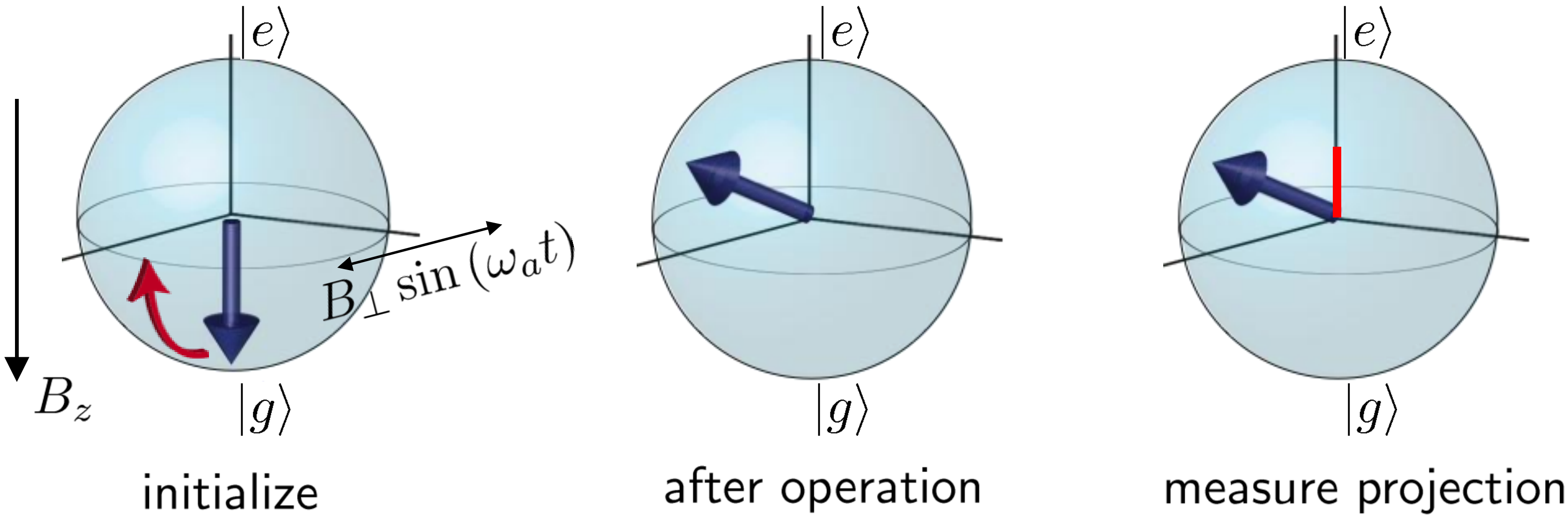
Single qubit control.

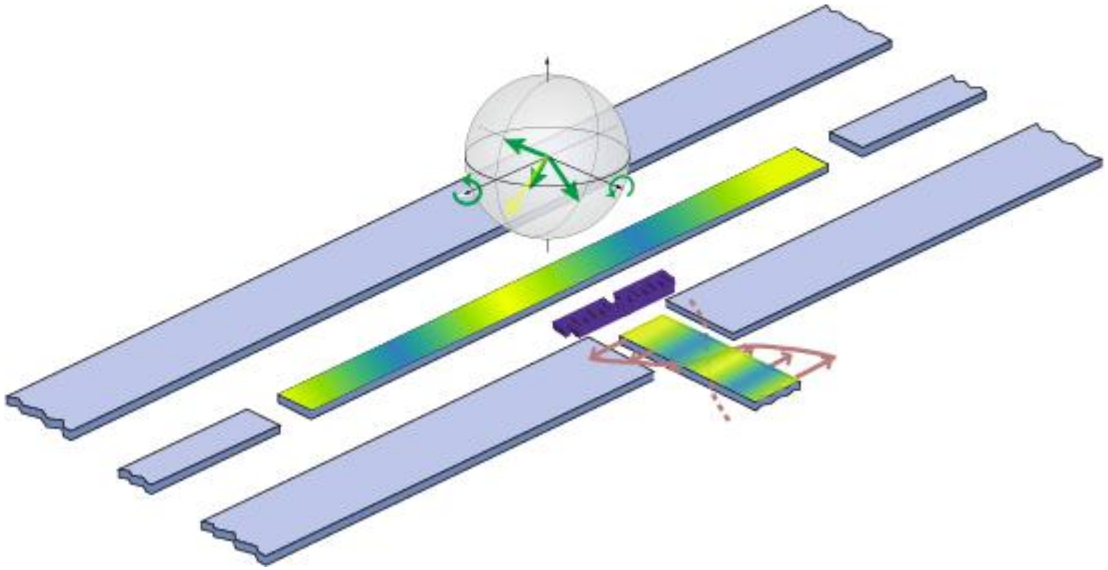
Coherent Control and Measurement



- qubit state represented on a Bloch sphere
- vary length, amplitude and phase of microwave pulse to control qubit state

Qubit control

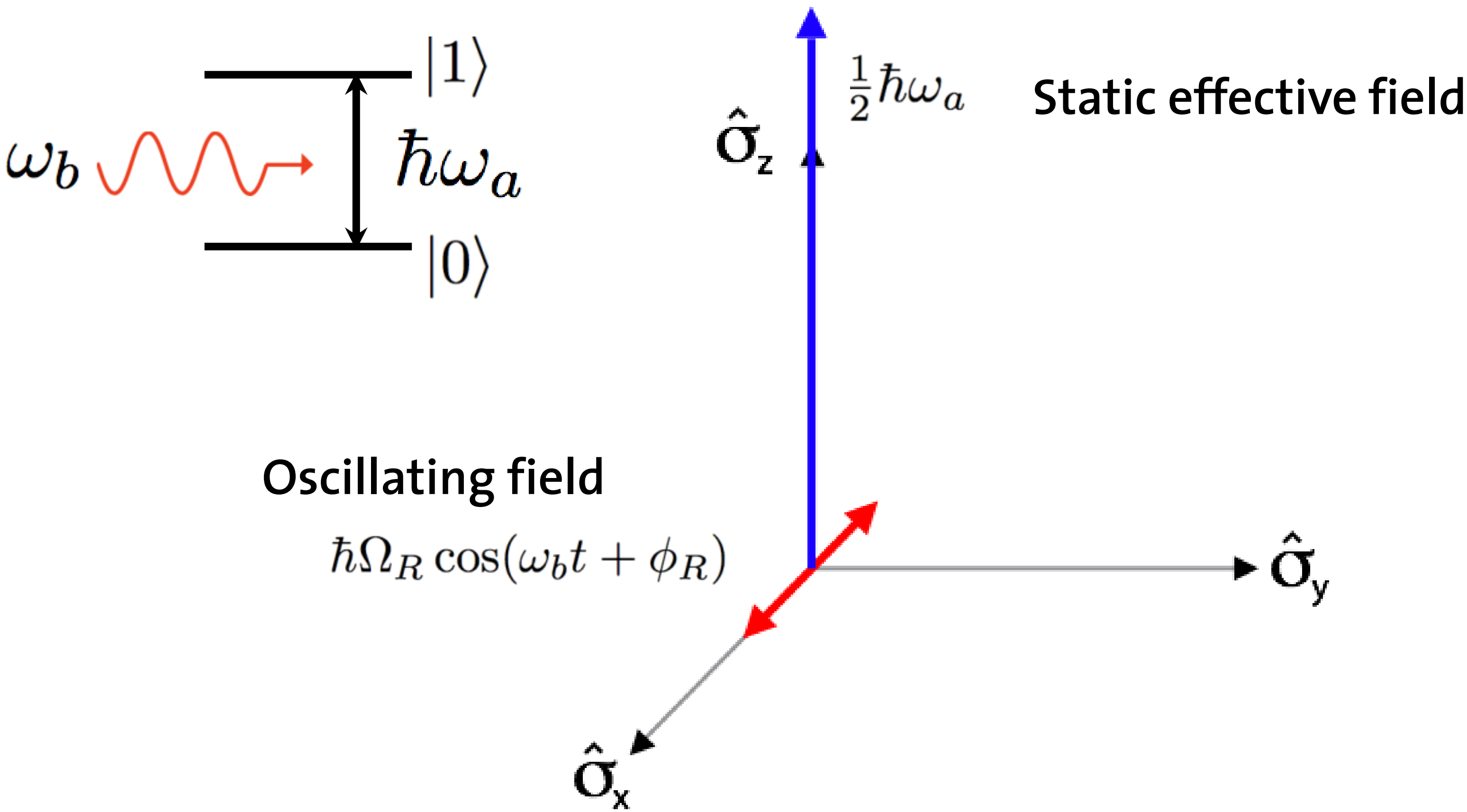
- apply microwave signal through resonator input
- or through side-gate



- time-dependent Hamiltonian for state manipulation

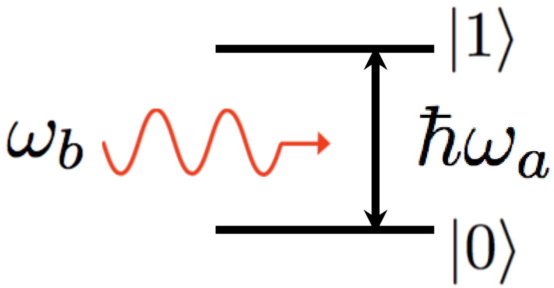
$$\hat{H} = \frac{1}{2}\hbar\omega_a\hat{\sigma}_z + \hbar\Omega_R\cos(\omega_b t + \phi_R)\hat{\sigma}_x$$

Qubit with dipole coupled electric field

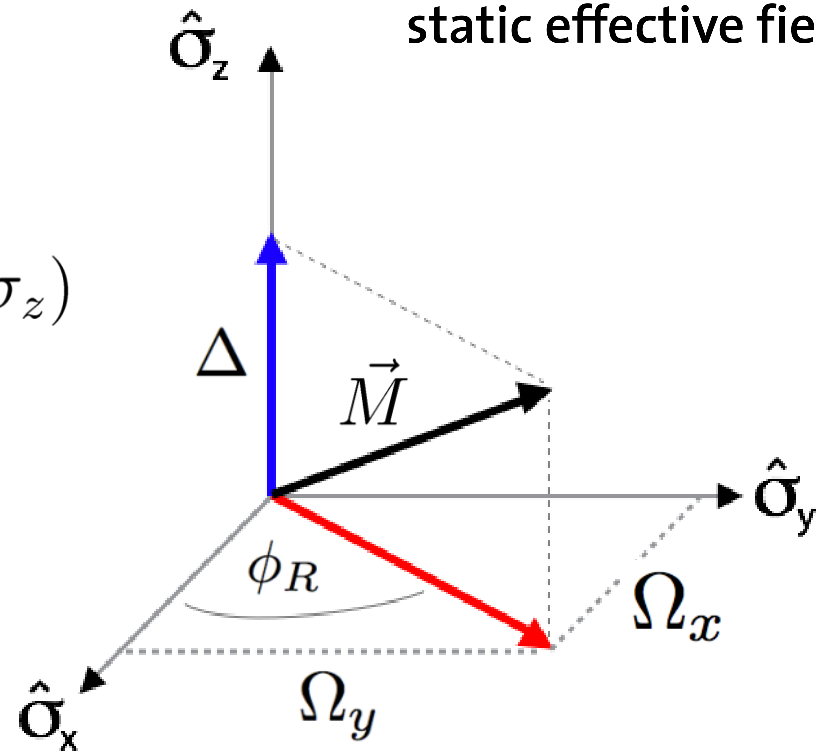


$$\hat{H} = \frac{1}{2}\hbar\omega_a\hat{\sigma}_z + \hbar\Omega_R\cos(\omega_b t + \phi_R)\hat{\sigma}_x$$

