

# Quantum Interfaces based on Superconducting Electronic Circuits

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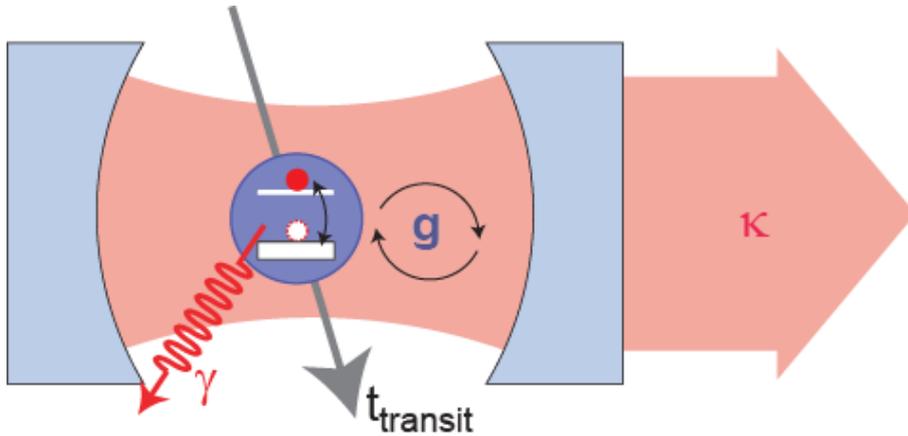
Material from colleagues at:

Yale, CEA Saclay, UCSB, Chalmers, TU Delft



Eidgenössische Technische Hochschule Zürich SWISS NATIONAL SCIENCE FOUNDATION  
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# Cavity QED with Superconducting Circuits



coherent interaction of photons with quantum two-level systems ...

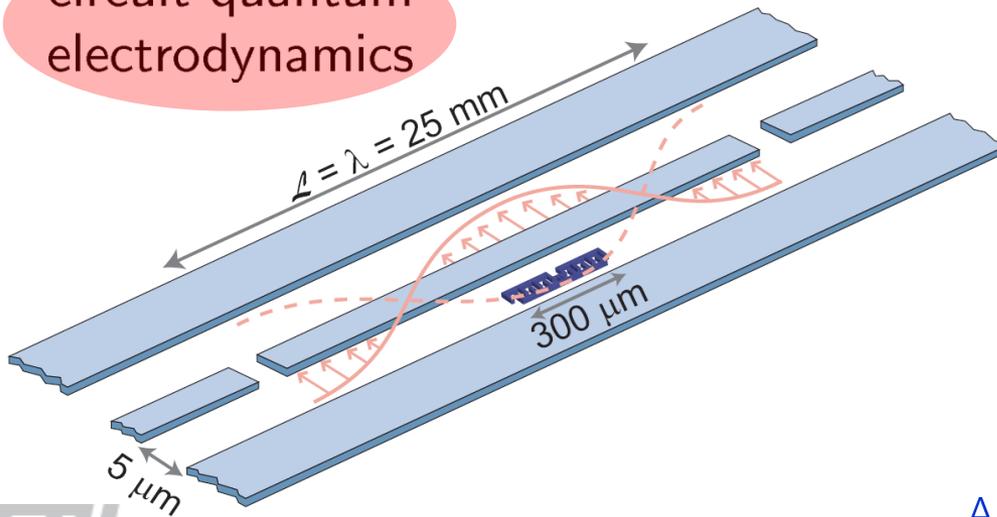
J. M. Raimond *et al.*, *Rev. Mod. Phys.* **73**, 565 (2001)

S. Haroche & J. Raimond, *oup Oxford* (2006)

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

... in superconducting circuits

circuit quantum electrodynamics



Properties:

