Exploring Quantum Physics with Superconducting Qubits

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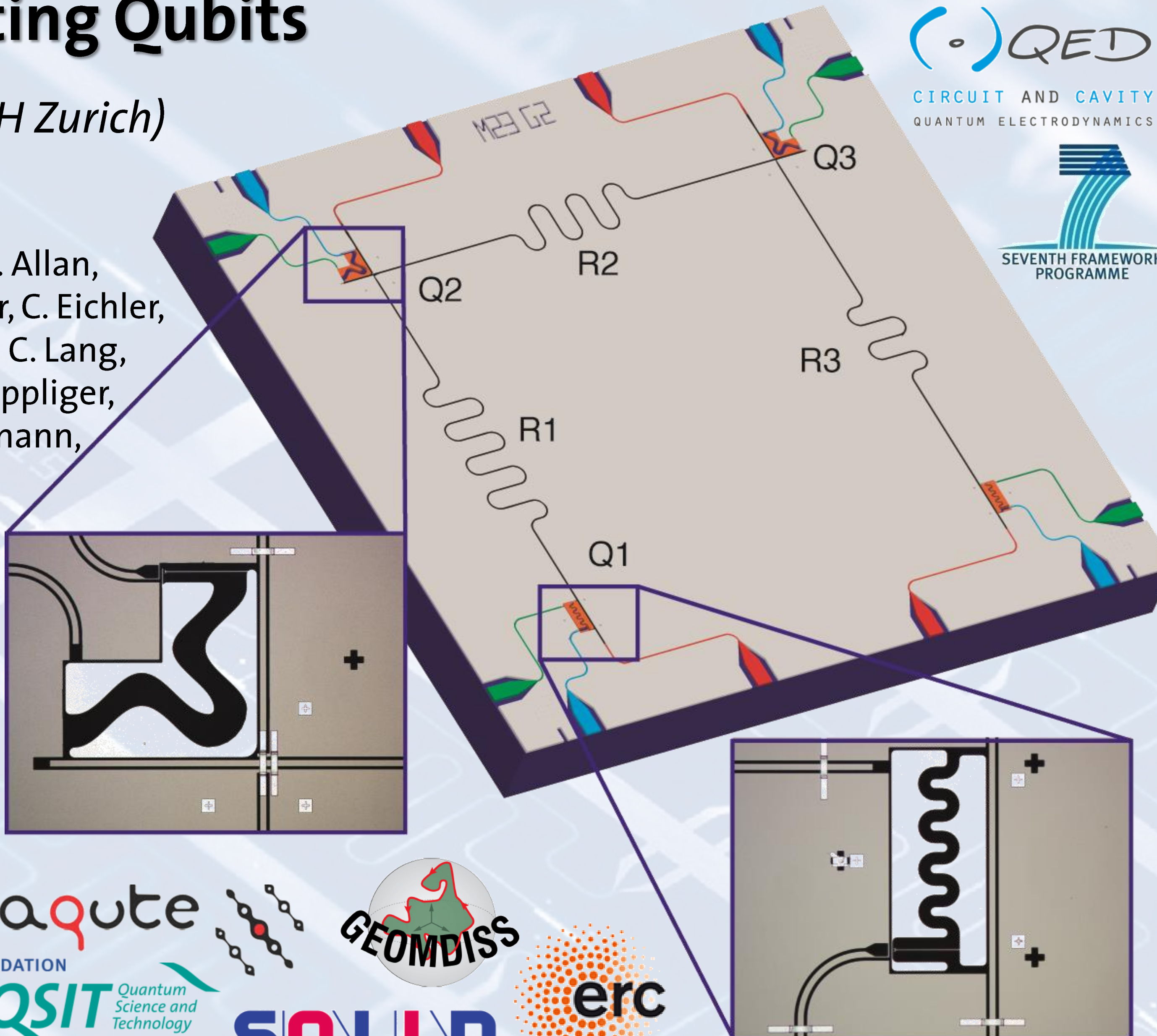
www.qudev.ethz.ch

(•)QED
CIRCUIT AND CAVITY
QUANTUM ELECTRODYNAMICS

SEVENTH FRAMEWORK PROGRAMME



Team: A. Abdumalikov, M. Allan, J. Basset, M. Baur, S. Berger, C. Eichler, A. Fedorov, S. Filipp, T. Frey, C. Lang, P. Kurpiers, J. Mlynek, M. Oppliger, M. Pechal, G. Puebla-Hellmann, Y. Salathe, M. Stammeier, L. Steffen, T. Thiele, A. van Loo *(ETH Zurich)*

Collaborations with:
A. Blais *(Sherbrooke, Canada)*, M. da Silva *(Raytheon, USA)*,
K. Ensslin, T. Ihn, F. Merkt, V. Wood *(ETH Zurich)*

FNSNF
Swiss National Science Foundation

aqute

GEOMDISS

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

QSIT Quantum Science and Technology
National Centre of Competence in Research

SOLID

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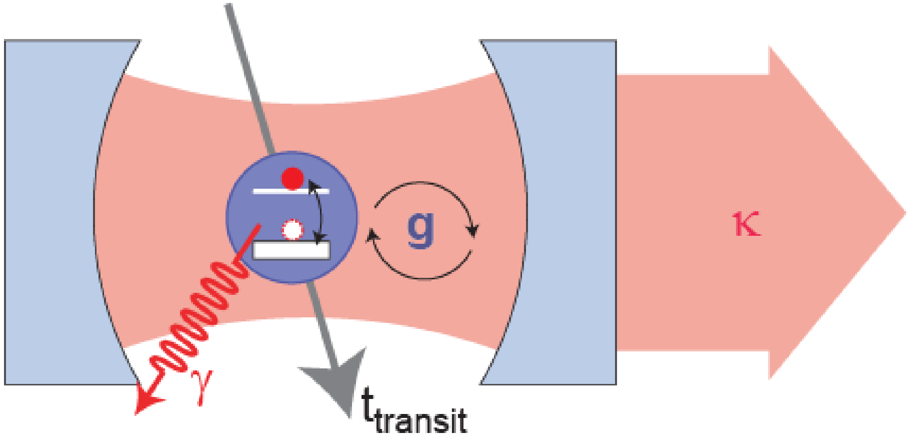
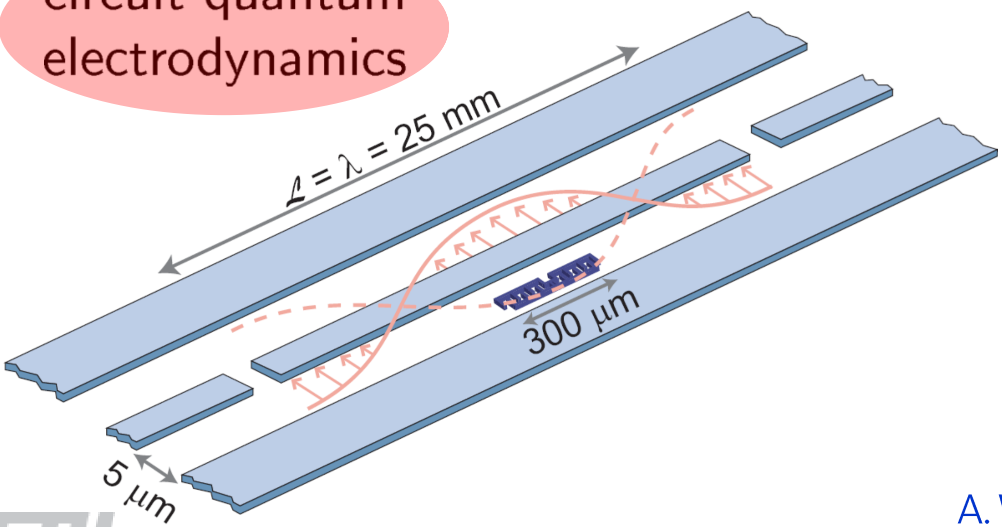
Cavity QED with Superconducting Circuits



coherent interaction of photons with quantum two-level systems ...

J. M. Raimond *et al.*, *Rev. Mod. Phys.* **73**, 565 (2001)
S. Haroche & J. Raimond, *OUP Oxford* (2006)
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

circuit quantum electrodynamics



Properties:

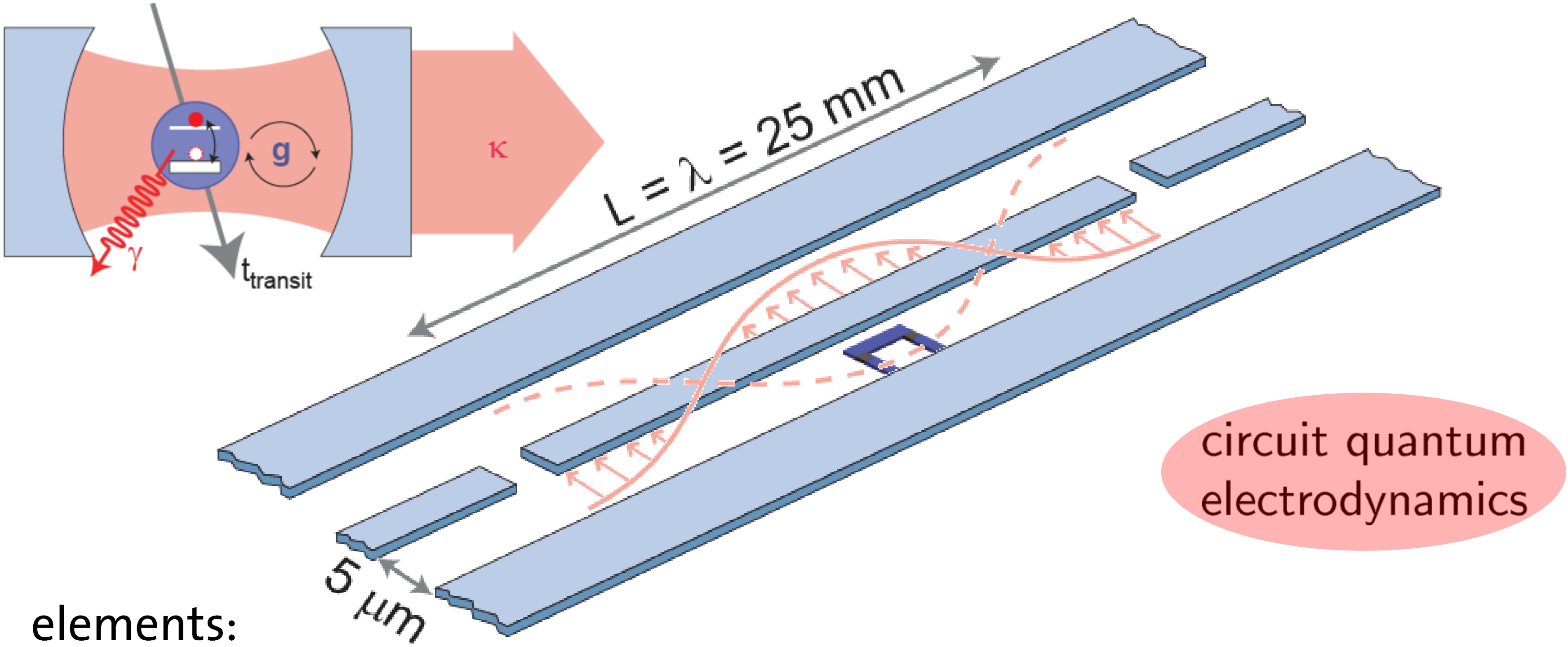
- strong coupling in solid state sys.
- ‘easy’ to fabricate and integrate

Research directions:

- quantum optics
- quantum information
- hybrid quantum systems

A. Blais, *et al.*, *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)
R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Cavity QED with Superconducting Circuits

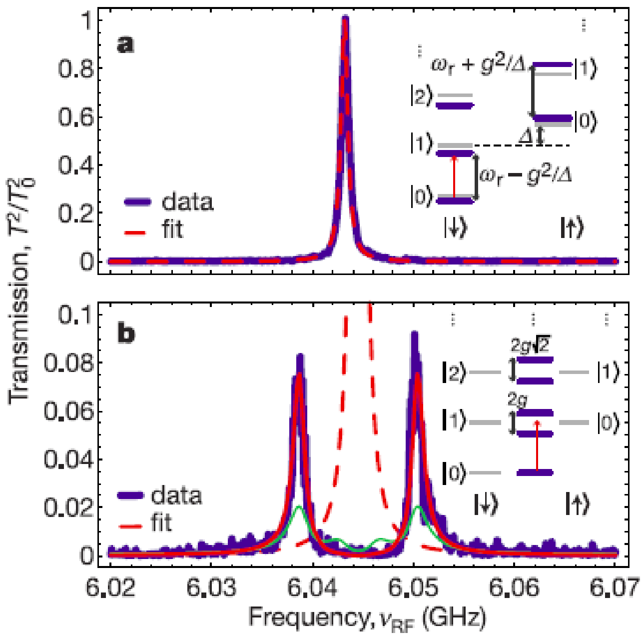


elements:

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_o and long photon life time $1/\kappa$
- the atom: a superconducting qubit with large dipole moment d and long coherence time $1/\gamma$ and fixed position ...
- ... or any microscopic/macroscopic quantum element or ensemble thereof with an appreciable dipole moment

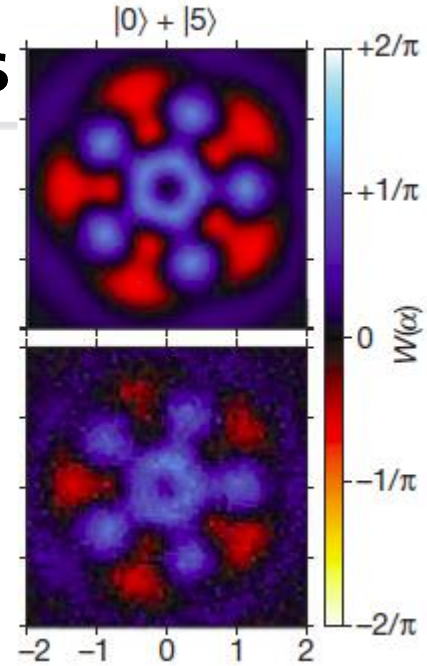
A. Blais, *et al.*, *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)
R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Quantum Optics with Supercond. Circuits



Strong Coherent Coupling

Chiorescu *et al.*, *Nature* **431**, 159 (2004)
Wallraff *et al.*, *Nature* **431**, 162 (2004)
Schuster *et al.*, *Nature* **445**, 515 (2007)

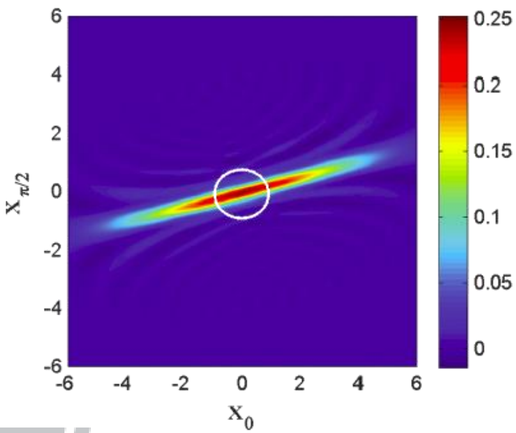


Root n Nonlinearities

Fink *et al.*, *Nature* **454**, 315 (2008)
Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)
Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

Microwave Fock and Cat States

Hofheinz *et al.*, *Nature* **454**, 310 (2008)
Hofheinz *et al.*, *Nature* **459**, 546 (2009)
Kirchmair *et al.*, *Nature* **495**, 205 (2013)

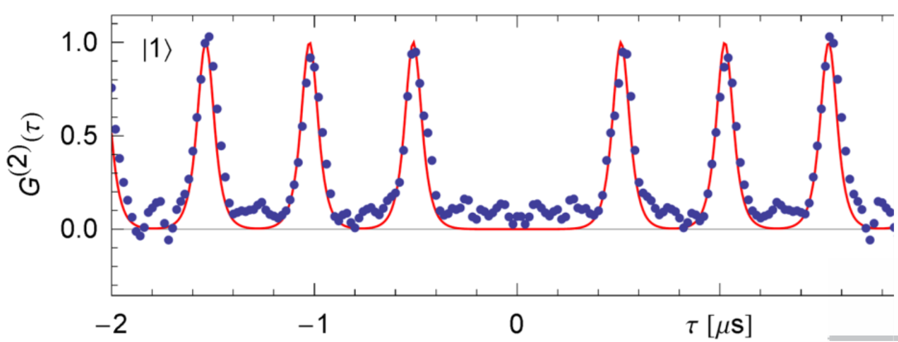


Parametric Amplification & Squeezing

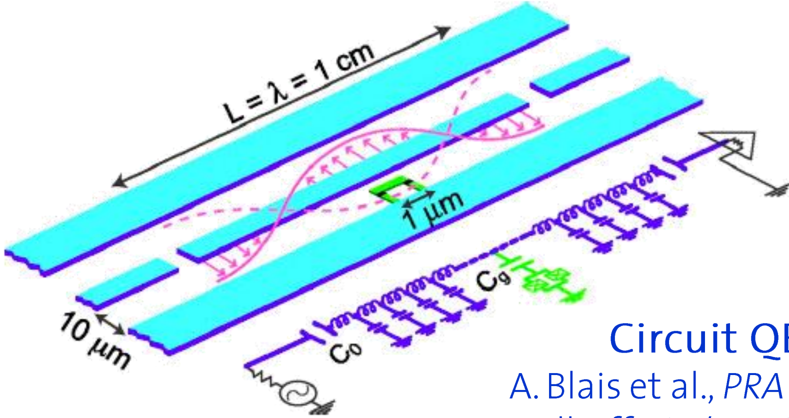
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 928 (2008)

Single Photons & Correlations

Houck *et al.*, *Nature* **449**, 328 (2007)
Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

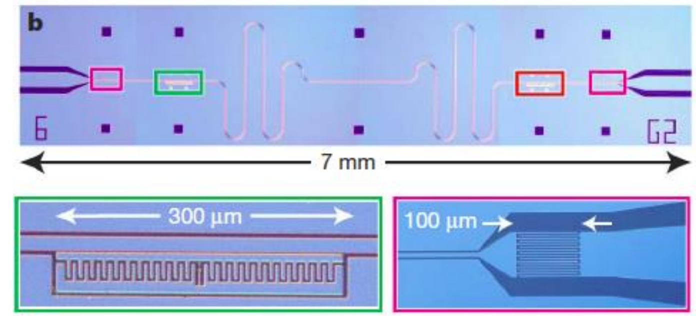


Quantum Computing with Superconducting Circuits



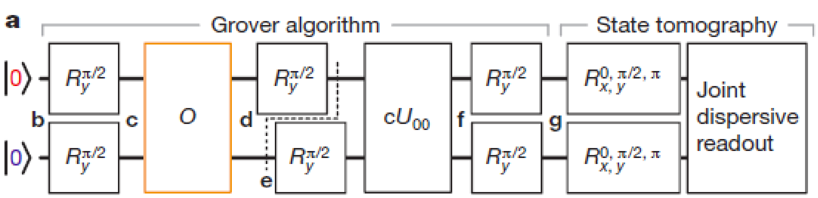
Circuit QED Architecture

A. Blais et al., *PRA* **69**, 062320 (2004)
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)



Resonator as a Coupling Bus

M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)
H. Majer *et al.*, *Nature* **449**, 443 (2007)

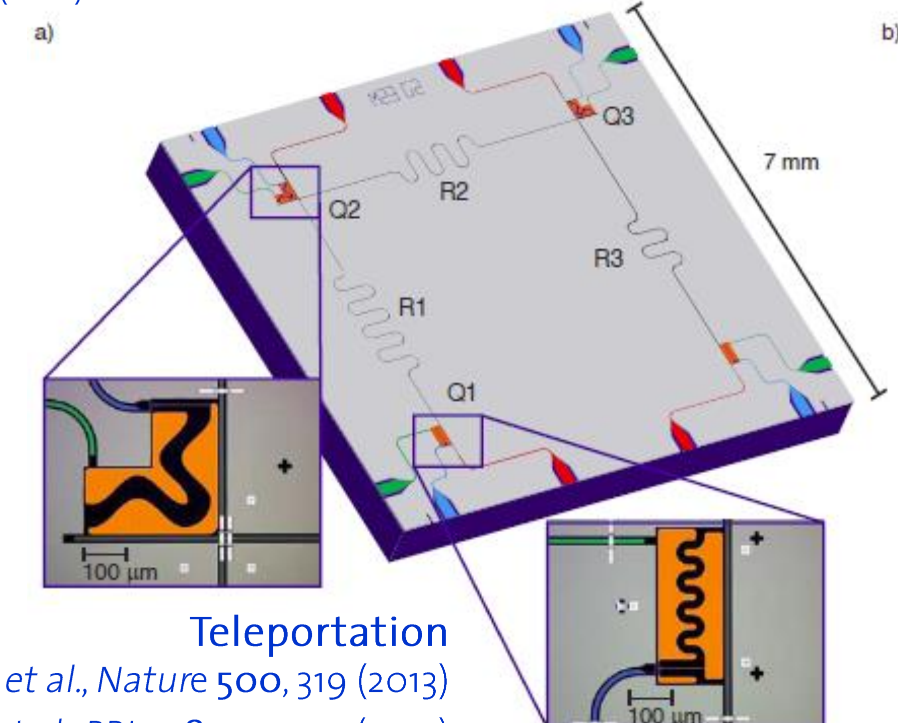


Deutsch, Grover Algorithms

L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)
L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)

Toffoli Gates & Error Correction

A. Fedorov *et al.*, *Nature* **481**, 170 (2012)
M. Reed *et al.*, *Nature* **481**, 382 (2012)



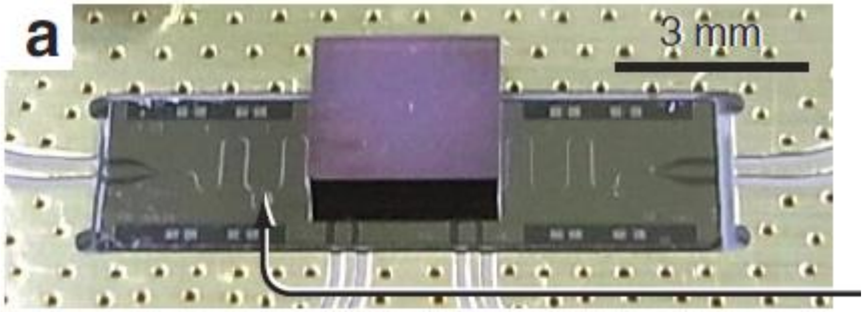
Teleportation

L. Steffen *et al.*, *Nature* **500**, 319 (2013)
M.. Baur *et al.*, *PRL* **108**, *040502* (2012)

Hybrid Systems with Superconducting Circuits

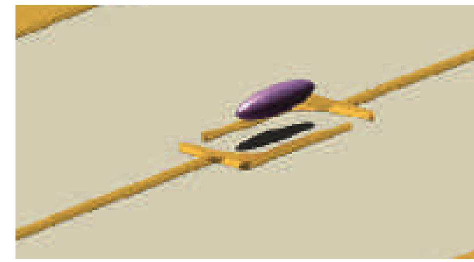
Spin Ensembles: e.g. NV centers

D. Schuster *et al.*, *PRL* **105**, 140501 (2010)
Y. Kubo *et al.*, *PRL* **105**, 140502 (2010)



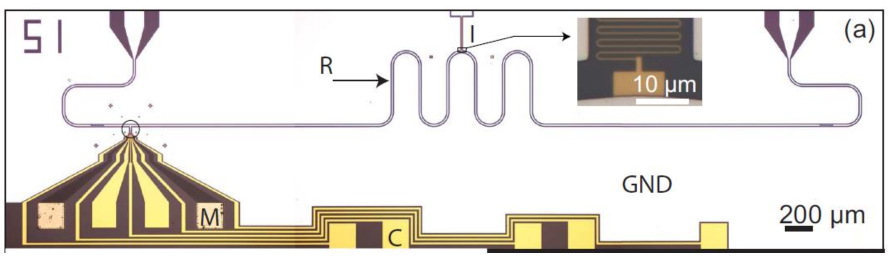
Polar Molecules, Rydberg, BEC

P. Rabl *et al*, *PRL* **97**, 033003 (2006)
A. Andre *et al*, *Nat. Phys.* **2**, 636 (2006)
D. Petrosyan *et al*, *PRL* **100**, 170501 (2008)
J. Verdu *et al*, *PRL* **103**, 043603 (2009)



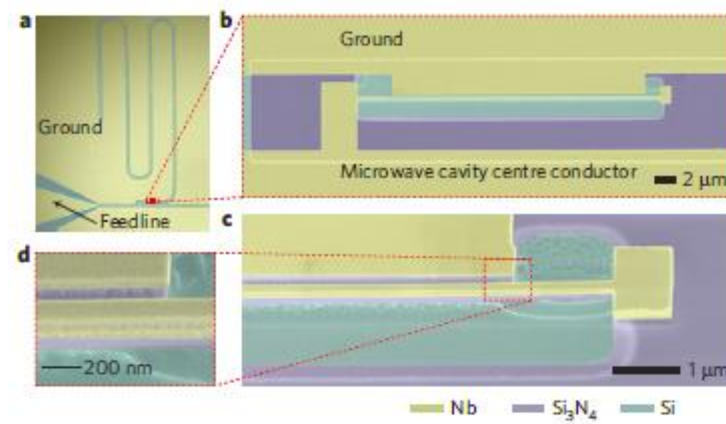
CNT, Gate Defined 2DEG, or nanowire Quantum Dots

M. Delbecq *et al.*, *PRL* **107**, 256804 (2011)
T. Frey *et al.*, *PRL* **108**, 046807 (2012)
K. Petersson *et al.*, *arXiv*:1205.6767 (2012)



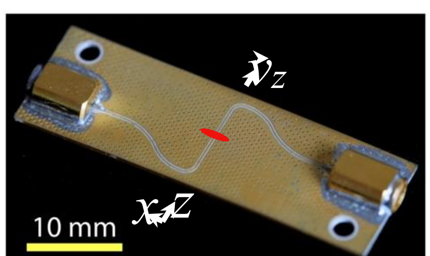
Nano-Mechanics

J. Teufel *et al.*, *Nature* **475**, 359 (2011)
X. Zhou *et al.*, *Nat. Phys.* **9**, 179(2013)



Rydberg Atoms

S. Hogan *et al.*, *PRL* **108**, 063004 (2012)



... and many more

Lecture Topics

I. Quantum Mechanics of Superconducting Electronic Circuits
   - Electronic Harmonic Oscillators
   - Josephson Junctions and Superconducting Qubits
   - Circuit Quantum Electrodynamics (QED)

II. Exploring Matter/Light Interactions in Circuit QED

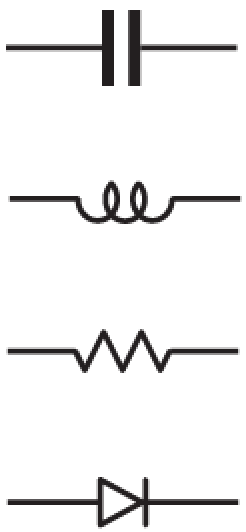
III. Characterizing Propagating Microwave Photons

IV. Teleportation in Superconducting Circuits

I. Quantum Mechanics of Superconducting Electronic Circuits

Conventional Electronic Circuits

basic circuit elements:

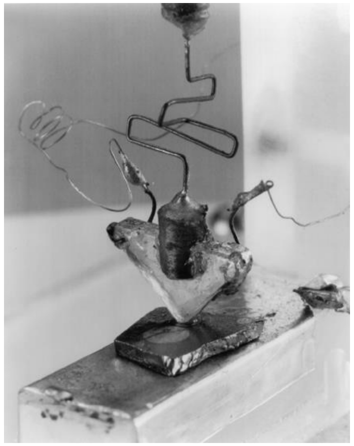


basis of modern information and communication technology

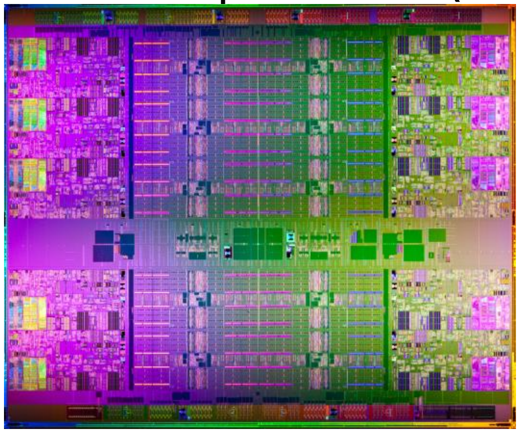
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



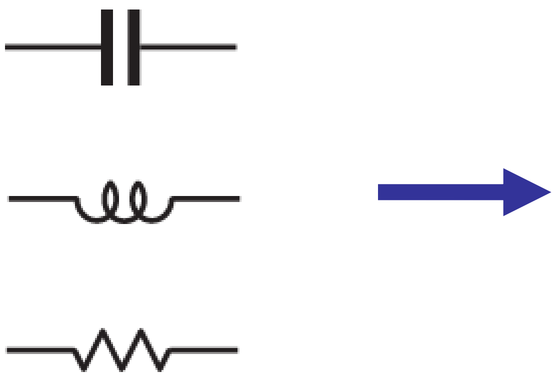
intel xeon processors (2011)



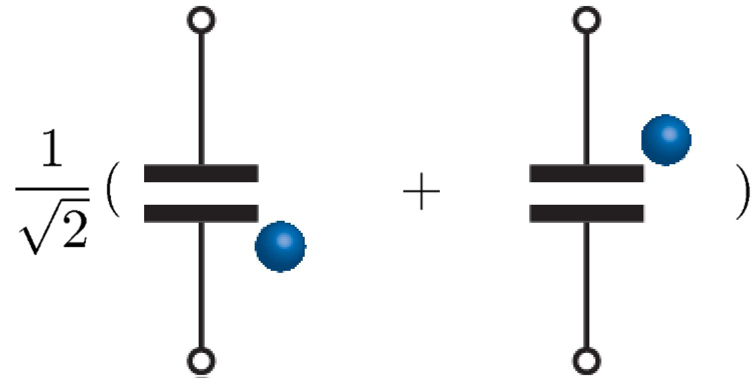
3.000.000.000 transistors
smallest feature size 32 nm
clock speed ~ 3 GHz
power consumption 10 W

Classical and Quantum Electronic Circuit Elements

basic circuit elements:



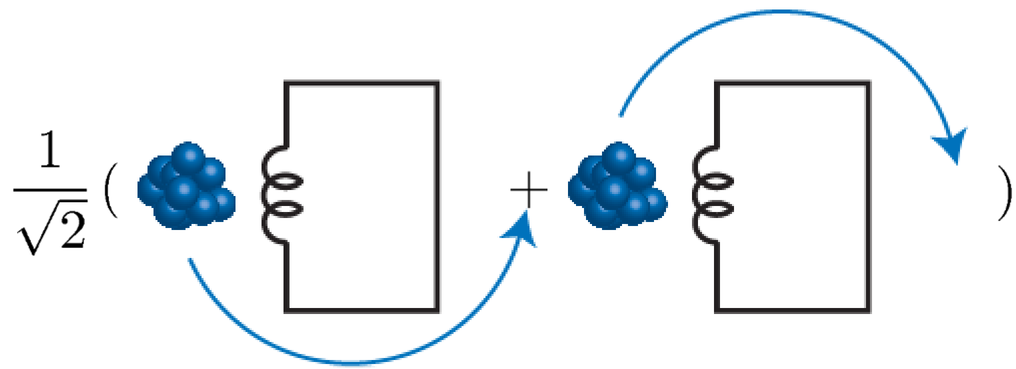
charge on a capacitor:



quantum superposition states of:

- charge q
- flux ϕ

current or magnetic flux in an inductor:

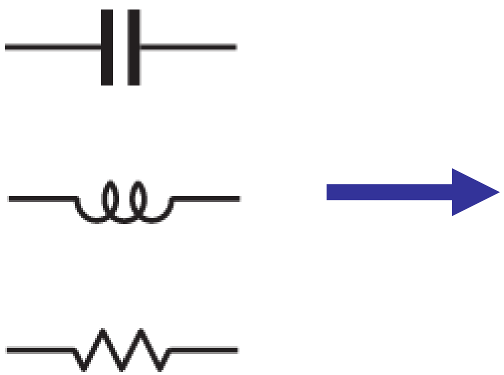


commutation relation (c.f. *x*, *p*):

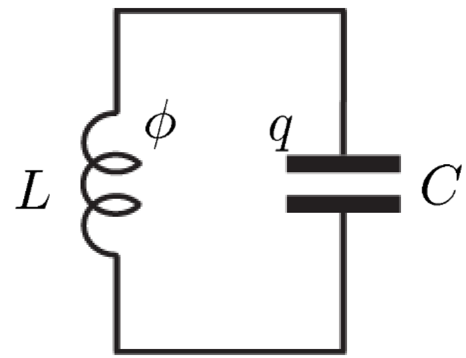
$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$

Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

classical physics:

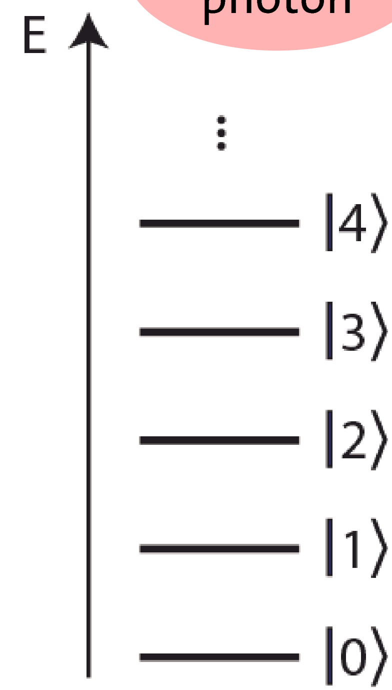
$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger \hat{a} + \frac{1}{2}) \qquad \left[\hat{\phi}, \hat{q}\right] = i\hbar$$

energy:

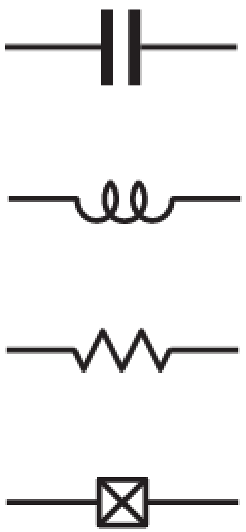
electronic photon



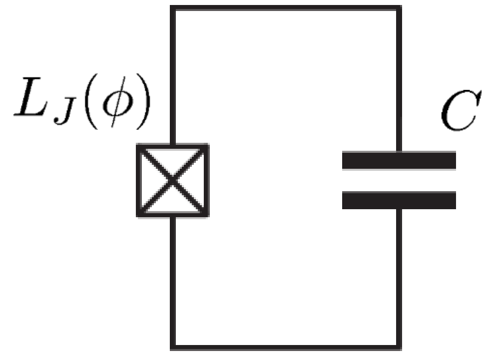
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Constructing Non-Linear Quantum Electronic Circuits

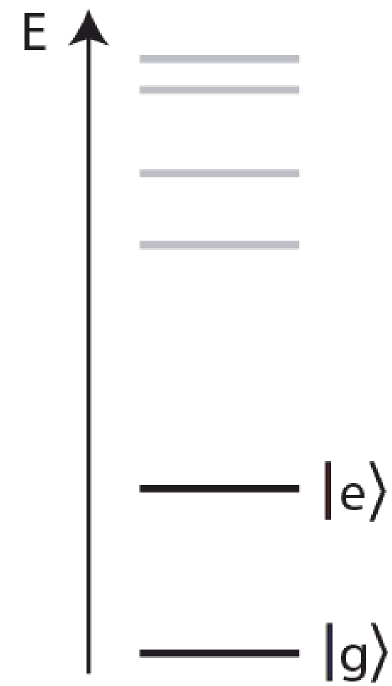
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



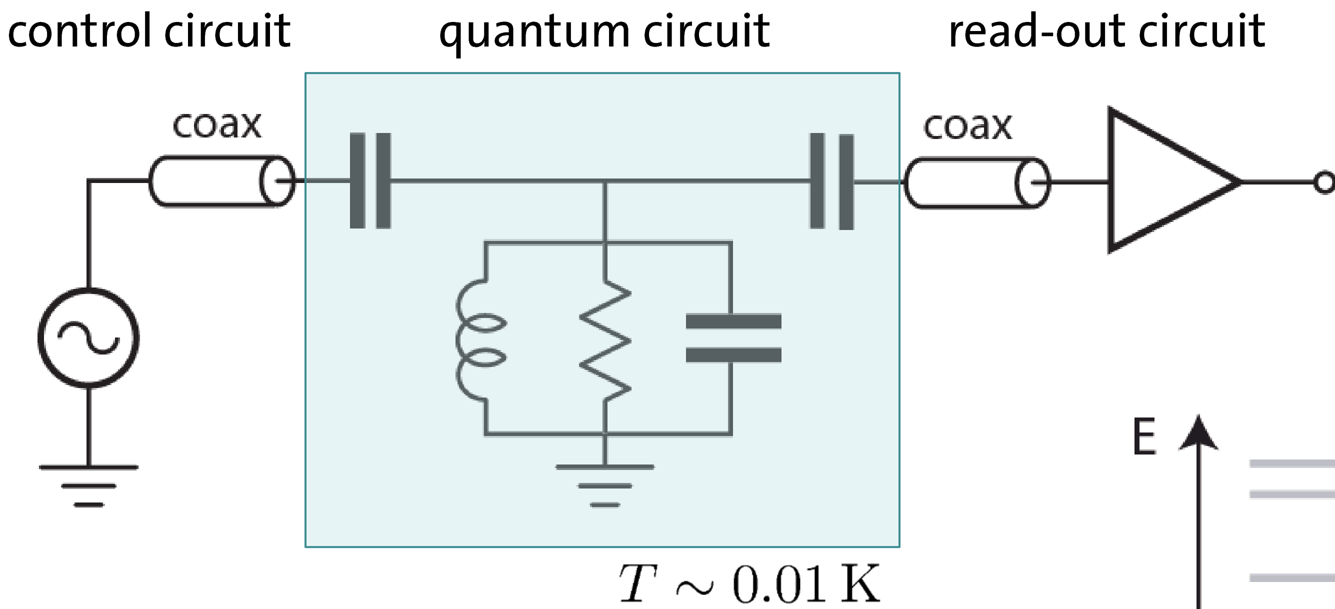
Josephson junction:
a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c}\frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic artificial atom

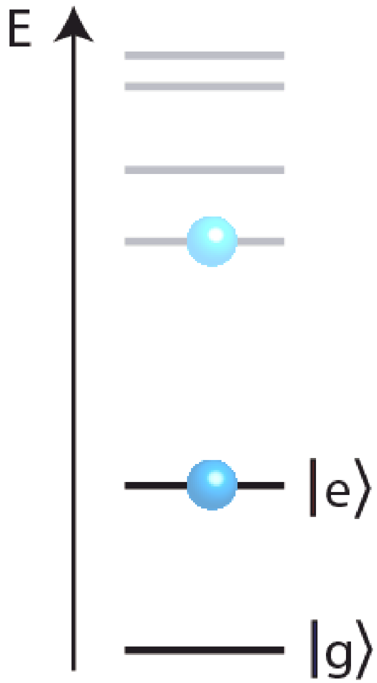
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

How to Operate Circuits in the Quantum Regime?



recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

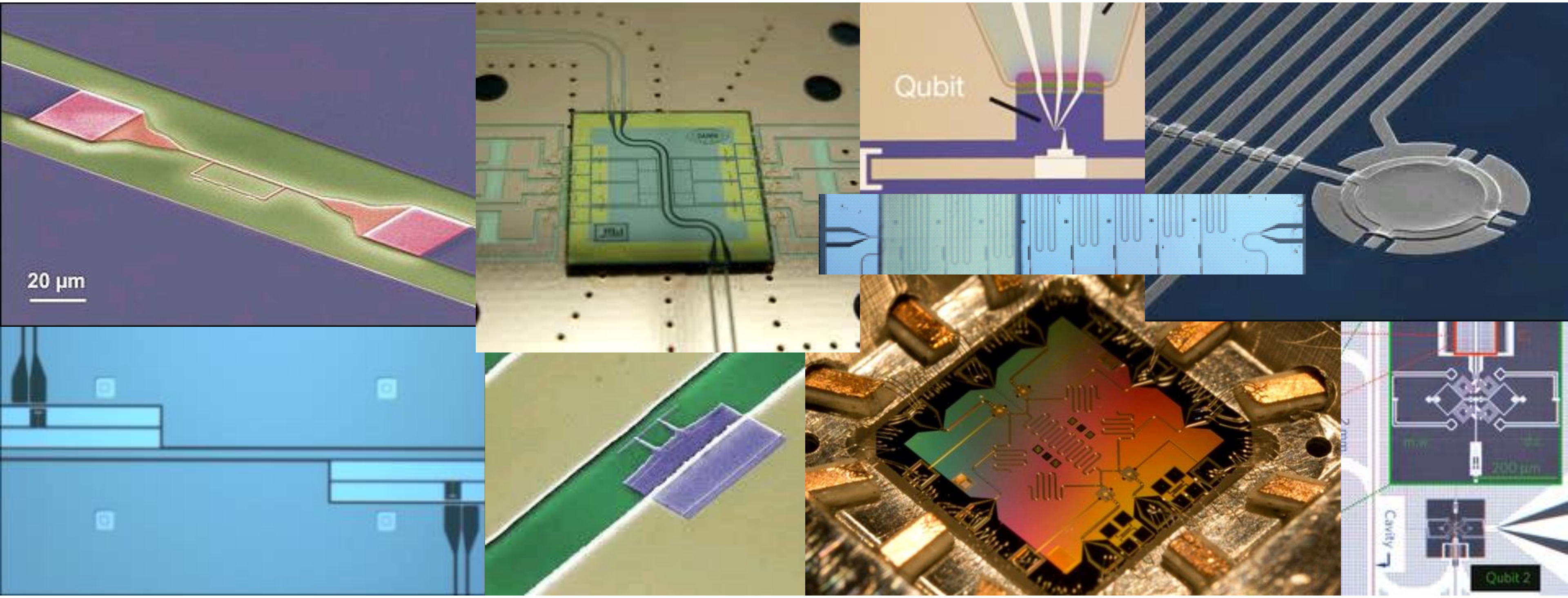


Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Superconducting Quantum Electronic Circuits

single or multiple superconducting qubits coupled to harmonic oscillators

- investigated in a few dozen labs around the world
- for basic science and applications



reviews:

R. J. Schoelkopf, S. M. Girvin, *Nature* **451**, 664 (2008)

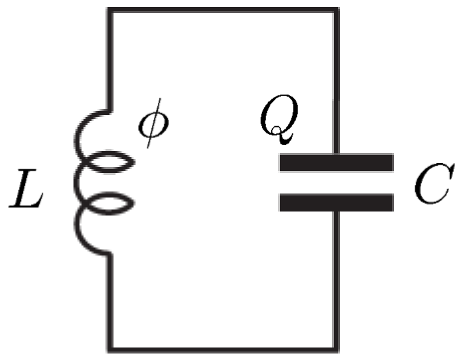
J. Clarke and F. Wilhelm, *Nature* **453**, 1031 (2008)

J. Q. You and F. Nori, *Nature* **474**, 589 (2011)

Electronic Harmonic Oscillators

Quantization of an Electronic Harmonic Oscillator

Harmonic LC oscillator:



$$\begin{aligned} Q &= CV && \text{Charge on capacitor} \\ \phi &= LI && \text{Flux in inductor} \\ V &= -L\dot{I} = -\dot{\phi} && \text{Voltage across inductor} \end{aligned}$$

Classical Hamiltonian:

$$H = \frac{CV^2}{2} + \frac{LI^2}{2} = \frac{Q^2}{2C} + \frac{\phi^2}{2L}$$

Conjugate variables:

$$\begin{aligned} \frac{\partial H}{\partial \phi} &= \frac{\phi}{L} = I = \dot{Q} \\ \frac{\partial H}{\partial Q} &= \frac{Q}{C} = V = -L\dot{I} = -\dot{\phi} \end{aligned}$$

Hamilton operator:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C}$$

Flux and charge operator:

$$\begin{aligned} \hat{\phi} &= \phi \\ \hat{Q} &= -i\hbar \frac{\partial}{\partial \phi} \end{aligned}$$

Commutation relation:

$$\left[\hat{\phi}, \hat{q}\right] = i\hbar$$

Creation and Annihilation Operators for Circuits

Hamilton operator of harmonic oscillator in second quantization:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2)$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$ Creation operator

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$ Annihilation operator

$$\hat{a}^\dagger\hat{a}\,|n\rangle = n\,|n\rangle$$ Number operator

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_C}}(\hat{a}^\dagger + \hat{a})$$ Charge/voltage operator $$\hat{V} = \frac{\hat{Q}}{C}$$

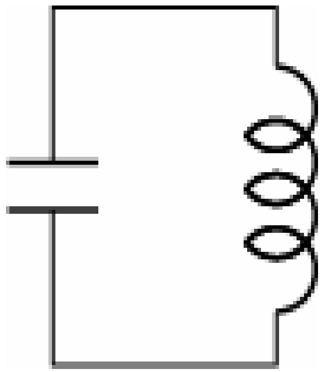
$$\hat{\phi} = i\sqrt{\frac{\hbar Z_C}{2}}(\hat{a}^\dagger - \hat{a})$$ Flux/current operator $$\hat{I} = \frac{\hat{\phi}}{L}$$

With characteristic impedance:

$$Z_C = \sqrt{\frac{L}{C}}$$

Superconducting Harmonic Oscillator

a simple electronic circuit:



- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm

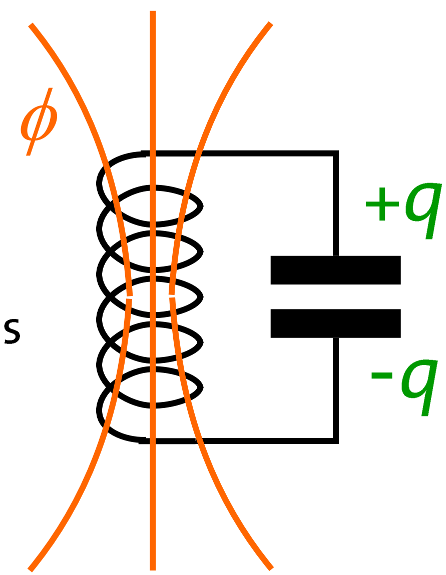
- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF

- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

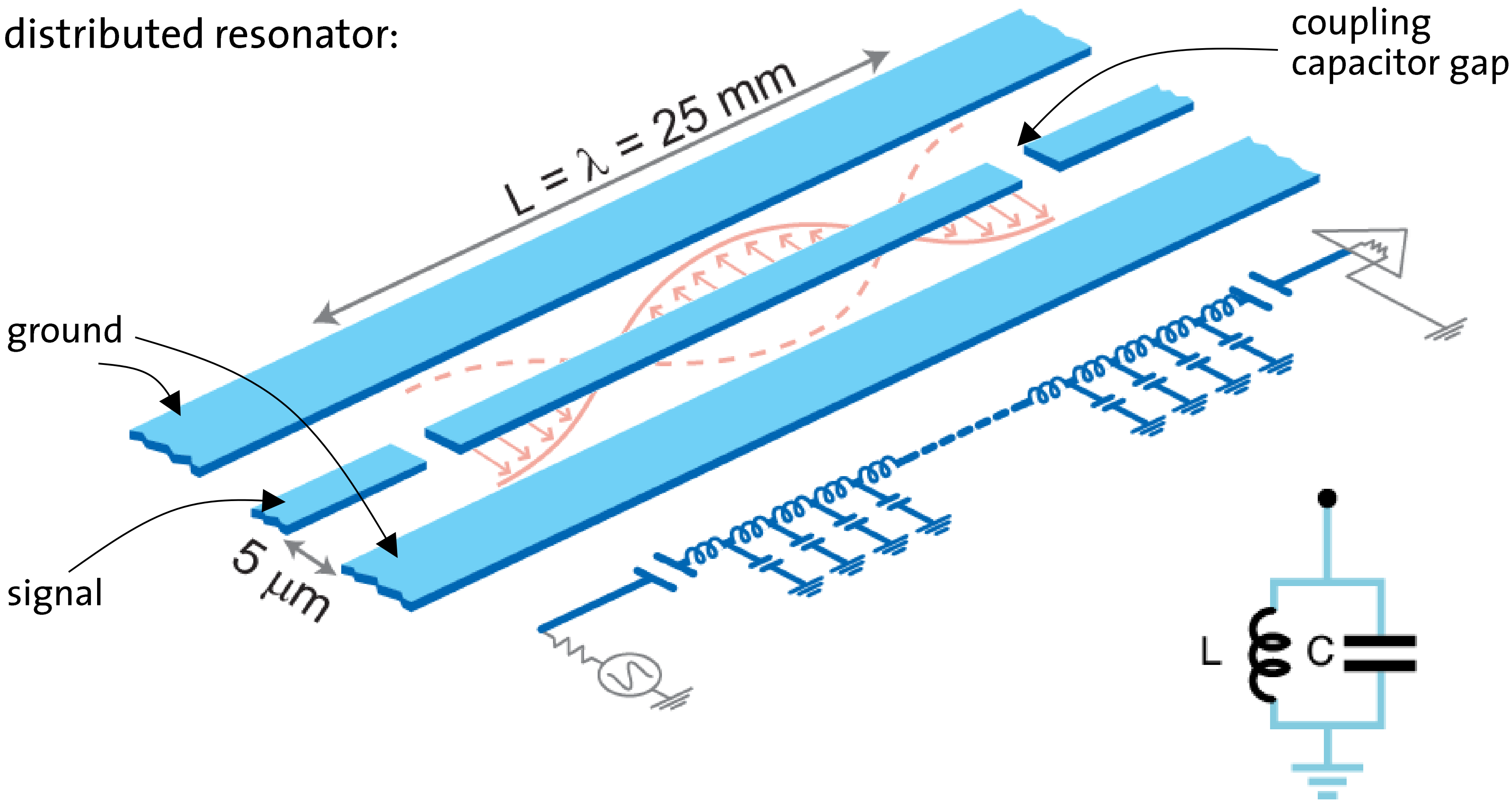
A Simple Electronic Harmonic Oscillator Circuit

LC resonator:



Realization of H.O.: Transmission Line Resonator

distributed resonator:

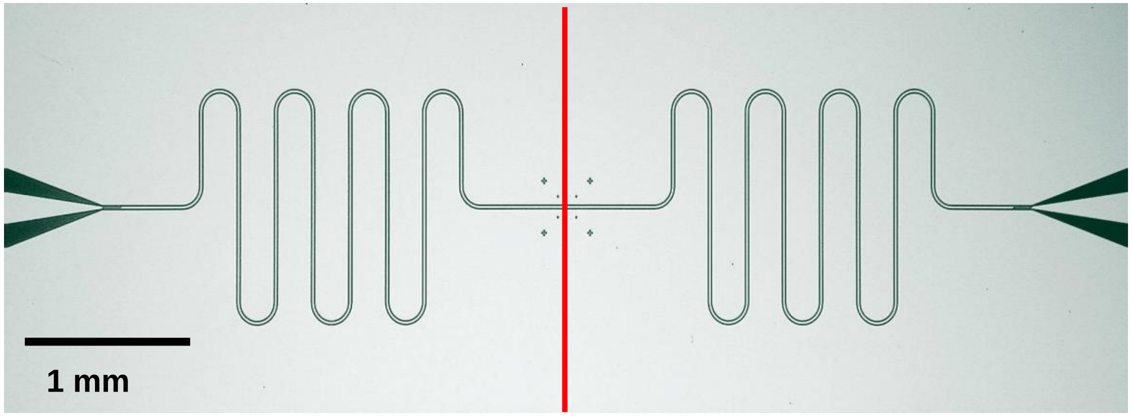


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

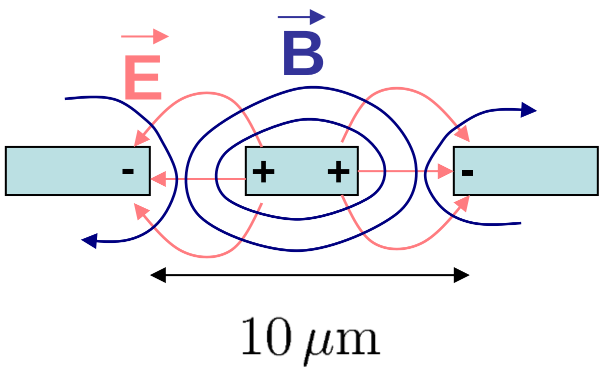
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED,
Journal of Applied Physics **104**, 113904 (2008)

Realization of Transmission Line Resonator

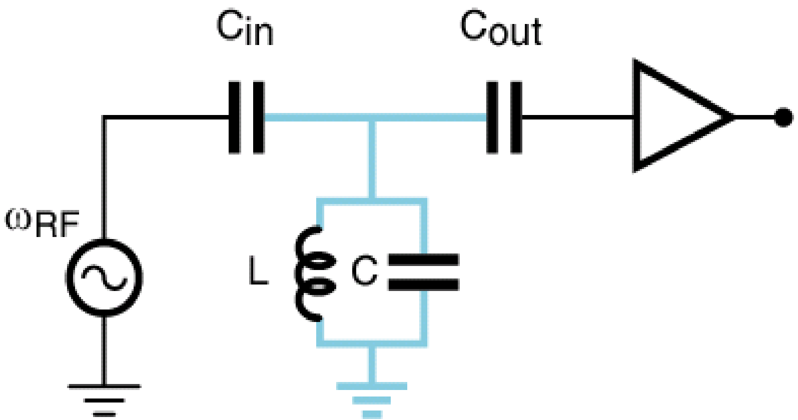
coplanar waveguide:



cross-section of transm. line (TEM mode):

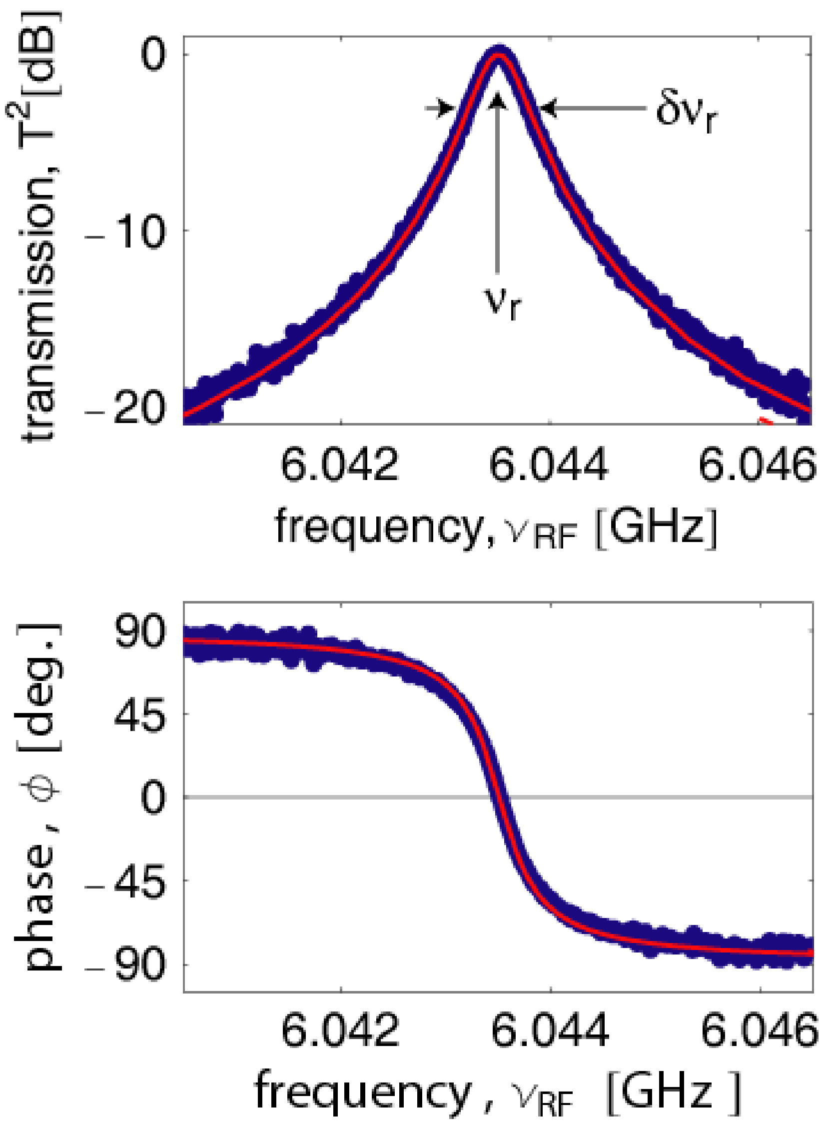


measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

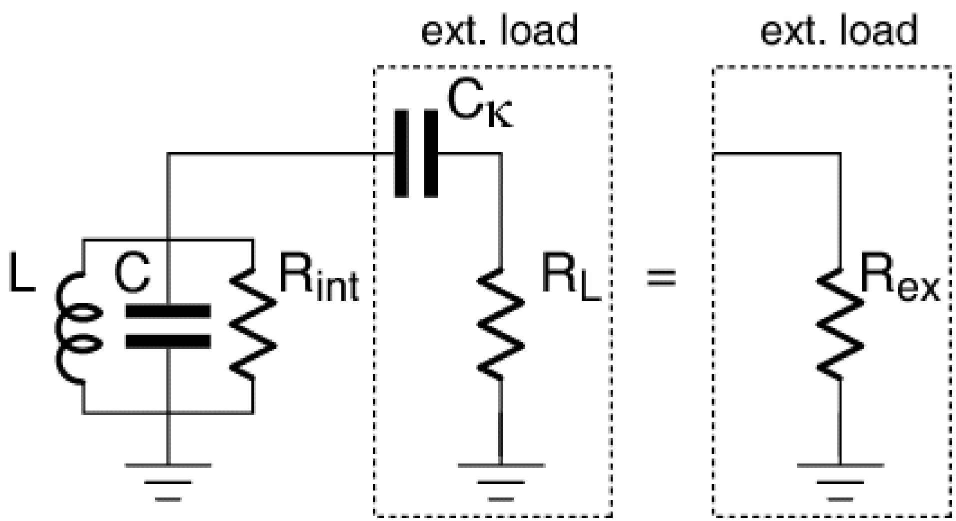
photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Internal and External Dissipation in an LC Oscillator



internal losses R_{int}:
conductor, dielectric

external losses R_{ext} :
radiation, coupling

Loading due to external circuit:

total effective resistance:

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_{\text{int}}} + \frac{1}{R^*_{\text{ext}}}$$

total effective capacitance:

$$C_{\text{tot}} = C_{\text{int}} + C^*_{\text{ext}}$$

energy decay time/rate:

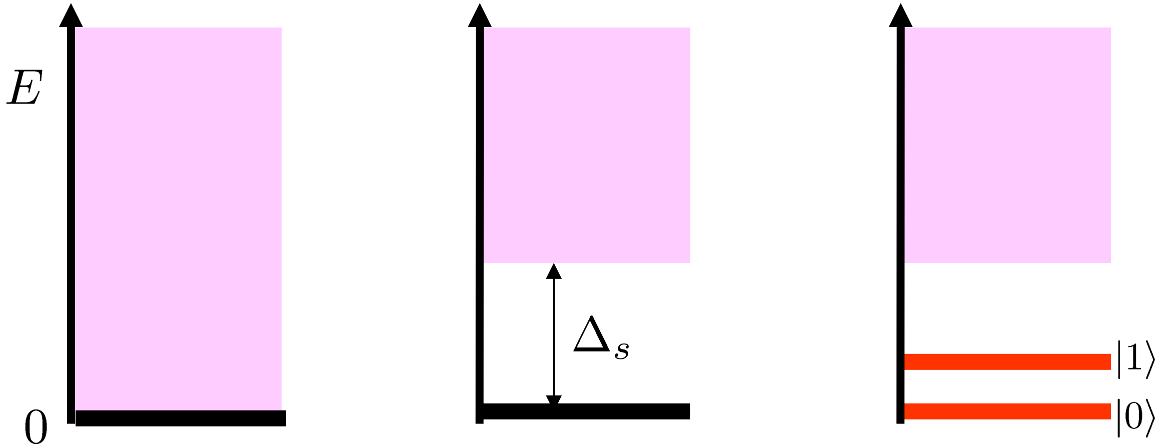
$$T_\kappa = \frac{1}{\kappa} = R_{\text{tot}} C_{\text{tot}}$$

External circuit induces:

decrease of total resistance
-> decrease of quality factor

increase of total capacitance
-> decrease of res. frequency

Why Superconductors?



normal metal | superconductor | How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

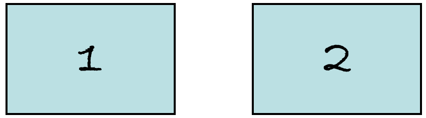
Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta/h$ = 725 GHz, T_c = 9.2 K
- Aluminum (Al): $2\Delta/h$ = 100 GHz, T_c = 1.2 K

Cooper pairs:
bound electron pairs

Bosons (S=0, L=0)

2 chunks of superconductors

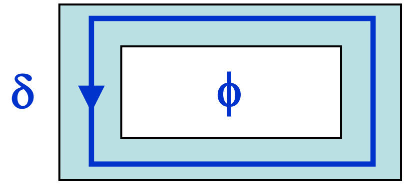


macroscopic wave function

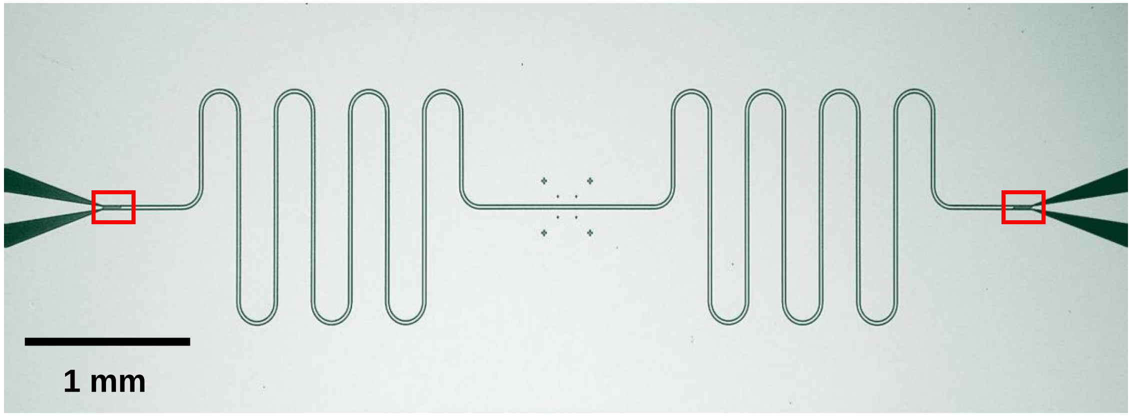
$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

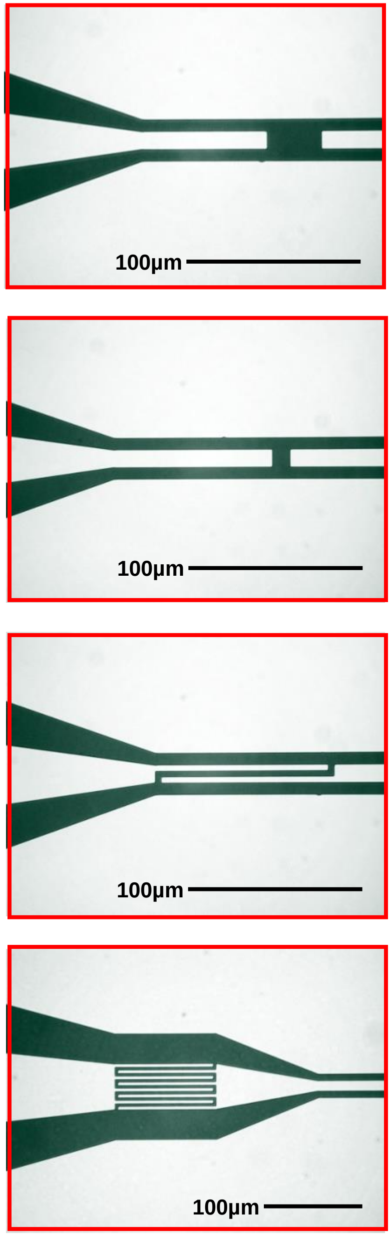
Cooper pair density n_i
and global phase δ_i

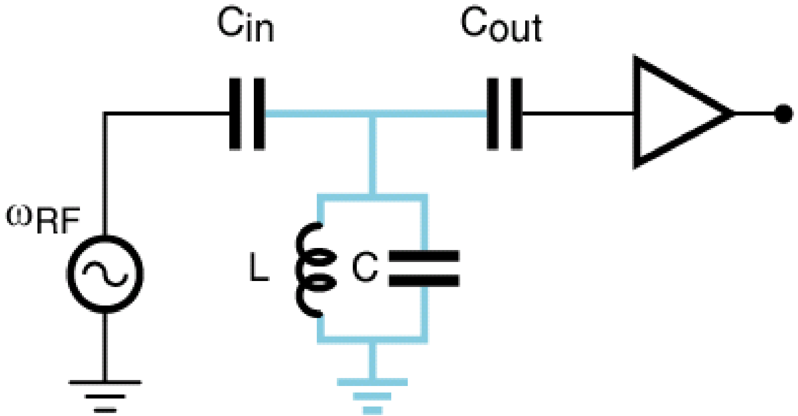
phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$



Controlling the Photon Life Time

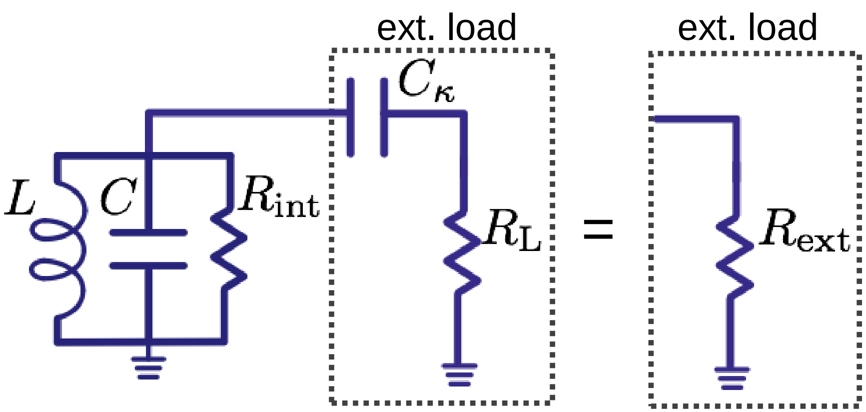


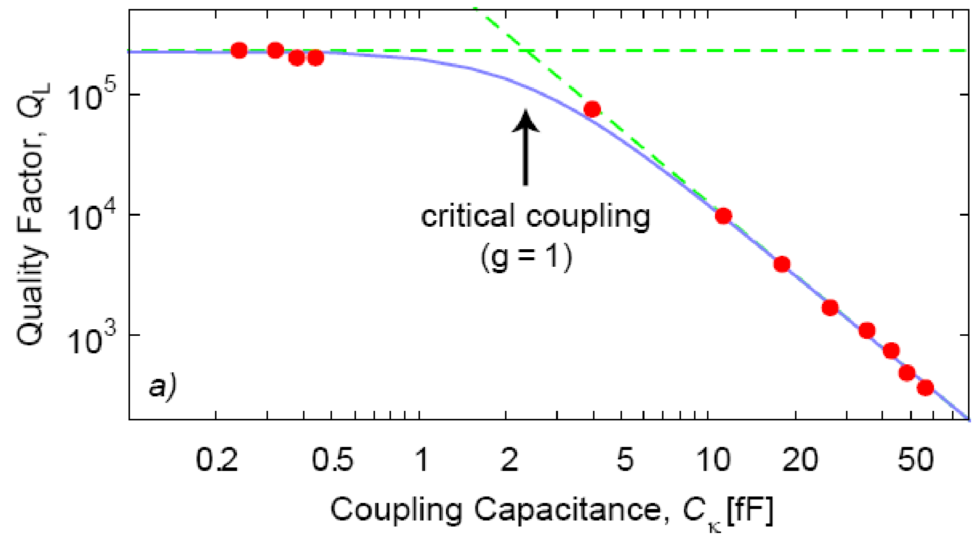


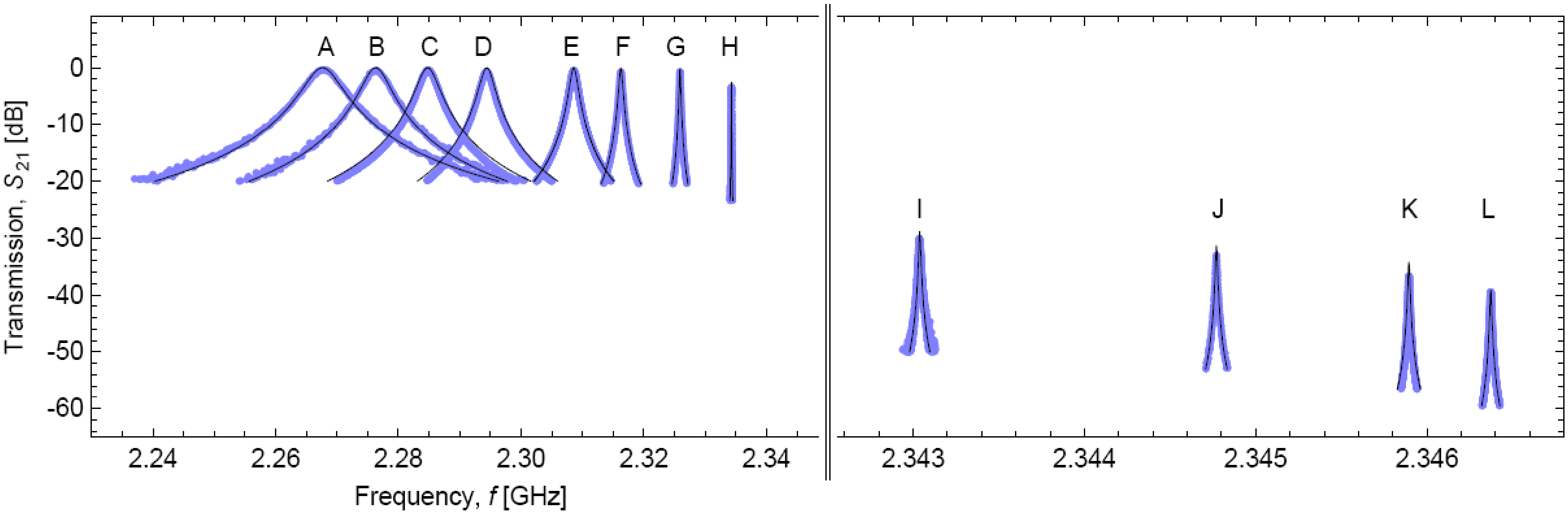


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Quality Factor Measurement

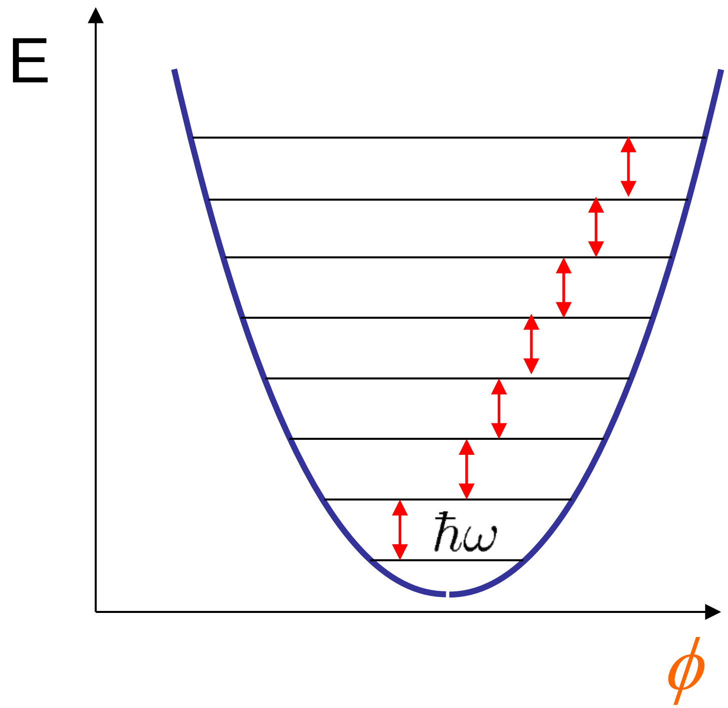






M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\rm th} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{\rm th} \rangle \sim 10^{-11}$$

How to Prove that a Harmonic Oscillator is Quantum?

measure:

- resonance frequency
- average charge (momentum)
- average flux (position)

all simple averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

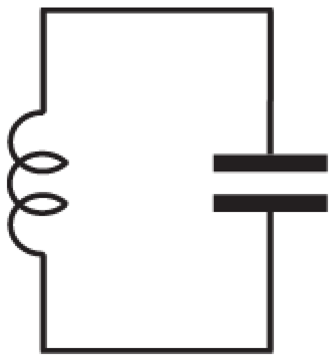
solutions:

- make oscillator non-linear in a controllable way
- measure higher order statistical properties

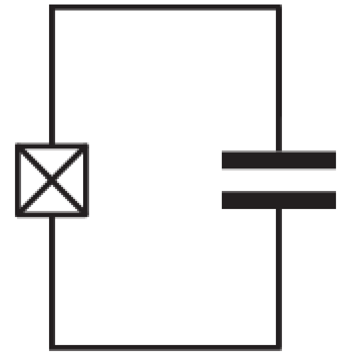
Josephson Junctions and Superconducting Qubits

Linear vs. Nonlinear Superconducting Oscillators

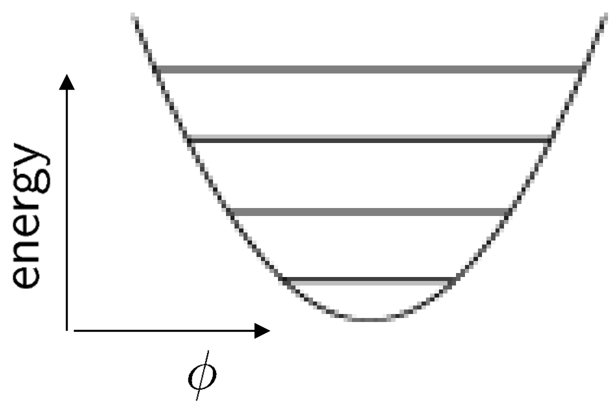
LC resonator:

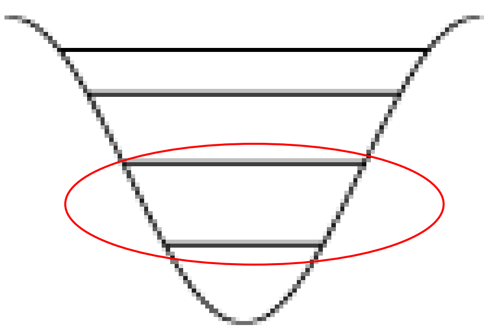


Josephson junction resonator:
Josephson junction = nonlinear inductor



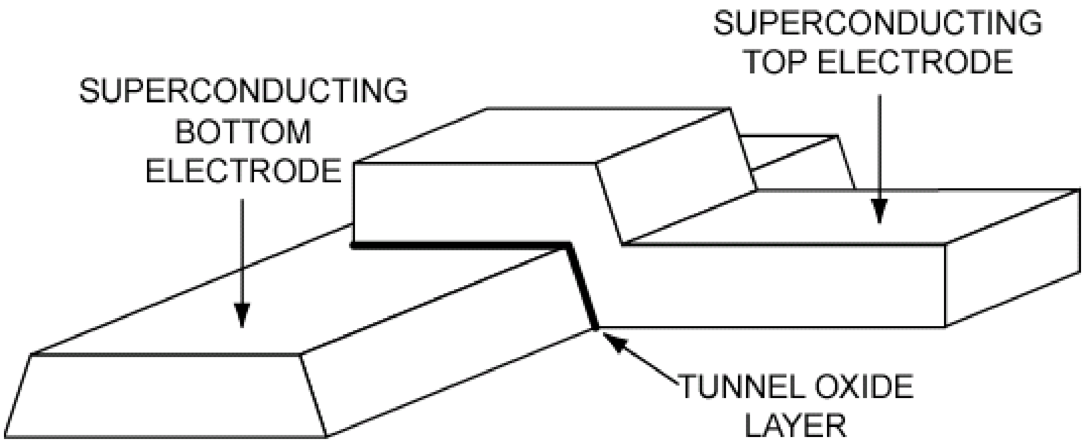
anharmonicity defines effective two-level system





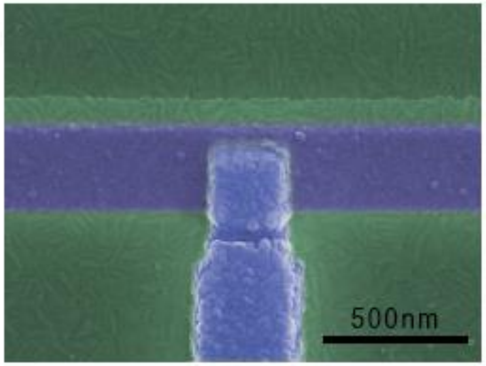
A Low-Loss Nonlinear Element

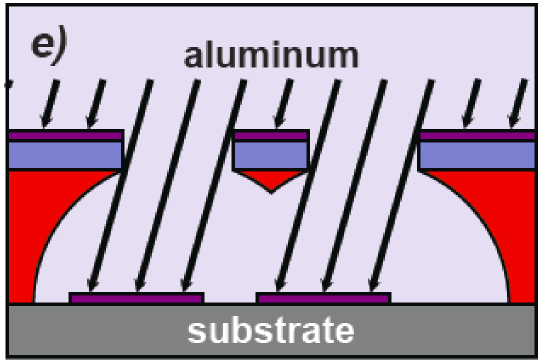
a (superconducting) Josephson junction:

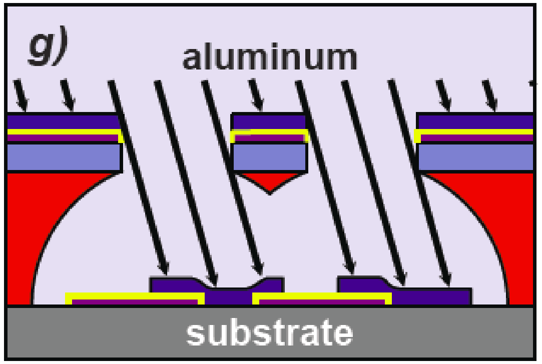


- superconductors: Nb, Al
- tunnel barrier: AlO_x

Josephson junction fabricated by shadow evaporation:



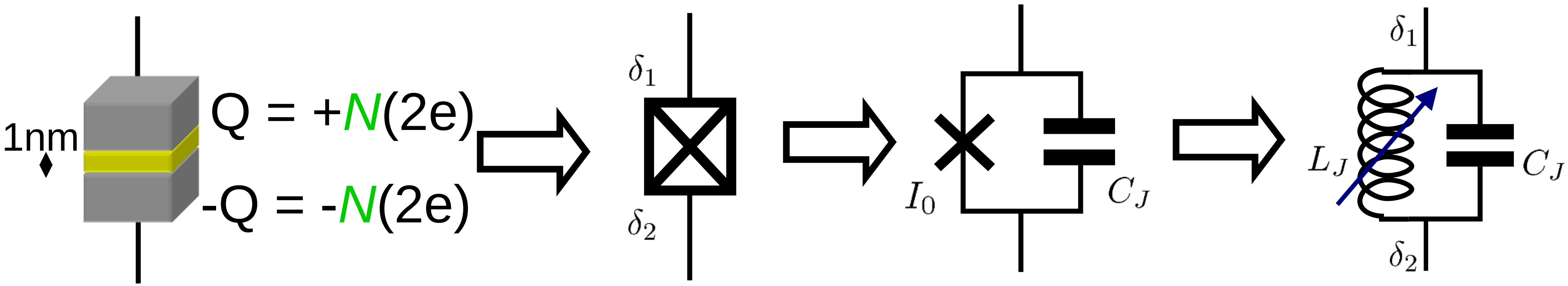




M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

The only non-linear resonator with no dissipation (BCS, $k_B T < \Delta$)



Tunnel junction parameters:

- Critical current I_o
- Junction capacitance C_J
- Internal resistance R_J

Josephson relations:

$$I = I_0 \sin \delta$$

$$V = \frac{\phi_0}{2\pi} \dot{\delta}$$

Flux quantum:

$$\phi_0 = \frac{h}{2e}$$

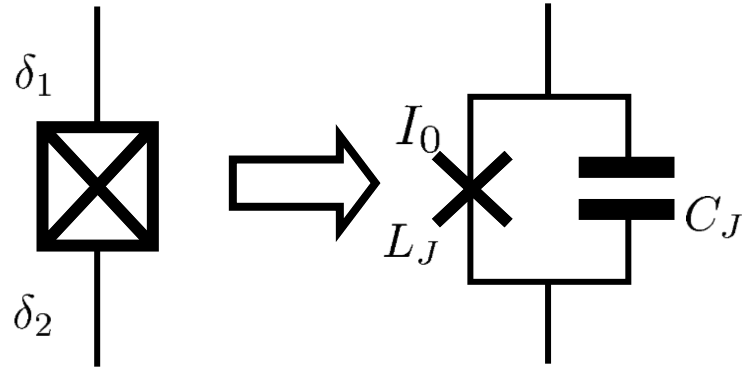
Phase difference:

$$\delta = \delta_2 - \delta_1$$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson Junction as an ideal Non-Linear Inductor

a nonlinear inductor without dissipation



Josephson relations:

$$I = I_0 \sin \delta = I_0 \sin \left[2\pi \phi(t)/\phi_0 \right]$$

$$V = \frac{\phi_0}{2\pi} \dot{\delta} = \dot{\phi}$$

nonlinear current/phase relation

gauge inv. phase difference: $\delta = \delta_2 - \delta_1 = 2\pi \phi(t)/\phi_0$

specific Josephson inductance L_{J0}

Josephson inductance:

$$V = -L_J \dot{I} = \frac{\phi_0}{2\pi I_0} \frac{1}{\cos \delta} \dot{I}$$

specific Josephson energy E_{J0}

Josephson energy:

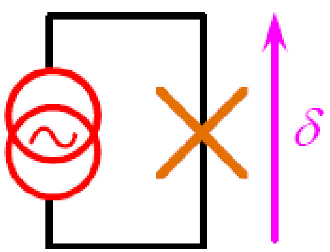
$$E_J = \int V I dt = \frac{I_0 \phi_0}{2\pi} \cos \delta$$

A Classification of Josephson Junction Based Qubits

How to make use in of Jospehson junctions in a qubit?

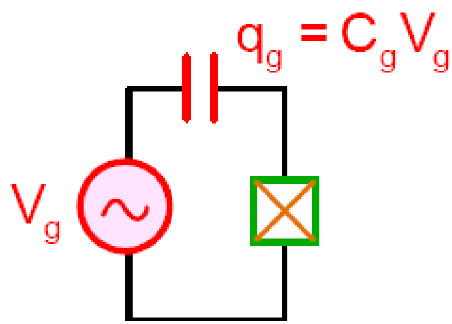
Common options of bias (control) circuits:

phase qubit



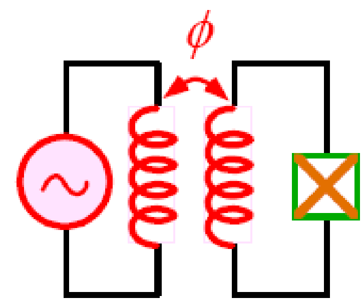
current bias

charge qubit
(Cooper Pair Box, Transmon)



charge bias

flux qubit

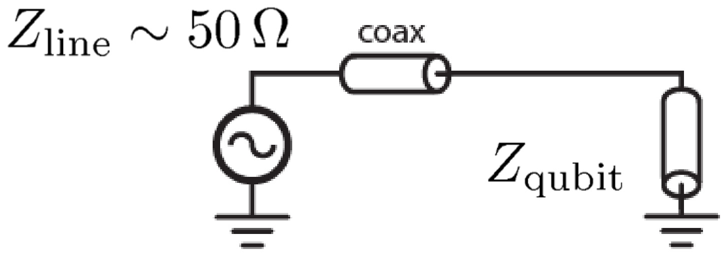


flux bias

How is the control circuit important?

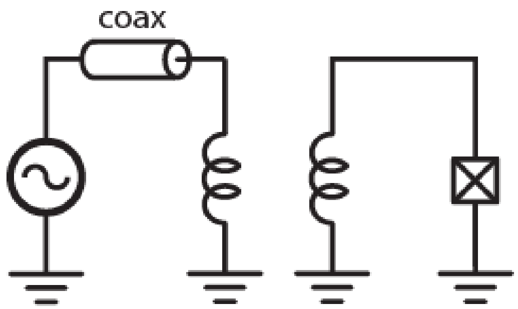
Control of Coupling to Electromagnetic Environment

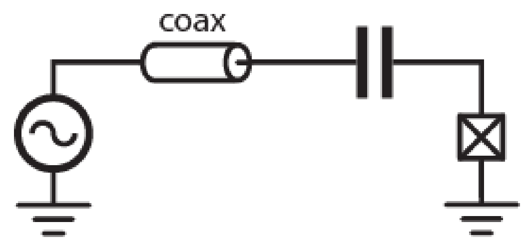
coupling to environment (bias wires):



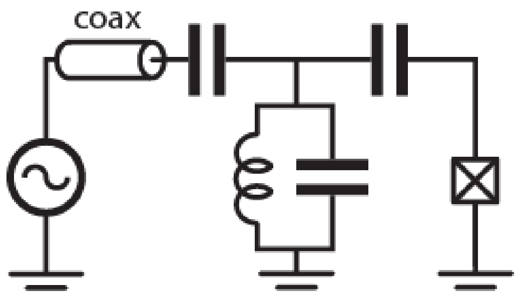
decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...





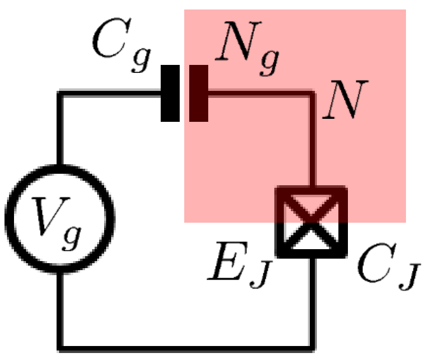
... or resonant impedance transformers



control spontaneous emission by circuit design

The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \frac{Q}{2e}$

continuous gate charge: $N_g = \frac{C_g V_g}{2e}$

total box capacitance $C_\Sigma = C_g + C_J$

Hamiltonian: $H = H_{\mathrm{el}} + H_{\mathrm{mag}}$

electrostatic part:

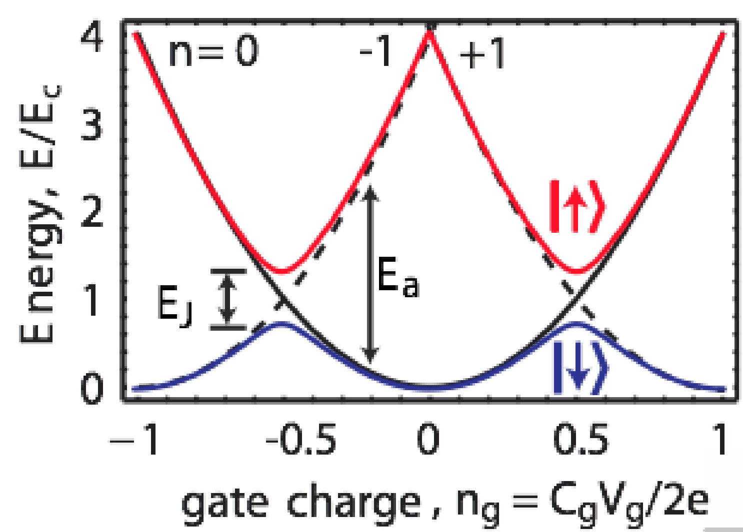
$$H_{\mathrm{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\mathrm{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator:

$$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** N:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 + E_J \cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

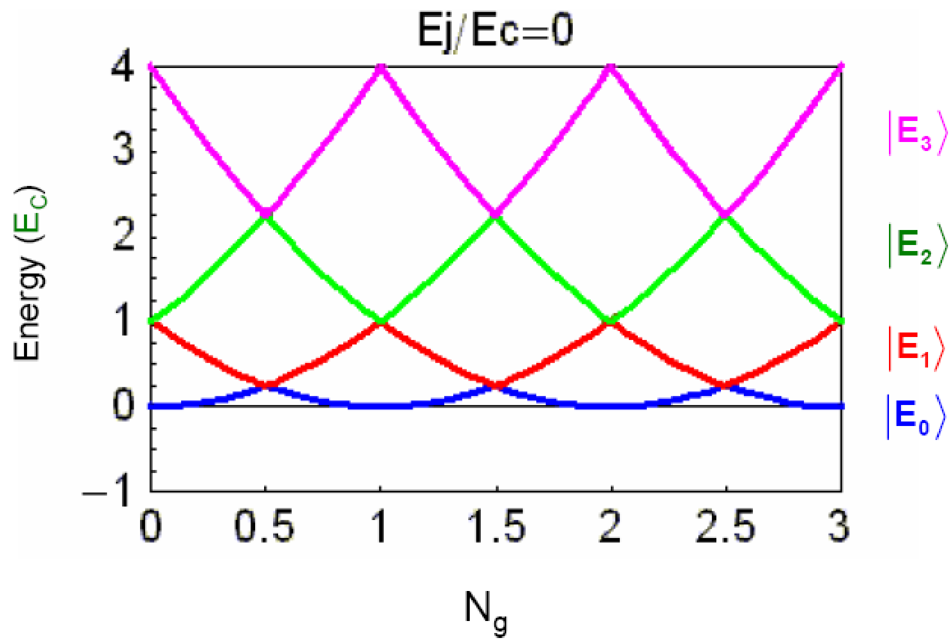
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

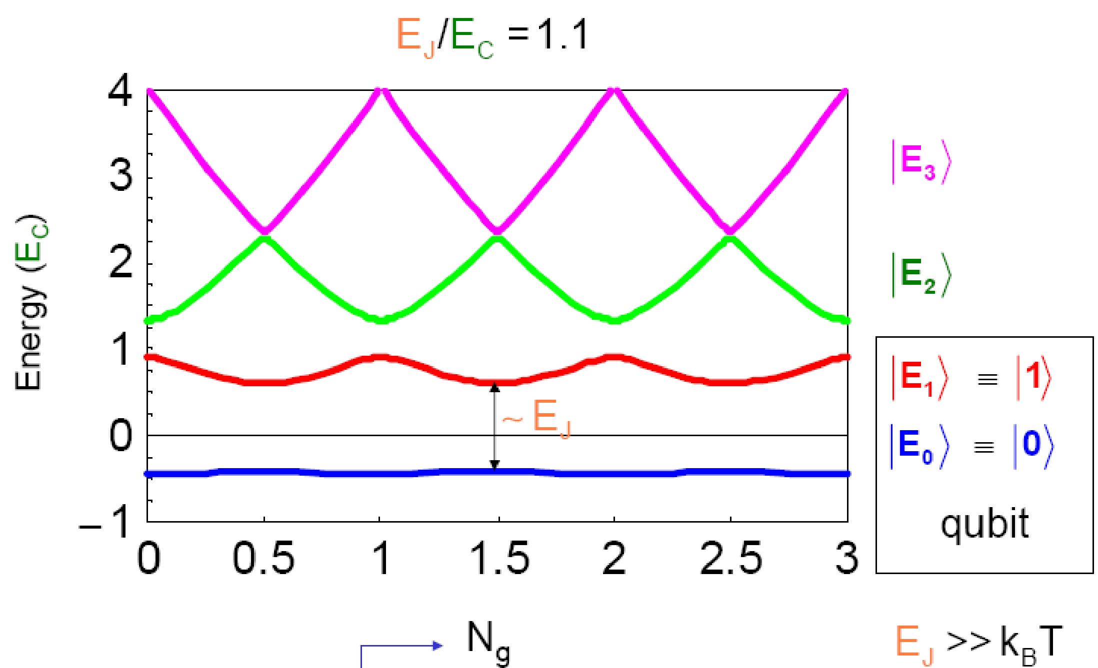
energy level diagram for E_J=0:

