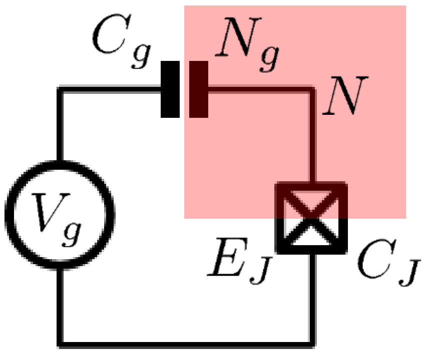
III. The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \frac{Q}{2e}$

continuous gate charge: $N_g = \frac{C_g V_g}{2e}$

total box capacitance $C_\Sigma = C_g + C_J$

Hamiltonian: $H = H_{\mathrm{el}} + H_{\mathrm{mag}}$

electrostatic part:

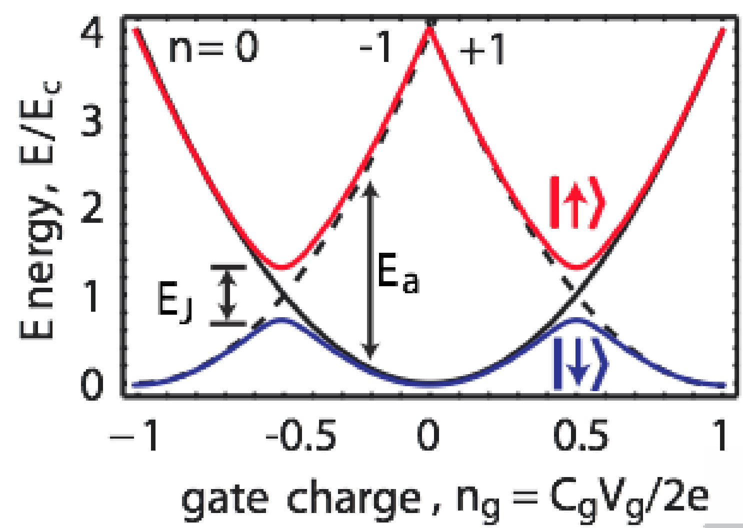
$$H_{\mathrm{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\mathrm{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator: $$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis: $$|\delta\rangle = \frac{1}{\sqrt{2\pi}}\sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** *N*:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 + E_J \cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

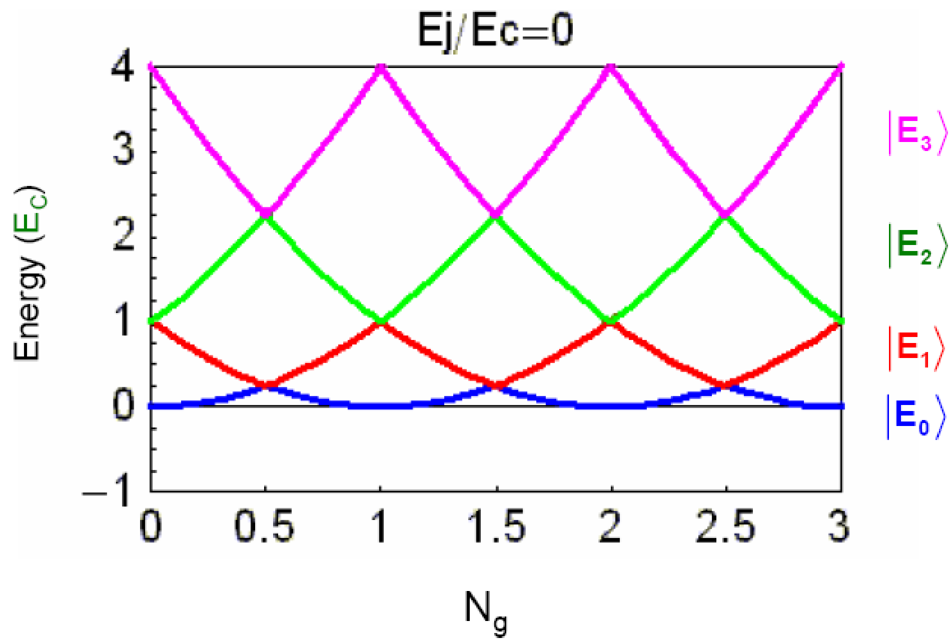
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

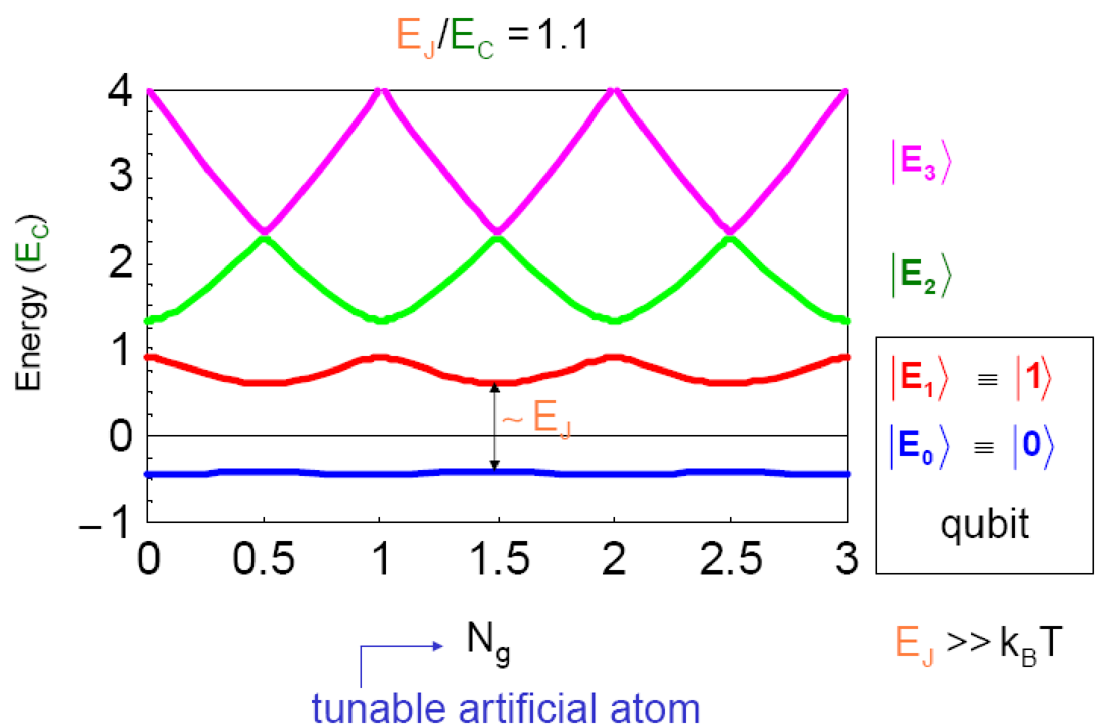
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g

