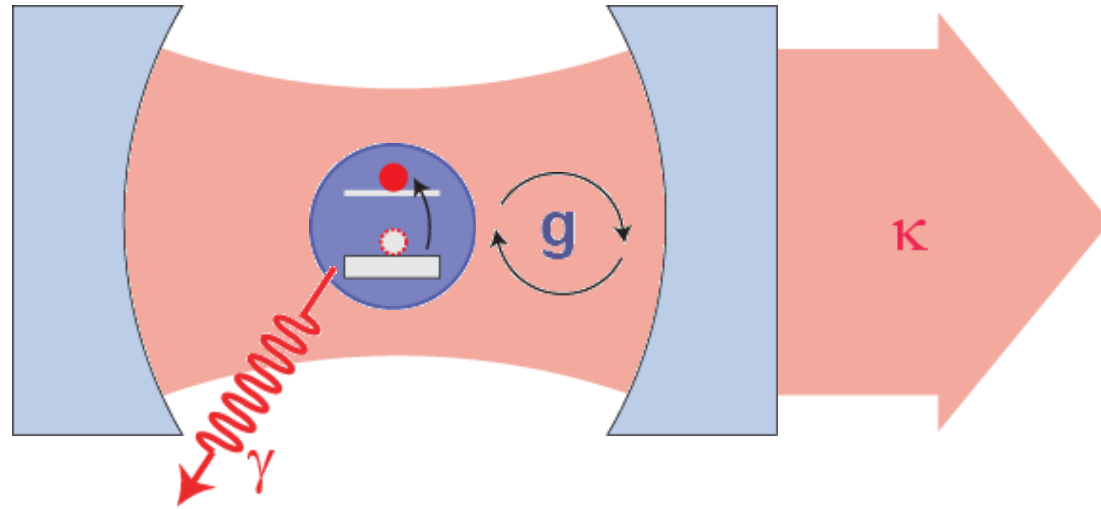


Coupling a Harmonic Oscillator to a Qubit

Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}}$)

Dressed States Energy Level Diagram

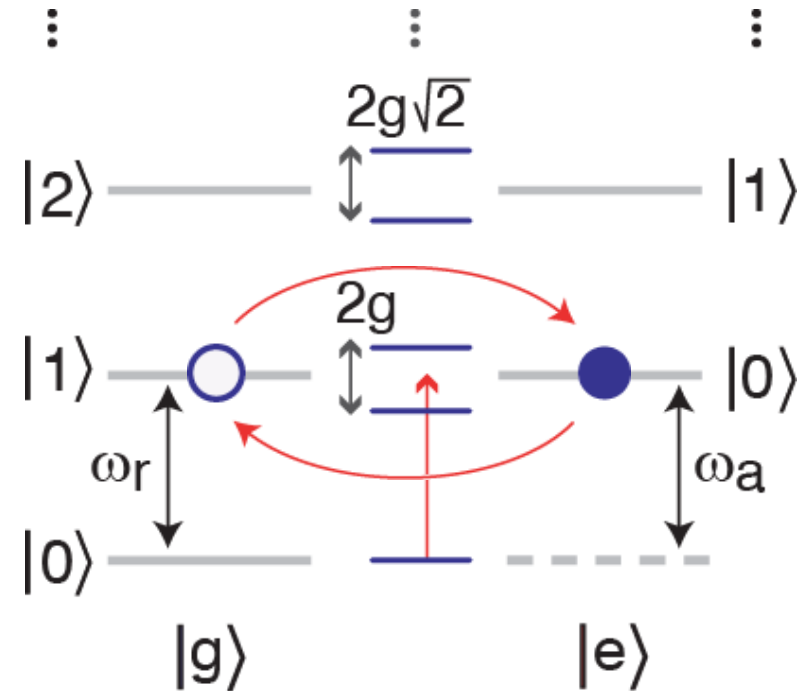
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



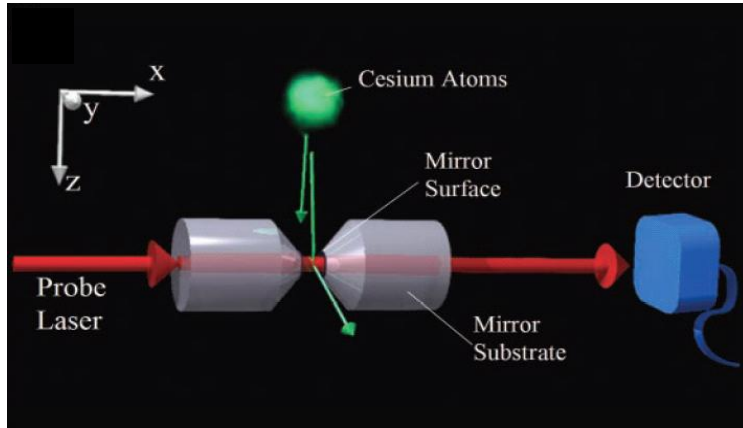
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

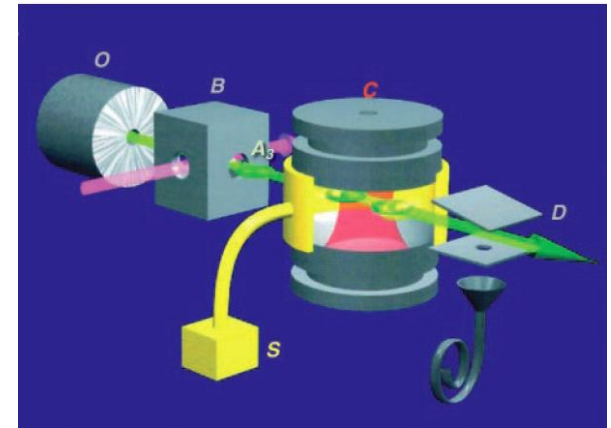
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

S. Haroche & J. Raimond, *Exploring the Quantum*, OUP Oxford (2006)

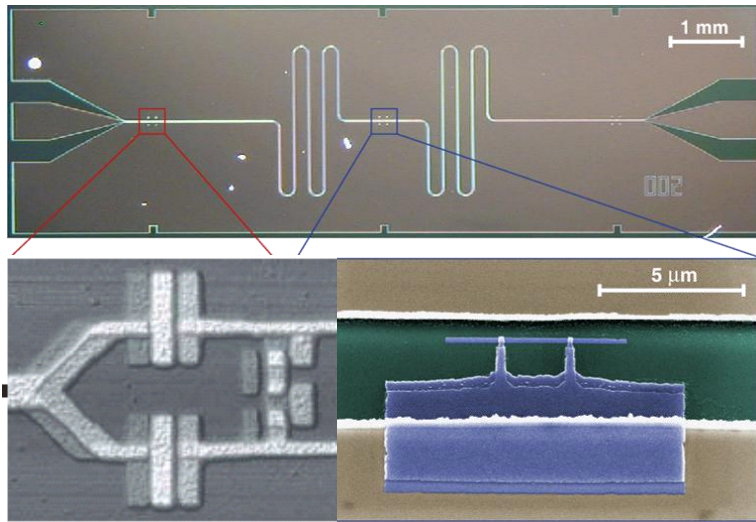
Cavity Quantum Electrodynamics (QED)



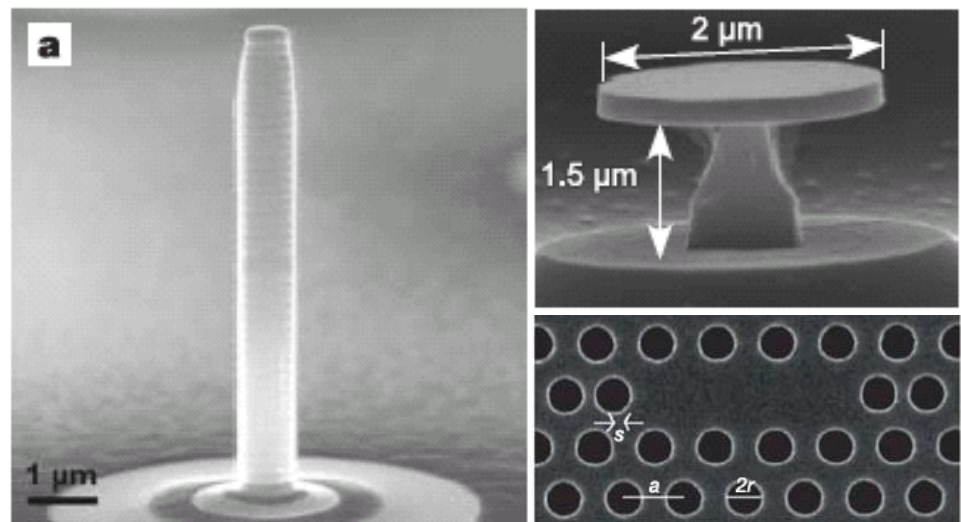
alkali atoms
MPQ, Caltech, ...



Rydberg atoms
ENS, MPQ, ...

