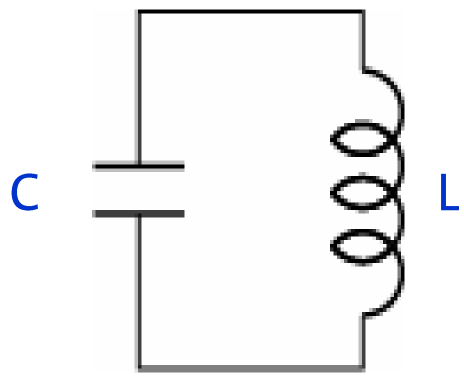
Superconducting Harmonic Oscillator

a simple electronic circuit:



- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm

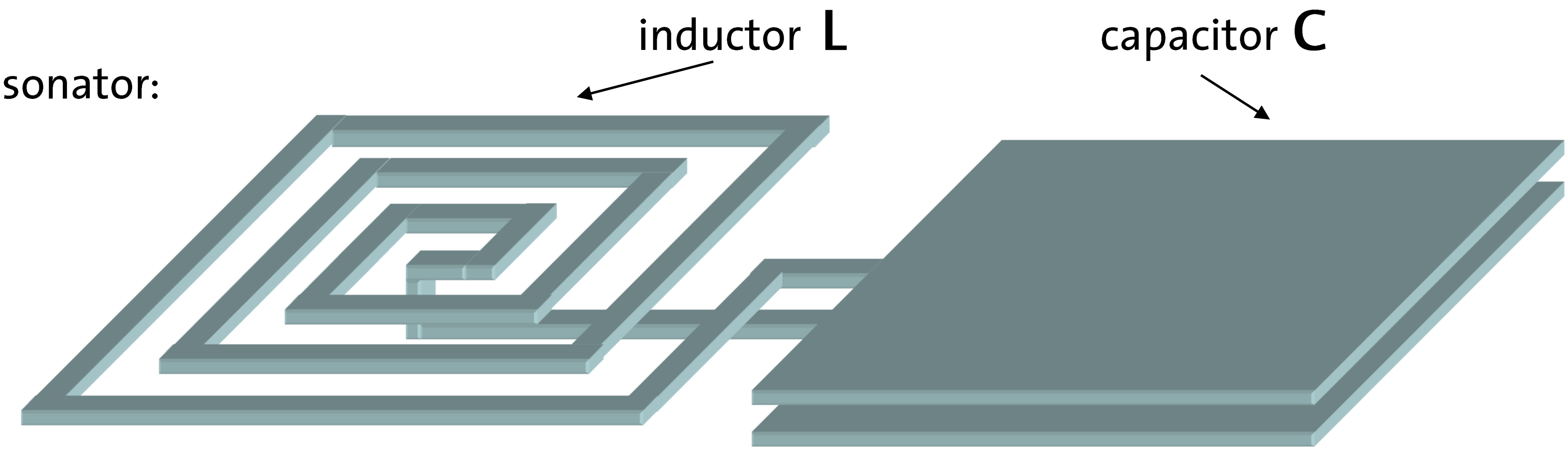
- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF

- resonance frequency

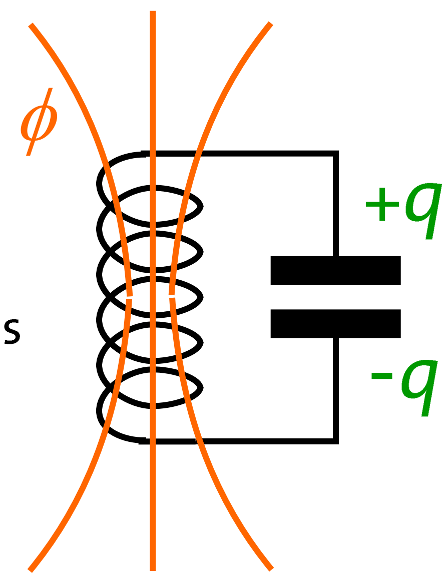
$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\text{GHz}$$

A Simple Electronic Harmonic Oscillator Circuit

LC resonator:



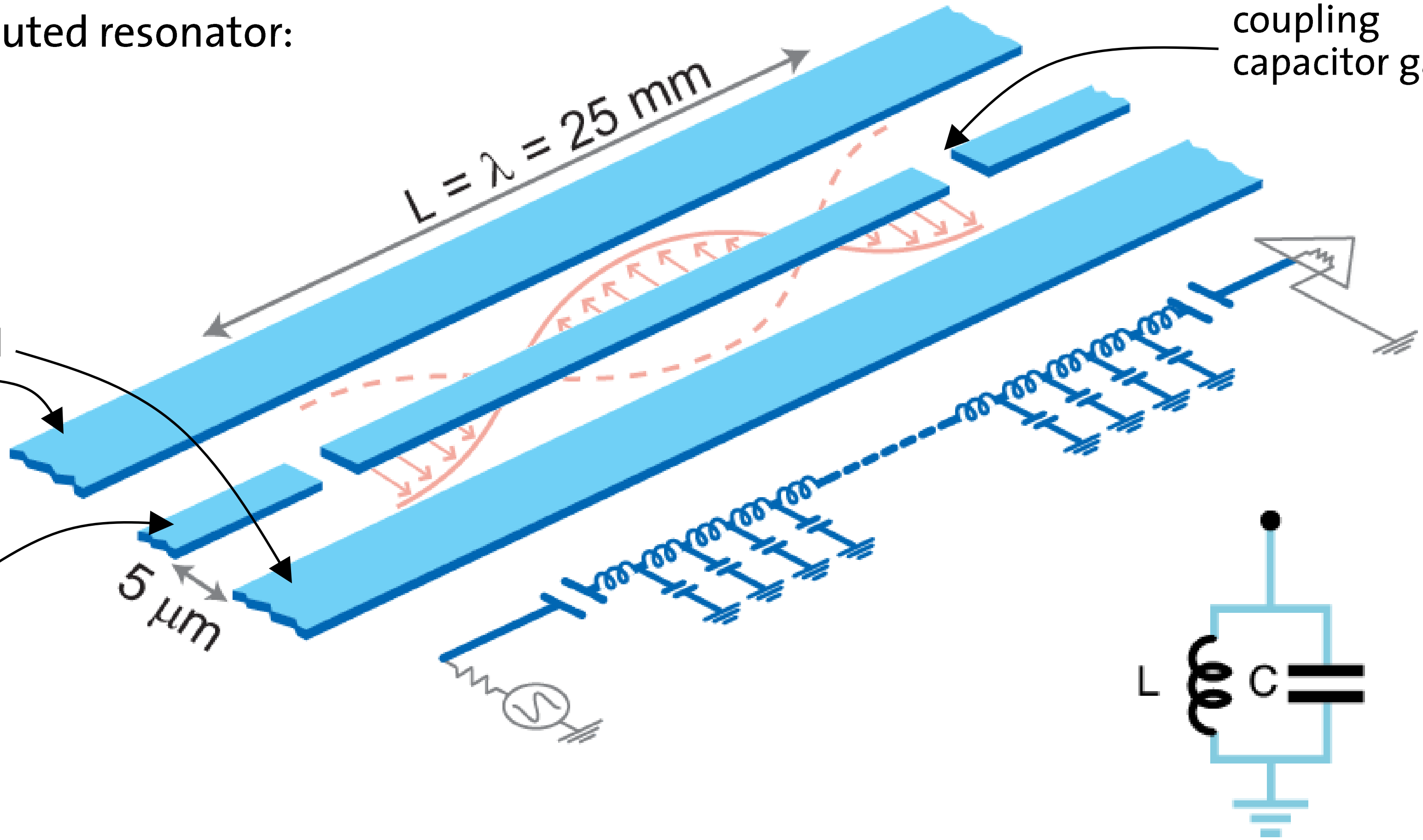
currents and magnetic fields



charges and electric fields

Realization of H.O.: Transmission Line Resonator

distributed resonator:

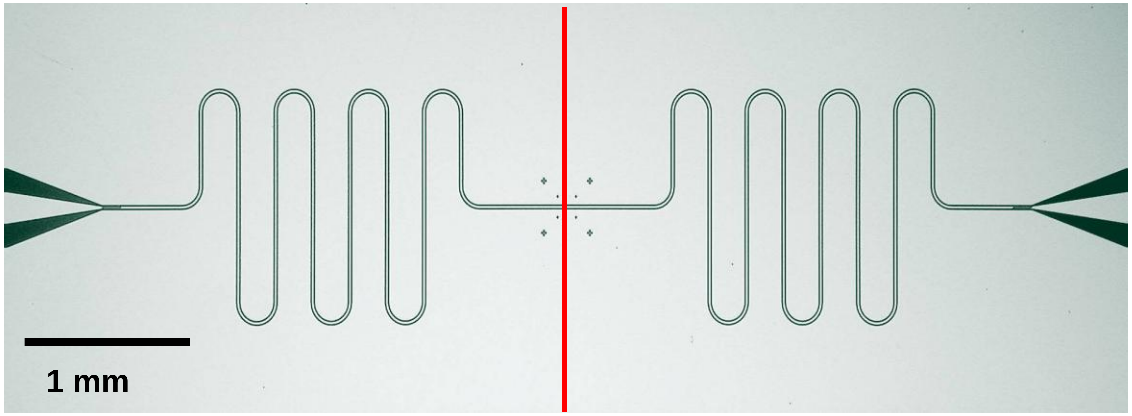


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

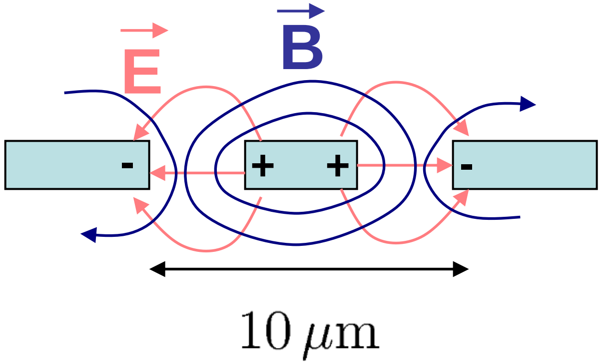
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