Rotating frame



Rotating frame:
static effective fields



$$H' = \frac{\hbar}{2}(\Omega_x\sigma_x + \Omega_y\sigma_y + \Delta\sigma_z)$$

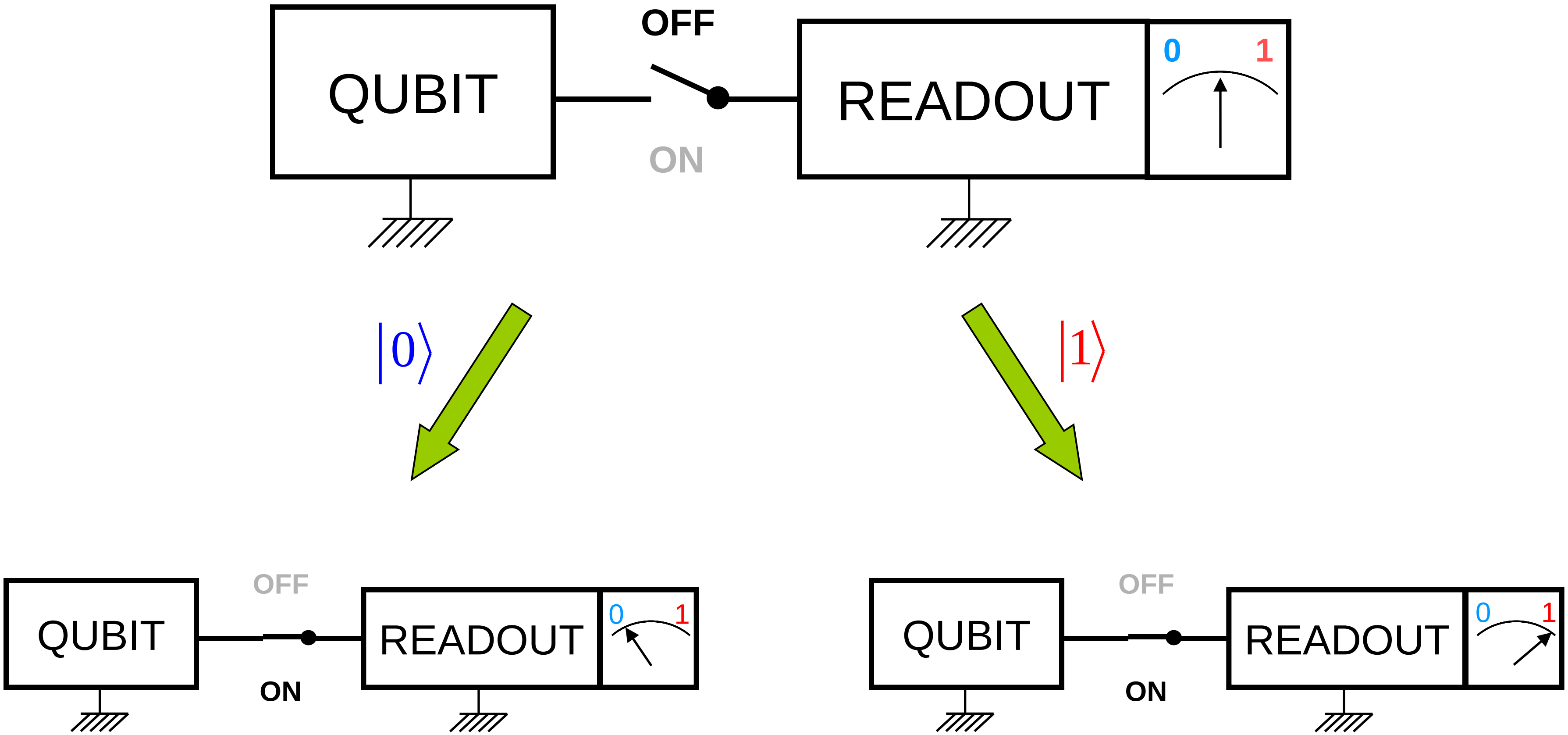
$$\equiv \frac{\hbar}{2}\vec{M}\cdot\vec{\sigma}$$

$$\vec{M} = (\Omega_x, \Omega_y, \Delta)$$

$$\Delta = \omega_a - \omega_b$$

Dispersive readout in circuit QED.

Qubit Read Out



desired: good on/off ratio
no relaxation in on state (QND)

Dressed States Energy Level Diagram

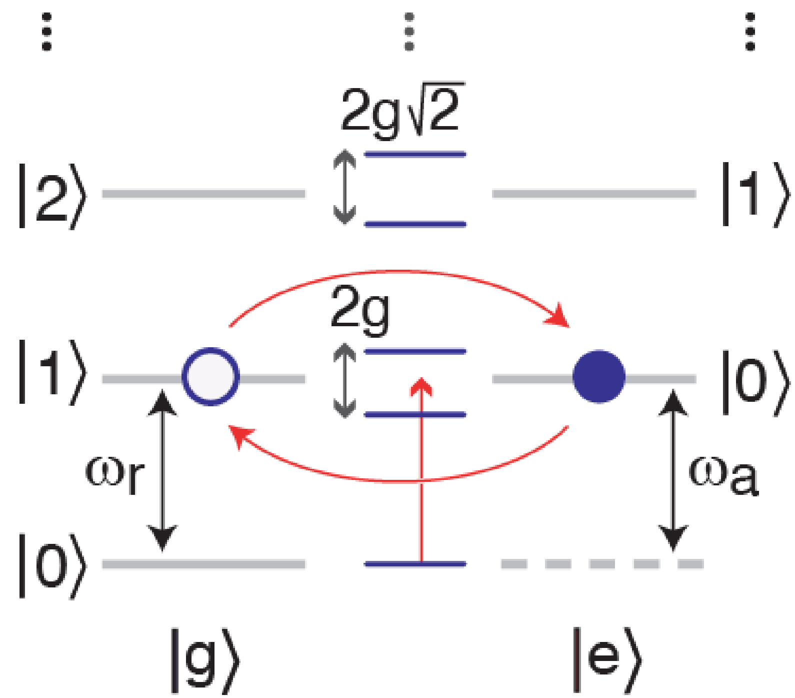
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$$



Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Non-Resonant (Dispersive) Interaction

approximate diagonalization: $|\Delta| = |\omega_a - \omega_r| \gg g$:

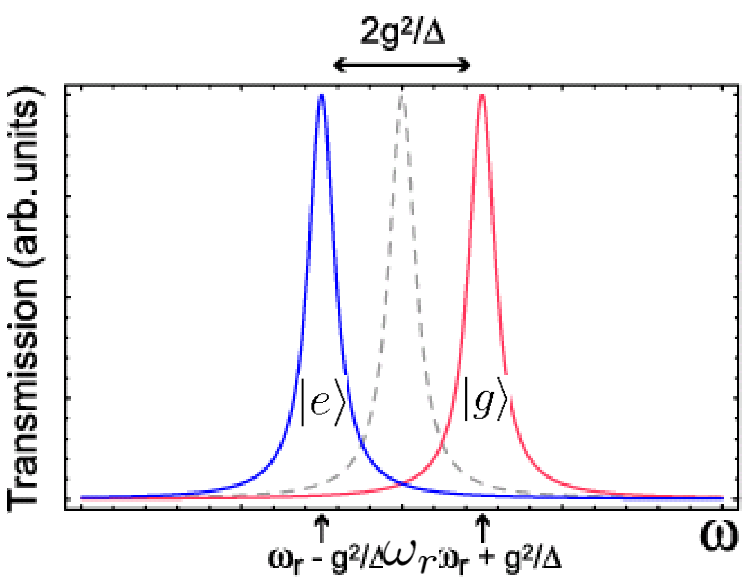
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{\hbar}{2}\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

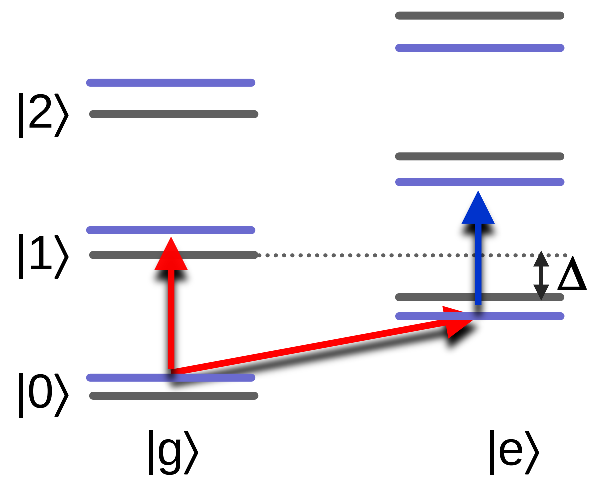
//

cavity frequency shift
and qubit ac-Stark shift

//

Lamb Shift



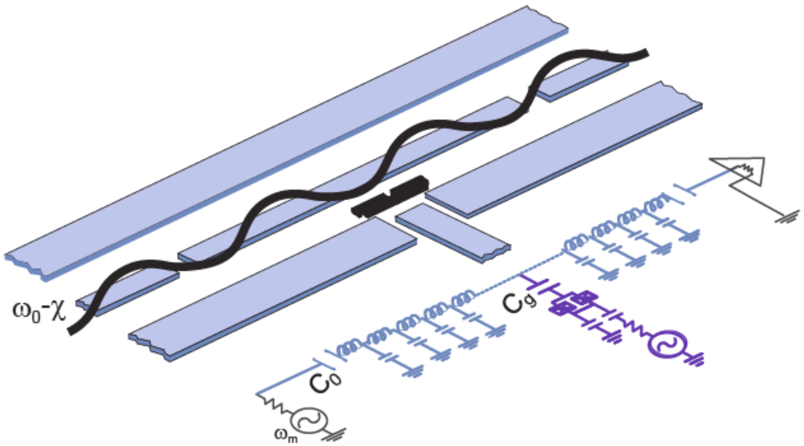


qubit detuned by Δ
from resonator

A. Blais *et al.*, *PRA* 69, 062320 (2004)

Circuit QED – read out of qubit state

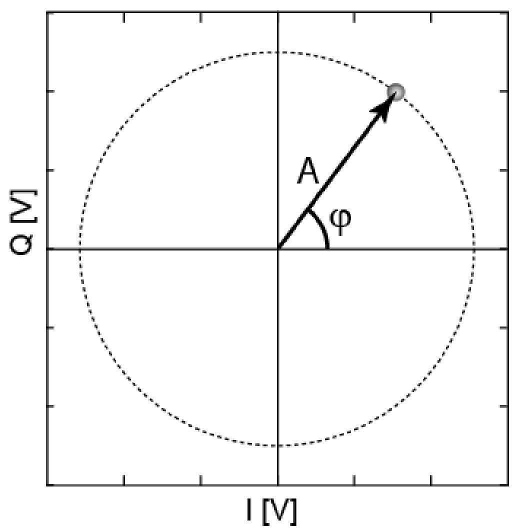
- transmission measurement to determine qubit state:



Phase sensitive measurement of transmitted microwave:

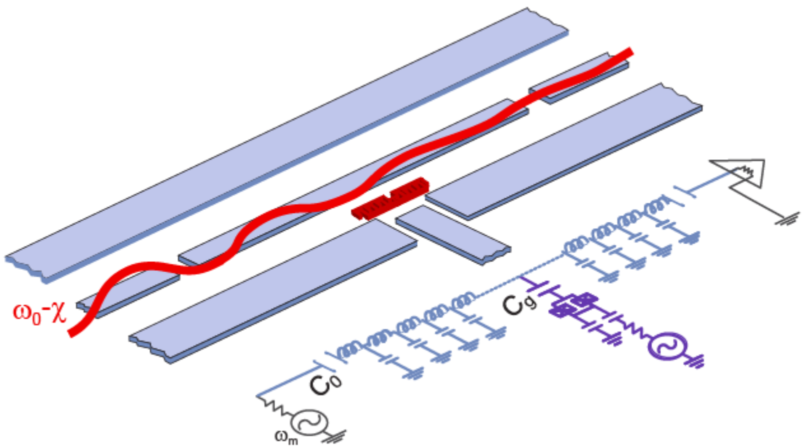
Voltage signal:

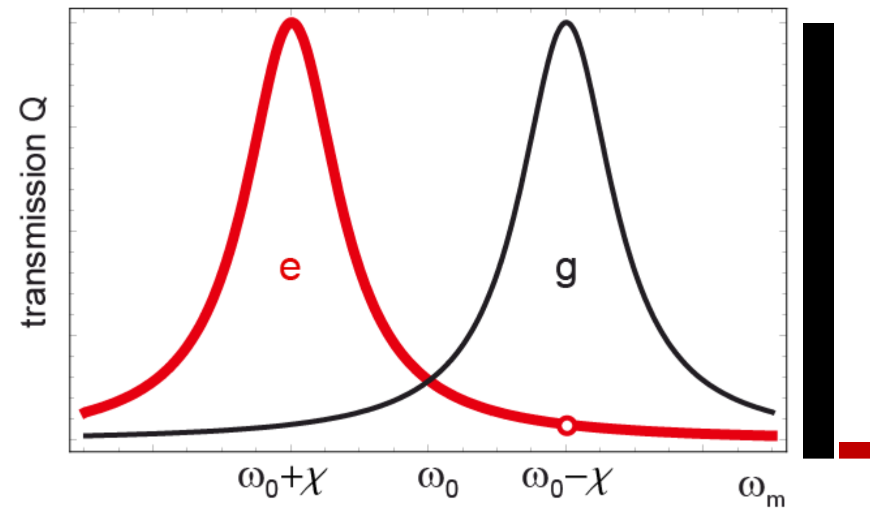
$$A(t)\sin(\omega_m t + \phi(t)) \equiv I(t)\sin\omega_m t + Q(t)\cos\omega_m t$$



Circuit QED – read out of qubit state

- transmission measurement to determine qubit state:



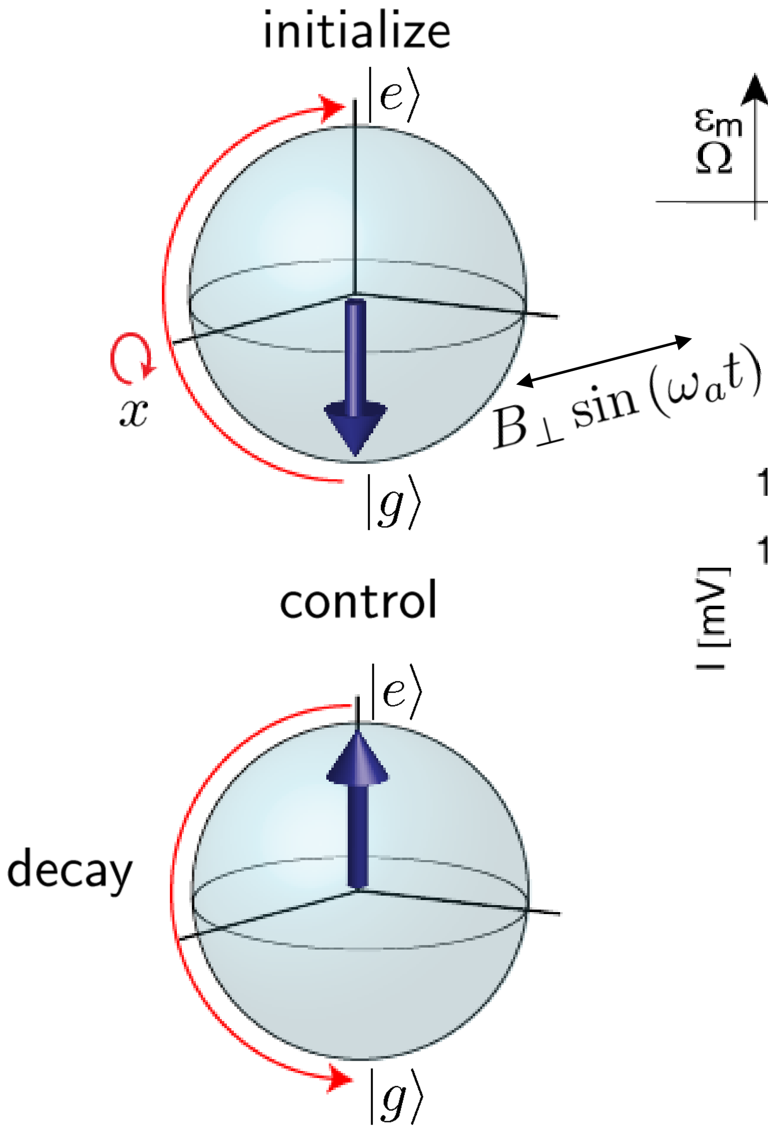


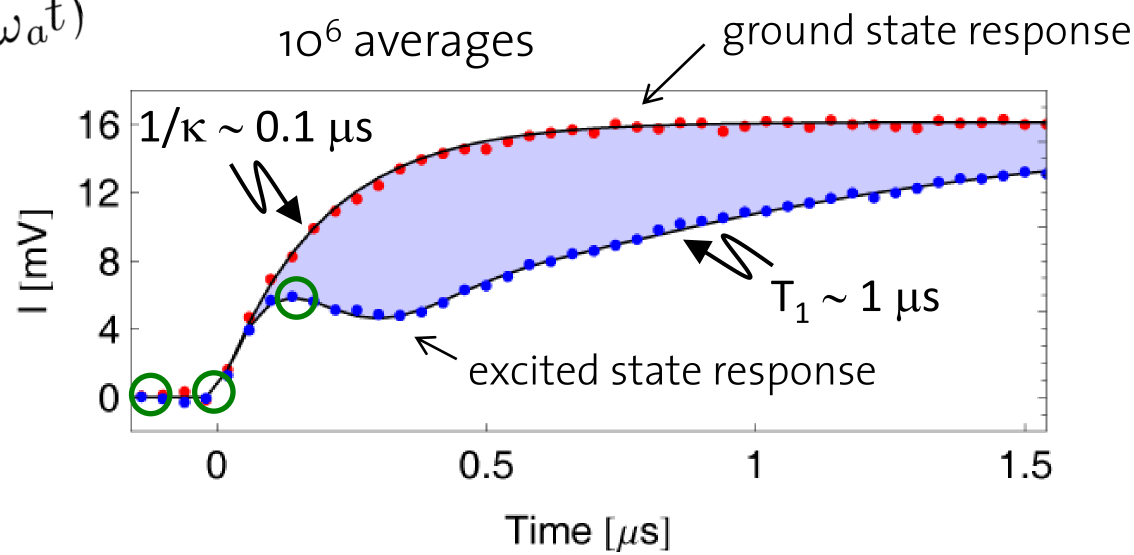
dispersive Hamiltonian:

$$H = \hbar(\omega_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi)\sigma_z$$

state-dependent frequency shift -> σ_z determined

Qubit Control and Readout

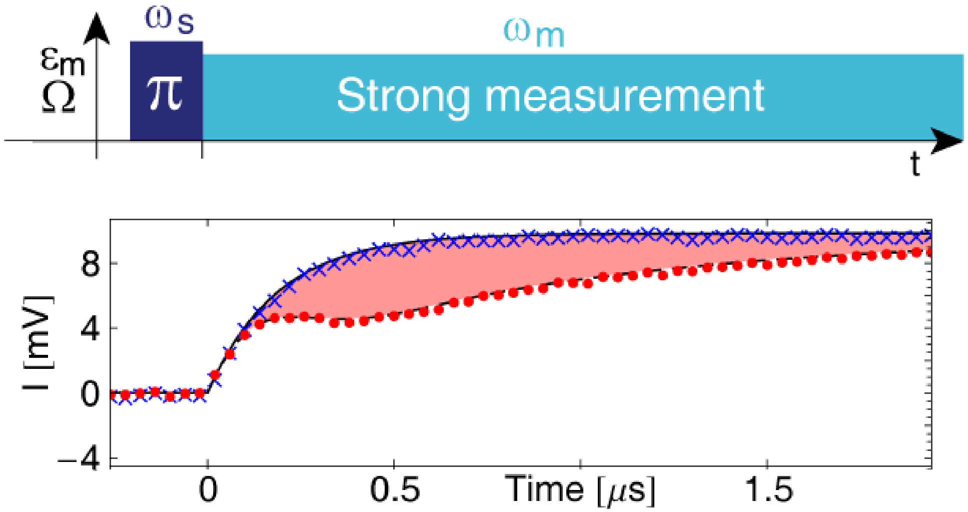




Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Time dependent measurements

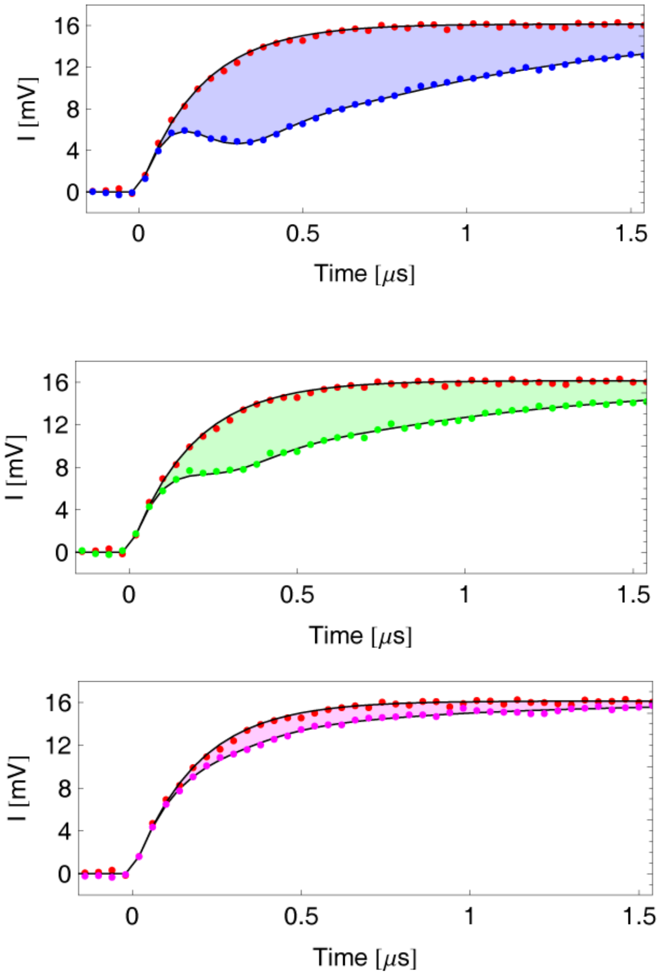
- excite qubit at t<o
- measure transmitted field quadratures (I, Q) with microwave drive at resonance ($\omega_m = \omega_r - \chi$)
- qubit in ground state: full resonator transmission (rise time given by κ)
- qubit in excited state: only partial transmission until qubit decays to ground state

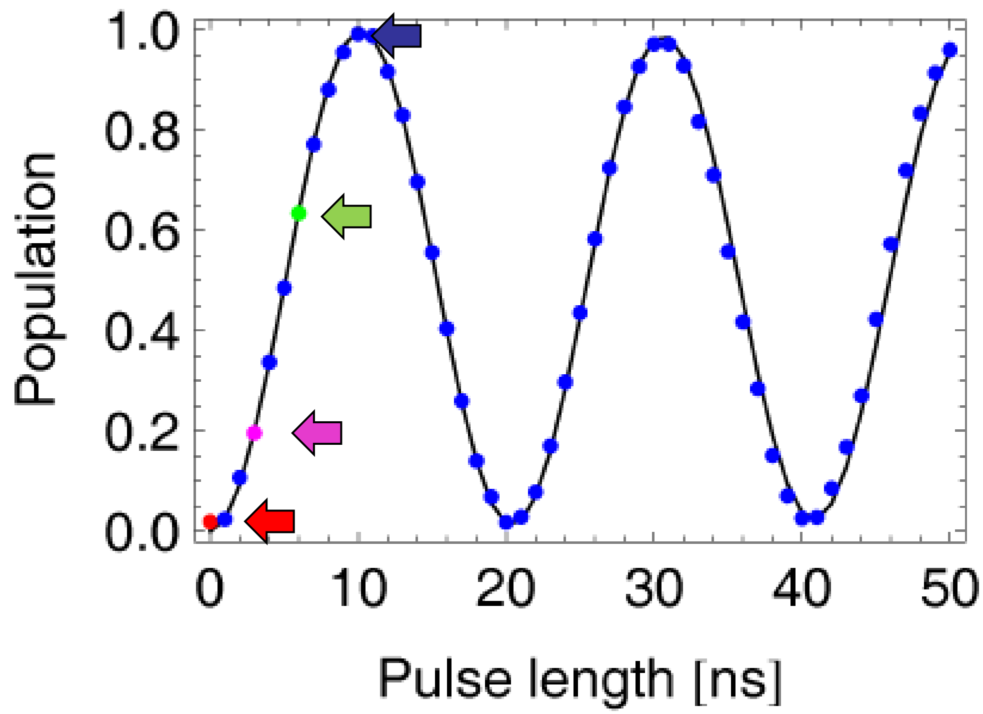


[Bianchetti *et al*. PRA 80 (2009)]

Population reconstruction

Area between curves is proportional to qubit state population:
[Bianchetti *et al.*, Phys. Rev. A 80, 043840 (2009)]



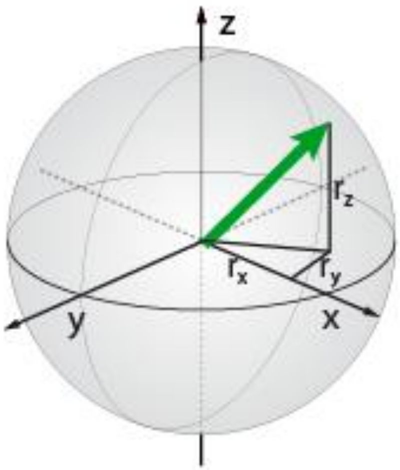


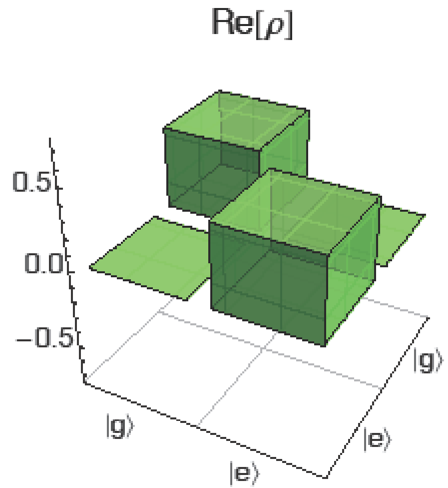
typical pulse length for π-pulse: 10ns

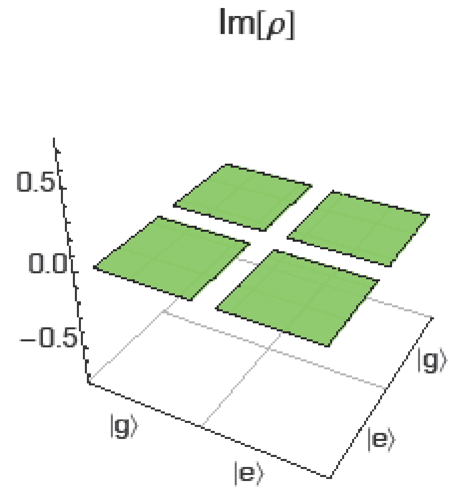
State reconstruction single qubit

3 measurements for 3 coefficients r_x, r_y, r_z of

$$\rho = \frac{1}{2}(\mathrm{id} + r_x\sigma_x + r_y\sigma_z + r_z\sigma_z)$$





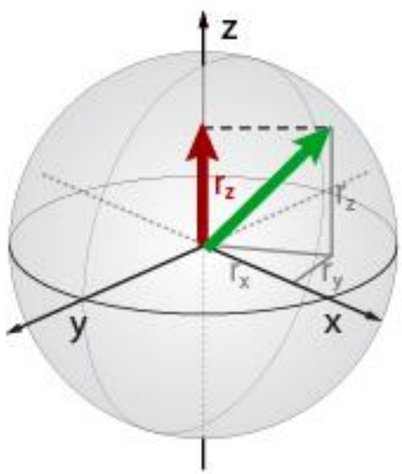


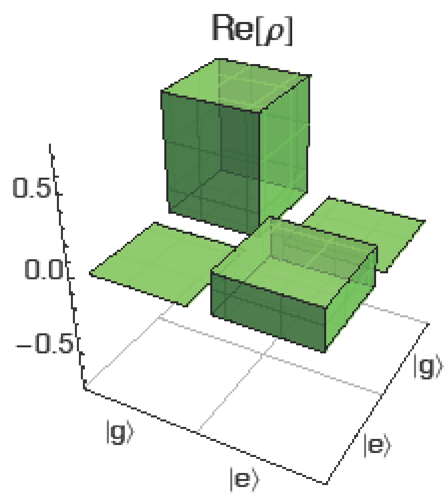
State reconstruction single qubit

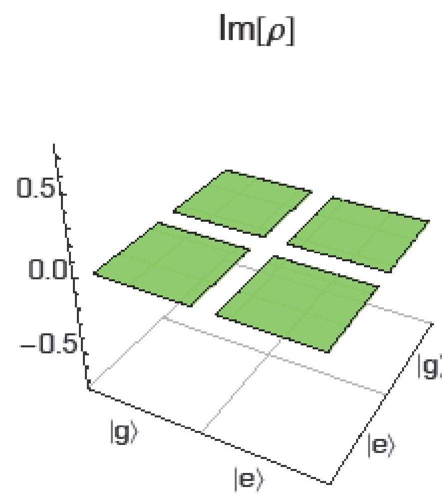
3 measurements for 3 coefficients r_x, r_y, r_z of

$$\rho = \frac{1}{2}(\mathrm{id} + r_x\sigma_x + r_y\sigma_z + r_z\sigma_z)$$

Measurement along z-axis: $r_z = \langle\sigma_z\rangle = \mathrm{Tr}[\rho\sigma_z]$