- energy bands are formed
- bands are periodic in N_g



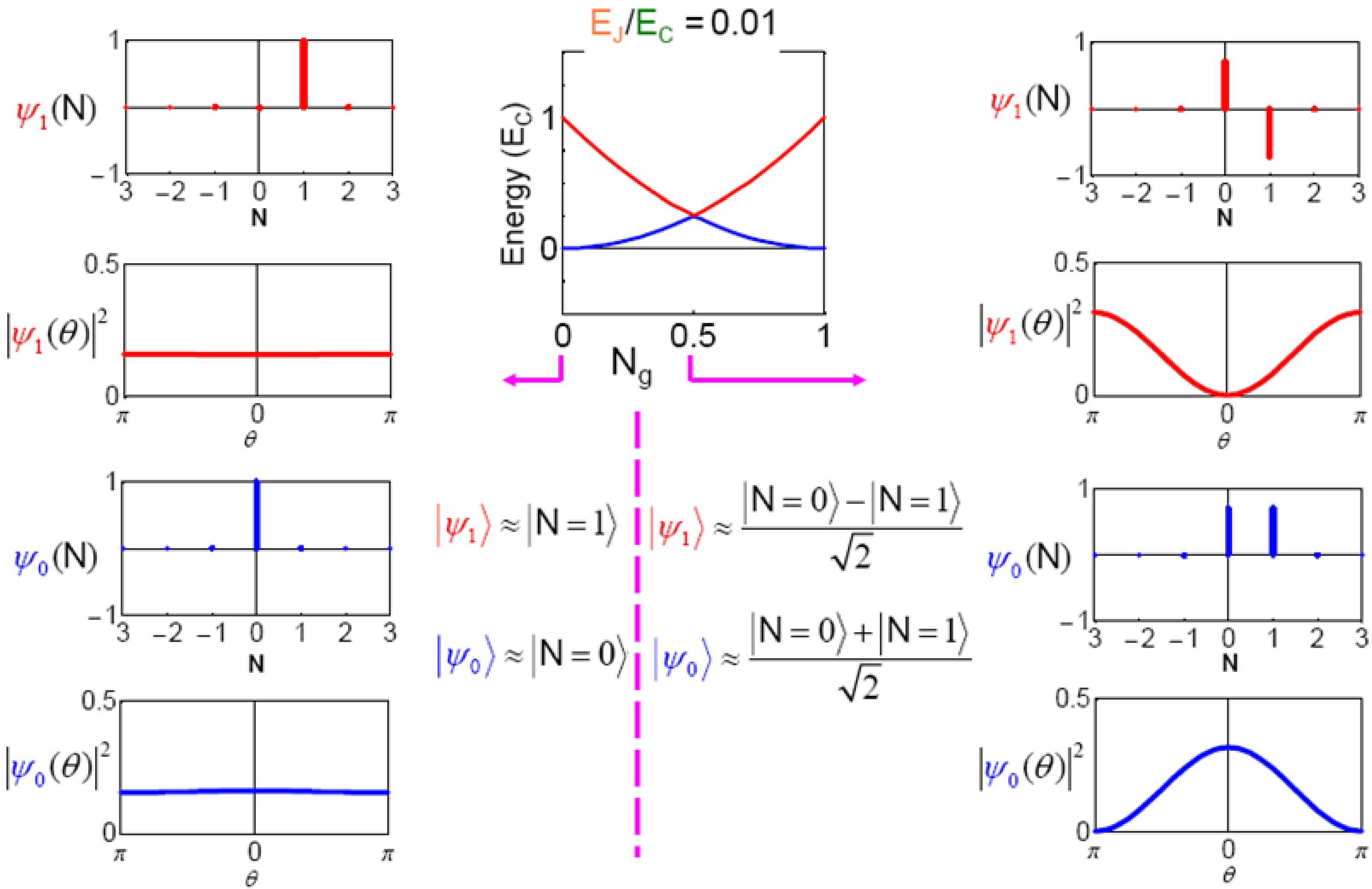
energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



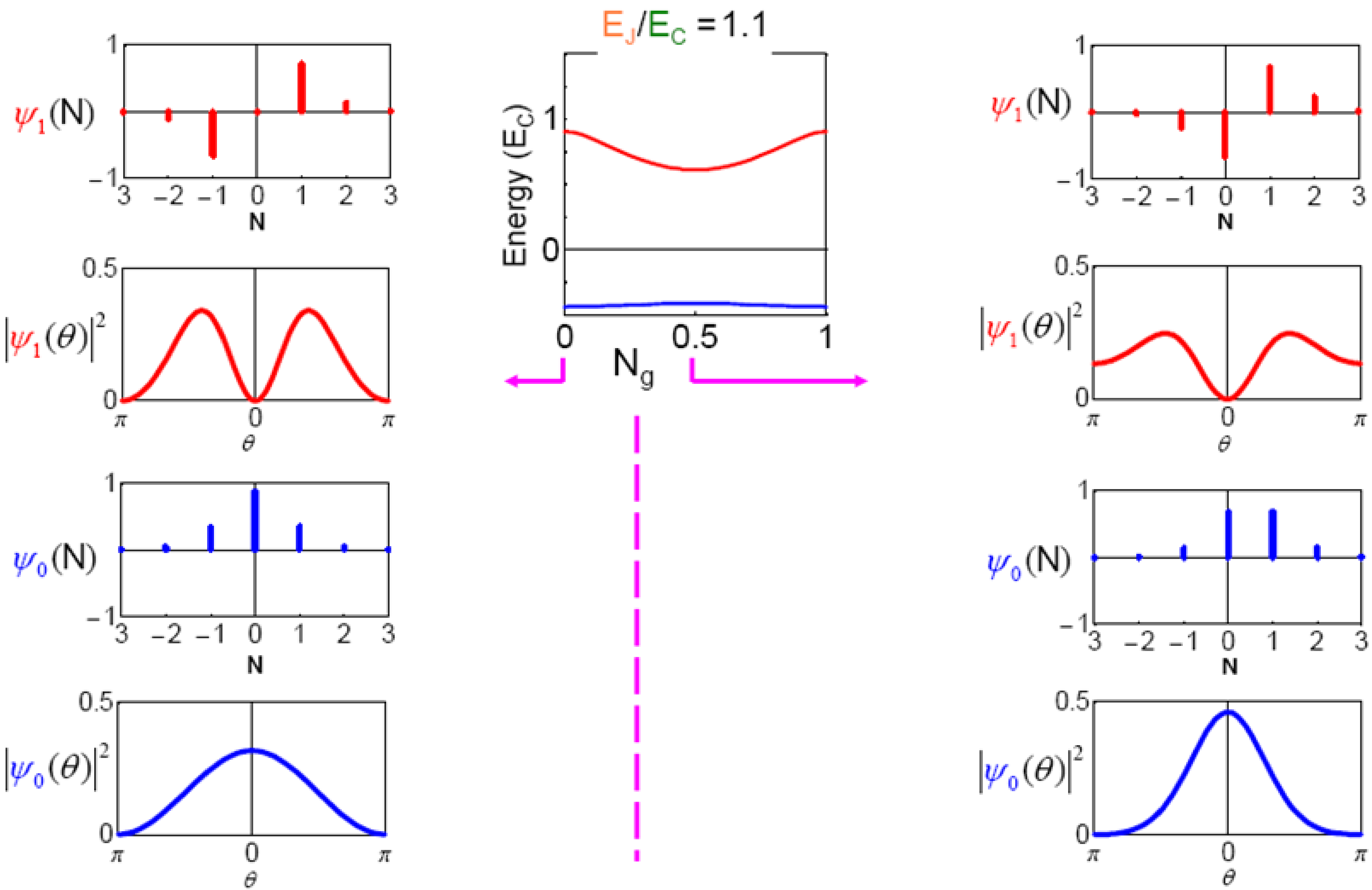
tunable artificial atom

Charge and Phase Wave Functions ($E_J \ll E_C$)



courtesy CEA Saclay

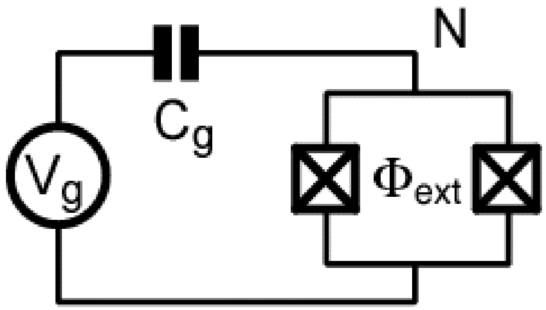
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

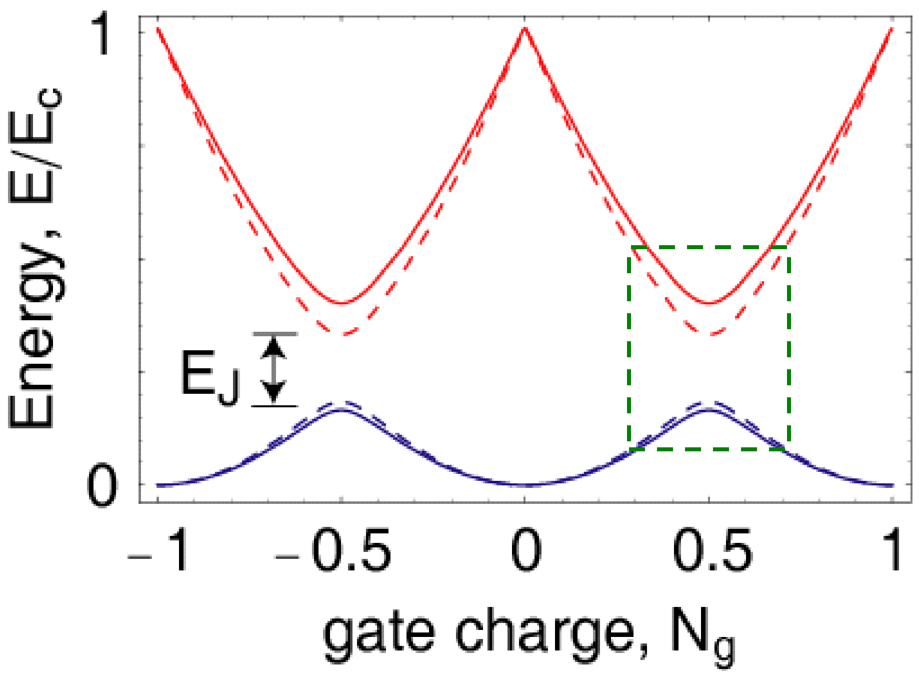
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max} \cos\left(\pi \frac{\phi_{\rm ext}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max} \cos\left(\pi \frac{\phi_{\rm ext}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\rm CPB} = \hat{H}_{\rm el} + \hat{H}_{\rm J} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

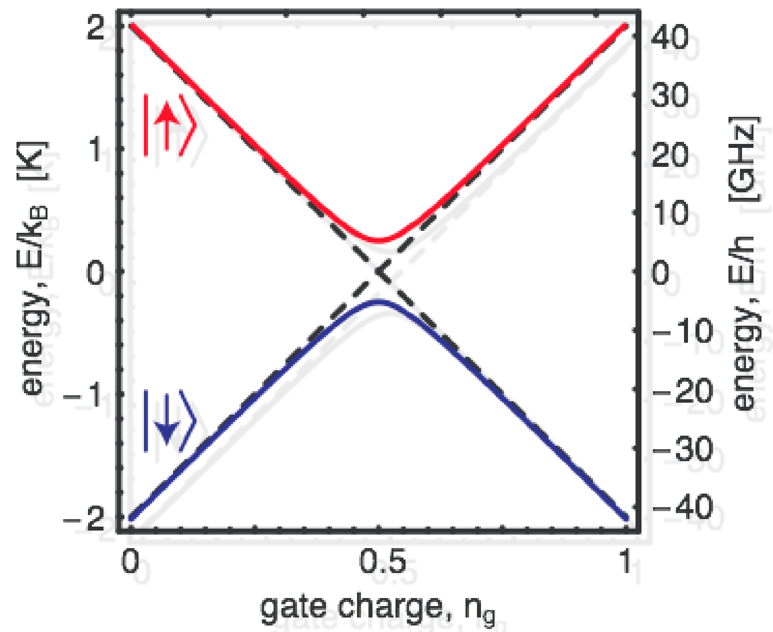
$$\hat{H}_{\rm CPB} = \sum_N \left[E_C(N - N_g)^2 |N\rangle\langle N| - \frac{E_{\rm J}}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\rm el}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

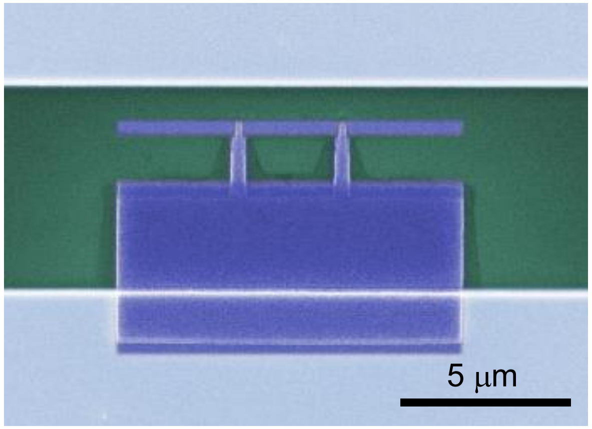


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

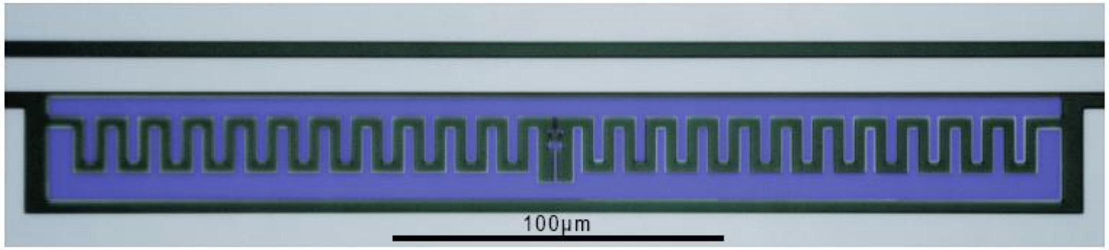
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

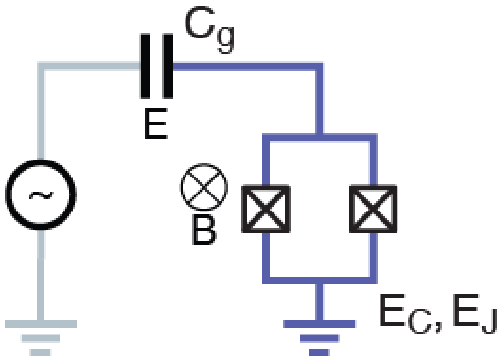
standard CPB:

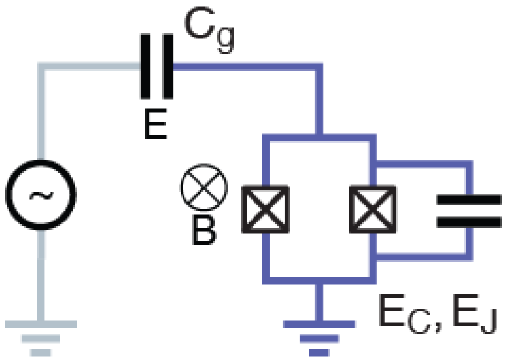


Transmon qubit:



circuit diagram:

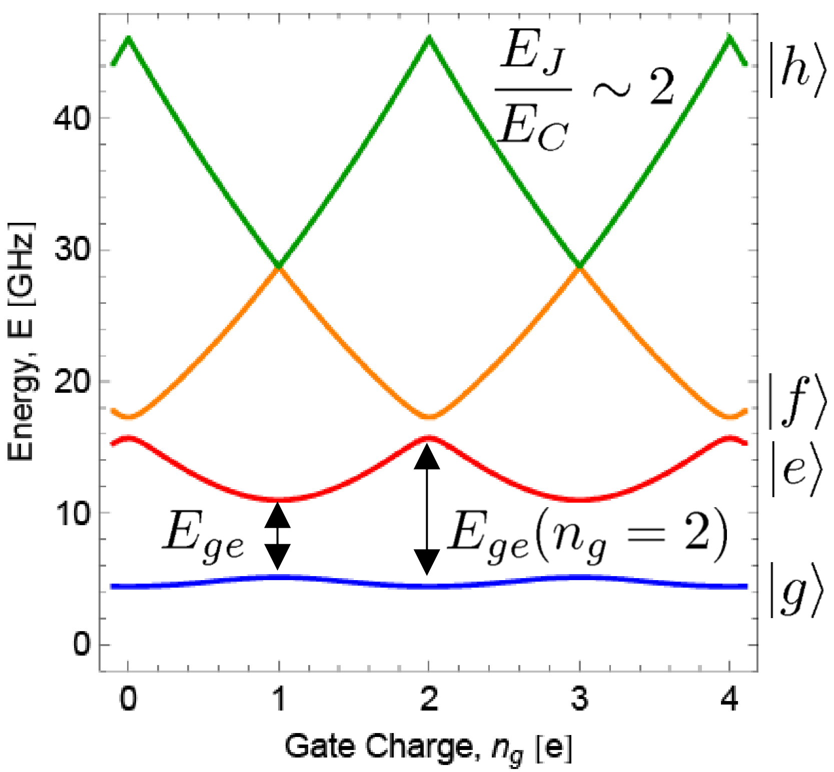




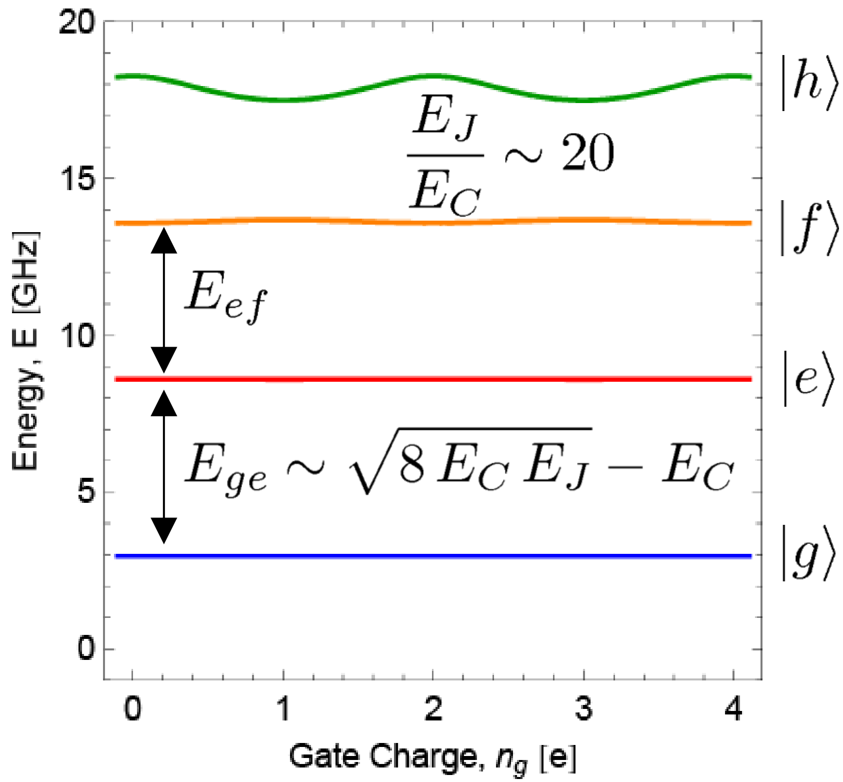
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity:

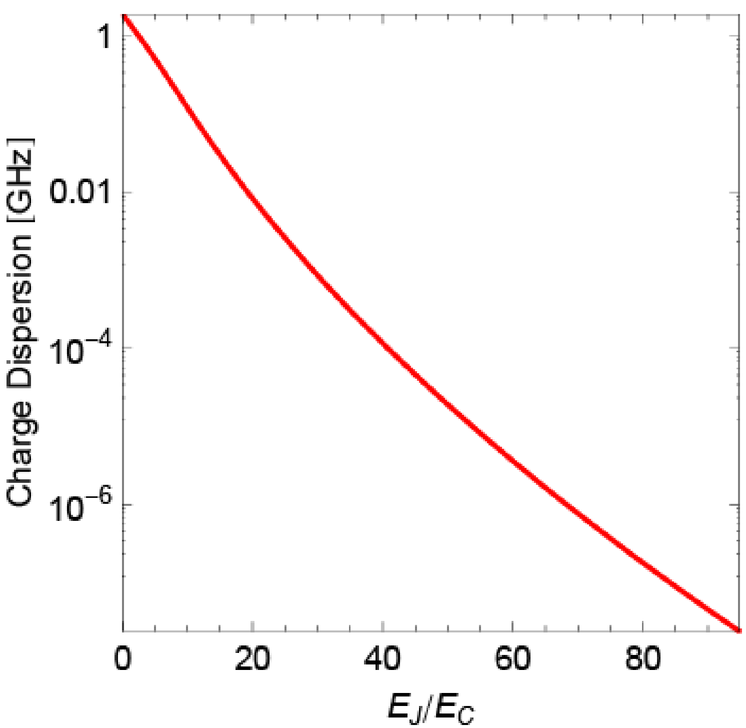
$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Dispersion and Anharmonicity

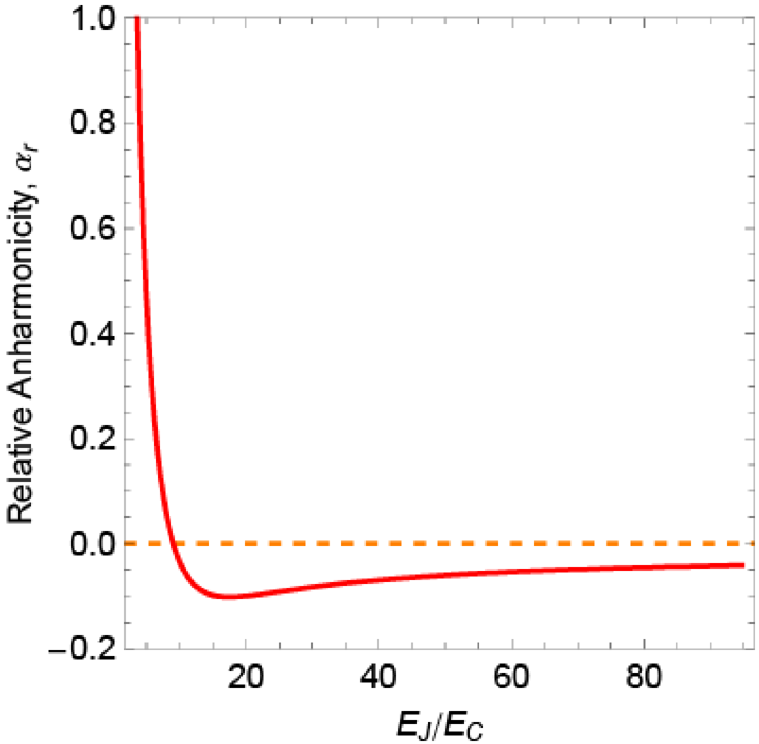
Charge dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$



Anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

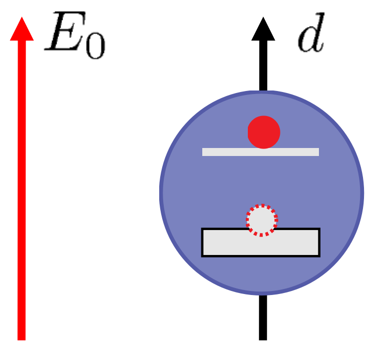


J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Circuit Quantum Electrodynamics (QED): Cavity QED with Superconducting Circuits

Controlling the Interaction of Photons and Qubits

challenging on the level of single particles



- dipole moment *d* in microscopic systems (usually small ~ ea_o)
- single photon fields E_o (small in 3D)
- photon/qubit interaction $\hbar g \sim dE_0$ (usually small)

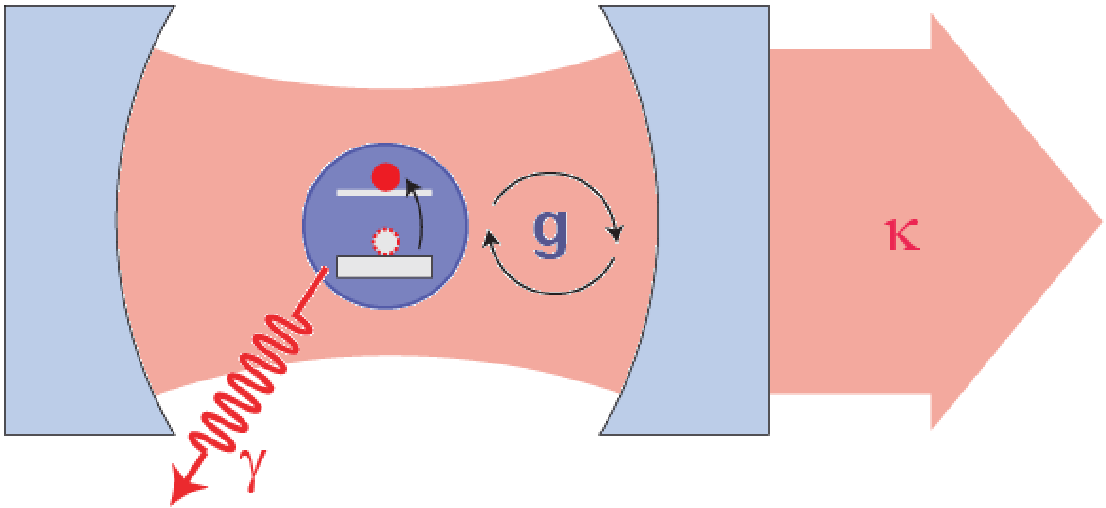
What to do?

- confine qubit and photon in a cavity (cavity QED)
- engineer qubit/light interaction in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa\ , 1/t_{\text{transit}}$

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Dressed States Energy Level Diagram

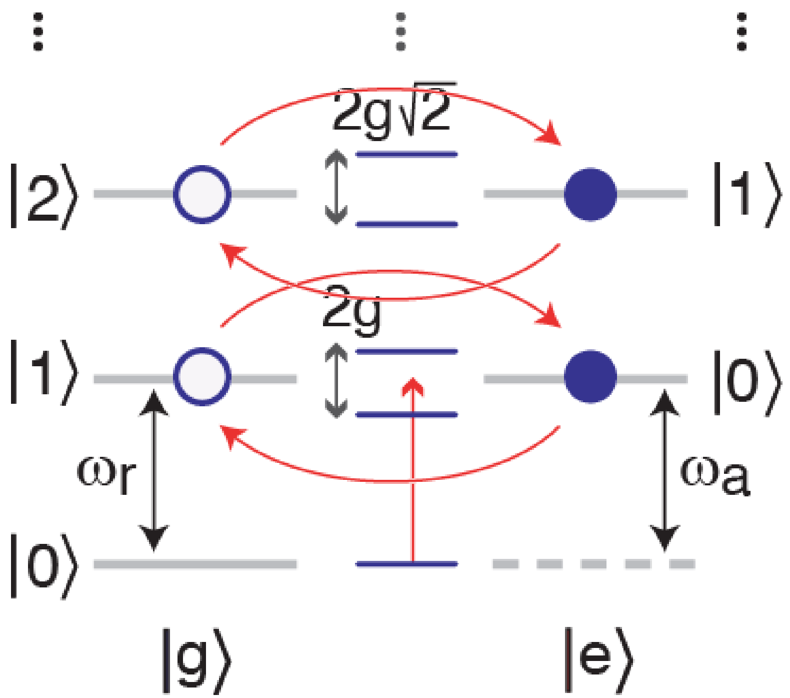
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



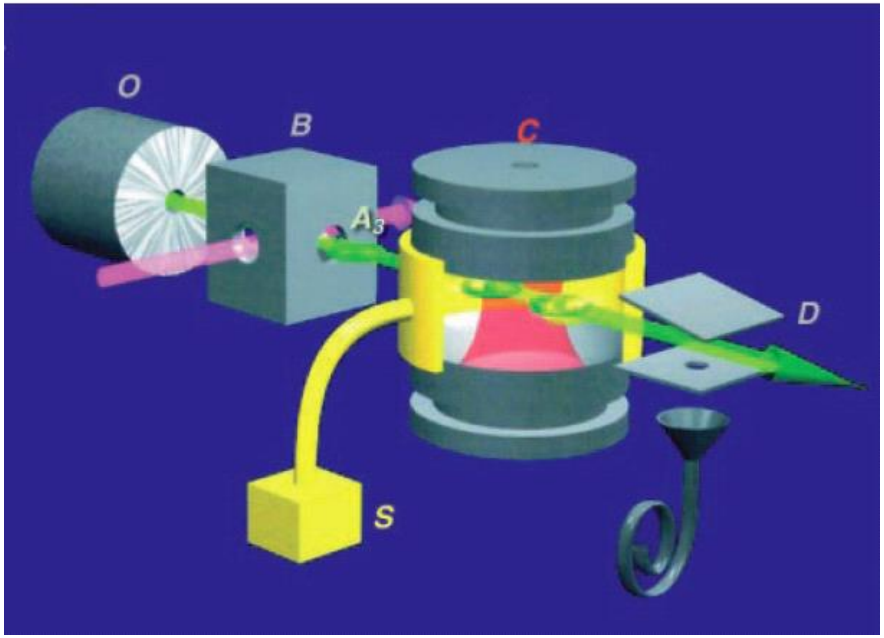
Jaynes-Cummings Ladder

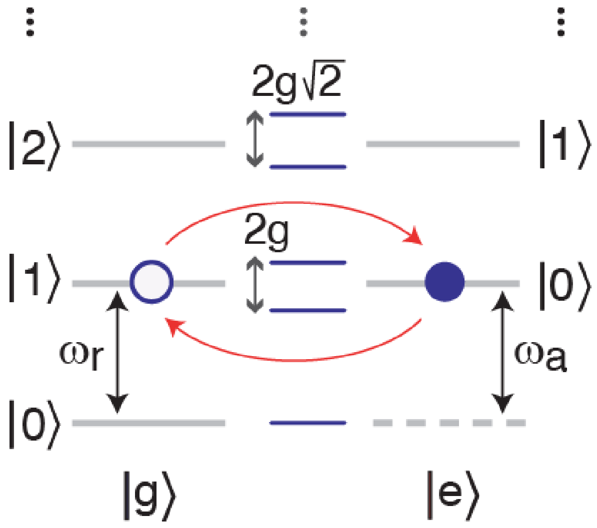
atomic cavity QED reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734-1738 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

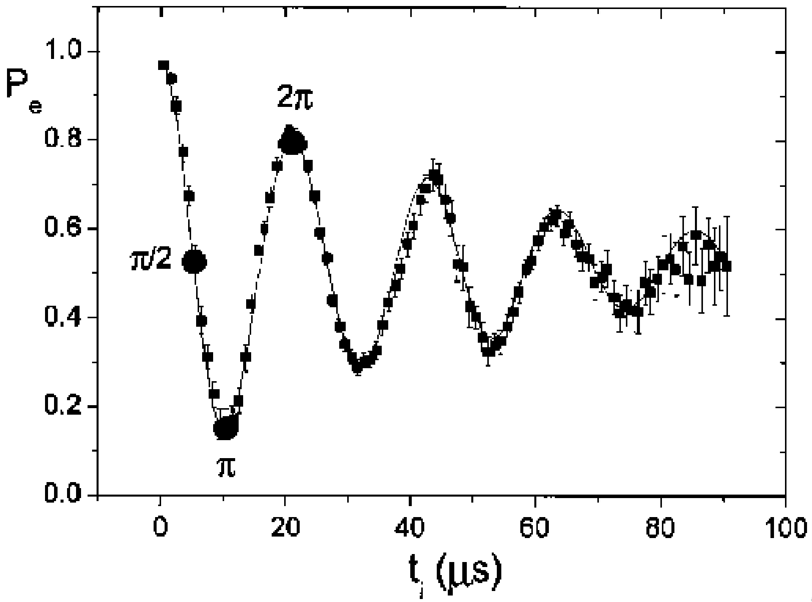
Vacuum Rabi Oscillations with Rydberg Atoms





with Rydberg atoms in microwave domain:

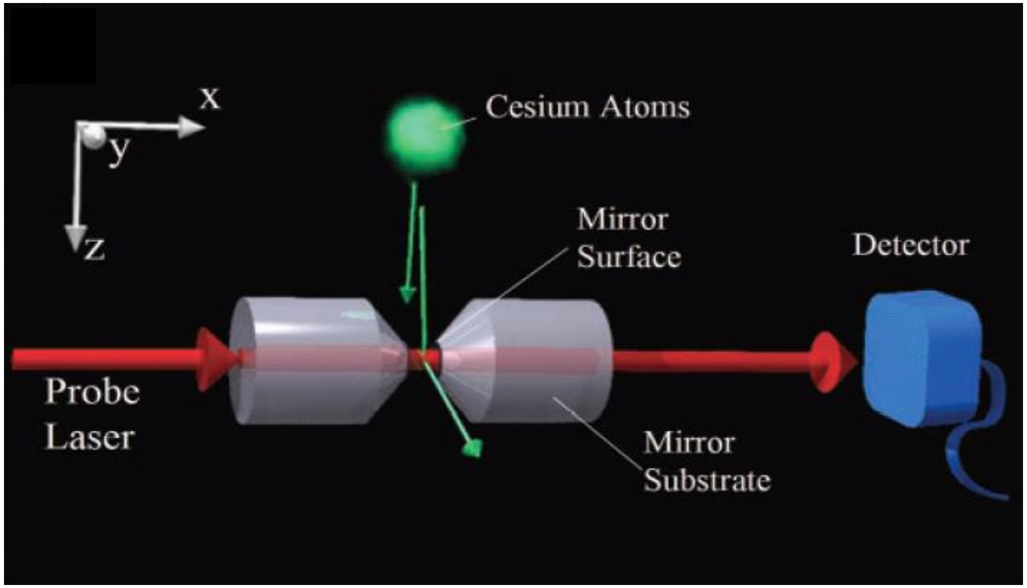
- large d

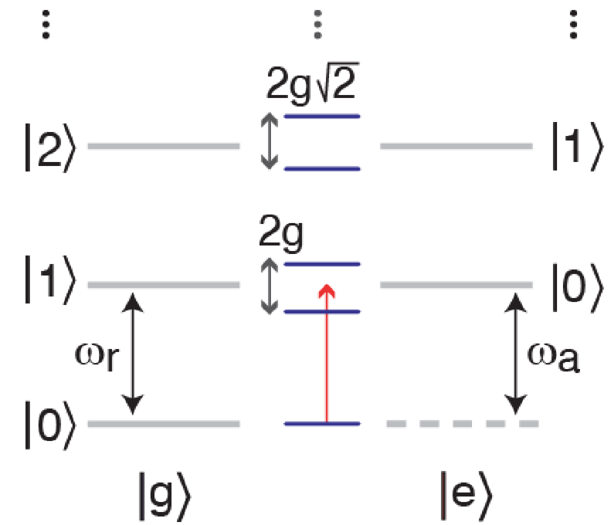


reviews:

Haroche, Raimond, *OUP Oxford* (2006)
Raimond, Brune, Haroche *RMP* **73**, 565 (2001)
this data: Brune *et al*, *PRL* **76**, 1800 (1996)

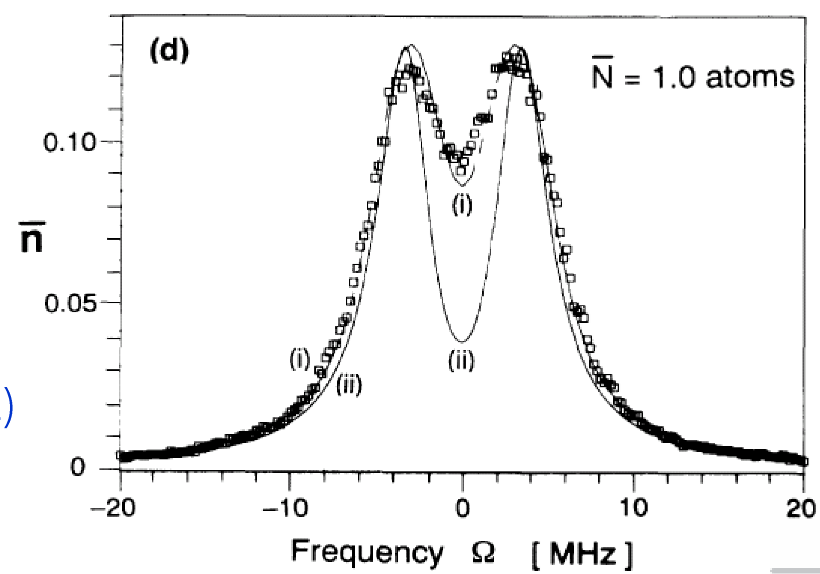
Vacuum Rabi Mode Splitting with Alkali Atoms





with alkali atoms in optical domain:

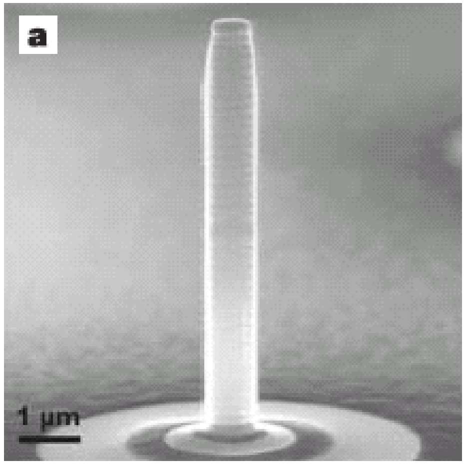
- large E_0



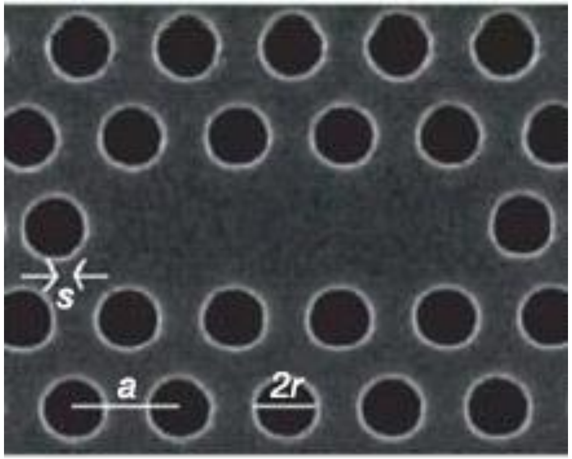
reviews:

Ye, Kimble, Katori, *Science* **320**, 1734 (2008)
Mabuchi, Doherty, *Science* **298**, 1372 (2002)
this data: Thompson, Rempe, Kimble *PRL* **68**, 1132 (1992)

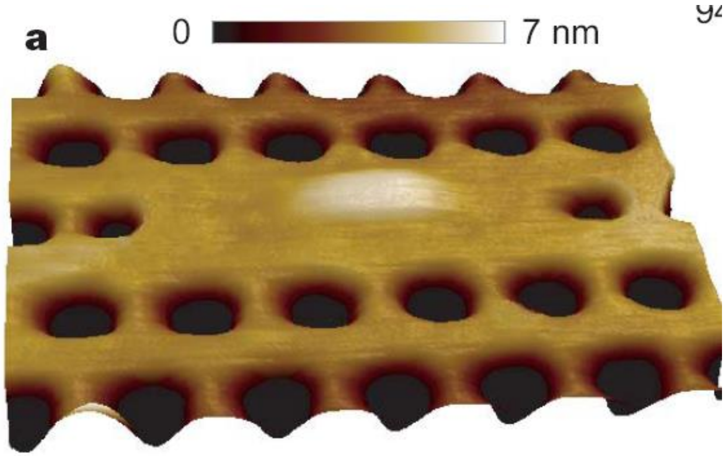
… e.g. with Semiconductors



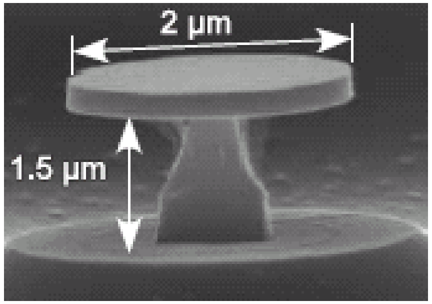
Wurzburg
*Nature **432**, 197 (2004)*



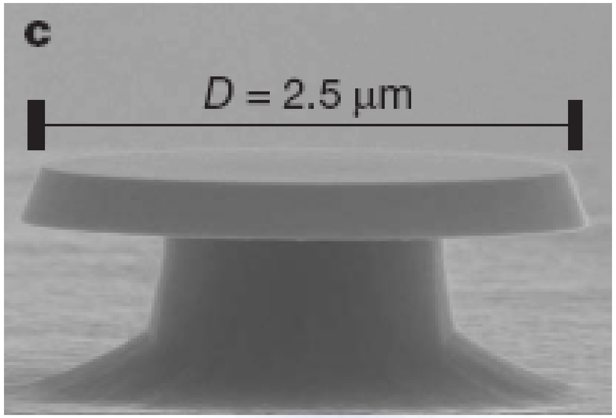
Arizona
*Nature **432**, 200 (2004)*



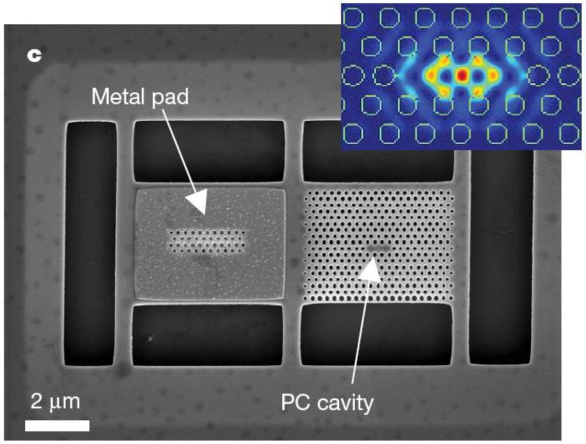
ETH Zurich
*Nature **445**, 896 (2007)*



Paris
PRL (2004)

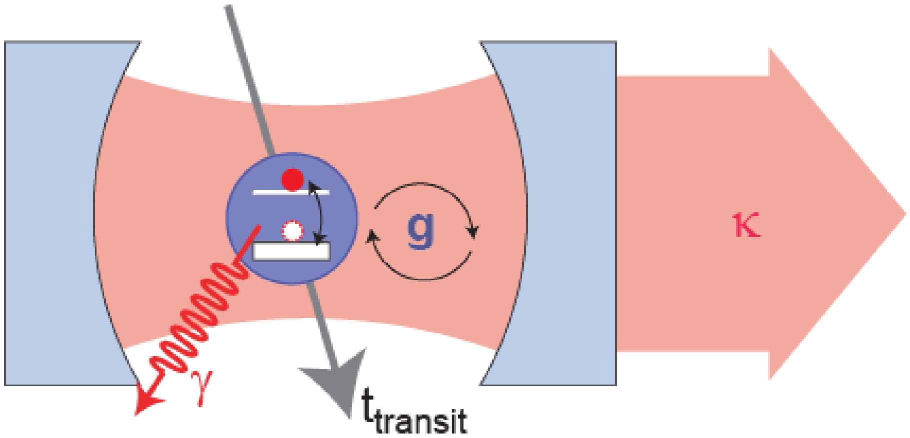


Caltech
*Nature **450**, 862 (2007)*



Stanford
*Nature **450**, 857 (2007)*

Proposals for Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

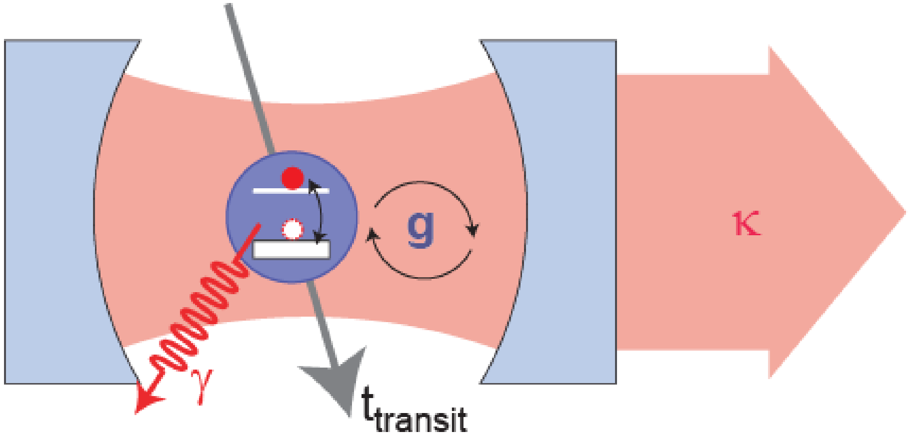
a number of approaches suggested at the time:

discrete LC circuits:
- Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
- O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).

large Josephson junctions:
- F. Marquardt and C. Bruder, Phys. Rev. B **63**, 054514 (2001).
- F. Plastina and G. Falci, Phys. Rev. B **67**, 224514 (2003).
- A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. **90**, 127901 (2003).

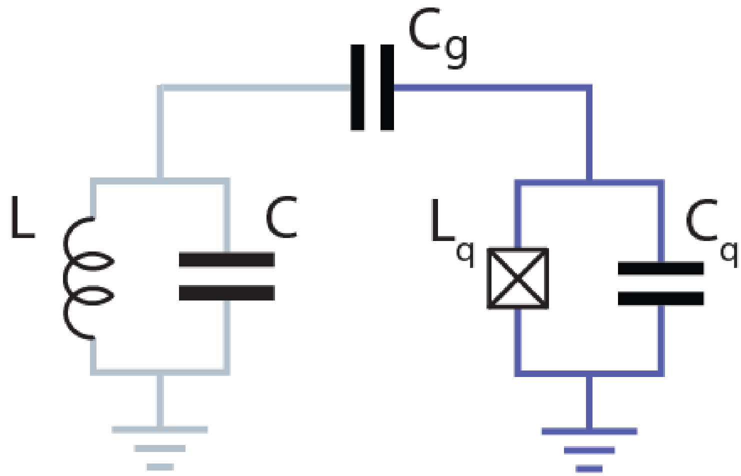
3D cavities:
- W. Al-Saidi and D. Stroud, Phys. Rev. B **65**, 014512 (2001).
- C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).
- J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).

Cavity QED with Superconducting Circuits



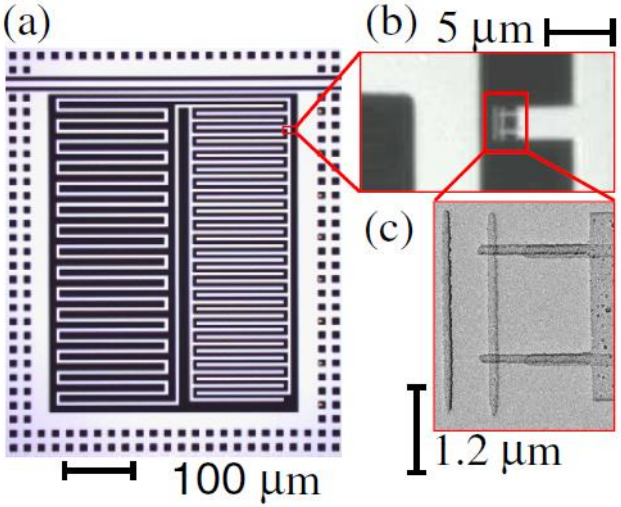
coherent quantum mechanics
with individual photons and qubits ...

