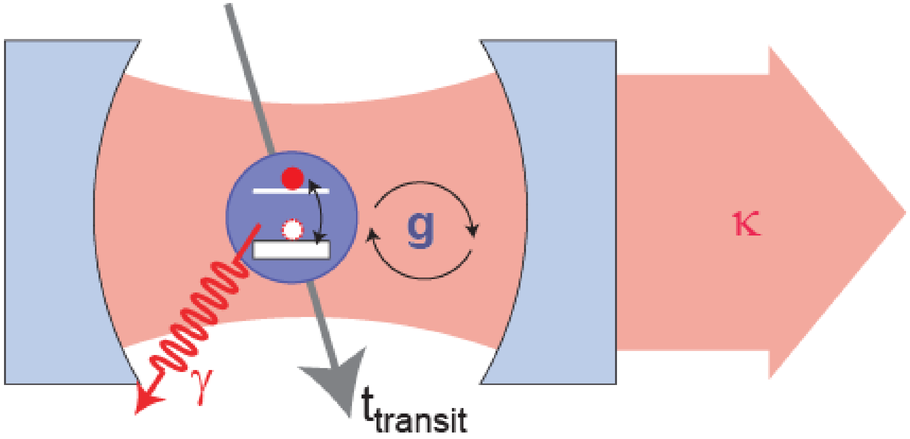
Proposals for Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

many potential approaches:

discrete LC circuits:

- Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
- O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).

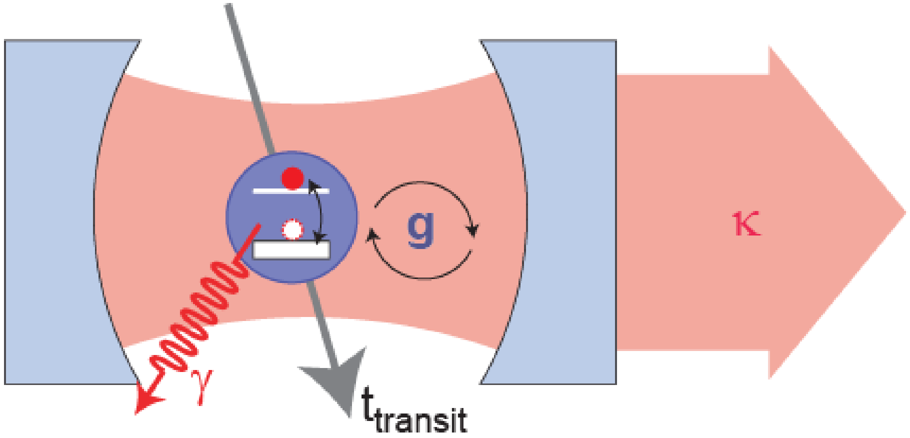
large junctions:

- F. Marquardt and C. Bruder, Phys. Rev. B **63**, 054514 (2001).
- F. Plastina and G. Falci, Phys. Rev. B **67**, 224514 (2003).
- A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. **90**, 127901 (2003).

3D cavities:

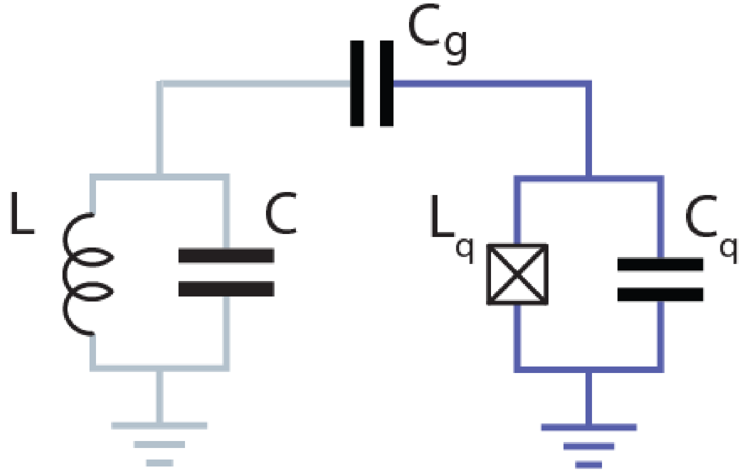
- W. Al-Saidi and D. Stroud, Phys. Rev. B **65**, 014512 (2001).
- C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).
- J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).

Cavity QED with Superconducting Circuits

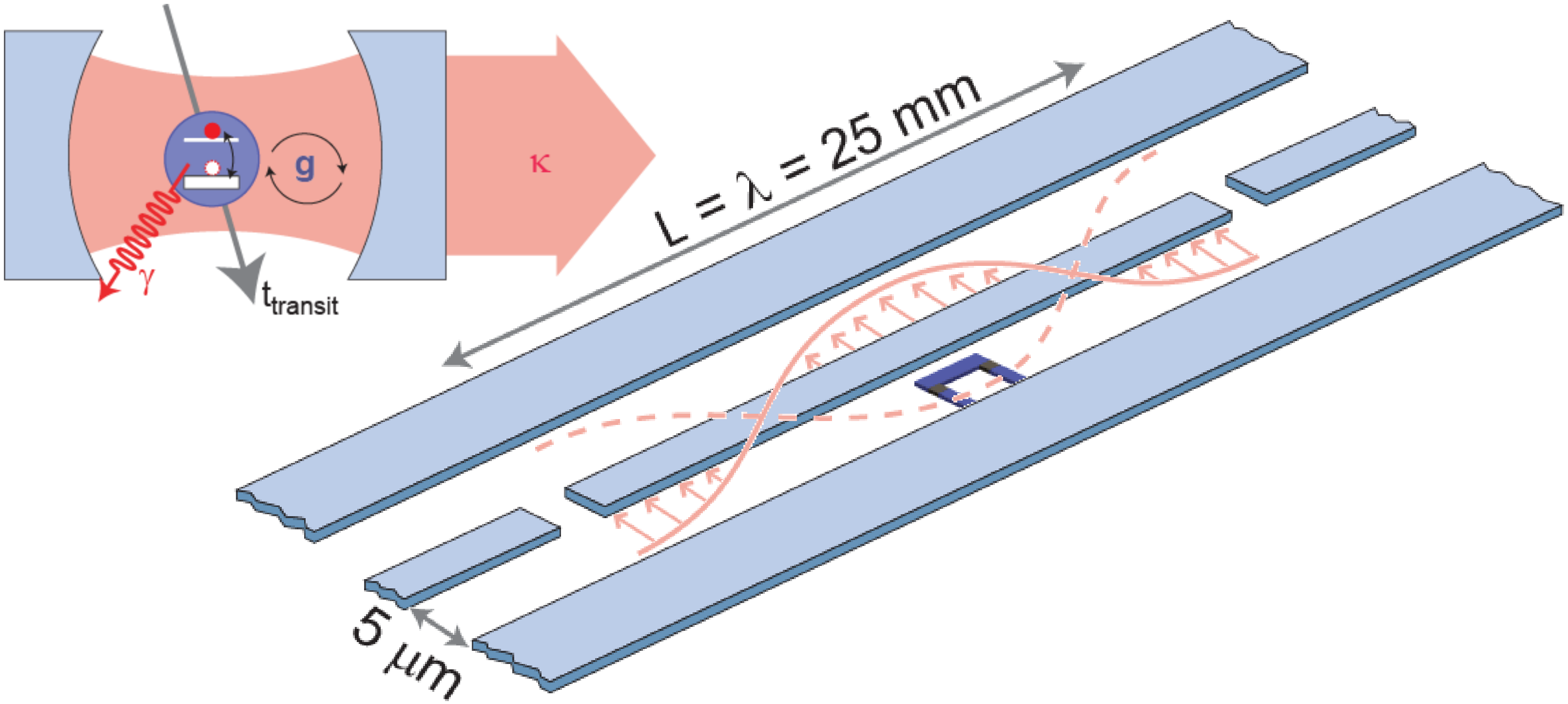


coherent quantum mechanics
with individual photons and qubits ...

... basic approach:



Circuit Quantum Electrodynamics



elements:

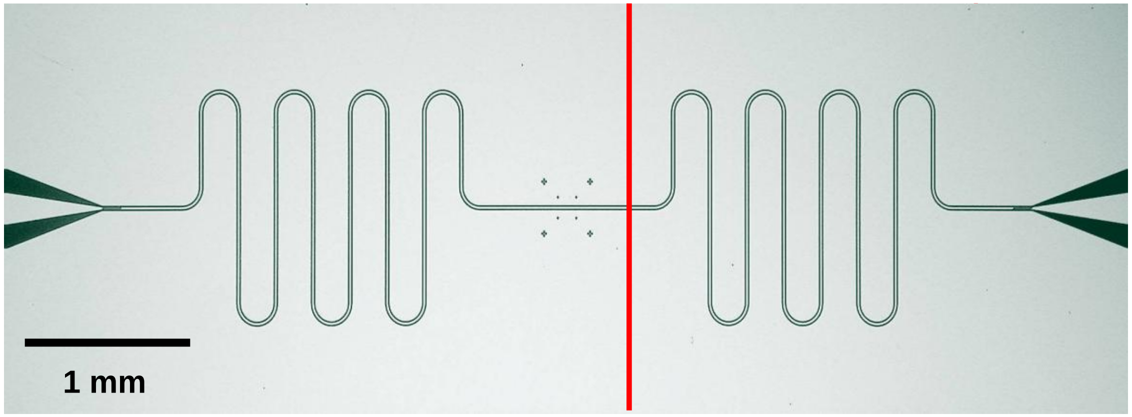
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a superconducting qubit with large dipole moment d and long coherence time $1/\gamma$ and fixed position

A. Blais, *et al.*, *PRA* **69**, 062320 (2004)

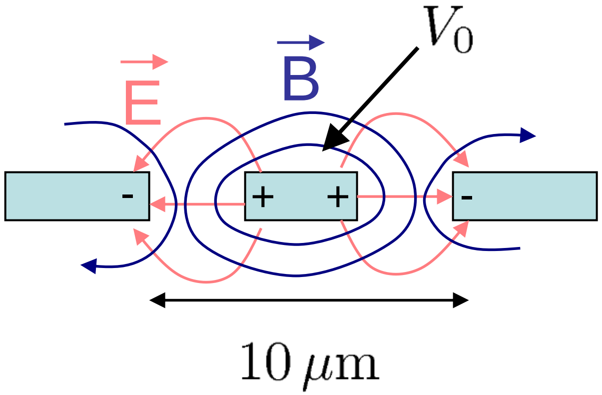
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Large Vacuum Field in 1D Cavity



optical microscope image of strip line resonator



electric field across resonator in vacuum state (*n*=*0*):

$$\int \epsilon_0 E_{0,\mathrm{rms}}^2 \, dV_{\mathrm{mod}} = \frac{\hbar\omega_r}{2}$$

harmonic oscillator

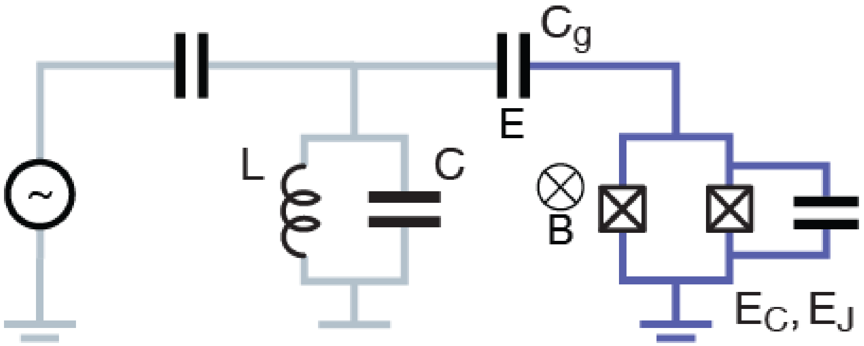
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

$$E_{0,\mathrm{rms}} \approx 0.2\,\mathrm{V/m}$$

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$

$\times 10^6$ larger than E_0
in 3D microwave cavity

Qubit/Photon Coupling



Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} \quad = \quad \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy N$_g$ = 1/2 and change of qubit basis (*z* to *x*, *x* to -*z*)

$$\hat{H} \quad = \quad \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

$$\hat{H}_g \quad = \quad \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \cancel{\hat{a}\hat{\sigma}}^- + \cancel{\hat{a}^\dagger\hat{\sigma}}^+ + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