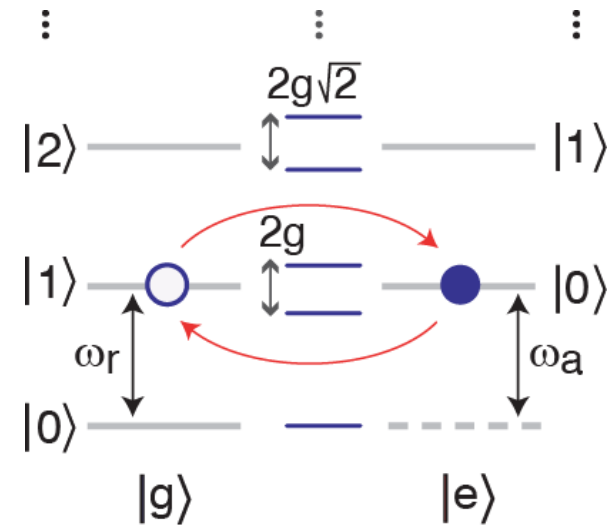
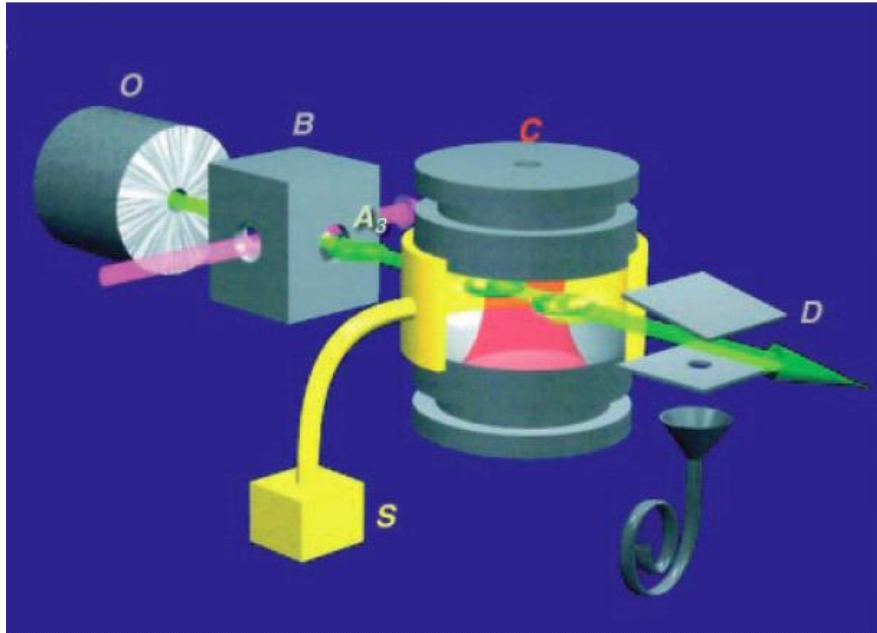


superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, ...

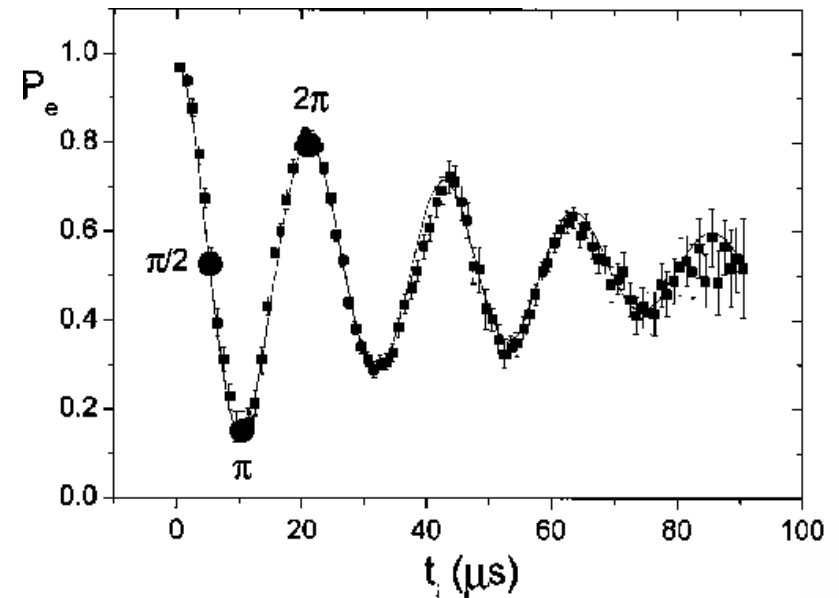


semiconductor quantum dots
Wurzburg, ETHZ, Stanford ...

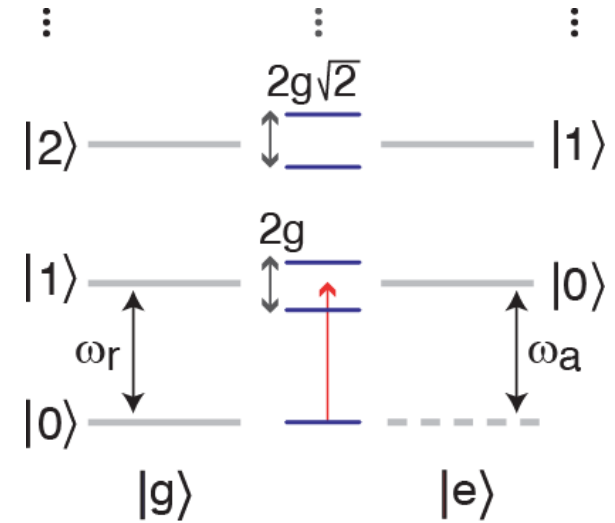
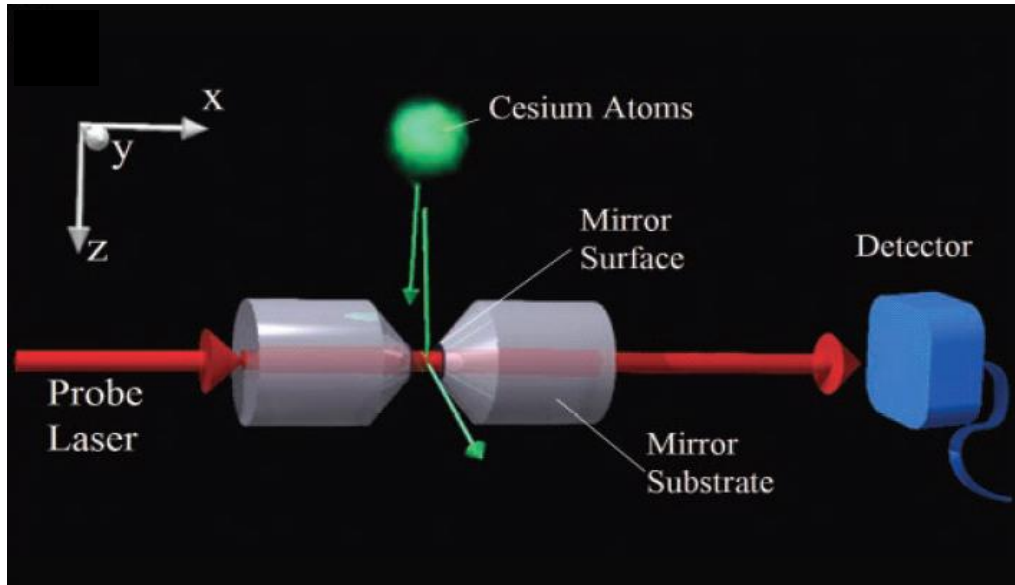
Vacuum Rabi Oscillations with Rydberg Atoms



Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
 P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)

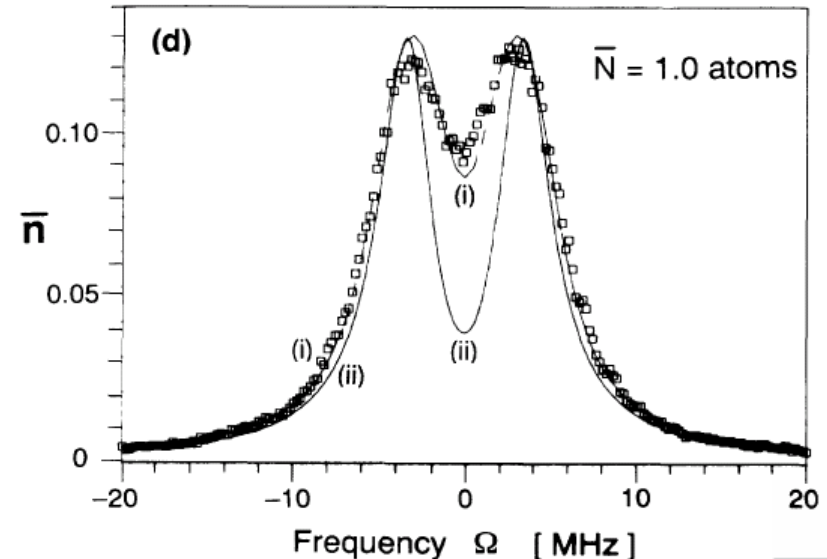


Vacuum Rabi Mode Splitting with Alkali Atoms

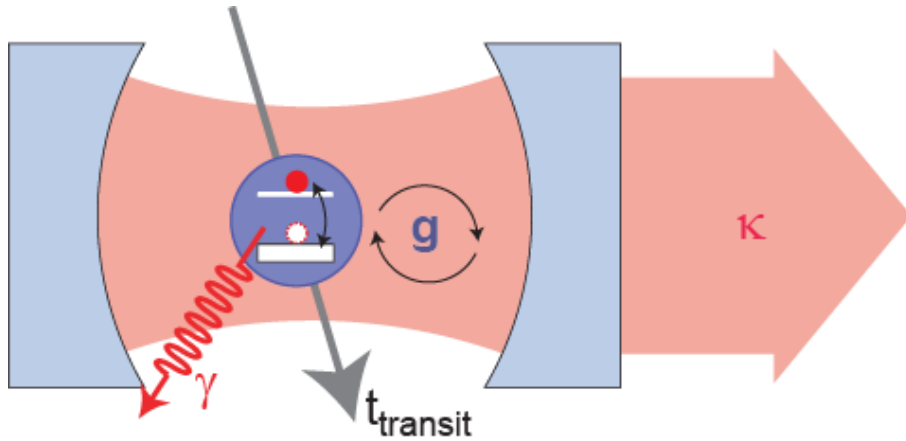


R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)