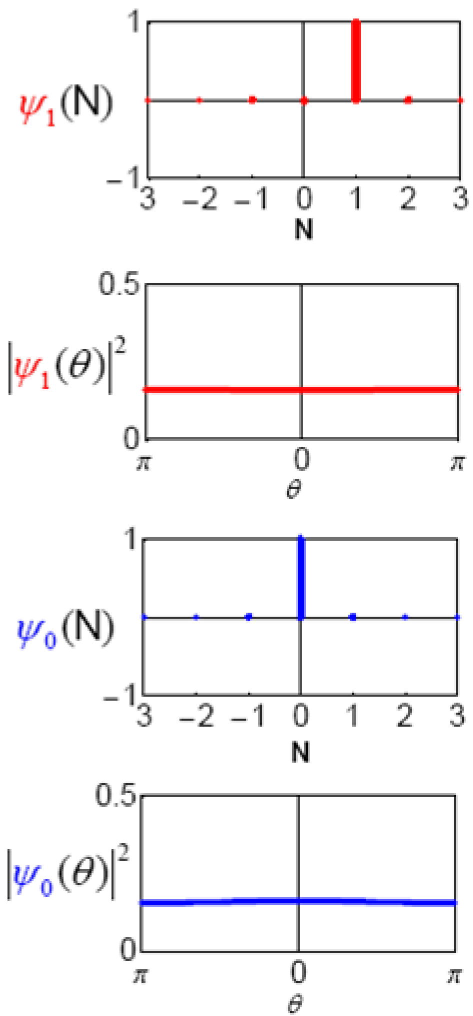


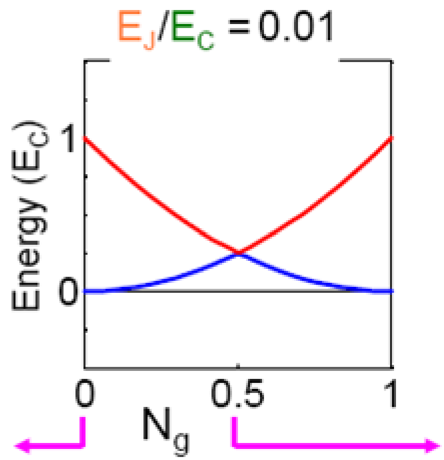
energy bands for finite E_J

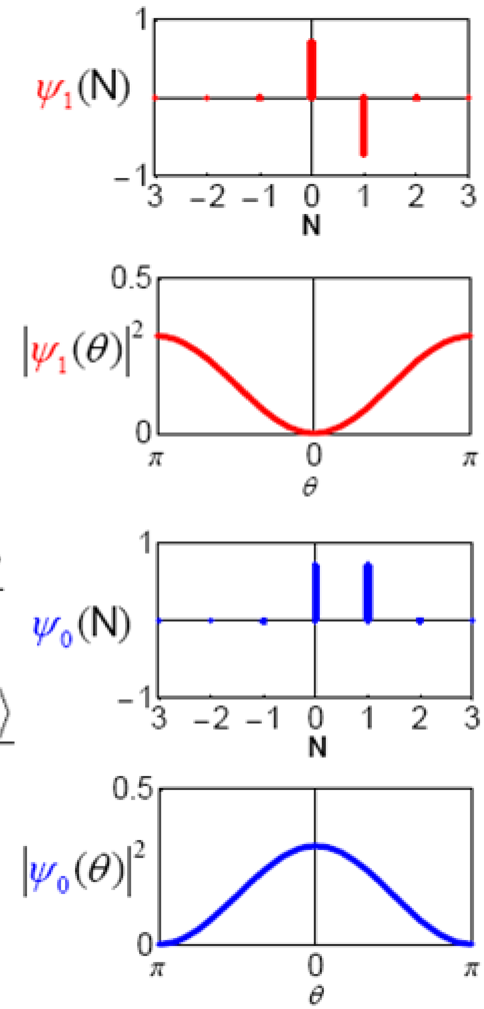
- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



Charge and Phase Wave Functions ($E_J << E_C$)