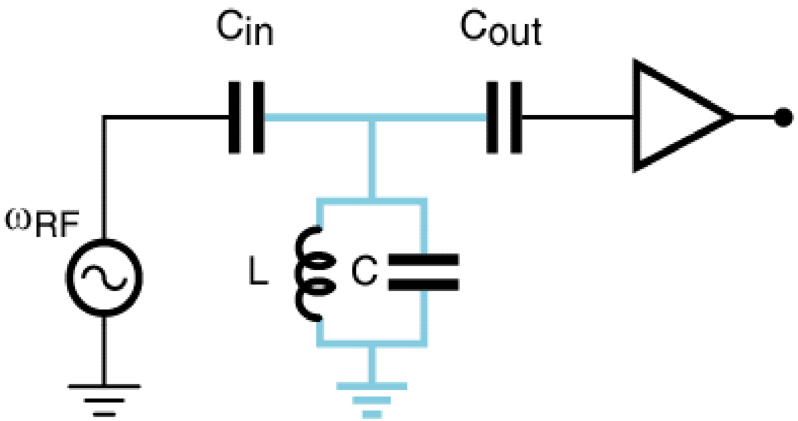
coplanar waveguide:



cross-section of transm. line (TEM mode):

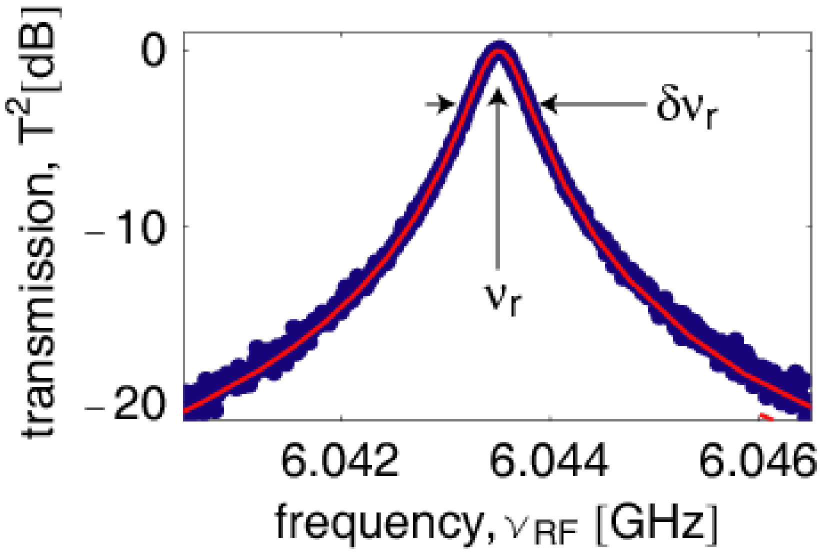


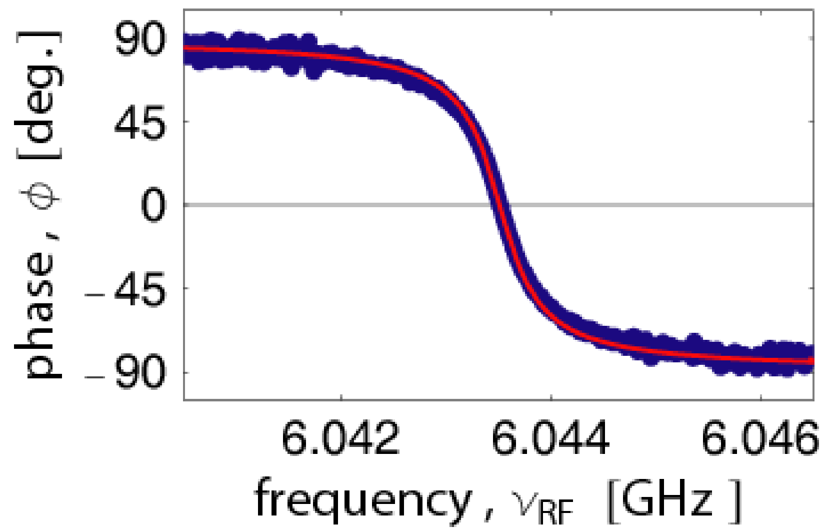
measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

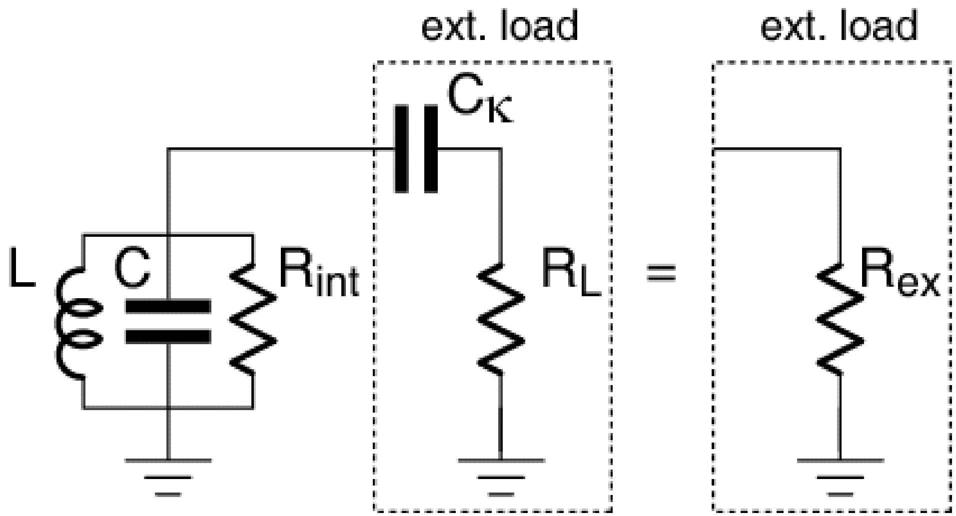
photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Internal and External Dissipation in an LC Oscillator



internal losses R_{int}:
conductor, dielectric

external losses R_{ext} :
radiation, coupling

Loading due to external circuit:

total effective resistance:

$$\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_{\mathrm{int}}} + \frac{1}{R^*_{\mathrm{ext}}}$$

total effective capacitance:

$$C_{\mathrm{tot}} = C_{\mathrm{int}} + C^*_{\mathrm{ext}}$$

energy decay time/rate:

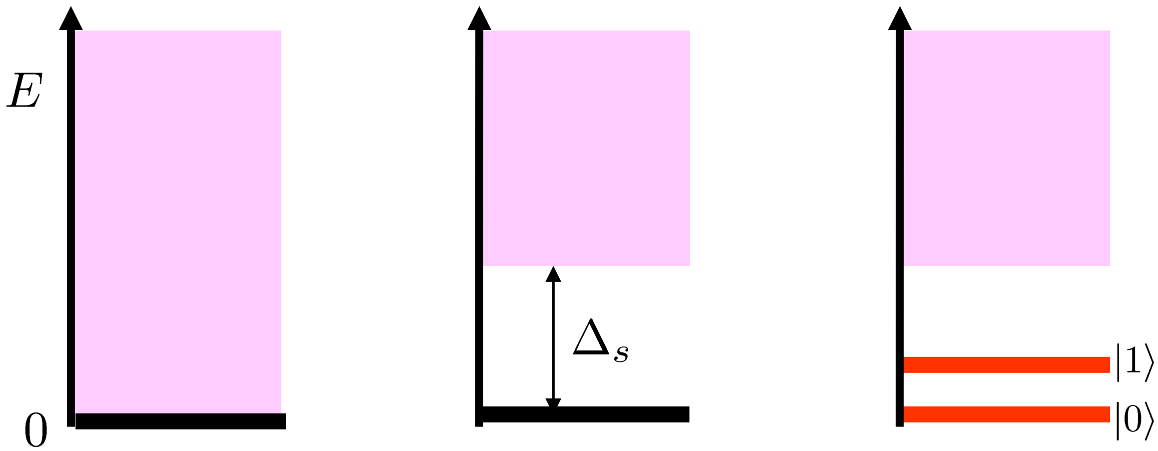
$$T_\kappa = \frac{1}{\kappa} = R_{\mathrm{tot}} C_{\mathrm{tot}}$$

External circuit induces:

decrease of total resistance
-> decrease of quality factor

increase of total capacitance
-> decrease of res. frequency

Why Superconductors?



normal metal — superconductor — How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

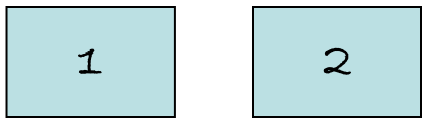
Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta/h$ = 725 GHz, T_c = 9.2 K
- Aluminum (Al): $2\Delta/h$ = 100 GHz, T_c = 1.2 K