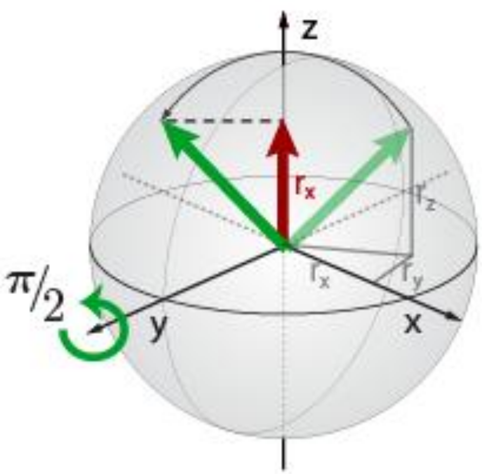
State reconstruction single qubit

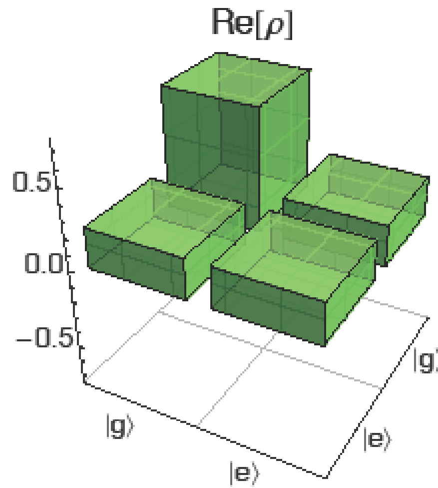
3 measurements for 3 coefficients r_x, r_y, r_z of

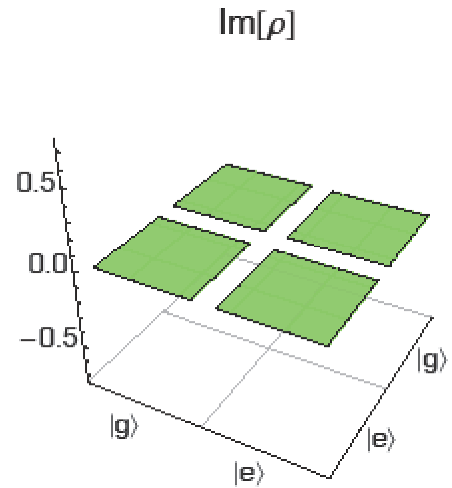
$$\rho = \frac{1}{2}(\mathrm{id} + r_x\sigma_x + r_y\sigma_z + r_z\sigma_z)$$

Measurement along z-axis: $r_z = \langle \sigma_z \rangle = \mathrm{Tr}[\rho\sigma_z]$

Rotation + measurement: $r_x = \langle \sigma_x \rangle = \mathrm{Tr}[\left(\frac{\pi}{2}\right)_y \rho \left(\frac{\pi}{2}\right)_{-y} \sigma_z]$







State reconstruction single qubit

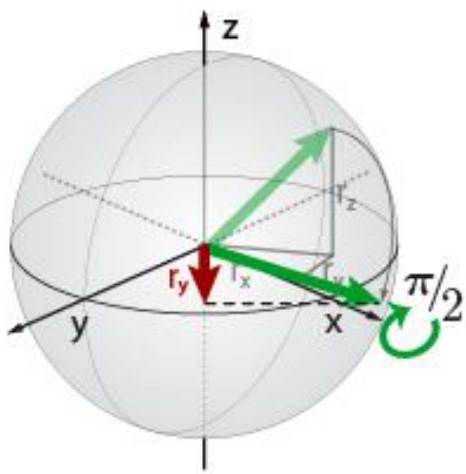
3 measurements for 3 coefficients r_x, r_y, r_z of

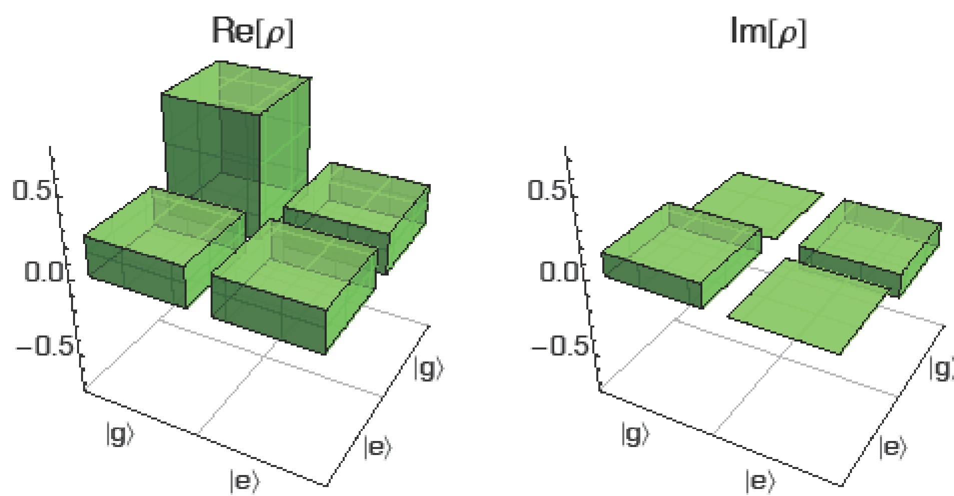
$$\rho = \frac{1}{2}(\mathrm{id} + r_x\sigma_x + r_y\sigma_z + r_z\sigma_z)$$

Measurement along z-axis: $r_z = \langle\sigma_z\rangle = \mathrm{Tr}[\rho\sigma_z]$

Rotation + measurement: $r_x = \langle\sigma_x\rangle = \mathrm{Tr}[\left(\frac{\pi}{2}\right)_y \rho \left(\frac{\pi}{2}\right)_{-y} \sigma_z]$

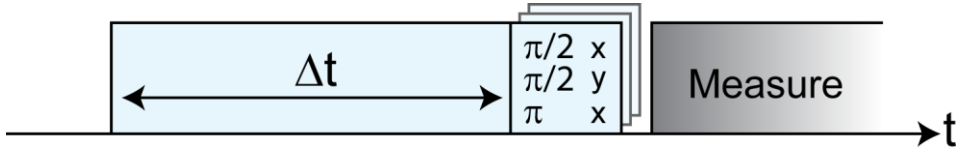
Rotation + measurement: $r_y = \langle\sigma_y\rangle = \mathrm{Tr}[\left(\frac{\pi}{2}\right)_x \rho \left(\frac{\pi}{2}\right)_{-x} \sigma_z]$



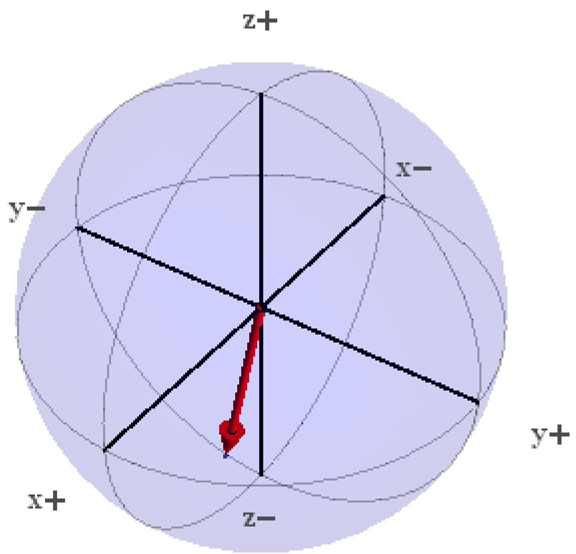


Control and Tomographic Read-Out of Single Qubit

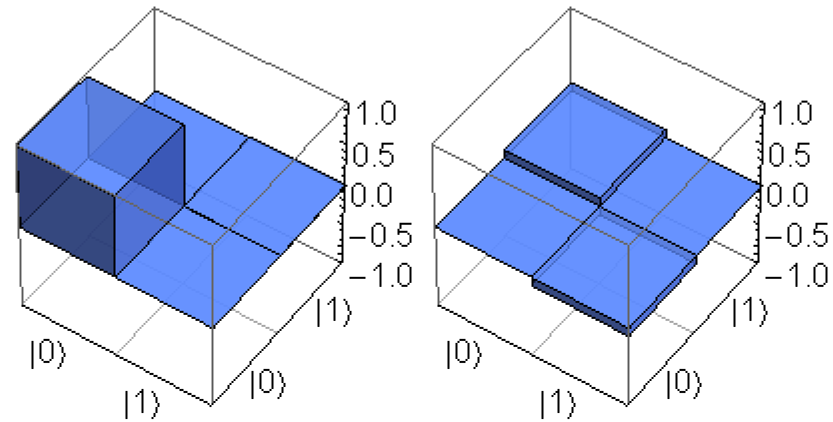
Rabi rotation pulse sequence:



experimental Bloch vector:



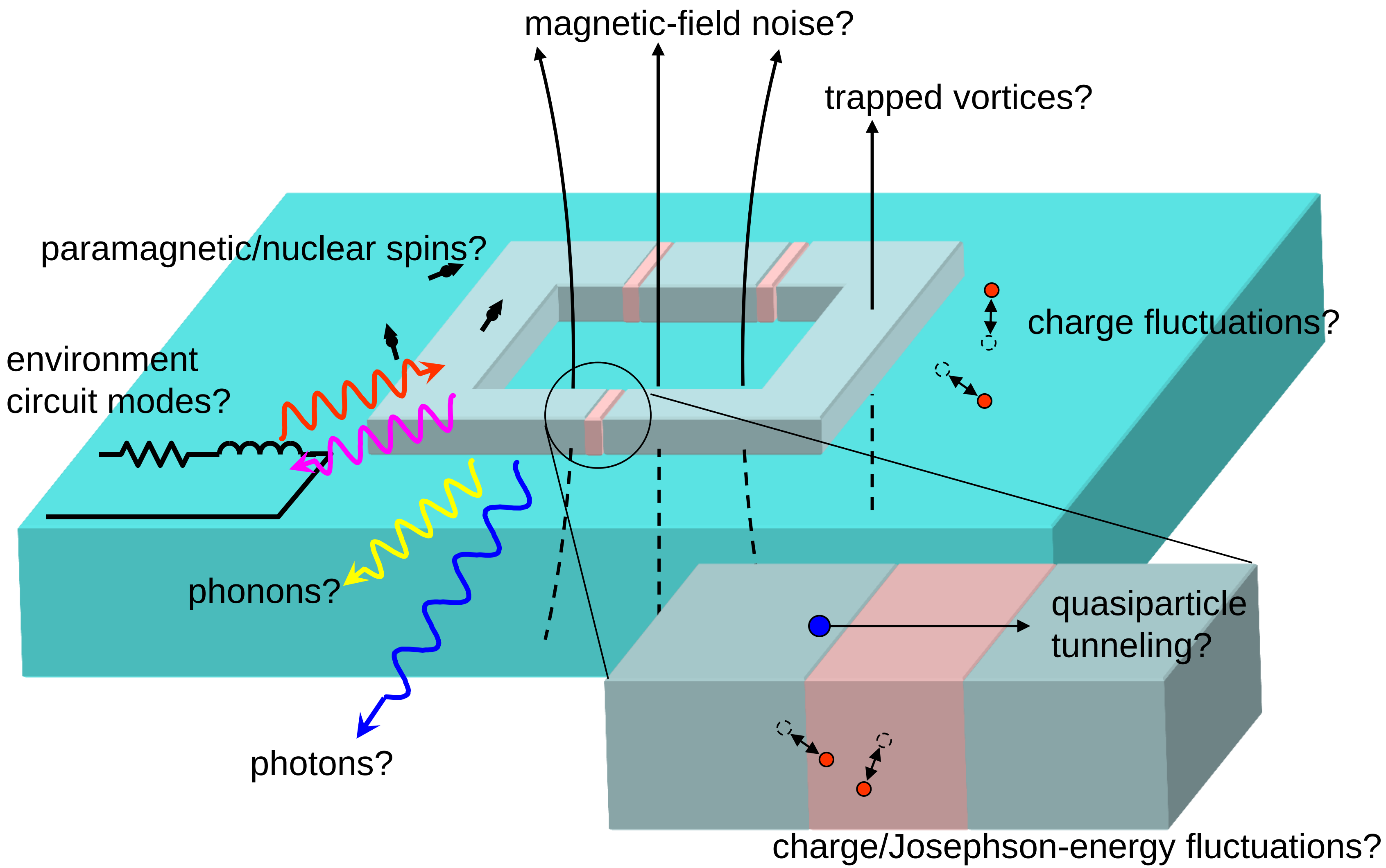
experimental density matrix:



$\Delta t = 0$ ns

L. Steffen *et al.*, Quantum Device Lab, ETH Zurich (2008)

Sources of Decoherence

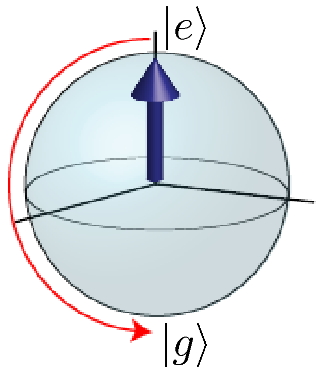


G. Ithier *et al.*, *Phys. Rev. B* **72**, 134519 (2005)

Relaxation and dephasing (T_1 and T_2)

- T_1: energy relaxation time

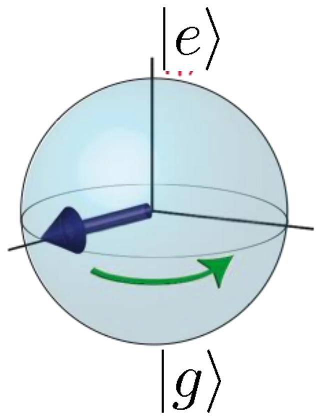
depolarization, decay



perturbation orthogonal to quantization axis ($\propto \sigma_{x,y}$); e.g. fast charge fluctuations causing transitions

- T_2: dephasing time

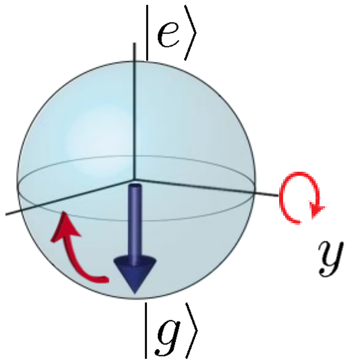
dephasing

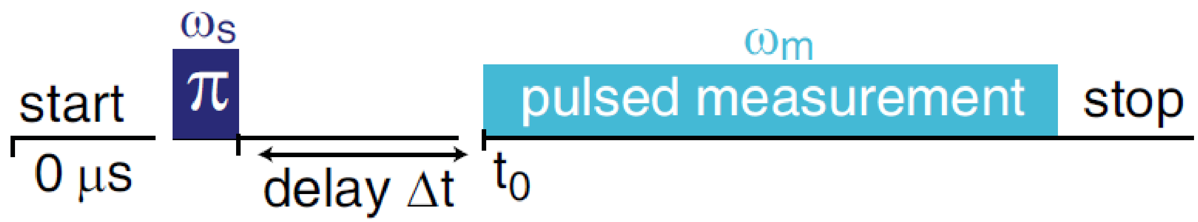


slow perturbation along quantization axis ($\propto \sigma_z$); e.g. magnetic flux noise causing phase randomization

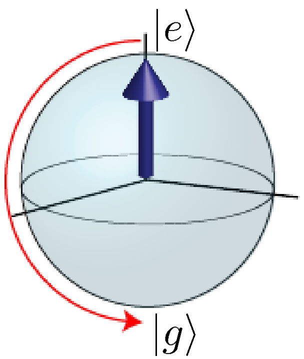
Relaxation Time (T1) Measurement

pulse scheme:

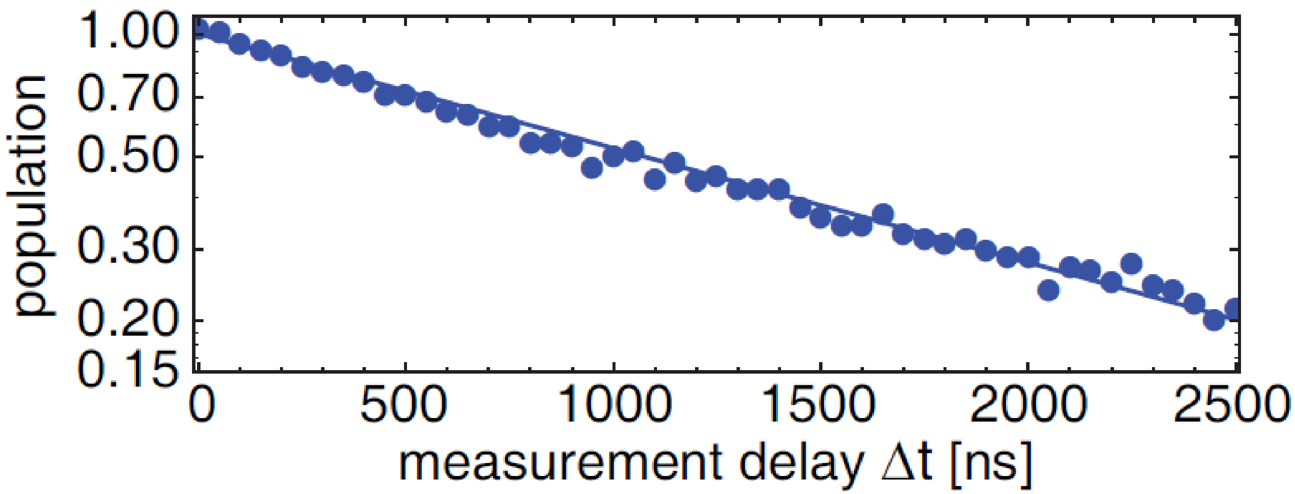




decay



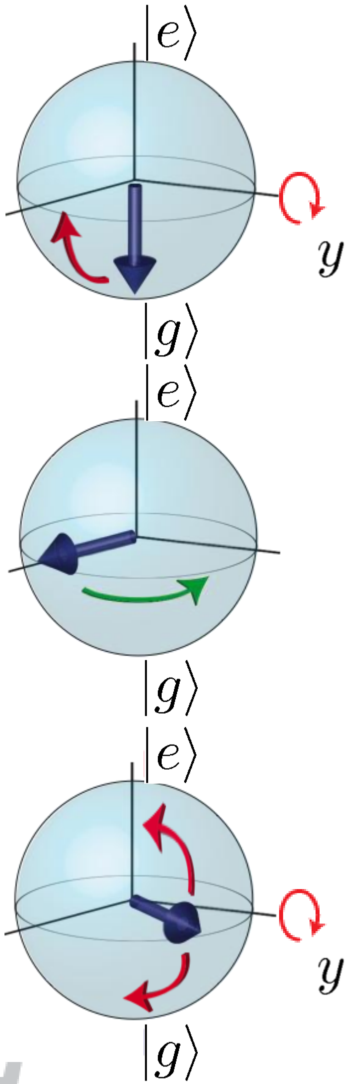
T_1=1.2 us

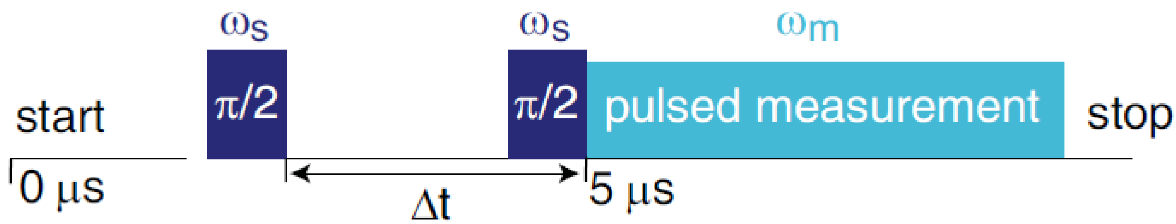


R. Bianchetti, QUDEV, ETH Zurich (2010)

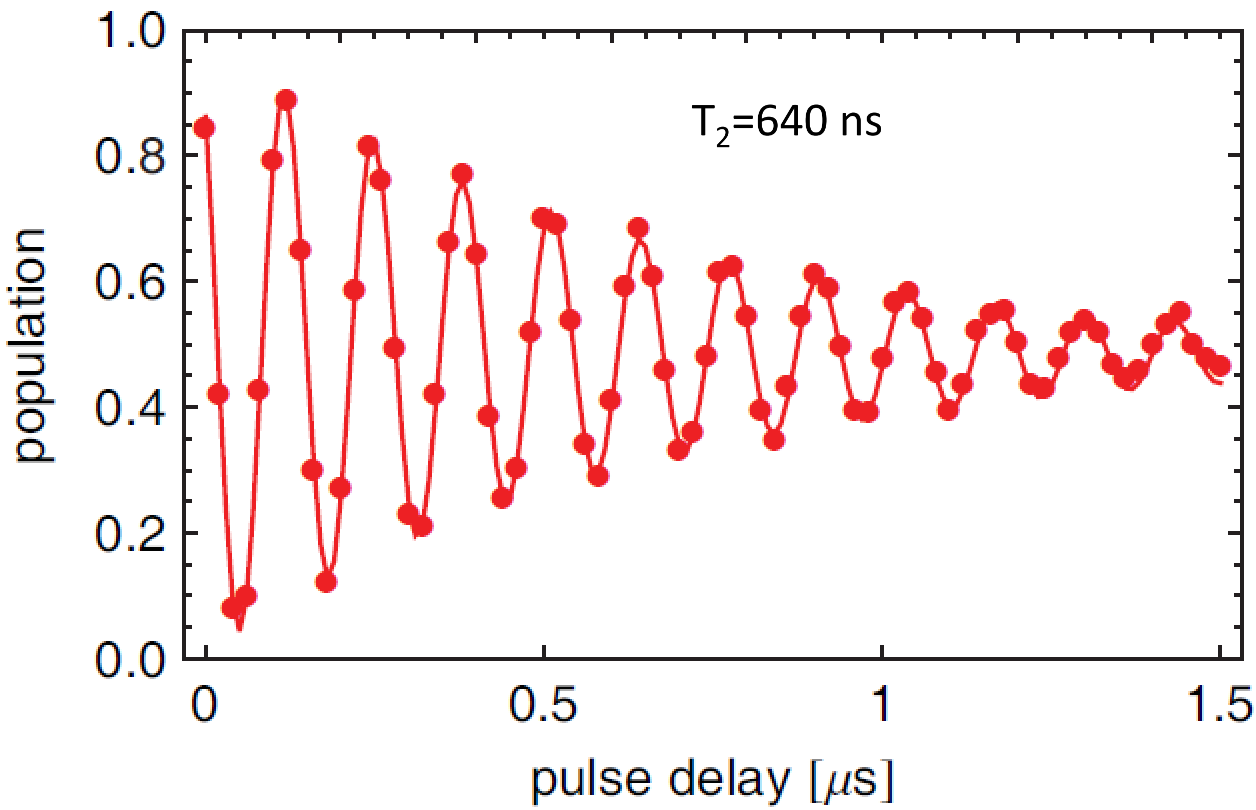
Coherence Time (T2) Measurement: Ramsey Fringes

pulse scheme:





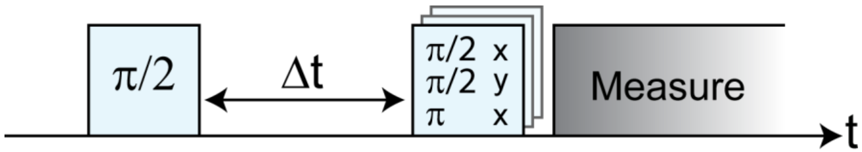
Ramsey fringes:

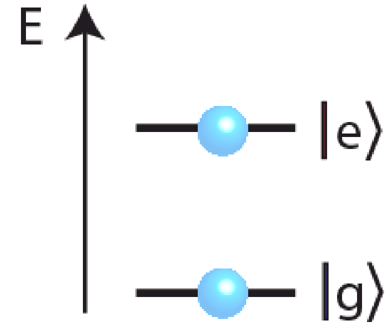


R. Bianchetti, QUDEV, ETH Zurich (2010)

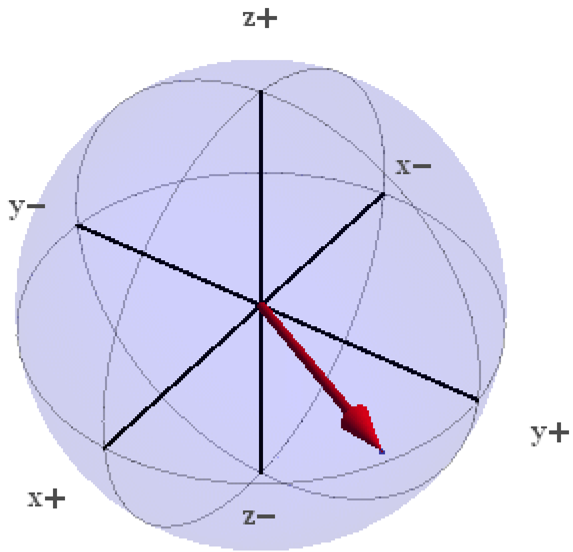
Tomography of Ramsey Experiment

pulse sequence:

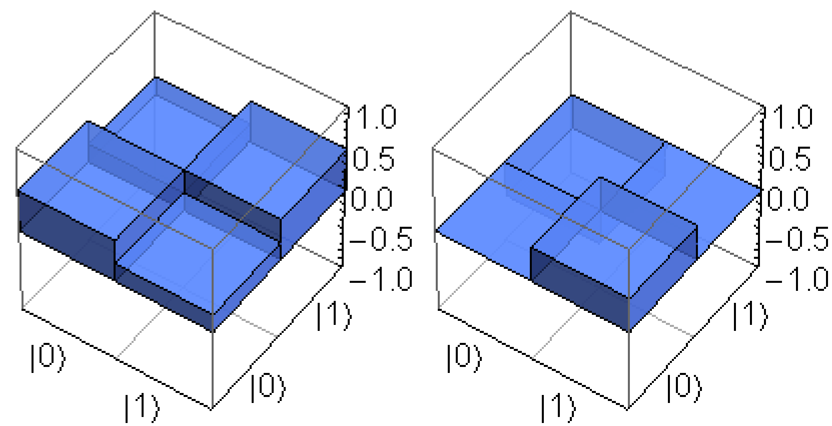




experimental Bloch vector:



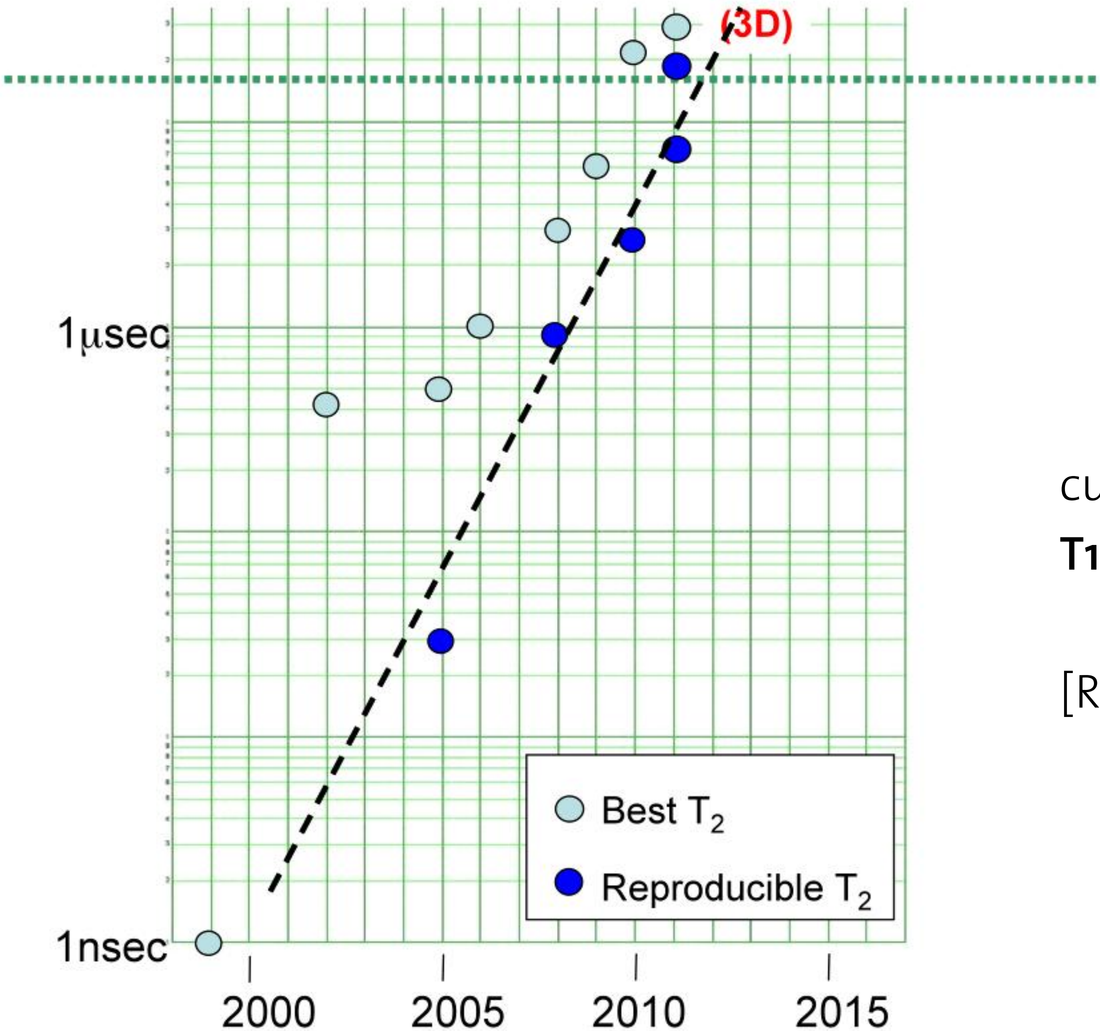
experimental density matrix:



$\Delta t = 0$ ns

L. Steffen *et al.*, Quantum Device Lab, ETH Zurich (2008)

Evolution of T2 – coherence of superconducting qubits



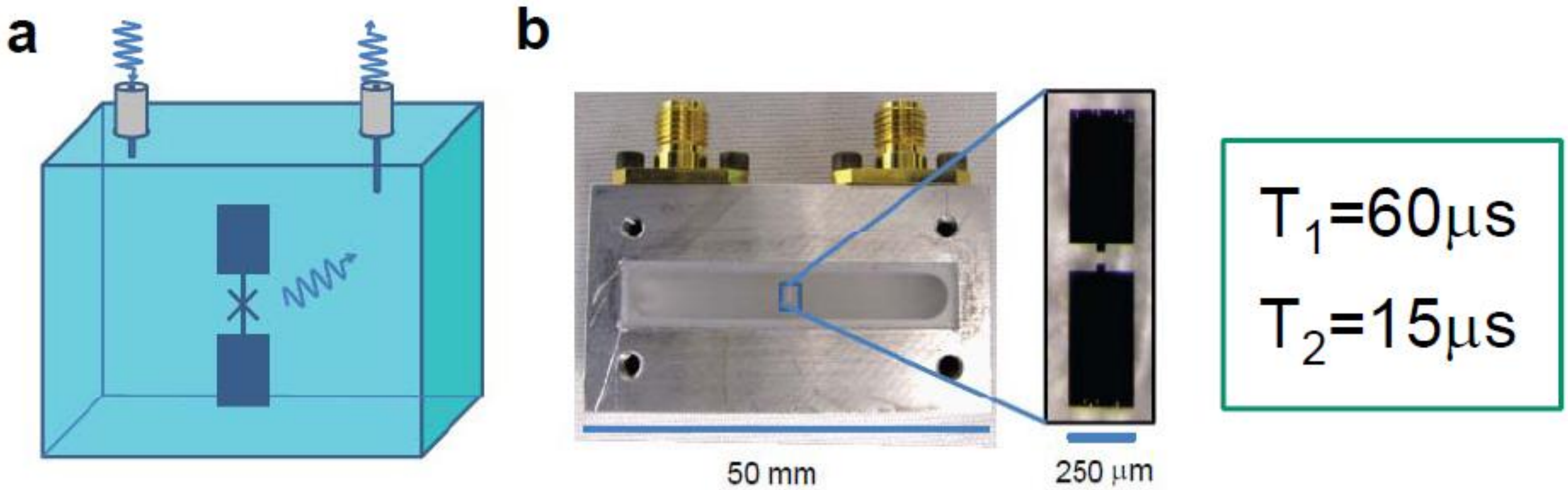
currently best T1/T2 (transmon):

T1=70us & T2=92us

[Rigetti *et al.*, PRB 86 (2012)

Recent trends – transmon in 3D cavity

Transmon in a three-dimensional cavity: lifetimes (T1) ~ 100μs



H. Paik et al., PRL (2012)

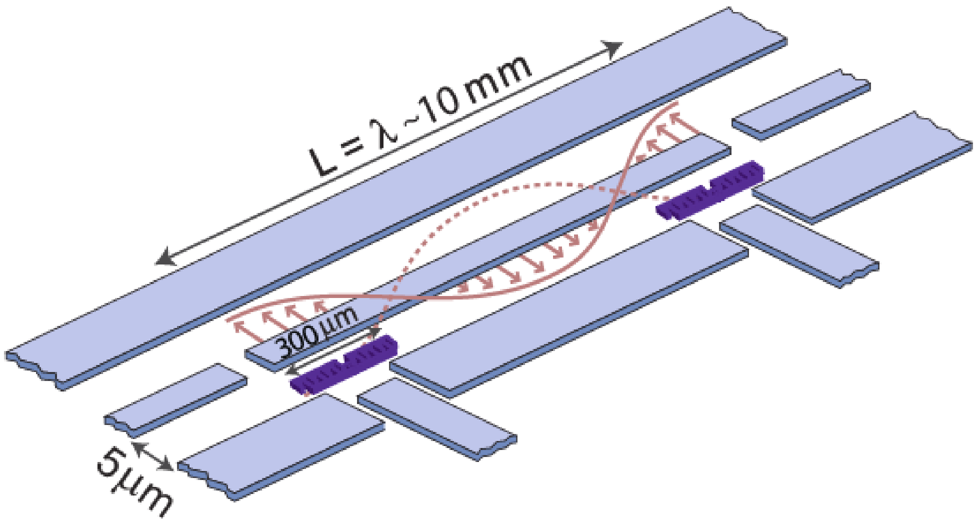
Questions: Tunabiliy? Scalability?

Coupling Superconducting Qubits – Two-qubit operations.

Entangling two distant qubits

transmission line resonator can be used as a **'quantum bus'**

to create **entangled** states

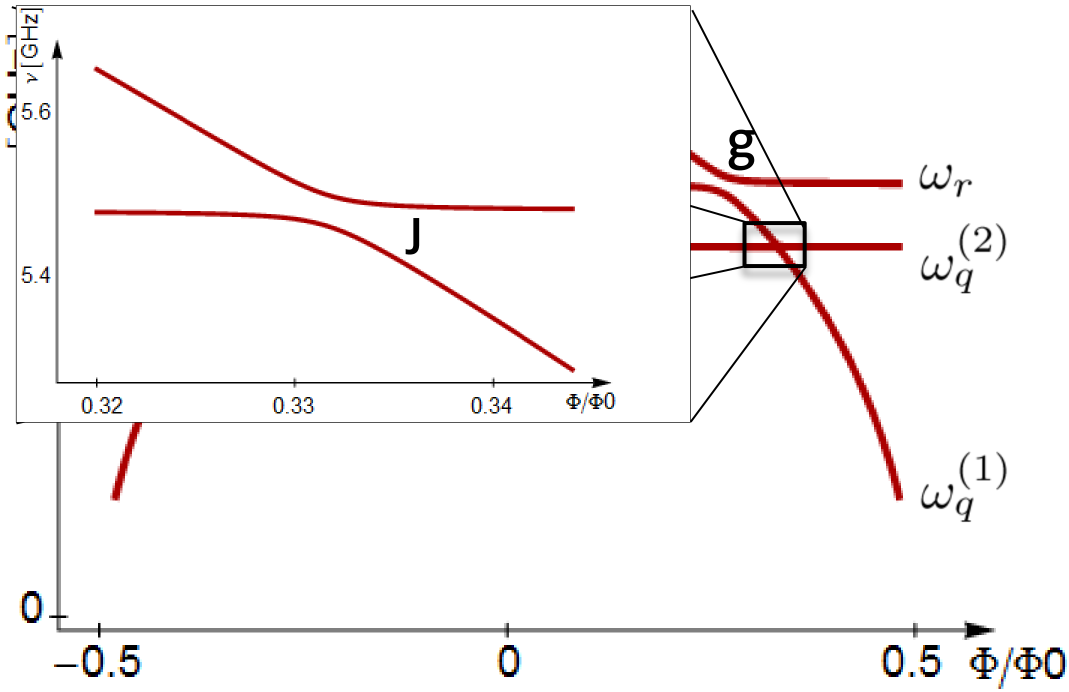


Dispersive two-qubit J-coupling – Energy levels

qubit 1: transition frequency: $\omega_q \approx \sqrt{8E_C E_J} = \sqrt{8E_C E_{J,max} \left|\cos(\pi\Phi/\Phi_0)\right|}$

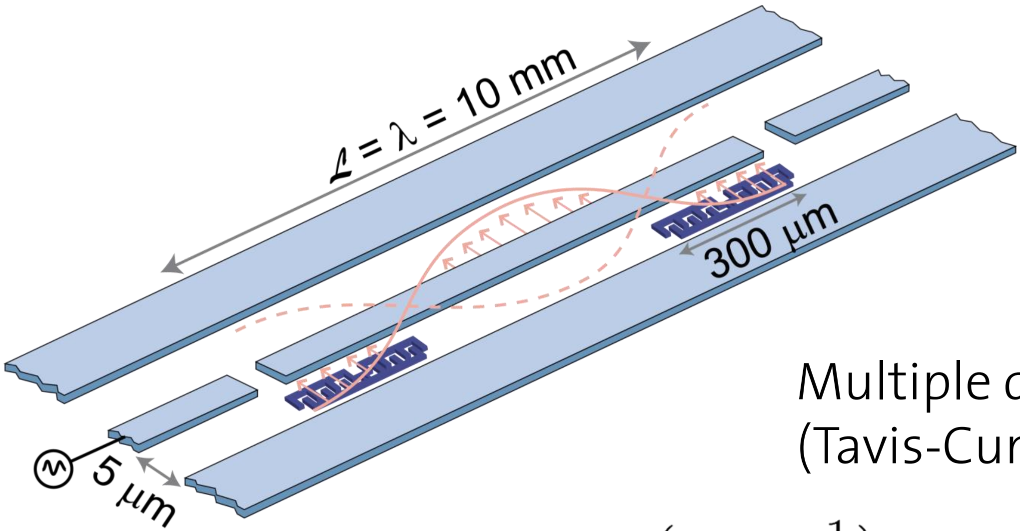
qubit 2: constant frequency (5.5 GHz)

resonator:
- direct coupling (g ~ 130 MHz)
- mediated J-coupling (J ~ 20 MHz)



[Majer *et al.*, Nature 449 (2007)]

