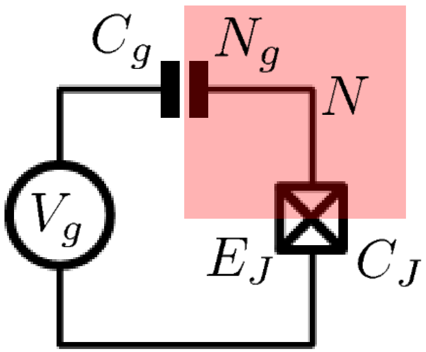
The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \dfrac{Q}{2e}$

continuous gate charge: $N_g = \dfrac{C_g V_g}{2e}$

total box capacitance $C_\Sigma = C_g + C_J$

Hamiltonian: $H = H_{\mathrm{el}} + H_{\mathrm{mag}}$

electrostatic part:

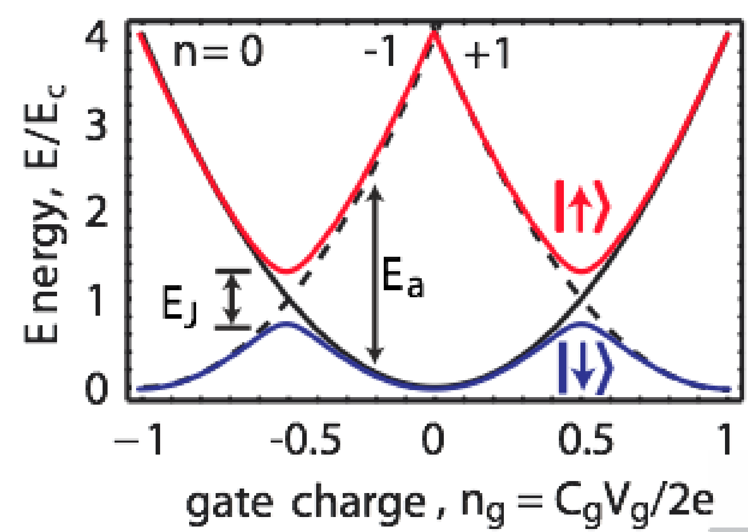
$$H_{\mathrm{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\mathrm{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator:

$$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** N:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 + E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 + E_J \cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

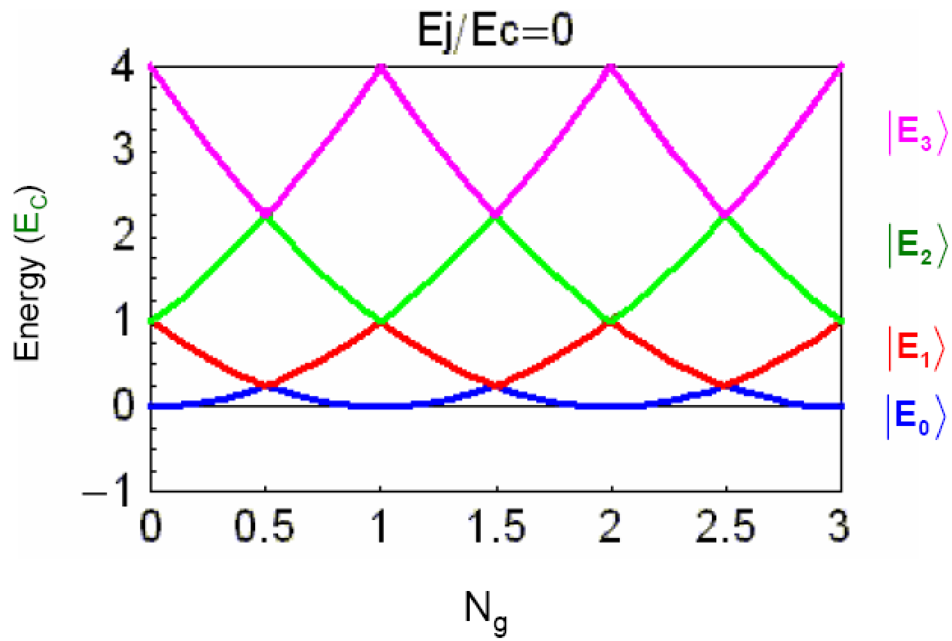
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

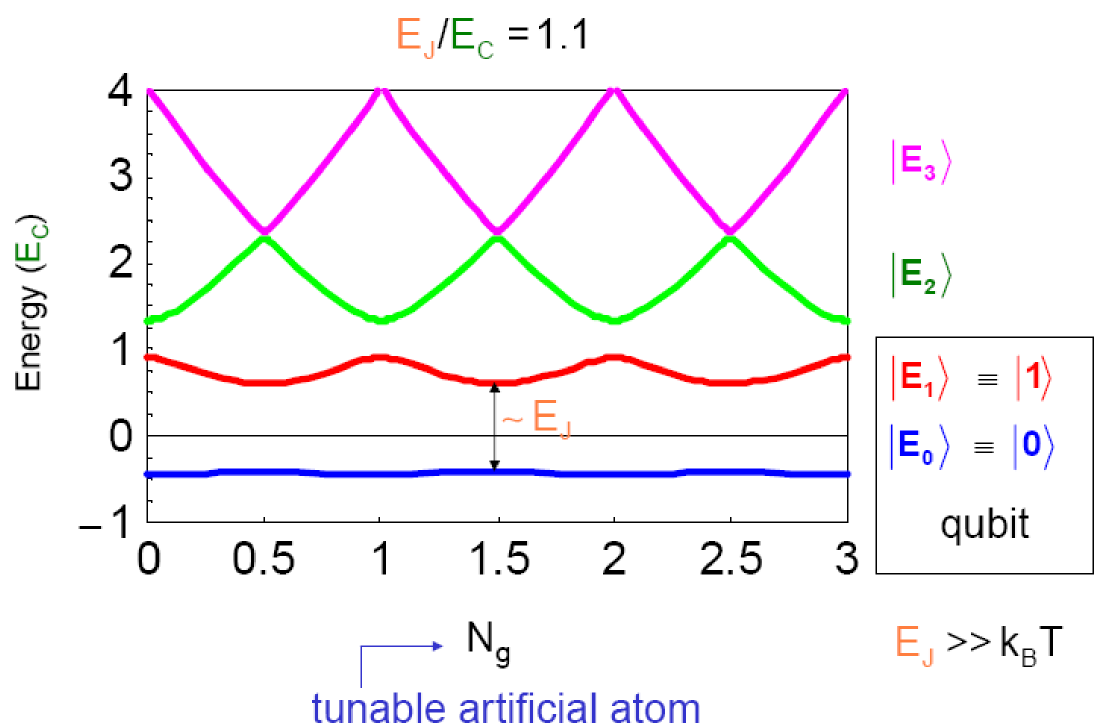
energy level diagram for E_J=0:

- energy bands are formed
- bands are periodic in N_g

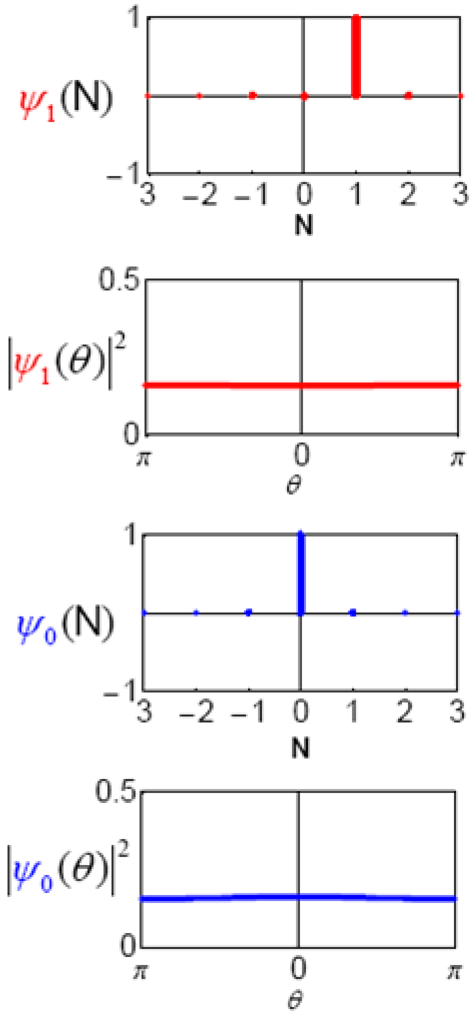


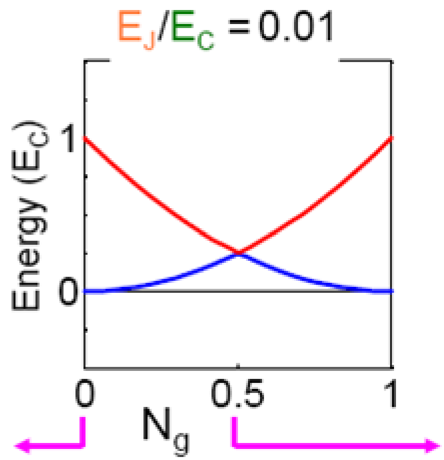
energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