Cooper pairs:
bound electron pairs

Bosons (S=0, L=0)

2 chunks of superconductors

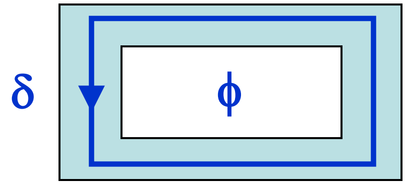


macroscopic wave function

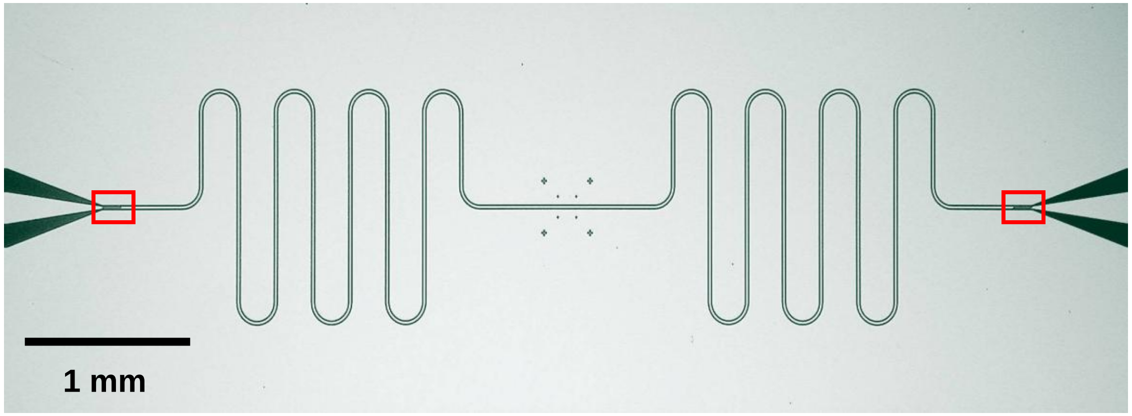
$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

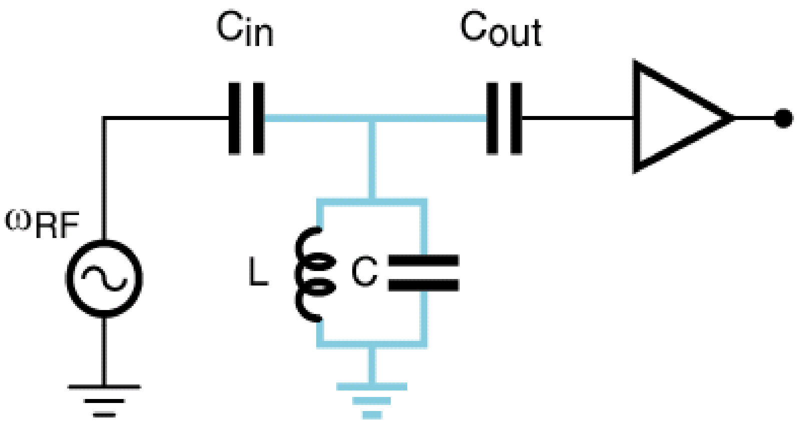
Cooper pair density n_i
and global phase δ_i

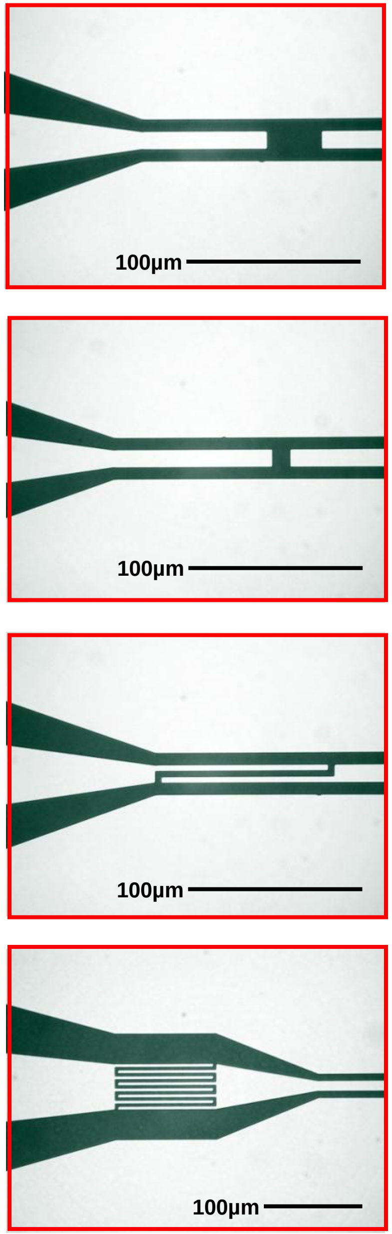
phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$



Controlling the Photon Life Time



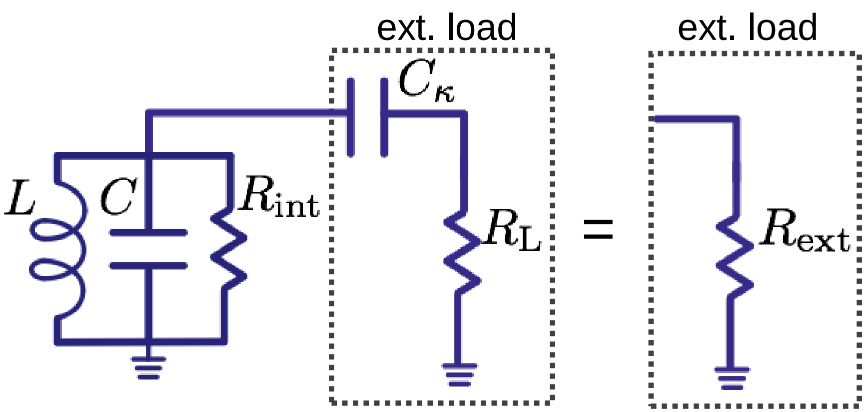


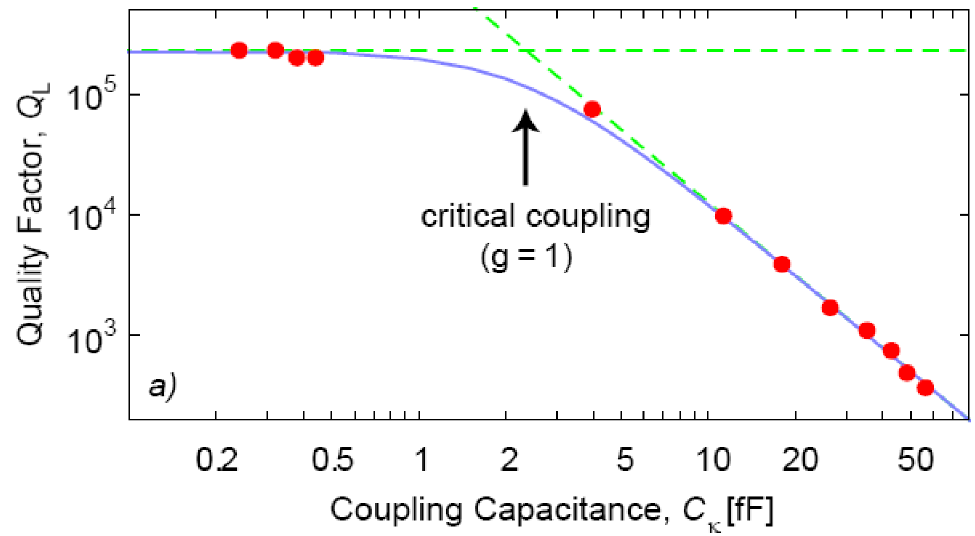


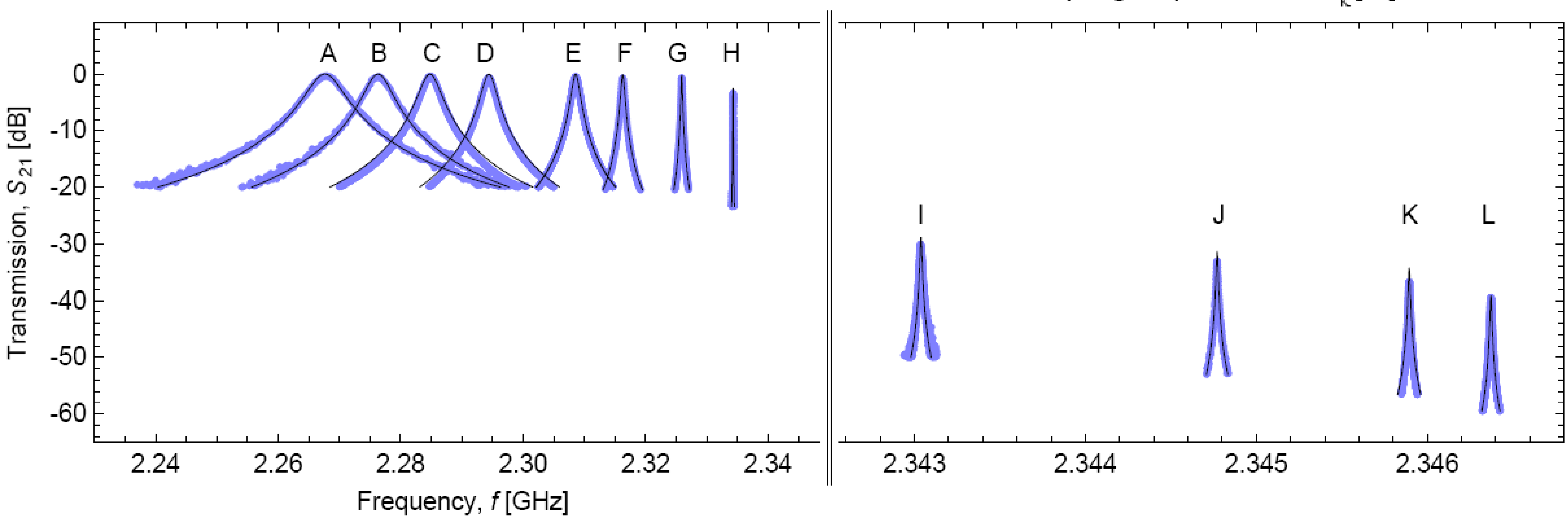
photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Quality Factor Measurement