Cavity Quantum Electrodynamics (QED) – 2 qubits



Multiple qubits coupled to single mode (Tavis-Cummings Hamiltonian):

$$H = \underbrace{\hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)}_{H_r} + \sum_i \frac{\hbar\omega_{ge}^{(i)}}{2}\sigma_z^{(i)} + \sum_i \hbar g_i (a^\dagger \sigma_-^{(i)} + a\sigma_+^{(i)})$$

dispersive regime (2 qubits): $\Delta = |\omega_{ge} - \omega_r| >> g$

$$H = H_r + \hbar \sum_{i=1,2} \frac{\omega_{ge}^{(i)} + \chi^{(i)}}{2}\sigma_z^{(i)} + \hbar J \left(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_+^{(2)}\sigma_-^{(1)}\right)$$

flip-flop interaction mediated by virtual photons

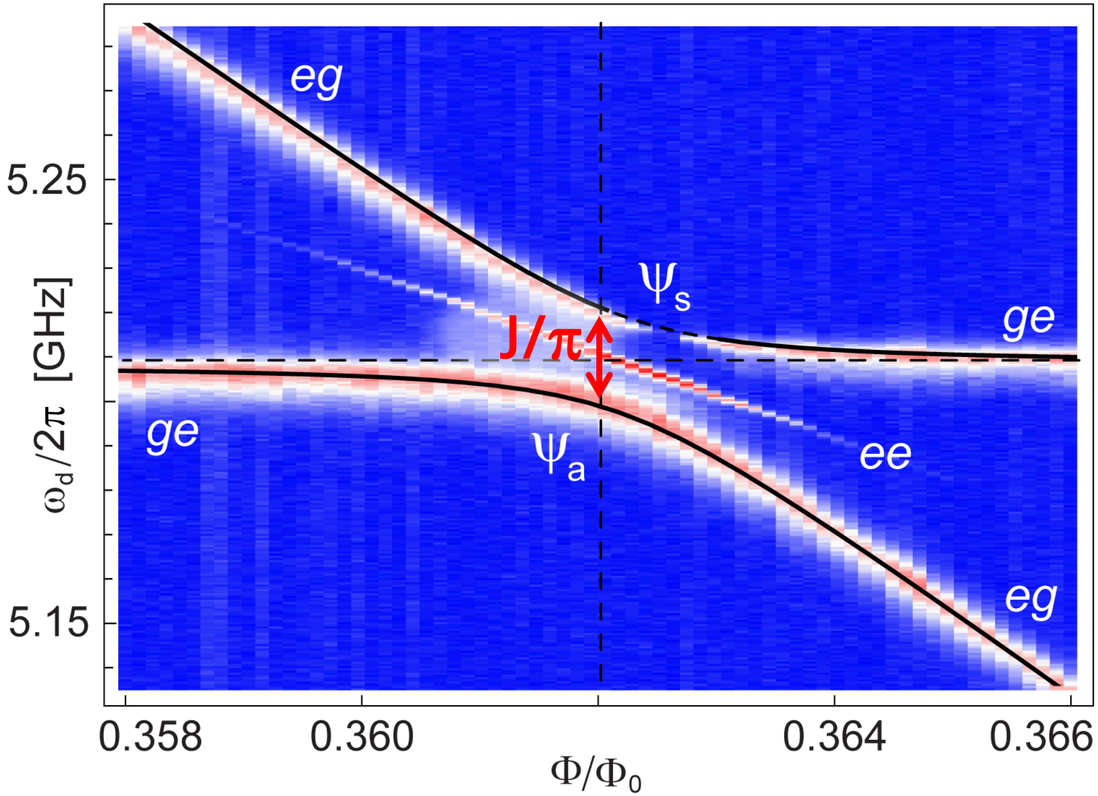
coupling strength: $J = \frac{g_1 g_2}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$

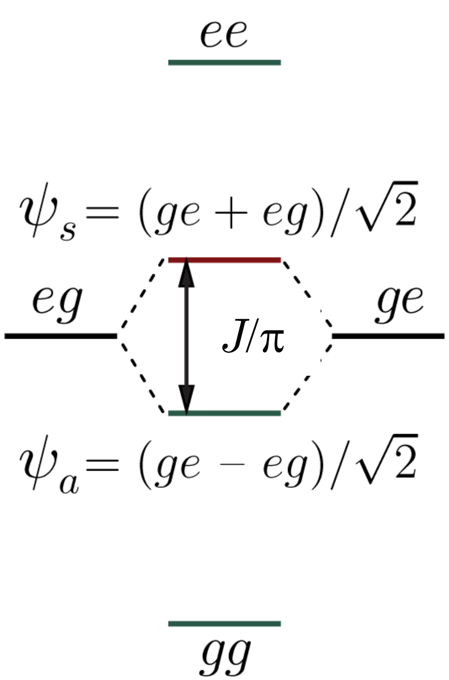
[Majer *et al.*, Nature **449** (2007); Filipp *et al.*, PRA 83, 063827 (2011)]

Avoided level crossing

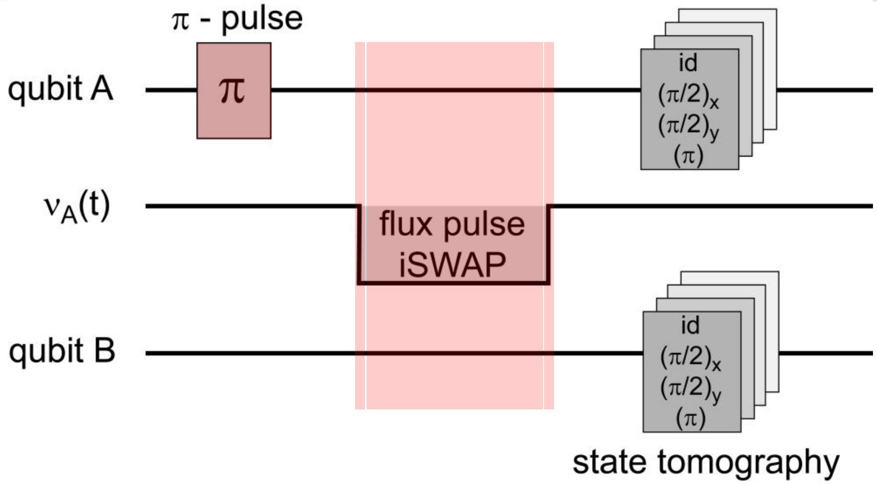
qubit A swept across resonance with fixed qubit B

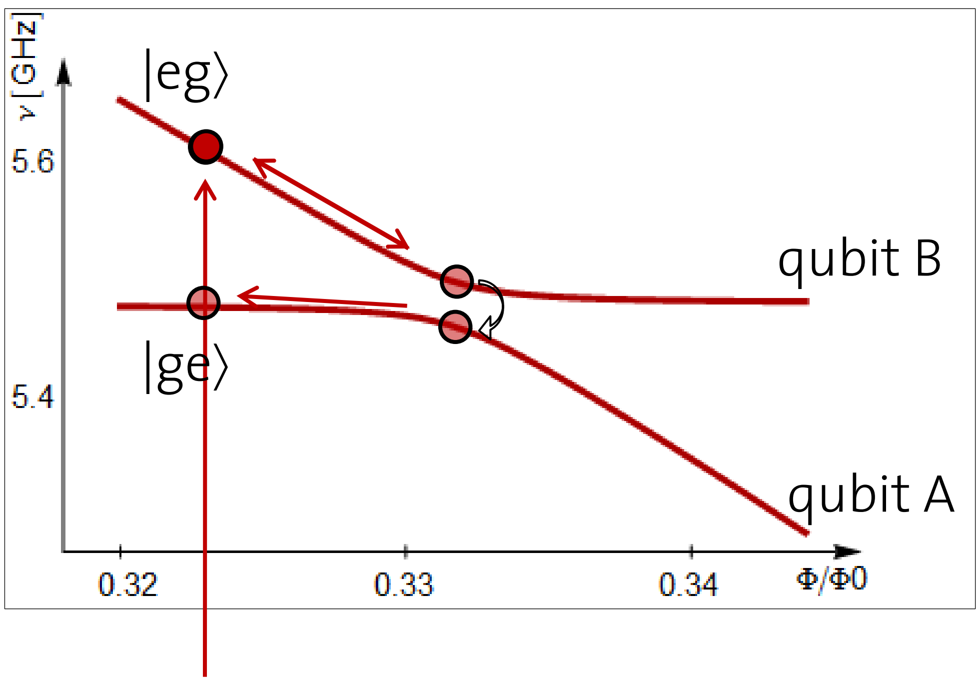
cavity mediated coupling leads to an avoided crossing

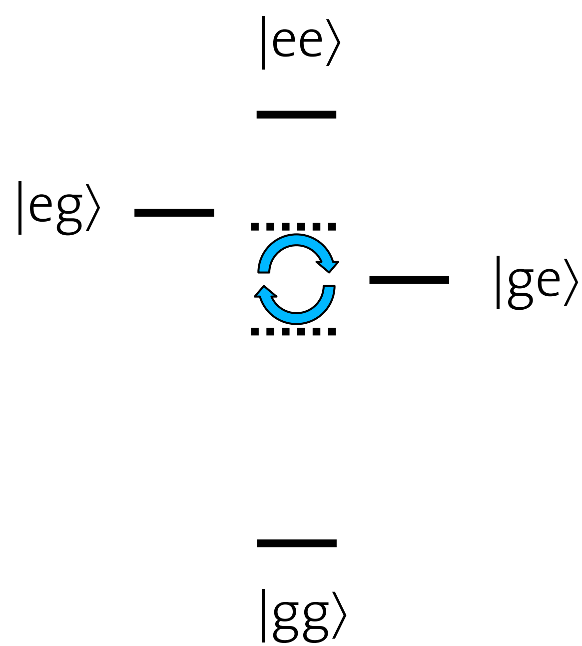




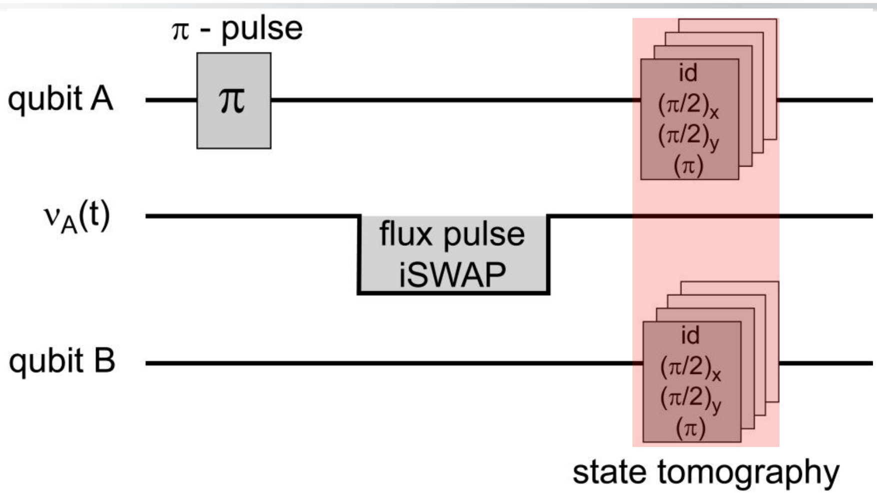
2-qubit gate: √iSWAP gate using ge ↔ eg transitions







2-qubit gate: iSWAP gate using ge $\leftrightarrow$ eg transitions

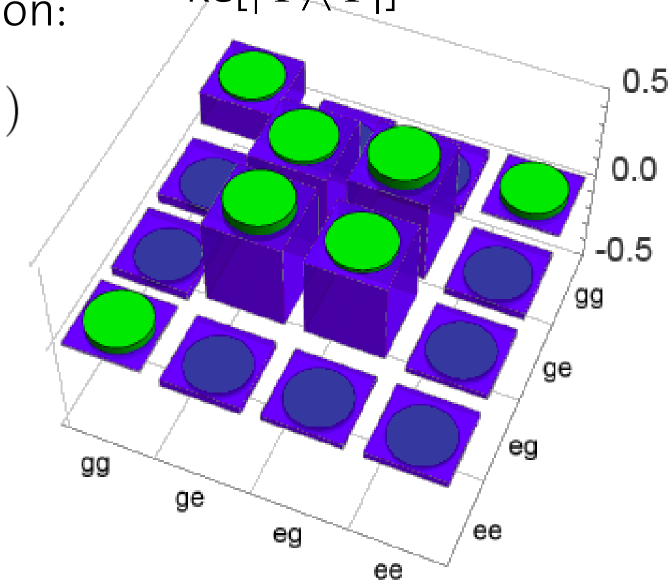


$$|gg\rangle \xrightarrow{\pi_A} |eg\rangle$$
$$\xrightarrow{\sqrt{iSWAP}} \frac{1}{\sqrt{2}}\left(|eg\rangle - i|ge\rangle\right)$$

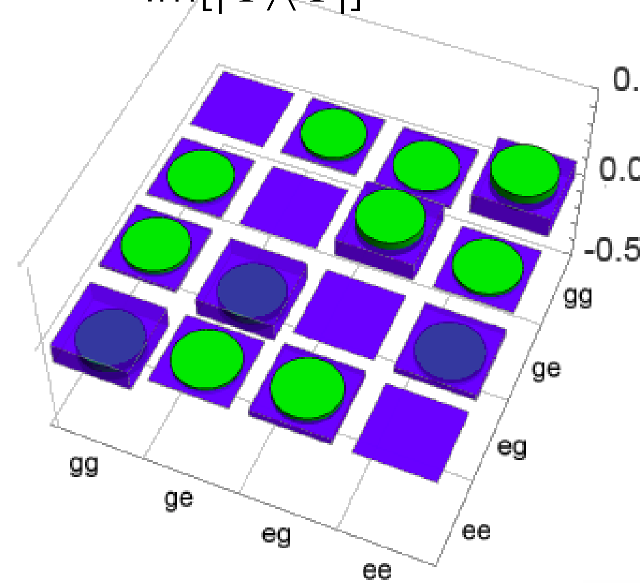
+ local phase transformation:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$$

Re[|Ψ⟩⟨Ψ|]



Im[|Ψ⟩⟨Ψ|]



Characterisation of two-qubit state:

Is it sufficient to measure single qubit observables

$$\sigma_x \otimes 1,\ \sigma_y \otimes 1,\ \sigma_z \otimes 1$$

and

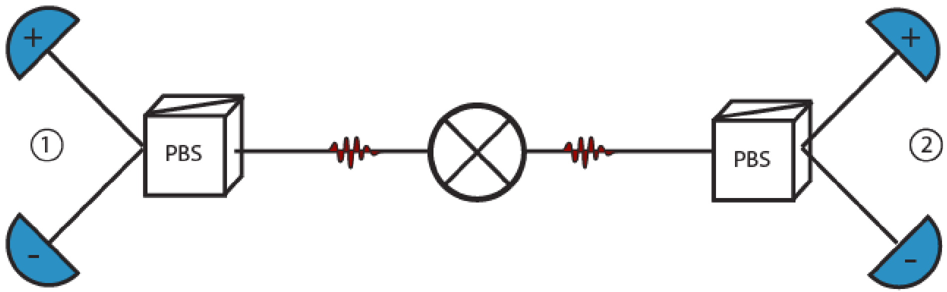
$$1 \otimes \sigma_x,\ 1 \otimes \sigma_y,\ 1 \otimes \sigma_z$$

to fully reconstruct any arbitrary **two**-qubit state?

