Nature* **500**, 319 (2013)

Feed-Forward in the Teleportation Protocol

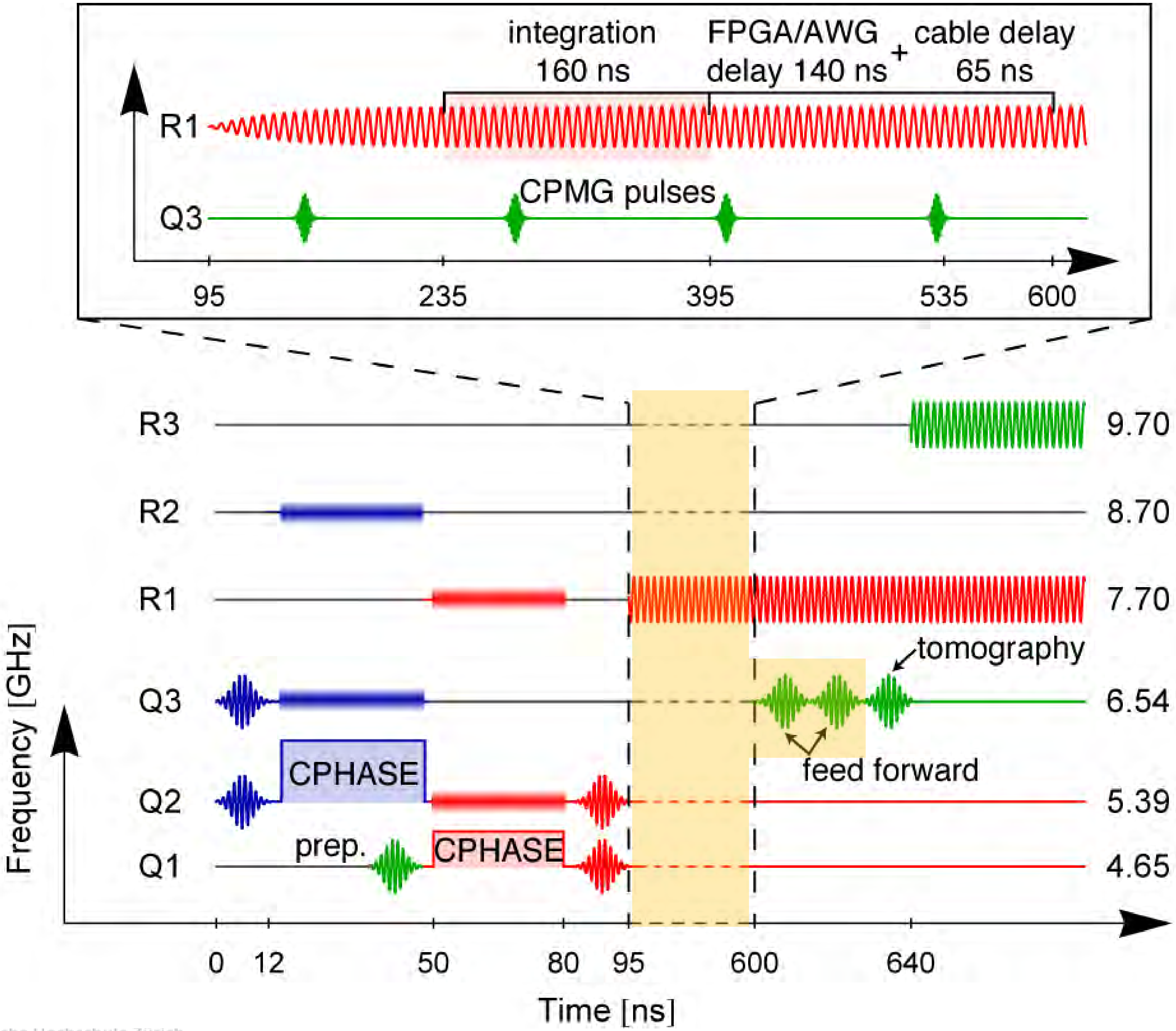


render protocol deterministic:

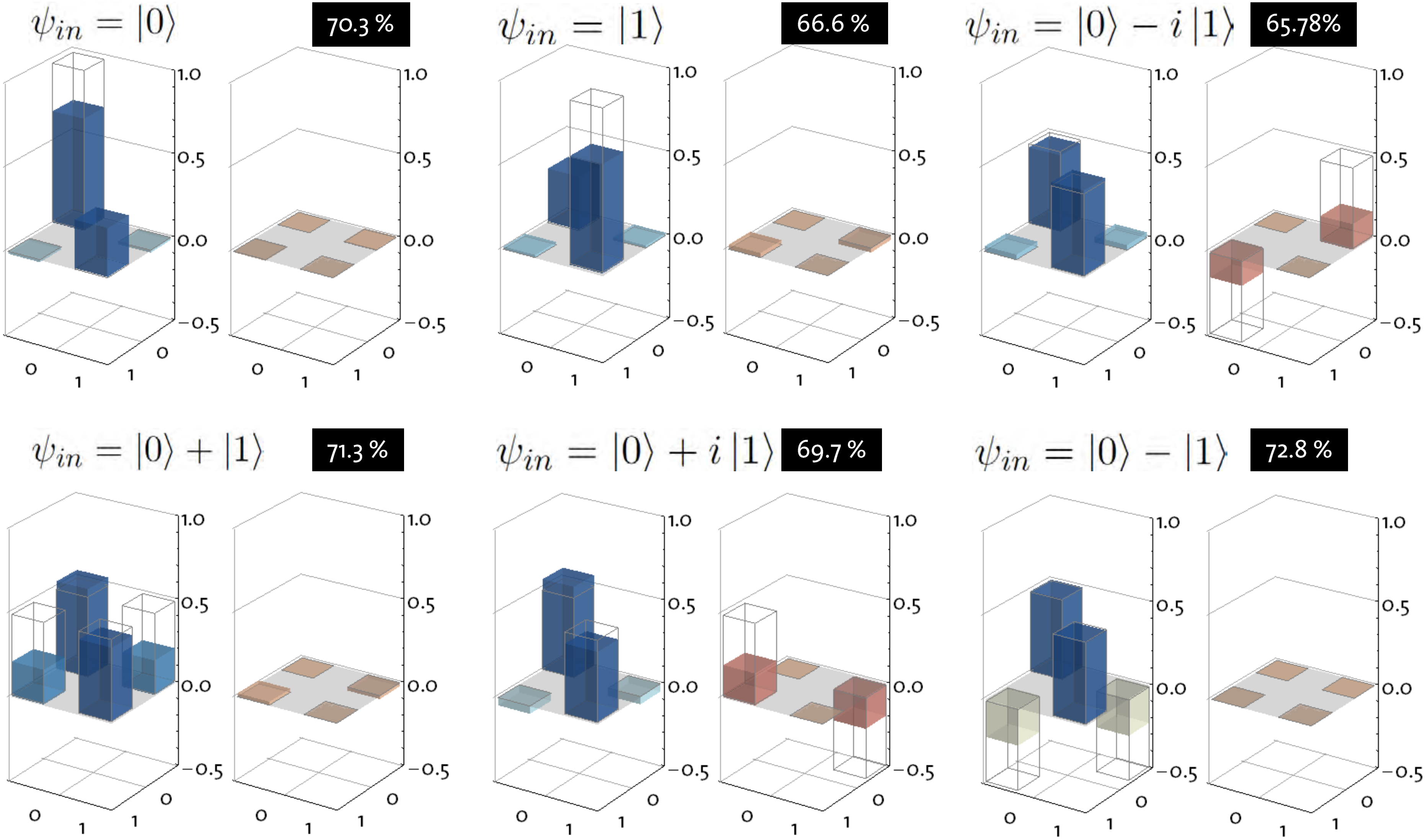
- Bell measurement (4 states)
- feed-forward
- completed in ~ 500 ns with FPGA based electronics

Steffen *et al.*, *Nature* **500**, 319 (2013)