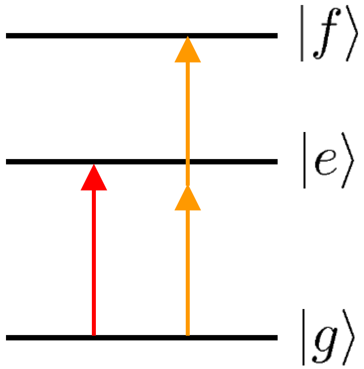
Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma}2e\sqrt{\frac{\hbar\omega_r}{2C}}$

Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1\ldots300\,\mathrm{MHz}$

$$g \gg [\kappa, \gamma]\ \text{possible!}$$

Spectroscopy of Transmon Qubit

one and two-photon spectroscopy:



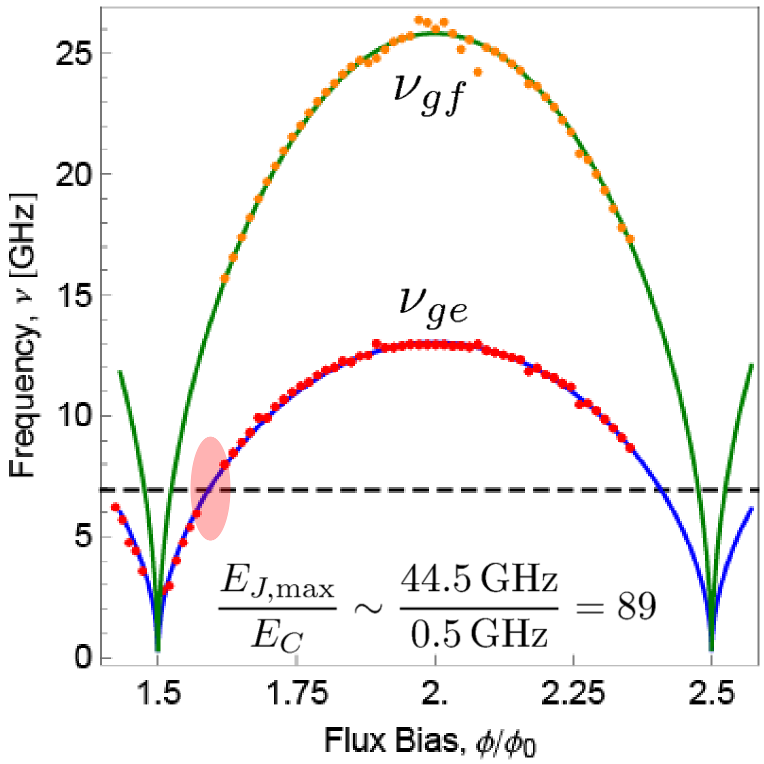
$|g\rangle \rightarrow |e\rangle$ transition:

$$\nu_{ge} = (E_e - E_g)/h$$

$|g\rangle \rightarrow |f\rangle$ transition:

$$2\,\nu_{gf} = (E_f - E_g)/h$$

flux dependence of energy levels:



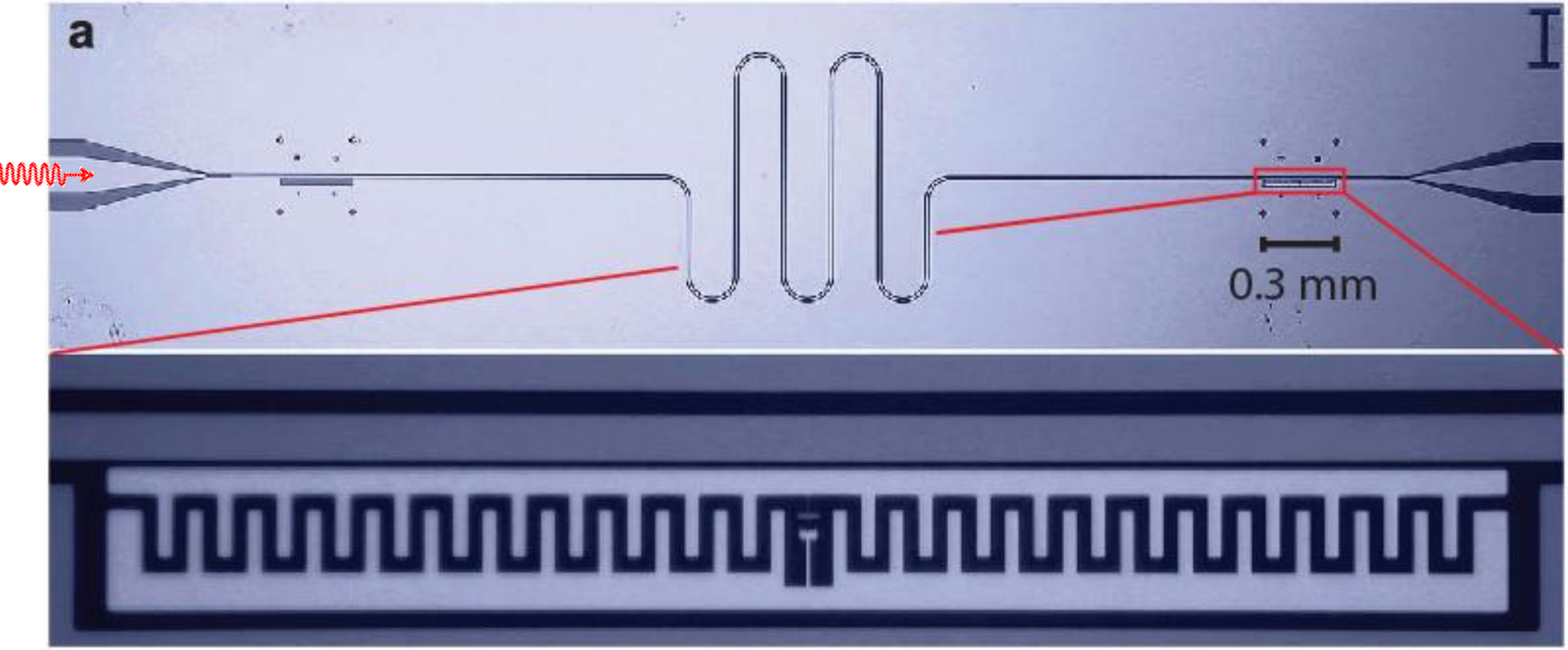
$$\frac{E_{J,\max}}{E_C} \sim \frac{44.5\,\mathrm{GHz}}{0.5\,\mathrm{GHz}} = 89$$

tune qubit into resonance

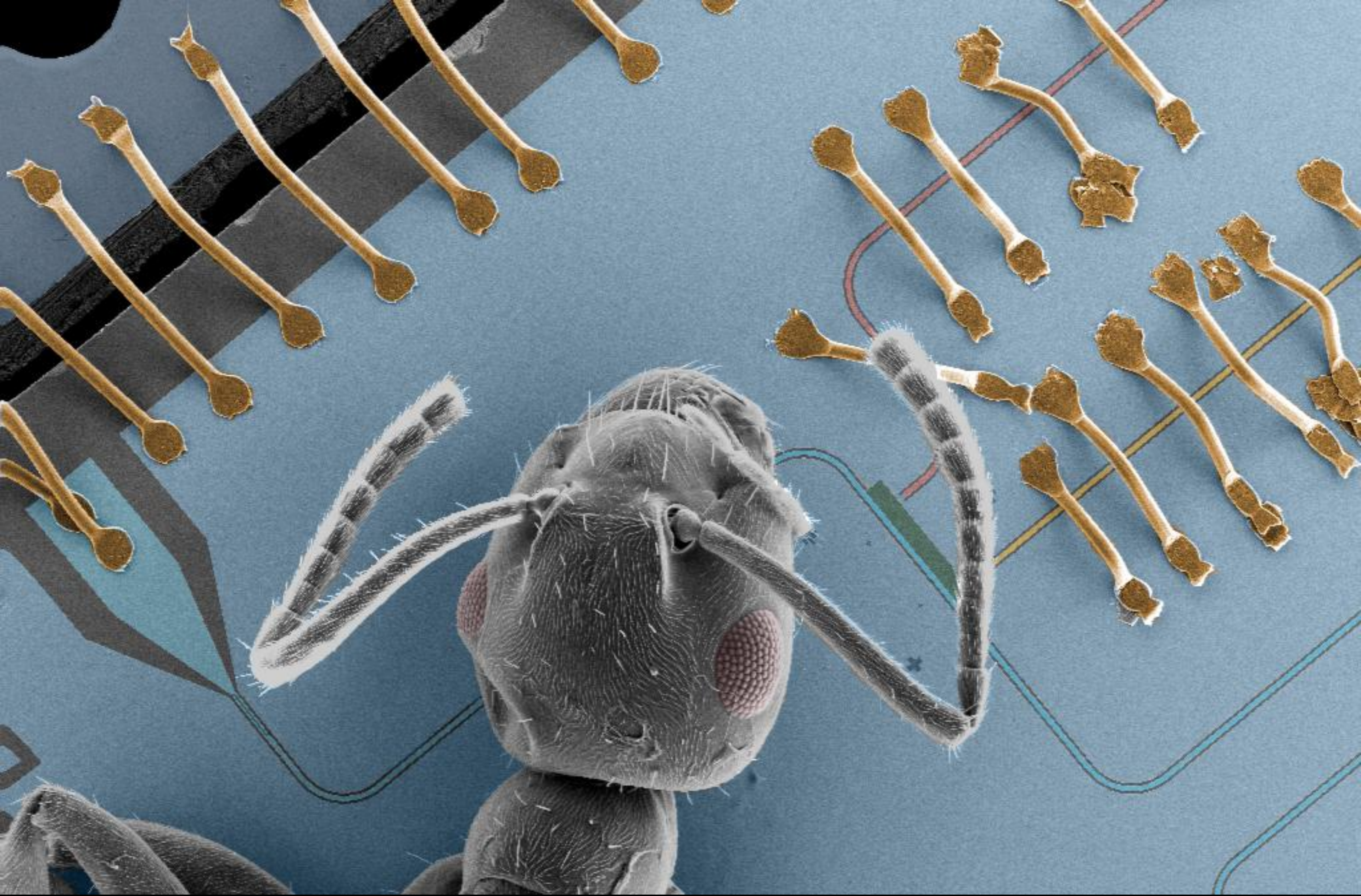
M. Baur, J. Fink (Quantum Device Lab, ETHZ, 2007)

more transmon experiments: J. Schreier *et al.* *PRB* **77**, 180502 (2008)