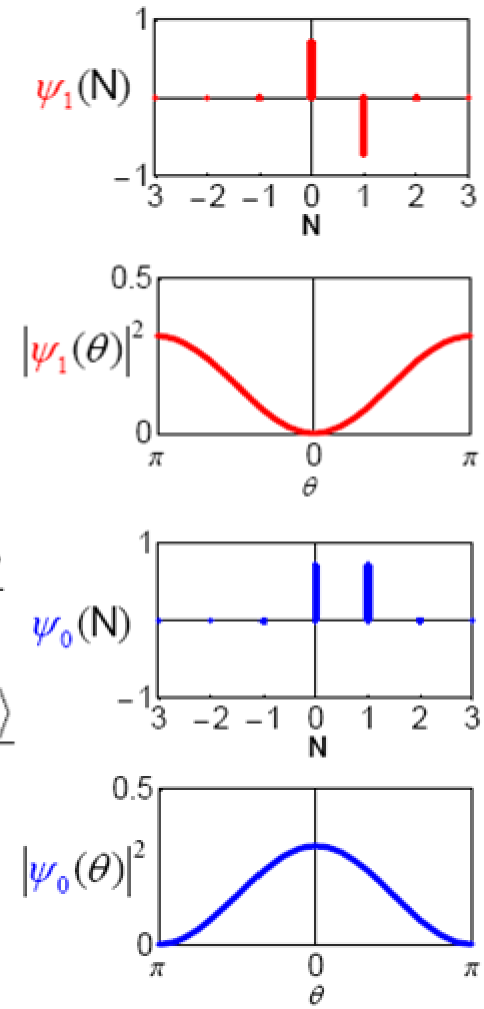
Charge and Phase Wave Functions ($E_J << E_C$)





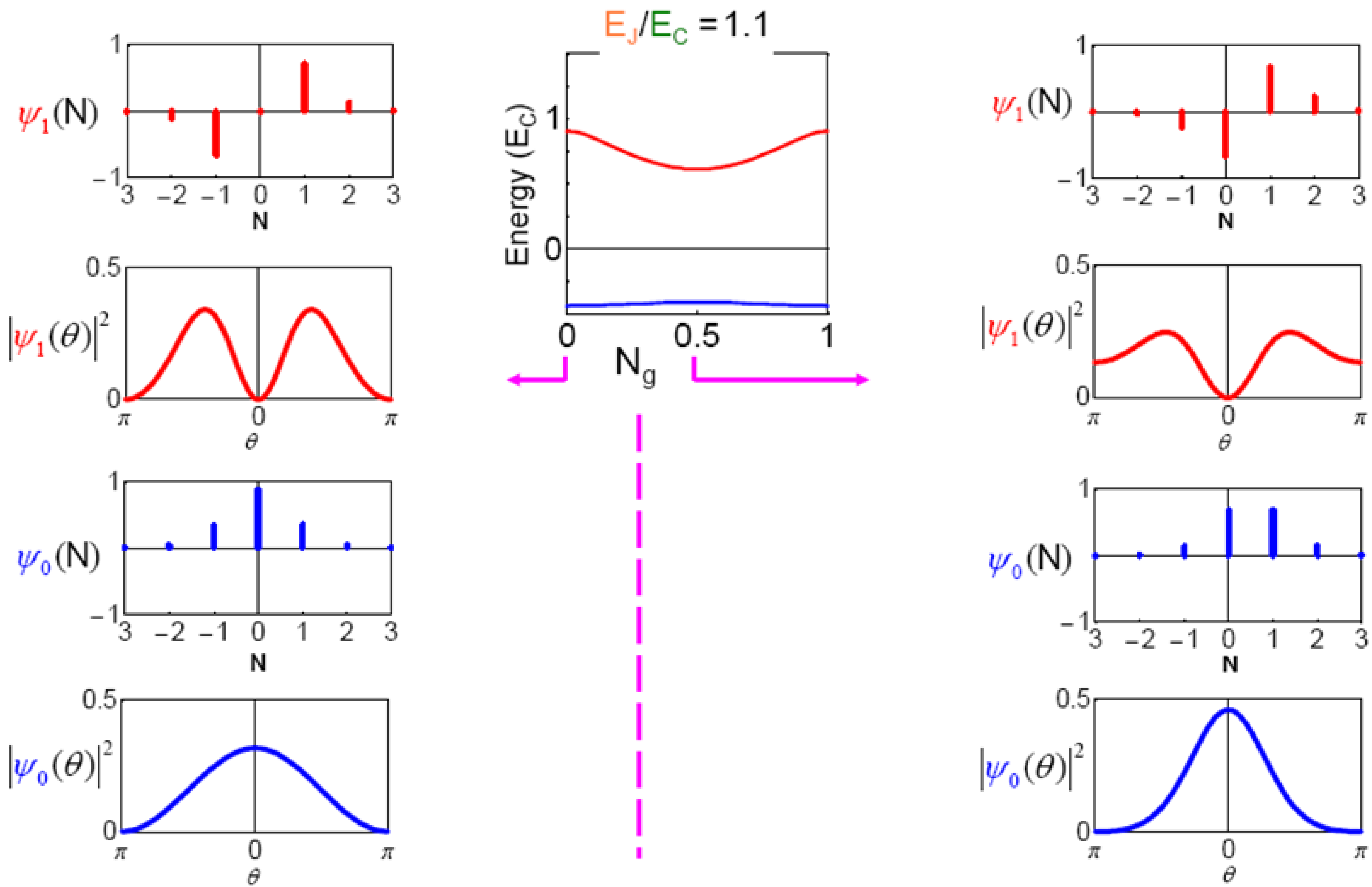
$$|\psi_1\rangle \approx |N=1\rangle \qquad |\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx |N=0\rangle \qquad |\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$



courtesy CEA Saclay

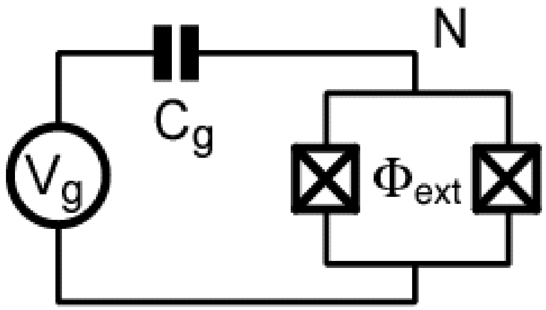
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

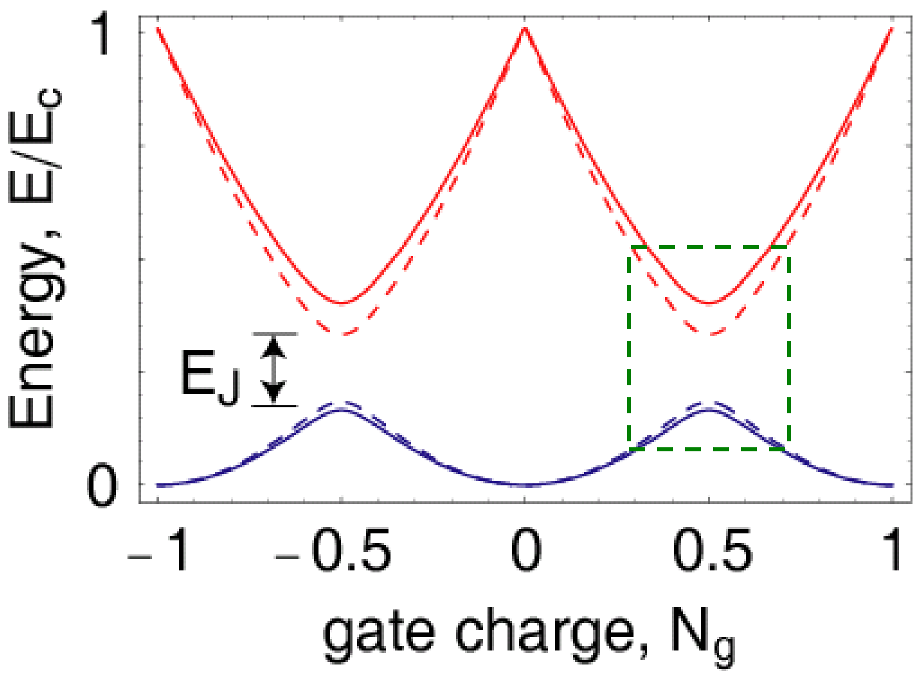
split Cooper pair box in perpendicular field