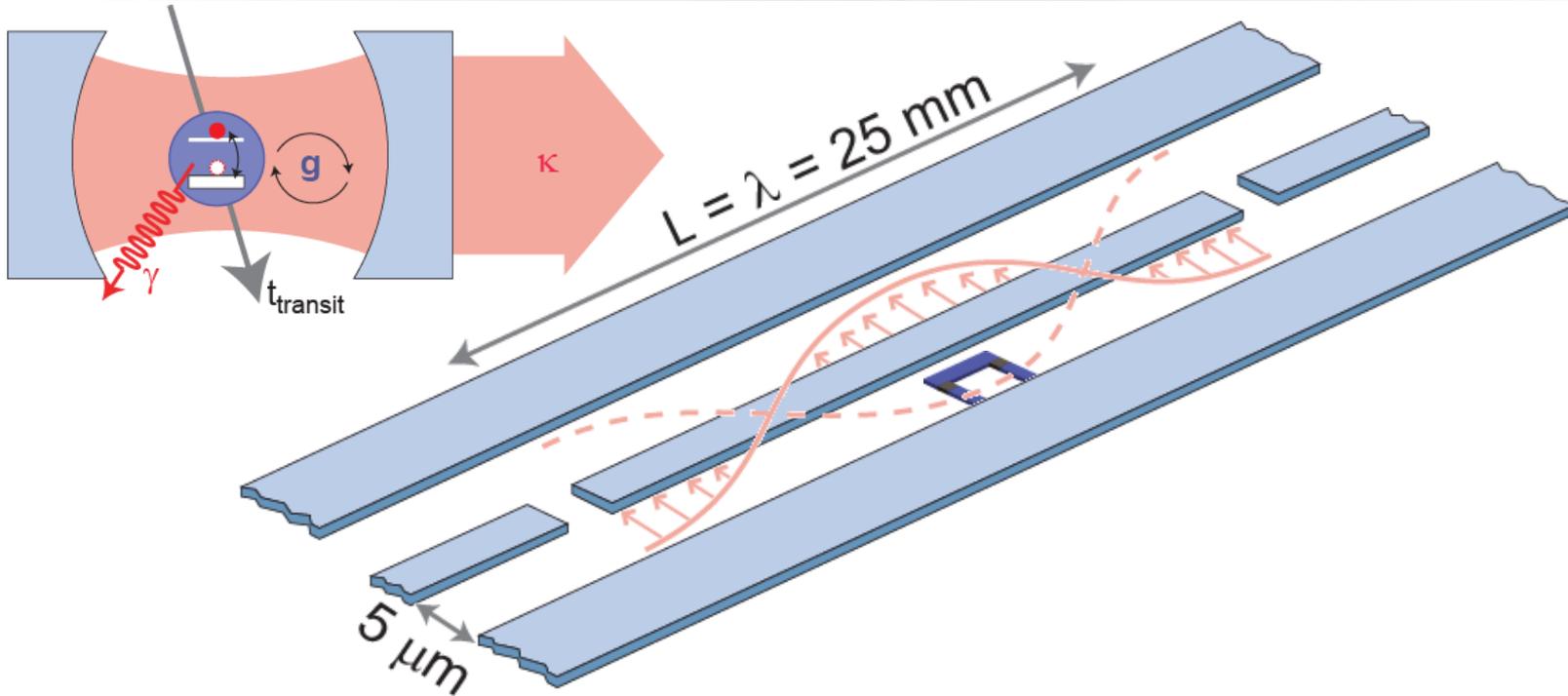
- solid state based
- large field per photon
- 'easy' to fabricate and integrate
- suitable for quantum interfaces

A. Blais, *et al.*, *PRA* **69**, 062320 (2004)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

# Circuit Quantum Electrodynamics



elements:

- the cavity: a superconducting 1D transmission line resonator with **large vacuum field**  $E_0$  and **long photon life time**  $1/\kappa$
- the artificial atom: a superconducting qubit with **large dipole moment**  $d$  and **long coherence time**  $1/\gamma$  and **fixed position**

A. Blais, et al., *PRA* **69**, 062320 (2004)

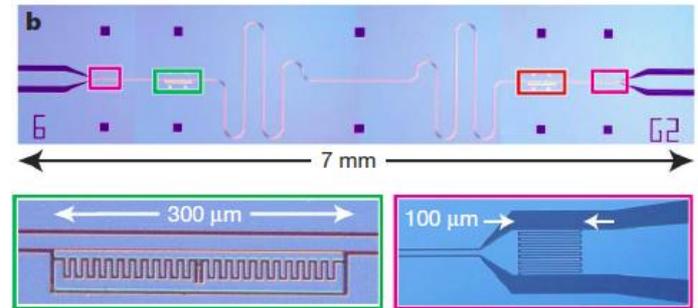
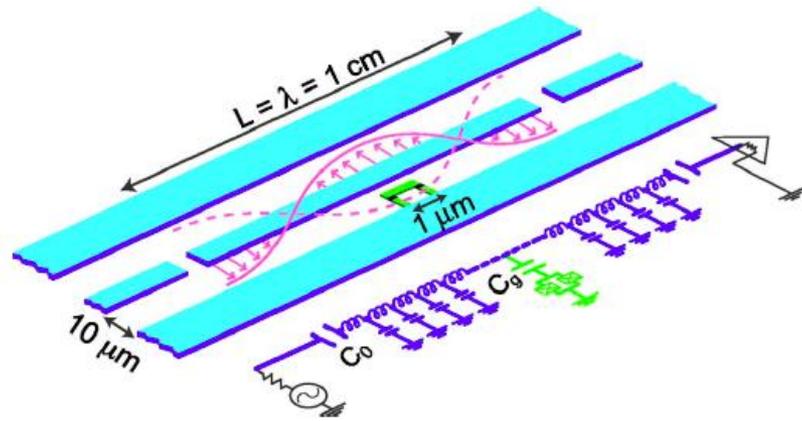
A. Wallraff et al., *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