Pulse Scheme



Tomography of Teleported States with Feed-Forward



$\psi_{in} = |0\rangle$ 70.3 %

$\psi_{in} = |1\rangle$ 66.6 %

$\psi_{in} = |0\rangle - i\,|1\rangle$ 65.78%

$\psi_{in} = |0\rangle + |1\rangle$ 71.3 %

$\psi_{in} = |0\rangle + i\,|1\rangle$ 69.7 %

$\psi_{in} = |0\rangle - |1\rangle$ 72.8 %

Average state fidelity of **69.5±0.1 %**

classical limit: 66.7 %

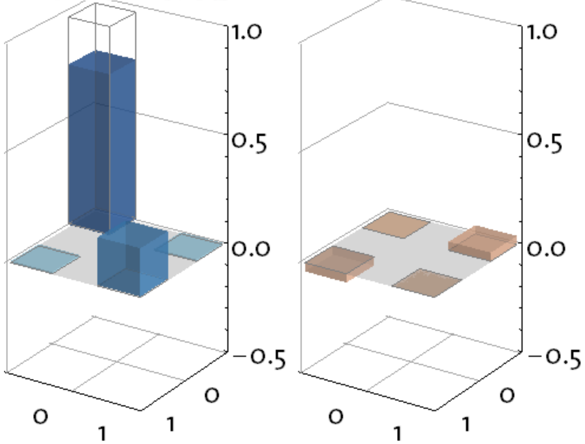
Steffen *et al.*, *Nature* **500**, 319 (2013)