... basic approach:



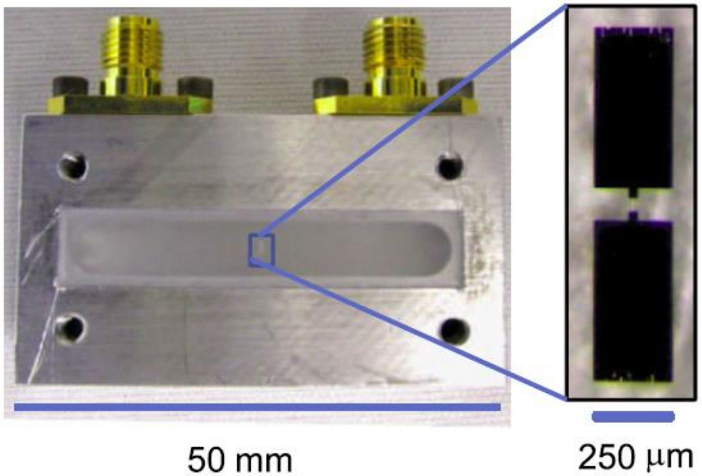
Circuit QED and its Different Realizations

lumped element resonator:



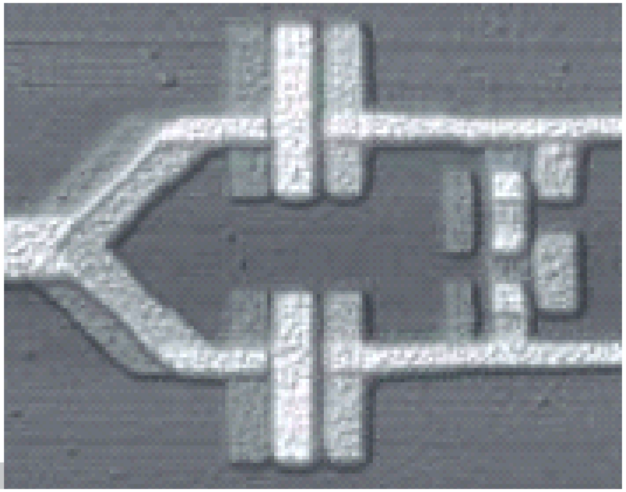
Z. Kim *et al.*, *PRL* **106**, 120501 (2011)

3D cavity:



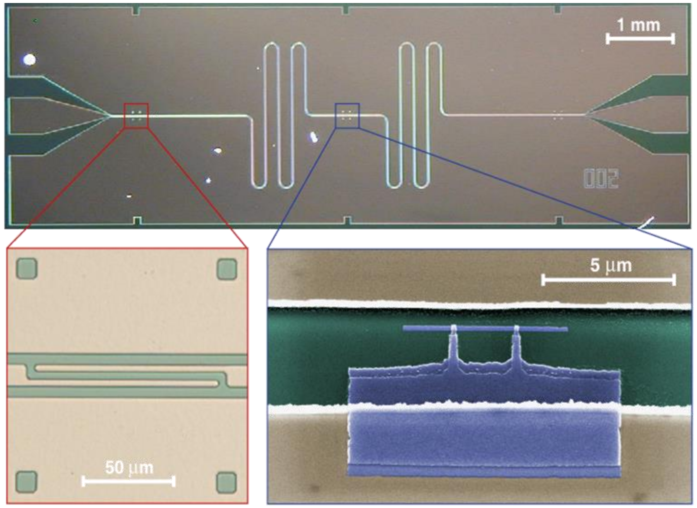
H. Paik *et al.*, *PRL* **107**, 240501 (2011)

weakly nonlinear junction:



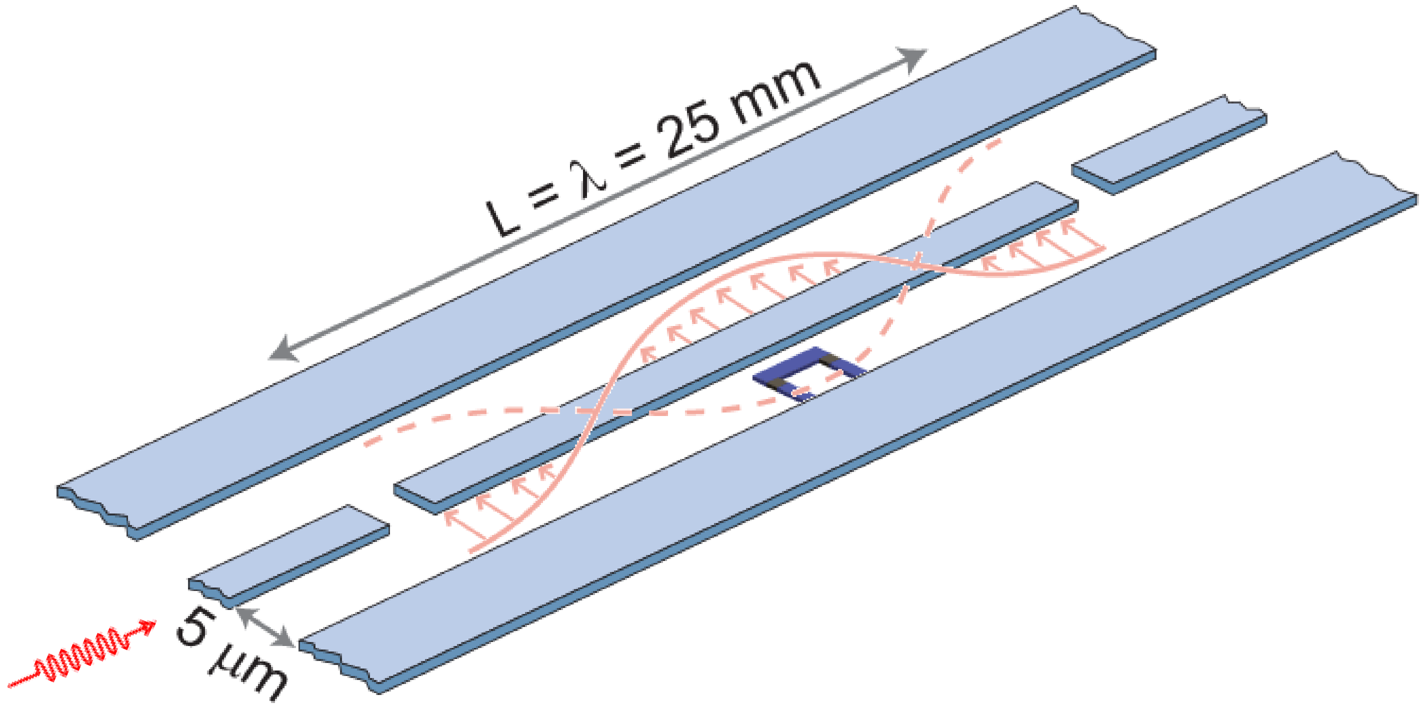
I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

planar transmission line resonator:



A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Circuit Quantum Electrodynamics

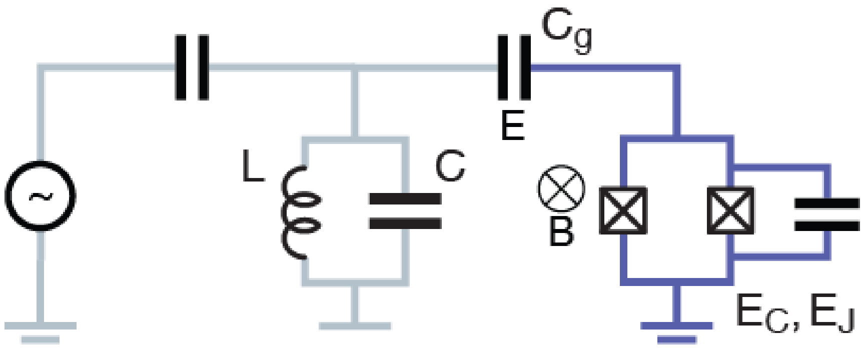


elements:

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_o and long photon life time $1/\kappa$
- the artificial atom: a superconducting qubit with large dipole moment d and long coherence time $1/\gamma$ and fixed position

A. Blais *et al.*, *PRA* **69**, 062320 (2004)

Qubit/Photon Coupling



Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy N_g = 1/2 and change of qubit basis (*z* to *x*, *x* to -*z*)

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

$$\hat{H}_g = \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^- + \hat{a}^\dagger\hat{\sigma}^+ + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

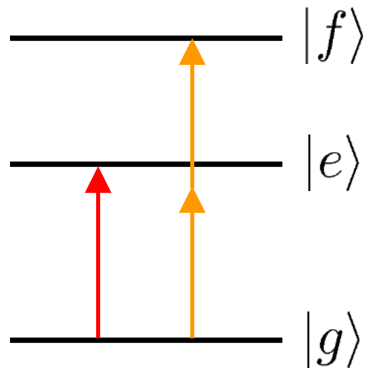
Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma} 2e\sqrt{\frac{\hbar\omega_r}{2C}}$

Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1\ldots300\,\mathrm{MHz}$

$$g \gg [\kappa, \gamma] \text{ possible!}$$

Spectroscopy of Transmon Qubit

one and two-photon spectroscopy:



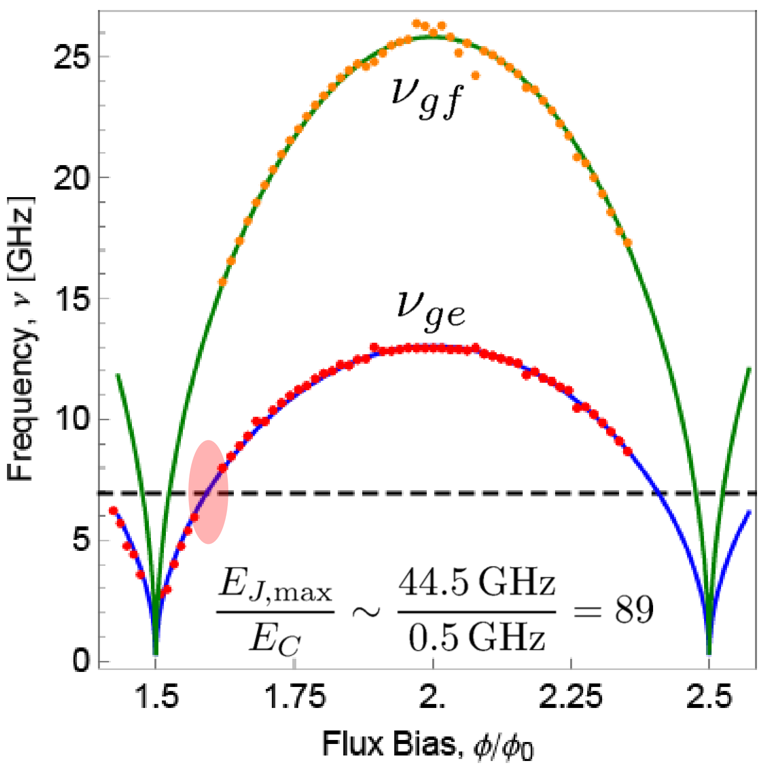
$|g\rangle \rightarrow |e\rangle$ transition:

$$\nu_{ge} = (E_e - E_g)/h$$

$|g\rangle \rightarrow |f\rangle$ transition:

$$2\,\nu_{gf} = (E_f - E_g)/h$$

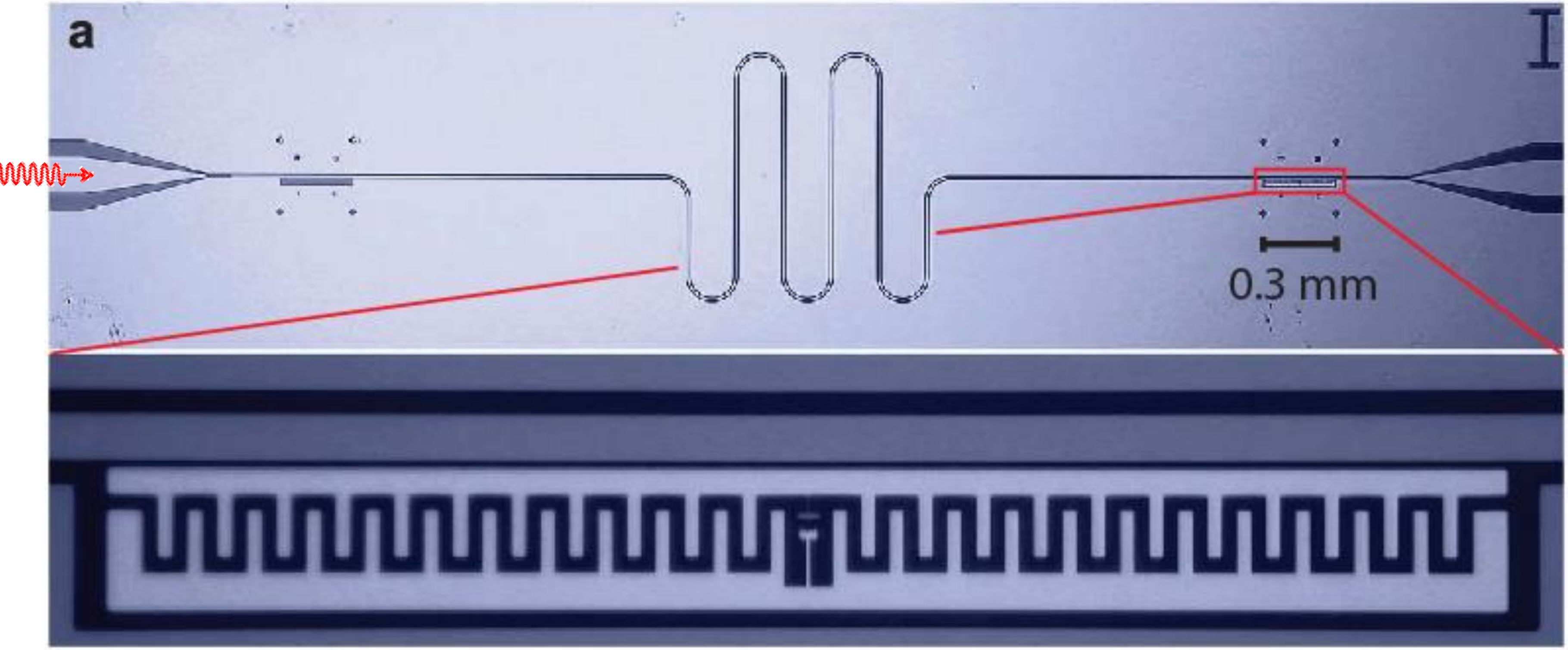
flux dependence of energy levels:



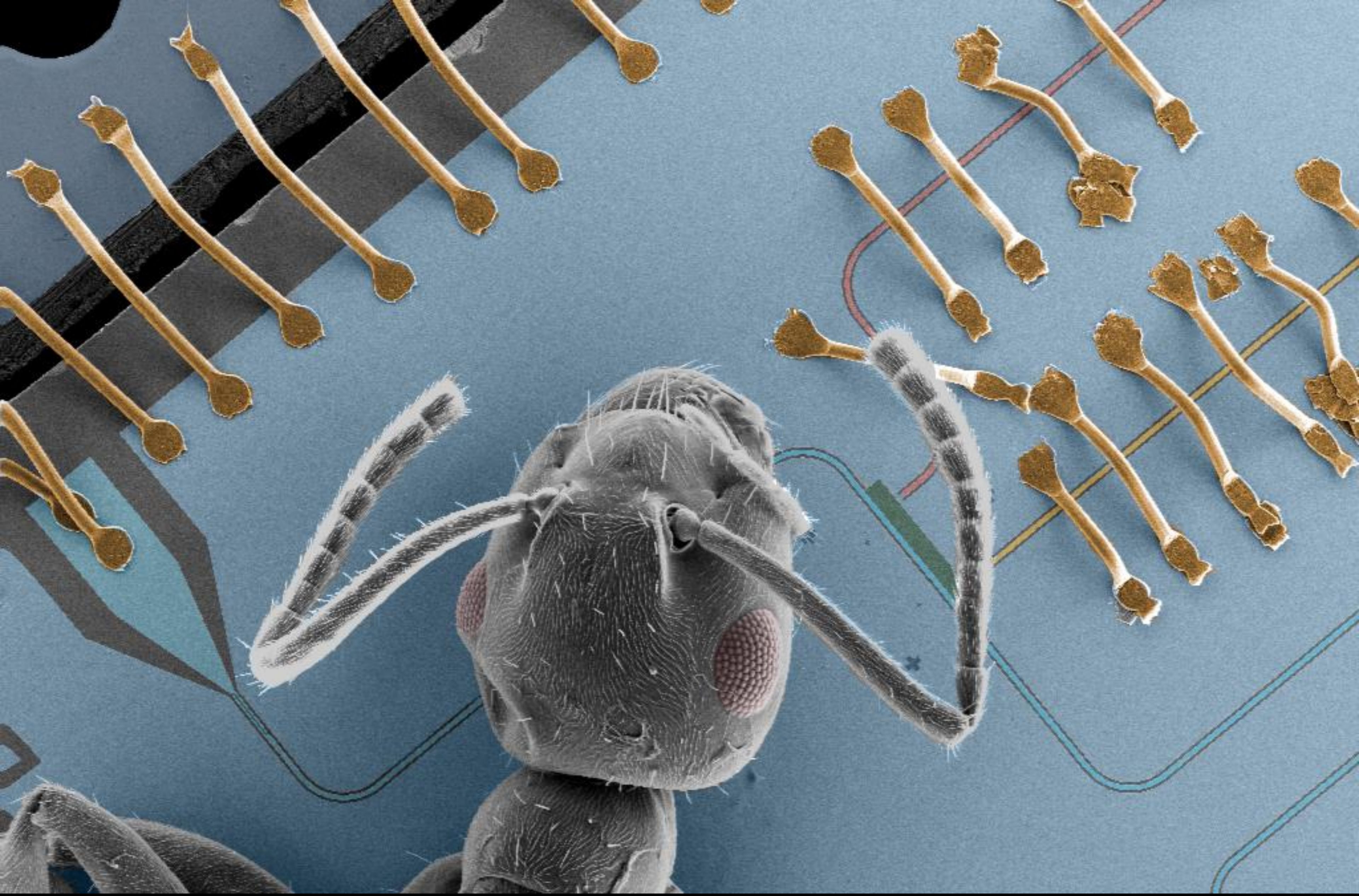
tune qubit into resonance

M. Baur, J. Fink (Quantum Device Lab, ETHZ, 2007)
more transmon experiments: J. Schreier *et al.* *PRB* **77**, 180502 (2008)

Realization



device fabrication: L. Frunzio et al., IEEE Trans. on Appl. Supercon. **15**, 860 (2005)



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

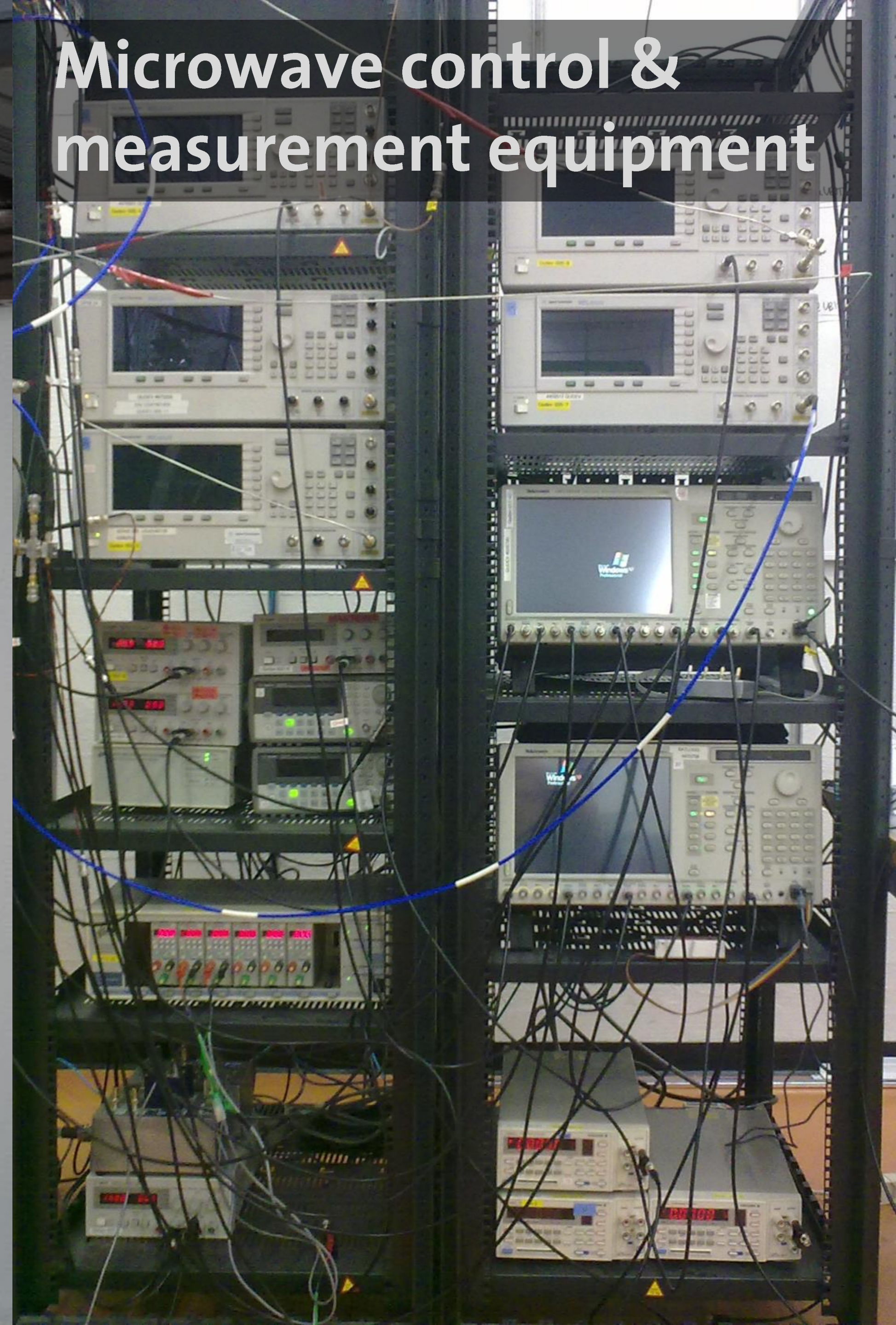
J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

Cryostate for temperatures down to 0.02 K

Microwave control & measurement equipment



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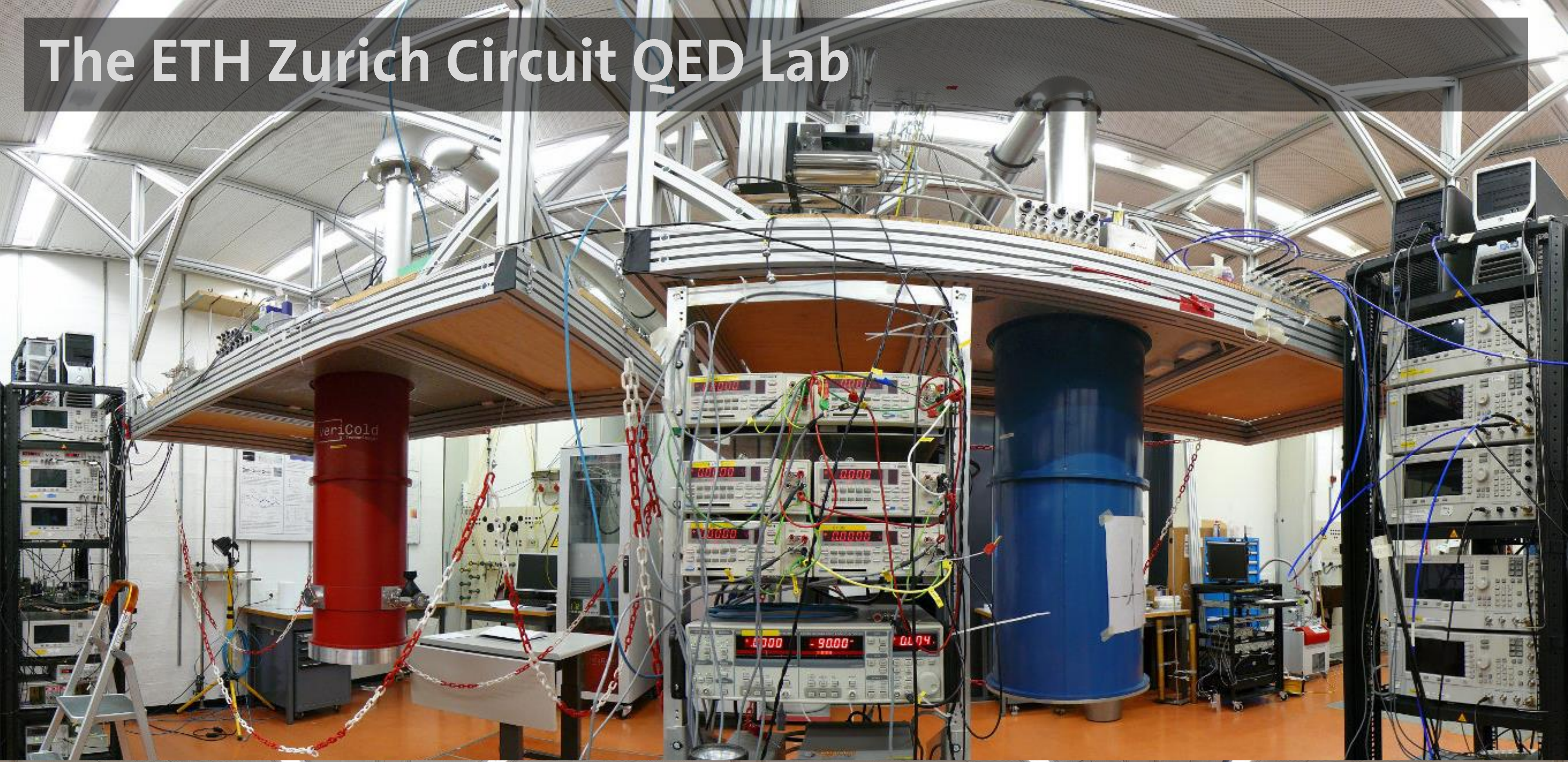
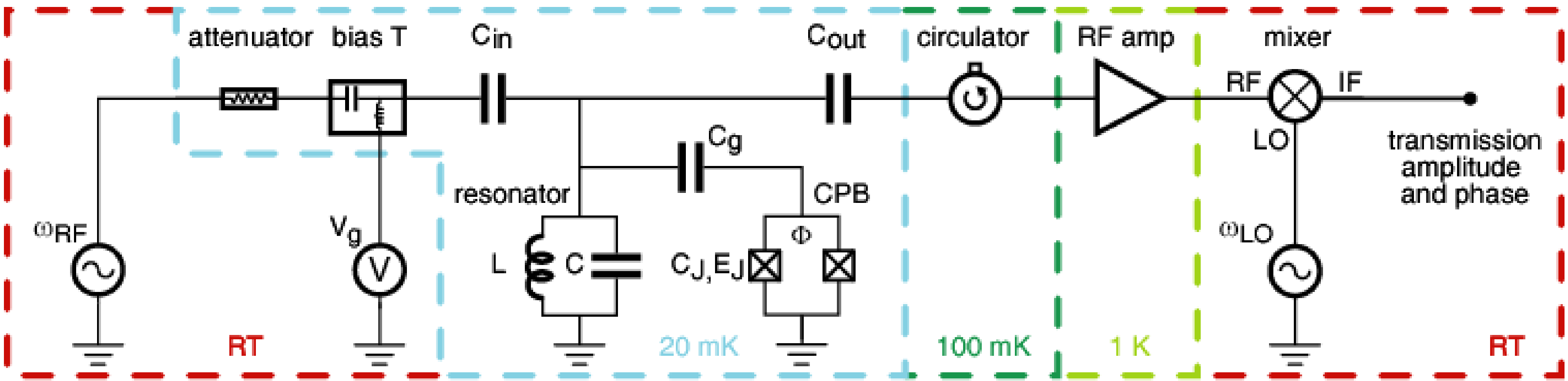
Swiss Federal Institute of Technology Zurich

The ETH Zurich Circuit QED Lab



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

How to do the Measurement



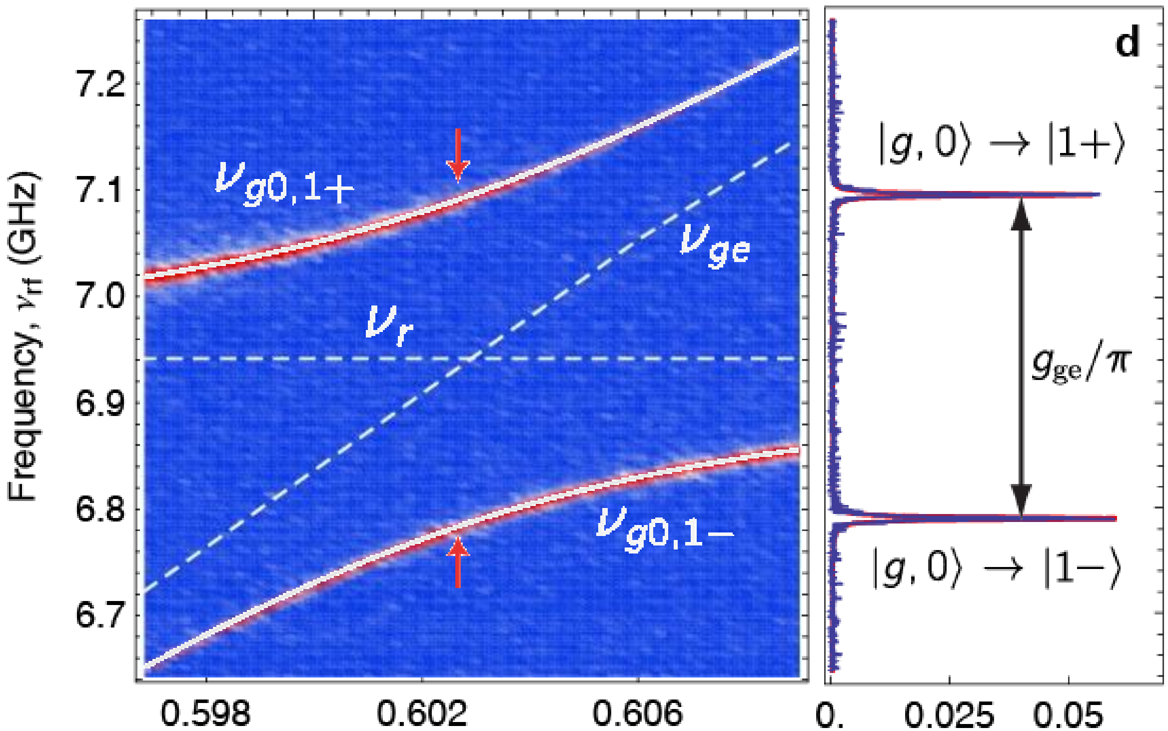
- prevent leakage of thermal photons (cold attenuators and circulators)
- average power to be detected ($\omega_r/2\pi$ = 6 GHz, $\kappa/2\pi$ = 1 MHz)

$$\langle n = 1 \rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

- efficient with cryogenic low noise HEMT amplifier T_N = 6 K

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

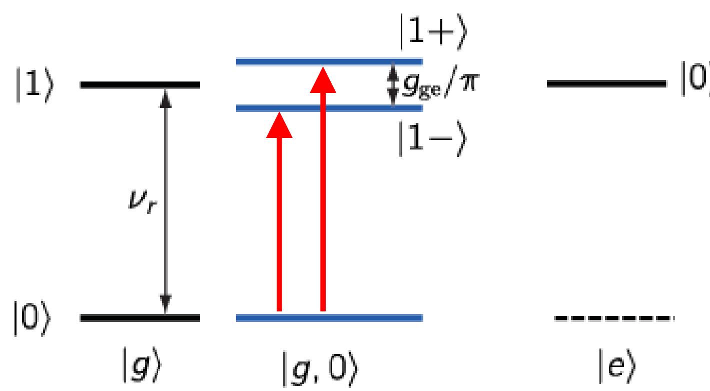


very strong coupling:

$$g_{ge}/\pi = 308\ \mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

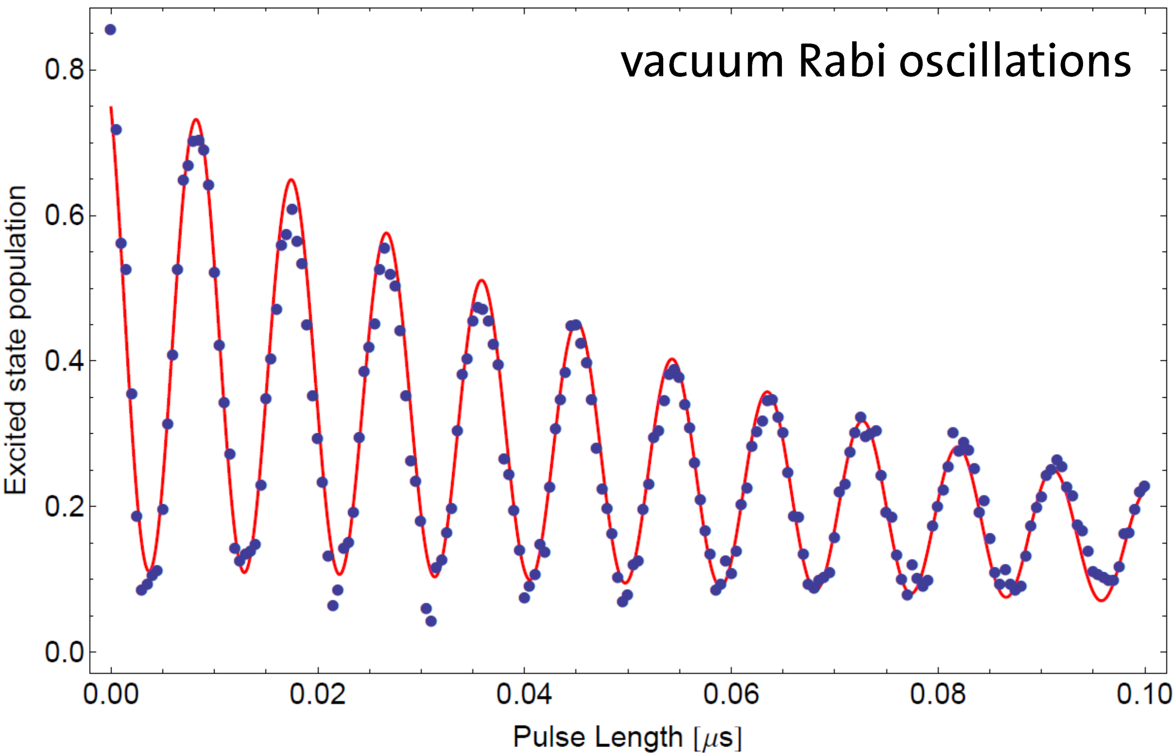
first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):



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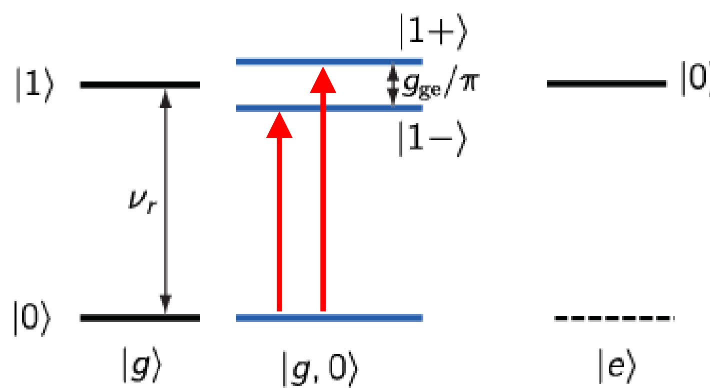

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this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

II. Exploring Matter Light Interaction in Circuit QED

Quantum Optics with Circuit QED ... a number of examples

Vacuum Rabi Mode Splitting
Wallraff *et al.*, *Nature* **431**, 162 (2004)

Coherent Flux-Qubit / SQUID Coupling
Chiorescu *et al.*, *Nature* **431**, 159 (2004)

Quantum AC-Stark & Lamb Shift
Schuster *et al.*, *Nature* **445**, 515 (2007)
Fragner *et al.*, *Science* **322**, 1357 (2008)

Fock and Arbitrary Photon States
Hofheinz *et al.*, *Nature* **454**, 310 (2008)
Hofheinz *et al.*, *Nature* **459**, 546 (2009)

Root n Nonlinearity
Fink *et al.*, *Nature* **454**, 315 (2008)
Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

Two Photon Nonlinearities
Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)

Parametric Amplification
Castellanos *et al.*, *Nat. Phys.* **4**, 928 (2008)
Bergeal *et al.*, *Nature* **465**, 64 (2010)

Ultrastrong Coupling
Niemczyk *et al.*, *Nat. Phys.* **6**, 772 (2010)

Cooling and Amplification
Grajcar *et al.*, *Nat. Phys.* **4**, 612 (2008)

Single-Photon Kerr Effect
Kirchmair *et al.*, *Nature* **495**, 205 (2013)

Single Photon Source
Houck *et al.*, *Nature* **449**, 328 (2007)

Single Qubit MASER
Astafiev *et al.*, *Nature* **449**, 588 (2007)

Single Qubit Resonance Fluorescence
Astafiev *et al.*, *Science* **327**, 840 (2010)

QND Measurement of Single Photon
Johnson *et al.*, *Nat. Phys.* **6**, 663 (2010)

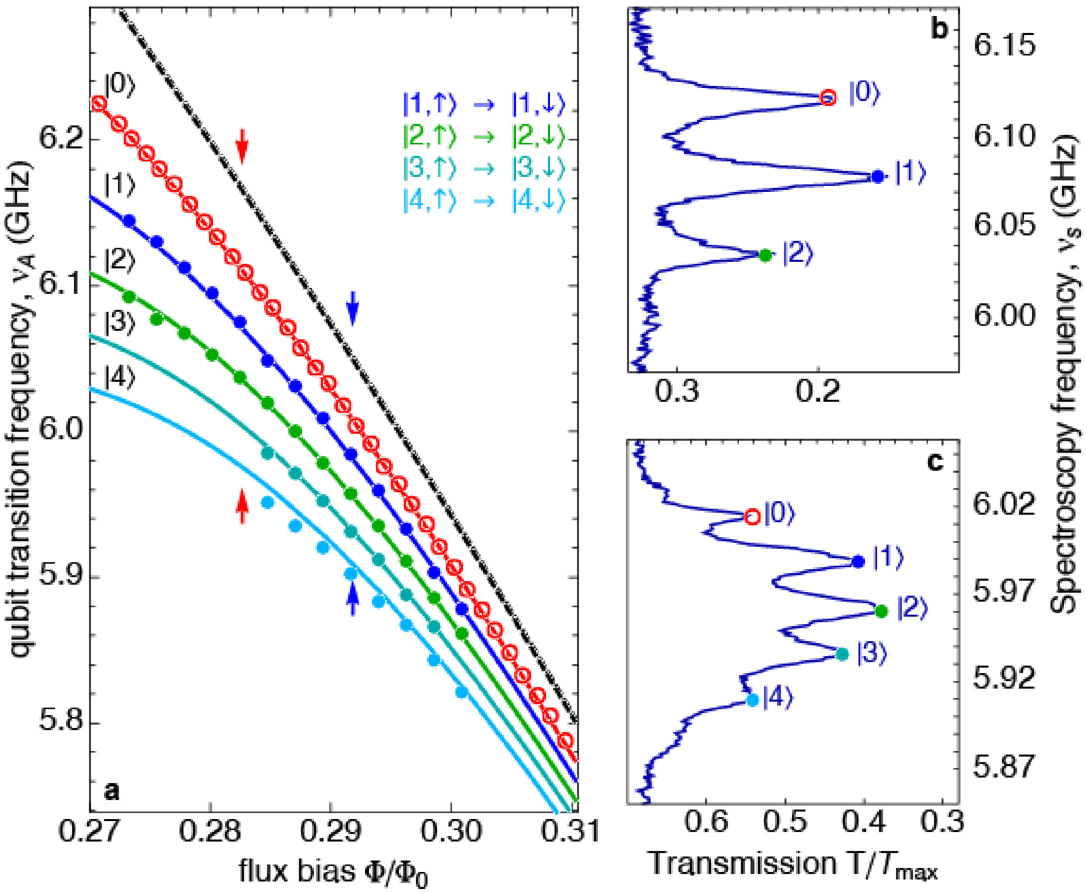
Correlation Function Measurements
Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

Hong-Ou-Mandel Effect
Lang *et al.*, *Nat. Phys.*, in print (2013)

Observation of Quantum Nonlinearities in Circuit QED

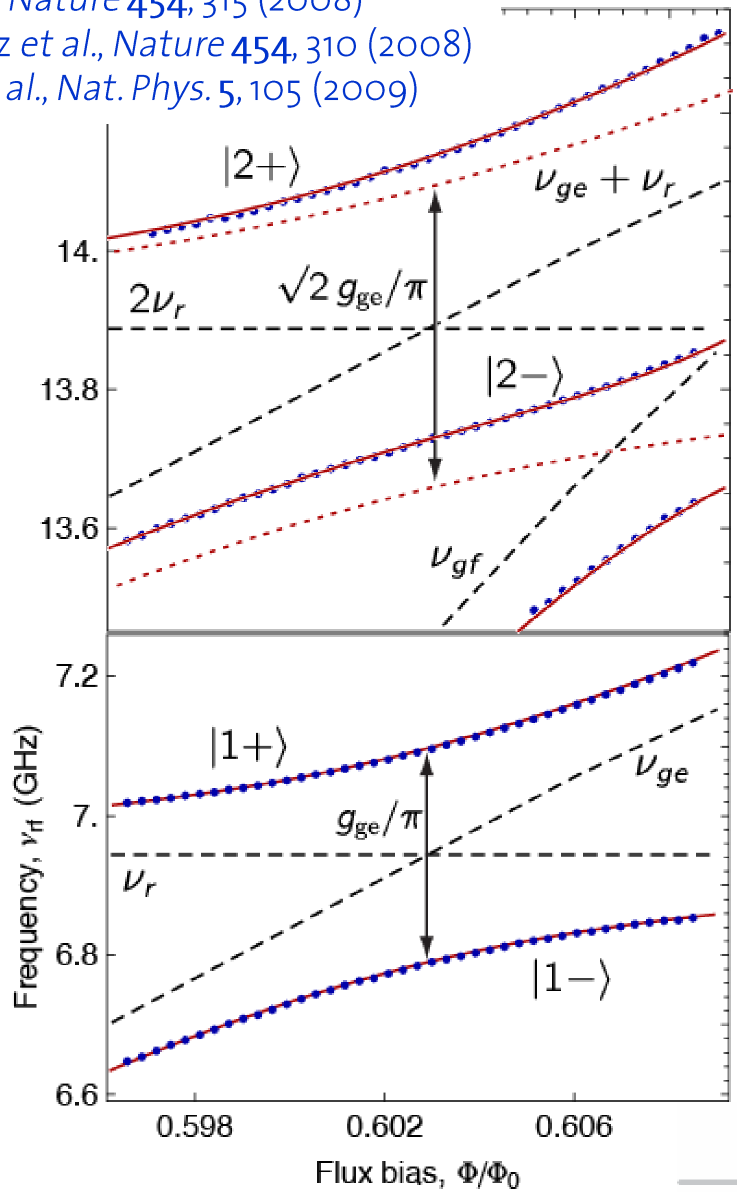
Strong Dispersive Coupling: Number-Splitting and the Lamb Shift

D. Schuster *et al.*, *Nature* **445**, 515 (2007)
A. Fragner *et al.*, *Science* **322**, 1357 (2008)

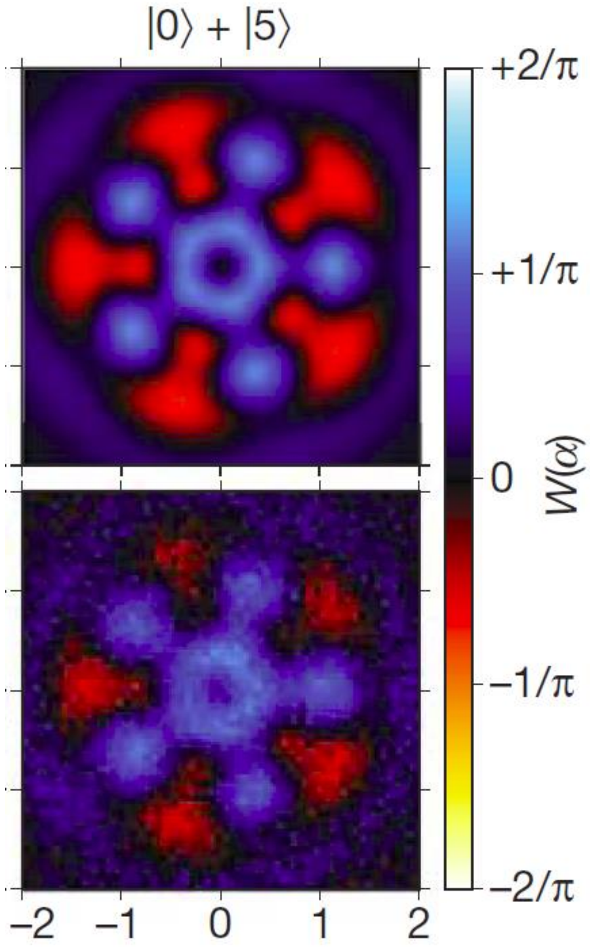


Root n Nonlinearities

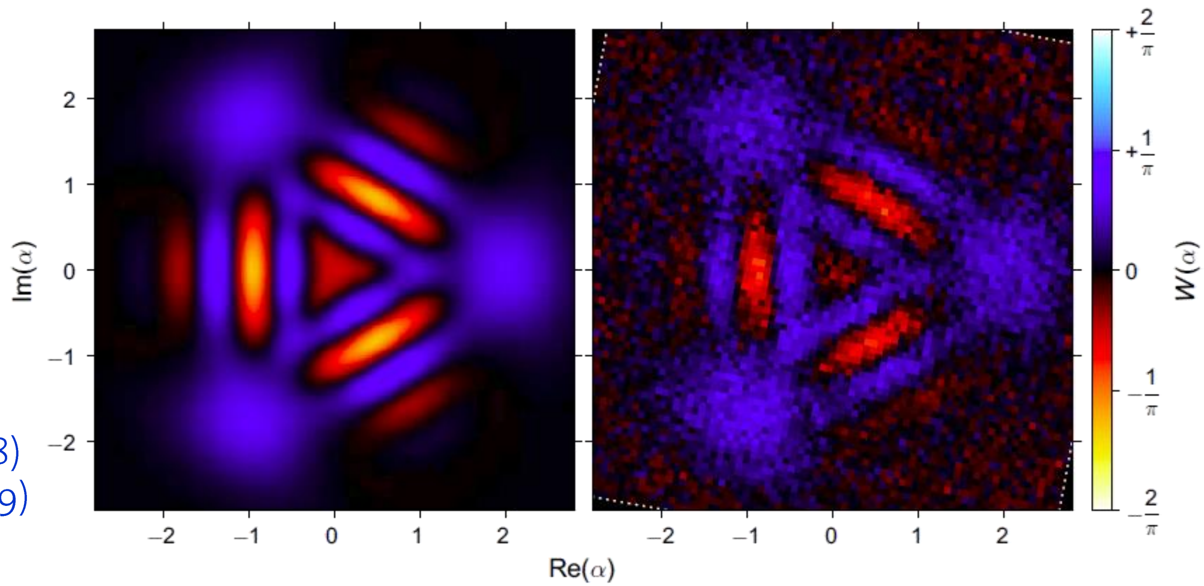
J. Fink *et al.*, *Nature* **454**, 315 (2008)
M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)