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\frac{\phi_{\rm ext}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\phi_{\rm ext}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\text{CPB}} = \hat{H}_{\text{el}} + \hat{H}_{\text{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

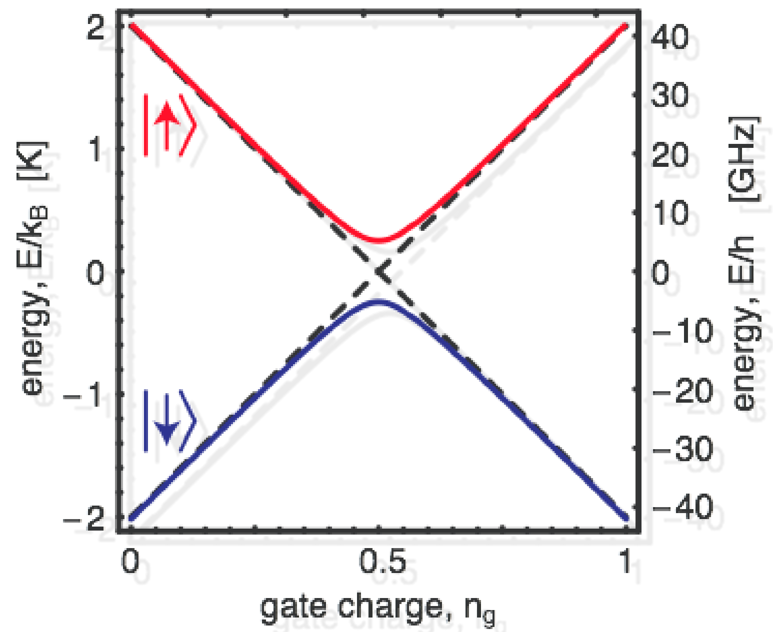
$$\hat{H}_{\text{CPB}} = \sum_N \left[E_C(N - N_g)^2 |N\rangle\langle N| - \frac{E_{\text{J}}}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\text{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

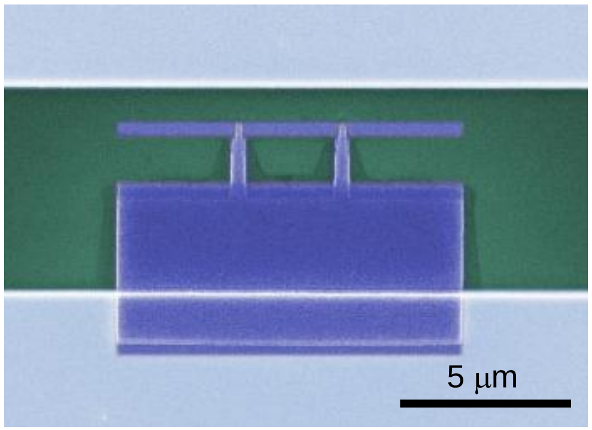


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

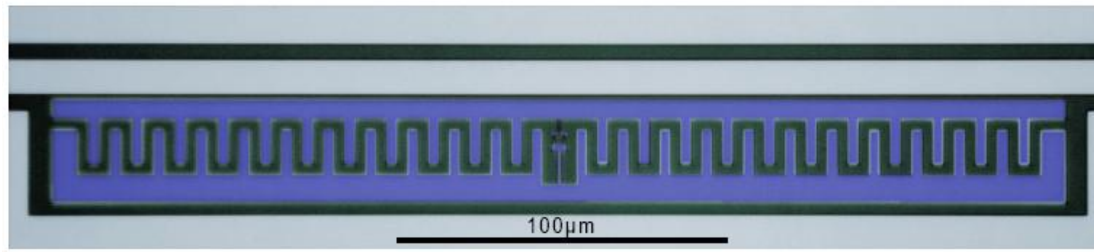
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

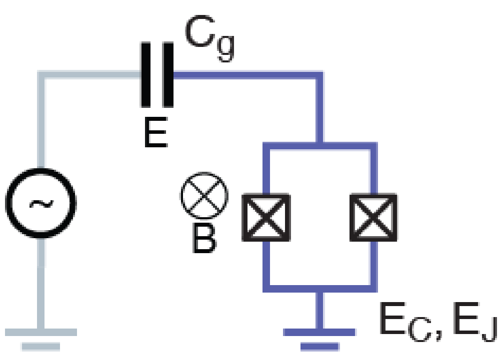
standard CPB:

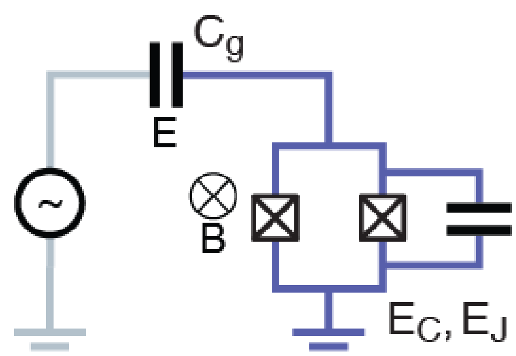


Transmon qubit:



circuit diagram:

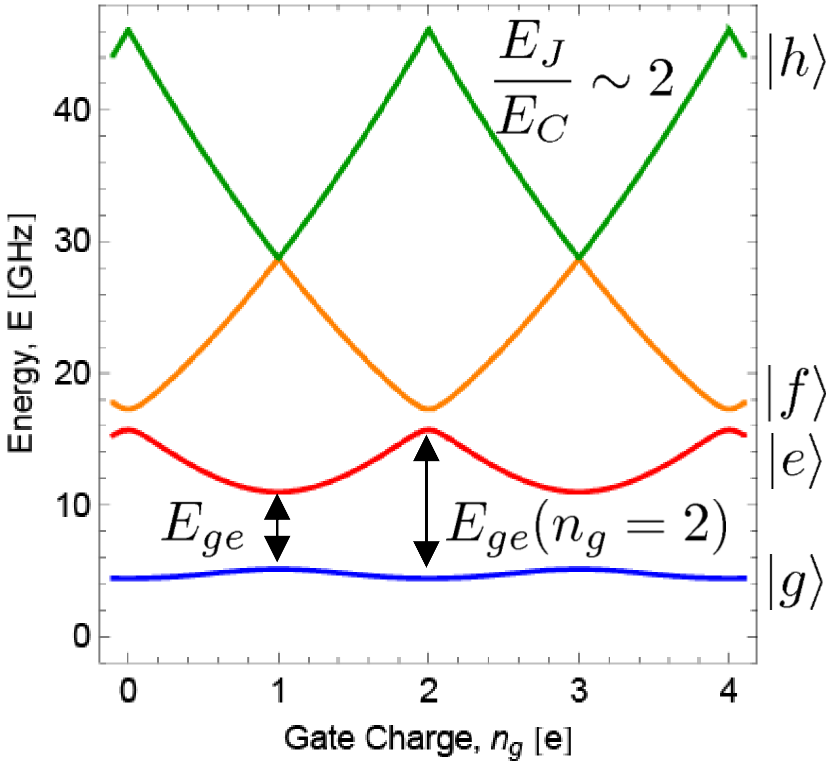




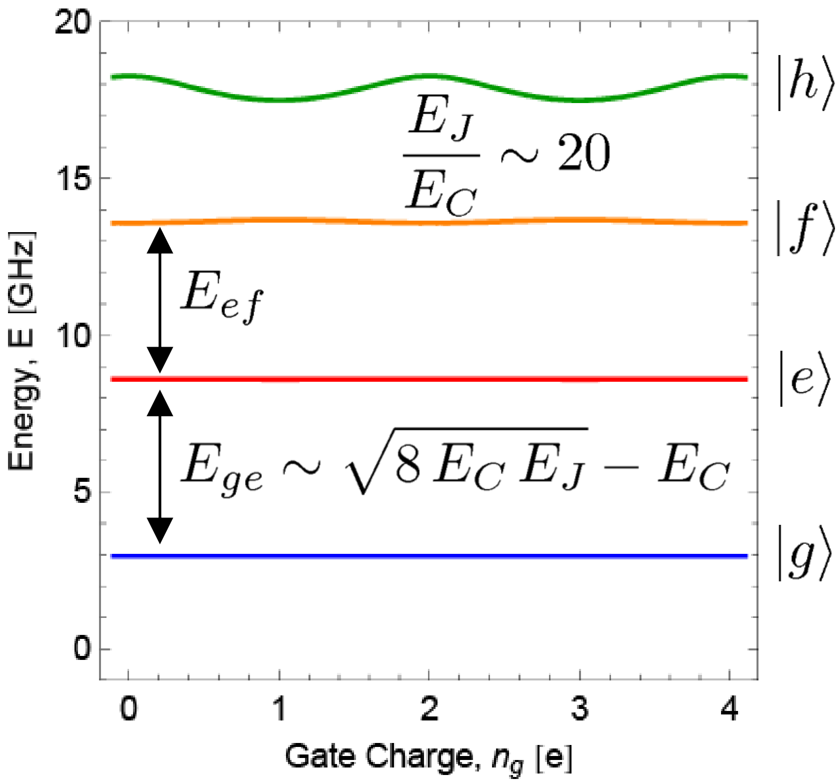
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity:

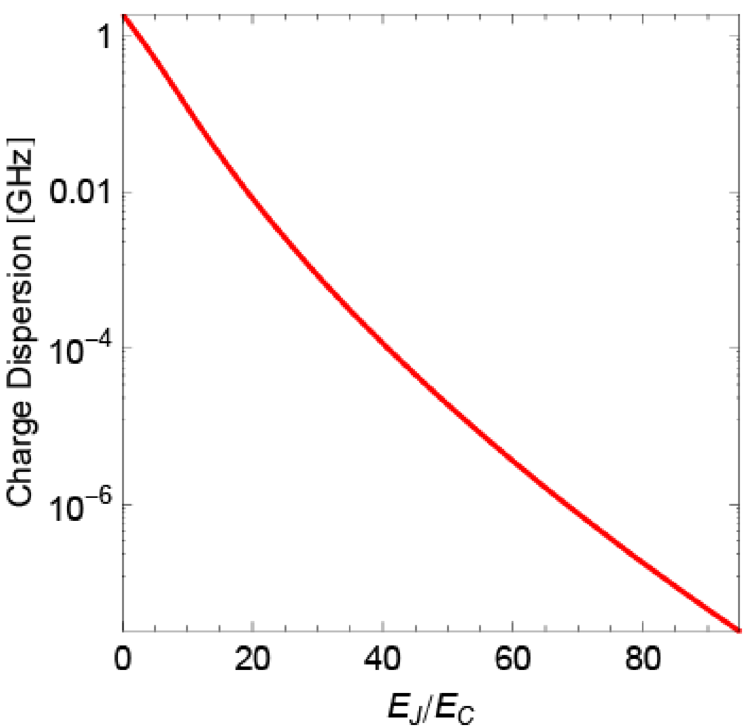
$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Dispersion and Anharmonicity

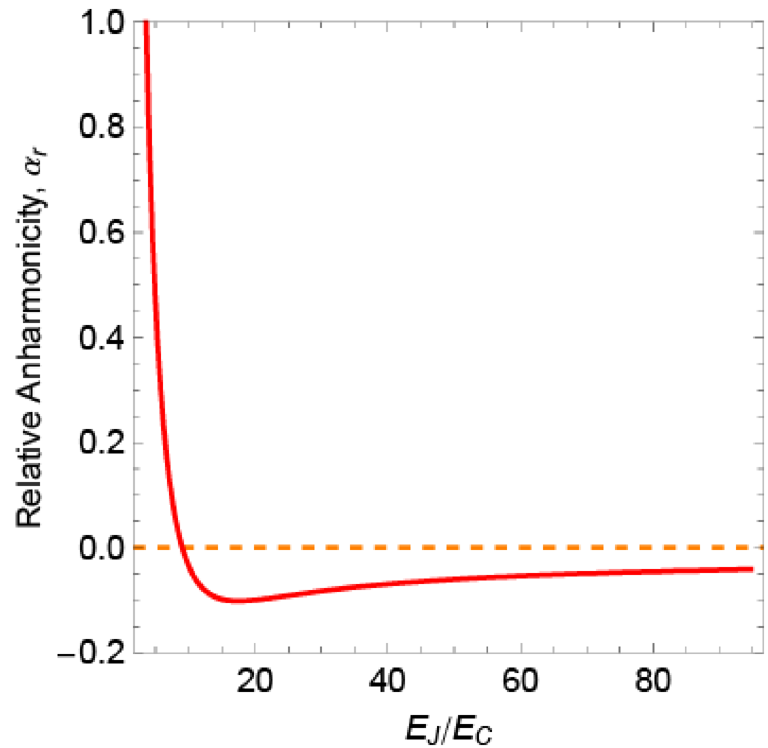
Charge dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$



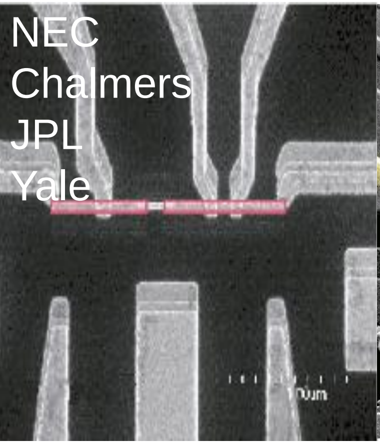
Anharmonicity:

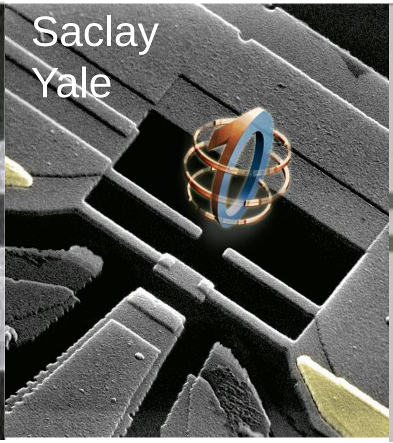
$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

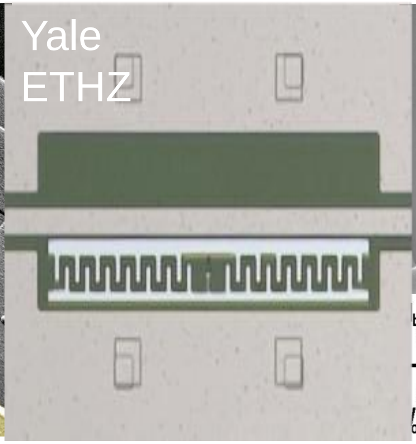


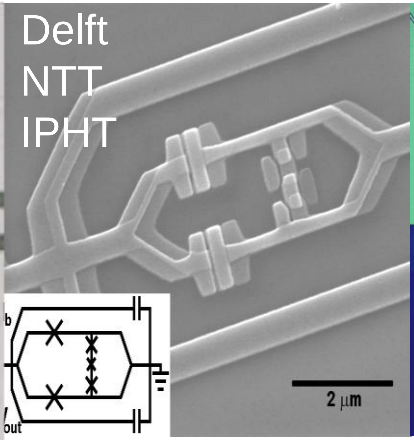
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Realizations of Superconducting Artificial Atoms