# Quantum Computing with Superconducting Circuits

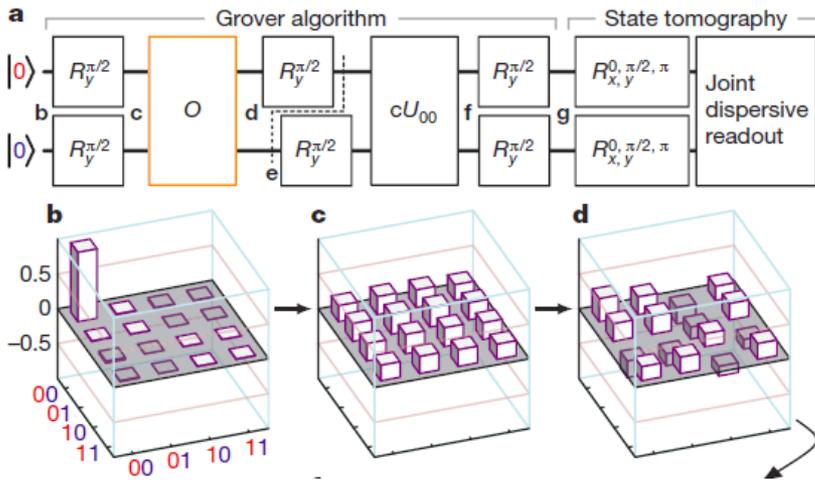
## Circuit QED Architecture

A. Blais et al., *PRA* **69**, 062320 (2004)  
 A. Wallraff et al., *Nature* **431**, 162 (2004)  
 M. Mariani et al., *Science* **334**, 61 (2011)



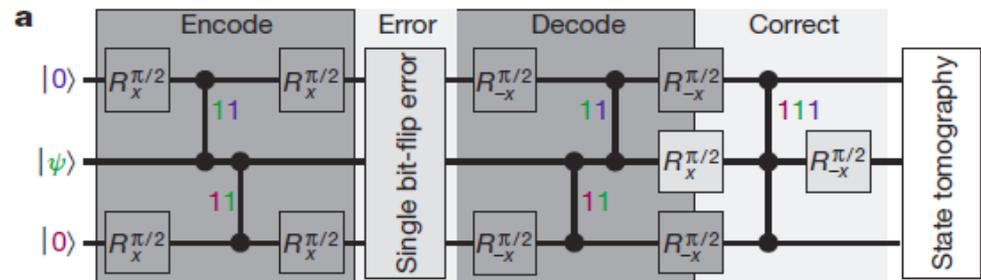
## Resonator as a Coupling Bus

M. Sillanpaa et al., *Nature* **449**, 438 (2007)  
 H. Majer et al., *Nature* **449**, 443 (2007)



## Deutsch, Grover Algorithms

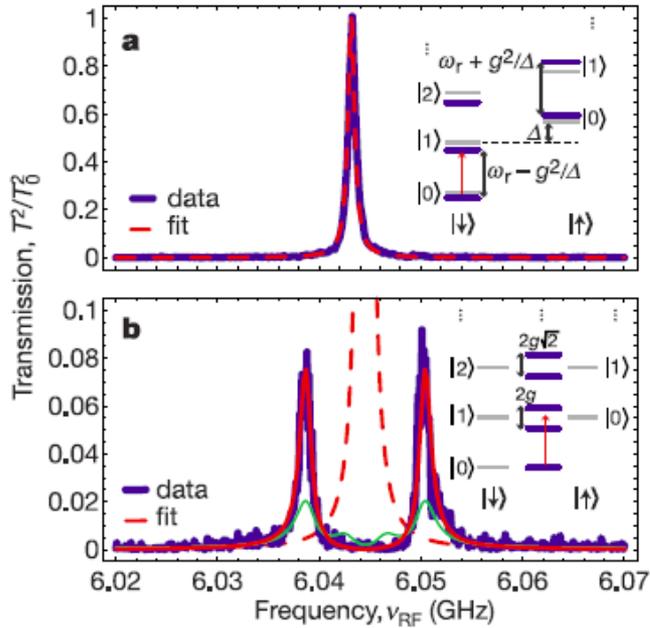
L. DiCarlo et al., *Nature* **460**, 240 (2009)  
 L. DiCarlo et al., *Nature* **467**, 574 (2010)