Realization

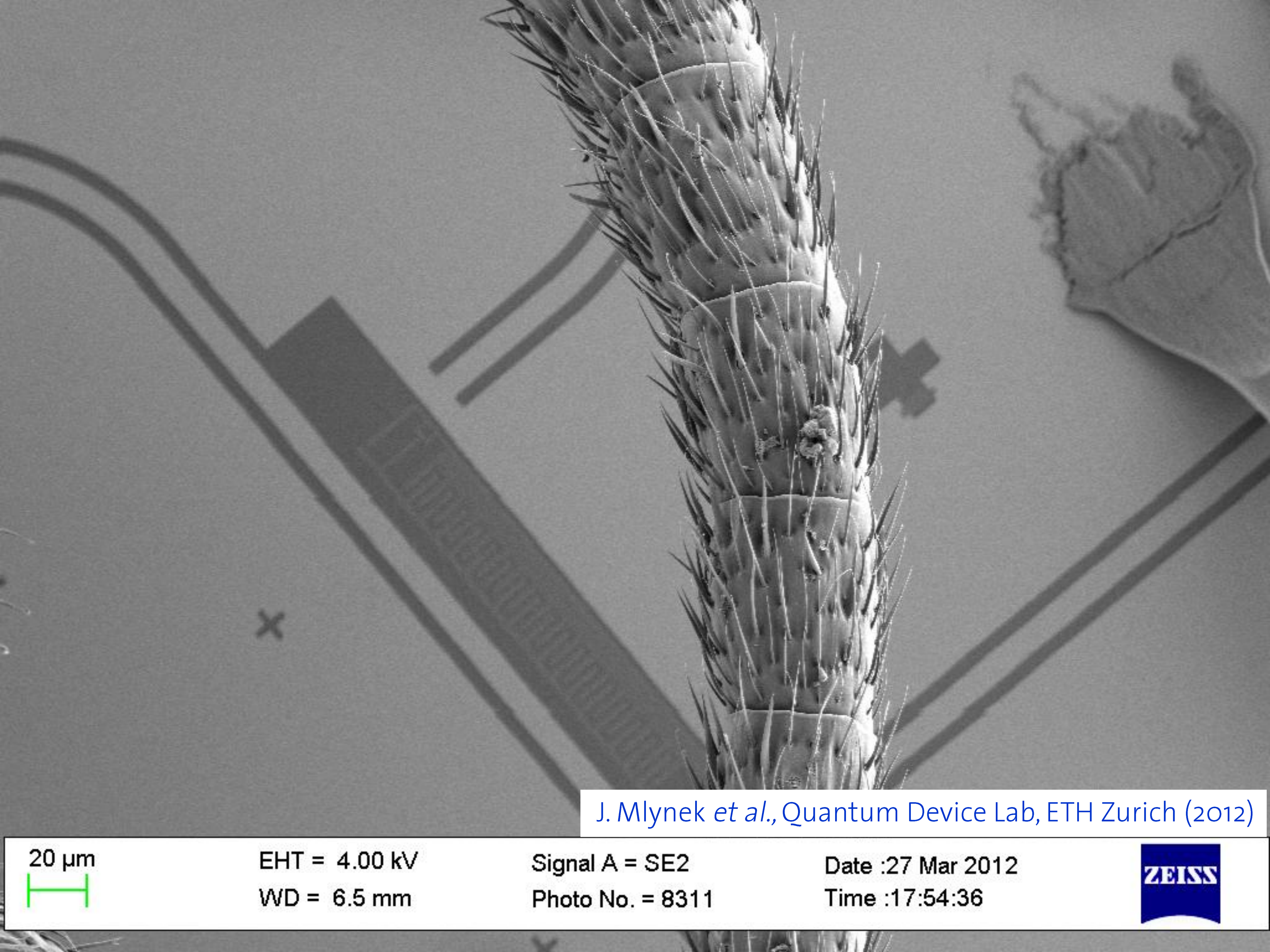


device fabrication: L. Frunzio et al., IEEE Trans. on Appl. Supercon. **15**, 860 (2005)

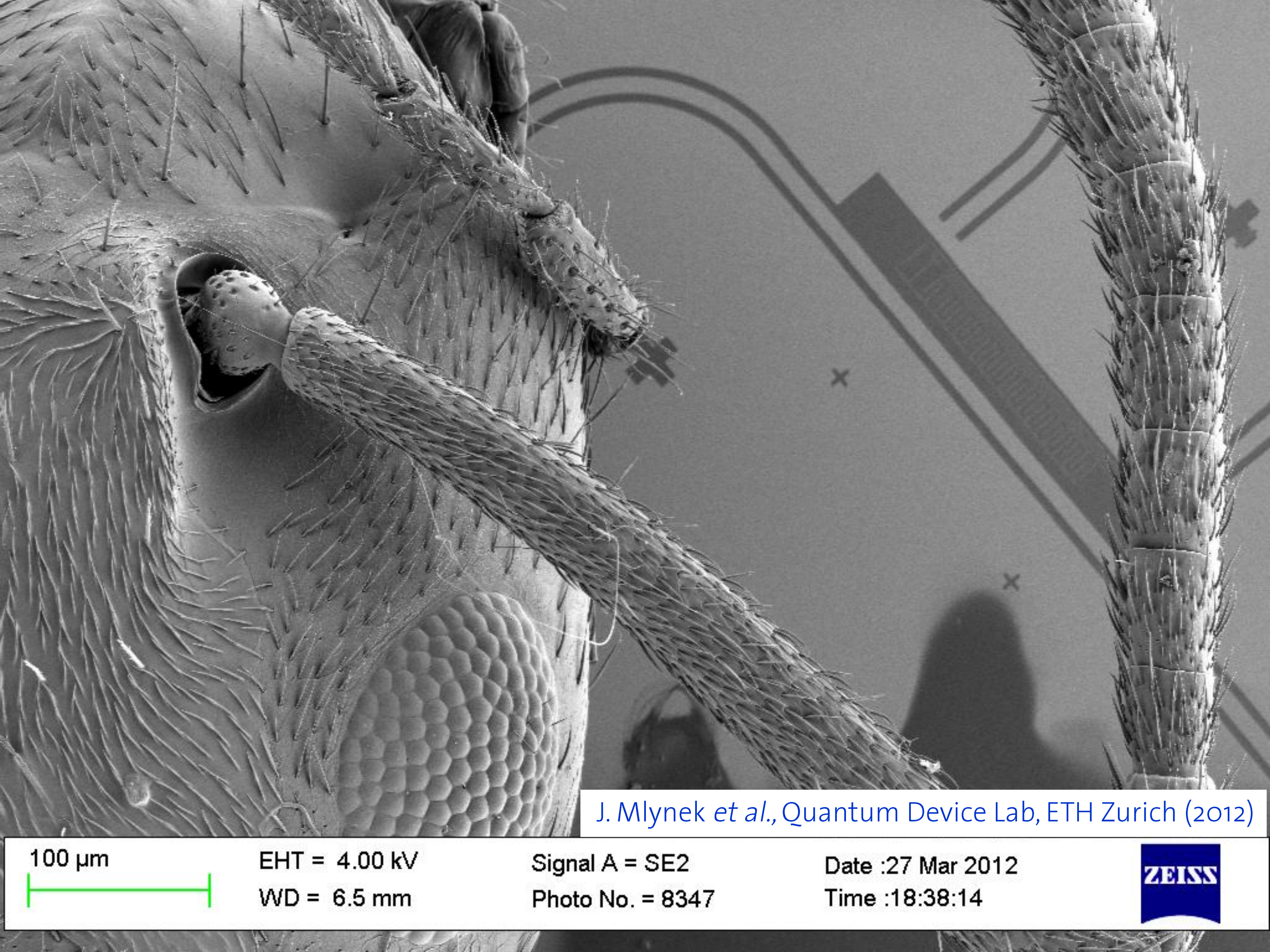


Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)



J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

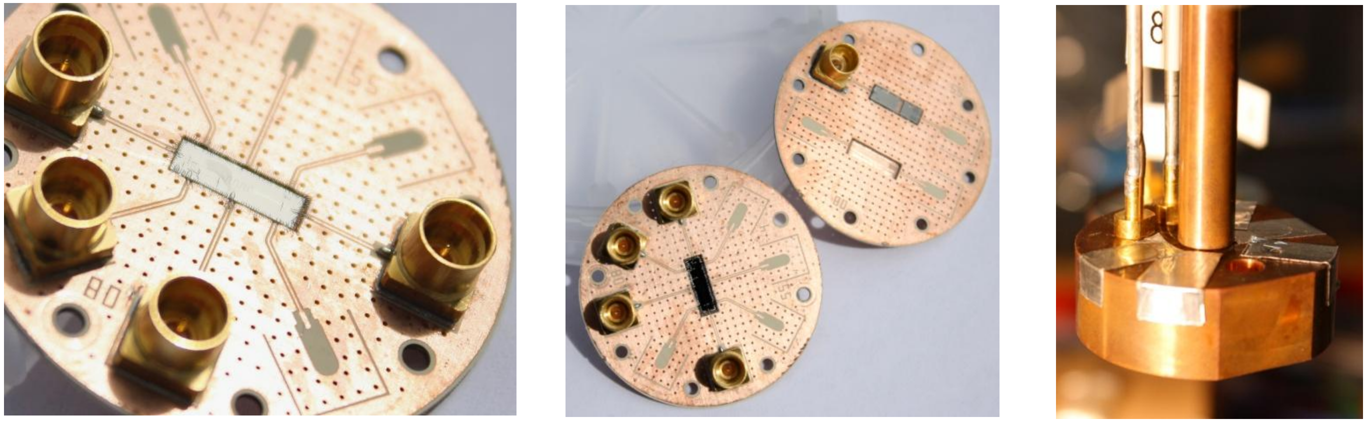


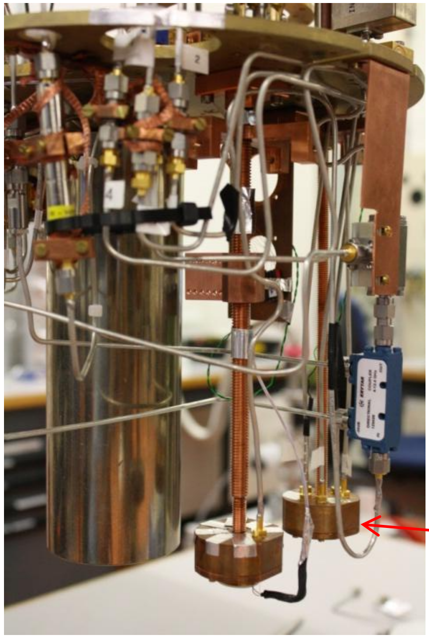
J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