$|\psi_1\rangle \approx |N=1\rangle$

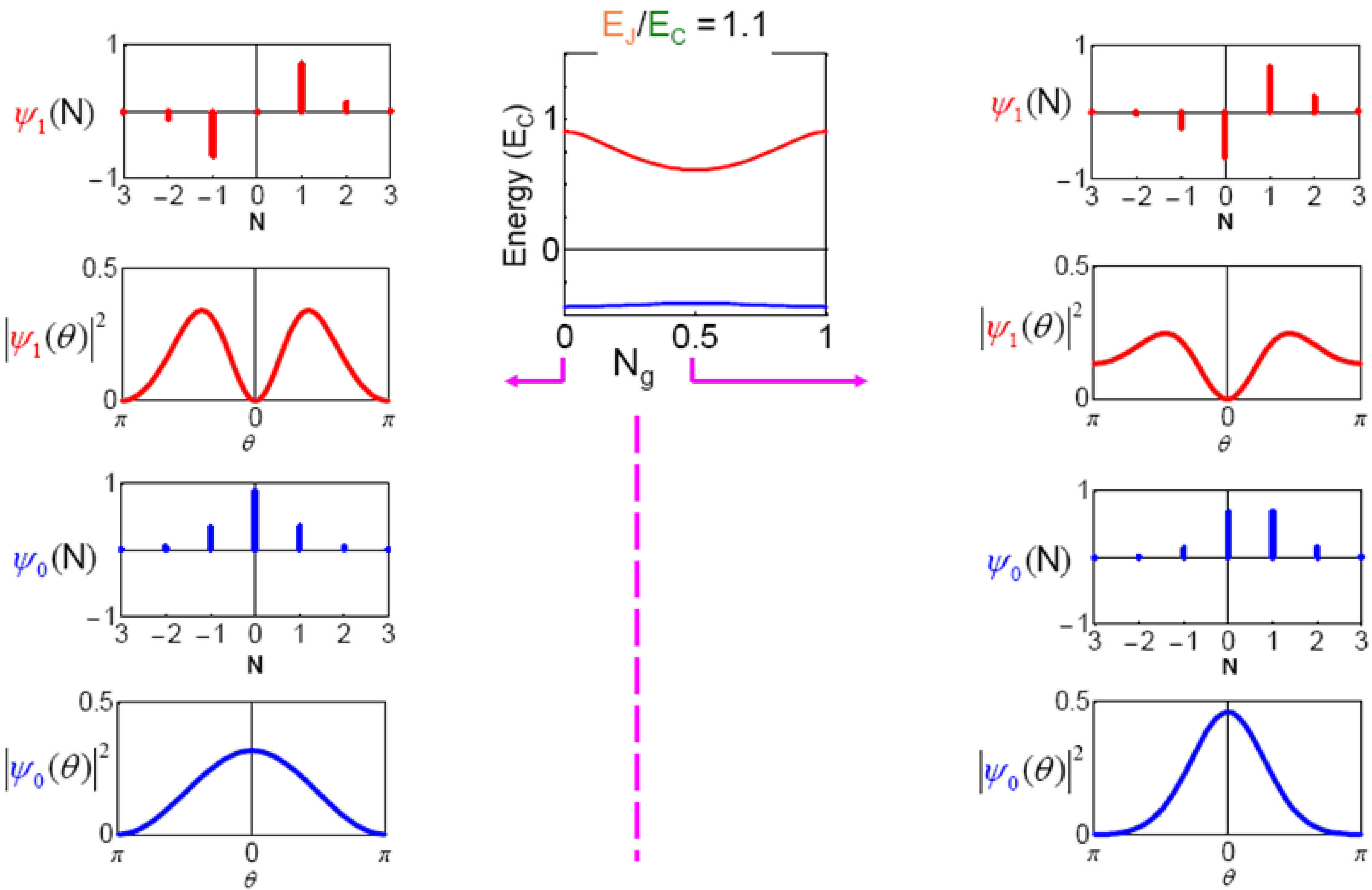
$|\psi_0\rangle \approx |N=0\rangle$

$$|\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$

courtesy CEA Saclay

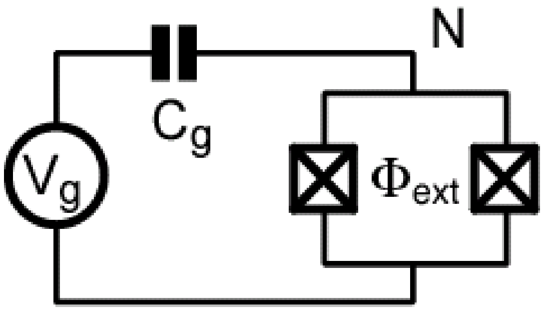
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

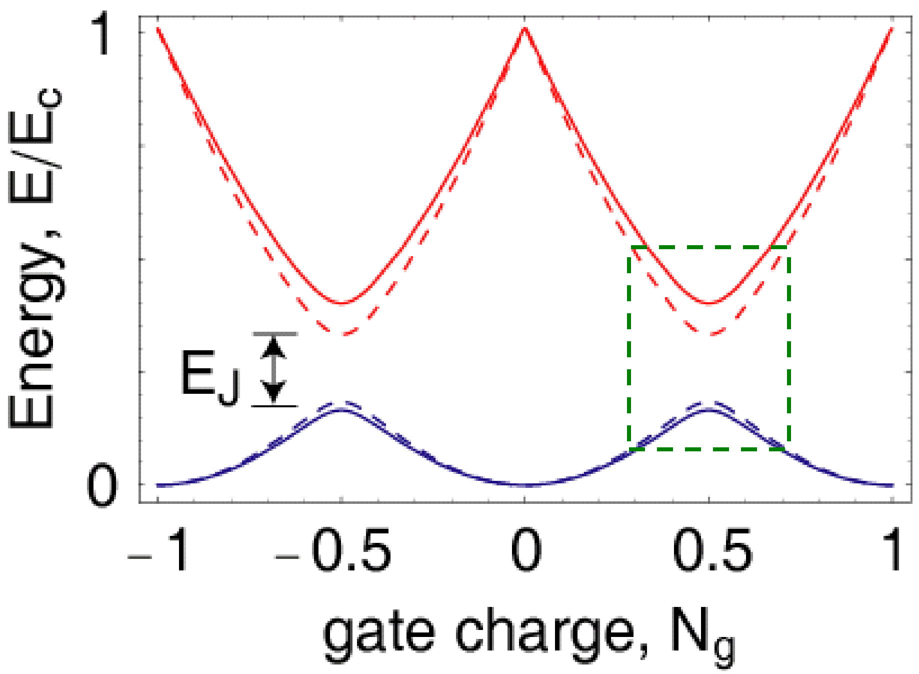
split Cooper pair box in perpendicular field



$$H = E_C\left(N - N_g\right)^2 - E_{J,\max}\cos\left(\pi\frac{\phi_{\rm ext}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\phi_{\rm ext}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

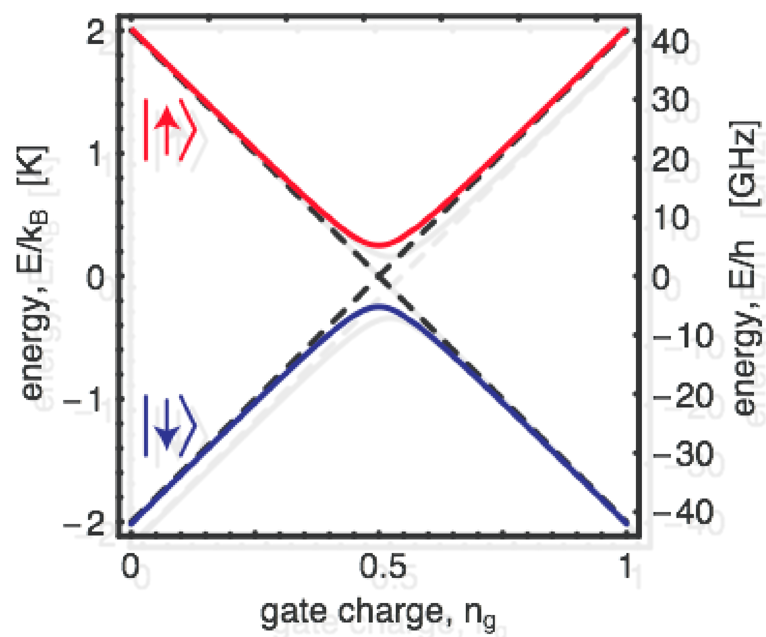
$$\hat{H}_{\mathrm{CPB}} = \sum_N \left[E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\mathrm{J}}}{2} \left(\left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\mathrm{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