Intra-Cavity Fock States and their Superpositions



- controlled creation via qubit state preparation and resonant interaction with resonator
- Wigner tomography using resonant qubit/field interaction and qubit read-out
- creation of three component cat-states with coherent fields

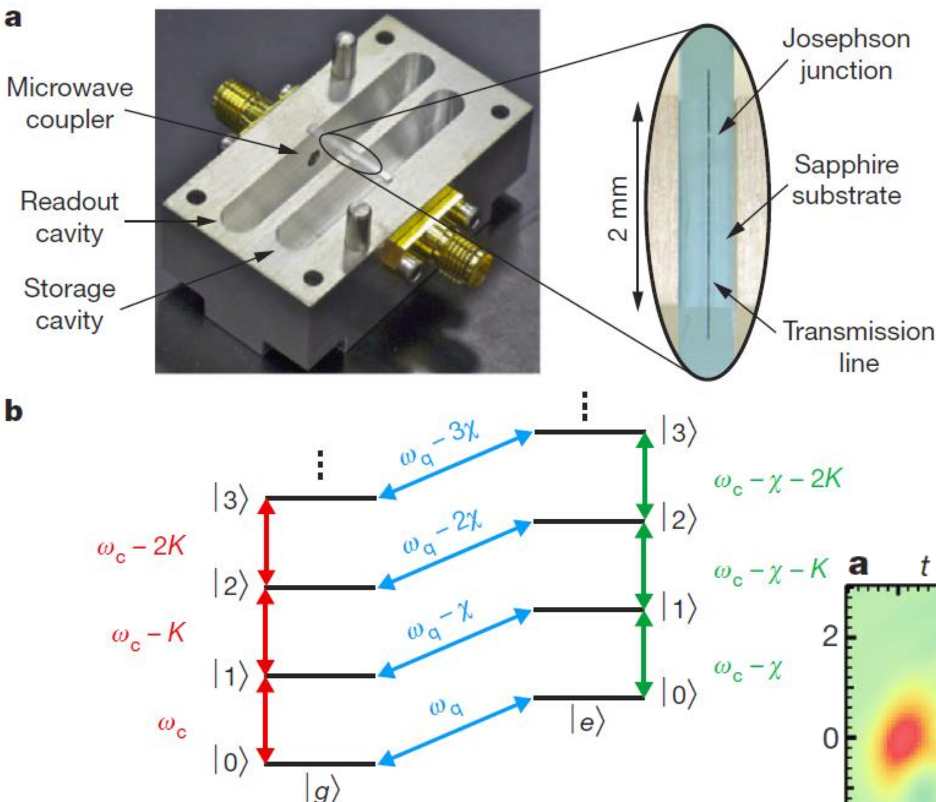


Fock and Arbitrary Photon States

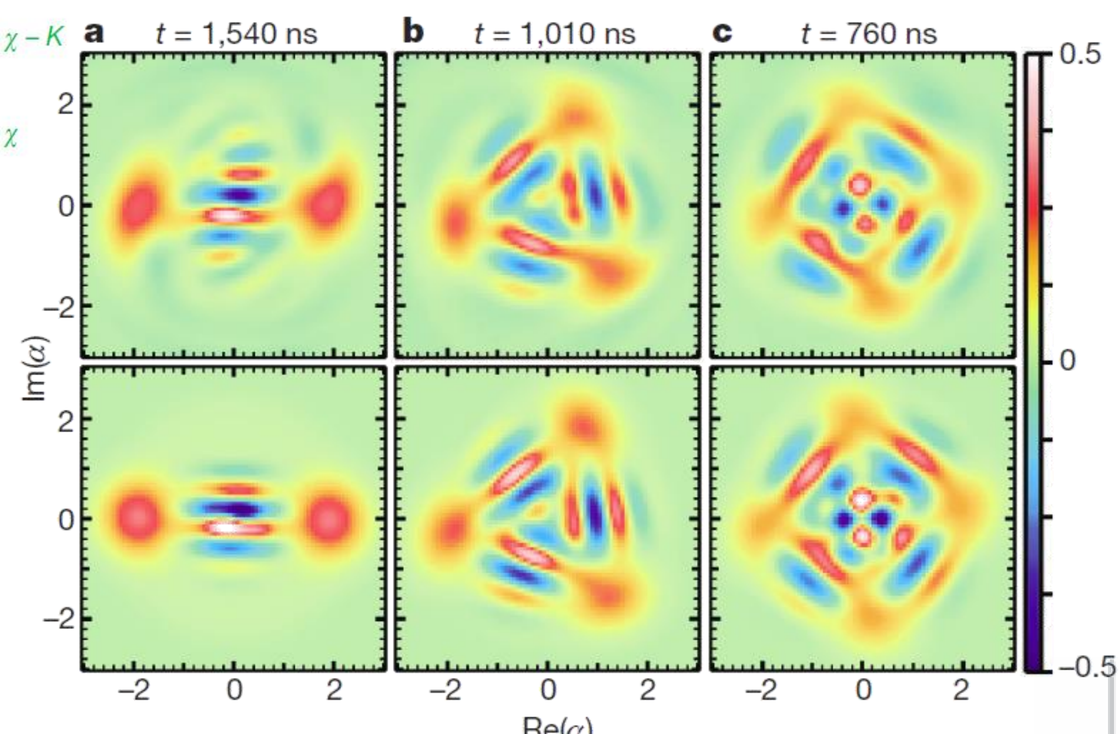
M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)

M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

Intra-Cavity Cat States Created by the Kerr Effect



- creation in cavity with Kerr non-linearity induced by a qubit
- Q-function measurement and Wigner tomography through projection into Fock basis



Single Photon Kerr Effect
Kirchmair *et al.*, *Nature* **495**, 205 (2013)

using QND photon number measurement:
Johnson *et al.*, *Nat. Phys.* **6**, 663 (2010)

Qubit Read-Out & Spectroscopy

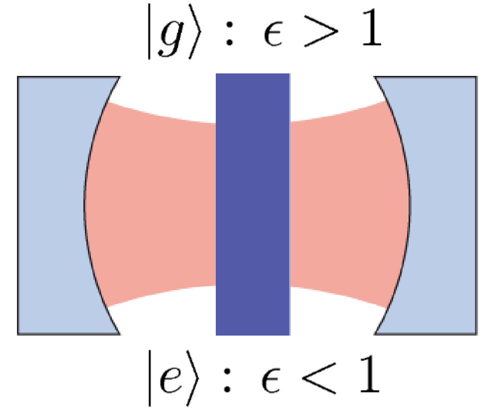
Non-Resonant Qubit-Photon Interaction

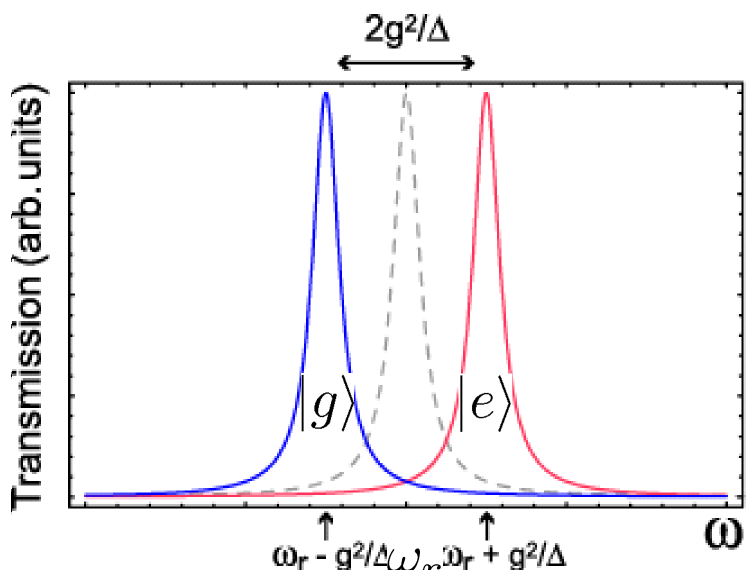
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

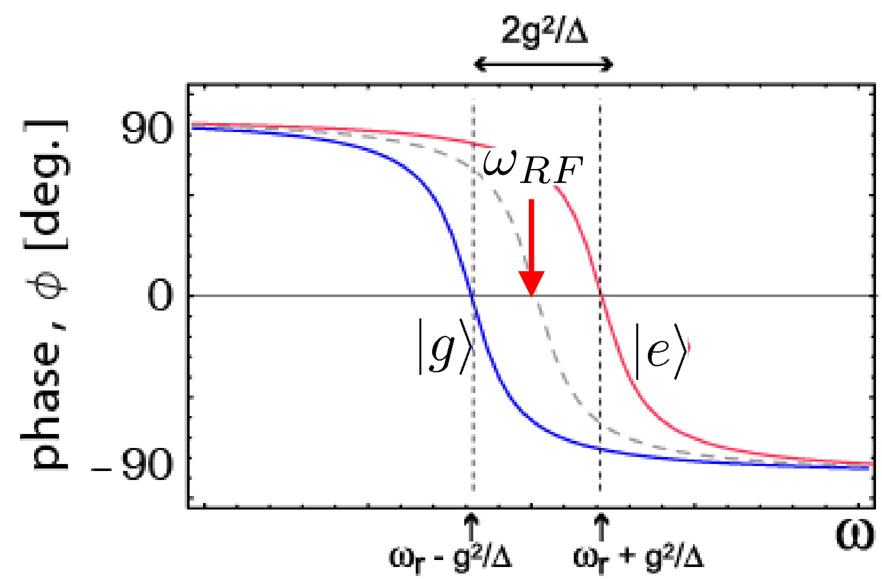
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

cavity frequency shift
and qubit ac-Stark shift

Lamb shift

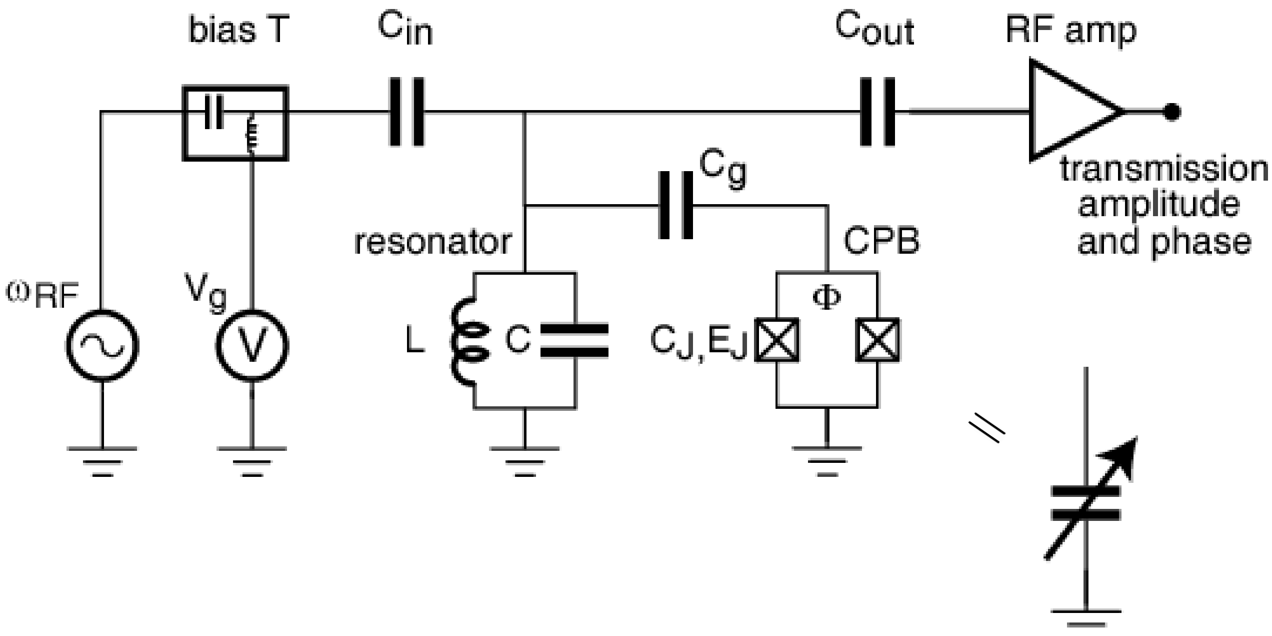






A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Measurement Technique



- measurement of microwave transmission amplitude T and phase ϕ
- intra-cavity photon number controllable from $n \sim 10^3$ to $n \ll 1$

Non-Resonant Coupling for Qubit Readout

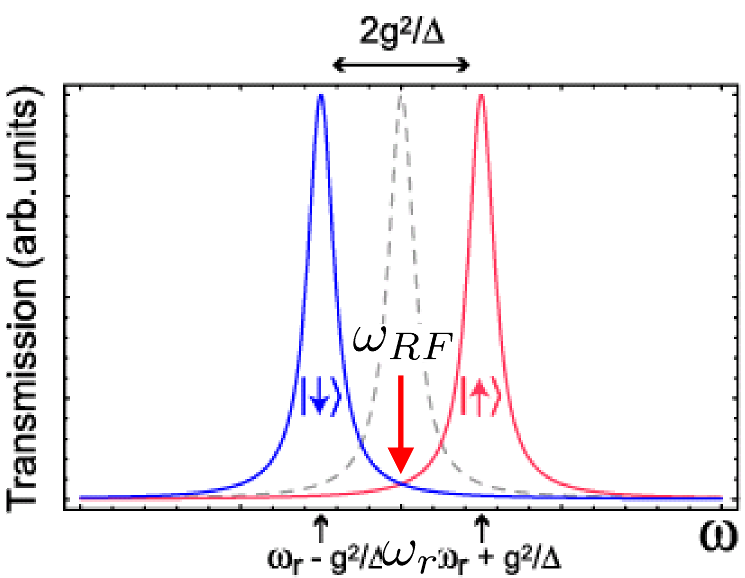
approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

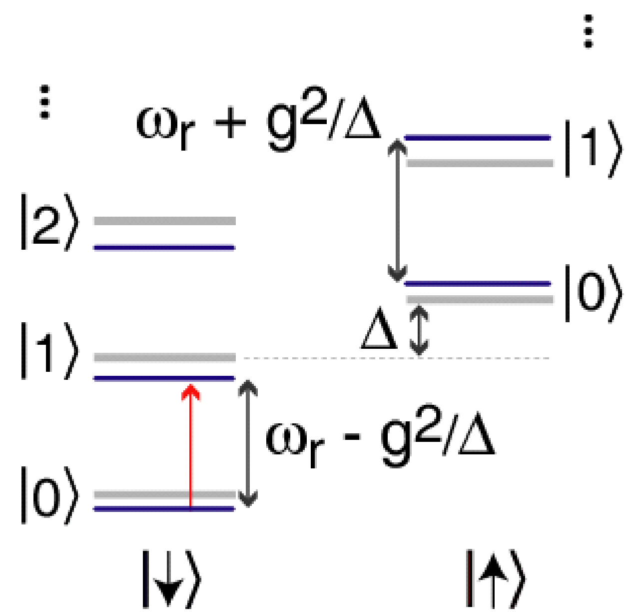
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z \right) a^\dagger a + \frac{1}{2}\hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

cavity frequency shift
and qubit ac-Stark shift

Lamb shift

dispersive level diagram:



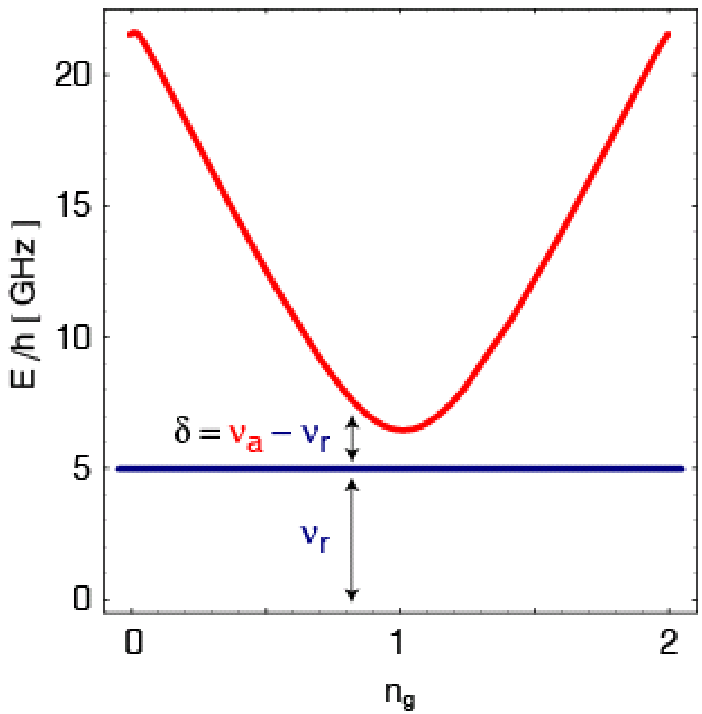


A. Blais *et al.*, *PRA* **69**, 062320 (2004)

Dispersive Shift of Resonance Frequency

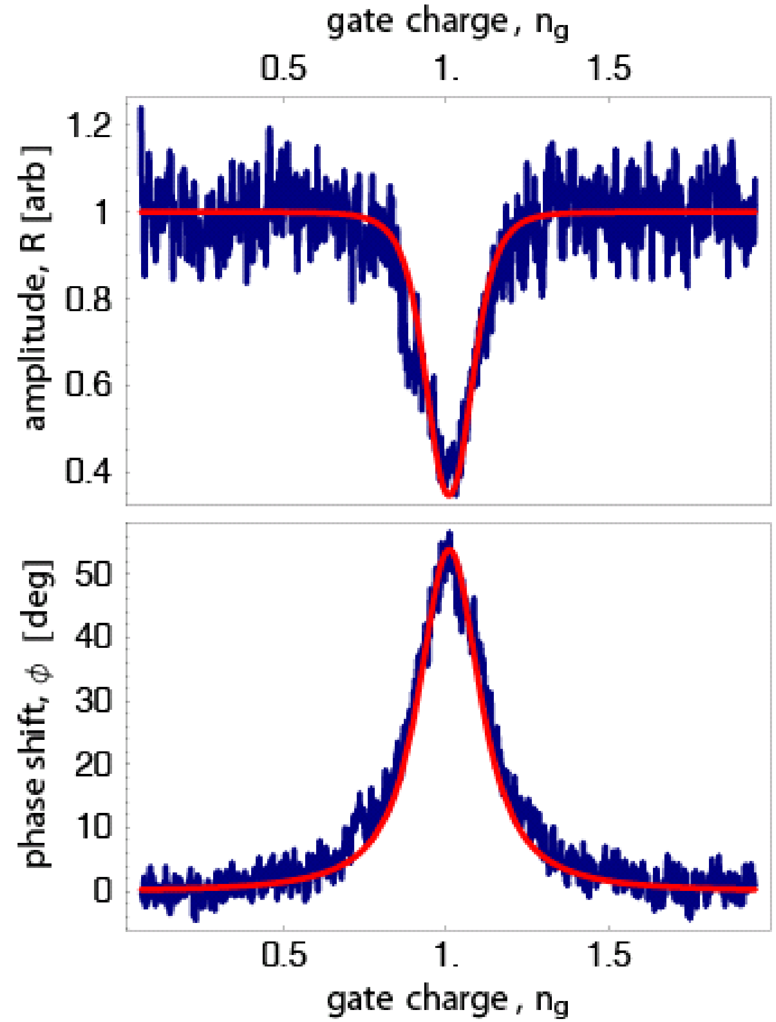
sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

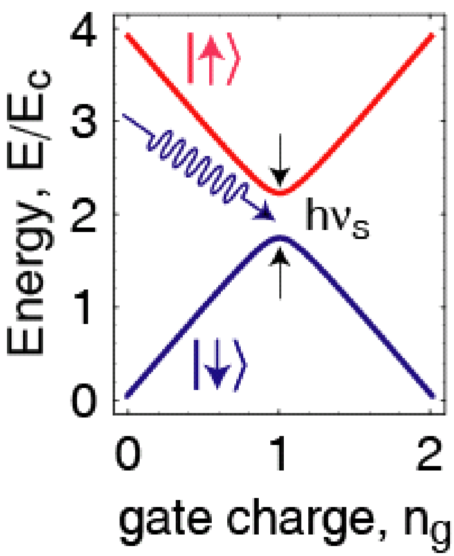


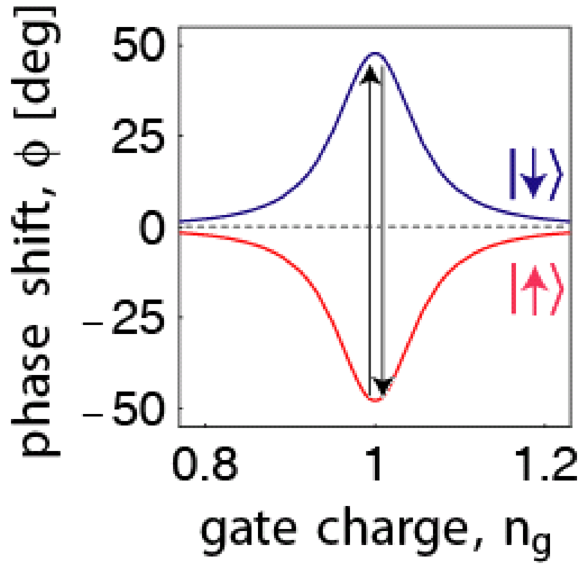
$$g/\pi = \nu_{\text{vac}} = 11\,\text{MHz}$$
$$\Delta(n_g = 1)/2\pi = 66\,\text{MHz}$$
$$n = 10$$

measured resonator transmission amplitude and phase:



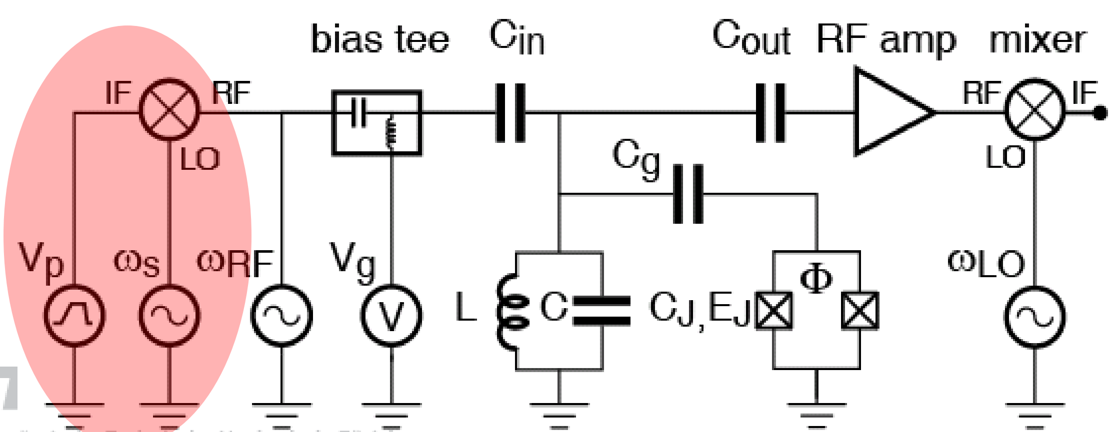
Coherent Control and Read-out in a Cavity





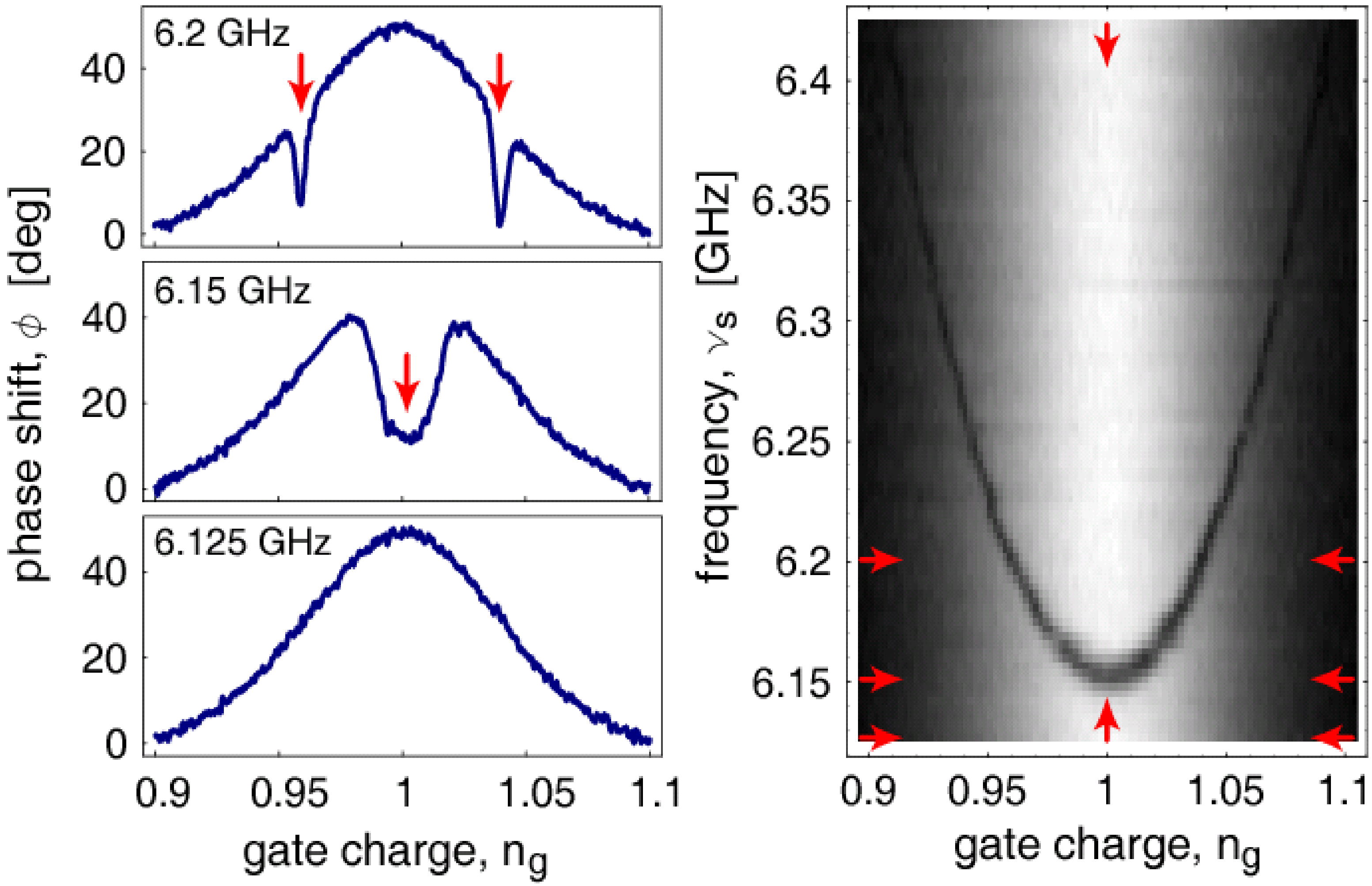
- apply resonant microwave pulse to qubit
- detect change of phase

realization:



- simultaneous control and measurement

CW Spectroscopy of Cooper Pair Box



detuning $\Delta_{\mathrm{r,a}}/2\pi \sim 100\,\mathrm{MHz}$

extracted: $E_J = 6.2\,\mathrm{GHz},\ E_C = 4.8\,\mathrm{GHz}$

D. I. Schuster et al., *Phys. Rev. Lett.* **94**, 123062 (2005)

III. Characterizing Propagating Microwave Photons with Linear Detectors

- Correlation Function Measurements
- Quantum State Tomography
- Qubit Photon Entanglement
- Two-Mode Entanglement

Exploring the Properties of Propagating Photons

quantum optics in the **visible**:

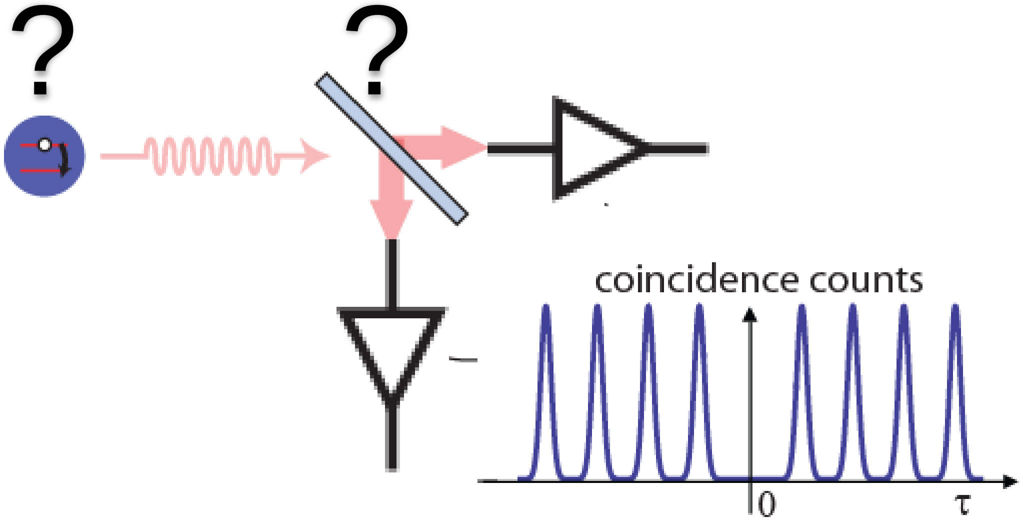
- single photon sources
- beam splitters
- photon counters

o.k. at **optical frequencies**

But in the **microwave domain**?

- smaller photon energy ...

$$\frac{\nu_{\mathrm{opt}}}{\nu_{\mu\mathrm{w}}} = \frac{500\,\mathrm{THz}}{5\,\mathrm{GHz}} = 10^5$$



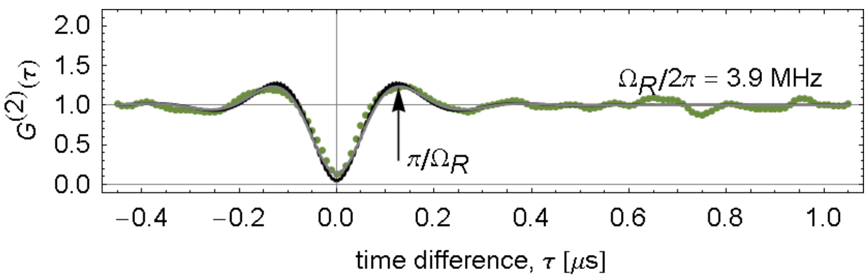
instead:

- linear amplifiers
- signal processing

J. Gabelli et al., *Phys. Rev. Lett.* **93**, 056801 (2004)
E. P. Menzel et al., *Phys. Rev. Lett.* **105**, 100401 (2010)
M. P. da Silva et al.. Phys. *Rev. A* **82**, 043804 (2010)
C. Eichler et al., *Phys. Rev. A* **86**, 032106 (2012)

Experiments with Propagating Quantum Microwaves

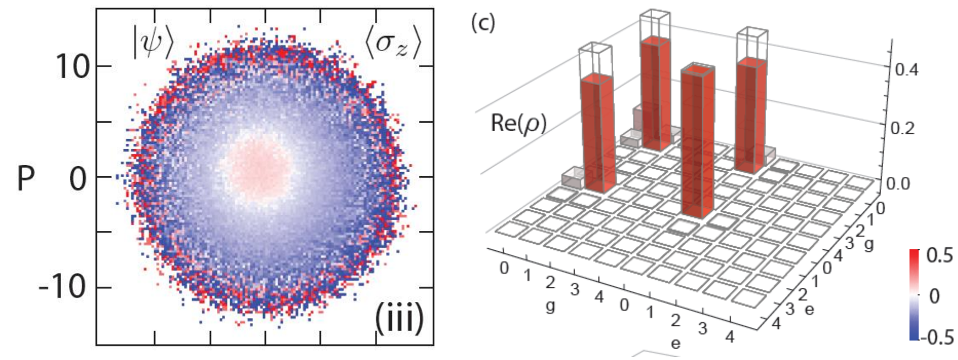
Single photon sources and their anti-bunching



Lang *et al.*, *PRL* **107**, 073601 (2011)

Bozyigit *et al.*, Nat. Phys **7**, 154 (2011)

Preparation and characterization of qubit-**propagating** photon entanglement

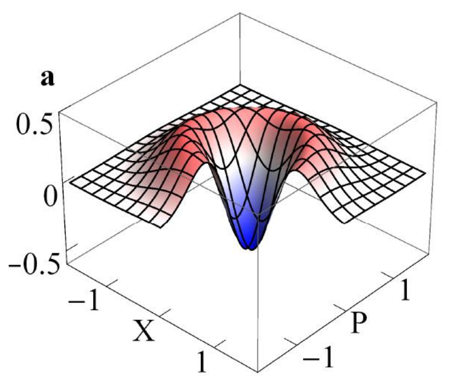


Eichler *et al.*, *PRL* **109**, *240501* (2012)

Eichler *et al.*, *PRA* **86**, 032106 (2012)

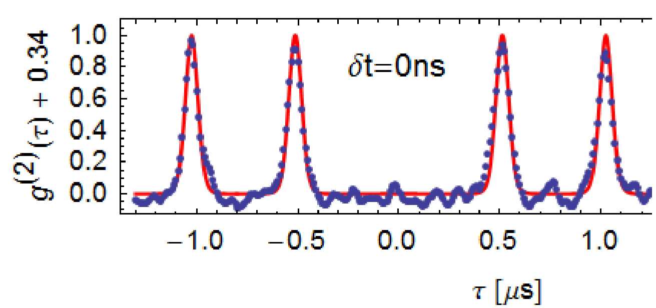
Wigner functions and full state tomography of propagating photons:

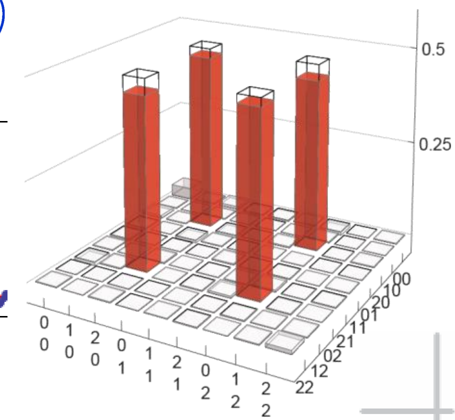
Eichler *et al.*, *PRL* **106**, 220503 (2011)



Hong-Ou-Mandel: Two-photon interference with coherences at microwave frequencies

Lang *et al.*, *Nat. Phys.* **9**, 345 *(2013)*





Propagating Quantum Microwaves

Correlation Function Measurements of Single Photons

Bozyigit *et al.*, Nat. Phys **7**, 154 (2011)
Lang *et al.*, *PRL* **107**, 073601 (2011)

Quantum State Tomography

Mallet *et al.*, *PRL* **106**, 220502 (2011)
Eichler *et al.*, *PRL* **106**, 220503 (2011)

Photon Routers

Hoi *et al.*, *PRL*, **107**, 073601 (2011)

Single Photon Detectors

Chen *et al.*, *PRL* **107**, 217401 (2011)

Positive P-Function/Dual Path Detection

Menzel *et al.* PRL **105**, 100401 (2010)
Eichler *et al.*, *PRA* **86**, 032106 (2012)

Photon/Qubit Entanglement

Eichler *et al.*, *Phys. Rev. Lett.* **109**, 240501 (2012)

Hong-Ou-Mandel 2-Photon Interference

Lang *et al.*, *Nat. Phys.* **9**, 345 *(2013)*

Thermal and Vacuum Noise

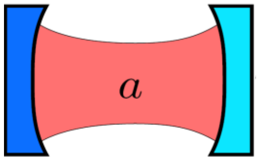
Mariantoni *et al.*, *PRL* **105**, 133601 (2010)

Squeezing & Two Mode Correlations

Castellanos *et al.*, *Nat. Phys.* **4**, 929 (2008)
Eichler *et al.*, *PRL* **107**, 113601 (2011)
Bergeal *et al.*, *PRL* **108**, 123902 (2012)
Flurin *et al.*, *Phys. Rev. Lett.* **109**, 183901 (2012)
Menzel *et al.*, *Phys. Rev. Lett.* **109**, 250502 (2012)

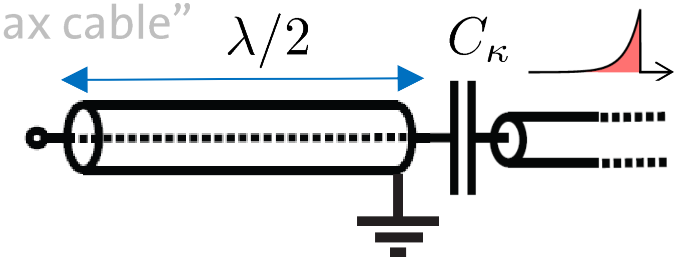
Quantum Microwave Sources: Components

cavity



=

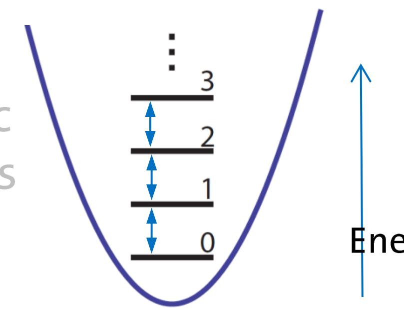
"piece of a coax cable"



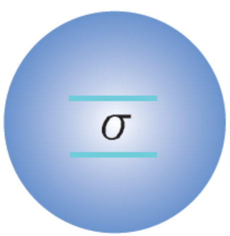
Radiation field stored inside:

$$H = \hbar\omega \ a^\dagger a$$

harmonic oscillators

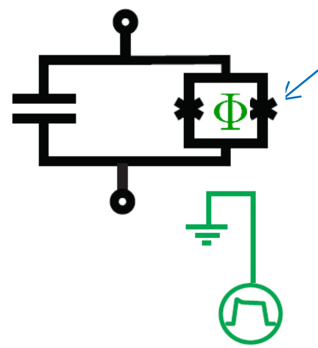


atom



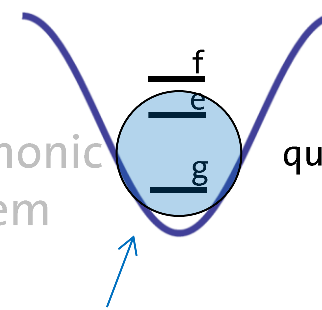
=

transmon



Josephson Junction = "Nonlinear Inductor"

anharmonic system



qubit

Change transition energy on **ns** timescales!

Quantum Microwave Sources: Chip and Operation

- Transmon qubit

  $T_1 = 1.1\,\mu s$

  $T_2 = 550\,ns$

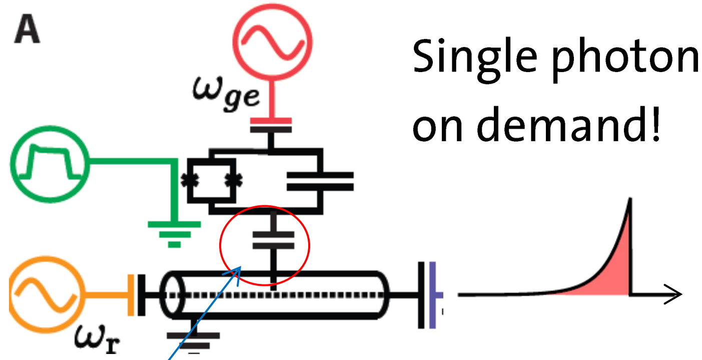
  $T_2^* = 220\,ns$

- Single sided resonator

  $1/\kappa = 25\,ns$

- Coupling strength

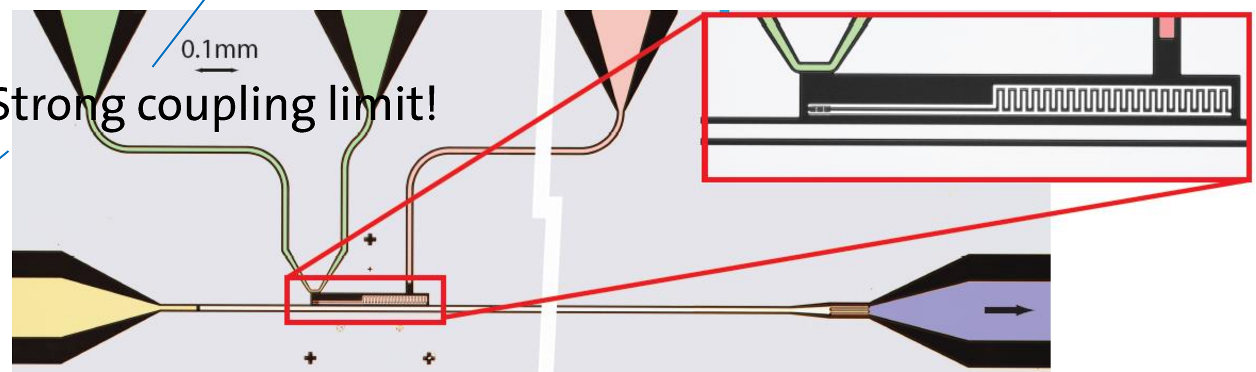
  $\pi/g = 7.7\,ns$



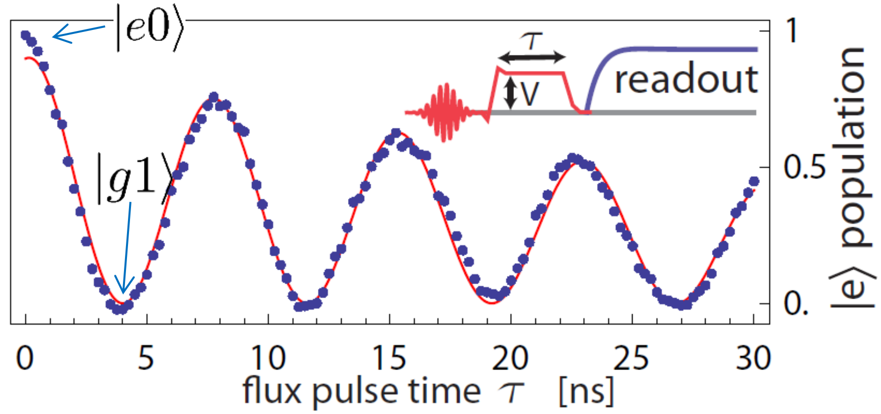
Single photons on demand!

Detection ?

Strong coupling limit!



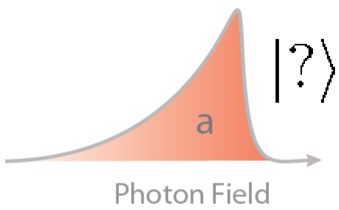
Vacuum Rabi oscillations



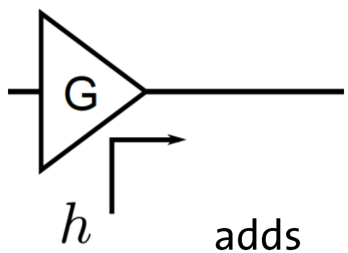
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Houck *et al.*, *Nature* **449**, 328 (2007), Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

Microwave Photon Field Detection

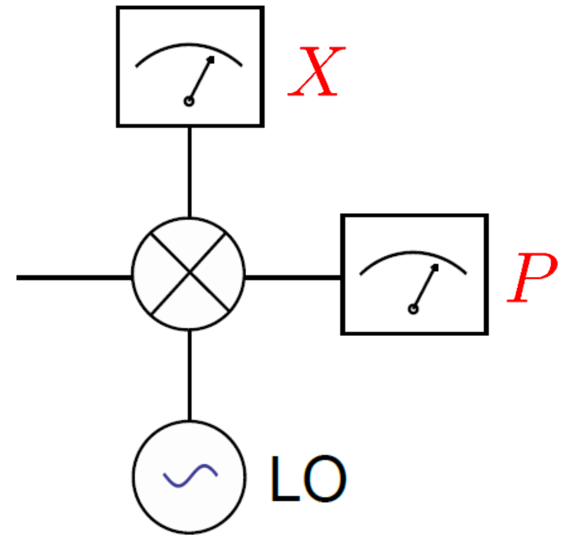


linear amplifier



adds vacuum/thermal noise

amplitude detector = ADC & digital filtering

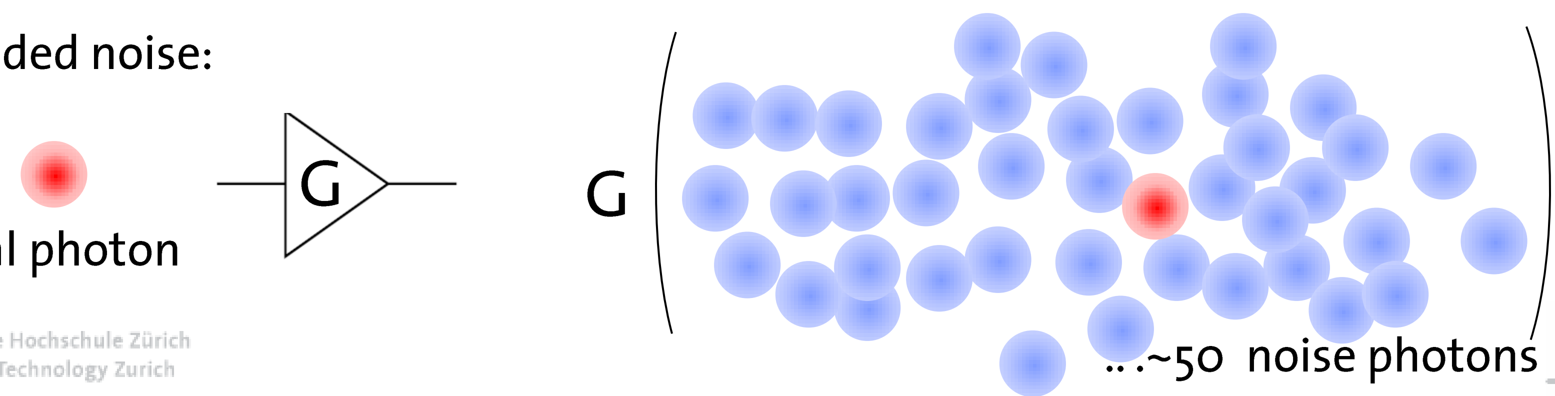


complex amplitude: $S = X + iP = a + h^\dagger$

"signal" → a, "noise" → $h^\dagger$

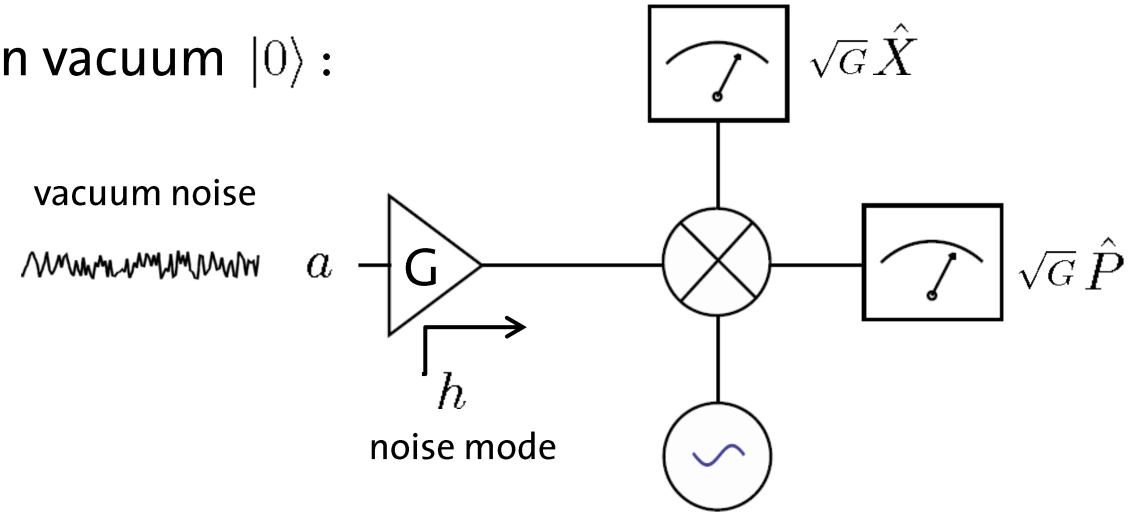
Eichler *et al.*, *PRA* **86**, 032106 (2012)
M. P. da Silva et al., *PRA* **82**, 043804 (2010).
C. M. Caves, *PRD* **26**, 1817 (1982).