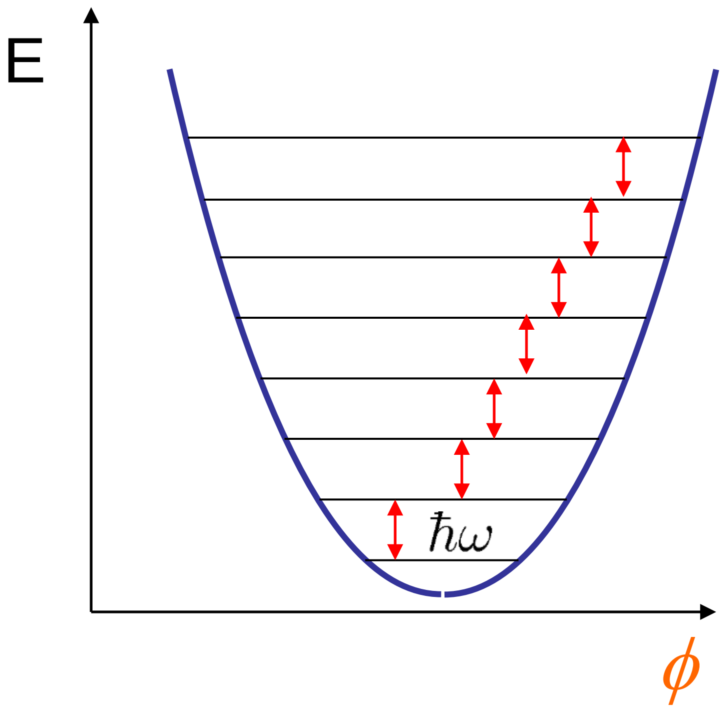






M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\mathrm{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{\mathrm{th}} \rangle \sim 10^{-11}$$

How to Prove that a Harmonic Oscillator is Quantum?

measure:

- resonance frequency
- average charge (momentum)
- average flux (position)

all simple averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

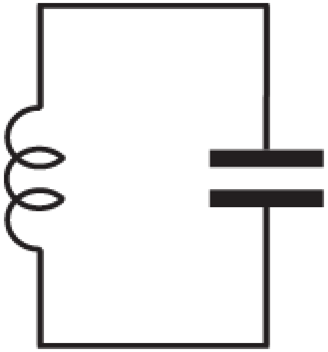
solutions:

- make oscillator non-linear in a controllable way
- measure higher order statistical properties

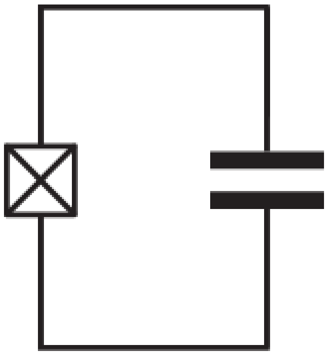
Nonlinear Electronic Oscillators

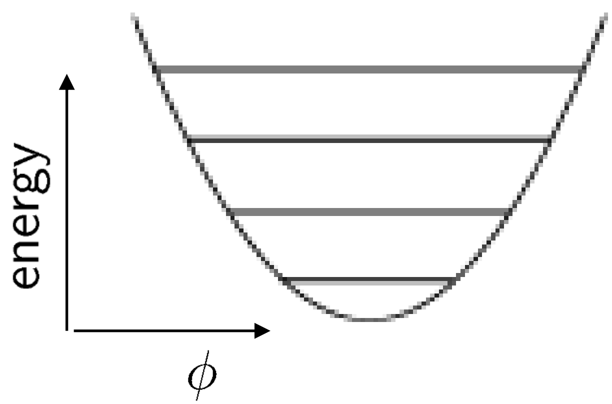
Linear vs. Nonlinear Superconducting Oscillators

LC resonator:

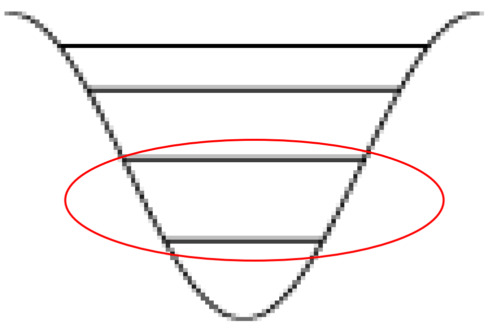


Josephson junction resonator:
Josephson junction = nonlinear inductor



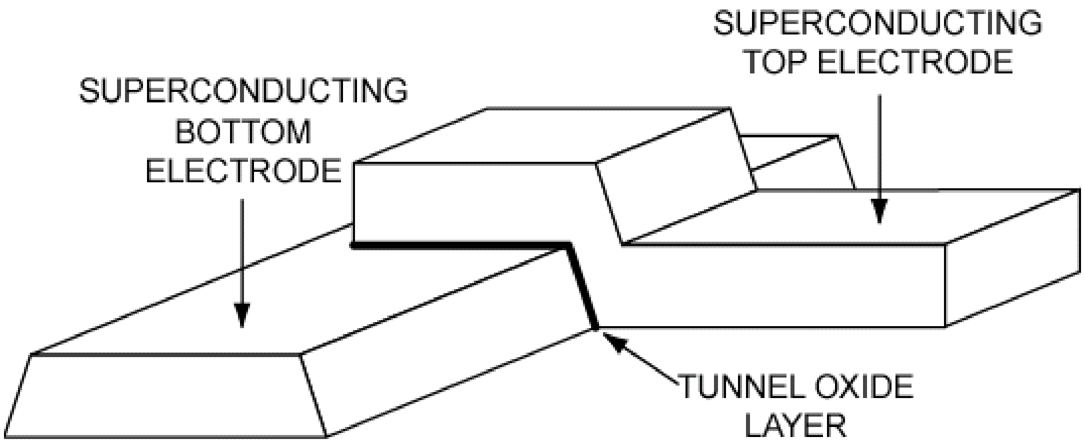


anharmonicity defines effective two-level system



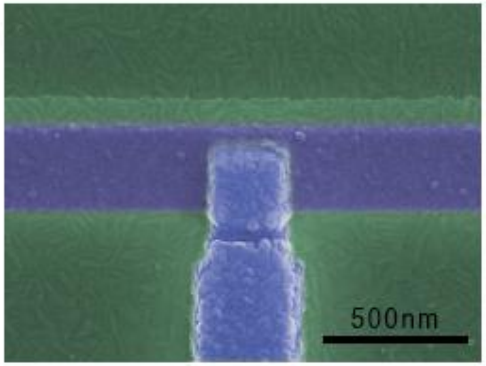
A Low-Loss Nonlinear Element

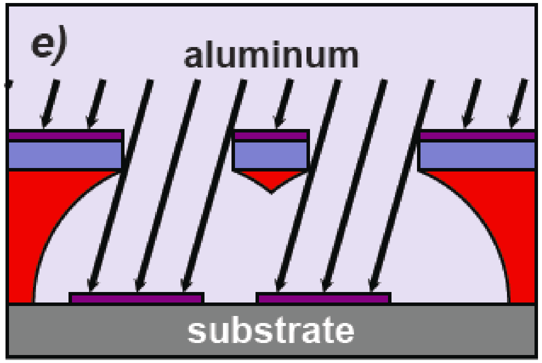
a (superconducting) Josephson junction:

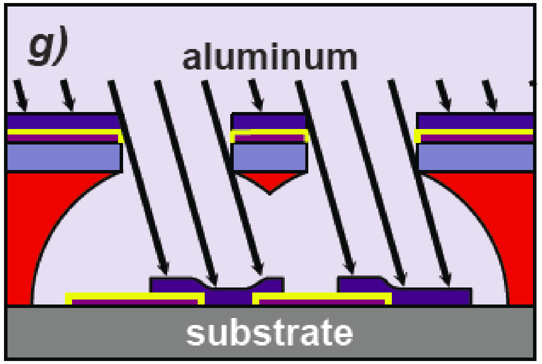


- superconductors: Nb, Al
- tunnel barrier: AlO_x

Josephson junction fabricated by shadow evaporation:



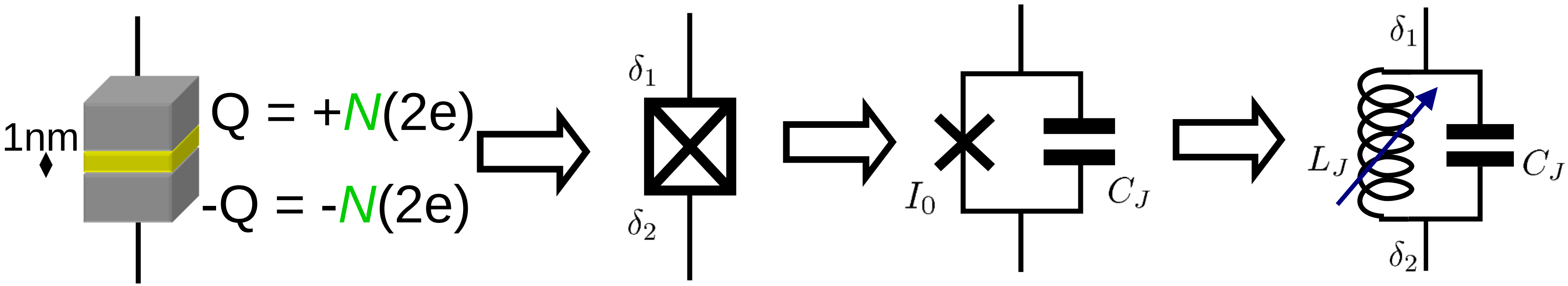




M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction parameters:

- critical current I_0
- junction capacitance C_J
- high internal resistance R_J

Josephson relations:

$$I = I_0 \sin\delta$$

$$V = \frac{\phi_0}{2\pi}\dot{\delta}$$

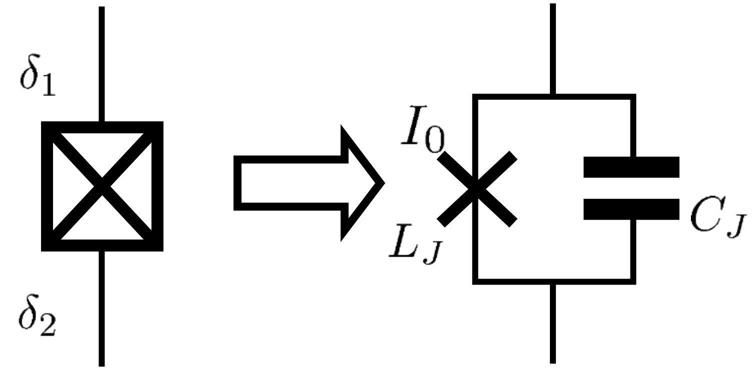
flux quantum: $\phi_0 = \frac{h}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson Junction as an ideal Non-Linear Inductor

a nonlinear inductor without dissipation



Josephson relations:

$$I = I_0 \sin\delta = I_0 \sin\left[2\pi\phi(t)/\phi_0\right]$$

$$V = \frac{\phi_0}{2\pi}\dot{\delta} = \dot{\phi}$$

nonlinear current/phase relation

gauge inv. phase difference:

$$\delta = \delta_2 - \delta_1 = 2\pi\phi(t)/\phi_0$$

specific Josephson inductance L_{J0}

Josephson inductance:

$$V = -L_J \dot{I} = \frac{\phi_0}{2\pi I_0}\frac{1}{\cos\delta}\dot{I}$$

specific Josephson energy E_{J0}

Josephson energy:

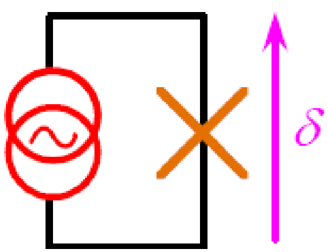
$$E_J = \int V I dt = \frac{I_0\phi_0}{2\pi}\cos\delta$$

A Classification of Josephson Junction Based Qubits

How to make use in of Jospehson junctions in a qubit?

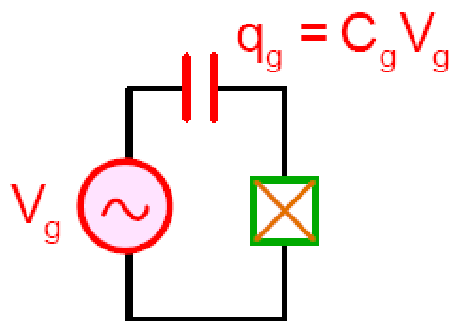
Common options of bias (control) circuits:

phase qubit



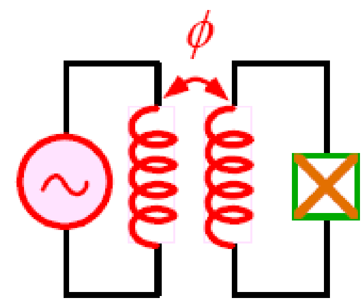
current bias

charge qubit
(Cooper Pair Box, Transmon)



charge bias

flux qubit



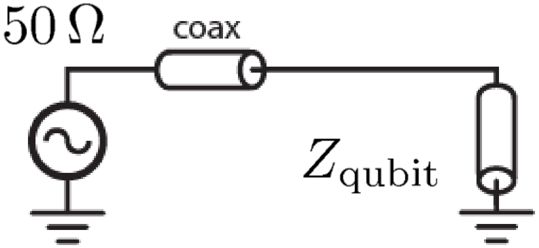
flux bias

How is the control circuit important?

Control of Coupling to Electromagnetic Environment

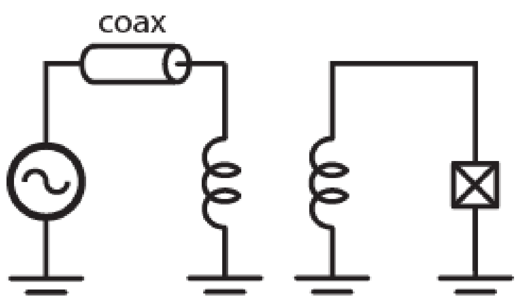
coupling to environment (bias wires):

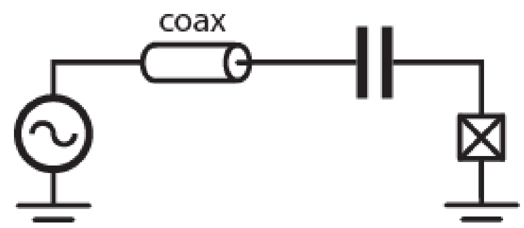
$Z_{\mathrm{line}} \sim 50\,\Omega$



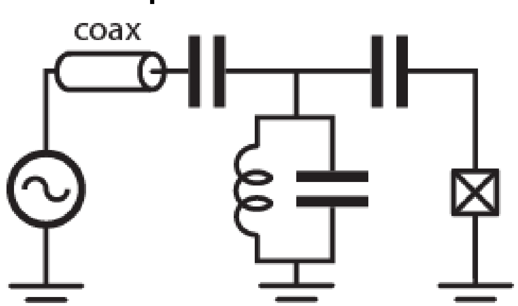
decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...





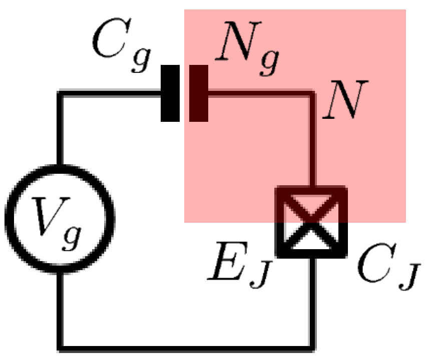
... or resonant impedance transformers



control spontaneous emission by circuit design

The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \dfrac{Q}{2e}$

continuous gate charge: $N_g = \dfrac{C_g V_g}{2e}$

Hamiltonian: $H = H_{\text{el}} + H_{\text{mag}}$

electrostatic part:

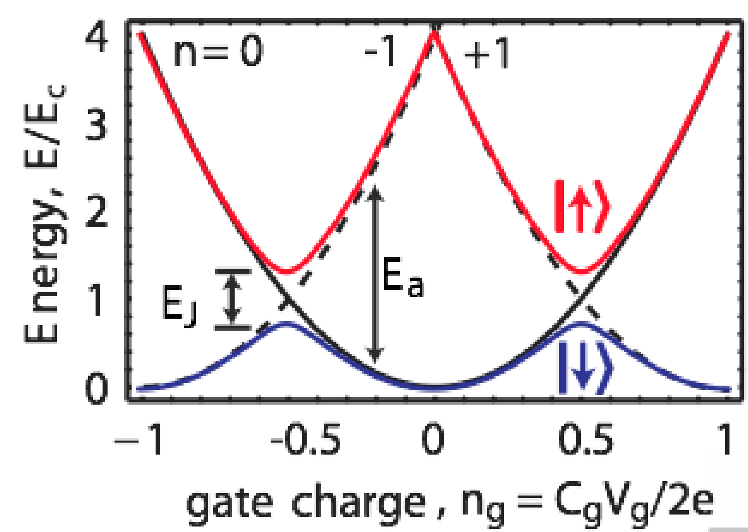
$$H_{\text{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\text{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator: $$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_m |N\rangle\langle N| = 1$$ completeness

$$\langle N|N\rangle = \delta_{nm}$$ orthogonality

phase operator: $$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis *N***:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n |\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 + E_J \cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar \frac{1}{2e} \frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