A. Boca, ... , J. McKeever, & H. J. Kimble
Phys. Rev. Lett. **93**, 233603 (2004)



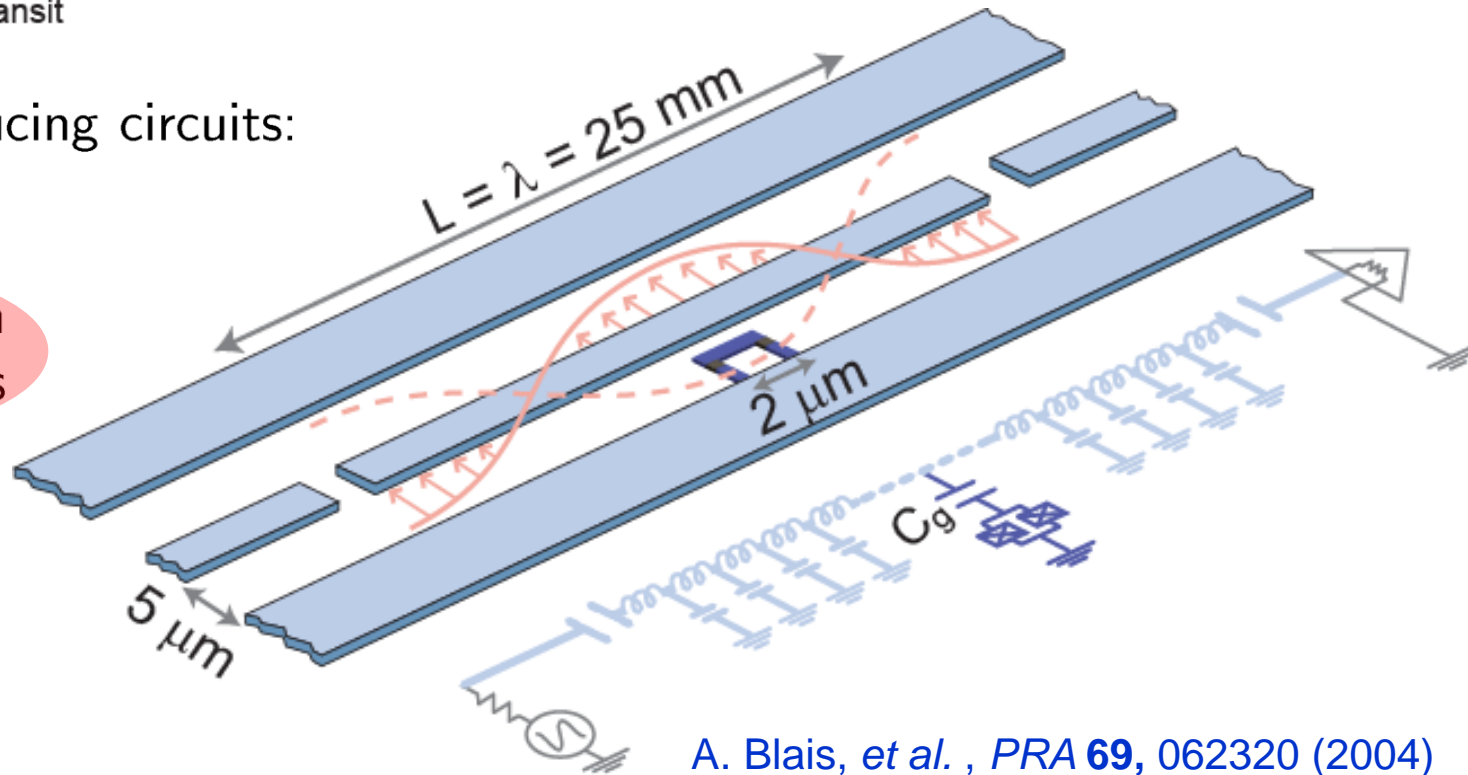
Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

... in superconducting circuits:

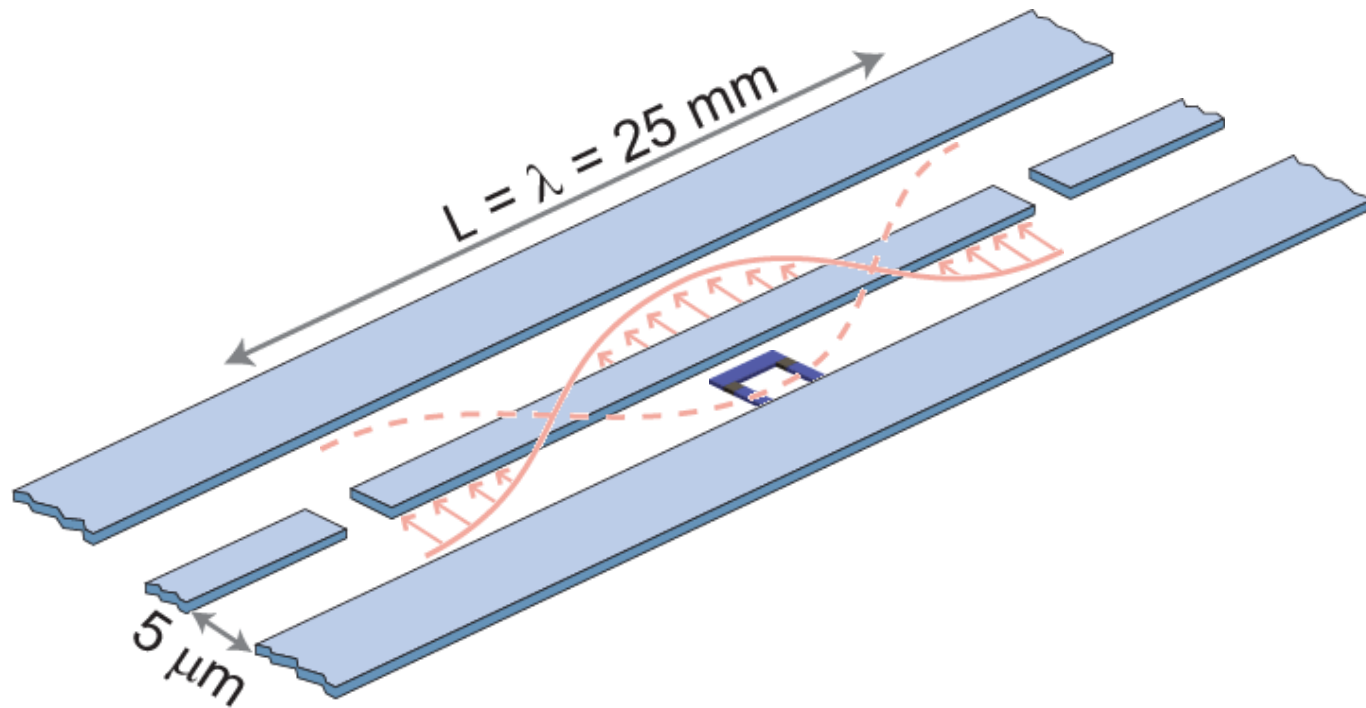
circuit quantum
electrodynamics



A. Blais, *et al.*, *PRA* **69**, 062320 (2004)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

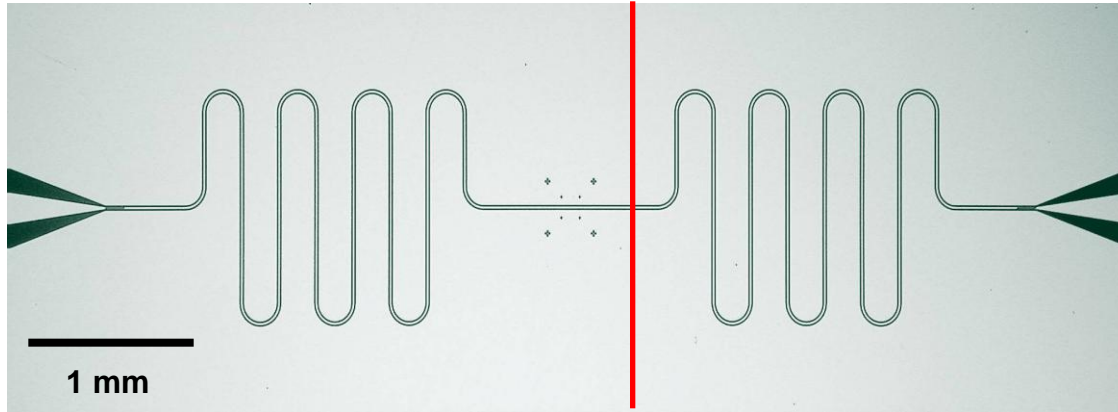
Circuit Quantum Electrodynamics



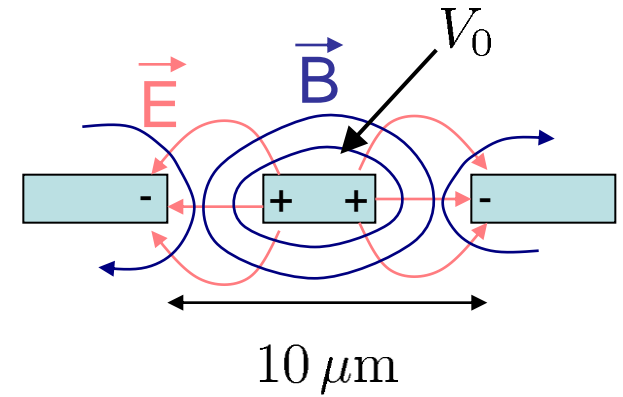
elements

- the cavity: a superconducting 1D transmission line resonator with **large vacuum field** E_0 and **long photon life time** $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with **large dipole moment** d and **long coherence time** $1/\gamma$