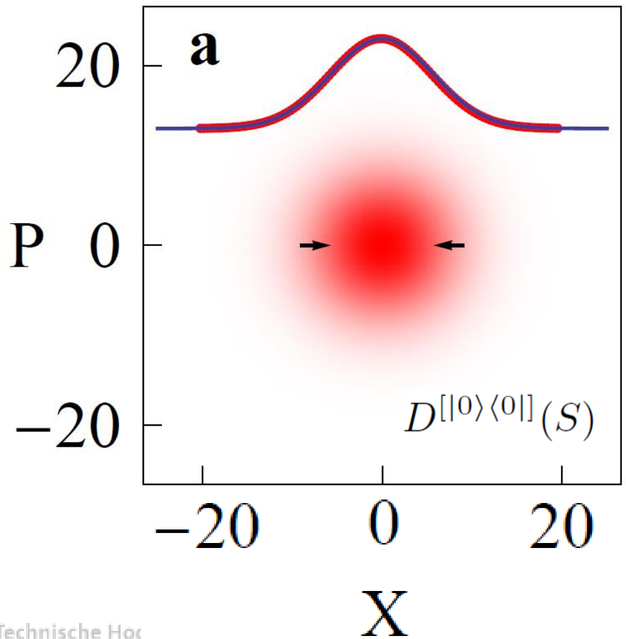
typical added noise:



Full Tomography of a Single Propagating Mode

1) prepare a in vacuum $|0\rangle$:





record histogram $D^{[|0\rangle\langle 0|]}(S)$

of measurement results $S/\sqrt{G} = X + iP$

normal distribution with variance

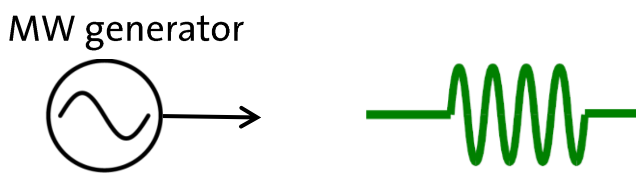
$$2\sigma^2 = \langle \hat{S}^\dagger \hat{S} \rangle / G = \frac{1}{G} \int d^2S \, D^{[|0\rangle\langle 0|]}(S)\, S^* S = 67$$

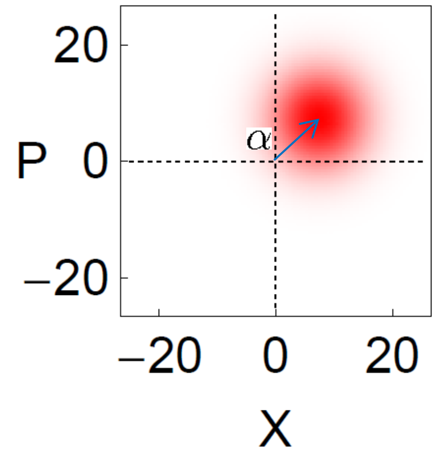
h introduces thermal noise

with mean photon number N_{noise}

Coherent State Histograms

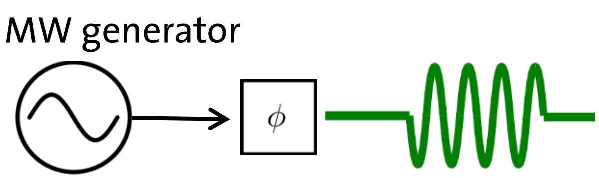
2) prepare a in coherent state $|\alpha\rangle$:

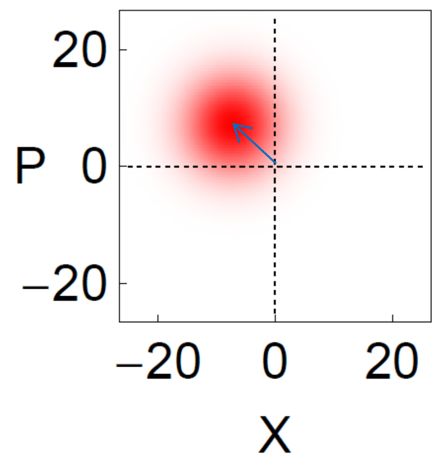




$$|\alpha| \approx 6.3$$
$$\Leftrightarrow$$
$$\langle a^\dagger a \rangle \approx 41 \sim N_{\mathrm{noise}}$$

3) rotate phase $|e^{i\phi}\alpha\rangle$:





Question: What can we learn about state when $\langle a^\dagger a \rangle \leq 1$?

Single Photon Source Histogram

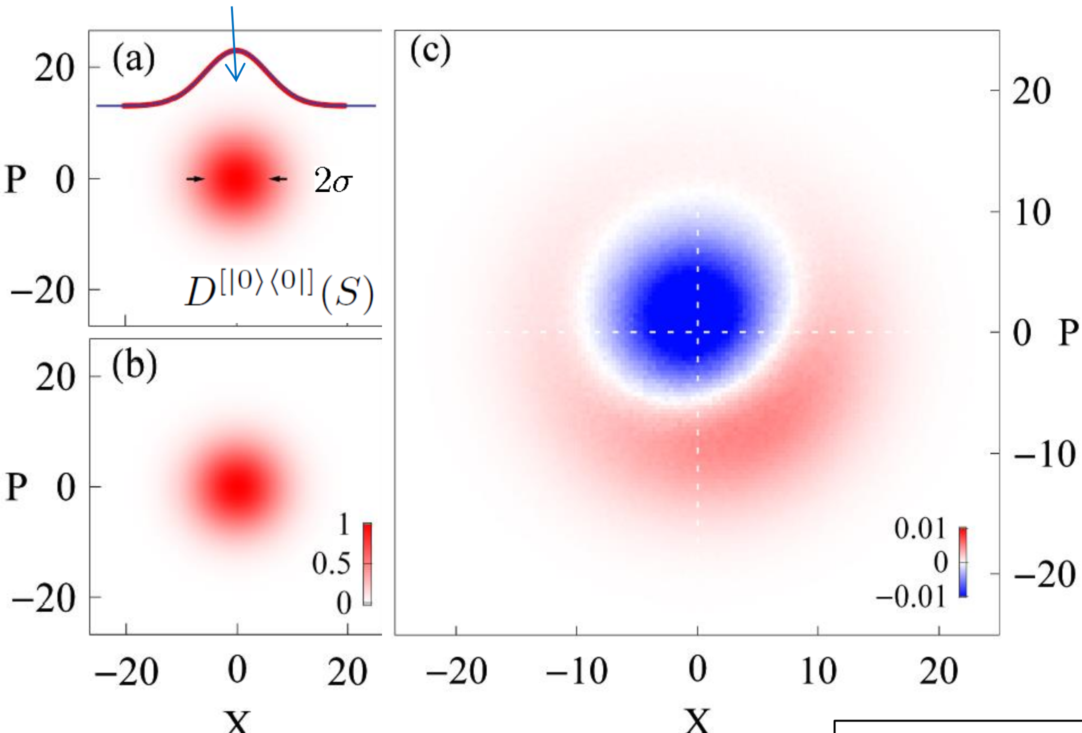
store 2D histogram $D^{[\rho]}(S)$ from $S/\sqrt{G} = X + iP$ measurement results:

corresponding phase space distribution

signal mode a in **vacuum**

Q - function of noise mode : Q_h

convolution with **P** – function of signal $Q_h * P_a$



subtracted histograms to visualize difference

signal mode a in **single photon Fock state**

separate noise h from signal a systematically!

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

Statistical Analysis of Histograms

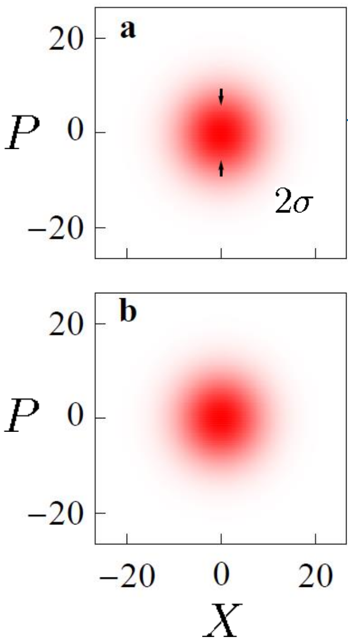
systematic mode separation:

histogram moments: $$\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_\rho = \int \mathrm{d}^2 S \; (S^*)^n S^m \; D^{[\rho]}(S)$$

1. calculate histogram moments

2. algebraic inversion

vacuum $|0\rangle$



Fock state $|1\rangle$

$$\frac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|0\rangle\langle 0|}}{G^{(n+m)/2}} = \langle h^n (h^\dagger)^m \rangle$$

$$\frac{\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_{|1\rangle\langle 1|}}{G^{(n+m)/2}} = \langle (h + a^\dagger)^n (h^\dagger + a)^m \rangle$$

$$\Rightarrow \langle (a^\dagger)^n a^m \rangle$$

reminder: $X + iP = S/\sqrt{G} = (a + h^\dagger)$

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

State Dependent Moments of Probability Distribution

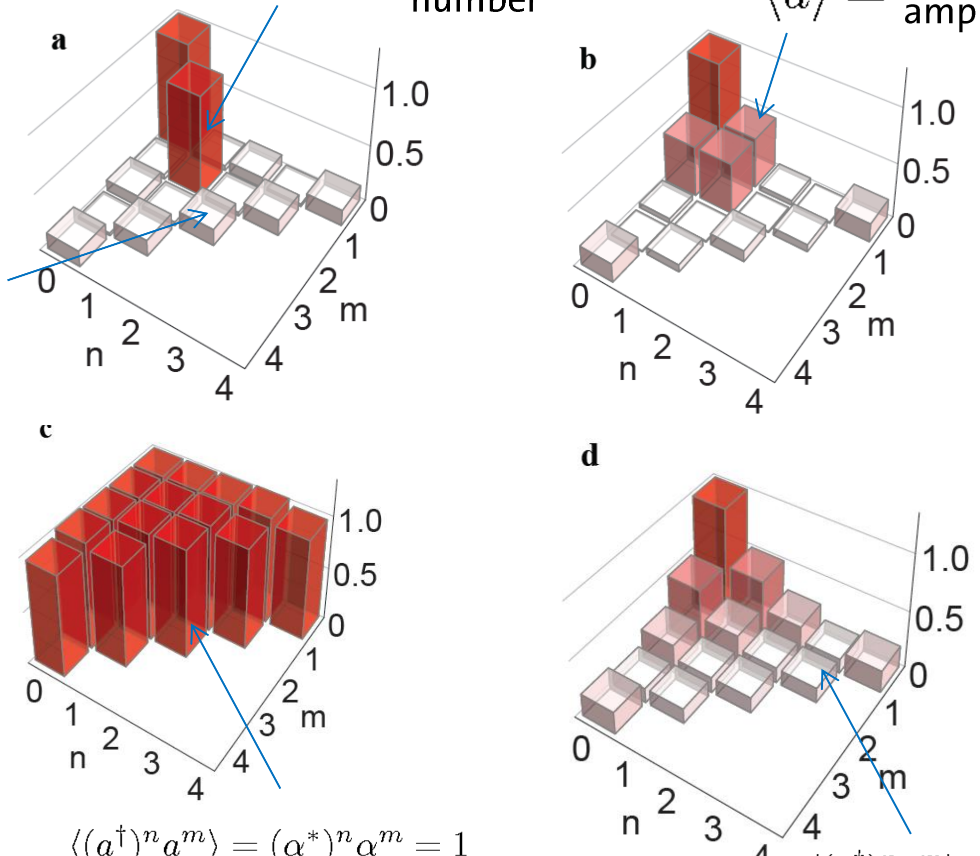
moments $|\langle (a^\dagger)^n a^m \rangle|$ for different prepared states:

Fock state $|1\rangle$

$\langle a^\dagger a \rangle$ = "mean photon number"

$\langle a \rangle$ = "mean amplitude"

$\langle (a^\dagger)^2 a^2 \rangle \approx 0$ "anti bunching"



superposition $\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

coherent state $|\alpha = 1\rangle$

$\langle (a^\dagger)^n a^m \rangle = (\alpha^*)^n \alpha^m = 1$

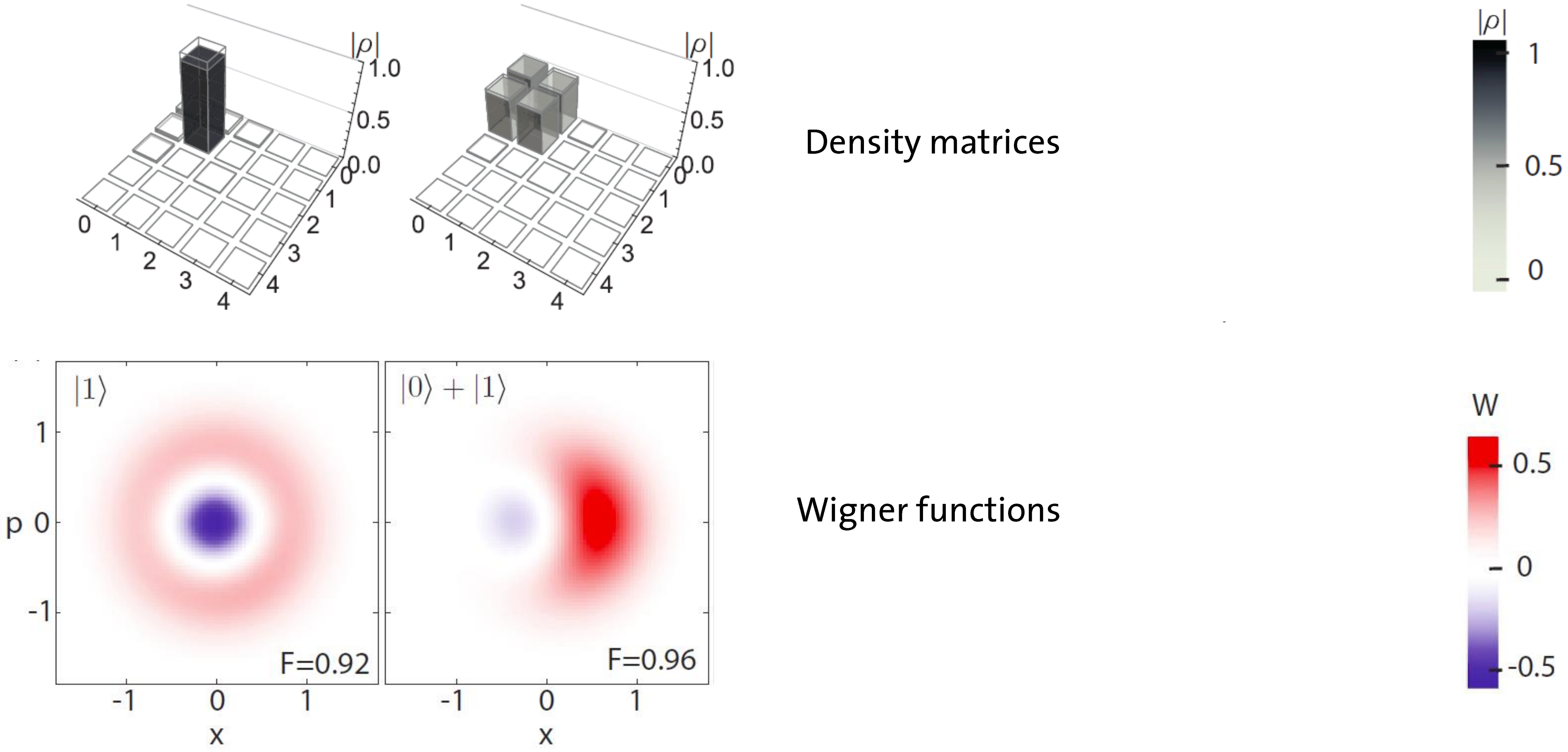
coherent state $|\alpha = 0.5\rangle$

$\langle (a^\dagger)^n a^m \rangle = (1/2)^{n+m}$

C. Eichler, et al., *PRL* **106**, 220503 (2011), *PRA* **86**, 032106 (2012)

Reconstruct Density Matrices and Wigner functions…

… for **propagating multi-photon** Fock states and their superpositions:

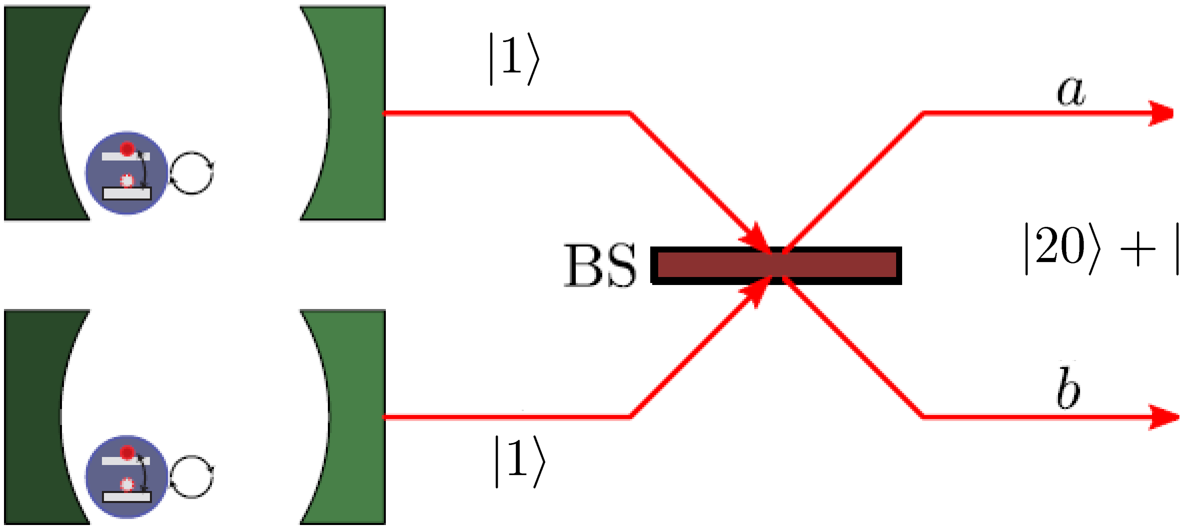


measured using near-quantum-limited parametric amplifier

C. Eichler, et al., *PRL* **106**, 220503 (2011)
C. Eichler et al., *PRA* **86**, 032106 (2012)

Hong-Ou-Mandel Experiments with Microwaves

Measure field – field correlations in two spatial modes:



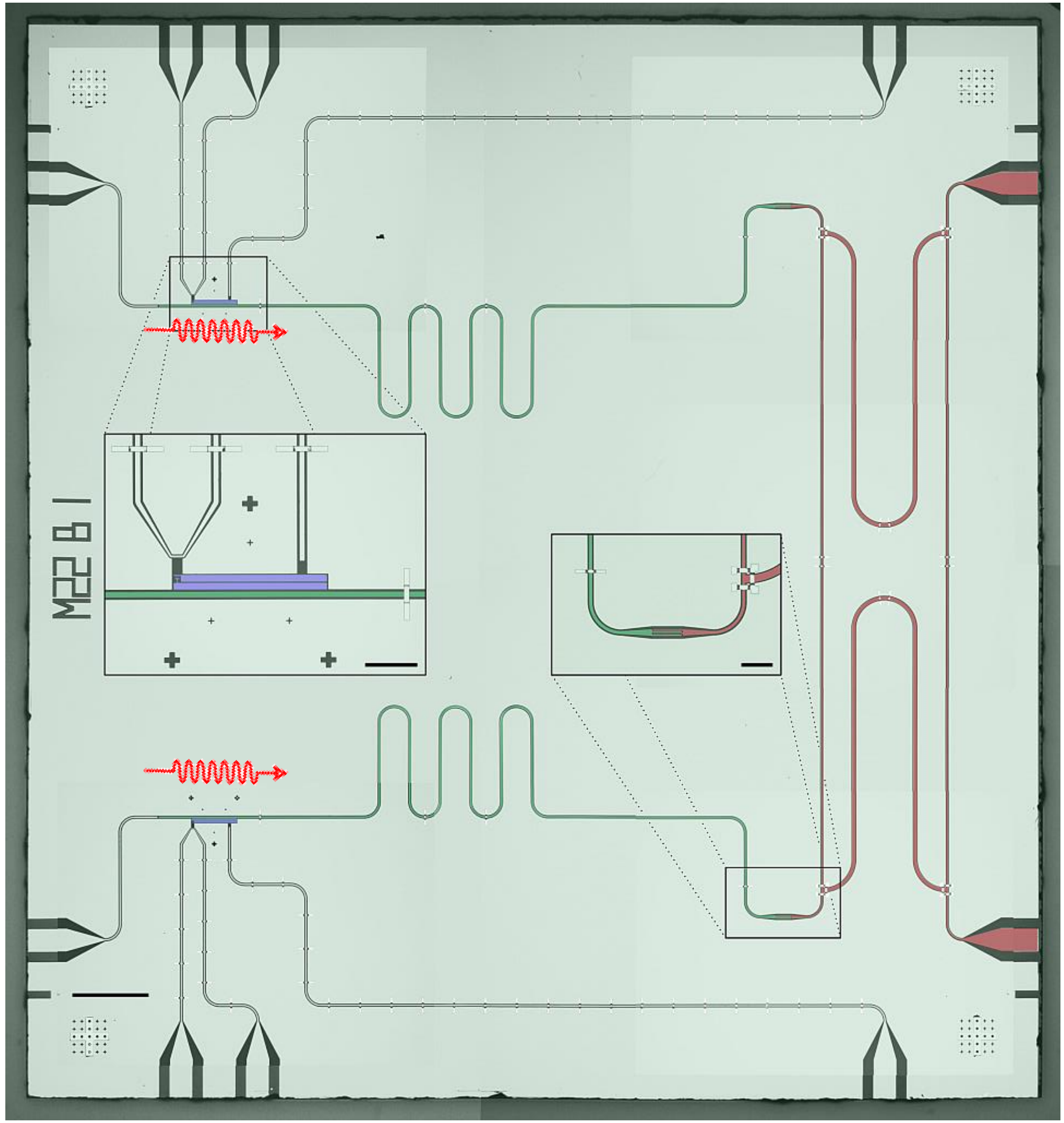
Measure

$$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$$

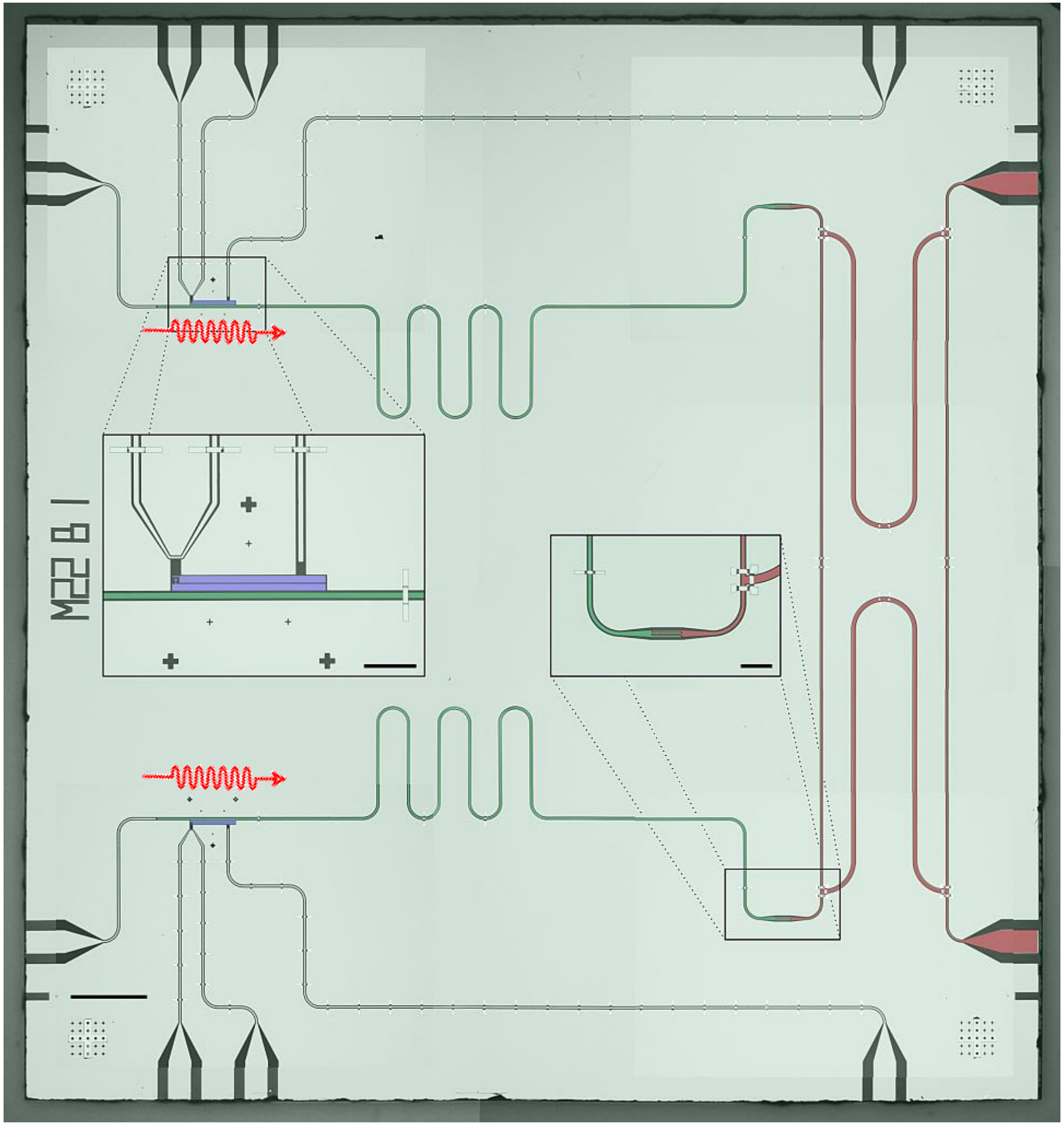
... from recorded **4D histograms:** $D(X_a, P_a, X_b, P_b)$

C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

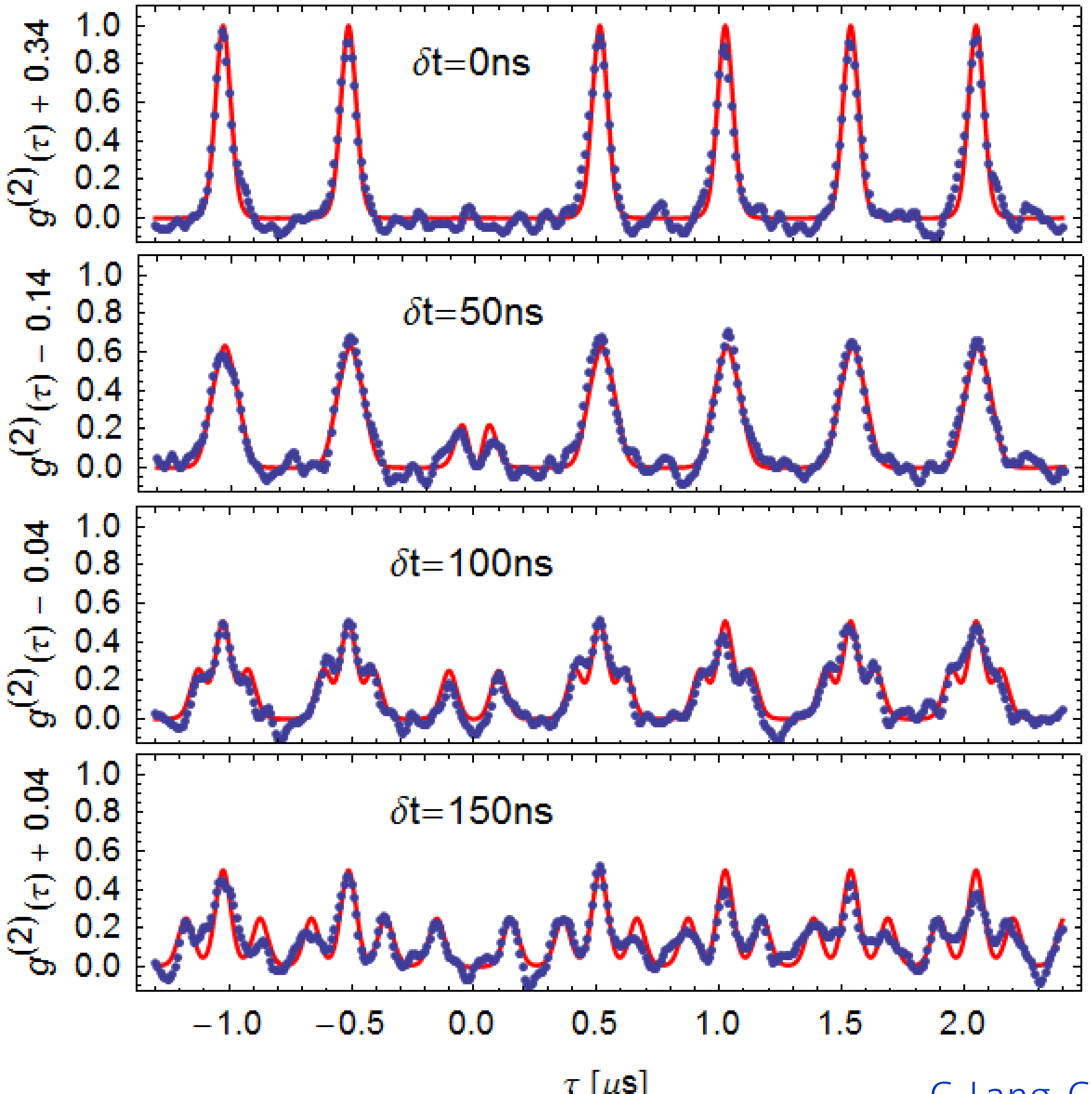
Two Single Photon Sources and Beam Splitter



Two Single Photon Sources and Beam Splitter



Hong-Ou-Mandel $g^{(2)}(\tau)$ for Microwave Photons



Observations:

- Photon-Pair anti-bunching

For $\tau > 0$:

- Broadening of satellite peaks
- Triple-peak structure of satellite peaks
- Full recovery of double-peak at $\tau \approx 0$

C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

Learning More: Two-Channel Tomography

Idea: Measure 4D histogram and evaluate relevant photon statistics

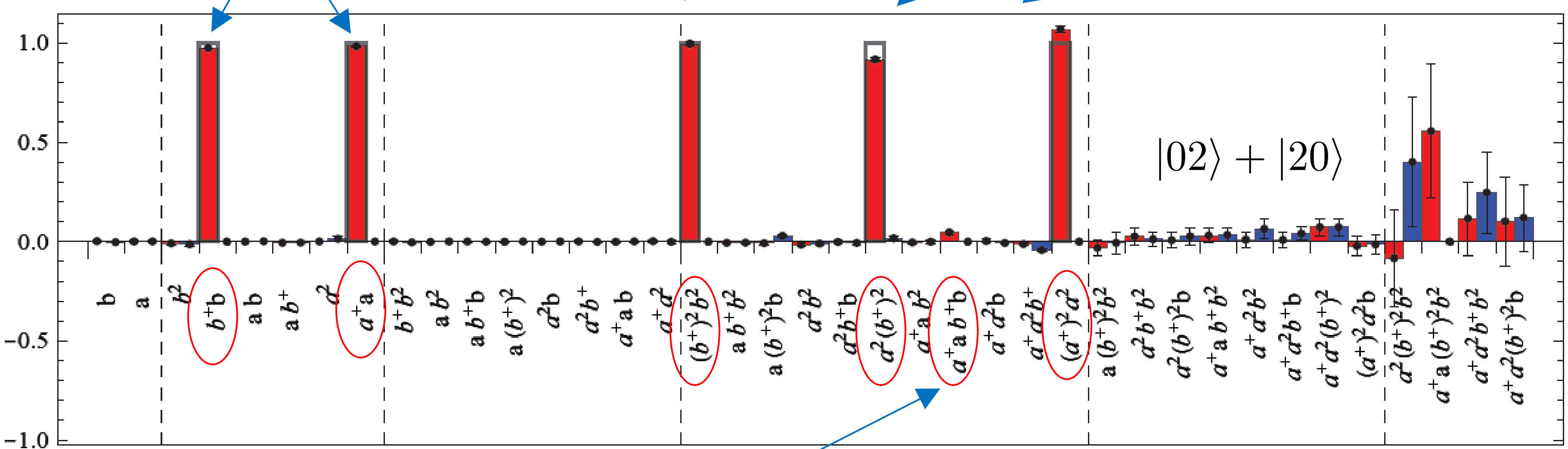
$$D_{\mathrm{ON}}(X_a, P_a, X_b, P_b)$$
$$D_{\mathrm{OFF}}(X_a, P_a, X_b, P_b)$$

analogous to 1-channel case → $\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$

1 average photon in each channel

2-photon bunching

Quantum superposition!



never simultaneous "click" in both channels

C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

Density Matrix Displaying Two-Mode Entanglement

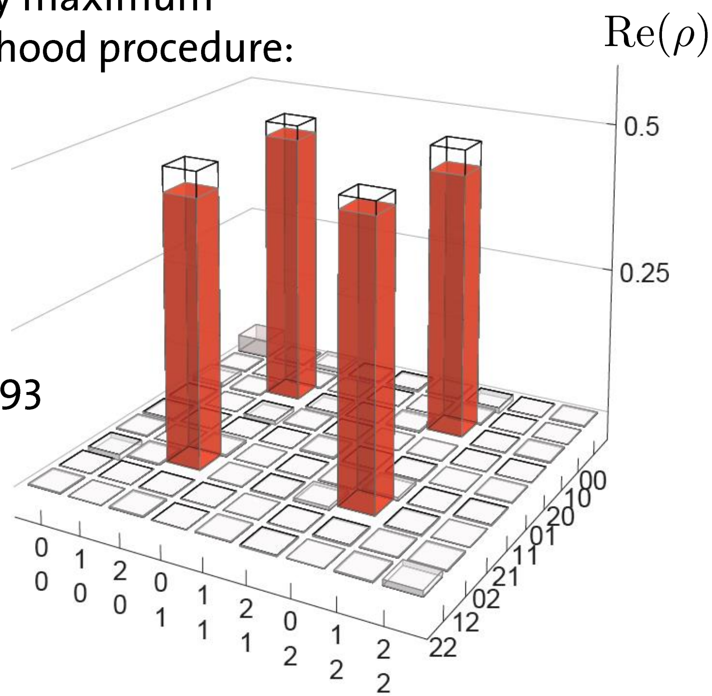
Density matrix reconstruction:

$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$ —linear map→ $\langle nm|\rho|kl\rangle$

moments

Fock space density matrix

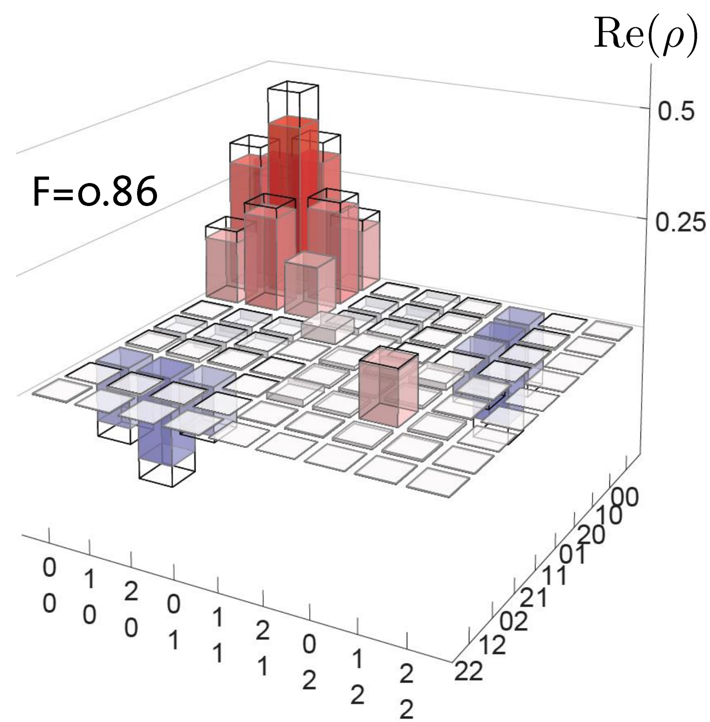
Apply maximum likelihood procedure:



F=0.93

$|02\rangle + |20\rangle$

F=0.86

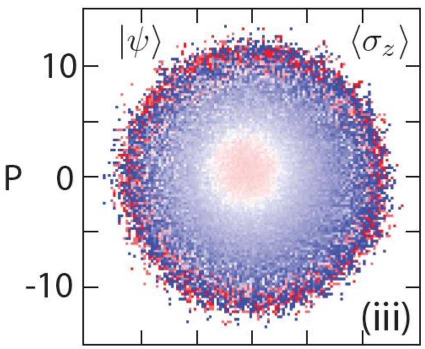


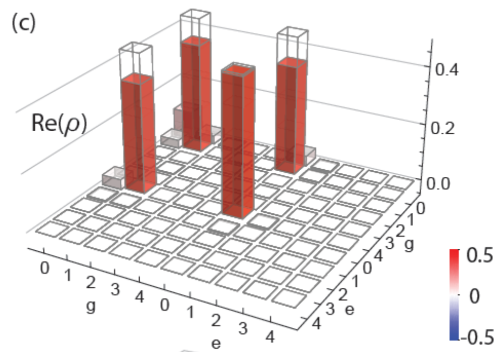
$|00\rangle + \sqrt{2}|10\rangle + (|20\rangle - |02\rangle)\sqrt{2}$

C. Lang, C Eichler *et al.*, *Nat. Phys.* **9**, 345 (2013)

Experiments with Propagating Quantum Microwaves

Preparation and characterization of qubit-**propagating** photon entanglement





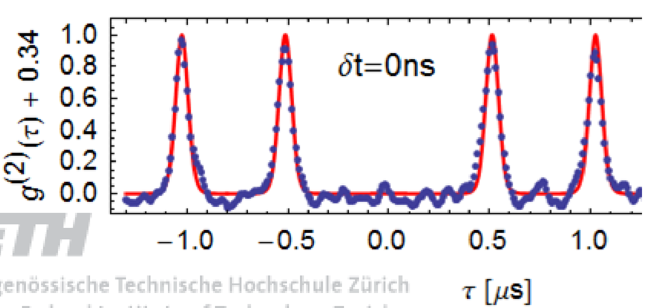
Eichler *et al., PRL* **109**, *240501* (2012)
Eichler *et al., PRA* **86**, 032106 (2012)

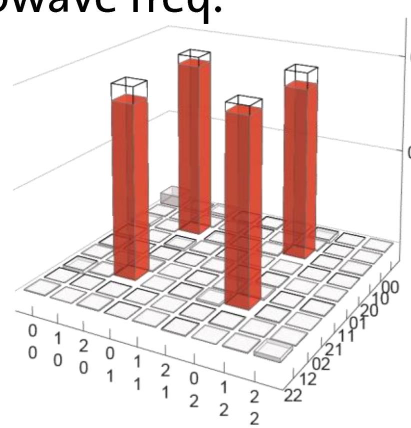
Prospects:

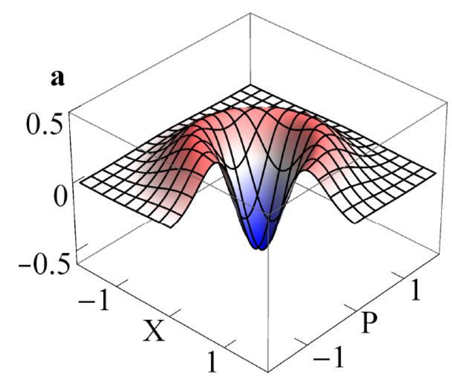
- linear & non-linear optics
  -> photon/photon logic gates & interferometers
- explore quantum communication over *mm*, *cm* and *m* distances
- characterize microwave radiation emission from novel sources (e.g. nano-structures)
  -> ready to use correlation and tomography measurements

Hong-Ou-Mandel: Two-photon interference in msrmnt of coherences at microwave freq.

Lang *et al.*, *Nat. Phys.* **9**, 345 (2013)

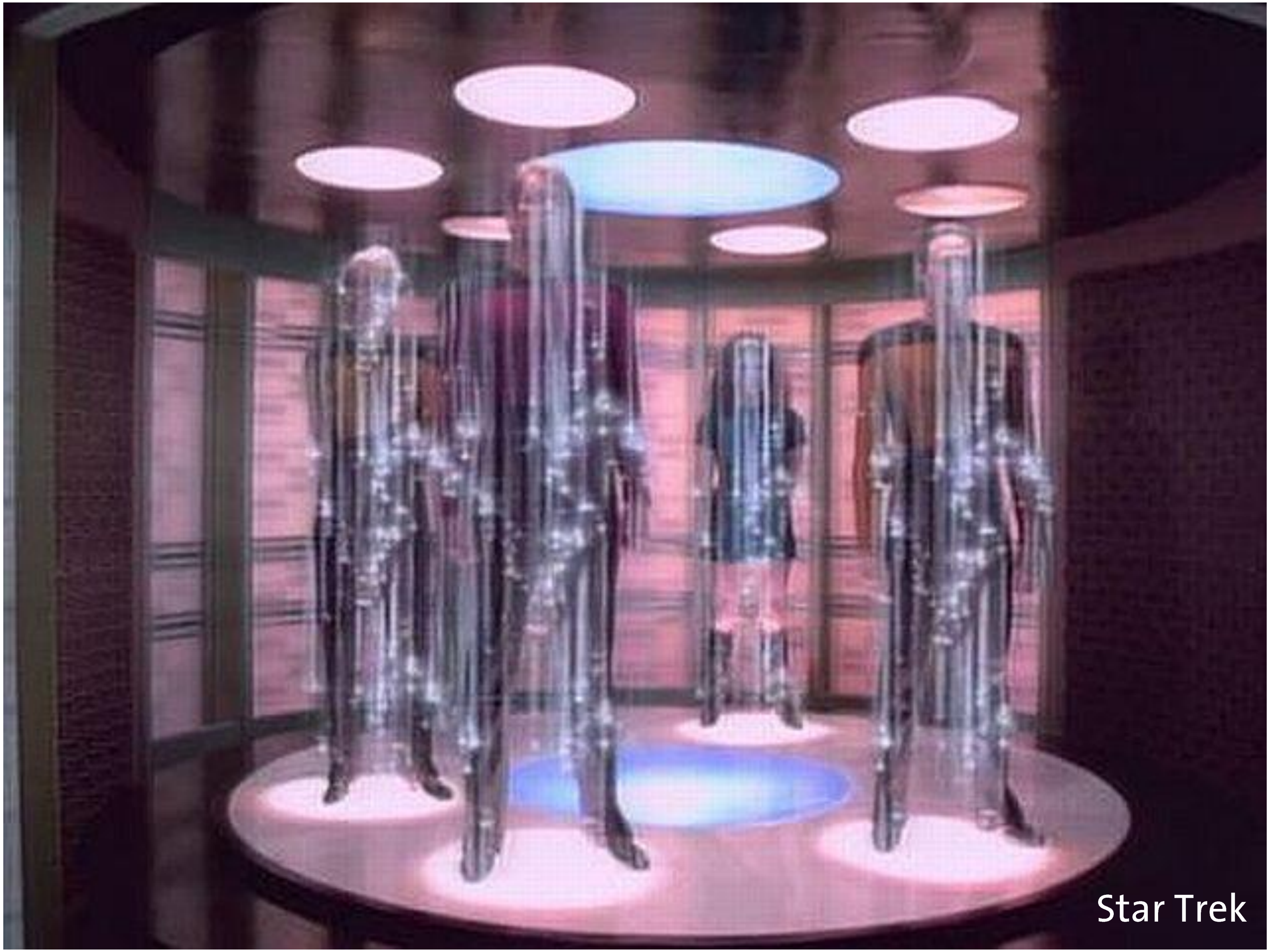






IV. Teleportation

Teleportation (what one may wish for)



Teleportation

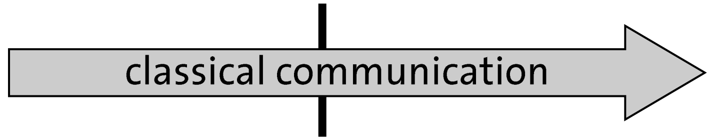
Task:

- transfer unknown state of qubit (A) from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)
- a small quantum computer
- classical communication

Alice



Bob

Features:

- exploits non-local quantum correlations
- uses many essential ingredients required for realizing a quantum computer: important benchmark to pass!

Applications:

- quantum repeaters
- simplification of quantum circuits
- universal computation

Has only been demonstrated for photons and ions. But work on solid state realizations is progressing!