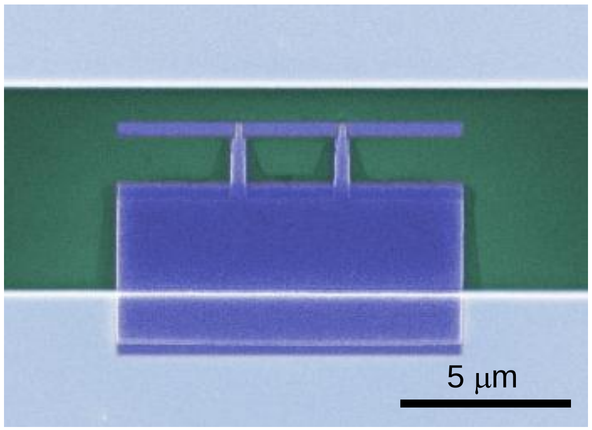


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

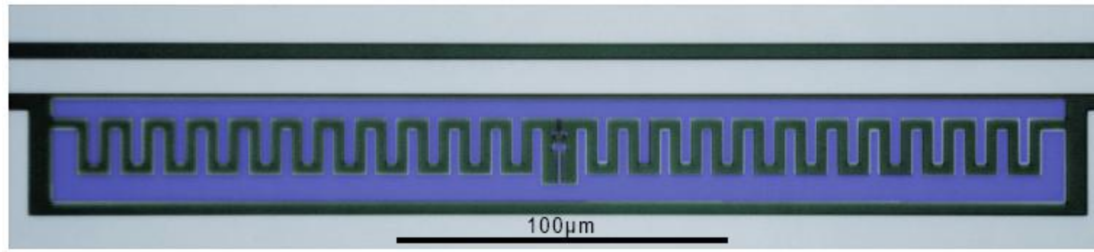
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

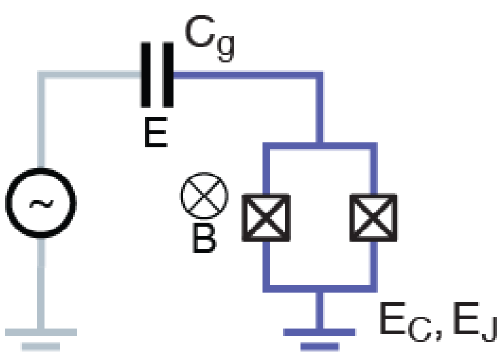
standard CPB:

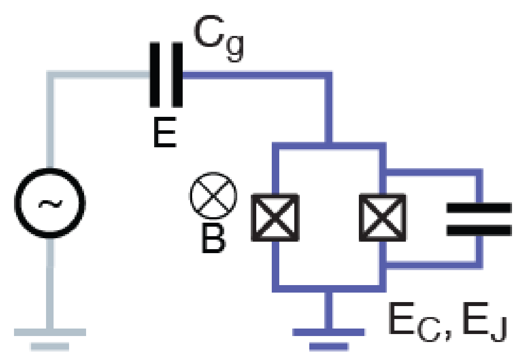


Transmon qubit:



circuit diagram:

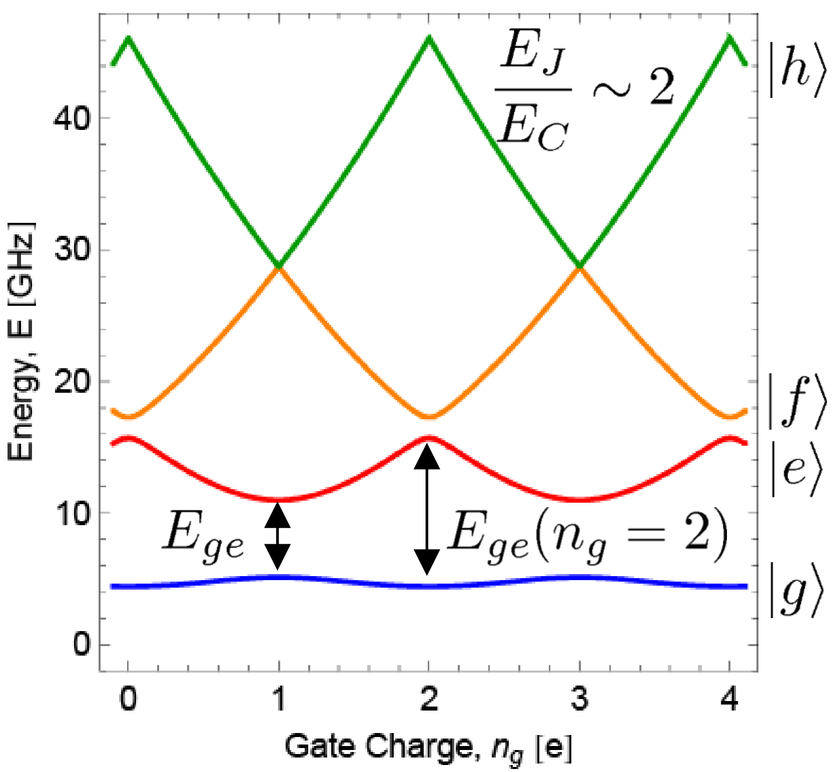




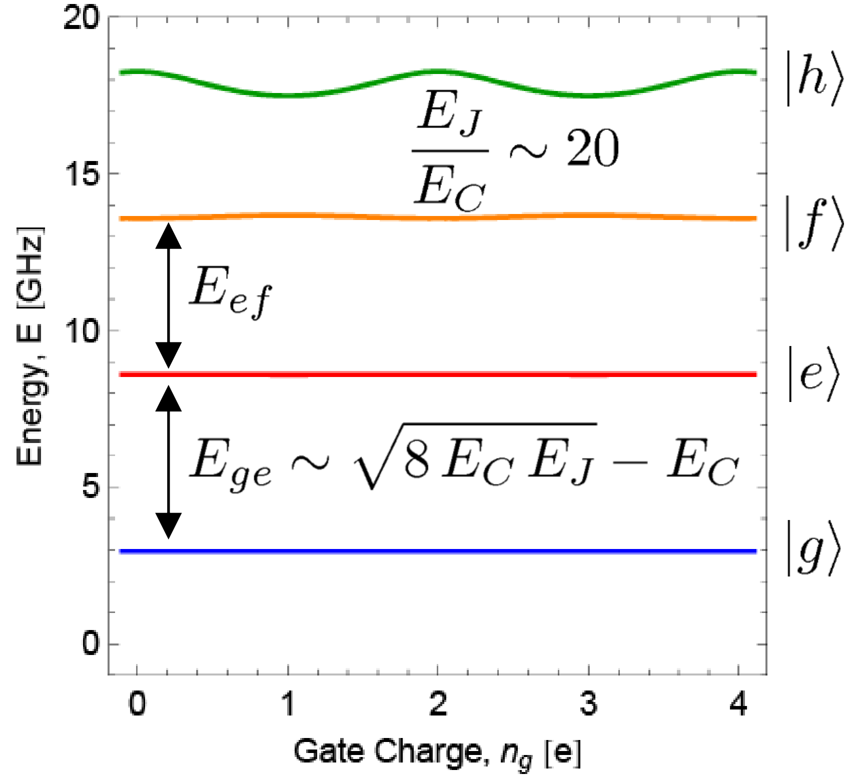
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity:

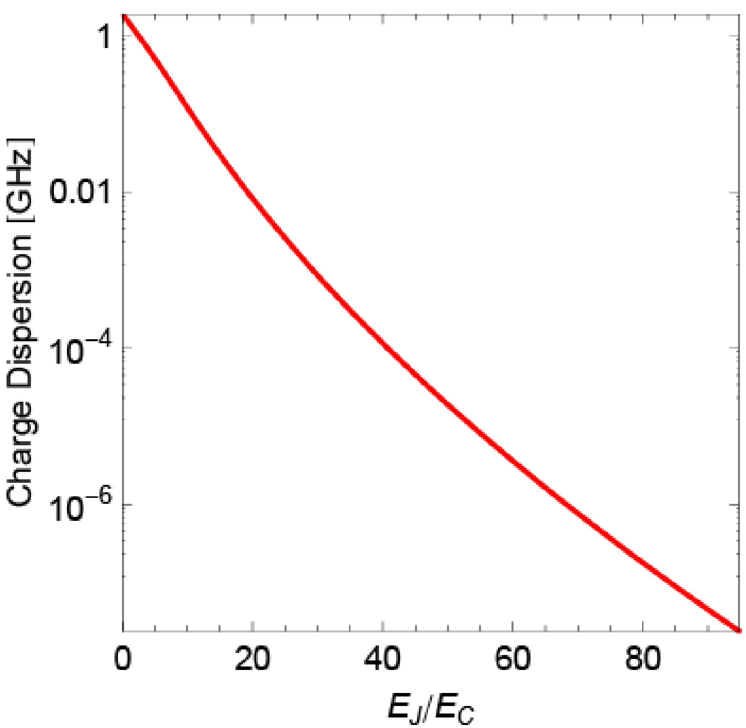
$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Dispersion and Anharmonicity

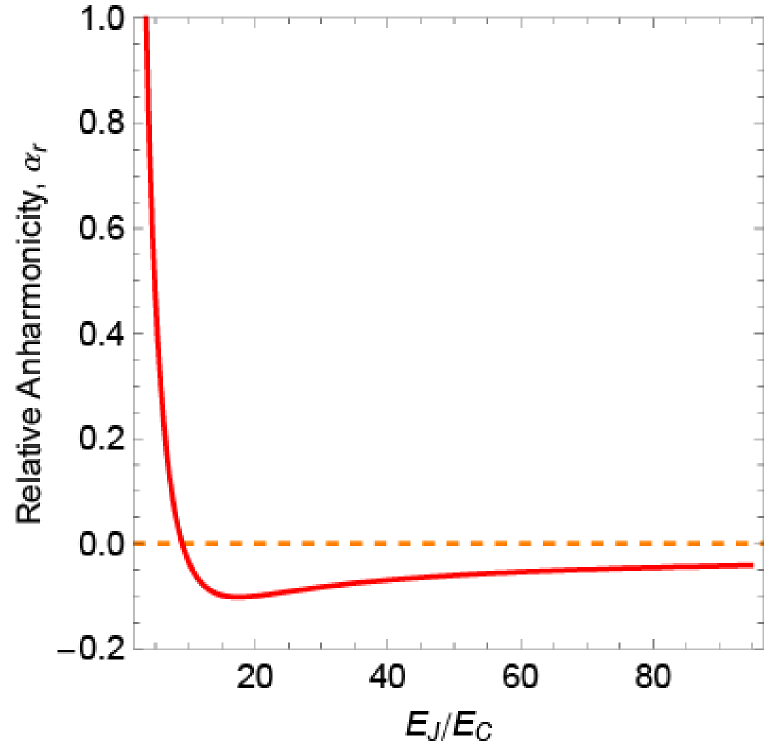
Charge dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$



Anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

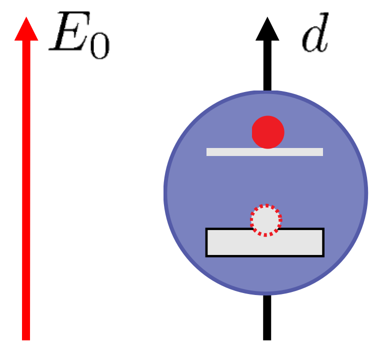


J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

IV. Circuit Quantum Electrodynamics (QED): Cavity QED with Superconducting Circuits

Controlling the Interaction of Light and Matter

challenging on the level of single (artificial) atoms and single photons



- dipole moment d (usually small ~ ea_o)
- single photon fields E_o (small in 3D)
- photon/atom interaction $\hbar g \sim dE_0$ (usually small)