Tomography of Teleported States with Feed-Forward

averaged readout of qubit 3

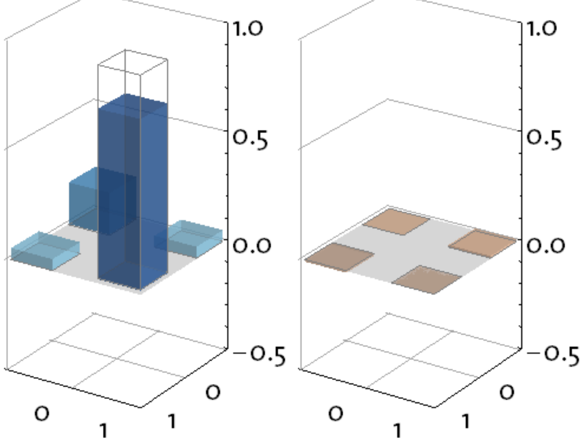
$\psi_{in} = |0\rangle$

77.5 %



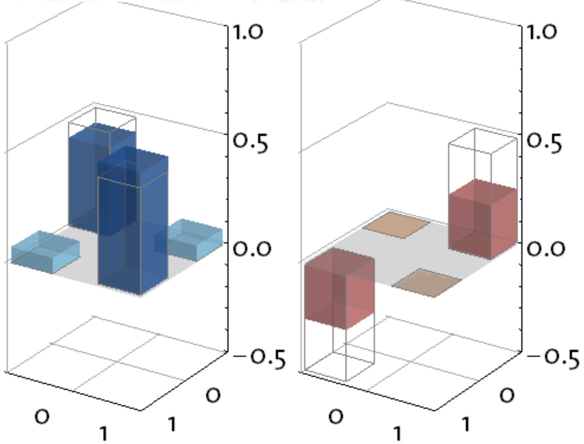
$\psi_{in} = |1\rangle$

79.9 %



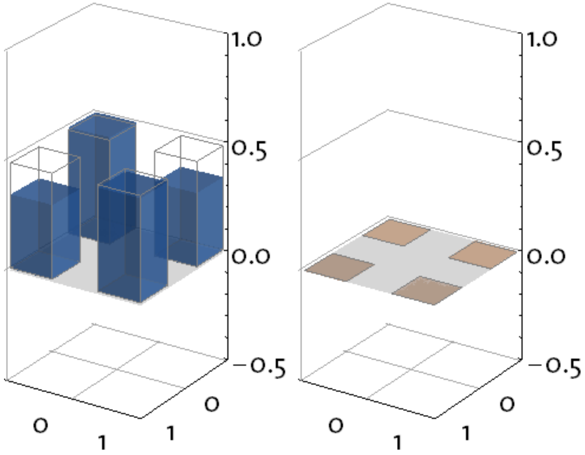
$\psi_{in} = |0\rangle - i\,|1\rangle$

76.2 %



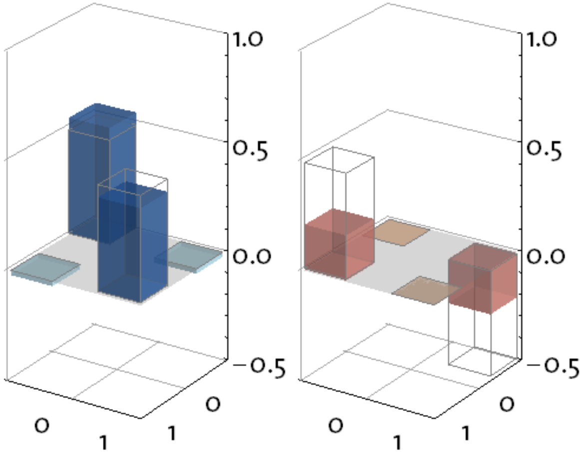
$\psi_{in} = |0\rangle + |1\rangle$

85.3 %



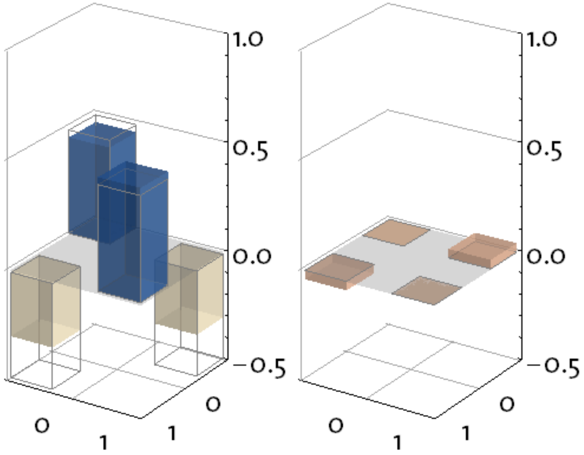
$\psi_{in} = |0\rangle + i\,|1\rangle$

71.2 %