Measurement Setup

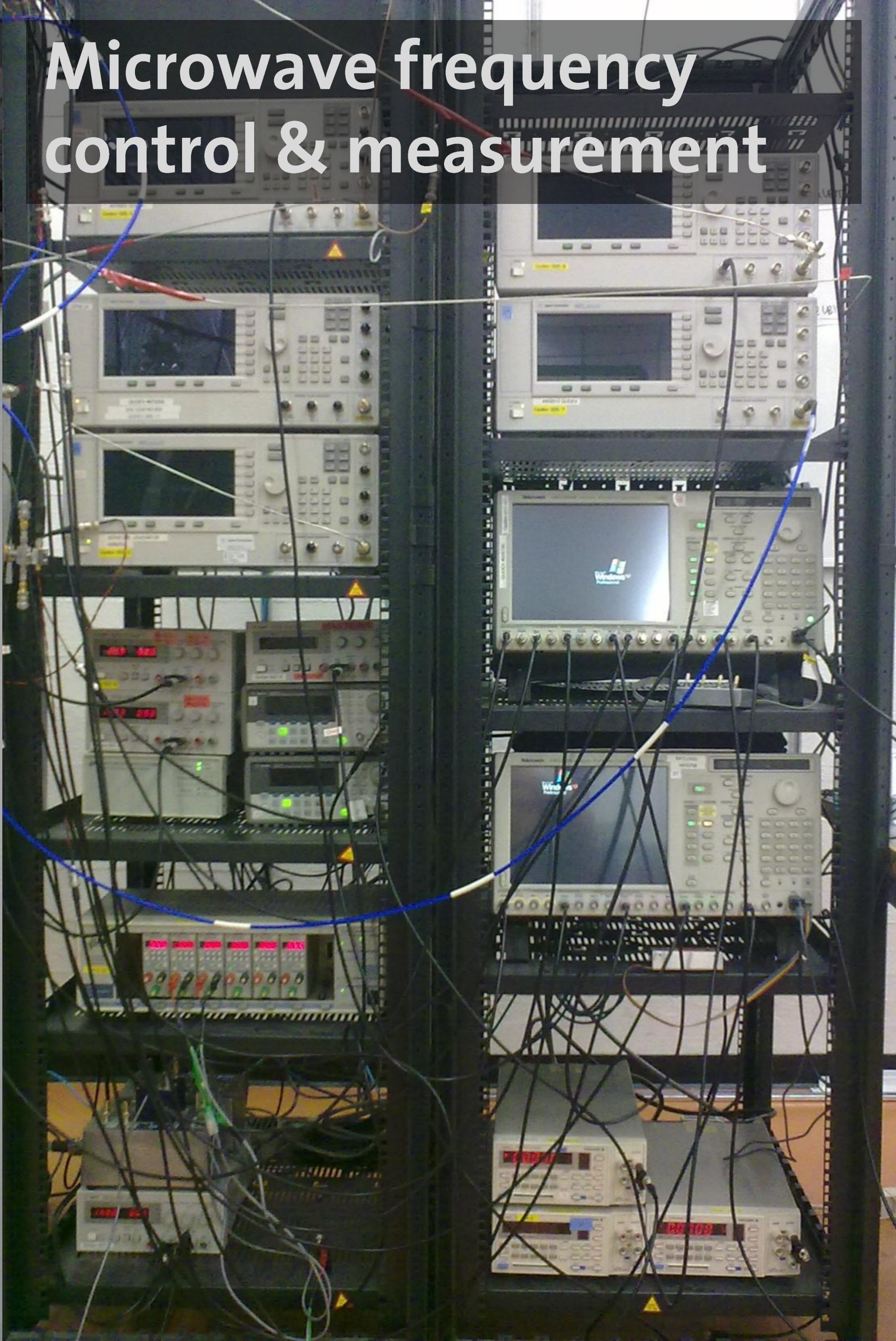




transmon & resonator

Cryostate for temperatures down to 0.02 K

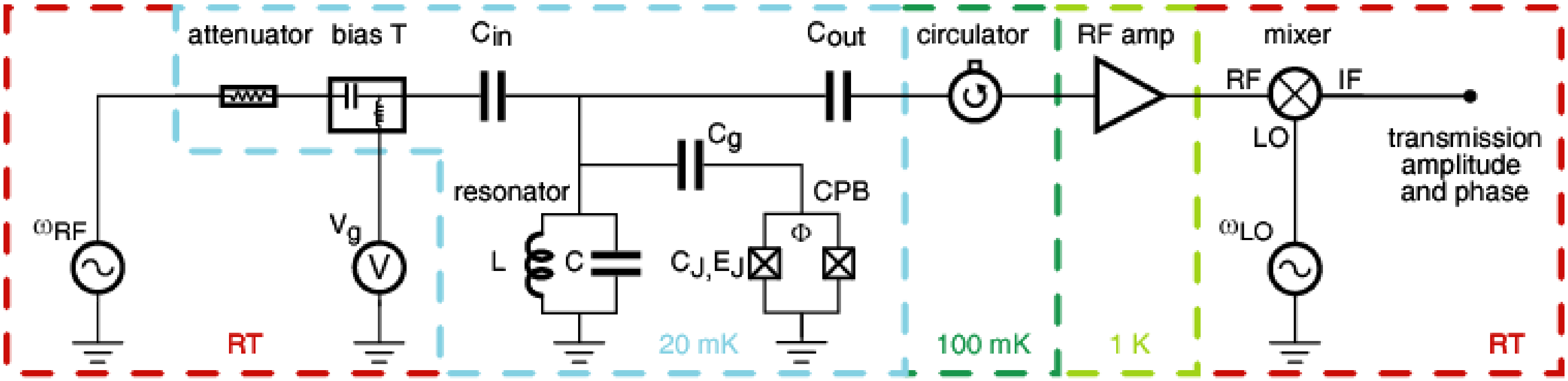
Microwave frequency control & measurement



The ETH Zurich Circuit QED Lab

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

How to do the Measurement



- prevent leakage of thermal photons (cold attenuators and circulators)
- average power to be detected ($\omega_r/2\pi$ = 6 GHz, $\kappa/2\pi$ = 1 MHz)

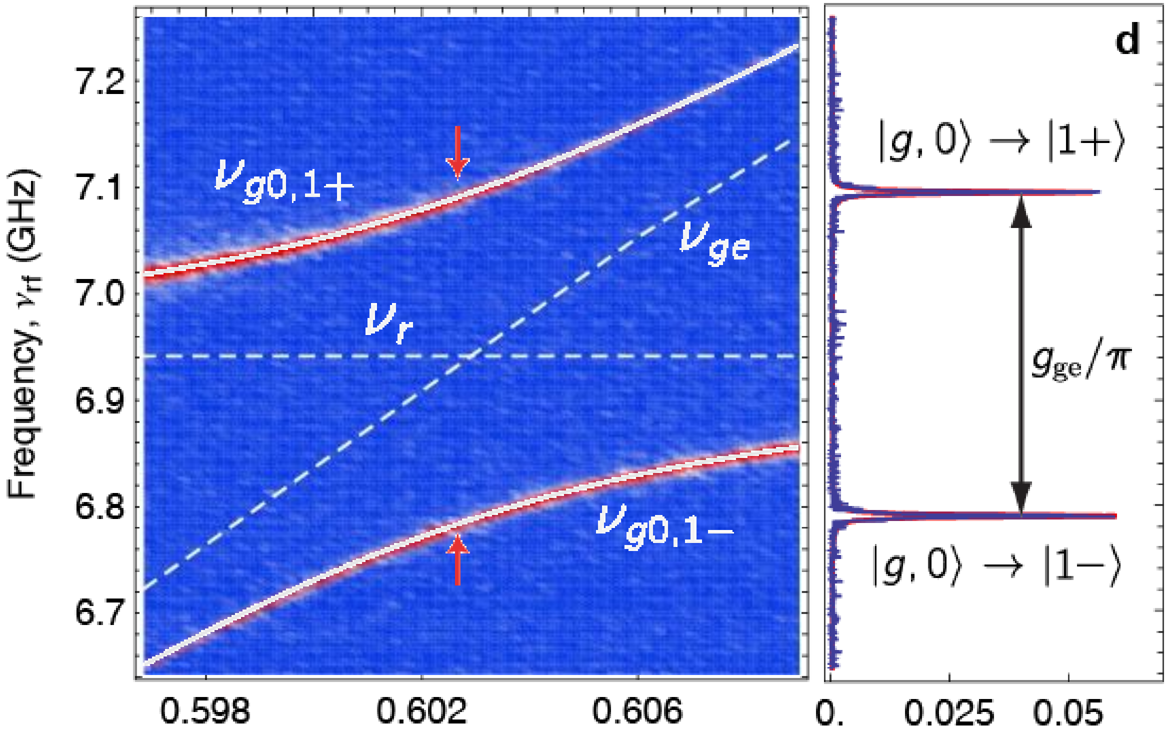
$$\langle n = 1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

- efficient with cryogenic low noise HEMT amplifier T_N = 6 K

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

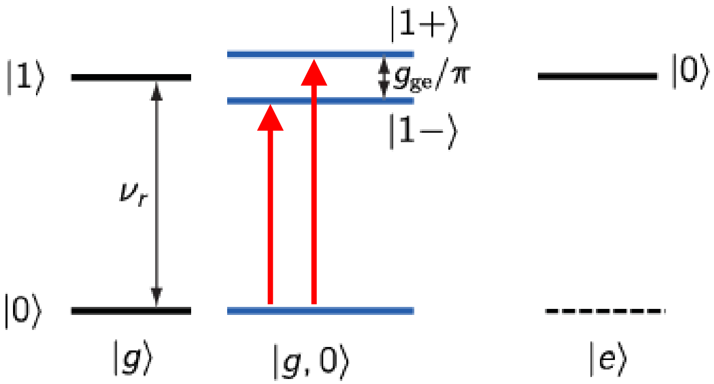
very strong coupling:



$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{\mathrm{ge}} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

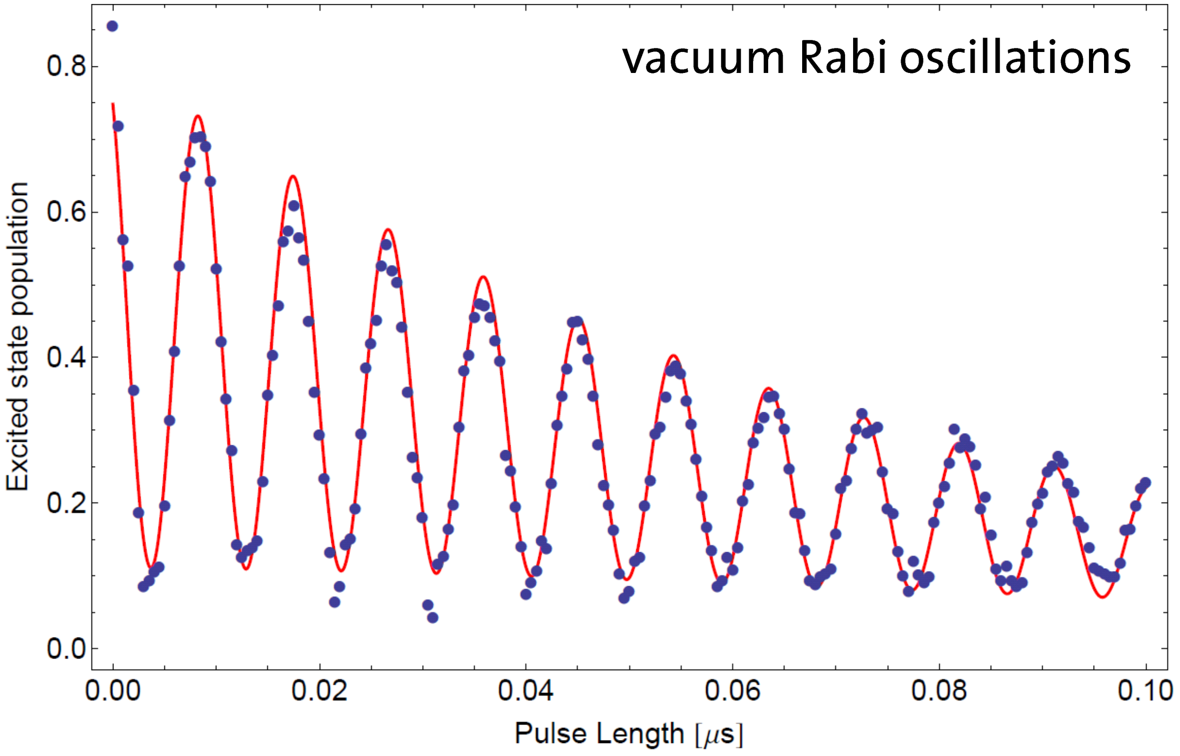
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n*=1):

very strong coupling:



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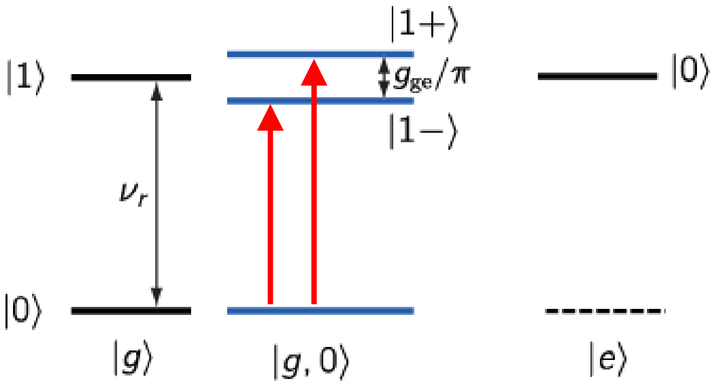


forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff et *al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Quantum Physics with Circuit QED ... some examples

Vacuum Rabi Mode Splitting
A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Coherent Flux-Qubit / SQUID Coupling
I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

Quantum AC-Stark Shift
D. Schuster *et al.*, *Nature* **445**, 515 (2007)

Lamb Shift
A. Fragner *et al.*, *Science* **322**, 1357 (2008)

Fock and Arbitrary Photon States
M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

Root n Nonlinearity
J. Fink *et al.*, *Nature* **454**, 315 (2008)

Two Photon Nonlinearities
F. Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)

Parametric Amplification
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 928 (2008)

Super Splitting and Root n Nonlinearity
L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

Ultrastrong Coupling
T. Niemczyk *et al.*, *Nat. Phys.* **6**, 772 (2010)

Single Photon Source
A. Houck *et al.*, *Nature* **449**, 328 (2007)

Single Qubit MASER
O. Astafiev *et al.*, *Nature* **449**, 588 (2007)

Single Qubit Resonance Fluorescence
O. Astafiev *et al.*, *Science* **327**, 840 (2010)

QND Measurement of Single Photon
B. Johnson *et al.*, *Nat. Phys.* **6**, 663 (2010)

Correlation Function Measurements
D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

Cooling and Amplification
M. Grajcar *et al.*, *Nat. Phys.* **4**, 612 (2008)

Quantum Algorithms & Entangled States
L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)
L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)
A. Fedorov *et al.*, *Nature* **481**, 170 (2012)
M. Reed *et al.*, *Nature* **481**, 382 (2012)

Quantum Bus
M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)
H. Majer *et al.*, *Nature* **449**, 443 (2007)
M. Mariantoni *et al.*, *Nat.Phys.* **7**, 287 (2011)
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)

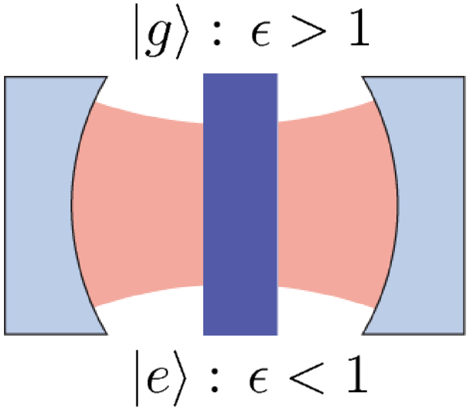
Non-Resonant Qubit-Photon Interaction

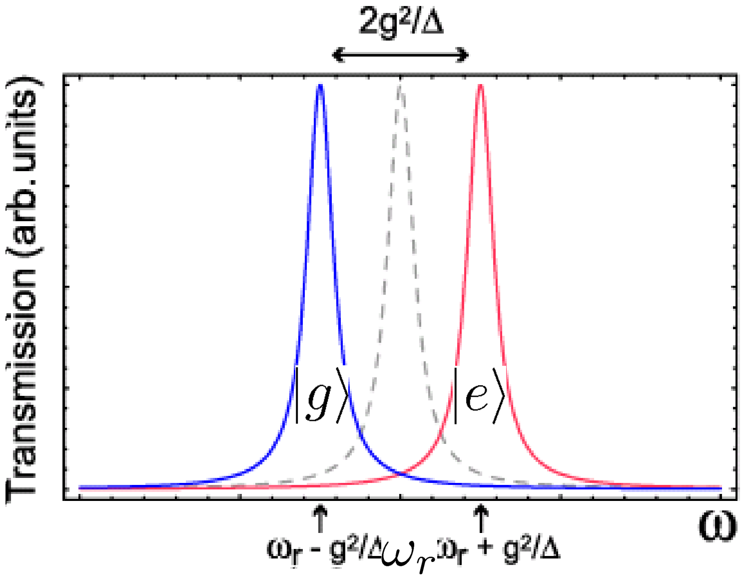
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

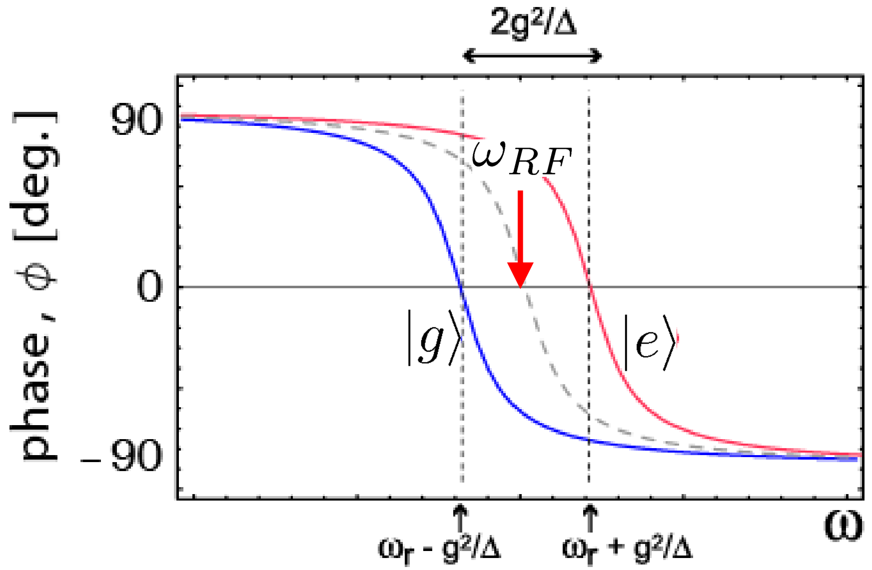
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//
cavity frequency shift
and qubit ac-Stark shift

//
Lamb shift



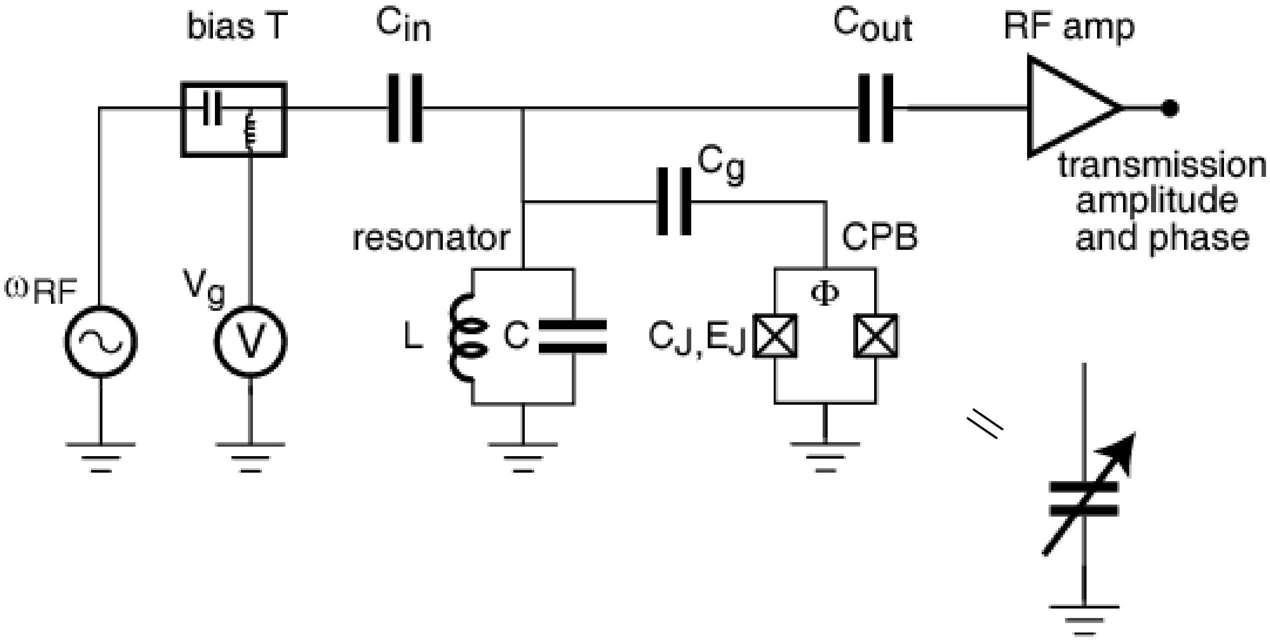




A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Qubit Read-Out & Spectroscopy

Measurement Technique



- measurement of microwave transmission amplitude T and phase ϕ
- intra-cavity photon number controllable from $n \sim 10^3$ to $n \ll 1$

Non-Resonant Coupling for Qubit Readout

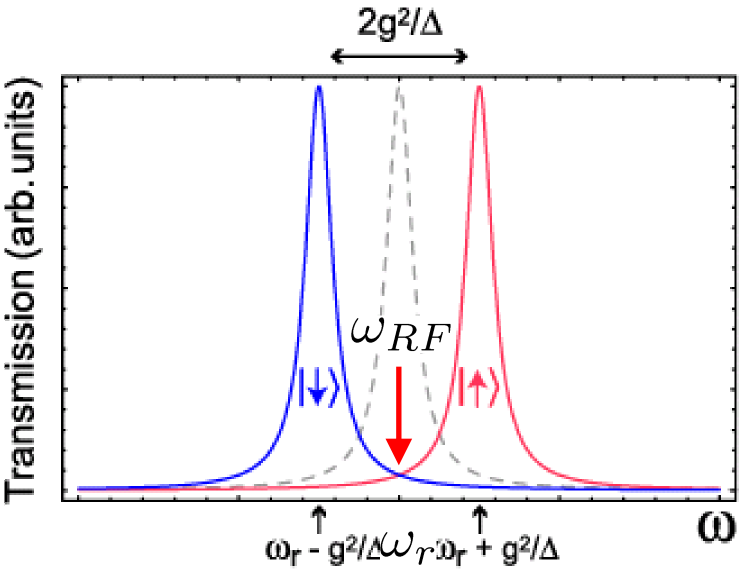
approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

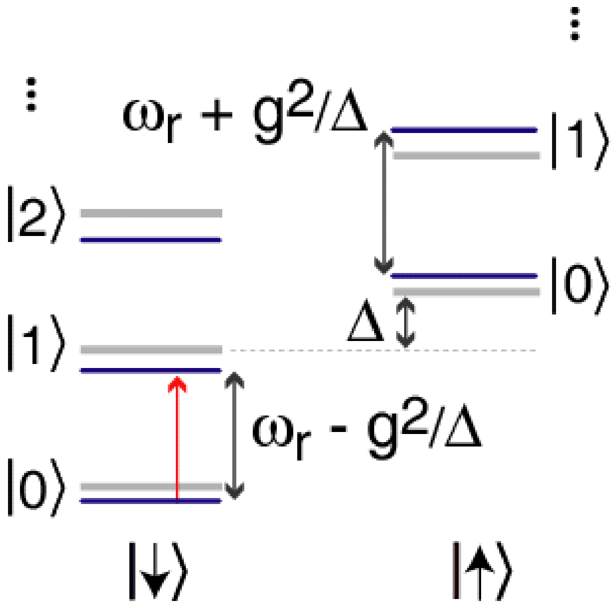
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z \right) a^\dagger a + \frac{1}{2}\hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

cavity frequency shift
and qubit ac-Stark shift

Lamb shift

dispersive level diagram:



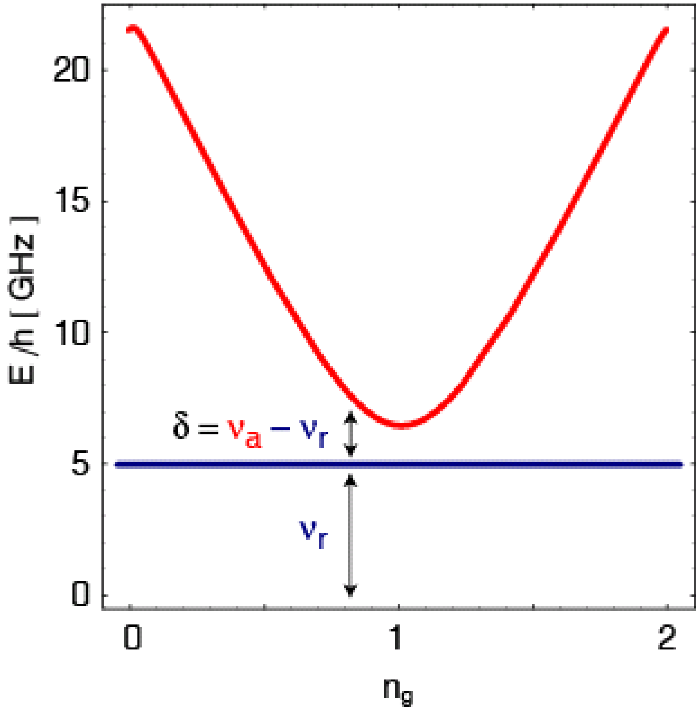


A. Blais *et al.*, *PRA* **69**, 062320 (2004)

Dispersive Shift of Resonance Frequency

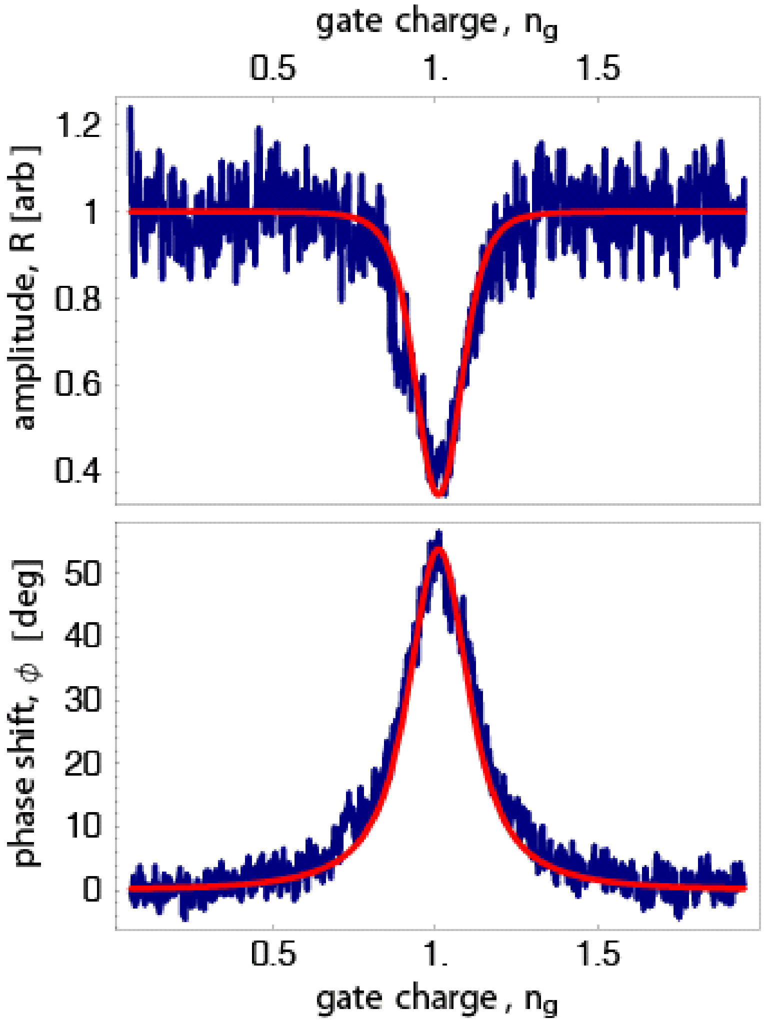
sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

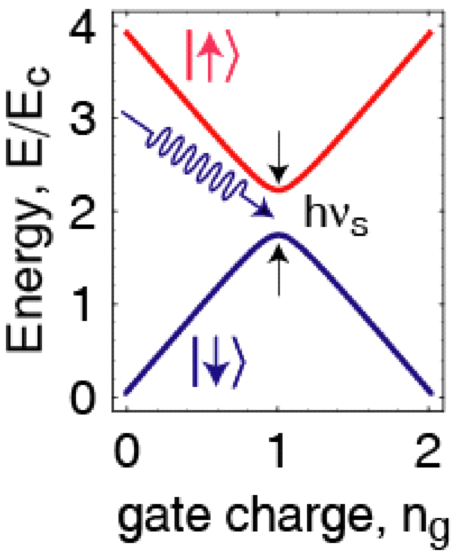


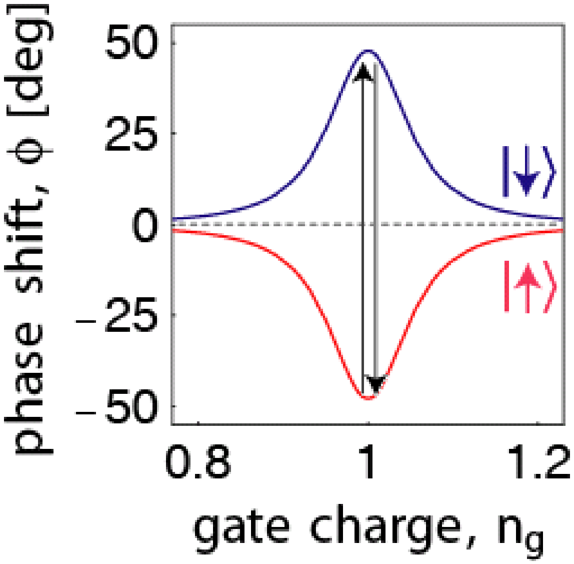
$$\begin{aligned} g/\pi = \nu_{\text{vac}} &= 11\,\text{MHz} \\ \Delta(n_g = 1)/2\pi &= 66\,\text{MHz} \\ n &= 10 \end{aligned}$$

measured resonator transmission amplitude and phase:



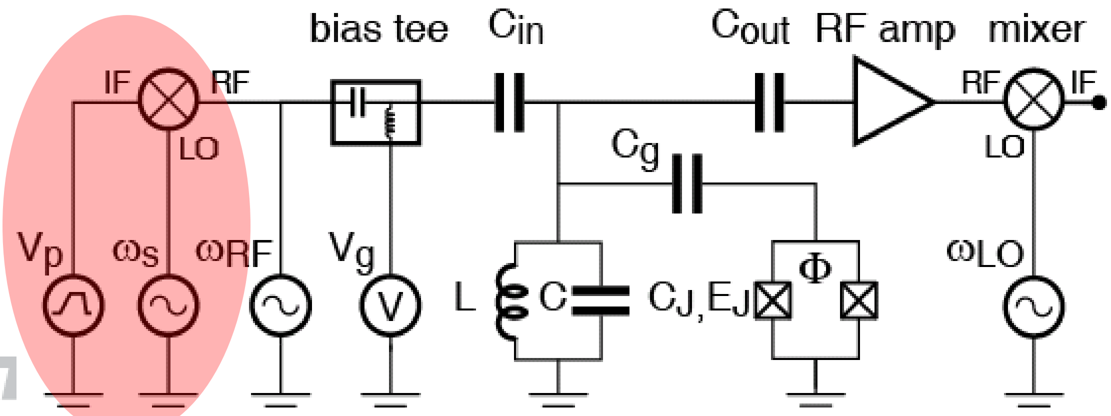
Coherent Control and Read-out in a Cavity





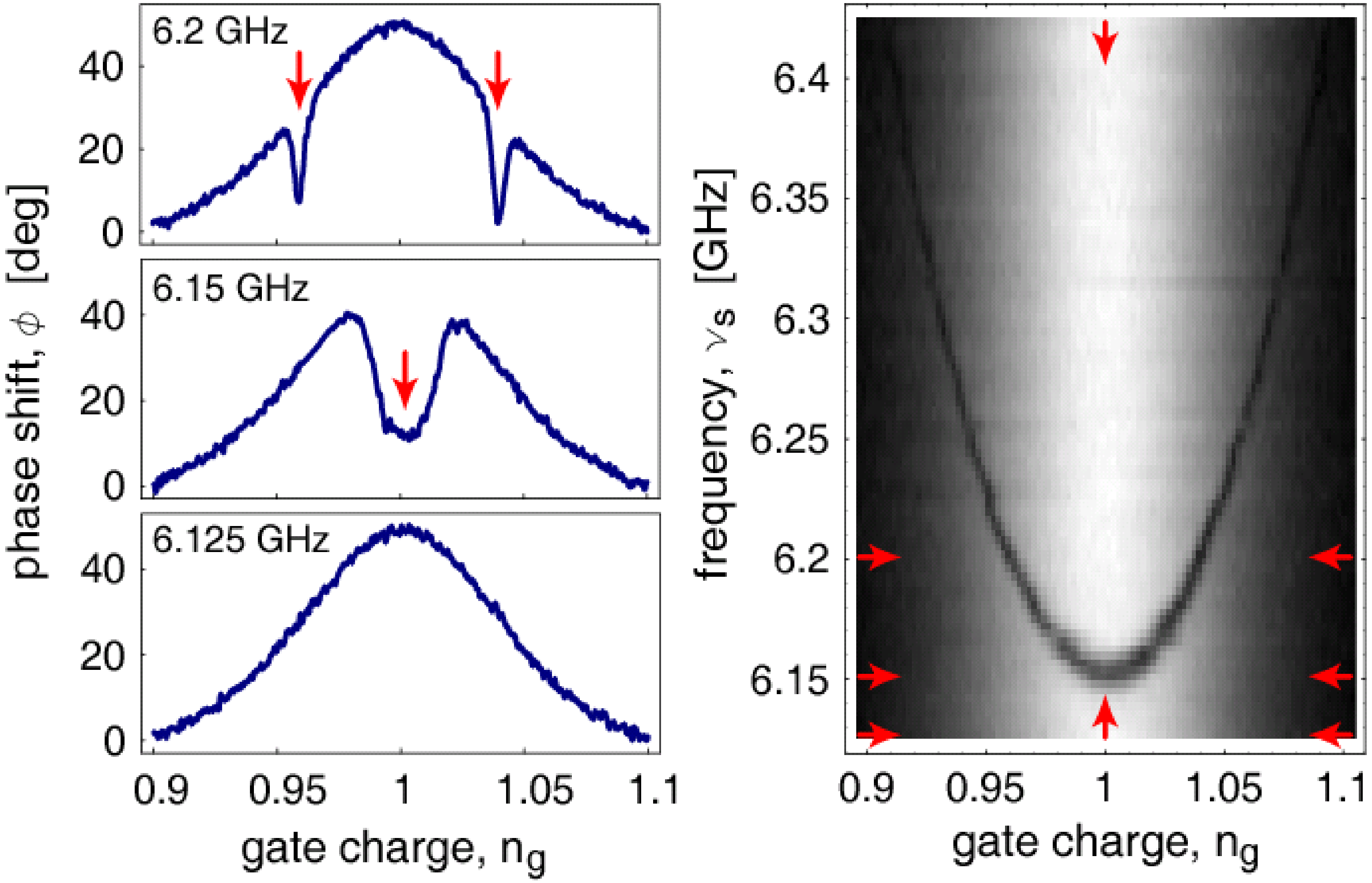
- apply resonant microwave pulse to qubit
- detect change of phase

realization:



- simultaneous control and measurement

CW Spectroscopy of Cooper Pair Box



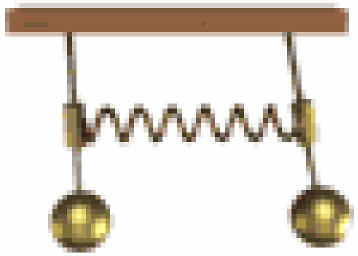
detuning $\Delta_{\mathrm{r,a}}/2\pi \sim 100\,\mathrm{MHz}$ extracted: $E_J = 6.2\,\mathrm{GHz},\ E_C = 4.8\,\mathrm{GHz}$

D. I. Schuster et al., *Phys. Rev. Lett.* **94,** 123062 (2005)

Probing Field Quantization on a Chip ...