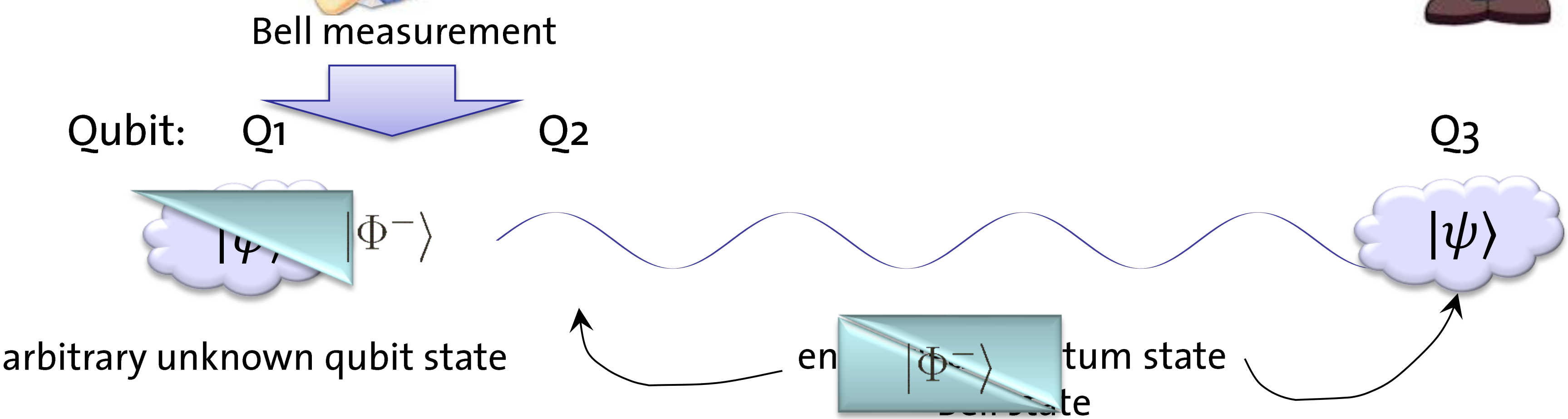
Teleportation Protocol

Task:
- transfer unknown quantum state from Alice to Bob

Resources:
- a pair of entangled qubits (B+C)

Alice

Bob



Teleportation Protocol

Task:

- transfer unknown quantum state from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)
- classical communication

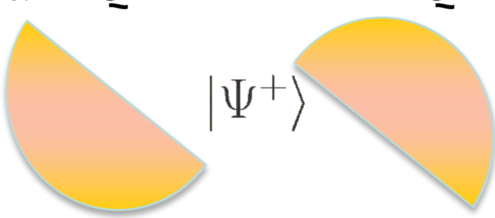
Alice

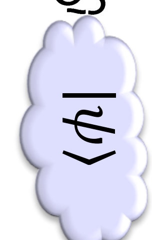
O1

classical communication →

Bob

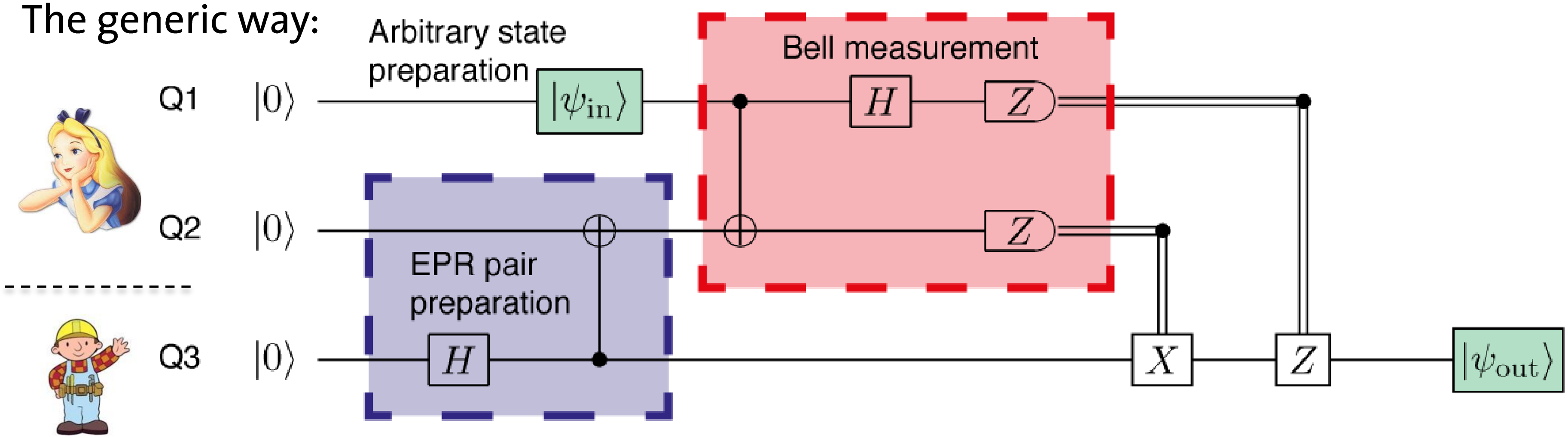
Qubit: Q1 Q2 Q3





Implementation of the Teleportation Protocol

The generic way:



Hadamard

Rotation around Y-axis

Controlled NOT

Controlled phase gate

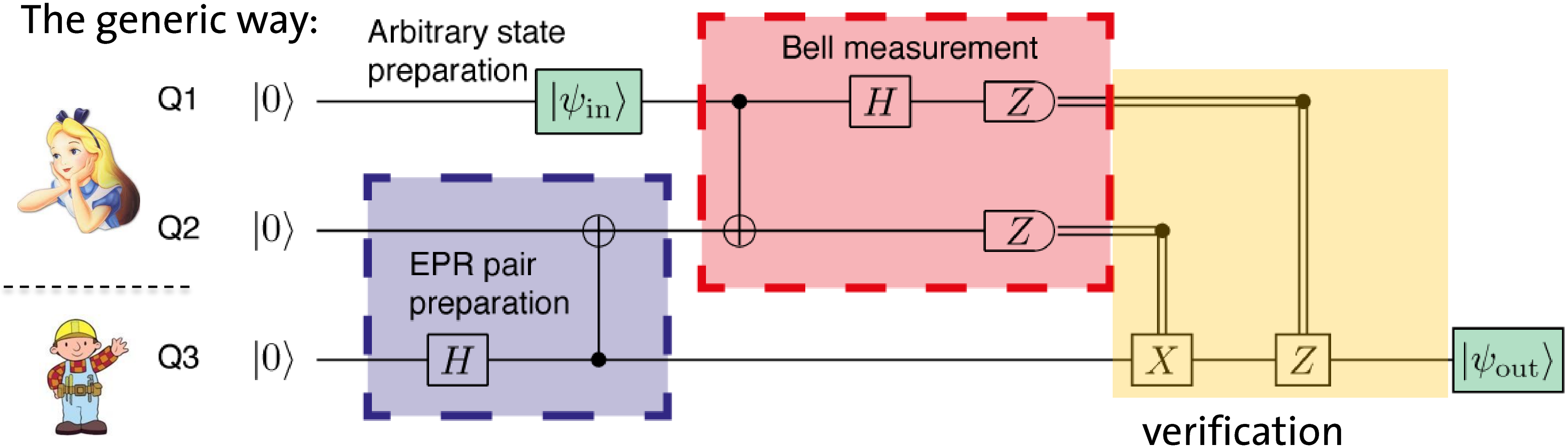
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measurement along Z-axis

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Implementation of the Teleportation Protocol

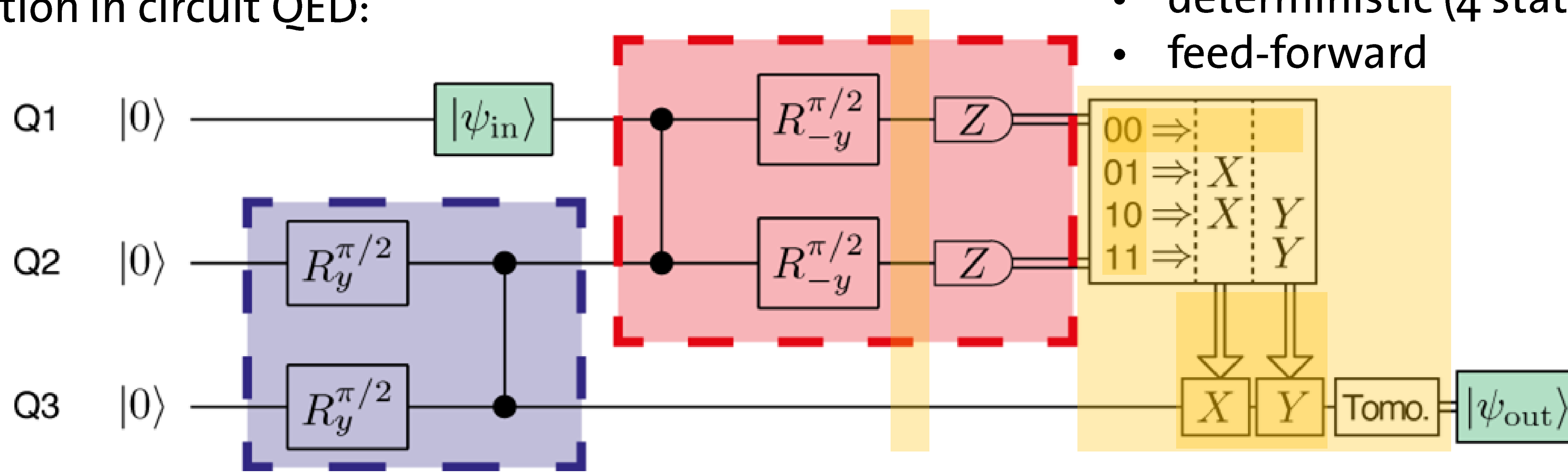
The generic way:



verification

- 3-qubit tomography
- post selection (1 state)
- deterministic (4 states)
- feed-forward

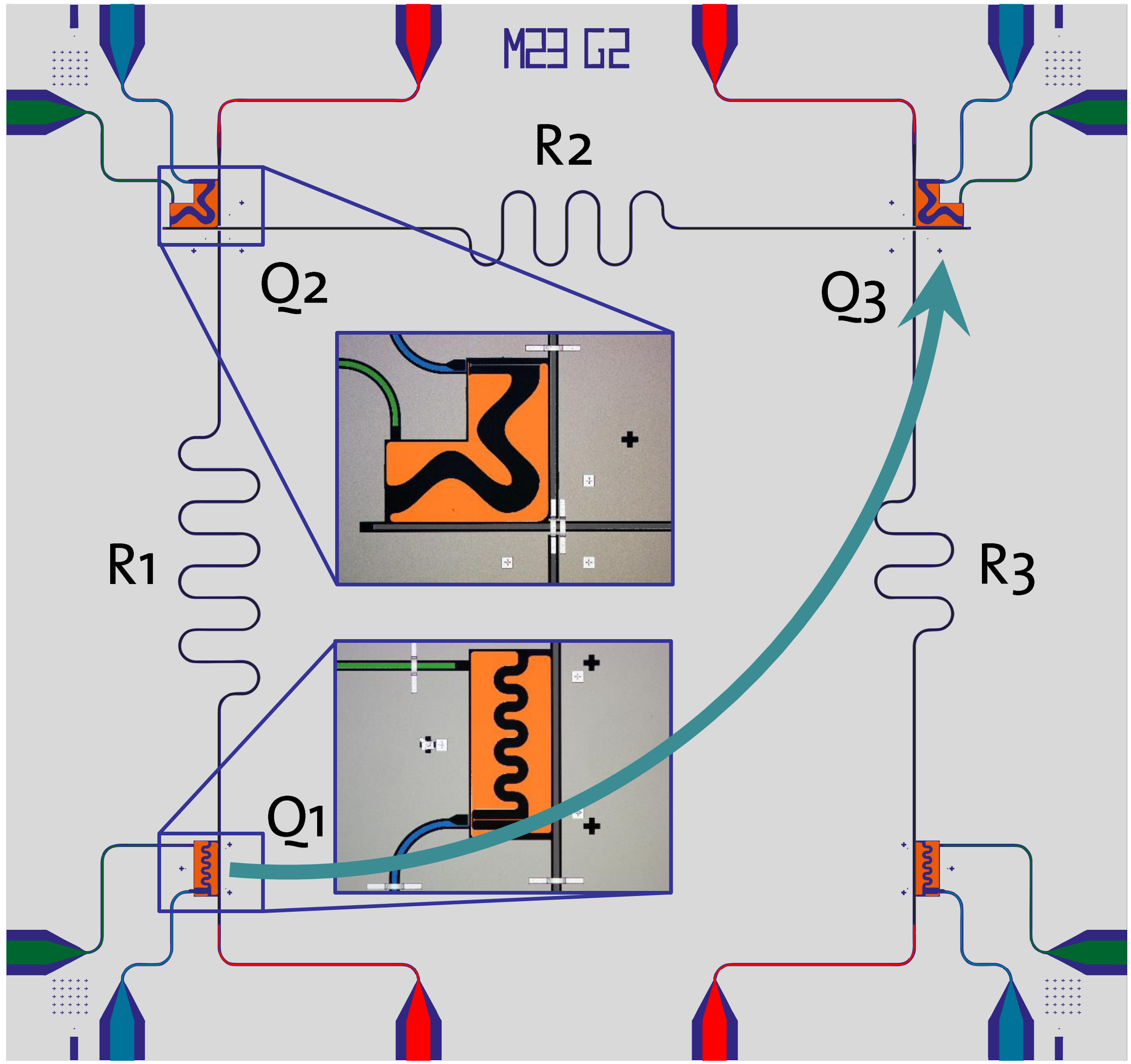
Realization in circuit QED:



The Sample

- 3 Resonators
- 3 Qubits
- single-qubit gates
- two-qubit gates (qubits in the same resonator)
- joint single-shot readout of qubits 1 & 2
- single-shot readout of qubit 3
- with two parametric amplifiers

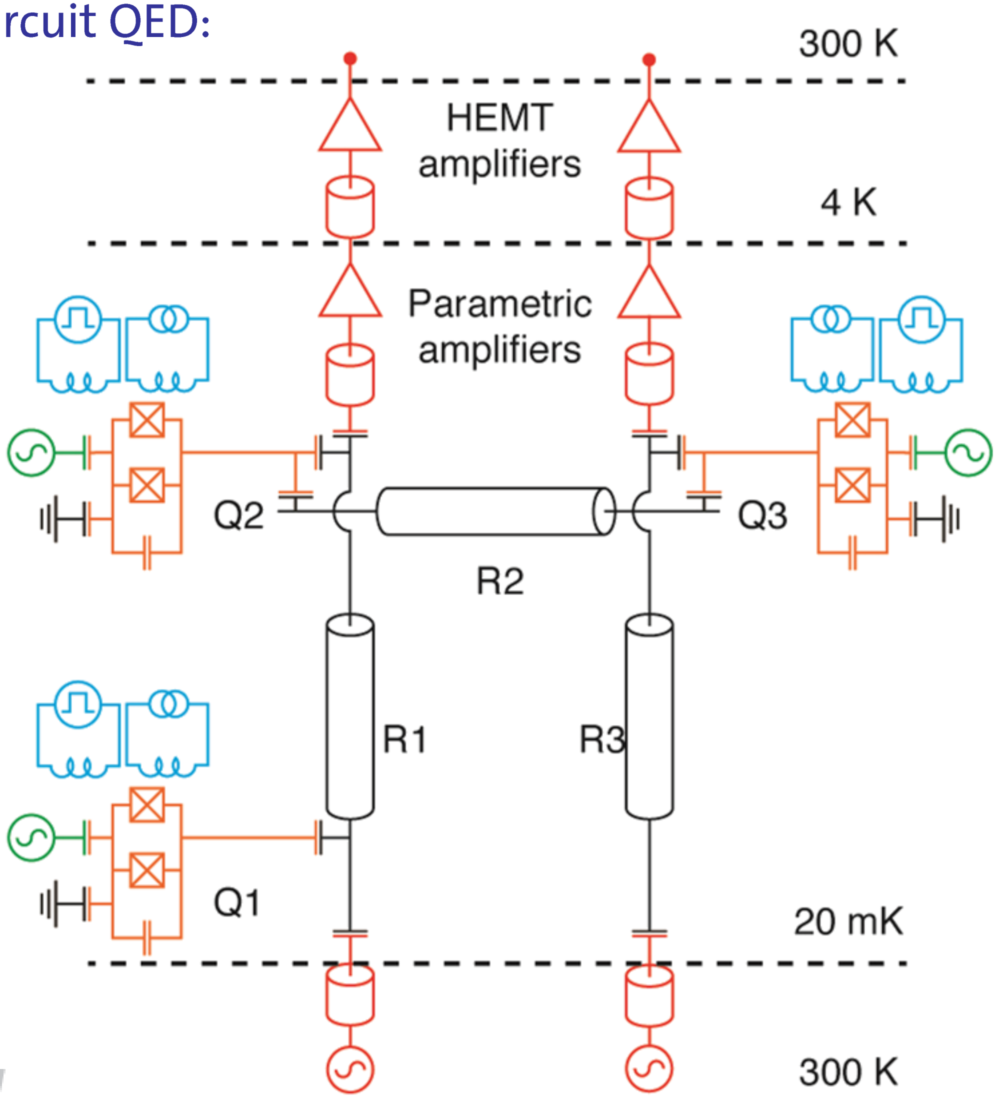
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006).
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008).
Eichler *et al.*, *PRL* **107**, 113601 (2011).



Steffen *et al.*, *Nature* **500**, 319 (2013)

The Circuit

12-port quantum device based on circuit QED:



Device highlights:

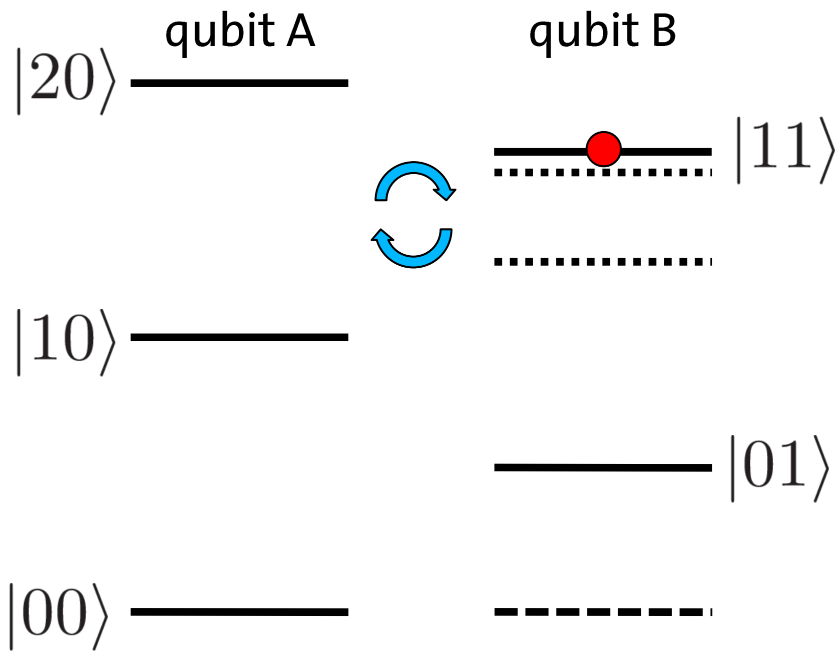
- 3 high-Q resonators
- 4 transmon qubits
- individual control of all qubits
- nearest neighbor interaction via quantum bus
- individual read-out for pairs of qubits 1-2 and 3-4 through resonators
- single-shot read-out using parametric amplifiers
- qubit separation ~ 10 mm
- cross-overs for resonators

Steffen *et al.*, *Nature* **500**, 319 (2013)


Quantum Teleportation with Superconducting Circuits
1 µeV ≙ 240 MHz
10 µeV ≙ 2.4 GHz
100 µeV ≙ 24 GHz
1 meV ≙ 240 GHz
10 meV ≙ 2.4 THz
QUDEV Phonelist
DELL

Universal Two-Qubit Phase Gate

Tune levels into resonance using magnetic field

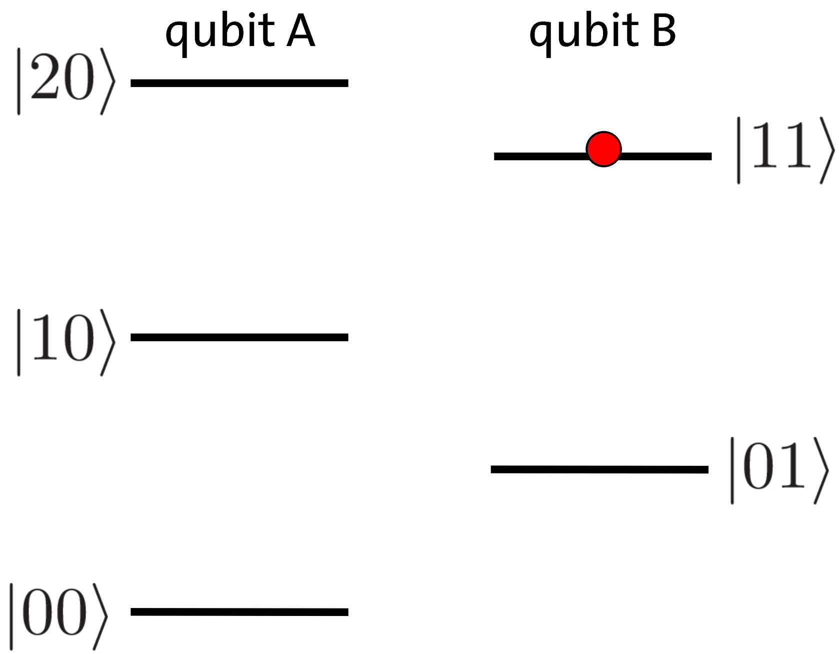


Interaction mediated by virtual photon exchange through resonator

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Controlled Phase Gate



How to verify the operation of this gate?

Universal two-qubit gate. Used together with single-qubit gates to create any quantum operation.

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$
$$|01\rangle \longrightarrow |01\rangle$$
$$|10\rangle \longrightarrow |10\rangle$$
$$|00\rangle \longrightarrow |00\rangle$$

C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Process Tomography: C-Phase Gate

arbitrary quantum process

$$\rho' = \mathcal{E}(\rho)$$

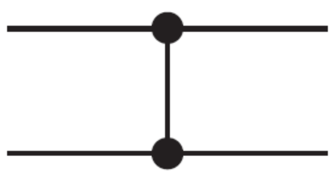
decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

$\{\tilde{E}_k\}$ is an operator basis

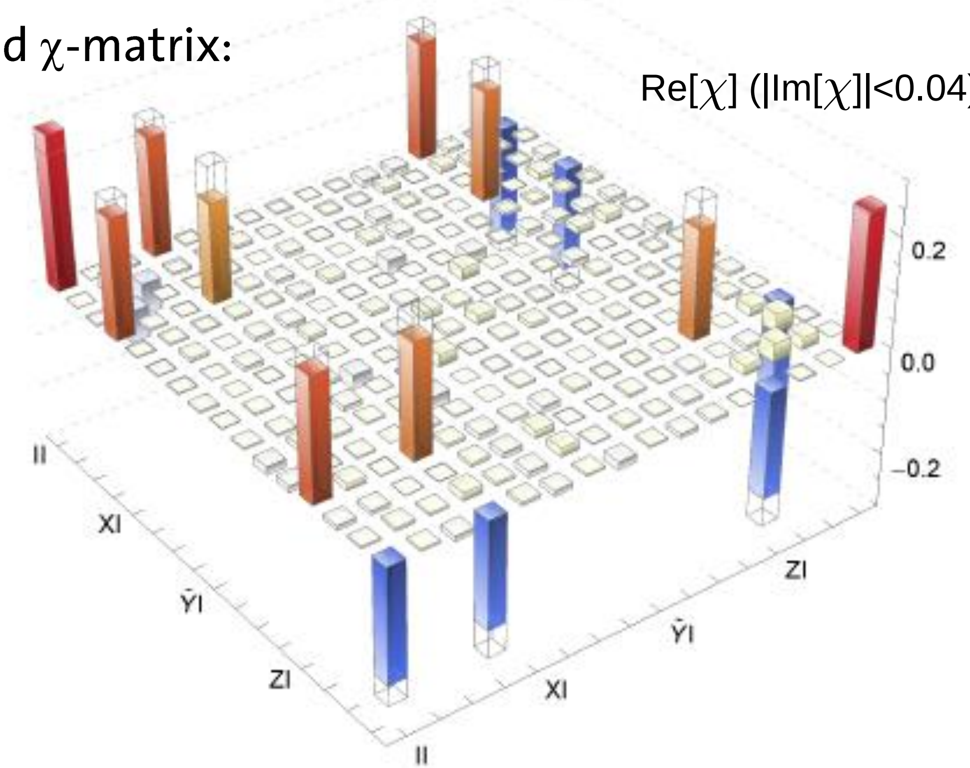
χ is a positive semi definite Hermitian matrix characteristic for the process

Controlled phase gate



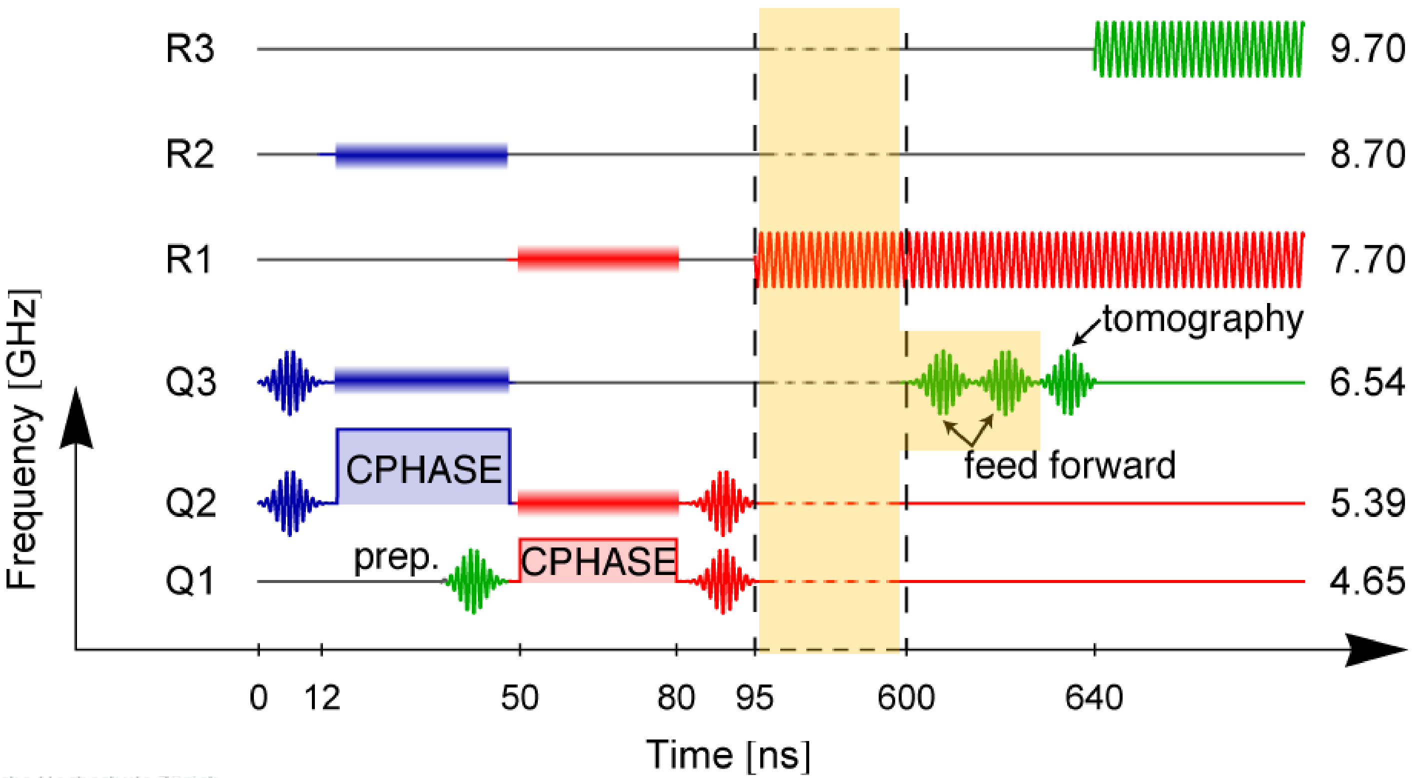
$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured χ-matrix:

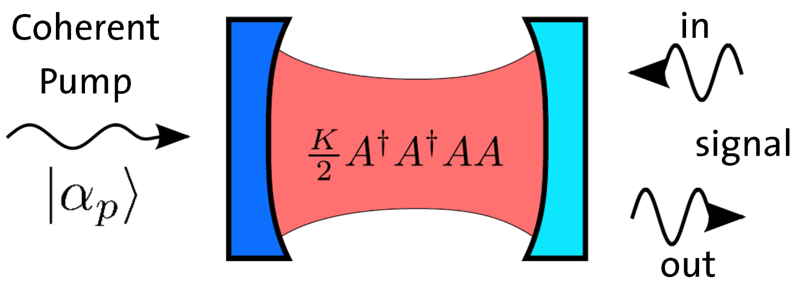


$$F = \mathrm{Tr}[\chi_{\mathrm{meas}}\chi_{\mathrm{ideal}}] > 0.90$$

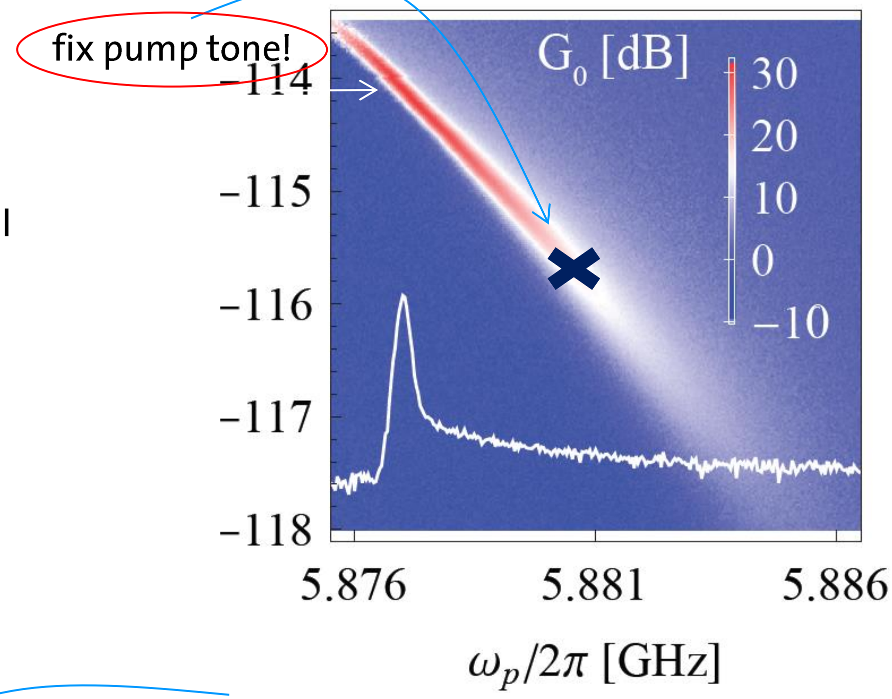
Pulse scheme



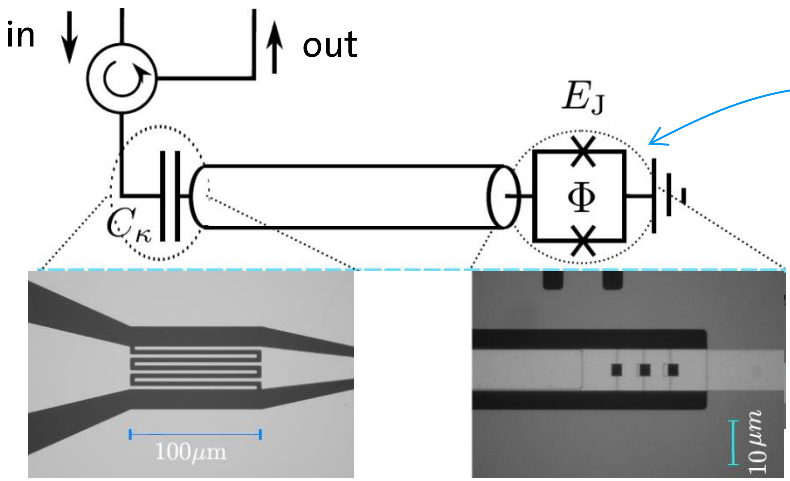
Near Quantum-Limited Parametric Amplifier



Measured Gain



Circuit QED implementation:

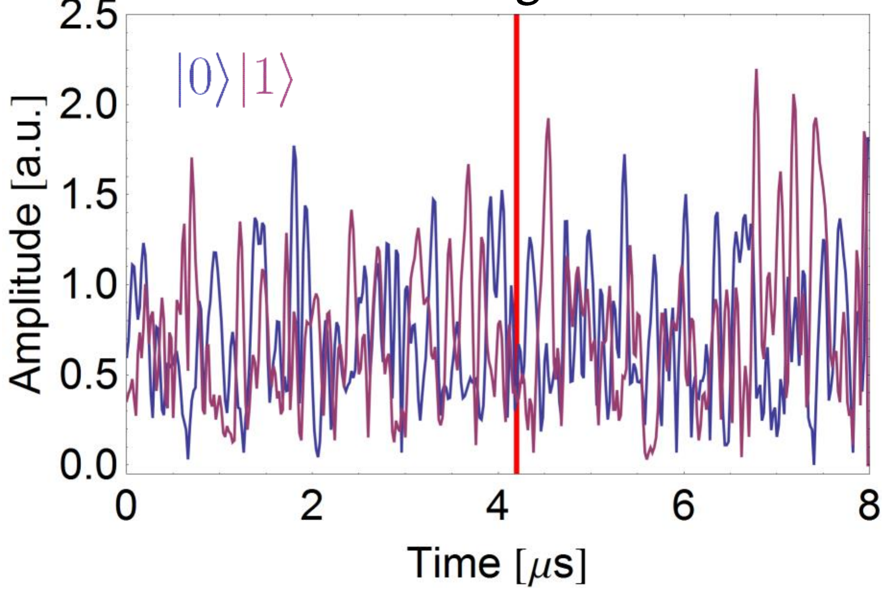


SQUID provides nonlinearity!

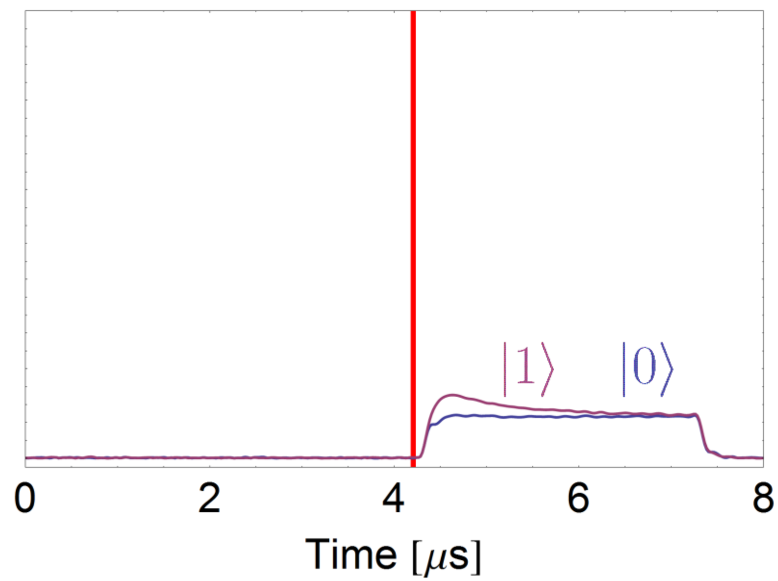
Caves, *Phys. Rev. D* **26**, 1817 (1982)
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)
Eichler et al., *Phys. Rev. Lett.* **107**, 113601 (2011)

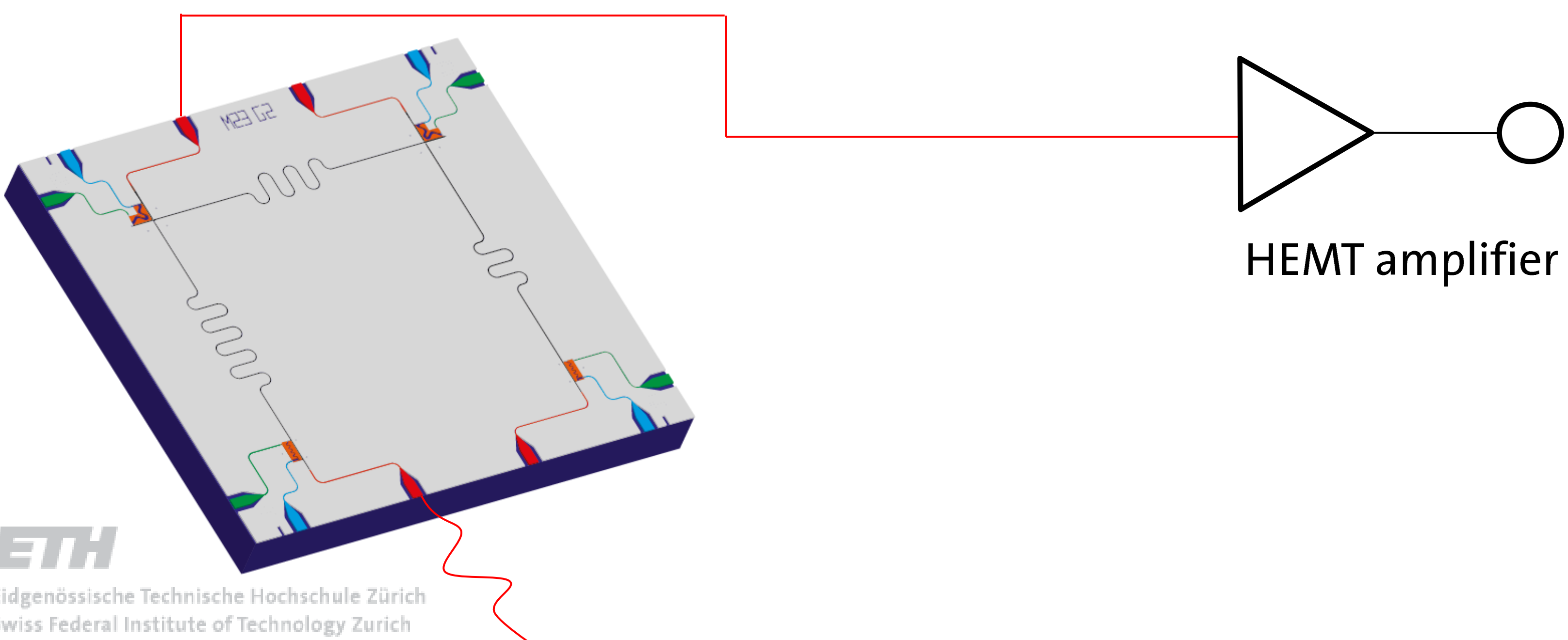
Readout

single measurement

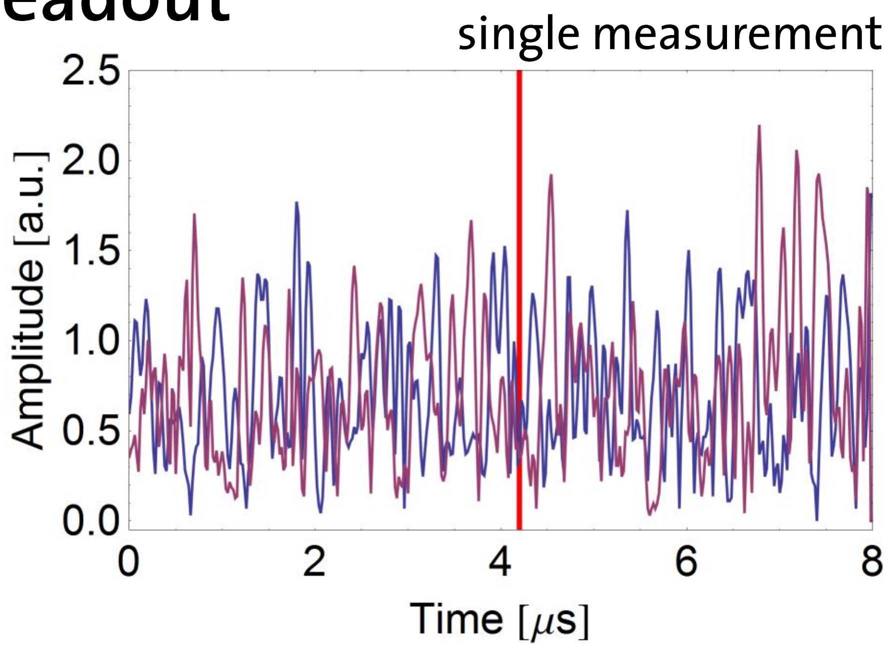


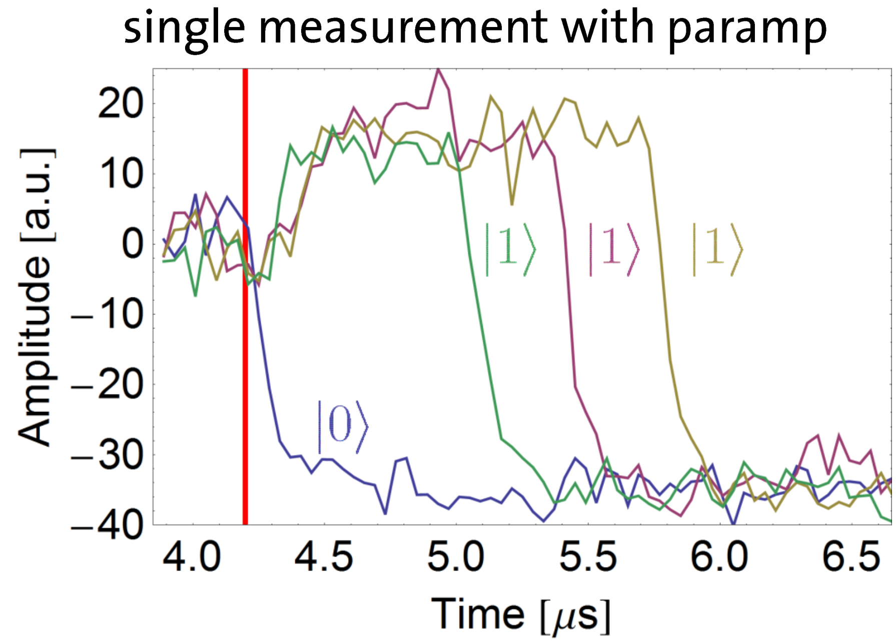
80’000 avg.