Vacuum Field in 1D Cavity



cross-section
of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

$$V_{0,\text{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1 \mu\text{V}$$

harmonic oscillator

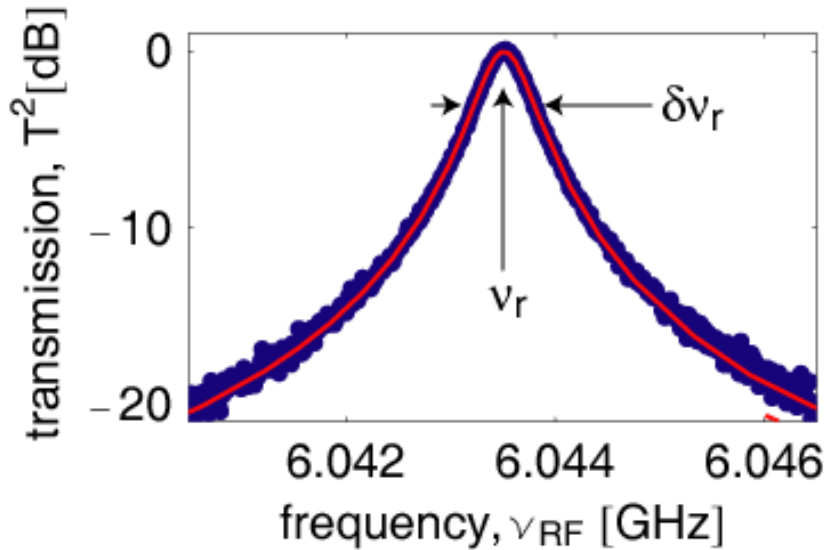
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

$$E_0 = \frac{V_{0,\text{rms}}}{b} \approx 0.2 \text{ V/m}$$

$\times 10^6$ larger than E_0
in 3D microwave cavity

for $\omega_r/2\pi \approx 6 \text{ GHz}$ ($C \sim 1 \text{ pF}$), $b \approx 5 \mu\text{m}$

Resonator Quality Factor and Photon Lifetime

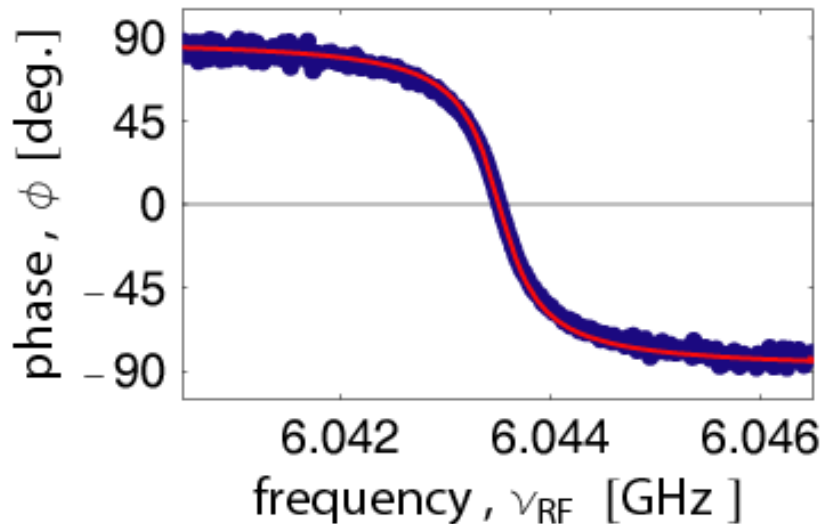


resonance frequency:

$$\nu_r = 6.04 \text{ GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$



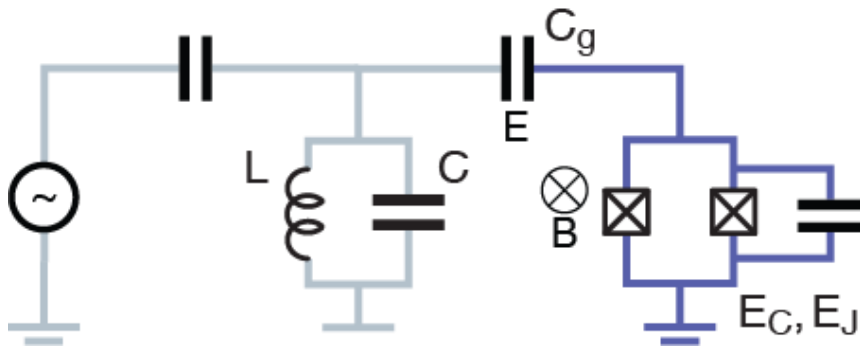
photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8 \text{ MHz}$$

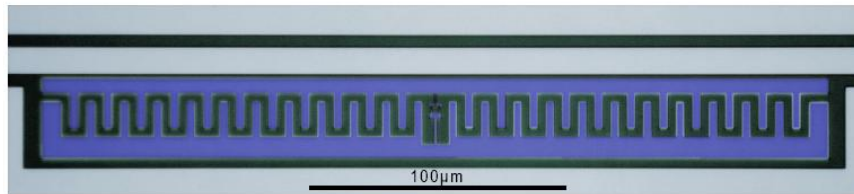
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200 \text{ ns}$$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = eV_{0,\text{rms}} \frac{C_g}{C_\Sigma}$$

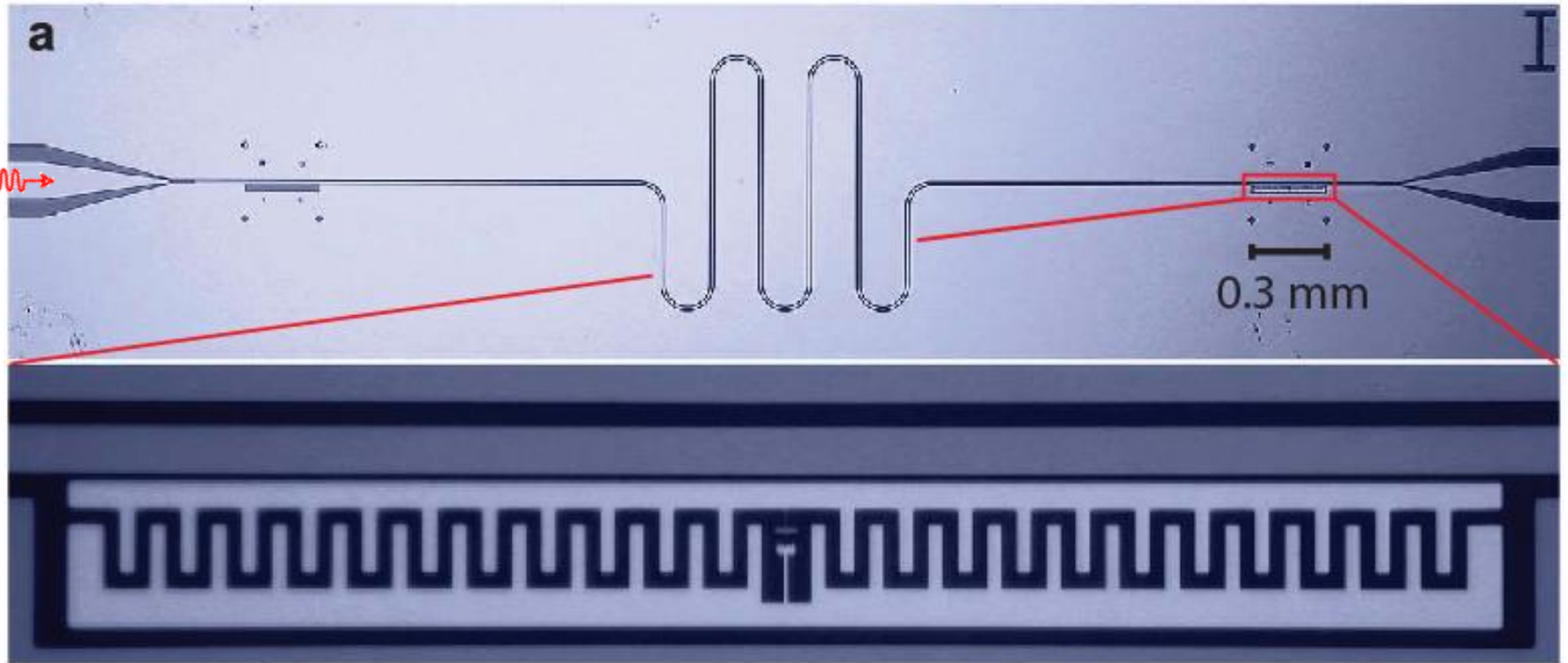
$$\Rightarrow \nu_{\text{vac}} = \frac{g}{\pi} \approx 1 \dots 300 \text{ MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \dots 10^4 ea_0$$

Circuit QED with One Photon



superconducting cavity QED circuit

Resonant Vacuum Rabi Mode Splitting ...