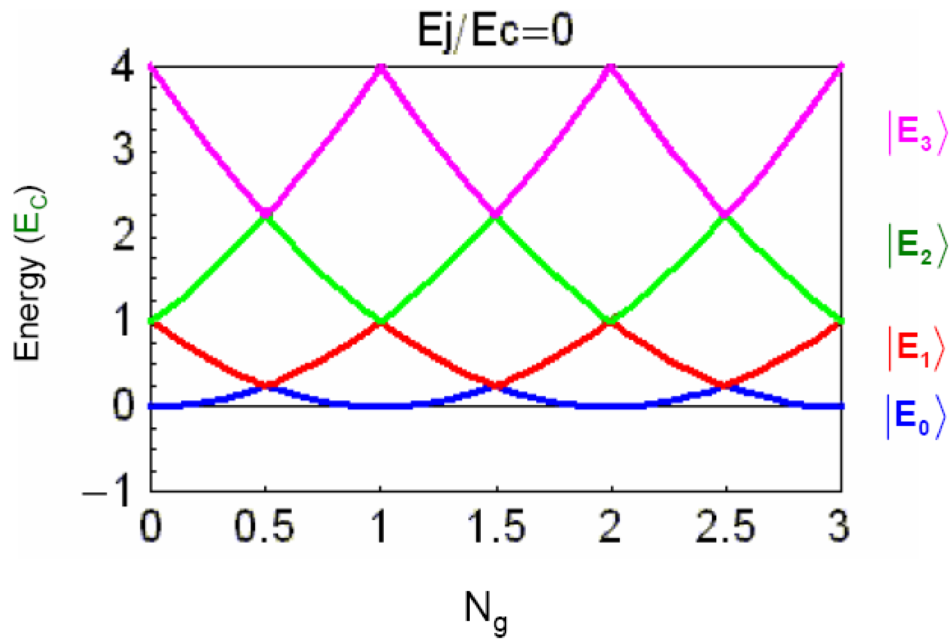
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n |\psi_n(\delta)\rangle$$

Energy Levels

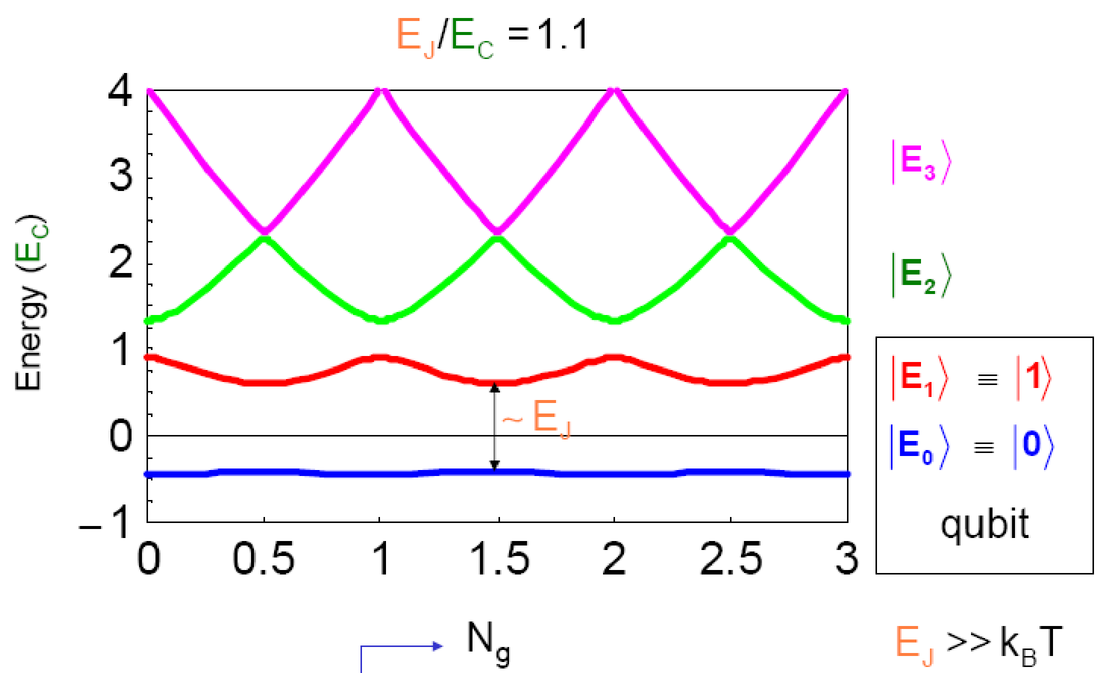
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g

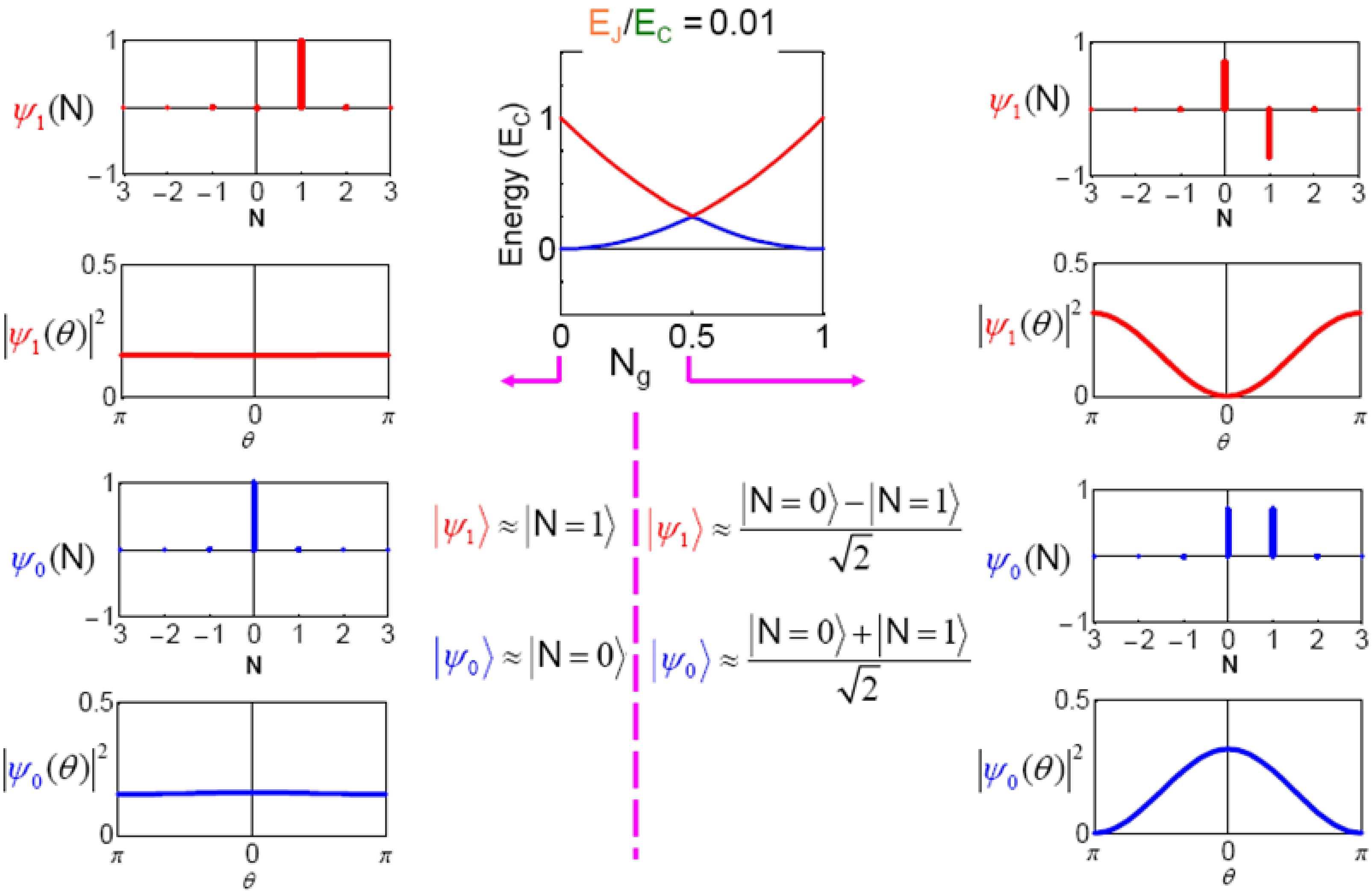


energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



Charge and Phase Wave Functions ($E_j << E_c$)