## Toffoli Gates & Error Correction

A. Fedorov et al., *Nature* **481**, 170 (2012)  
 M. Reed et al., *Nature* **481**, 382 (2012)

# Quantum Optics with Supercond. Circuits



## Strong Coherent Coupling

I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)  
 A. Wallraff *et al.*, *Nature* **431**, 162 (2004)  
 D. Schuster *et al.*, *Nature* **445**, 515 (2007)

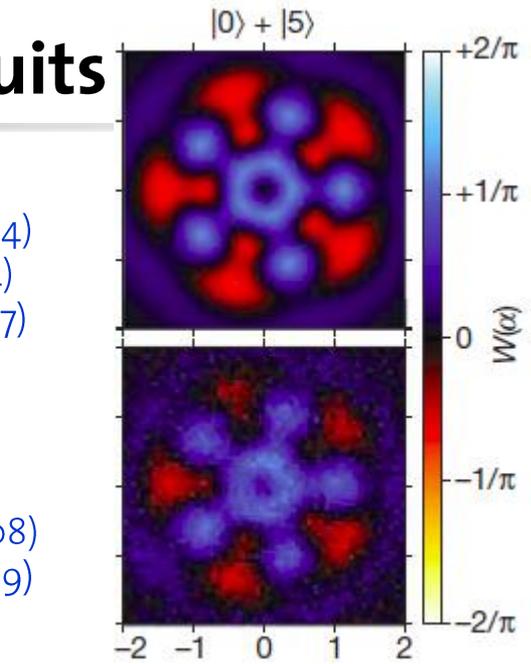
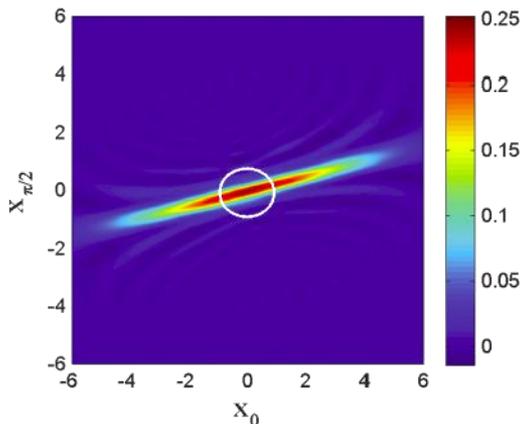
## Root n Nonlinearities

J. Fink *et al.*, *Nature* **454**, 315 (2008)  
 F. Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)  
 L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)



## Parametric Amplification & Squeezing

Castellanos-Beltran *et al.*,  
*Nat. Phys.* **4**, 928 (2008)

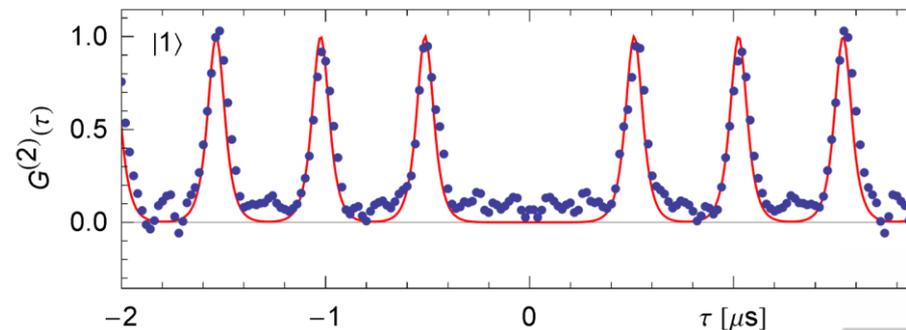


## Fock and Arbitrary Photon States

M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)  
 M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