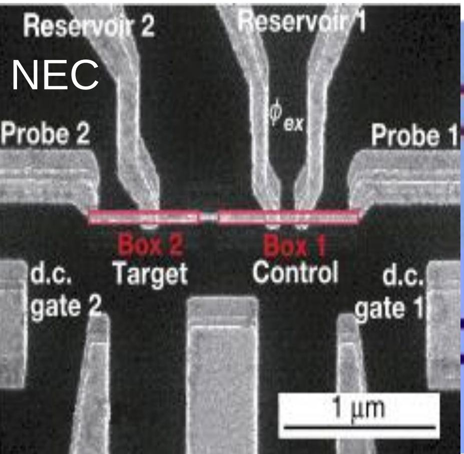


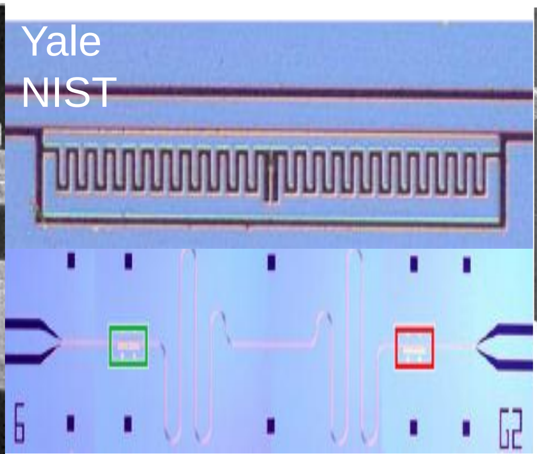


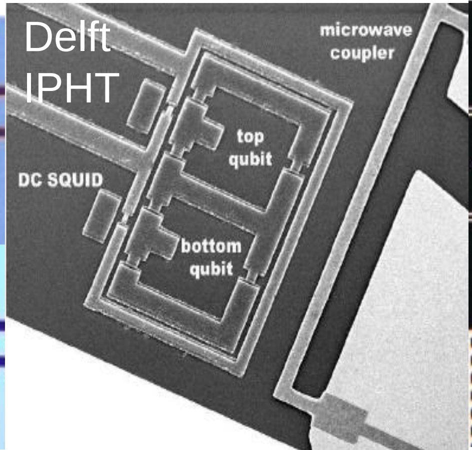
'artificial atoms' -- single superconducting qubits

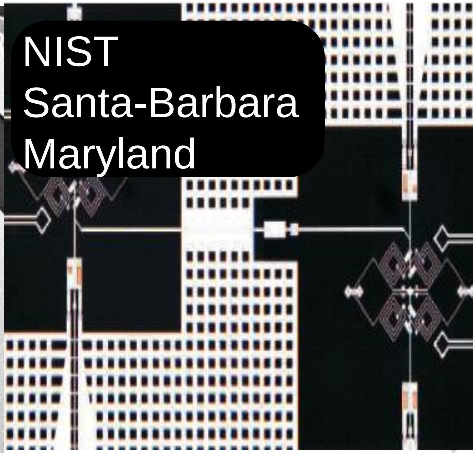
review:
J. Clarke and F. Wilhelm
Nature **453**, 1031 (2008)

'artificial molecules' -- coupled superconducting qubits





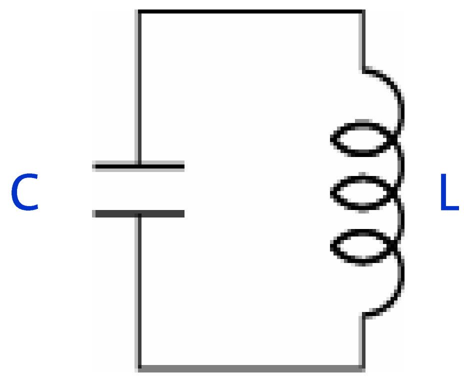




Realizations of Harmonic Oscillators

Superconducting Harmonic Oscillators

a simple electronic circuit:



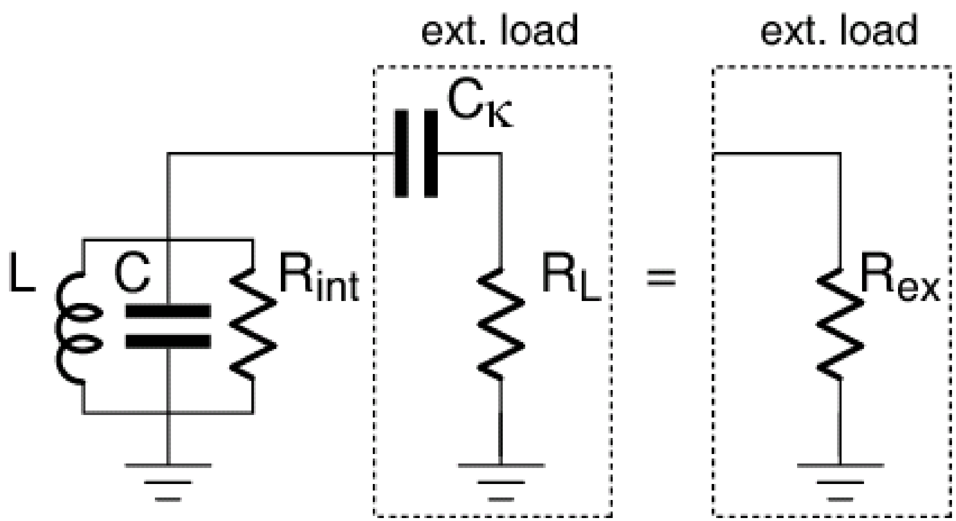
- typical inductor: $L = 1$ nH
- a wire in vacuum has inductance ~ 1 nH/mm

- typical capacitor: $C = 1$ pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx ($\varepsilon = 10$) of thickness 10 nm has a capacitance $C \sim 1$ pF

- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\text{ GHz}$$

Internal and External Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

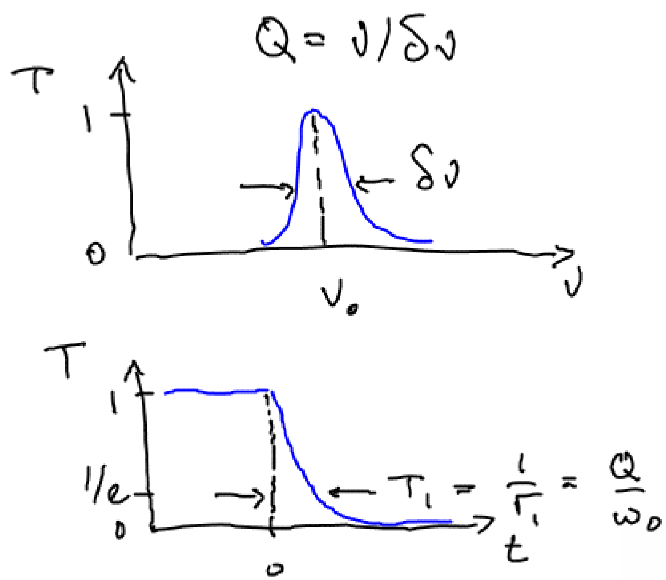
external losses: R_{ext}
radiation, coupling

total losses $$\frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}}$$

impedance $$Z = \sqrt{\frac{L}{C}}$$

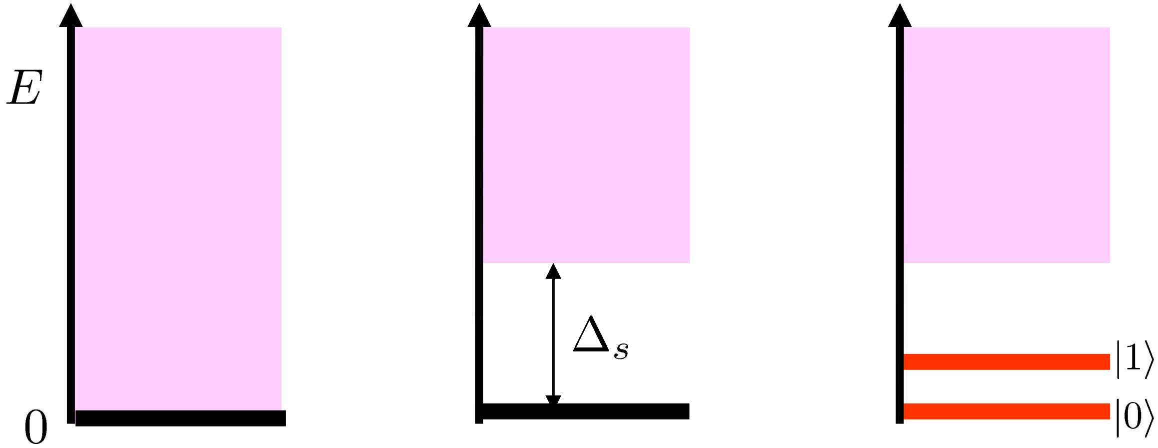
quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$



problem 2: internal and external dissipation

Why Superconductors?



normal metal superconductor How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

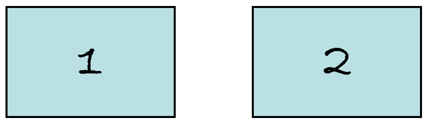
Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta/h$ = 725 GHz, T_c = 9.2 K
- Aluminum (Al): $2\Delta/h$ = 100 GHz, T_c = 1.2 K