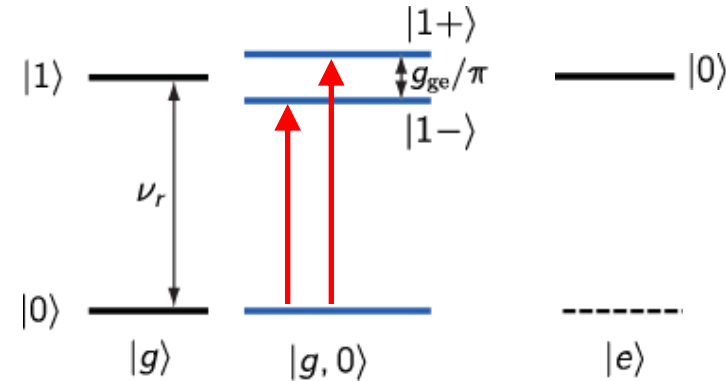
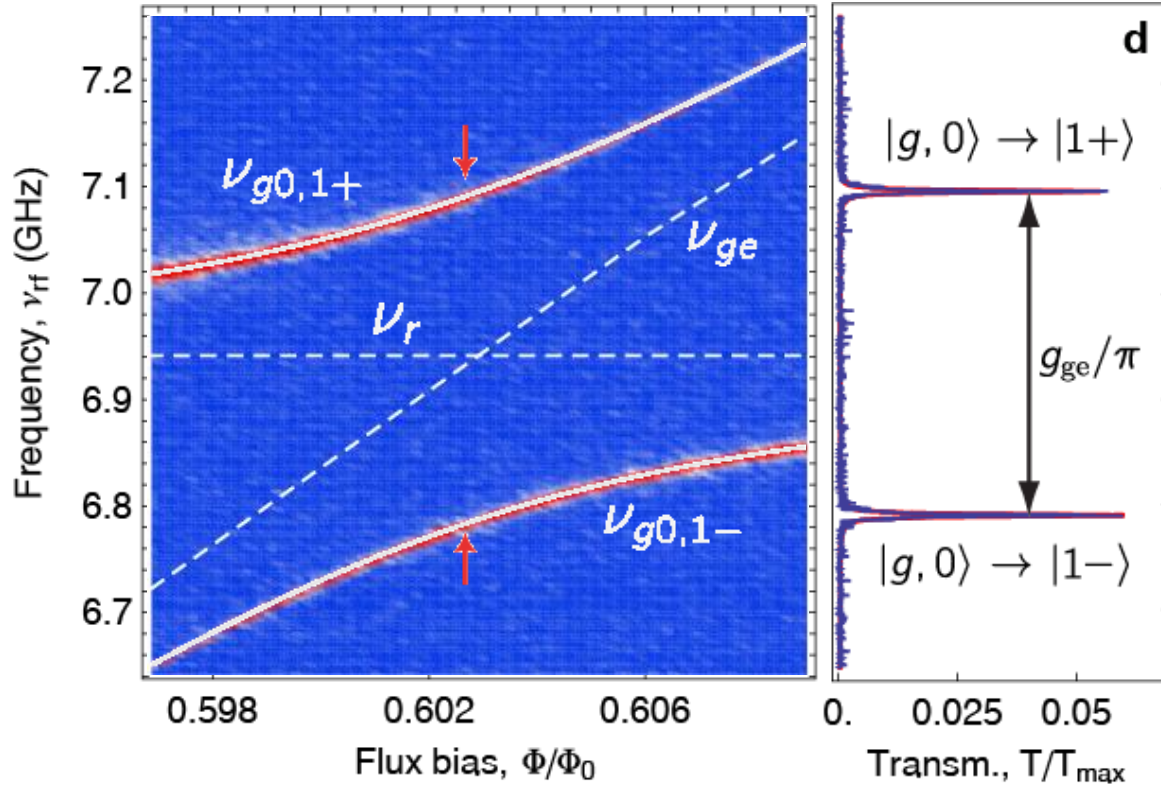
... with one photon ($n = 1$):

very strong coupling:

$$g_{ge}/\pi = 308 \text{ MHz}$$

$$\kappa, \gamma < 1 \text{ MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



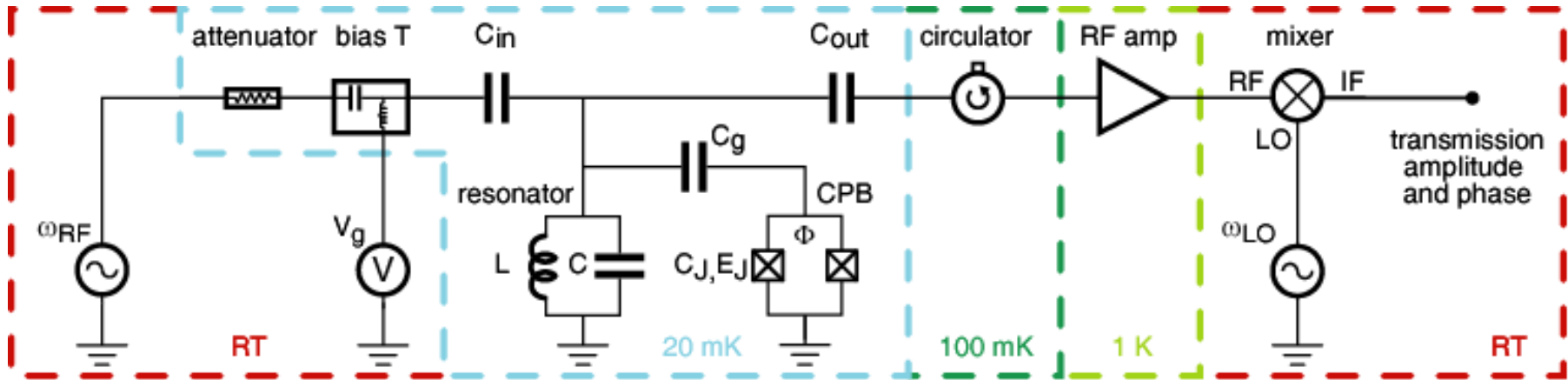
forming a 'molecule' of a qubit and a photon

$$|1\pm\rangle = (|g, 1\rangle \pm |e, 0\rangle) / \sqrt{2}$$

How to Measure Single Microwave Photons

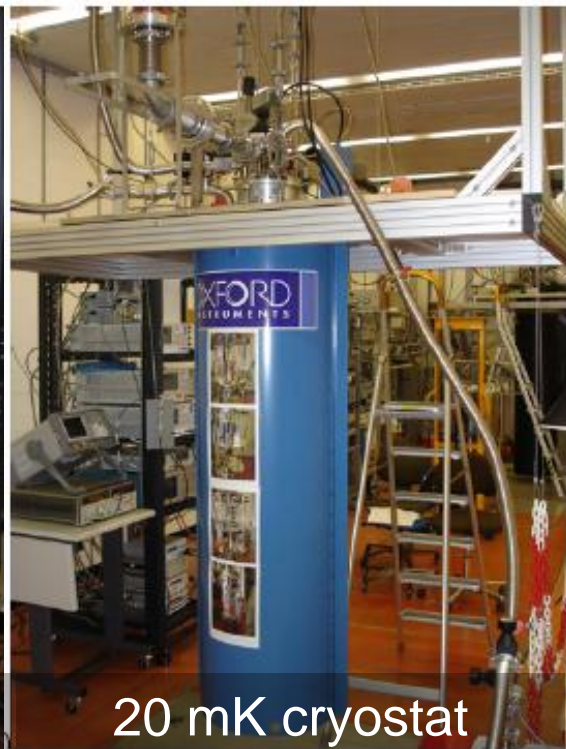
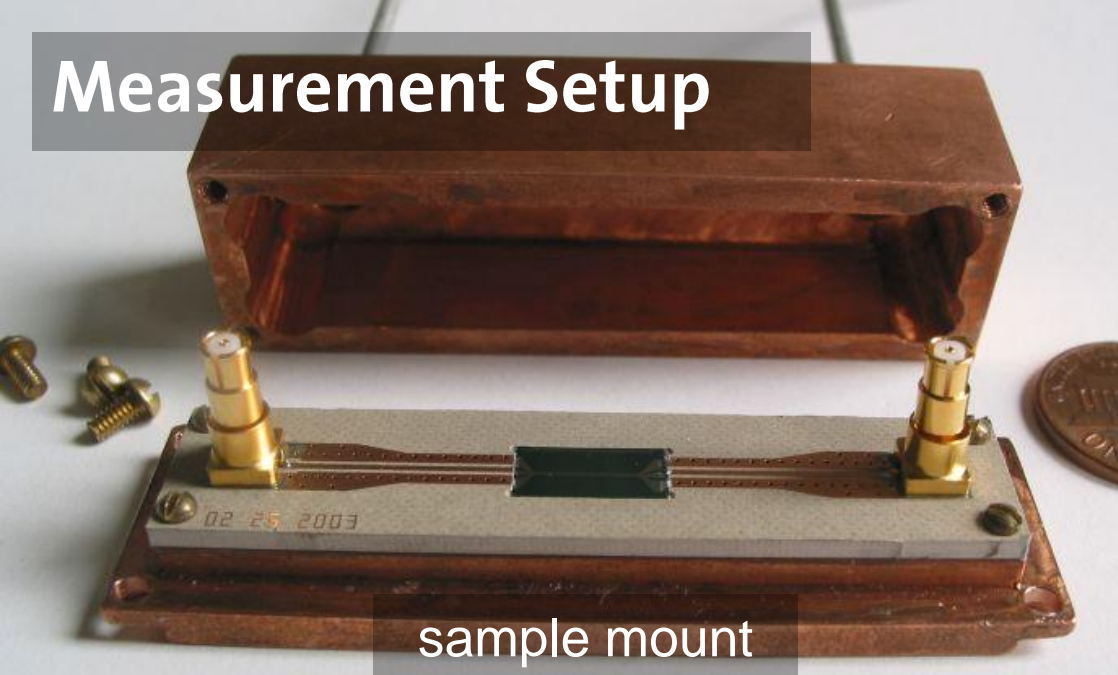
- average power to be detected

$$\rightarrow \langle n = 1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140 \text{ dBm} = 10^{-17} \text{ W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6 \text{ K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)