## Single Photons & Correlations

A. Houck *et al.*, *Nature* **449**, 328 (2007)  
 D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)



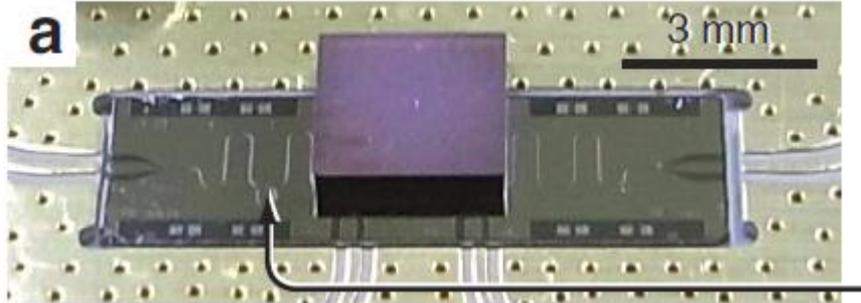
# Hybrid Systems with Superconducting Circuits

Proposals:

## Spin Ensembles: e.g. NV centers

D. Schuster *et al.*, *PRL* **105**, 140501 (2010)

Y. Kubo *et al.*, *PRL* **105**, 140502 (2010)



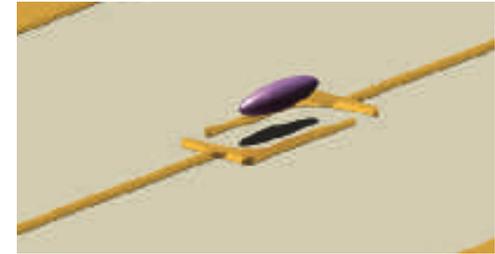
## Polar Molecules, Rydberg, BEC

P. Rabl *et al.*, *PRL* **97**, 033003 (2006)

A. Andre *et al.*, *Nat. Phys.* **2**, 636 (2006)

D. Petrosyan *et al.*, *PRL* **100**, 170501 (2008)

J. Verdu *et al.*, *PRL* **103**, 043603 (2009)

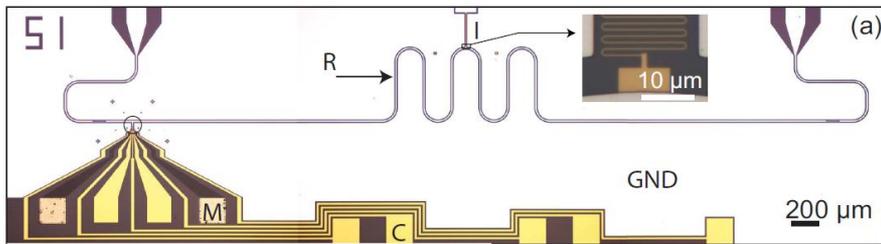


## CNT, Gate Defined 2DEG, or nanowire Quantum Dots

M. Delbecq *et al.*, *PRL* **107**, 256804 (2011)

T. Frey *et al.*, *PRL* **108**, 046807 (2012)

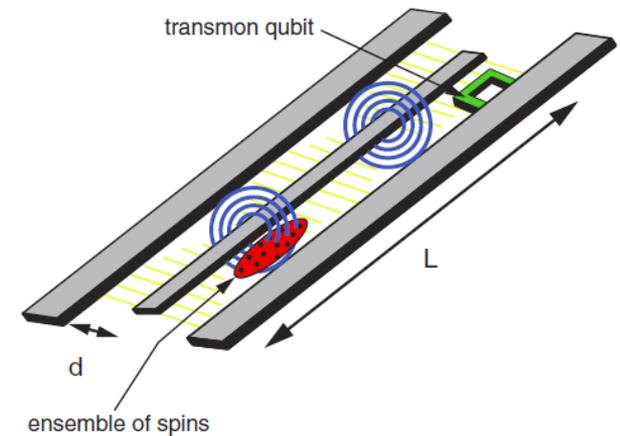
K. Petersson *et al.*, *arXiv*:1205.6767 (2012)



## Spin Ensembles

A. Imamoglu *et al.*, *PRL* **102**, 083602 (2009)

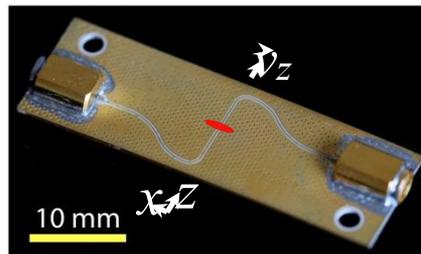
J. Wesenberg *et al.*, *PRL* **103**, 070502 (2009)



## Rydberg Atoms

S. Hogan *et al.*, *PRL* **108**,

063004 (2012)