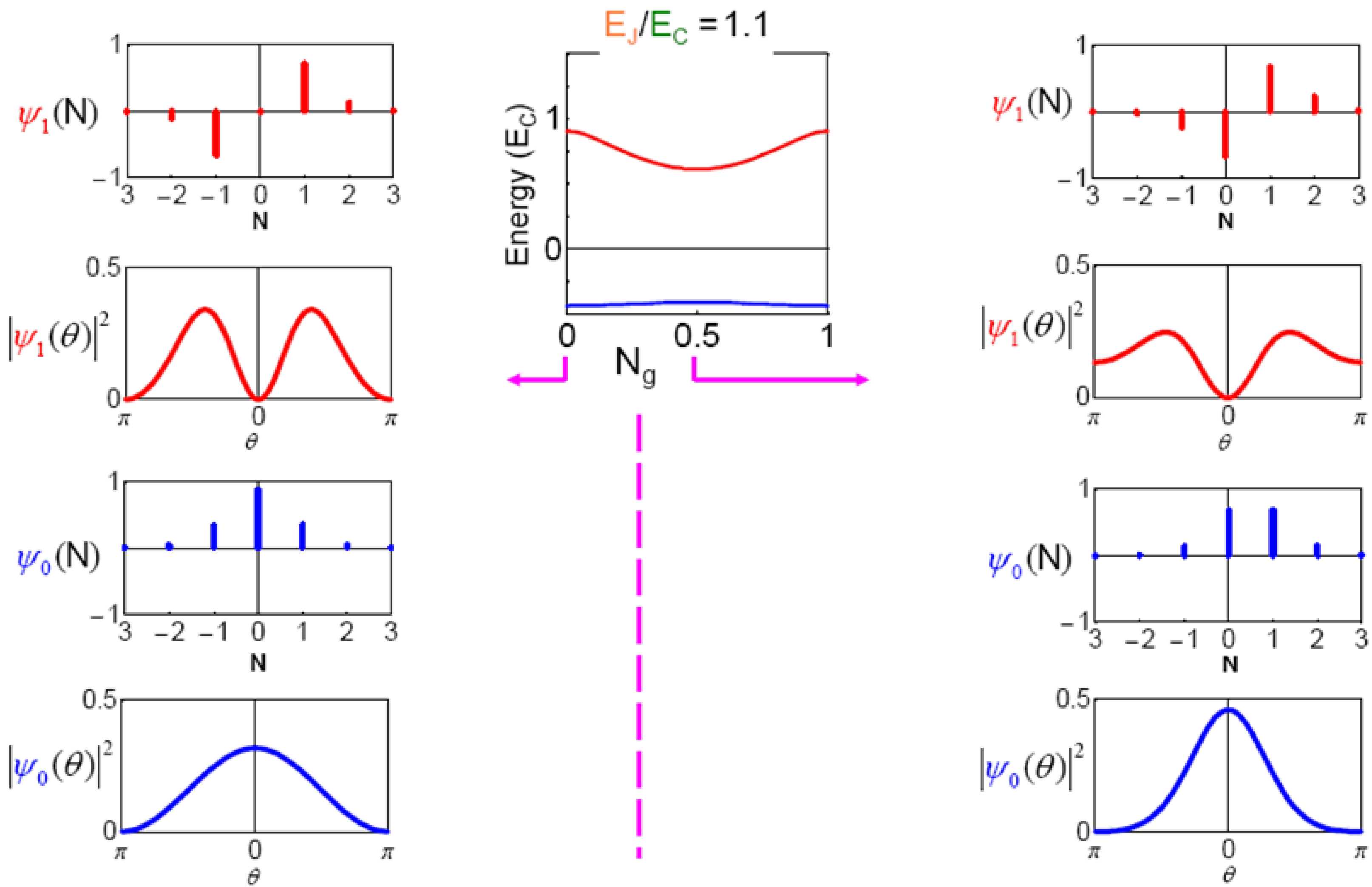


$$|\psi_1\rangle \approx |N=1\rangle \qquad |\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx |N=0\rangle \qquad |\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$

courtesy CEA Saclay

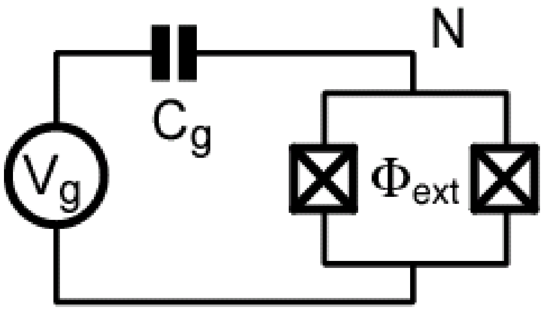
Charge and Phase Wave Functions ($E_j \sim E_c$)



courtesy CEA Saclay

Tuning the Josephson Energy

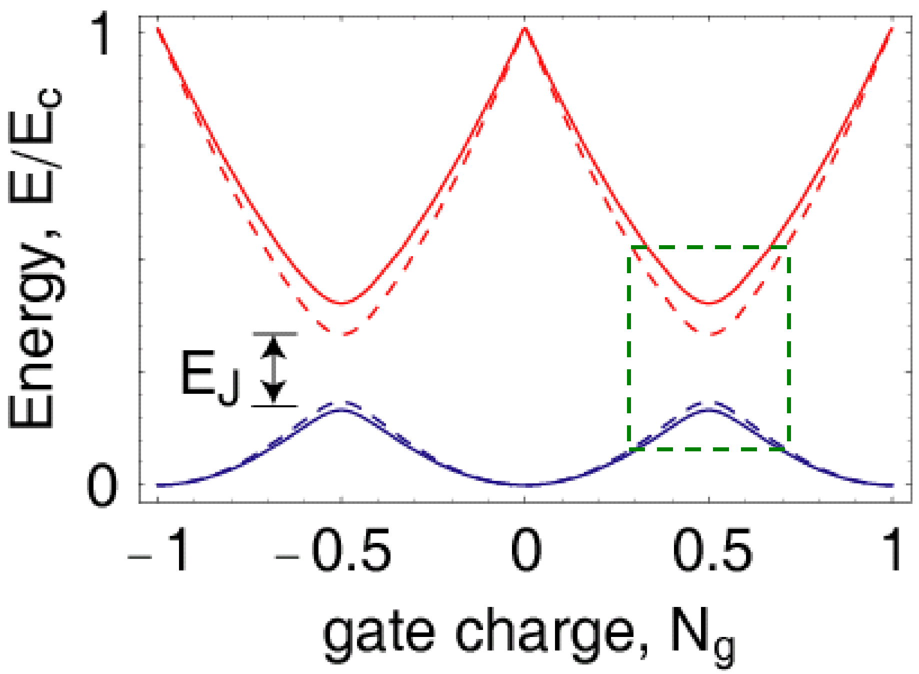
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two State Approximation

$$\hat{H}_{\text{CPB}} = \hat{H}_{\text{el}} + \hat{H}_{\text{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

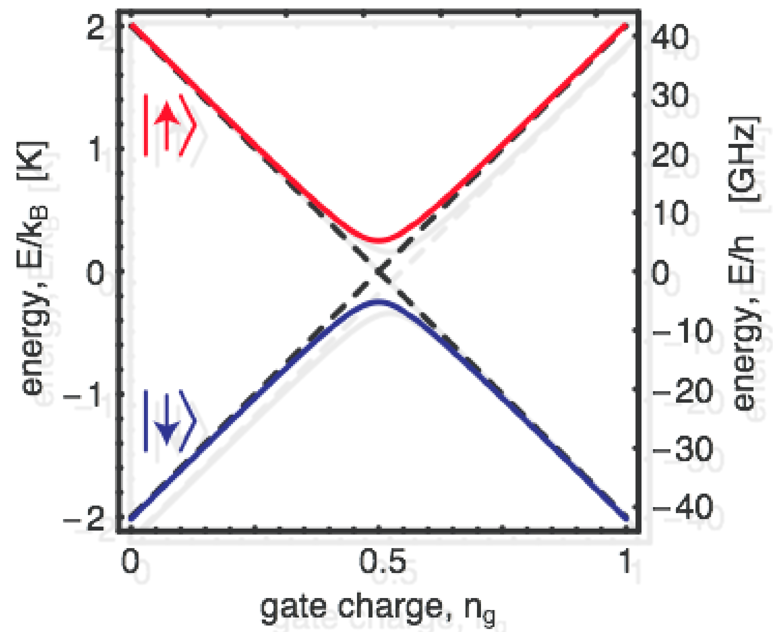
$$\hat{H}_{\text{CPB}} = \sum_N \left[E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\text{J}}}{2} \left(\left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\text{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

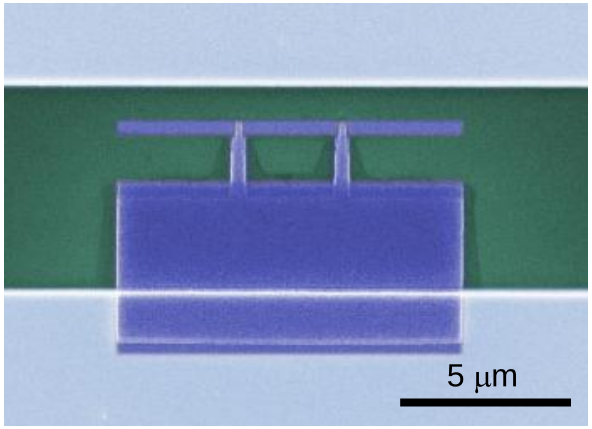


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

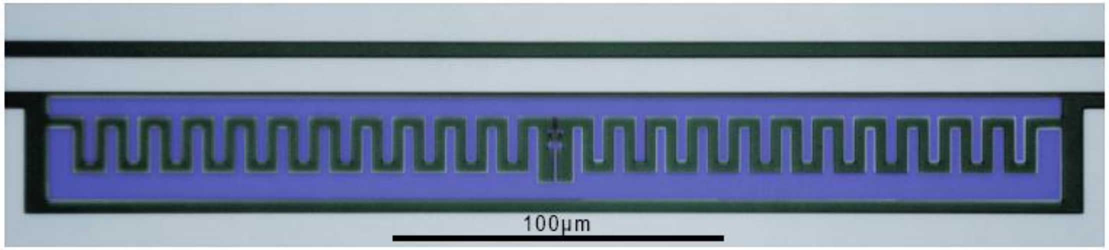
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