$\psi_{in} = |0\rangle - |1\rangle$

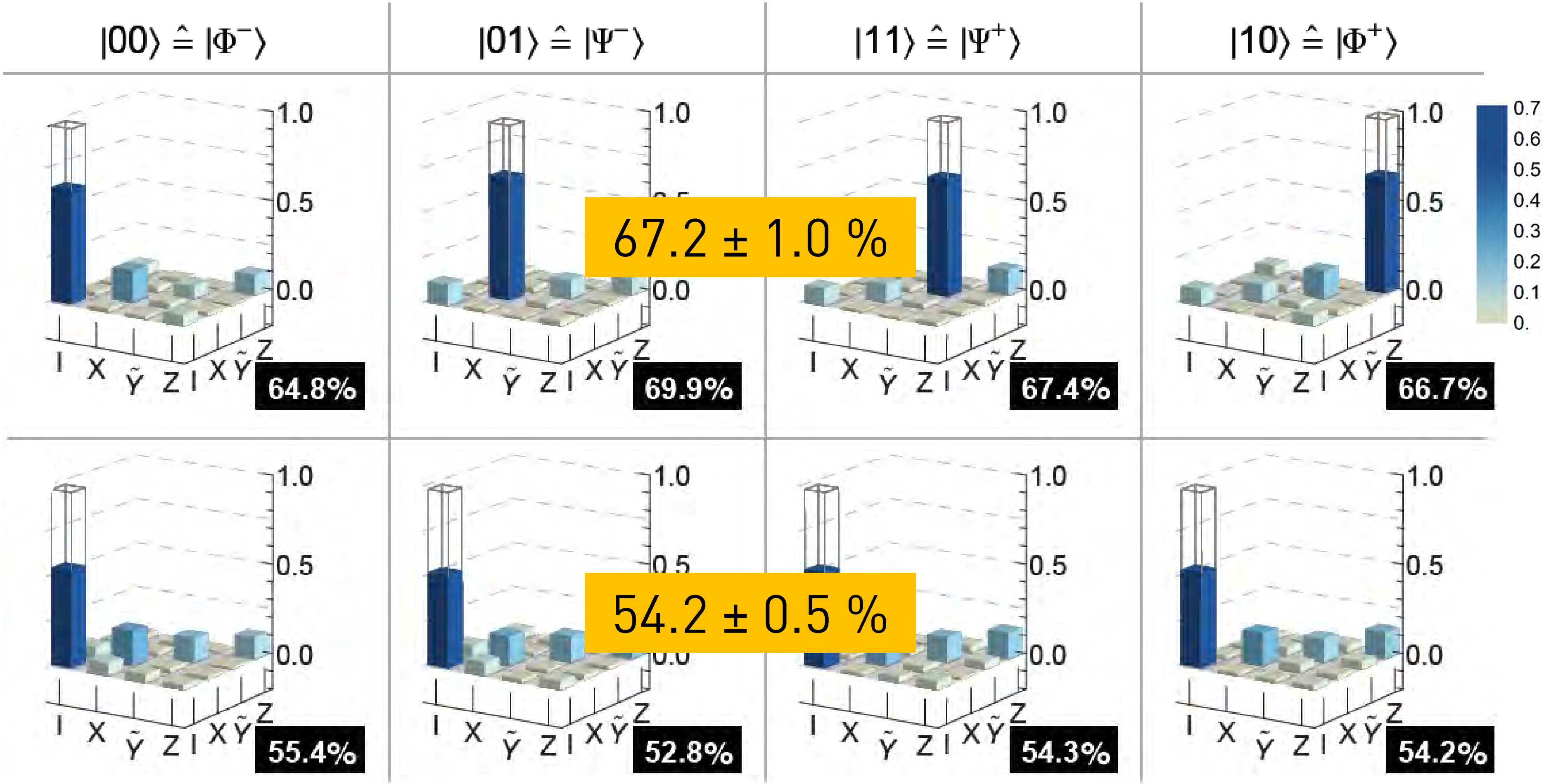
80.7 %



Average state fidelity of **78.5± 0.9%**

Steffen *et al.*, *Nature* **500**, 319 (2013)

Process Tomography – w/o and with Feed-Forward



67.2 ± 1.0 %

54.2 ± 0.5 %

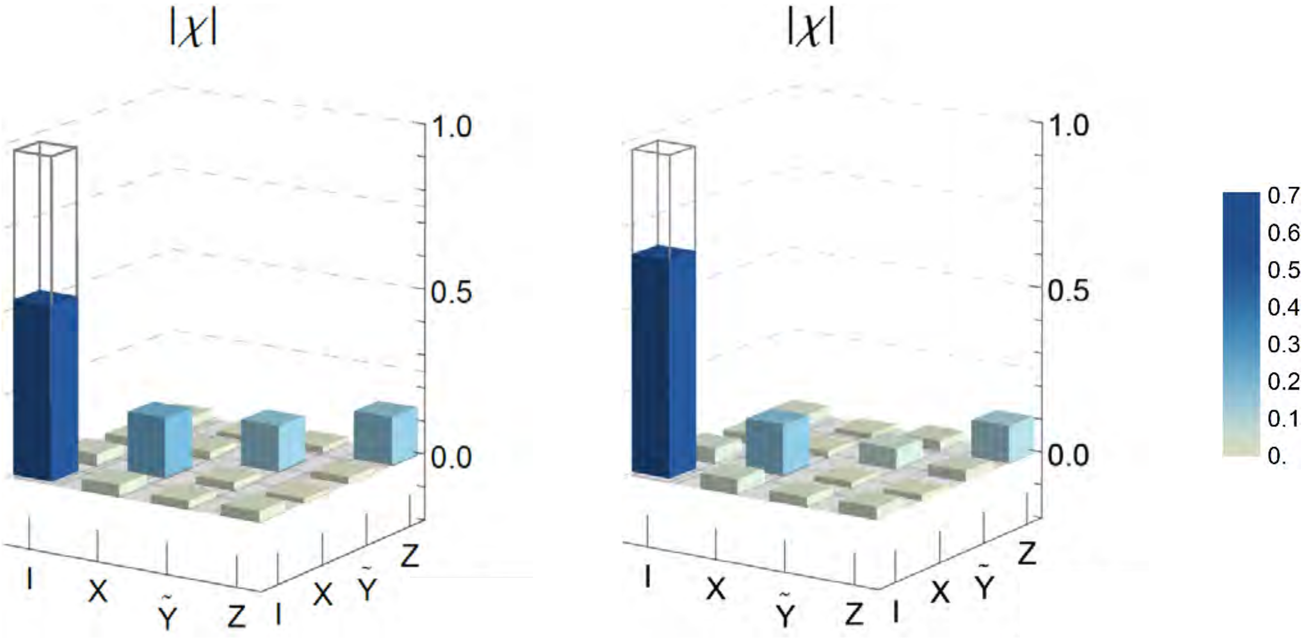
$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle =$ $|\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle =$ $|\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle =$ $|\psi_{\text{in}}\rangle$