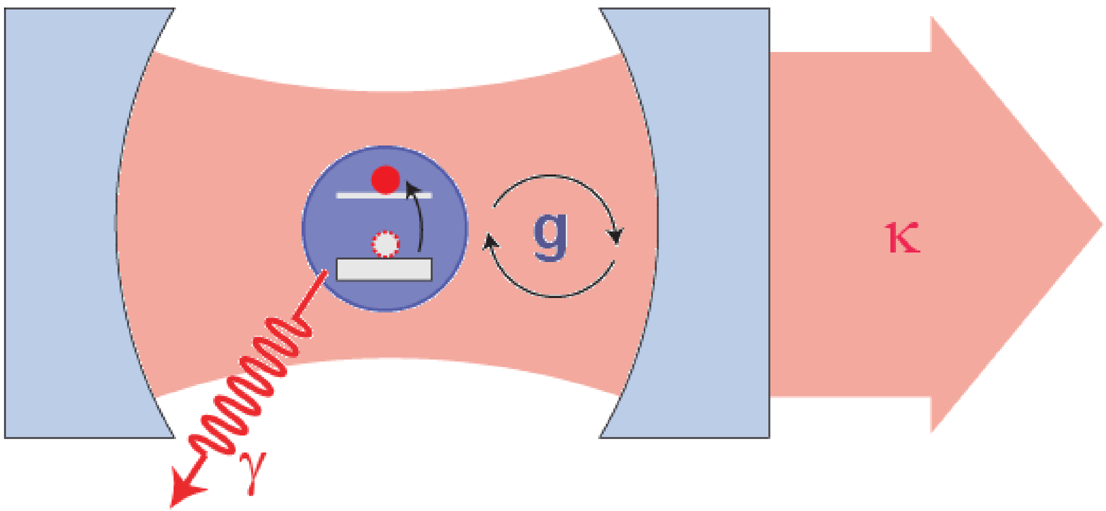
What to do?

- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa\ ,1/t_{\text{transit}}$

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

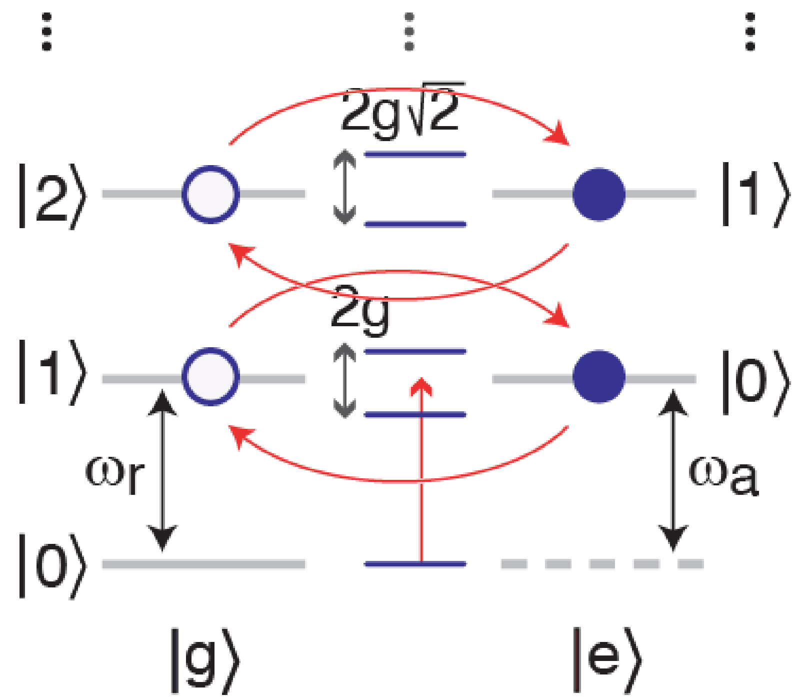
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



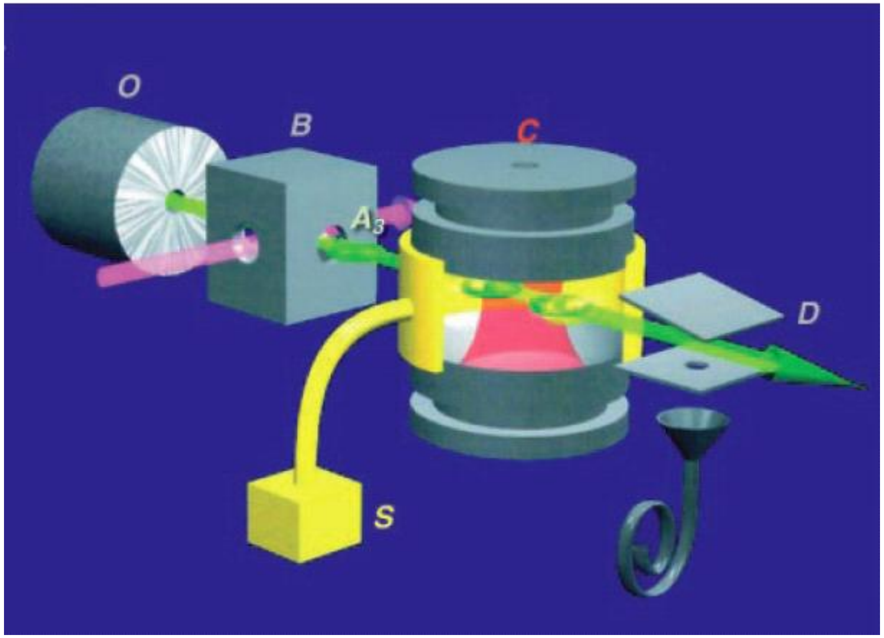
Jaynes-Cummings Ladder

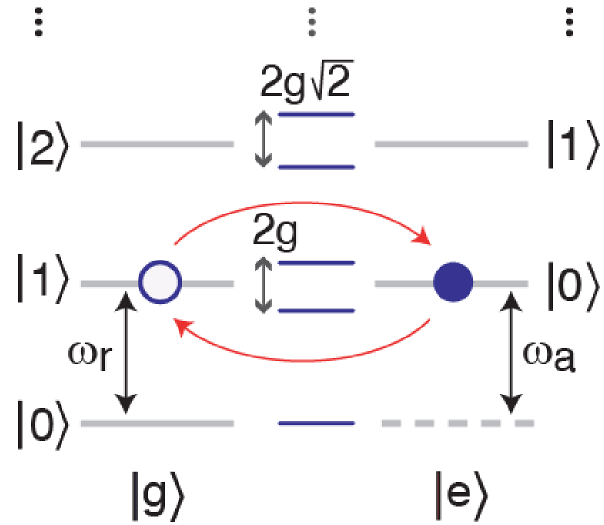
atomic cavity QED reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734-1738 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