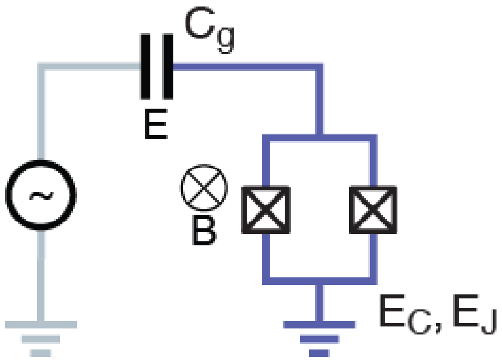
standard CPB:

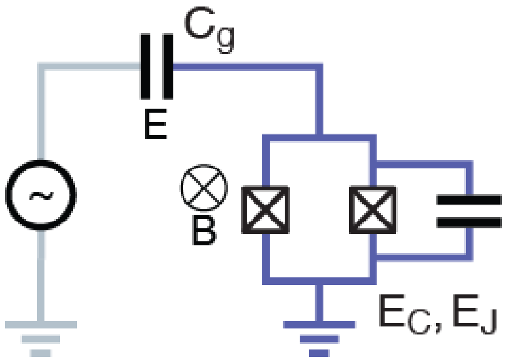


Transmon qubit:



circuit diagram:

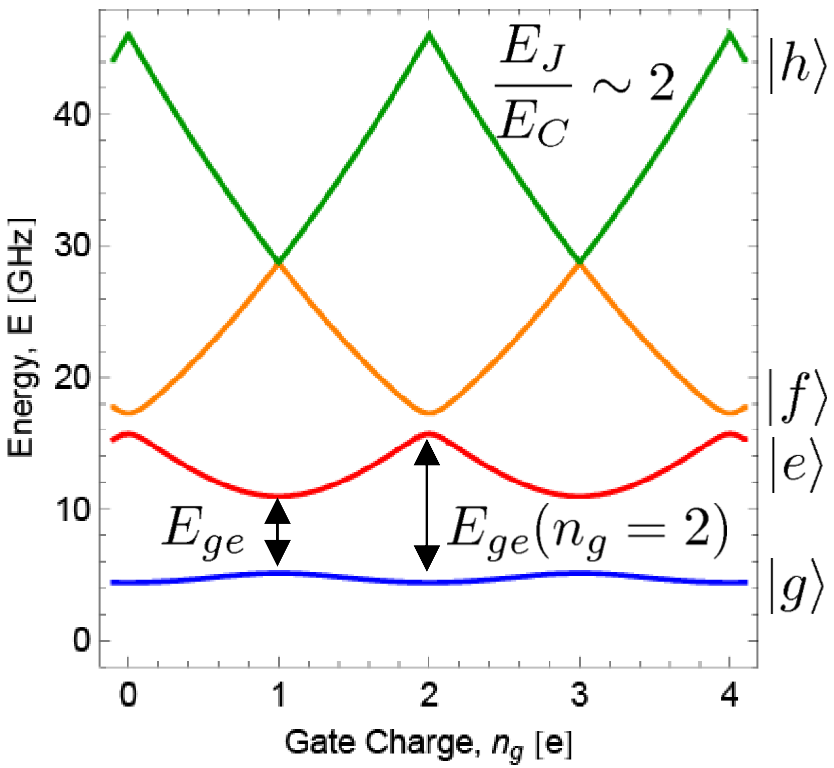




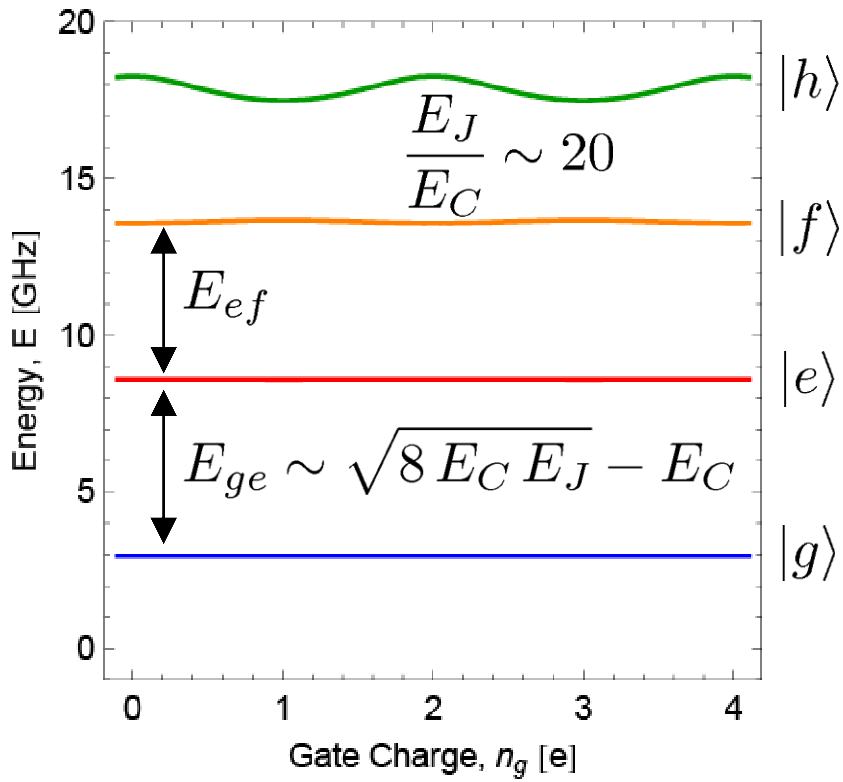
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity:

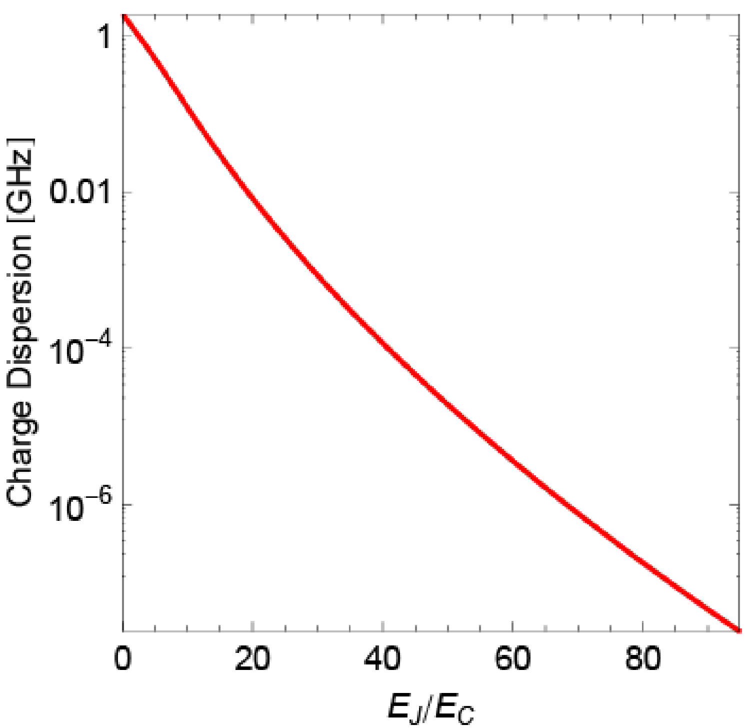
$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Dispersion and Anharmonicity

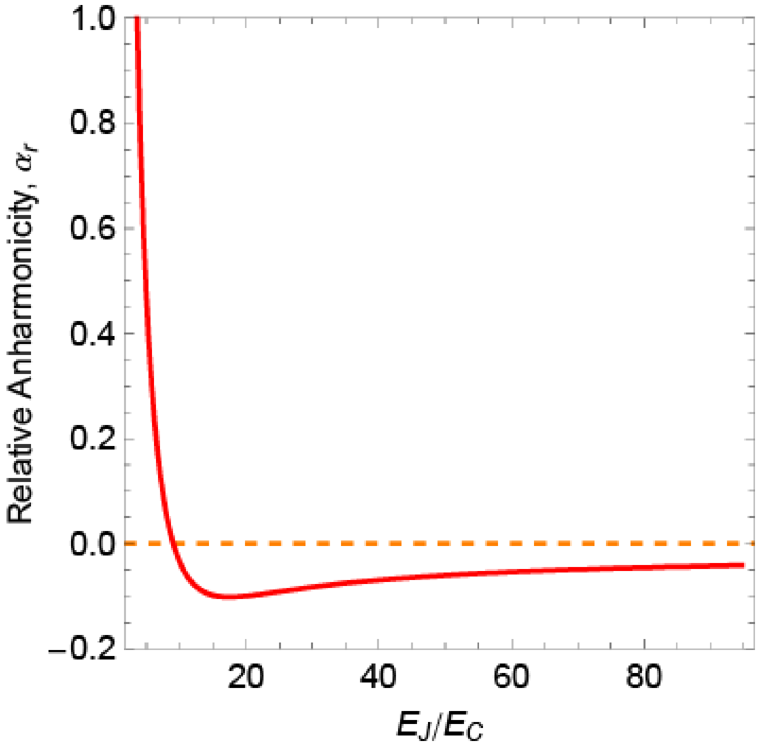
Charge dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$



Anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$



J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)