$X = \hat{\sigma}_x, \tilde{Y} = i\hat{\sigma}_y, Z =$

Classical limit: 50 %

$\mathcal{F}_p = (\mathcal{F}_s(d+1) - 1)/d$

Teleportation Process with Feed-Forward



Average process fidelity with single shot readout: 54.2 ± 0.1 %

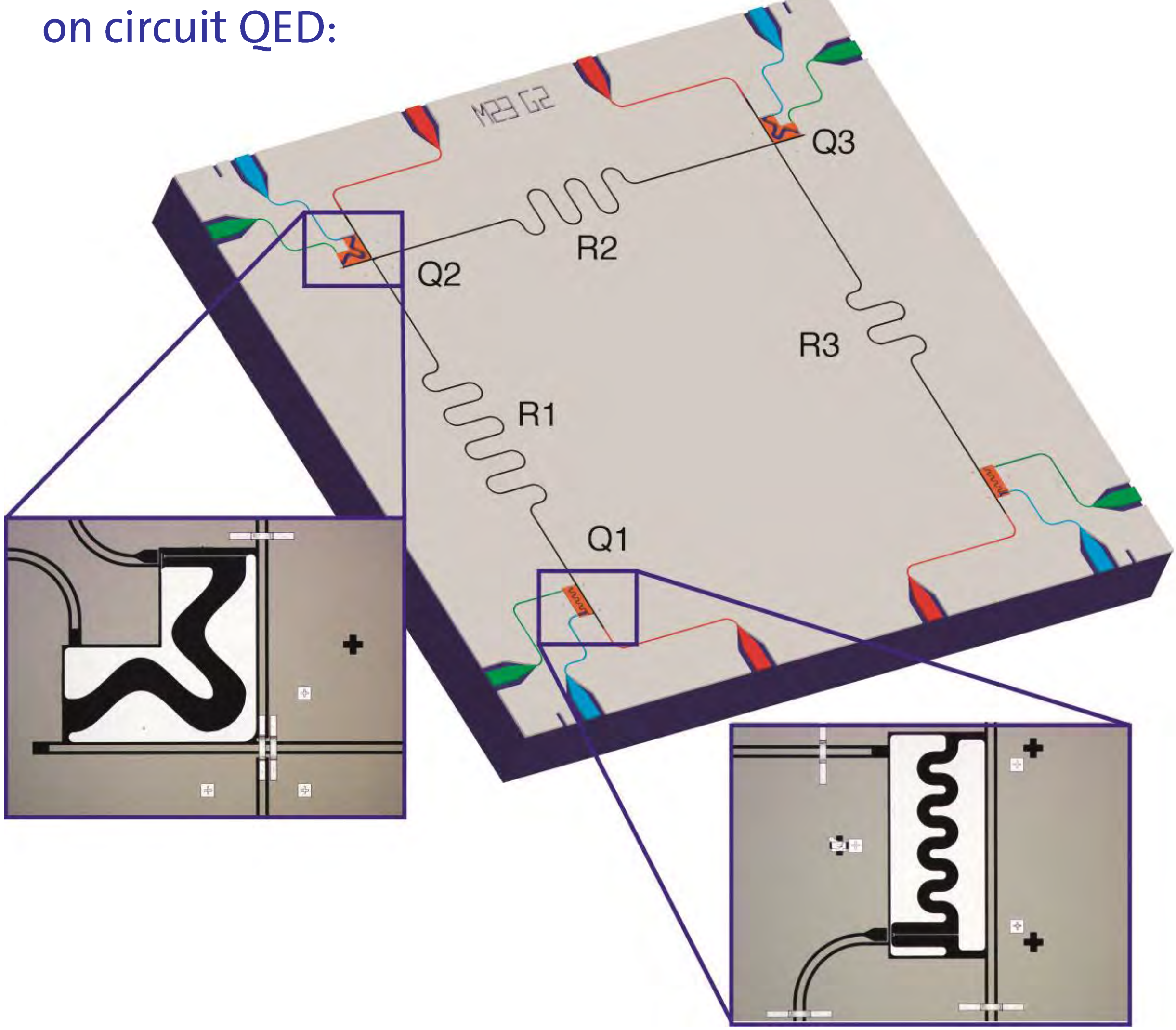
Average process fidelity with averaged readout: 67.7 ± 1.1 %