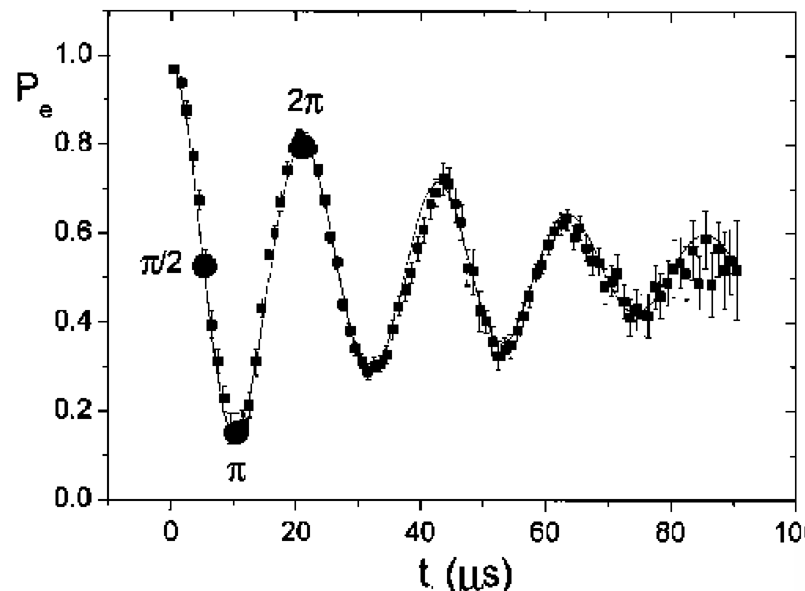
Vacuum Rabi Oscillations with Rydberg Atoms





with Rydberg atoms in microwave domain:

- large d

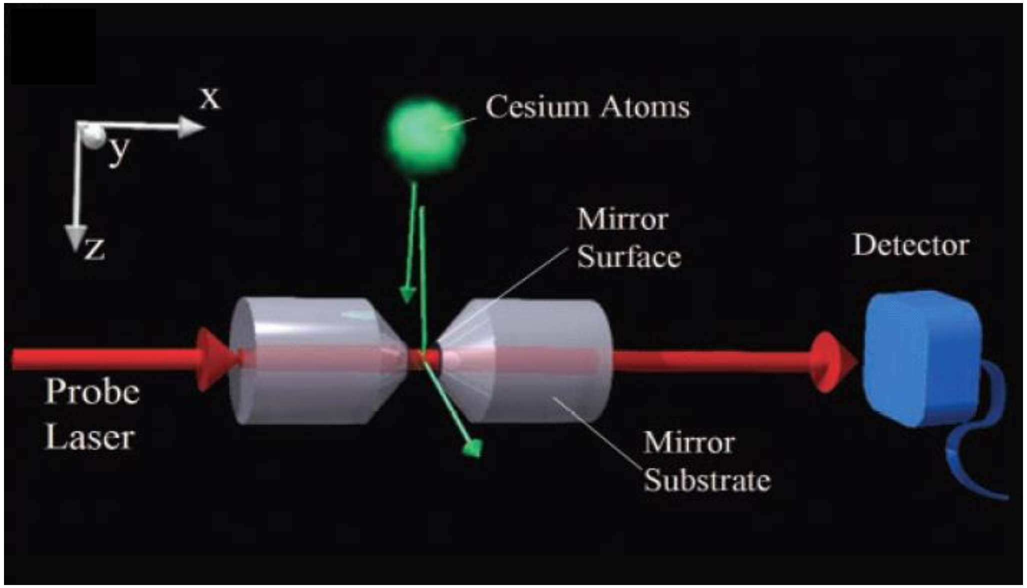


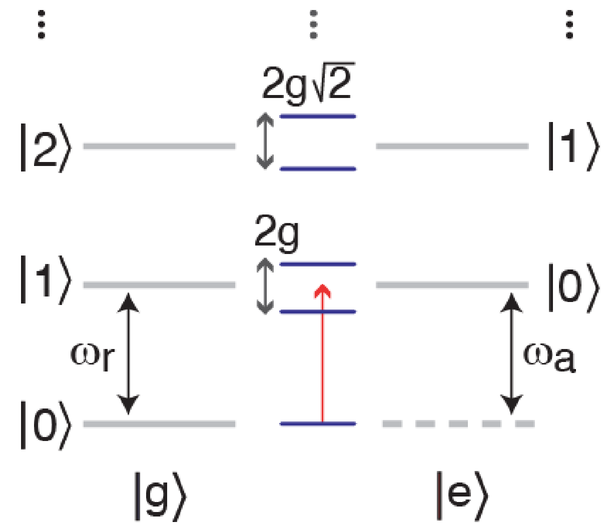
reviews:

S. Haroche & J. Raimond, *OUP Oxford* (2006)

J. M. Raimond, M. Brune, and S. Haroche *Rev. Mod. Phys.* 73, 565 (2001)

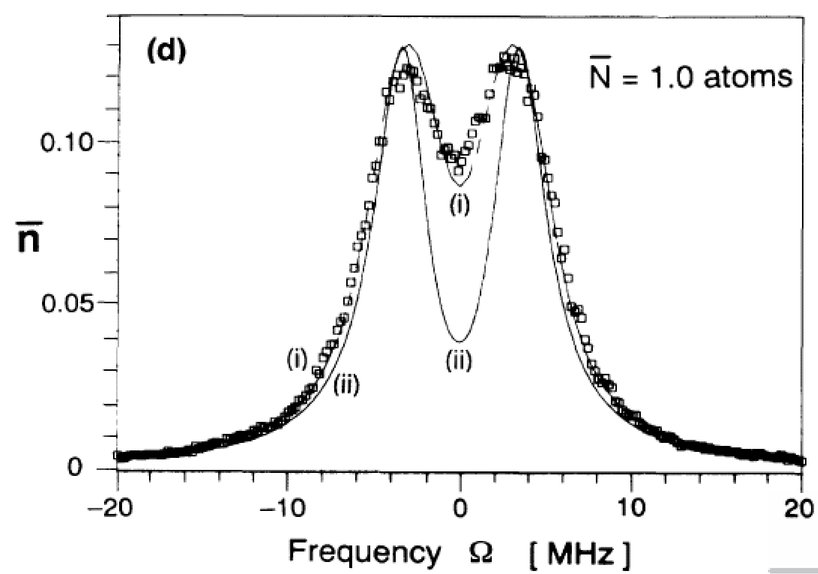
Vacuum Rabi Mode Splitting with Alkali Atoms





with alkali atoms in optical domain:

- large E_0

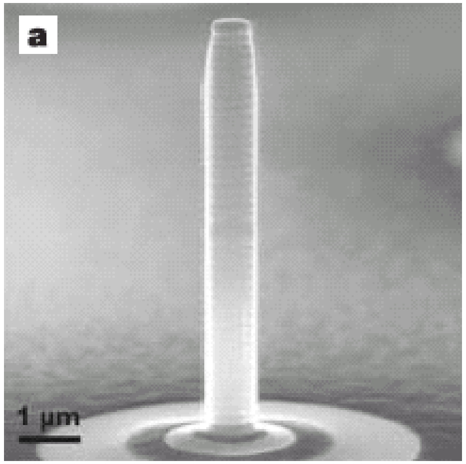


reviews:

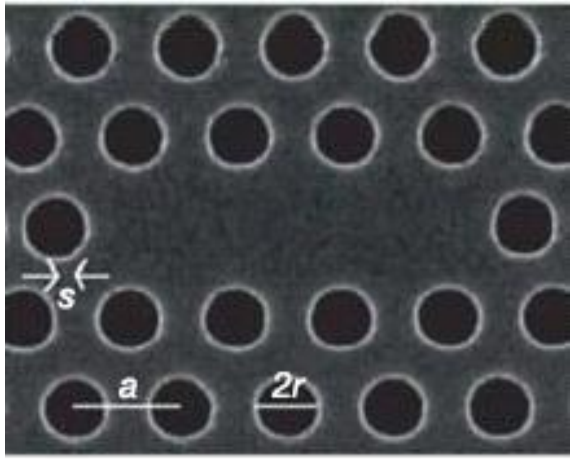
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

H. Mabuchi, A. C. Doherty, *Science* **298**, 1372 (2002)

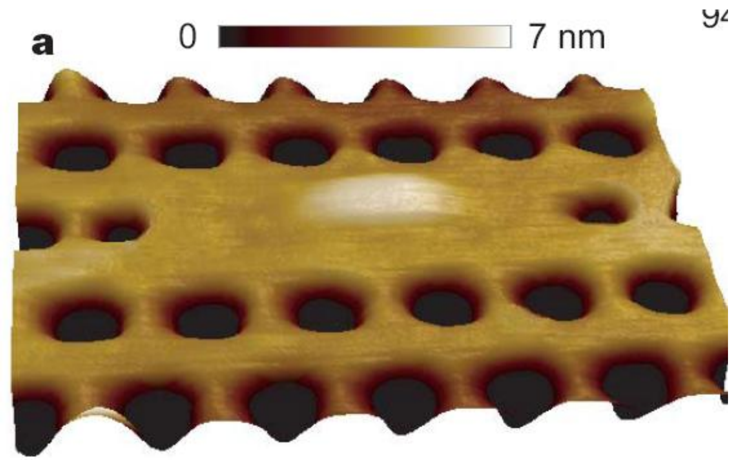
… and also with Semiconductors



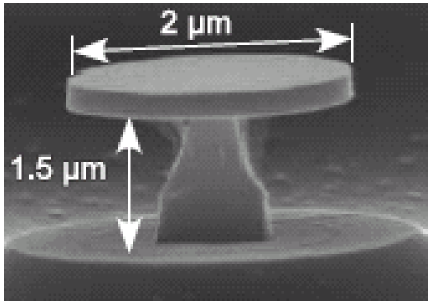
Wurzburg
*Nature **432**, 197 (2004)*



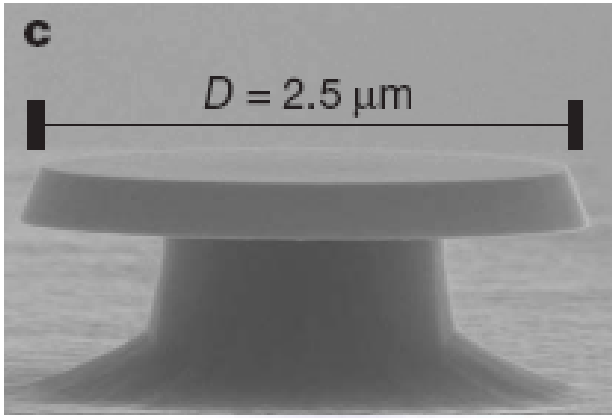
Arizona
*Nature **432**, 200 (2004)*



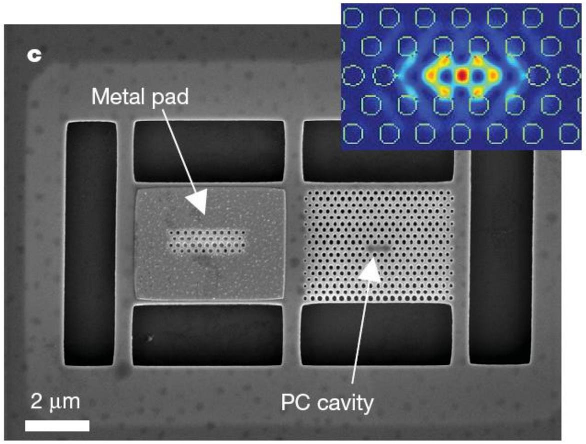
ETH Zurich
*Nature **445**, 896 (2007)*



Paris
PRL (2004)



Caltech
*Nature **450**, 862 (2007)*



Stanford
*Nature **450**, 857 (2007)*