... by measuring the quantum nonlinearity of the J-C ladder

What can be learned from a measurement of the vacuum Rabi mode splitting?

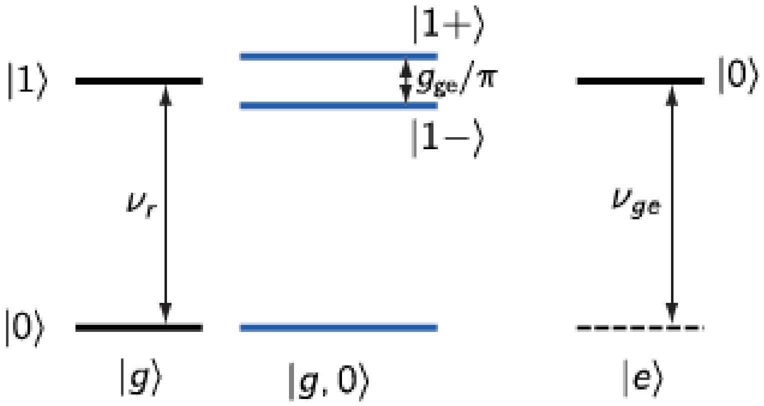


classical interpretation:

- coupled harmonic oscillators
- normal mode coupling

quantum effects:

- scaling of coupling g_{eff} with square root of the photon number n
- direct proof of field quantization



time-resolved data in atomic physics exps. (Haroche, Walther, ...) but no spectroscopic data until recently

$$|n\pm\rangle = \left(|g,n\rangle \pm |e,n-1\rangle\right)/\sqrt{2}$$

Climbing the Jaynes-Cummings Ladder

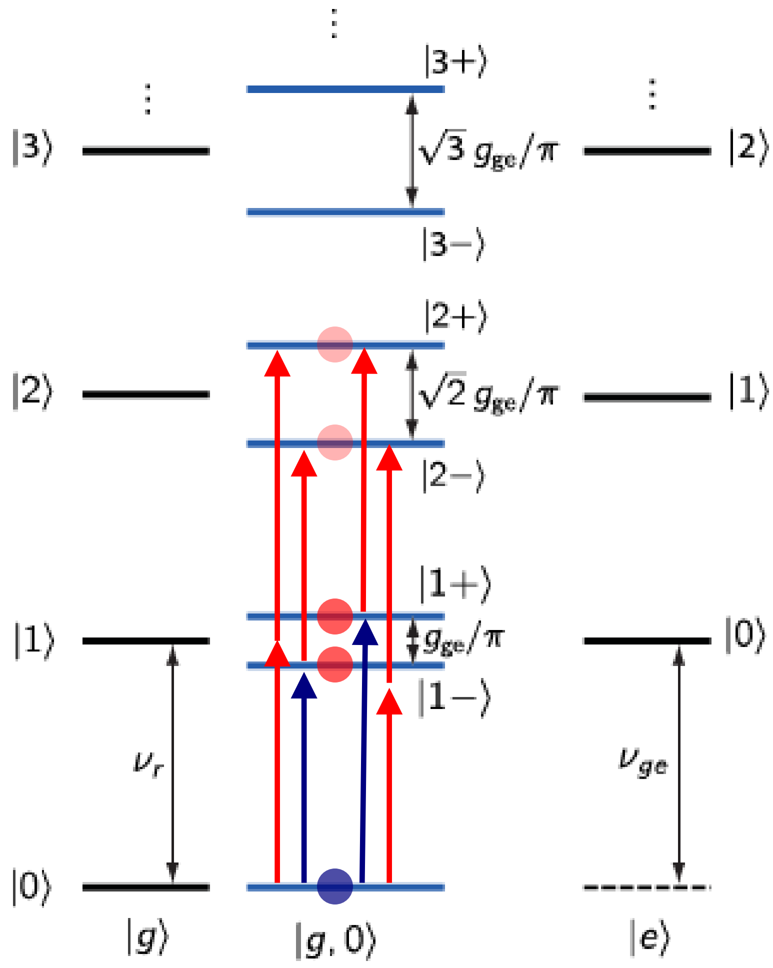
How to climb the ladder?

start on the lowest rung:

- cool to the ground state |g,0>

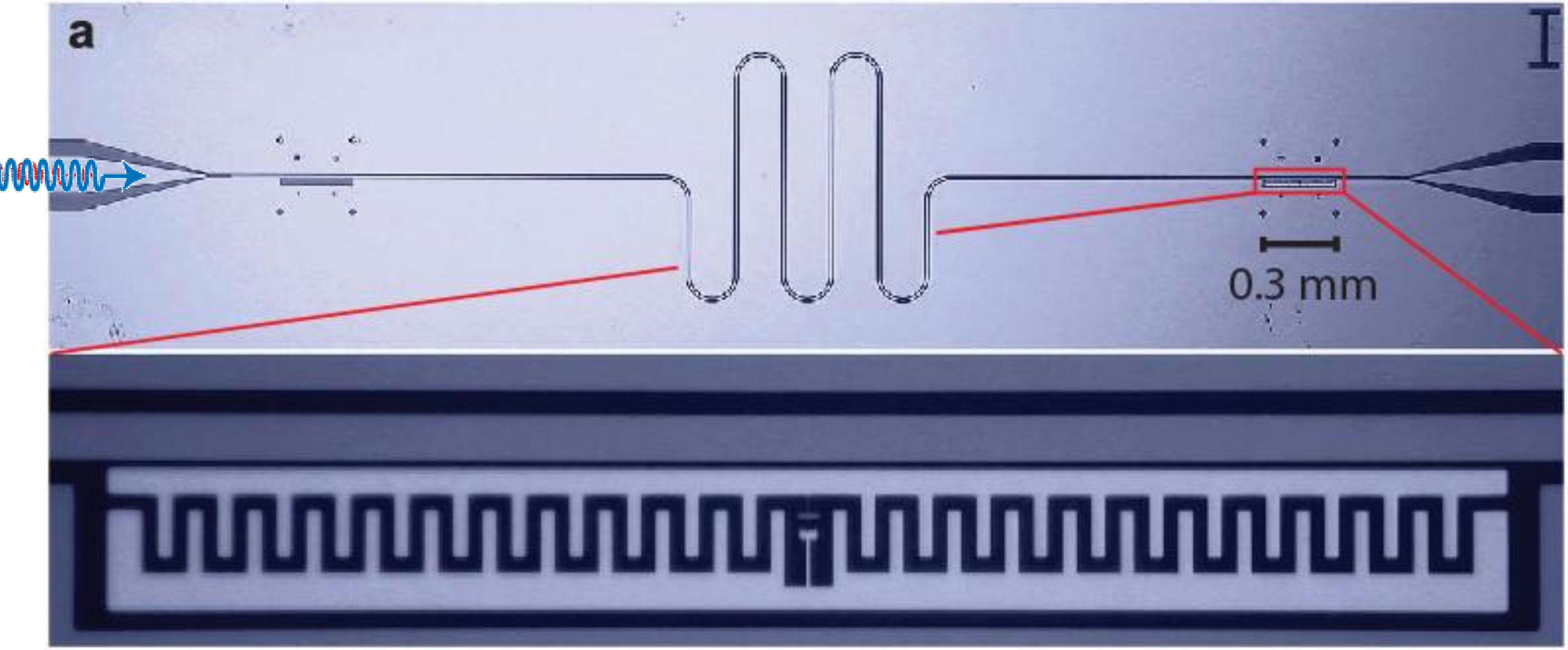
climb towards higher rungs:

- step by step:
  ‚pump & probe' excitation
  J. Fink *et al.*, *Nature* **454**, 315 (2008)
  I. Schuster *et al.*, *Nat. Phys.* **4**, 382 (2008)
  M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)
- many rungs at the same time:
  multi-photon excitation
  L. S. Bishop *et al.*, *Nature Phys.* **5**, 105 (2009)
- thermal excitation
  J. Fink *et al.*, *PRL* **105**, 163601 (2010)
  J. Fink *et al.*, *Physica Scripta* **T137**, 014013 (2009)
  with full control over phase:
  M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)