... and many more

# Lecture Topics

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- I. Quantum Mechanics of Superconducting Electronic Circuits
- II. Circuit Quantum Electrodynamics (QED)
- III. Exploring Matter/Light Interactions in Circuit QED
- IV. Characterizing Propagating Microwave Photons
- V. Interfaces between Superconducting Circuits and Quantum Dots or Rydberg Atoms

# Quantum Mechanics of Superconducting Electronic Circuits

# Conventional Electronic Circuits

basic circuit elements:

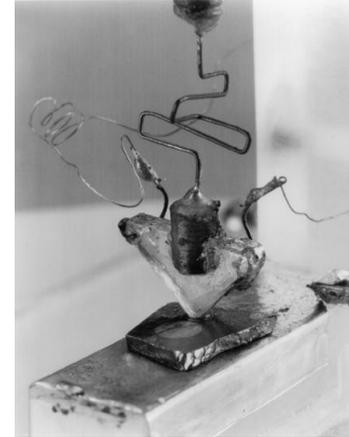


basis of modern  
information and  
communication  
technology

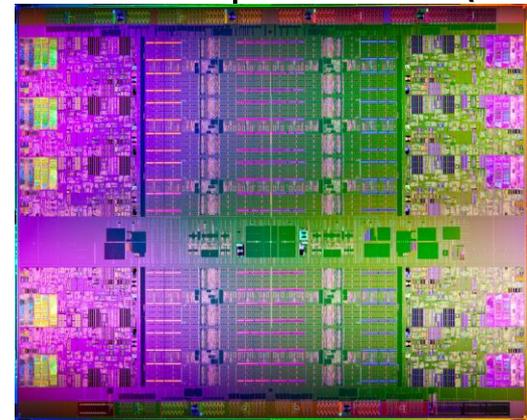
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



intel xeon processors (2011)



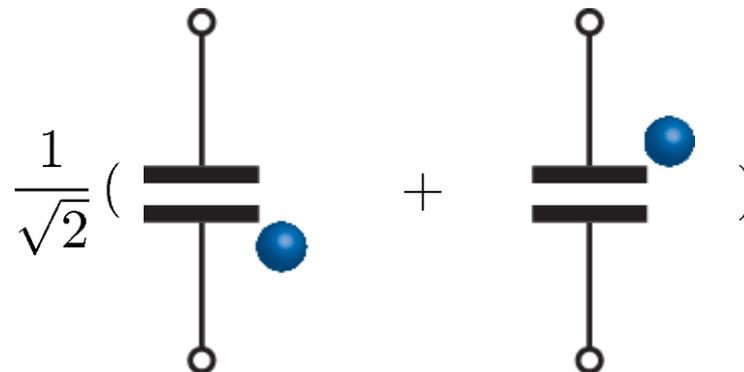
3.000.000.000 transistors  
smallest feature size 32 nm  
clock speed ~ 3 GHz  
power consumption 10 W

# Classical and Quantum Electronic Circuit Elements

basic circuit elements:



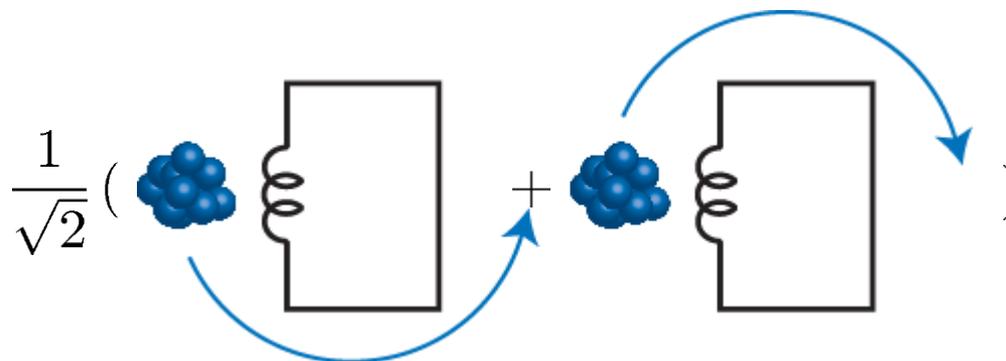
charge on a capacitor:



quantum superposition states of:

- charge  $q$
- flux  $\phi$

current or magnetic flux in an inductor:

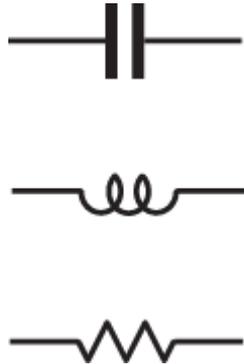


commutation relation (c.f.  $x, p$ ):