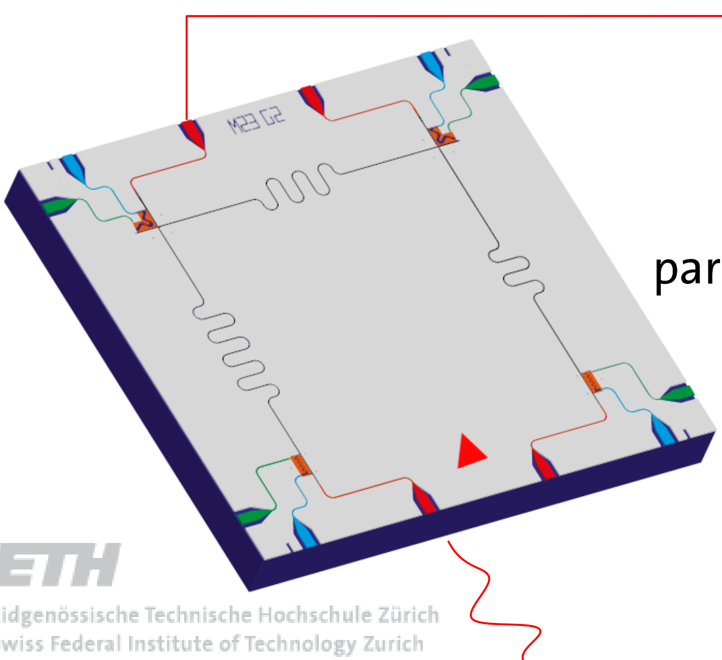


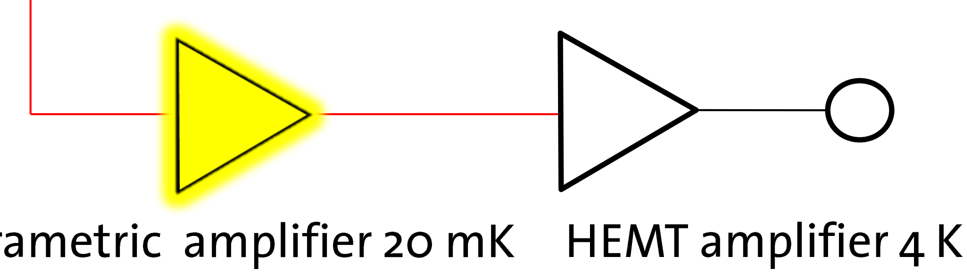


Readout





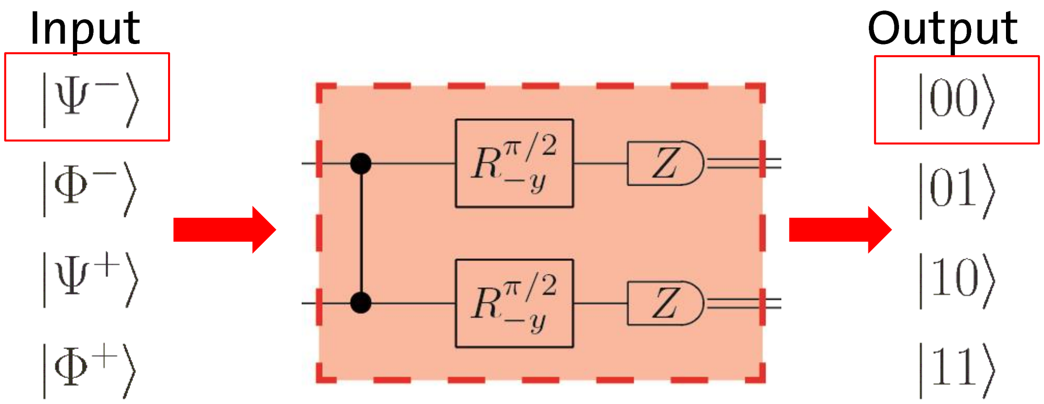


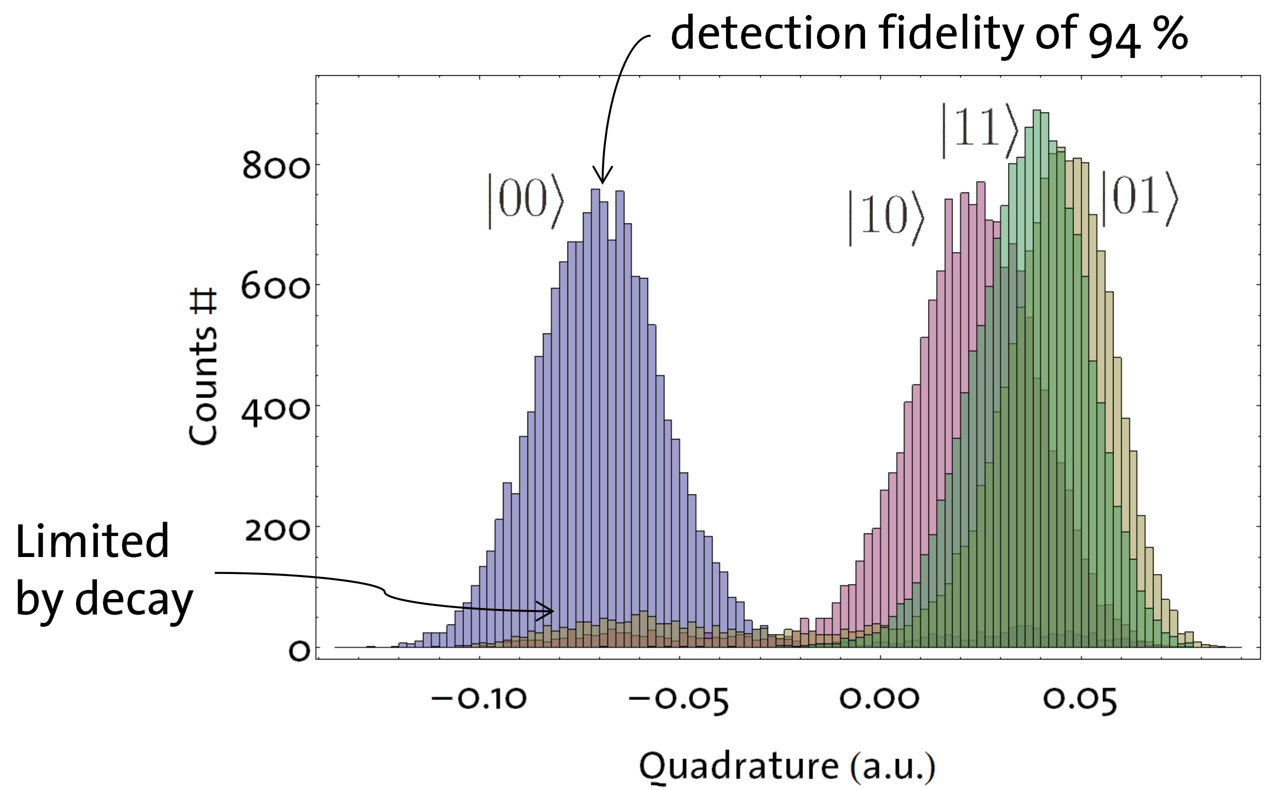


Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)
Eichler *et al.*, *PRL* **107**, 113601 (2011)

R. Vijay *et al*, *PRL* **106**, 110502 (2011)

Post-Selected Teleportation: Bell Measurement



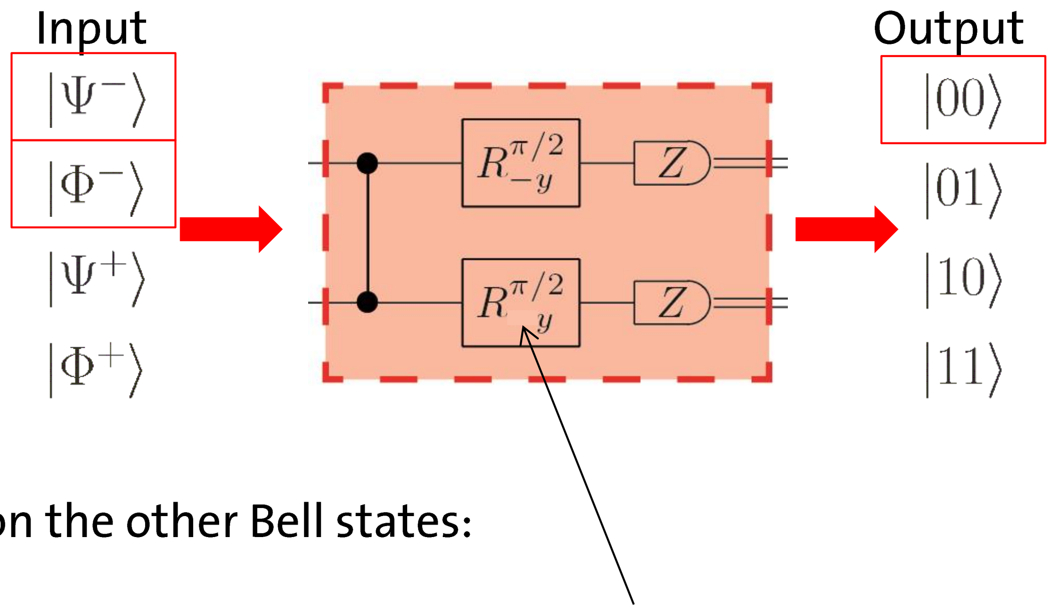


Operate parametric amplifier in phase sensitive mode

Maximize contrast of $|00\rangle$ to other states

Steffen *et al.*, *Nature* **500**, 319 (2013)

Post-Selecting Every State Individually

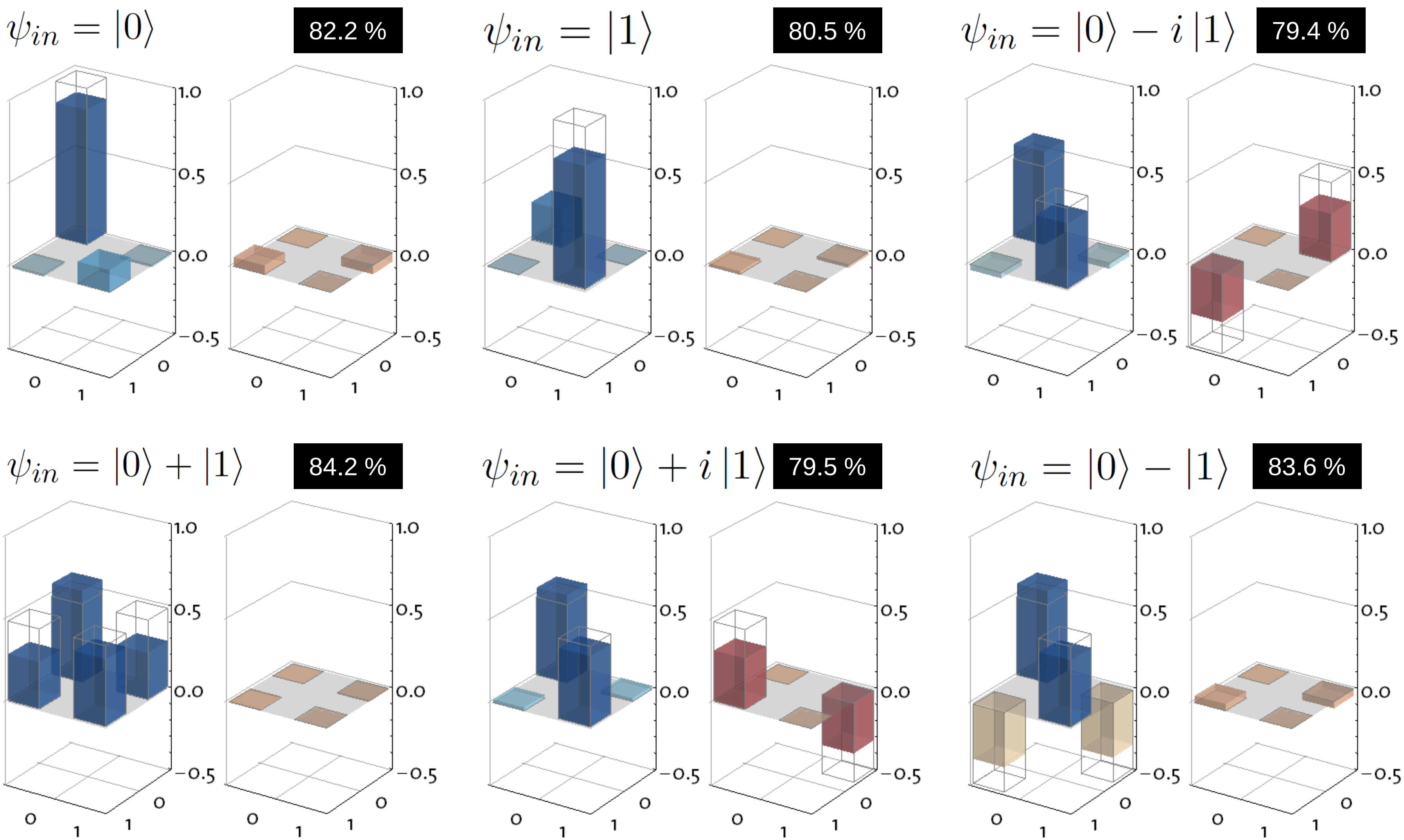


How to post-select on the other Bell states:

- Change the phases of the $\pi/2$ pulses
- Another Bell state is transformed to the $|00\rangle$ state
- Possibility to post-select on all four Bell states

Steffen *et al.*, *Nature* 500, 319 (2013)

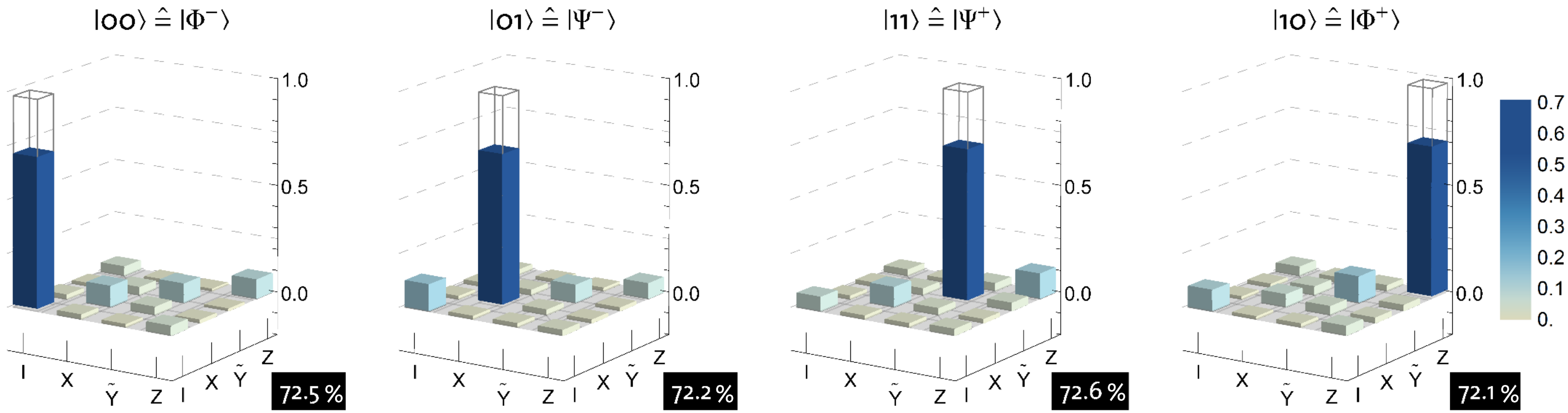
Tomography of Teleported States with Post-Selection



Steffen *et al.*, *Nature* 500, 319 (2013)

Process Tomography: Teleportation with Post-Selection

absolute value of process matrices $|\chi|$ for state transfer from qubit 1 to qubit 3:



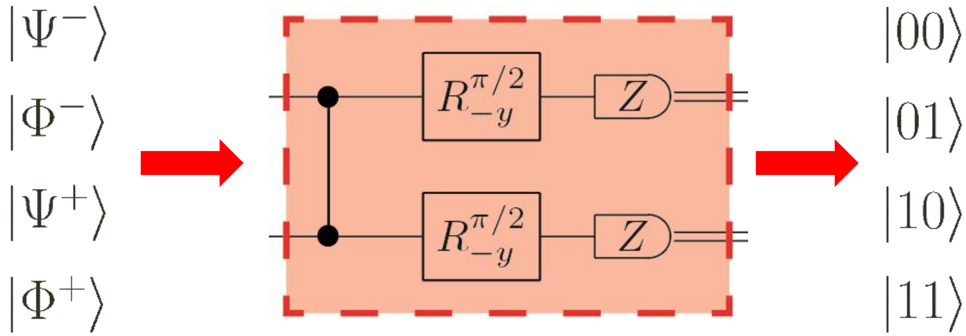
$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = X\,|\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = \tilde{Y}\,|\psi_{\text{in}}\rangle \qquad |\psi_{\text{out}}\rangle = Z\,|\psi_{\text{in}}\rangle$$

$$X = \hat{\sigma}_x, \tilde{Y} = i\hat{\sigma}_y, Z = \hat{\sigma}_z$$

Average process fidelity **72.3 ± 0.7 %**

Steffen *et al.*, *Nature* **500**, 319 (2013)

Deterministic Measurement of all 4 Bell States



- operate paramp in the phase preserving mode
- both quadratures are amplified equally

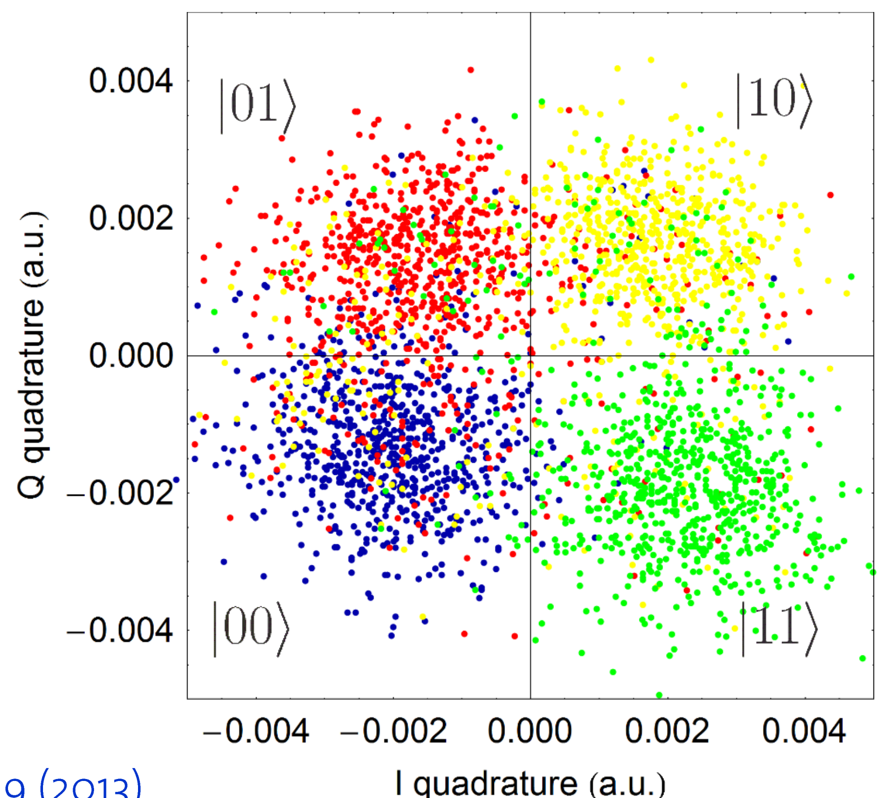
States are identified correctly with ~80% probability

upper left   upper right

identified as

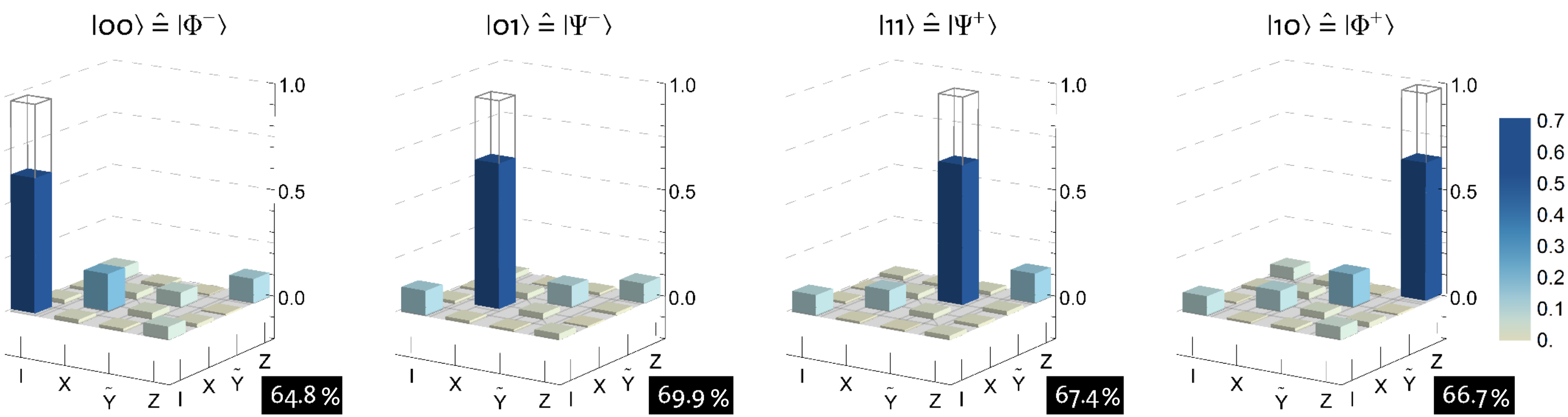
lower left   lower right

	$\lvert 00\rangle$	$\lvert 01\rangle$	$\lvert 10\rangle$	$\lvert 11\rangle$
$\lvert 00\rangle$	0.86	0.09	0.02	0.02
$\lvert 01\rangle$	0.14	0.73	0.04	0.09
$\lvert 10\rangle$	0.03	0.05	0.84	0.09
$\lvert 11\rangle$	0.08	0.10	0.09	0.73



Steffen *et al.*, *Nature* 500, 319 (2013)

Teleportation with Deterministic Bell Measurement



$|\psi_{\mathrm{out}}\rangle = |\psi_{\mathrm{in}}\rangle$ $\quad$ $|\psi_{\mathrm{out}}\rangle = X\,|\psi_{\mathrm{in}}\rangle$ $\quad$ $|\psi_{\mathrm{out}}\rangle = \tilde{Y}\,|\psi_{\mathrm{in}}\rangle$ $\quad$ $|\psi_{\mathrm{out}}\rangle = Z\,|\psi_{\mathrm{in}}\rangle$

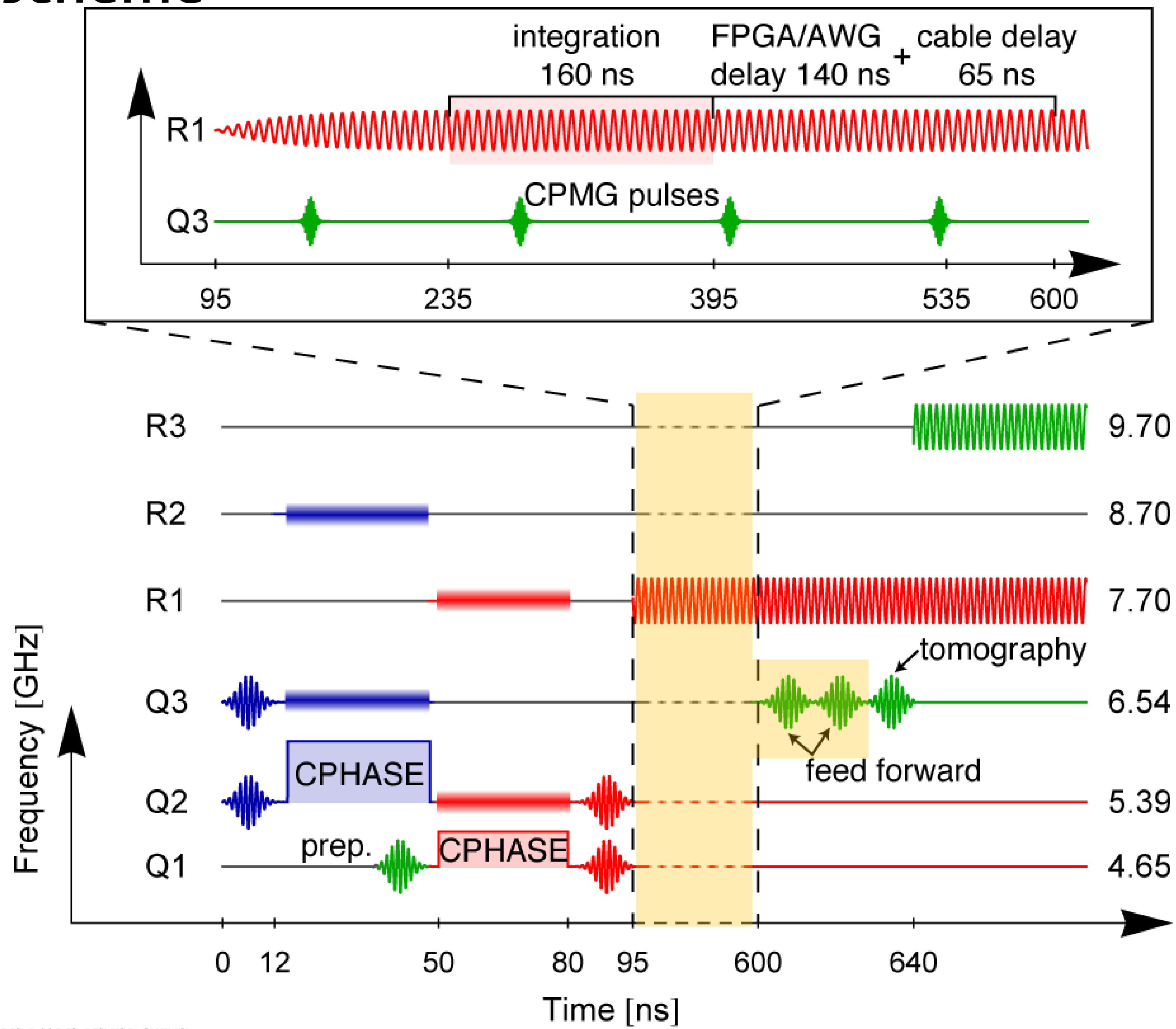
Average process fidelity **67.1 ± 0.5 %**

Average state fidelity **78.1 ± 0.9 %**

$$\mathcal{F}_p = (\mathcal{F}_s(d+1) - 1)/d$$

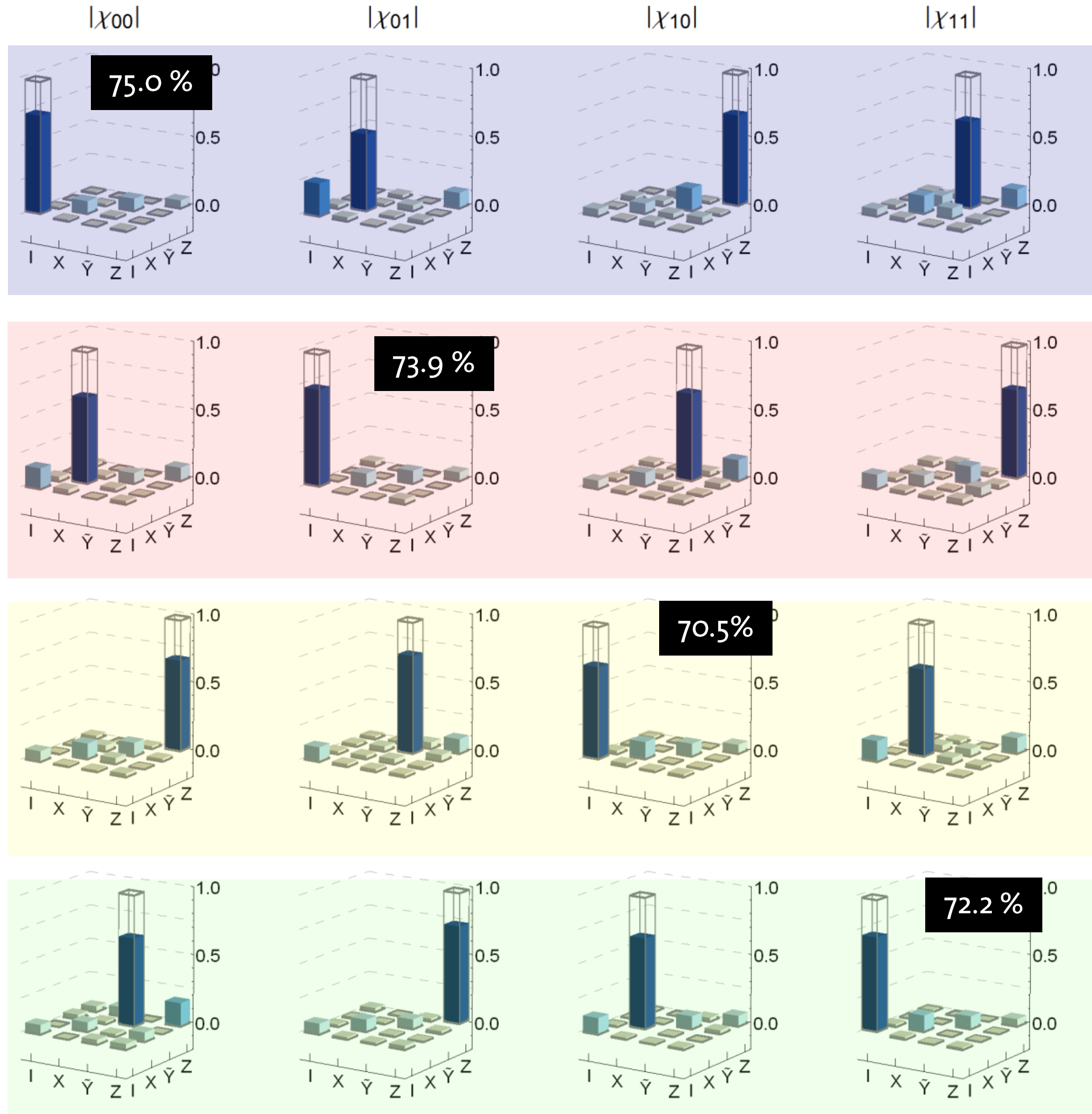
Steffen *et al.*, *Nature* **500**, 319 (2013)

Pulse scheme



Feed-Forward Characterization

process tomography for qubit 3 assuming input = $|\psi\rangle$



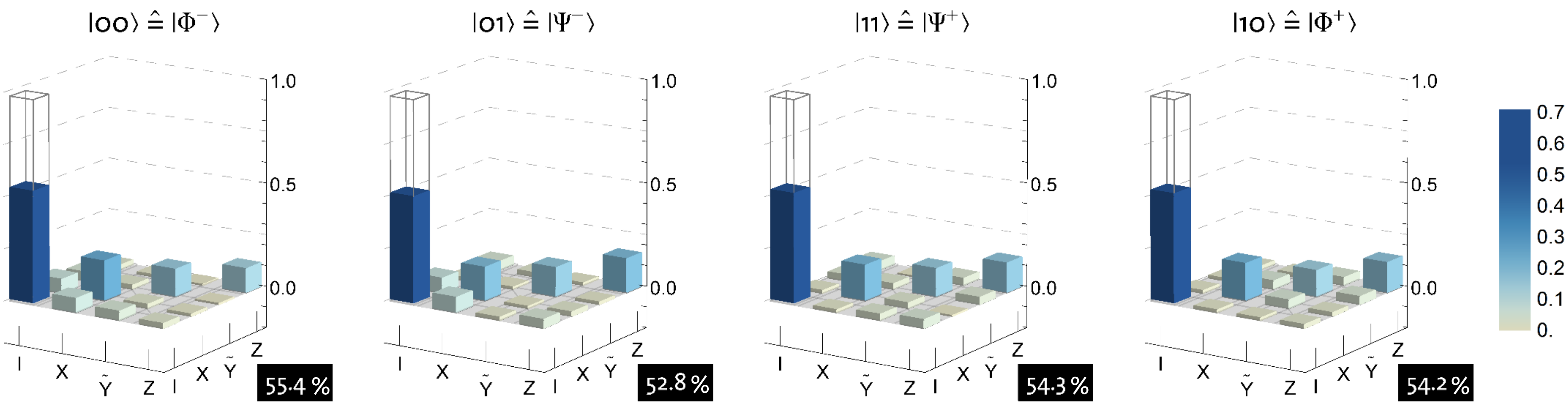
preparation:

- $|00\rangle \otimes |\psi\rangle$
- $|01\rangle \otimes \sigma_x |\psi\rangle$
- $|10\rangle \otimes \sigma_z |\psi\rangle$
- $|11\rangle \otimes i\sigma_y |\psi\rangle$

readout of qubit 1 and 2

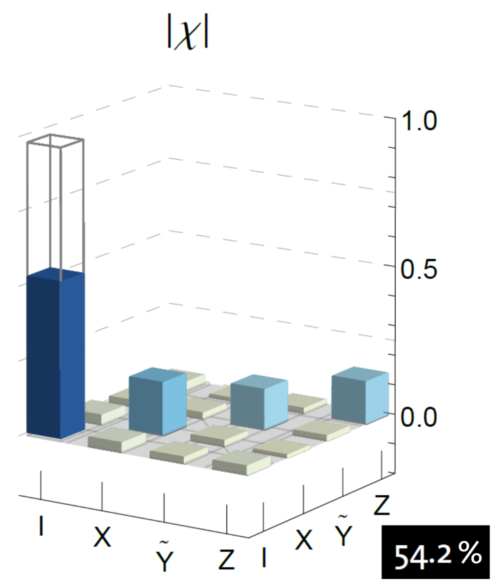
	$|00\rangle$	$|01\rangle$	$|10\rangle$	$|11\rangle$
$|00\rangle$	0.91	0.05	0.02	0.02
$|01\rangle$	0.1	0.81	0.03	0.05
$|10\rangle$	0.04	0.04	0.8	0.12
$|11\rangle$	0.06	0.03	0.11	0.8

Teleportation Process with Feed-Forward



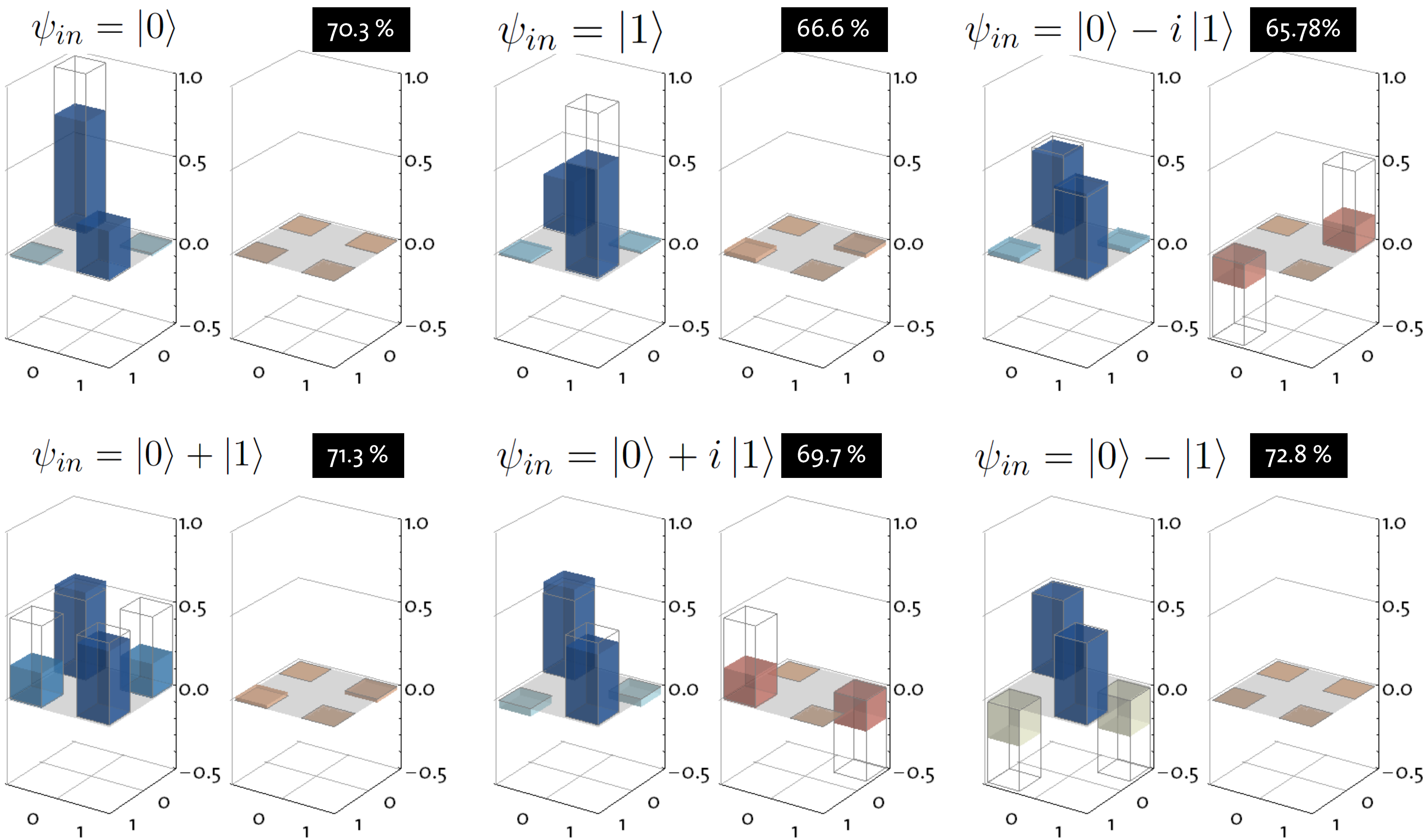
$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$ $|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$

Average process fidelity **54.2 ± 0.1 %**



Steffen *et al.*, *Nature* **500**, 319 (2013)

Tomography of Teleported States with Feed-Forward



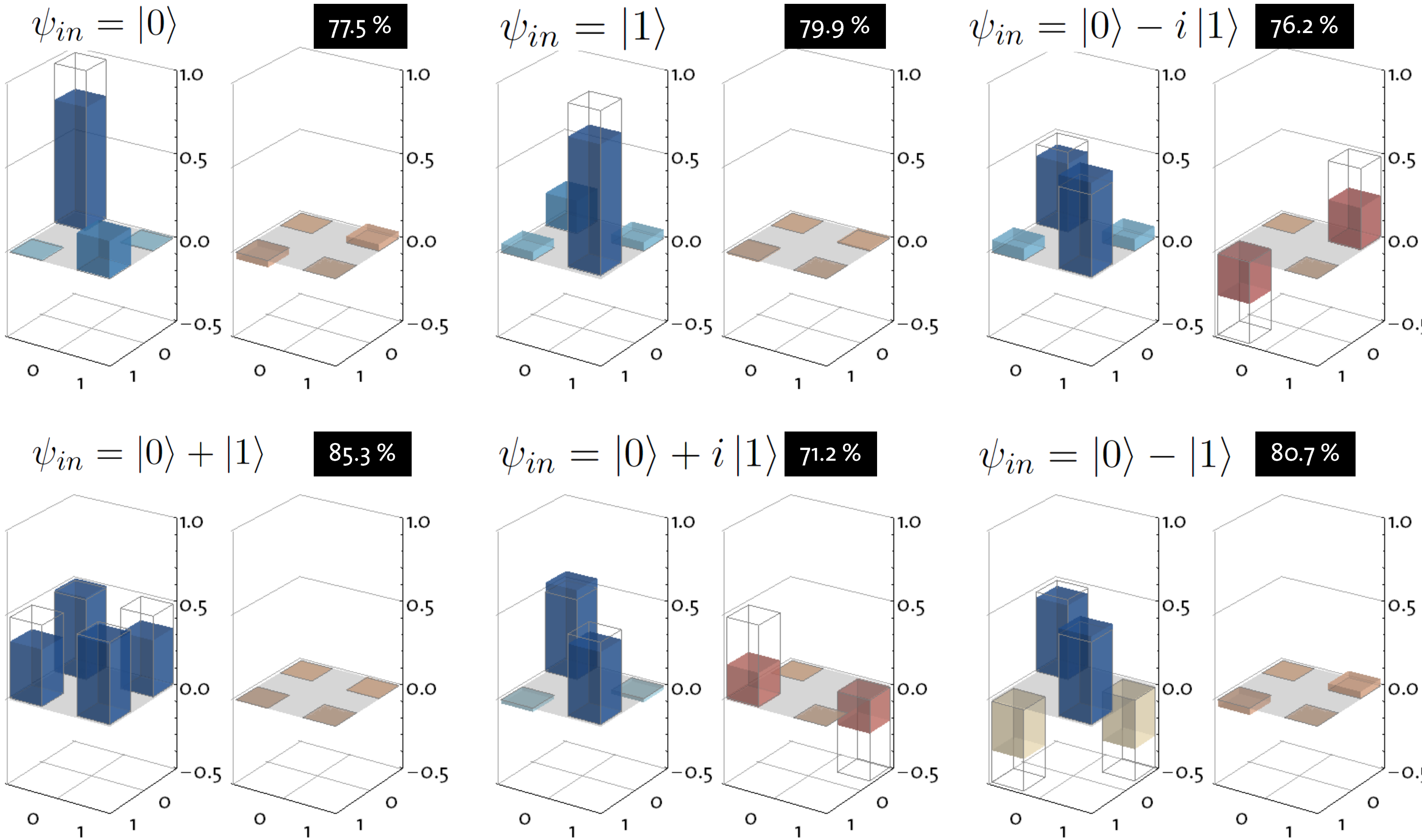
Average state fidelity of **69.5±0.1 %**

Steffen *et al.*, *Nature* 500, 319 (2013)

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Tomography of Teleported States with Feed-Forward

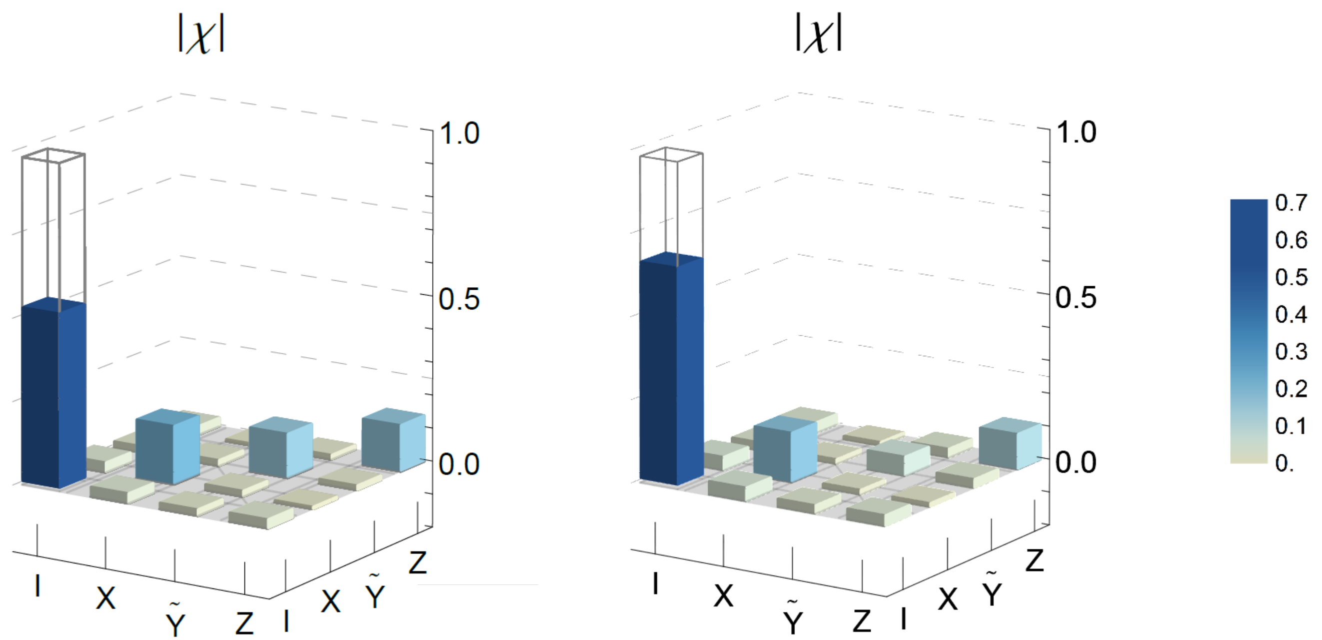
averaged readout of qubit 3



Average state fidelity of 78.5± 0.9%

Steffen *et al.*, *Nature* 500, 319 (2013)

Teleportation Process with Feed-Forward



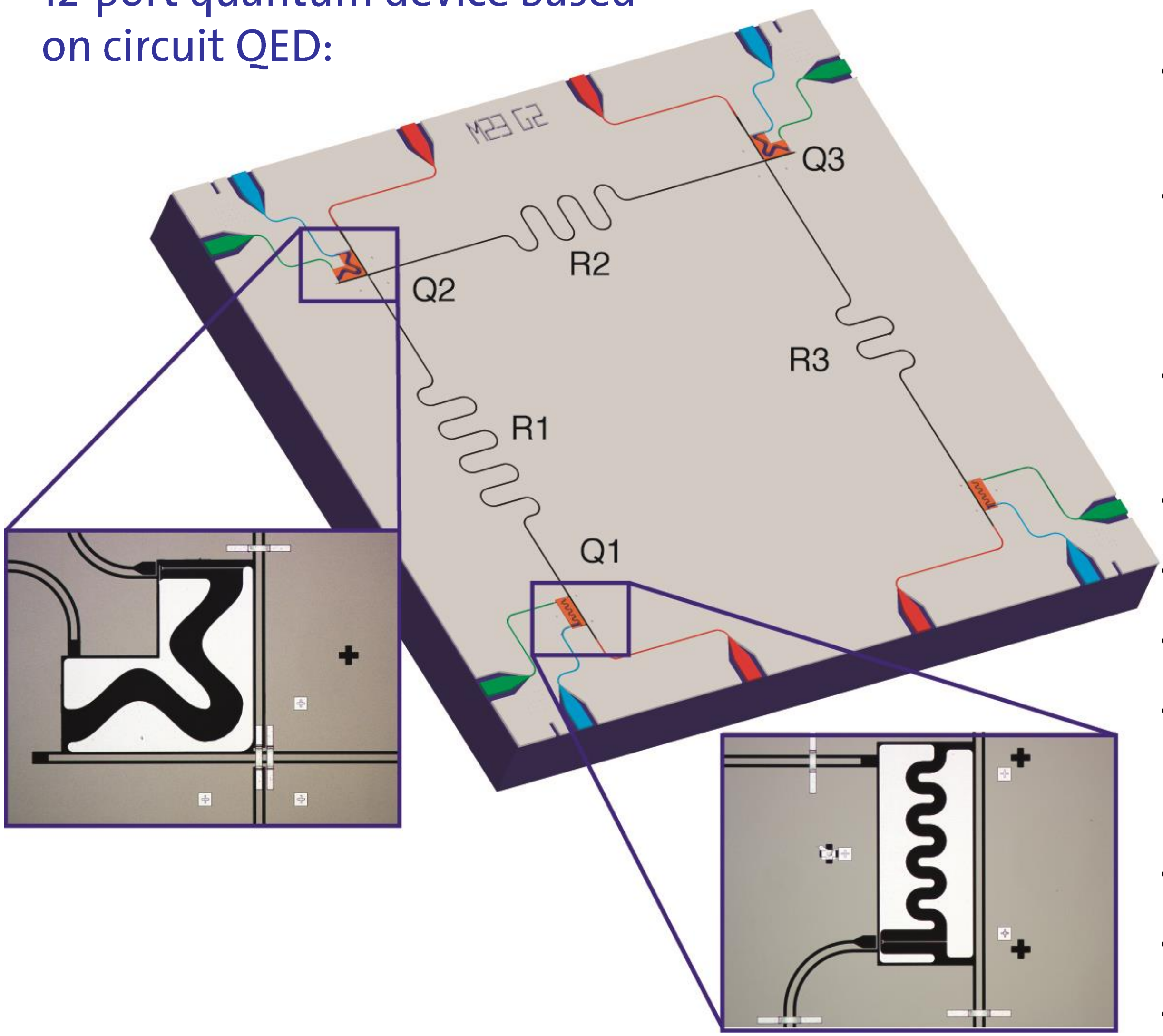
Average process fidelity with single shot readout: **54.2 ± 0.1 %**

Average process fidelity with averaged readout: **67.7 ± 1.1 %**

Steffen *et al.*, *Nature* **500**, 319 (2013)

Teleportation

12-port quantum device based on circuit QED:



Experimental highlights:

- teleportation in a (macroscopic) solid state system
- post-selection on either of 4 Bell states individually
- simultaneous and det. measurement of all 4 Bell states
- implementation of feed-forward
- fidelities > classical threshold
- O(Unity) success probability
- teleportation rate > 10 kHz
- distance ~ 10 mm

Next steps:

- improve fidelities
- increase distances
- apply teleportation

Steffen *et al.*, *Nature* **500**, 319 (2013)

The ETH Zurich Quantum Device Lab

Postdoc and PhD positions available

FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

QSIT Quantum Science and Technology
National Centre of Competence in Research

QED
CIRCUIT AND CAVITY
QUANTUM ELECTRODYNAMICS

aqute

SOLID

erc

GEOMDISS

Selected Circuit QED Publications

Circuit QED Proposal:

- Blais et al., *PRA* **69**, 062320 (2004)

Strong Coupling & Vacuum Rabi Mode Splitting:

- Wallraff et al., *Nature* **431**, 162 (2004)
- Fink et al., *Nature* **454**, 315 (2008)
- Fink et al., *PRL* **105**, 163601 (2010)

Tavis-Cummings Multi-Atom QED:

- Fink et al., *PRL* **103**, 083601 (2009)

AC-Stark & Lamb Shift, Autler-Townes and Mollow Transitions

- Schuster et al., *PRL* **94**, 123062 (2005)
- Gambetta et al., *PRA* **74**, 042318 (2006)
- Schuster et al., *Nature* **445**, 515 (2007)
- Fragner et al., *Science* **322**, 1357 (2008)
- Baur et al., *PRL* **102**, 243602 (2009)

Device Fabrication:

- Frunzio et al., *IEEE Trans. Appl. Sup.*. **15**, 860 (2005)
- Goeppl et al., *J. Appl. Phys.* **104**, 113904 (2008)

Geometric Phases:

- Leek et al., *Science* **318**, 1889 (2007)
- Pechal et al., *PRL* **108**, 170401 (2012)
- Abdumalikov et al., *Nature* **496**, 482 (2013)

One-, Two-, Three-Qubit Gates, Algorithms and Teleportation:

- Wallraff et al., *PRL* **95**, 060501 (2005)
- Blais et al., *PRA* **75**, 032329 (2007)
- Wallraff et al., *PRL* **99**, 050501 (2007)
- Majer et al., *Nature* **449**, 443 (2007)
- Leek et al., *PRB* **79**, 180511(R) (2009)
- Filipp et al., *PRL* **102**, 200402 (2009)
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