$$|n\pm\rangle = \left(|g,n\rangle \pm |e,n-1\rangle\right)/\sqrt{2}$$

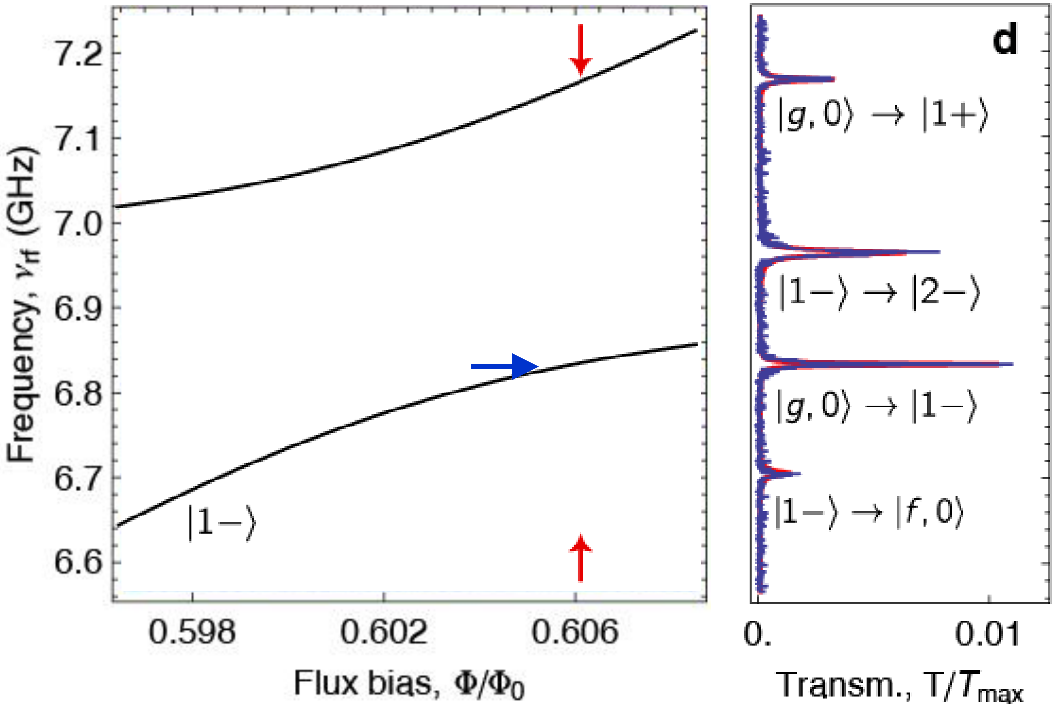
Two-Photon Pump and Probe Spectroscopy



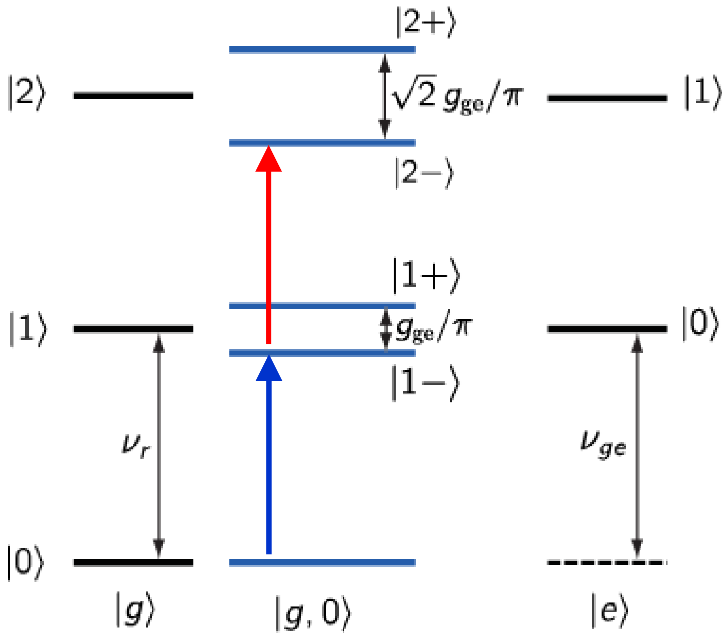
J. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. Leek, A. Blais, A. Wallraff,
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with two photons $(n = 2)$:



pump and probe: $|n-\rangle$



- $|n-\rangle \rightarrow |n+\rangle$ is weak

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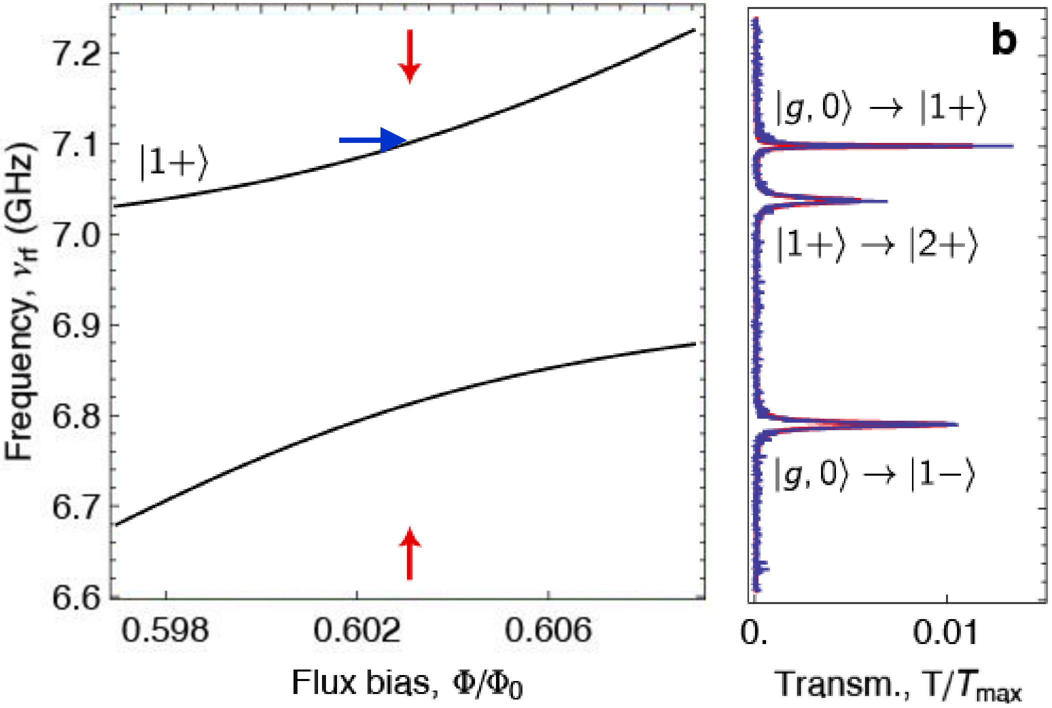
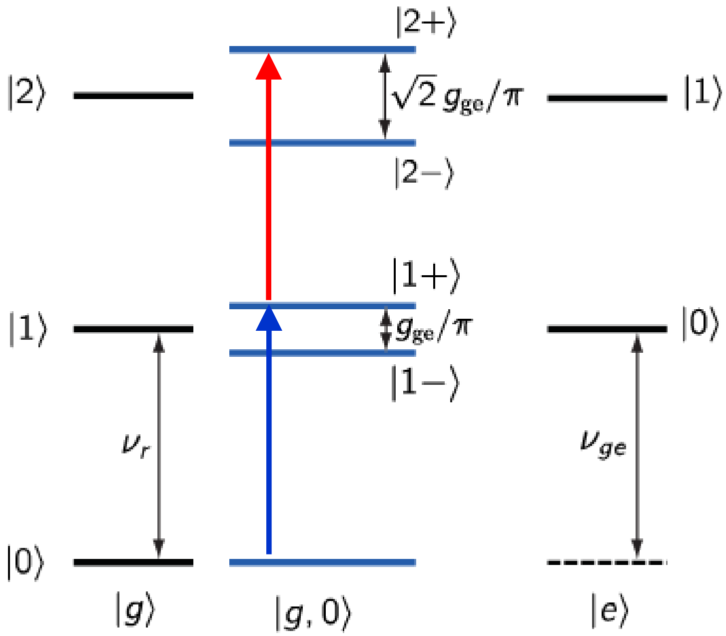
Nature (London) **454**, 315 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with two photons $(n = 2)$:



pump and probe: $|n+\rangle$



- $|n+\rangle \rightarrow |n-\rangle$ is weak

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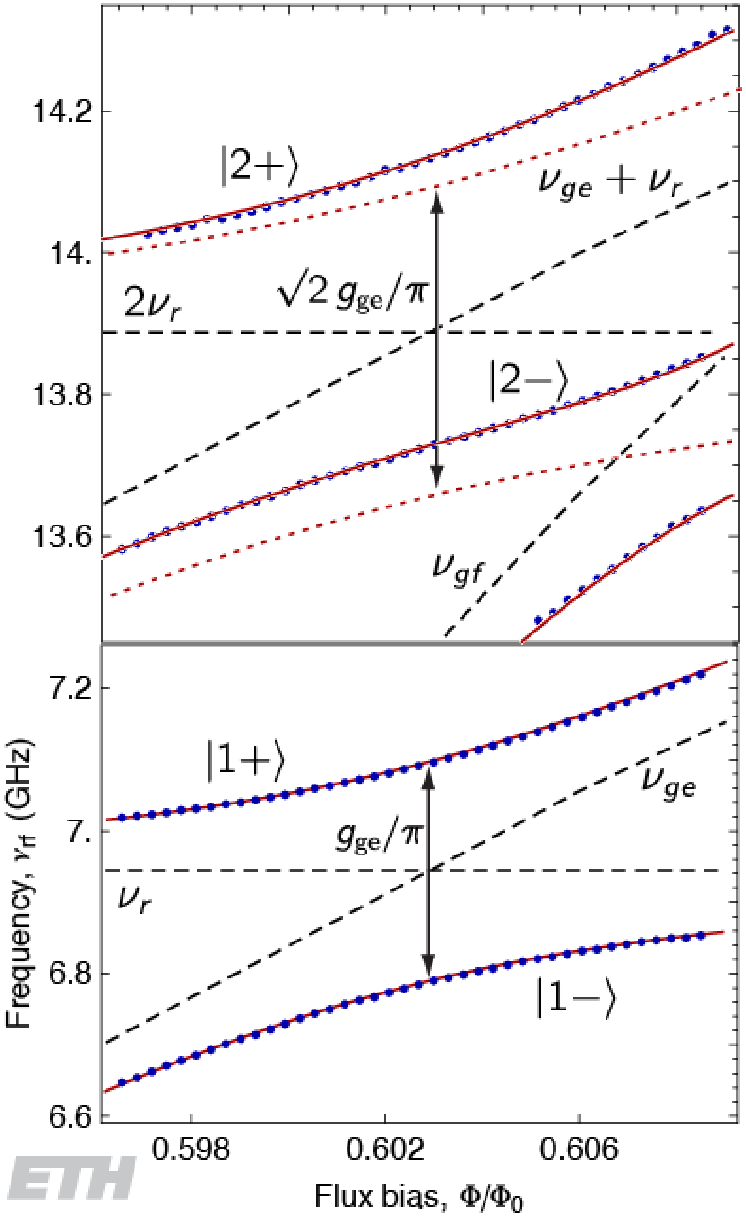
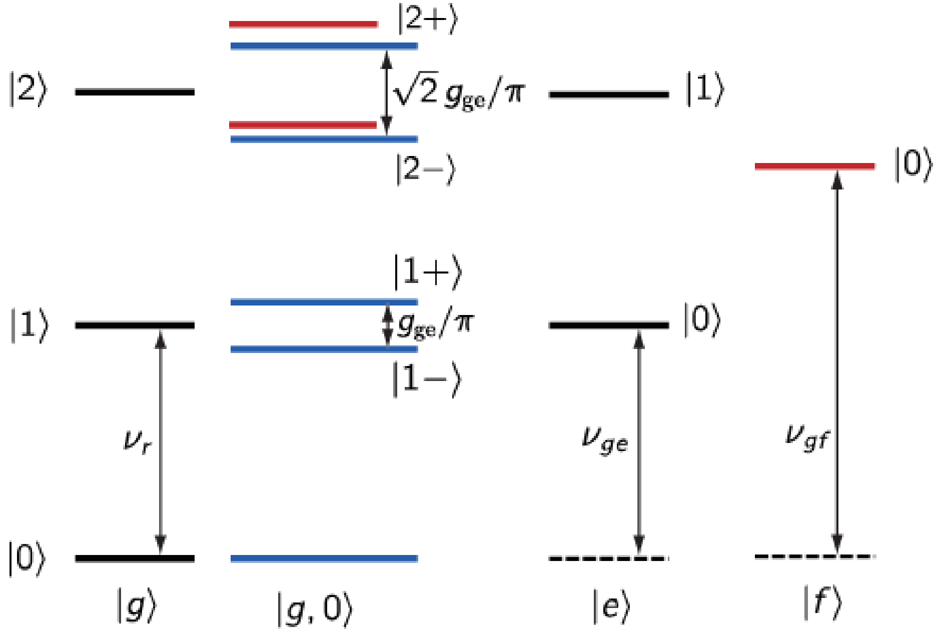
Sqrt(n) Quantum Nonlinearity



- energies reconstructed from pump + probe
- shifts due to 3rd qubit level $|f\rangle$
- full Hamiltonian yields good agreement
- clear spectroscopic demonstration of field quantization in cavity QED



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