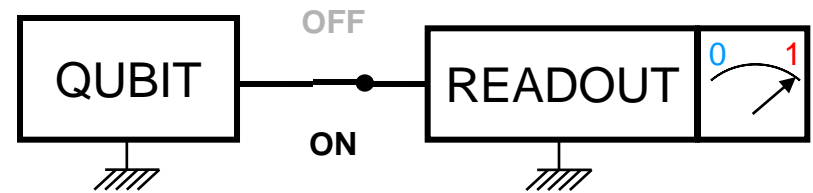
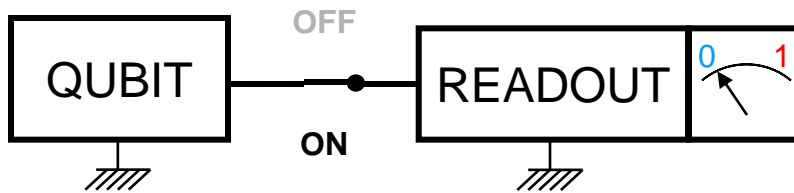
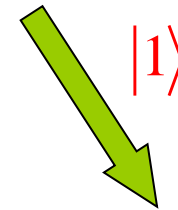
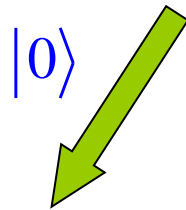
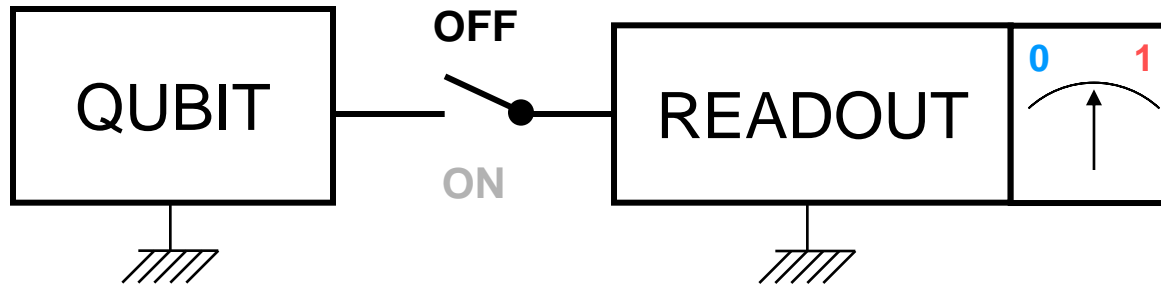
Measurement Setup



Read-Out ...

... of a superconducting charge qubit

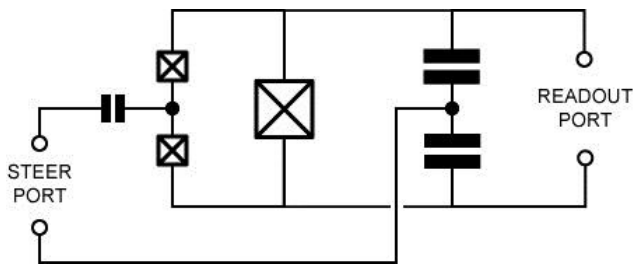
Qubit Read Out



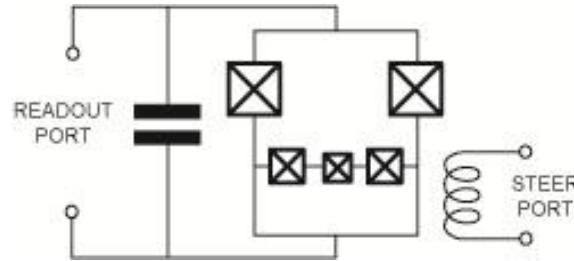
desired: good on/off ratio
no relaxation in on state (QND)

Read Out Strategies

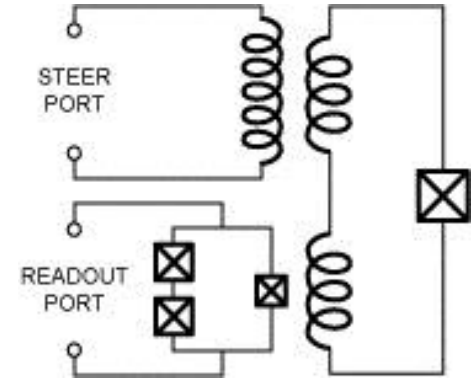
demolition measurements (switching/latching measurements)



Quantronium (Saclay, Yale)

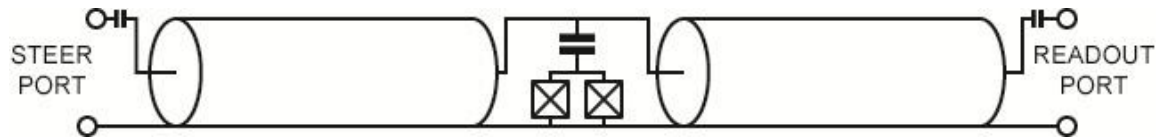


Flux Qubit (TU Delft, NEC)



Phase Qubit (NIST, UCSB)

quantum non-demolition (QND) measurements



Yale (circuit QED)

also: Chalmers, Delft, Yale (JBA)

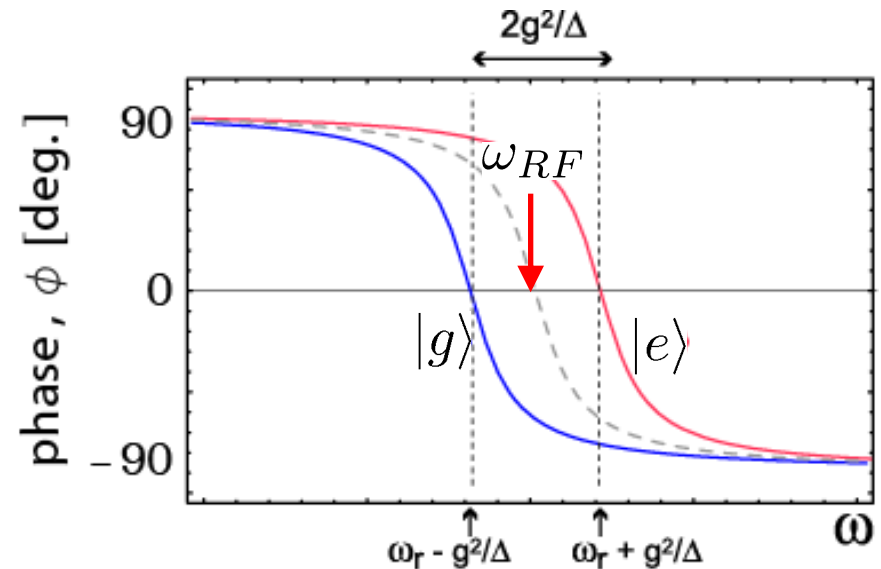
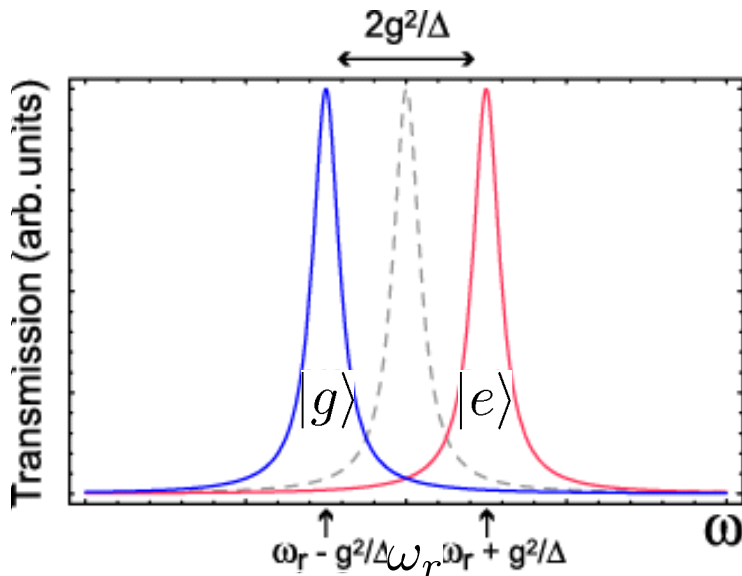
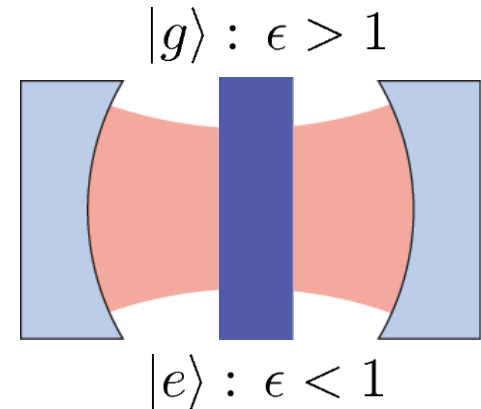
Non-Resonant Qubit-Photon Interaction

approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

//
cavity frequency shift
and qubit ac-Stark shift

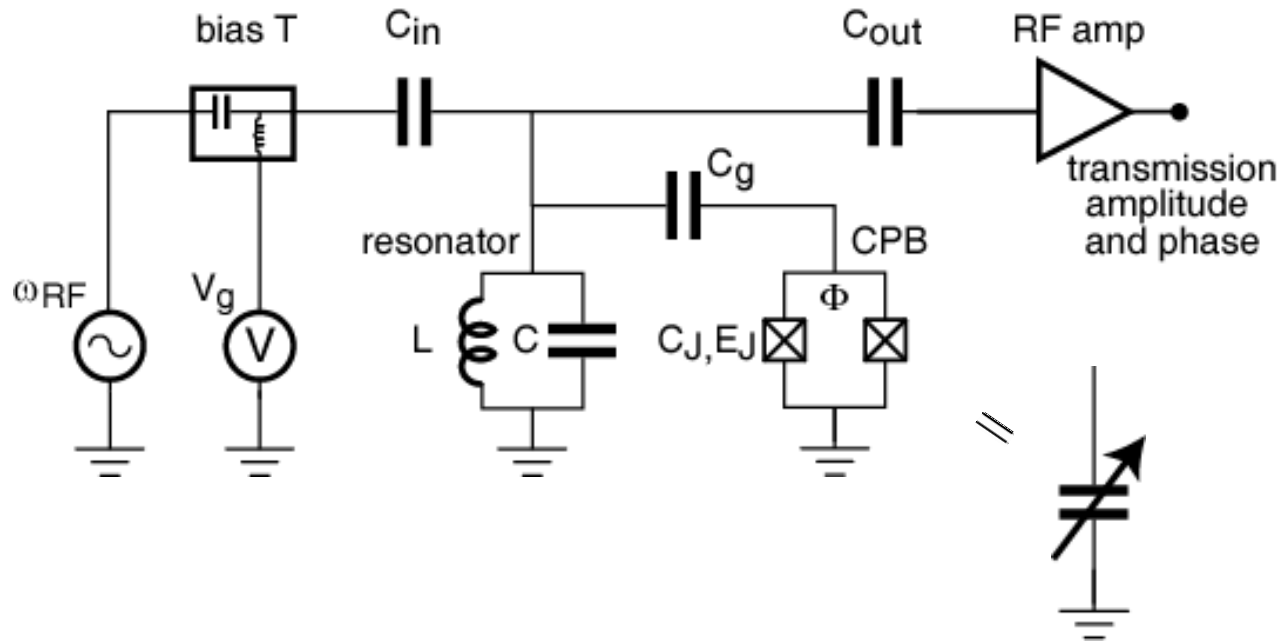
//
Lamb shift



Qubit Spectroscopy with Dispersive Read-Out ...