Cooper pairs:
bound electron pairs

Bosons (S=0, L=0)

2 chunks of superconductors

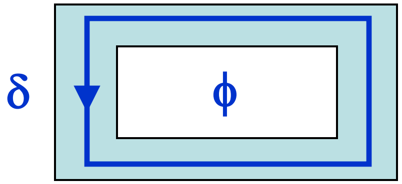


macroscopic wave function

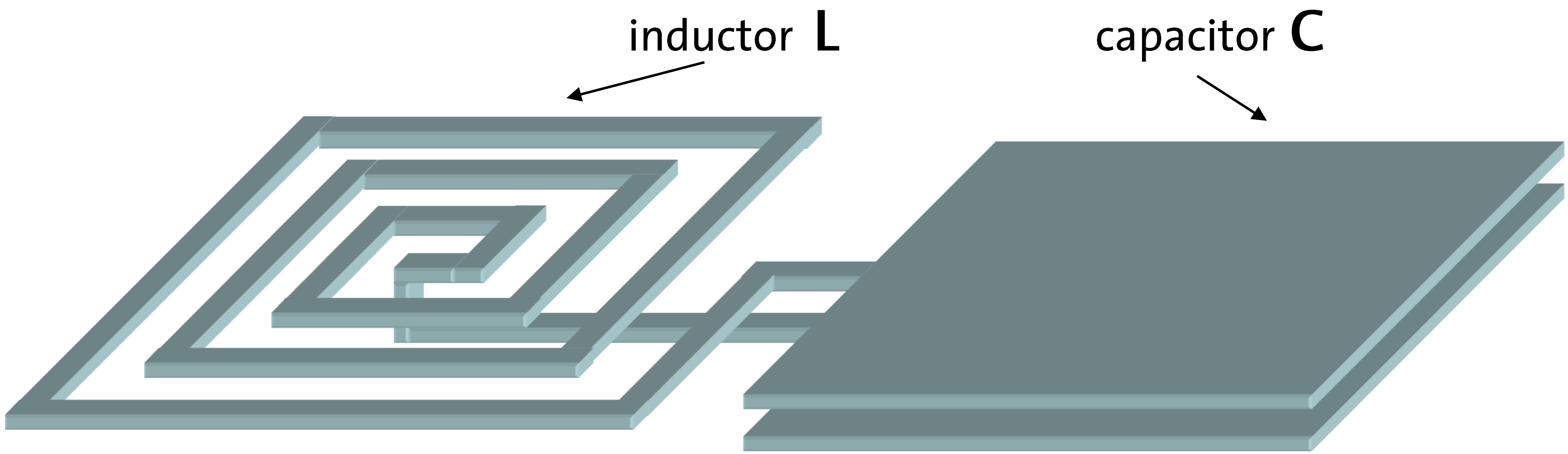
$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

Cooper pair density n_i
and global phase δ_i

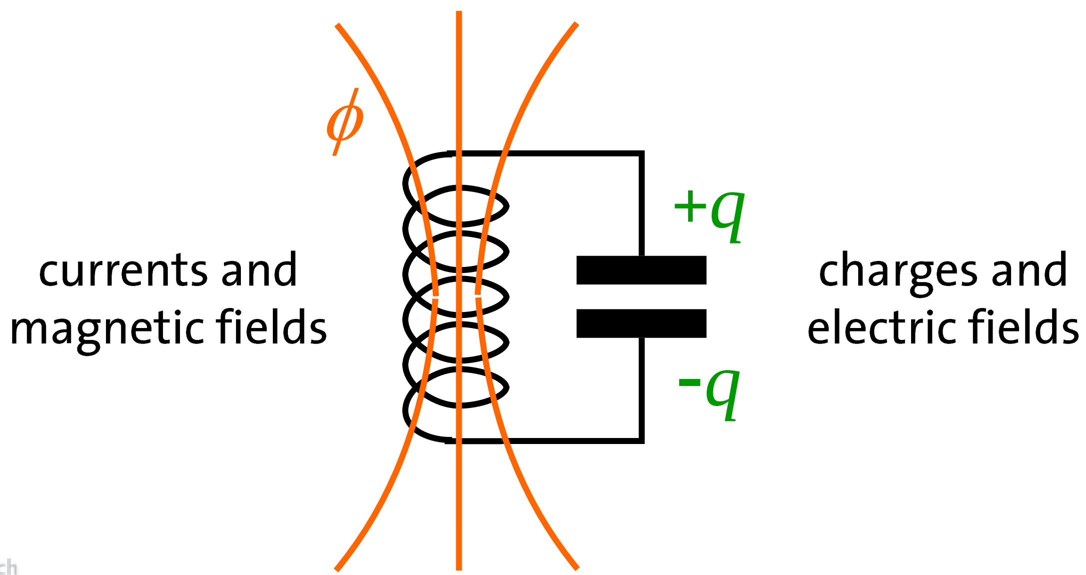
phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$



Realization of H.O.: Lumped Element Resonator

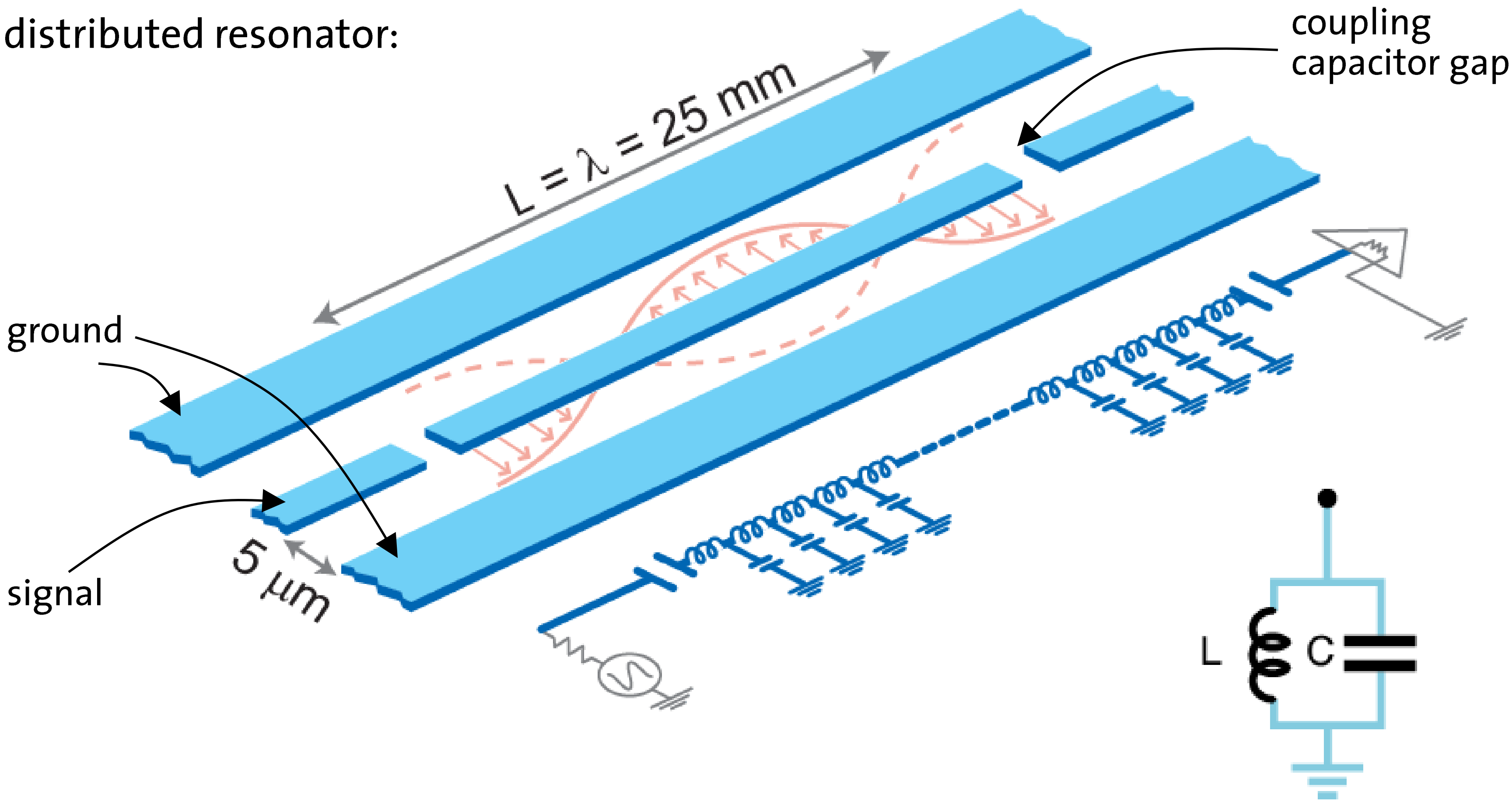


a harmonic oscillator



Realization of H.O.: Transmission Line Resonator

distributed resonator:

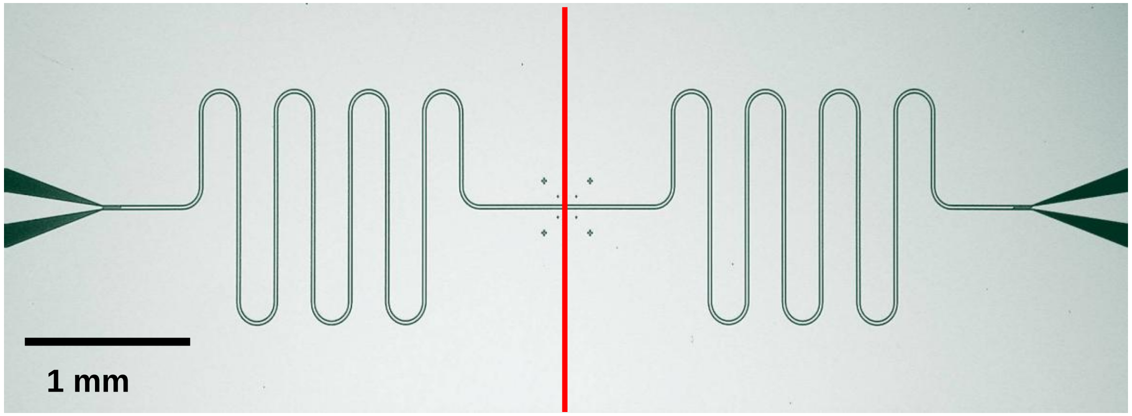


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

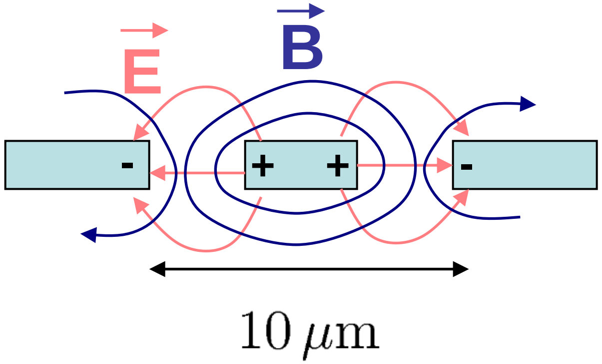
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

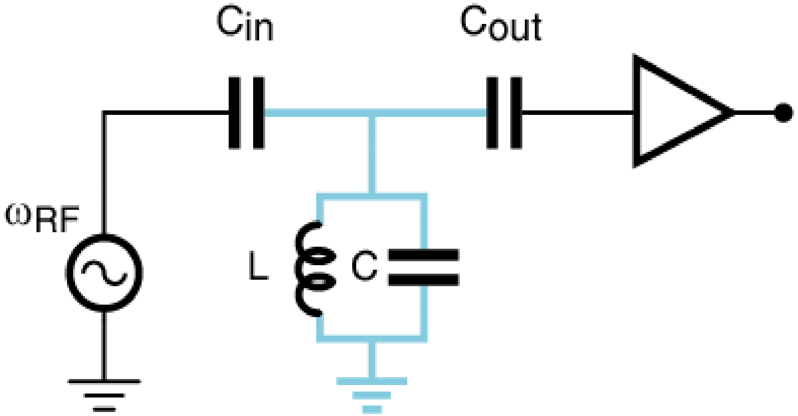
coplanar waveguide:



cross-section of transm. line (TEM mode):

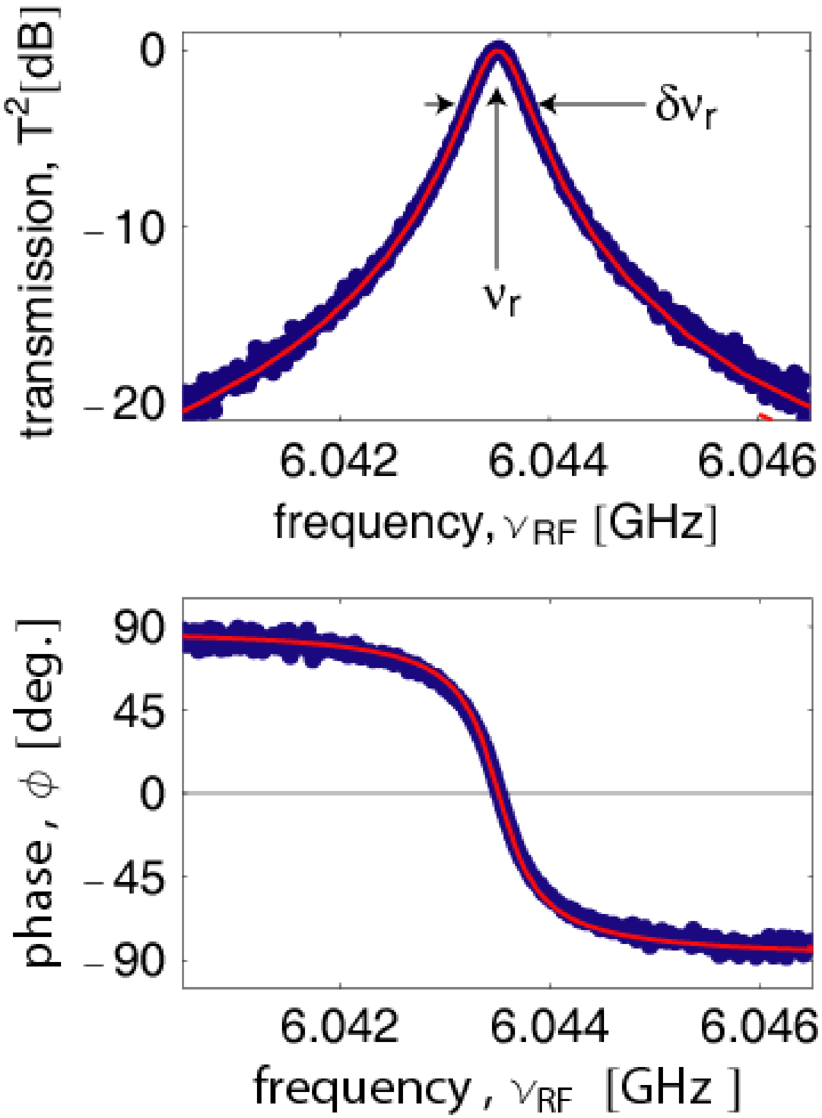


measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

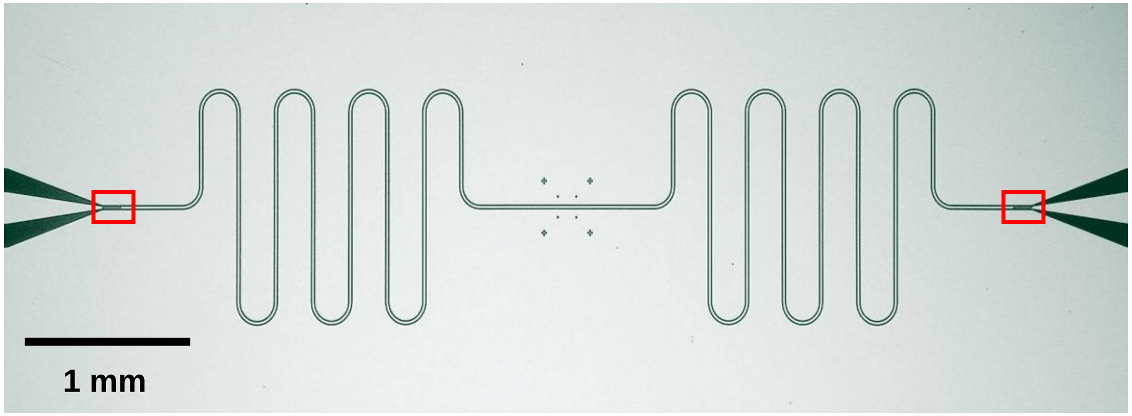
photon decay rate:

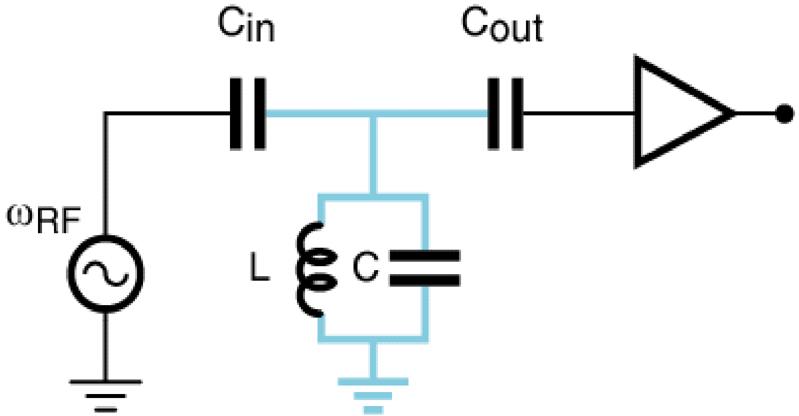
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

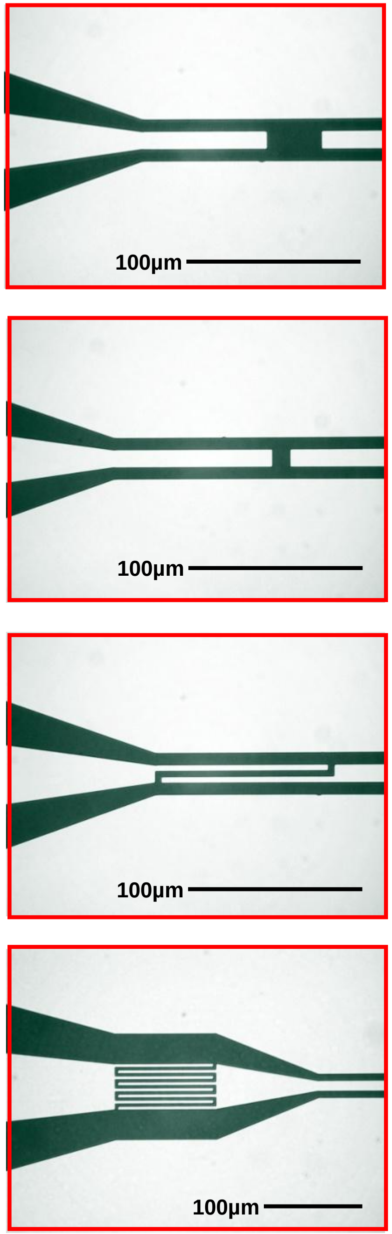
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Controlling the Photon Life Time

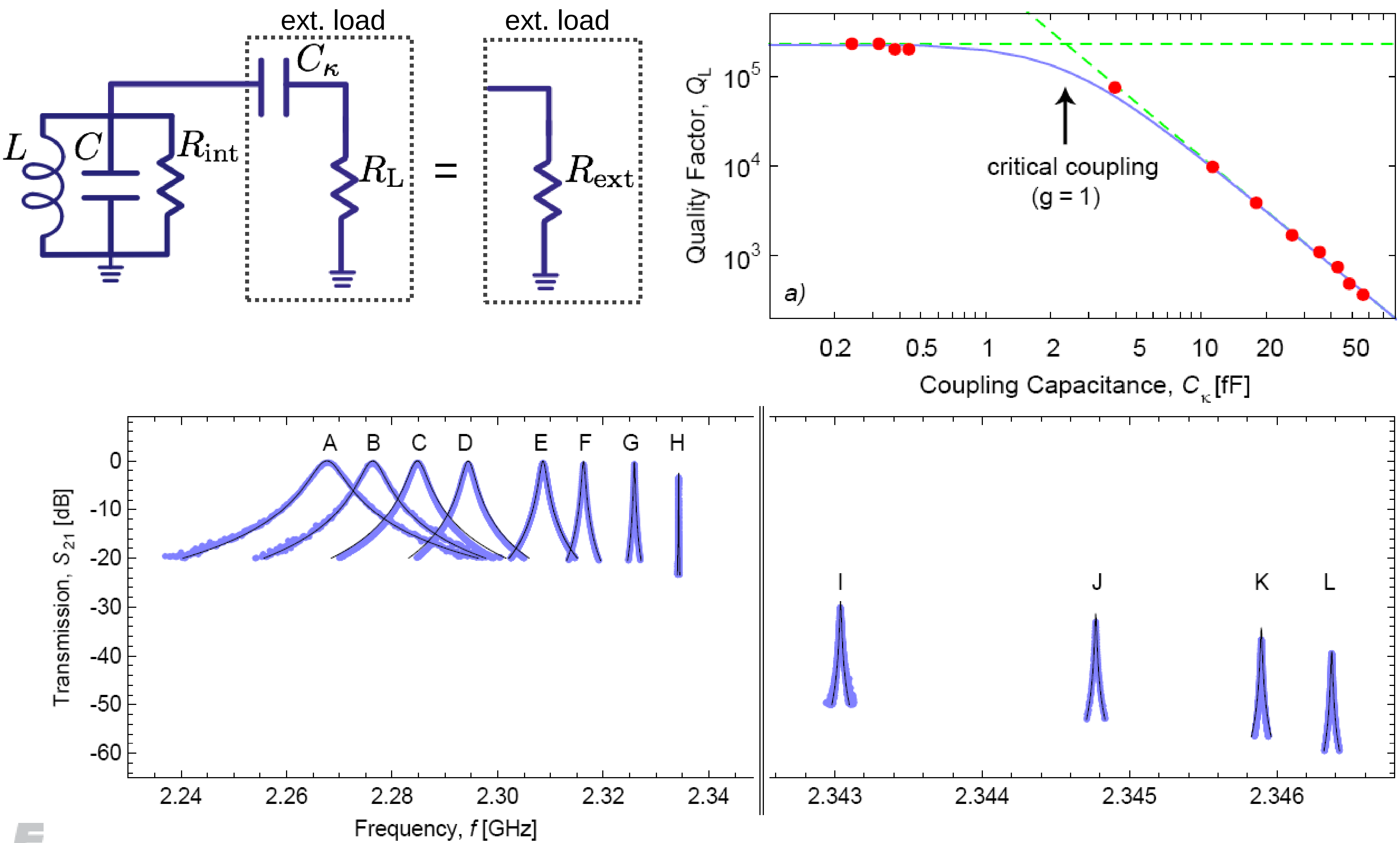






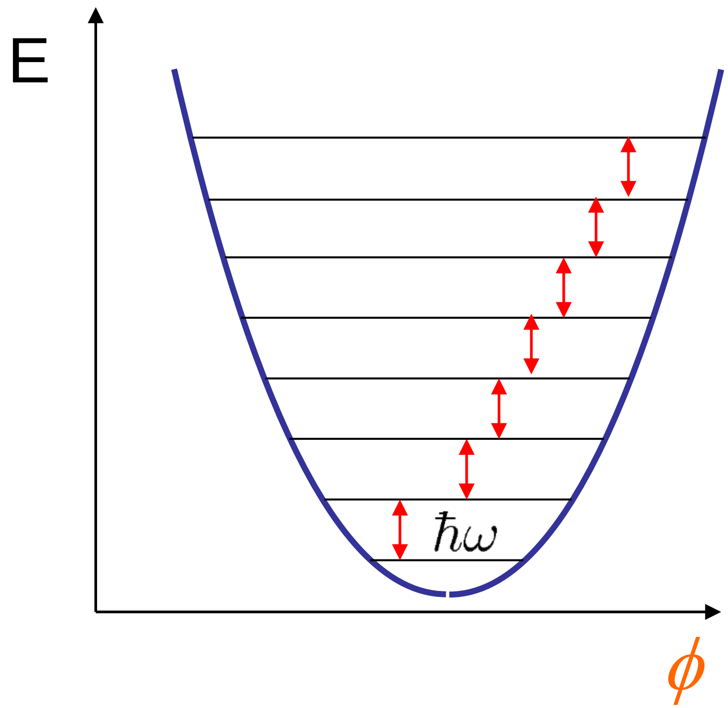
photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Quality Factor Measurement



M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\mathrm{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK

20 mK

$$\langle n_{\mathrm{th}} \rangle \sim 10^{-11}$$

How to Prove that a Harmonic Oscillator is Quantum?

measure:

- resonance frequency
- average charge (momentum)
- average flux (position)

all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

solution:

- make oscillator non-linear in a controllable way