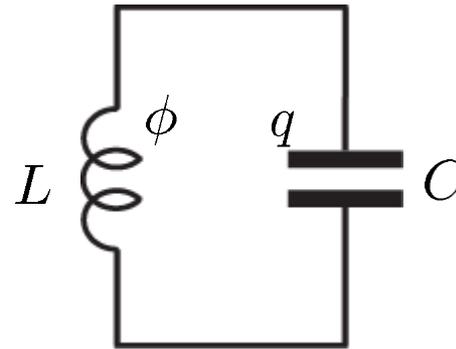
$$[\hat{\phi}, \hat{q}] = i\hbar$$

# Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5 \text{ GHz}$$

classical physics:

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

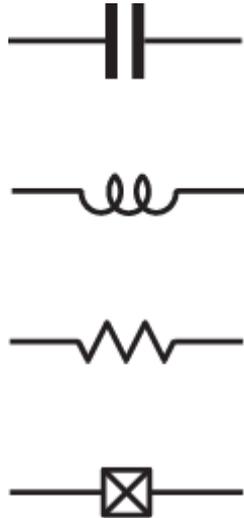
$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \quad [\hat{\phi}, \hat{q}] = i\hbar$$

energy:



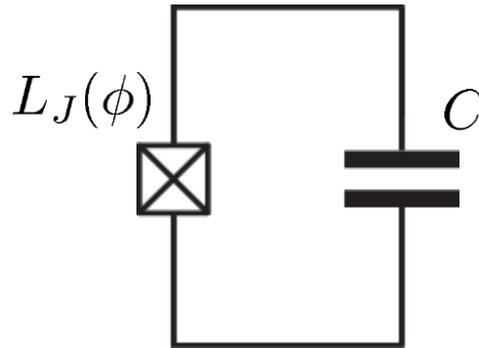
# Constructing Non-Linear Quantum Electronic Circuits

circuit elements:



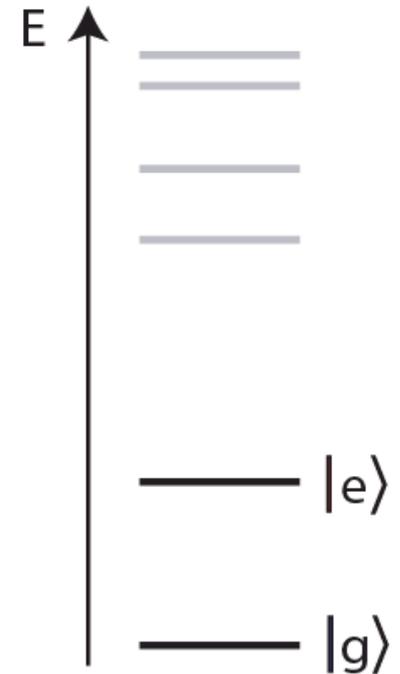
Josephson junction:  
a non-dissipative nonlinear  
element (inductor)

anharmonic oscillator:



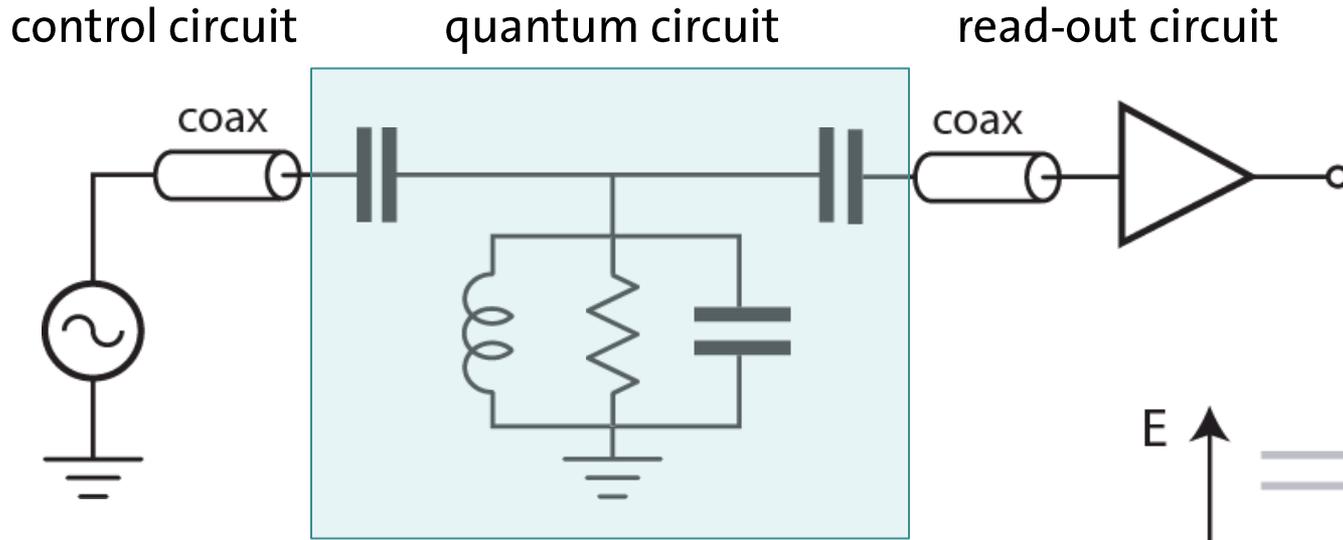
$$L_J(\phi) = \left( \frac{\partial I}{\partial \phi} \right)^{-1}$$
$$= \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