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Steffen *et al.*, *Nature* **500**, 319 (2013)

Teleportation

12-port quantum device based on circuit QED:



Experimental highlights:

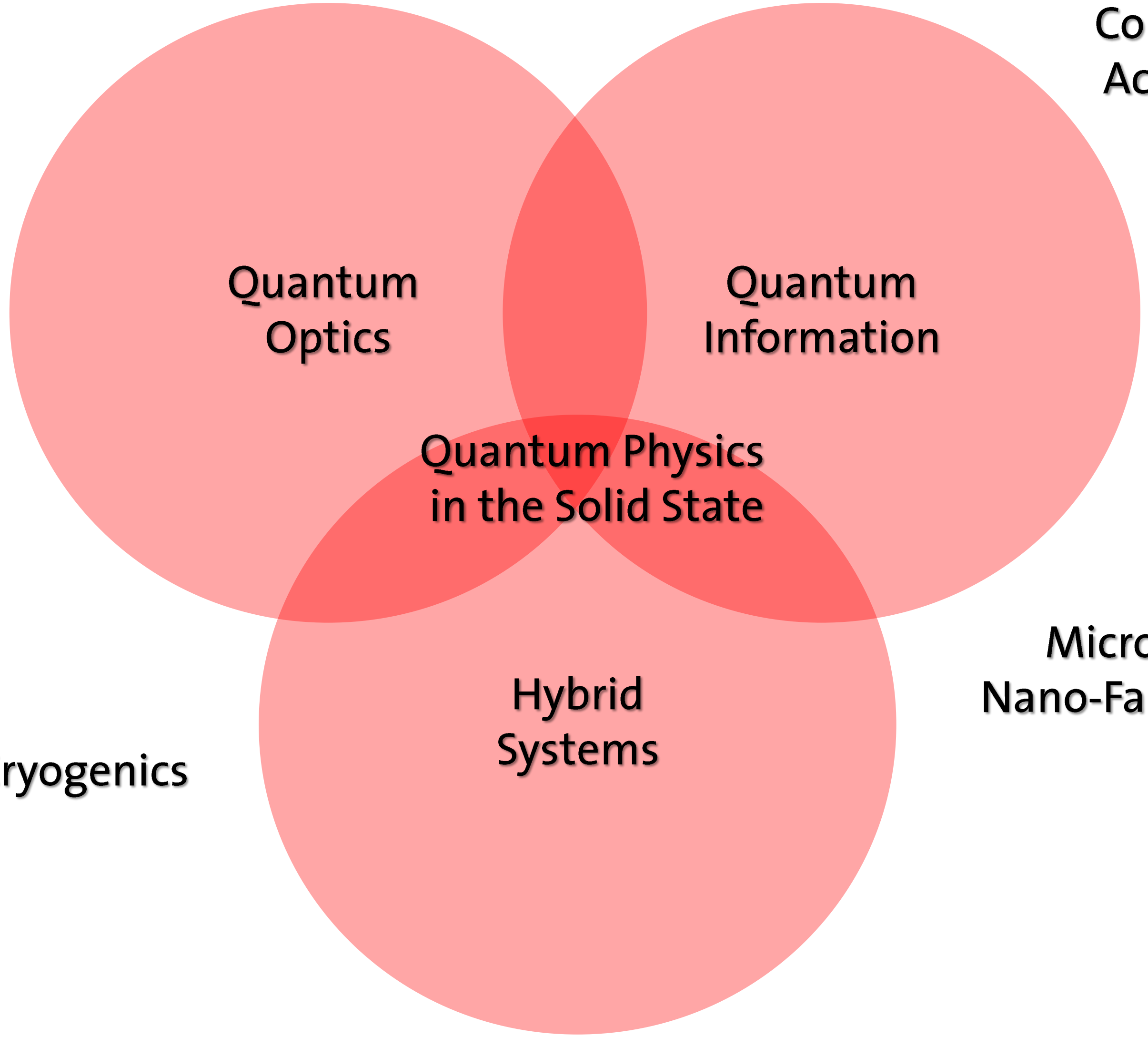
- teleportation in a (macroscopic) solid state system
- post-selection on either of 4 Bell states individually
- Simultaneous deterministic Bell measurement of all states
- implementation of feed-forward
- fidelities > classical threshold
- O(Unity) success probability
- teleportation rate > 10 kHz
- distance ~ 6 mm