... additional material

Measurement Technique

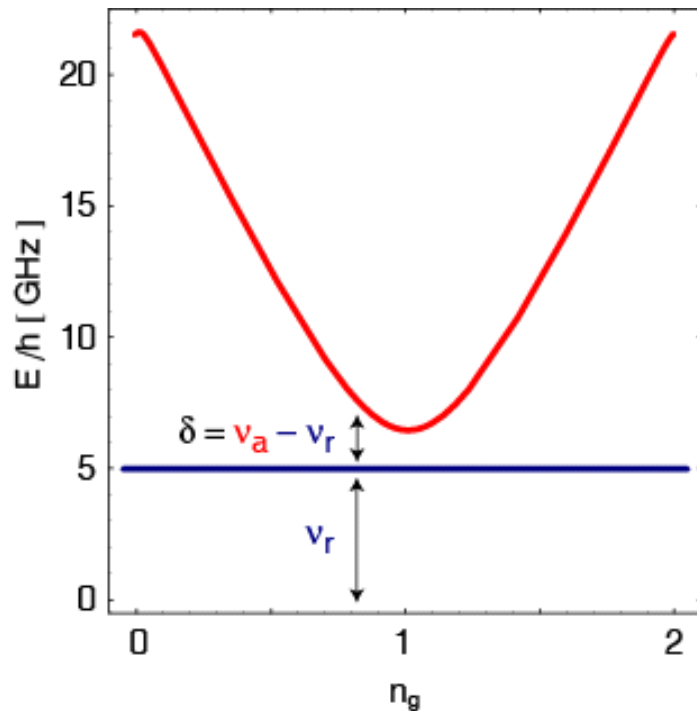


- measurement of microwave transmission amplitude T and phase ϕ
- intra-cavity photon number controllable from $n \sim 10^3$ to $n \ll 1$

Dispersive Shift of Resonance Frequency

sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

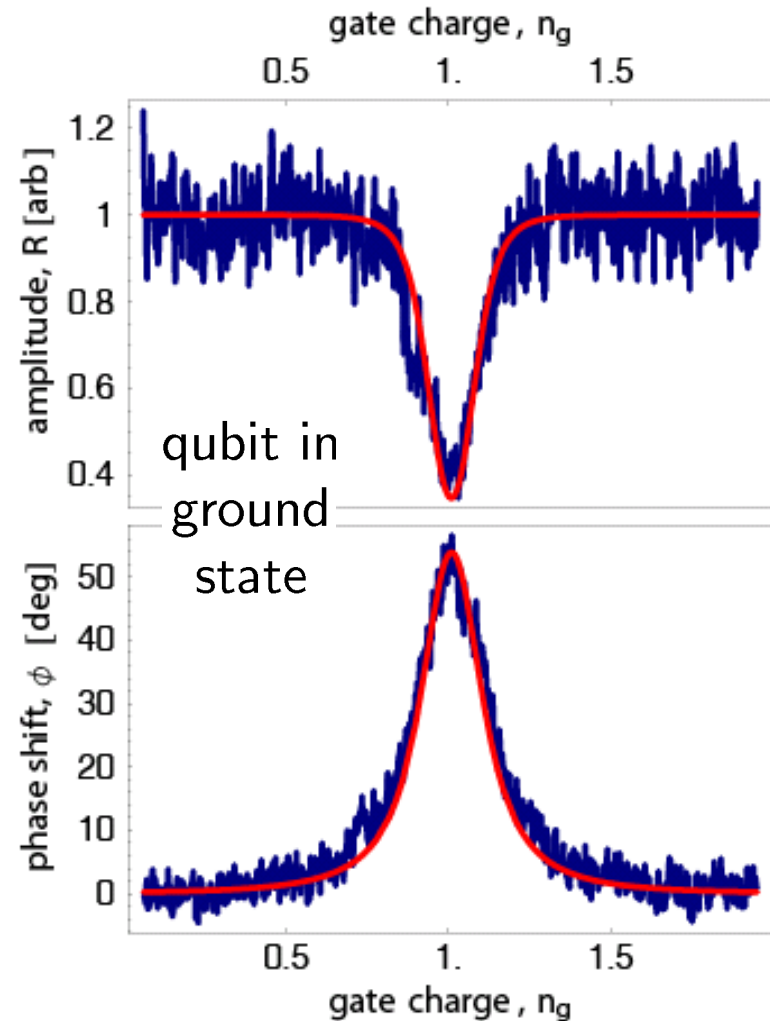


$$g/\pi = \nu_{\text{vac}} = 11 \text{ MHz}$$

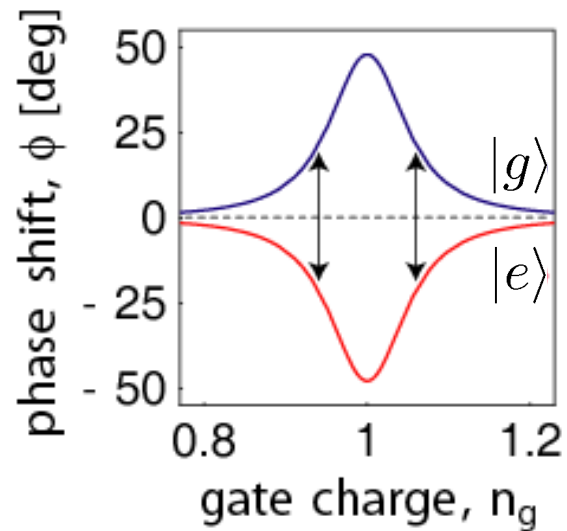
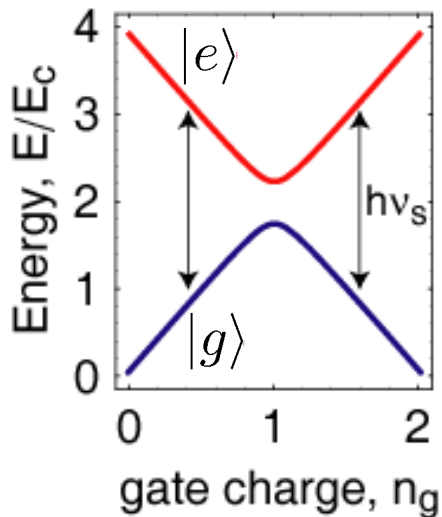
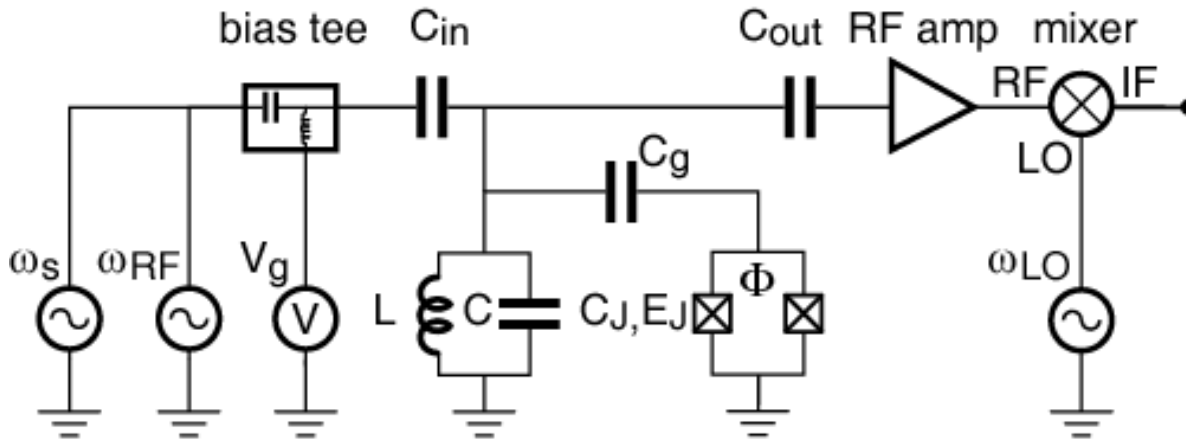
$$\Delta(n_g = 1)/2\pi = 66 \text{ MHz}$$