non-linear energy  
level spectrum:



electronic  
artificial atom

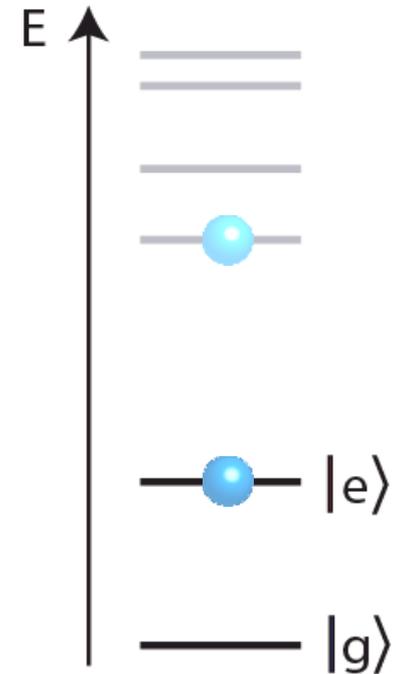
# How to Operate Circuits in the Quantum Regime?



recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

Can one actually build and operate such circuits?

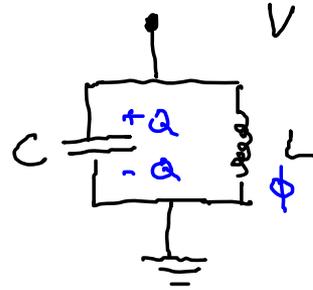




# Electronic Harmonic Oscillators

## Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L \frac{\partial I}{\partial t}$$

total energy (Hamiltonian):

$$H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} \frac{\phi^2}{L}$$

with the charge  $Q$  stored on the capacitor

$$Q = VC$$

a flux  $\phi$  stored in the inductor

$$\phi = LI$$

properties of Hamiltonian written in variables  $Q$  and  $\phi$ :

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L \frac{\partial I}{\partial t} = -\dot{\phi}$$

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

$Q$  and  $\phi$  are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

# Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation

$$[\hat{\phi}, \hat{Q}] = i\hbar$$



compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:

- charge  $Q$  corresponds to momentum  $p$
- flux  $\phi$  corresponds to position  $x$

$$[\hat{x}, \hat{p}] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}] = i\hbar$$

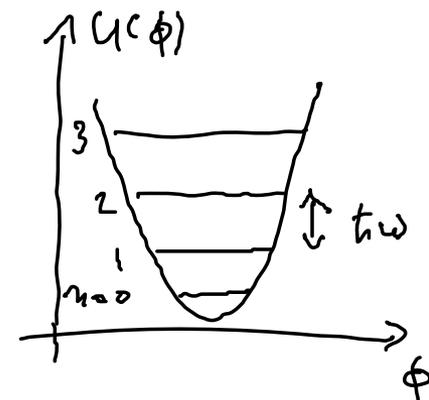
$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar \omega (a^\dagger a + \frac{1}{2})$$

with oscillator resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$



Raising and lowering operators:

$$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle ; \hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

$$a^\dagger a |n\rangle = n |n\rangle$$

number operator

in terms of  $Q$  and  $\phi$ :

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{Q} + i \hat{\phi})$$

with  $Z_c$  being the characteristic impedance of the oscillator

$$Z_c = \sqrt{\frac{L}{C}}$$

charge  $Q$  and flux  $\phi$  operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}} (a^\dagger + a)$$

$$\hat{\phi} = i \sqrt{\frac{\hbar Z_c}{2}} (a^\dagger - a)$$

**Exercise:** Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.