Next steps:

- use teleportation in alg.
- improve fidelities
- increase distances for quantum communication

Steffen *et al.*, *Nature* 500, 319 (2013)

Circuit QED Research



Analog and
Digital Electronics

Control and
Acquisition

Microwaves

Quantum
Optics

Quantum
Information

Quantum Physics
in the Solid State

Hybrid
Systems

Micro- and
Nano-Fabrication

Cryogenics

Measurement Technology

The ETH Zurich Quantum Device Lab

incl. undergrad and summer students

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

QSIT Quantum Science and Technology
National Centre of Competence in Research

(·) QED
CIRCUIT AND CAVITY
QUANTUM ELECTRODYNAMICS

FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

scaleQIT

erc

Want to work with us?
Postdoc and PhD positions are available!

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Selected Circuit QED Publications

Circuit QED Proposal:

- Blais et al., *PRA* **69**, 062320 (2004)

Strong Coupling & Vacuum Rabi Mode Splitting:

- Wallraff et al., *Nature* **431**, 162 (2004)
- Fink et al., *Nature* **454**, 315 (2008)
- Fink et al., *PRL* **105**, 163601 (2010)

Tavis-Cummings Multi-Atom QED:

- Fink et al., *PRL* **103**, 083601 (2009)

AC-Stark & Lamb Shift, Autler-Townes and Mollow Transitions

- Schuster et al., *PRL* **94,** 123062 (2005)
- Gambetta et al., *PRA* **74**, 042318 (2006)
- Schuster et al., *Nature* **445**, 515 (2007)
- Fragner et al., *Science* **322**, 1357 (2008)
- Baur et al., *PRL* **102**, 243602 (2009)

Device Fabrication:

- Frunzio et al., *IEEE Trans. Appl. Sup.*. **15**, 860 (2005)
- Goeppl et al., *J. Appl. Phys.* **104**, 113904 (2008)

Geometric Phases:

- Leek et al., *Science* **318**, 1889 (2007)
- Pechal et al., *PRL* **108**, 170401 (2012)
- Abdumalikov et al., *Nature* **496**, 482 (2013)

One-, Two-, Three-Qubit Gates, Algorithms and Teleportation:

- Wallraff et al., *PRL* **95**, 060501 (2005)
- Blais et al., *PRA* **75**, 032329 (2007)
- Wallraff et al., *PRL* **99**, 050501 (2007)
- Majer et al., *Nature* **449**, 443 (2007)
- Leek et al., *PRB* **79**, 180511(R) (2009)
- Filipp et al., *PRL* **102**, 200402 (2009)
- Leek et al., *PRL* **104**, 100504 (2010)
- Bianchetti et al., *PRL* **105**, 223601 (2010)
- Fedorov et al., *Nature* **481**, 170 (2012)
- Baur et al., *PRL* **108**, 040502 (2012)
- Steffen et al., *PRL* **108**, 260506 (2012)
- Steffen et al., *Nature* **500**, 319 (2013)

Review (gr.):

- Wallraff, *Physik Journal* **7 (12)**, 39 (Dez. 2008)

Additional Information: www.qudev.ethz.ch

Selected Circuit QED Publications (cont'd)

Itinerant Photons, Tomography, Photon Blockade, Correlation Functions, Qubit-Photon Entanglement, Hong-Ou-Mandel Effect:

- da Silva et al., *PRA* **82**, 043804 (2010)
- Bozyigit et al., *Nat. Phys.* **7**, 154(2011)
- Eichler et al., *PRL* **106**, 220503 (2011)
- Lang et al., *PRL* **106**, 243601 (2011)
- Eichler et al., *PRL* **107**, 113601 (2011)
- Eichler et al., *PRA* **86**, 032106 (2012)
- Eichler et al., *PRL* **109**, 240501 (2012)
- Lang et al., *Nat. Phys.* **9**, 345 (2013)

Interaction in 1D free space

- van Loo et al., *Science* **342**, 1494 (2013)

Hybrid Systems: Quantum Dots

- Frey et al., *PRL* **108**, 046807 (2012)
- Frey et al., *PRB* **86**, 115303 (2012)

Hybrid Systems: Rydberg Atoms

- Hogan et al., *PRL* **108**, 063004 (2012)

Additional Information: www.qudev.ethz.ch