$$n = 10$$

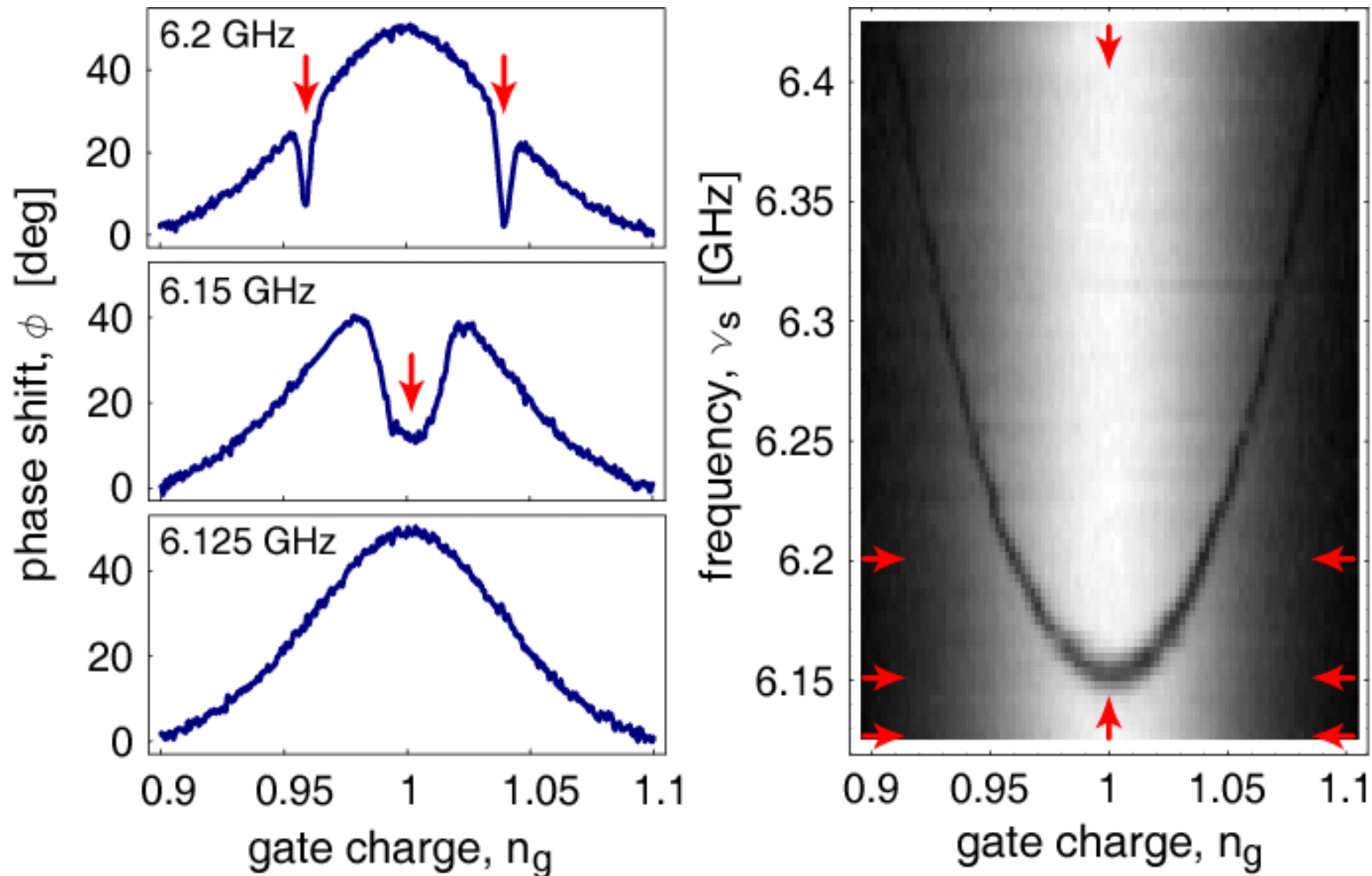
measured resonator transmission amplitude and phase:



Qubit Spectroscopy with Dispersive Read-Out



CW Spectroscopy of Cooper Pair Box



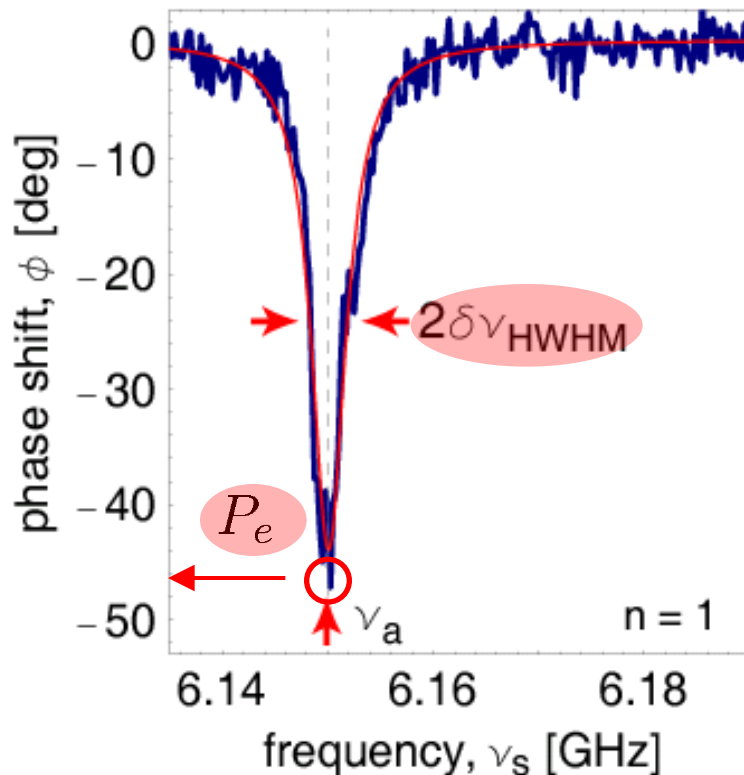
detuning $\Delta_{r,a}/2\pi \sim 100$ MHz

extracted: $E_J = 6.2$ GHz, $E_C = 4.8$ GHz

Line Shape

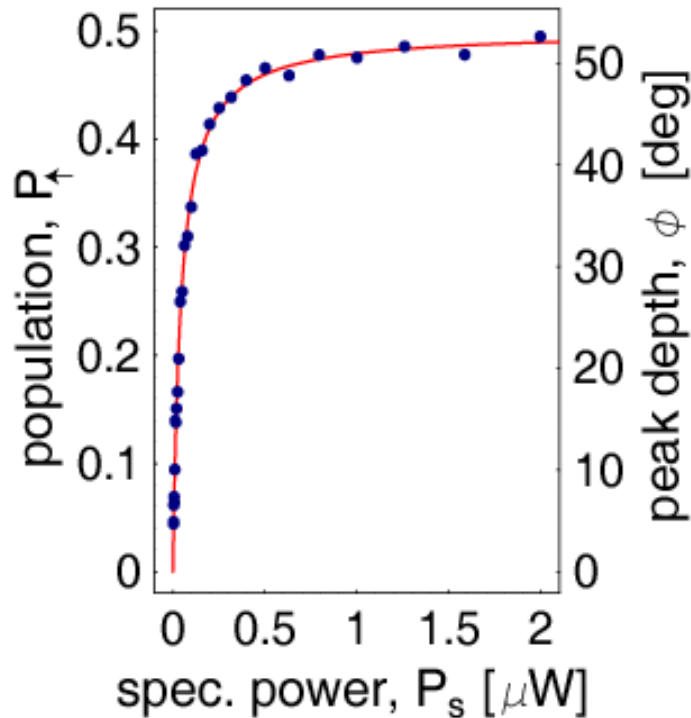
excited state population (steady-state Bloch equations):

$$P_e = 1 - P_g = \frac{1}{2} \frac{\Omega_R^2 T_1 T_2}{1 + (T_2 \Delta_{s,a})^2 + \Omega_R^2 T_1 T_2}$$



- fixed drive $P_s \propto \Omega_R^2 = n_s \omega_{\text{vac}}^2$
- varying $\Delta_{s,a} = \omega_s - \tilde{\omega}_a$
- weak continuous measurement ($n \sim 1$)
- at charge degeneracy ($n_g = 1$)

Excited State Population

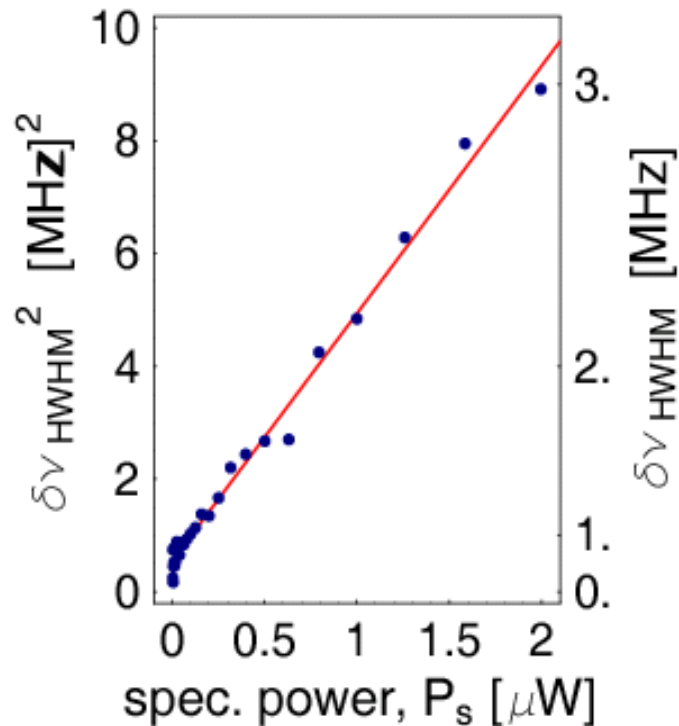


peak depth \rightarrow population (saturation):

$$P_e = 1 - P_g = \frac{1}{2} \frac{\Omega_R^2 T_1 T_2}{1 + \Omega_R^2 T_1 T_2}$$

D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Girvin, and R. J. Schoelkopf, *Phys. Rev. Lett.* **94**, 123062 (2005)

Line Width



line width \rightarrow coherence time:

$$2\pi\delta\nu_{\text{HWHM}} = \frac{1}{T'_2} = \sqrt{\frac{1}{T_2^2} + \Omega_R^2 \frac{T_1}{T_2}}$$

$\text{Min}(\delta\nu_{\text{HWHM}}) \sim 750 \text{ kHz} \rightarrow T_2 > 200 \text{ ns}$