1. Yes – it is sufficient.
2. No – more observables need to be measured.
3. Maybe.

Correlation measurement

[Photons: Weihs *et al.*, PRL **81** (1998); supercond. qubits: Steffen *et al.*, Science **313** (2006).



Correlation measurement with individual readout

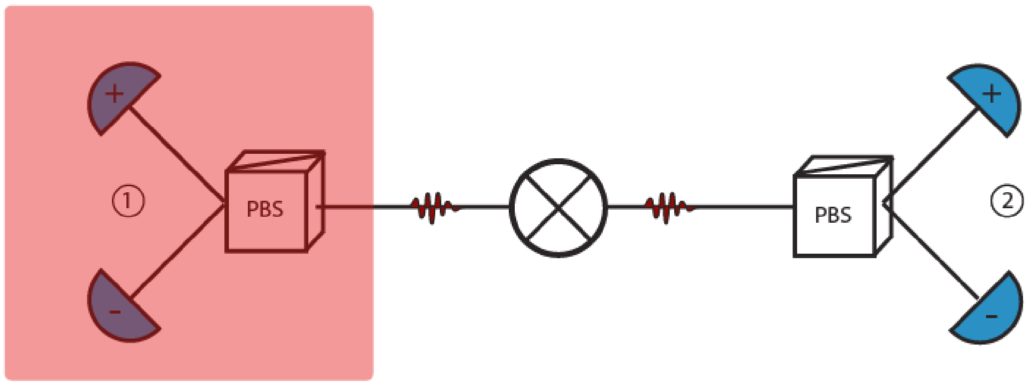


table of single shot values (±1):

k	$\sigma_z^{\,k} \quad 1$		
1	+1		
2	-1		
…	…		
K	-1		
$\langle \ldots \rangle = \frac{1}{K}\sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$		

Correlation measurement with individual readout

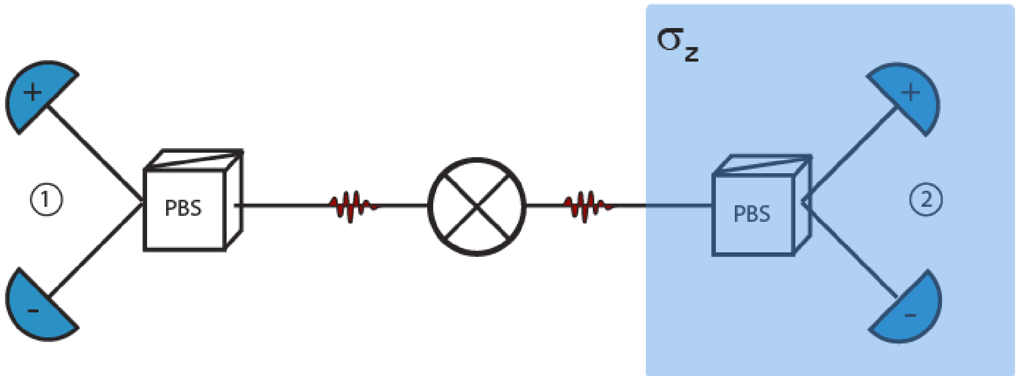


table of single shot values (±1):

k	$\sigma_z^k \quad 1$	$1 \quad \sigma_z^k$	
1	+1	+1	
2	-1	-1	
…	…	…	
K	-1	+1	
$\langle \ldots \rangle = \frac{1}{K} \sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$	$\langle 1 \otimes \sigma_z \rangle = 1/3$	

Correlation measurement with individual readout

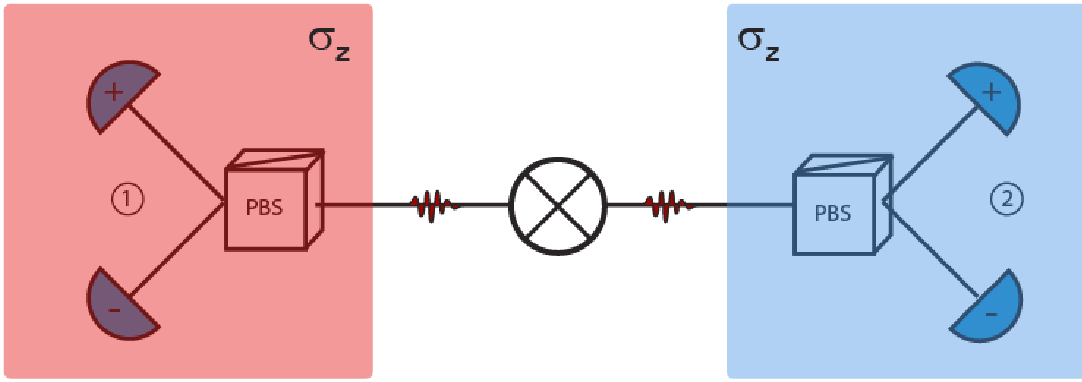
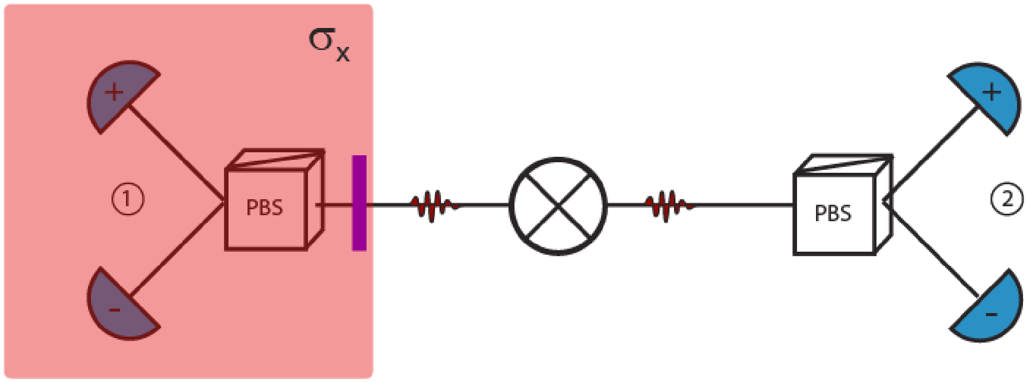


table of single shot values (±1):

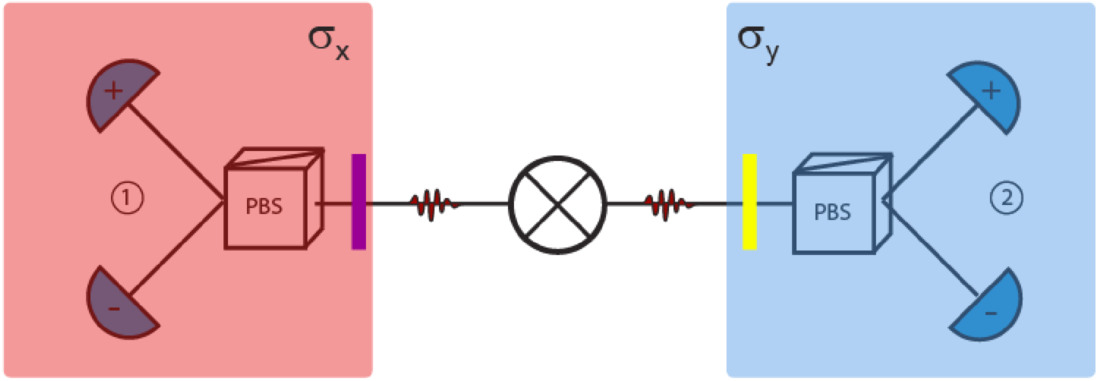
k	$\sigma_z^k \;\; 1$	$1 \;\; \sigma_z^k$	$\sigma_z^k \;\; \sigma_z^k$
1	+1	+1	(+1).(+1) = +1
2	-1	-1	(-1).(-1) = +1
…	…	…	…
K	-1	+1	(-1).(+1)=-1
$\langle \ldots \rangle = \frac{1}{K} \sum_k$	$\langle \sigma_z \otimes 1 \rangle = -1/3$	$\langle 1 \otimes \sigma_z \rangle = 1/3$	$\langle \sigma_z \otimes \sigma_z \rangle = 1/3$

Correlation measurement with individual readout



rotation of qubit: $\langle \sigma_x \ 1\rangle$, $\langle 1 \ \sigma_z\rangle$ and $\langle \sigma_x \ \sigma_z\rangle$ are measured

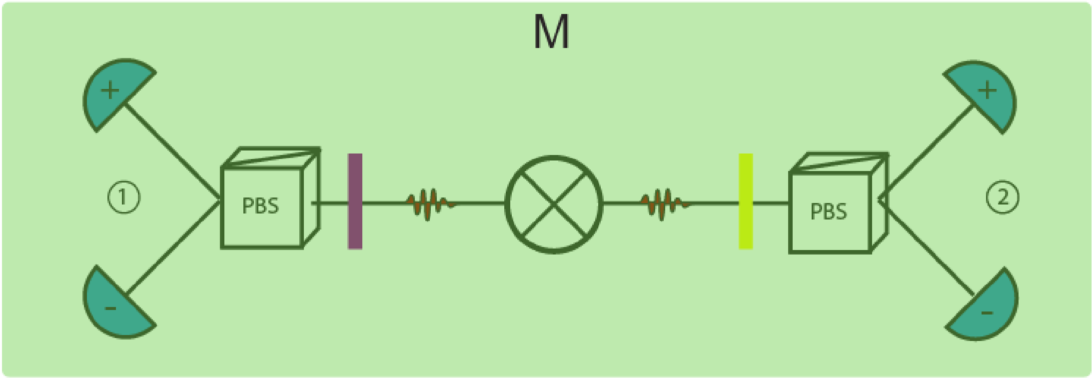
Correlation measurement with individual readout



or $\langle \sigma_x \; 1 \rangle$, $\langle 1 \; \sigma_y \rangle$ and $\langle \sigma_x \; \sigma_y \rangle$, a.s.o.

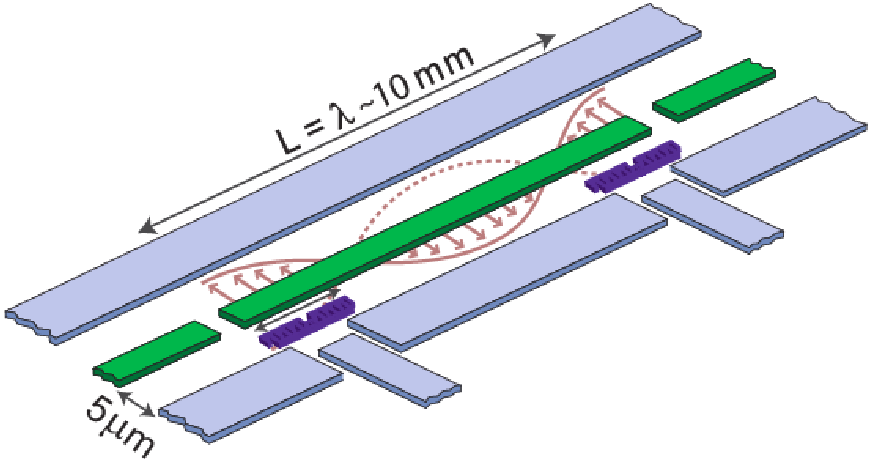
-> all combinations of $\{\sigma_x, \sigma_y, \sigma_z\}$ give full information about the state

Correlation measurement with joint readout



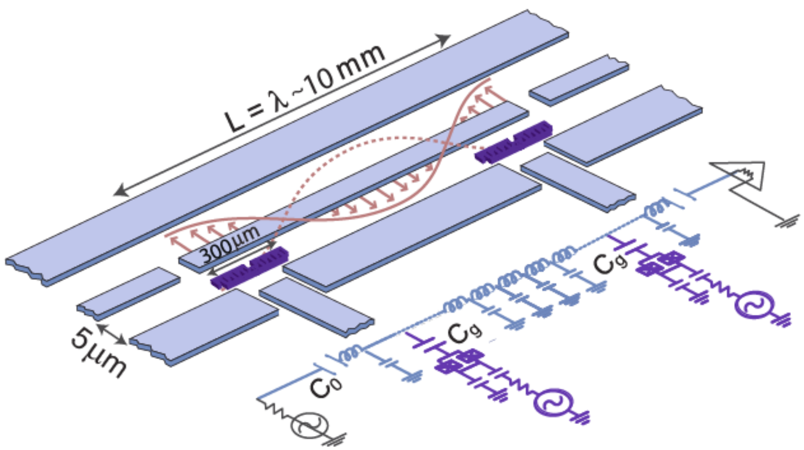
Circuit QED-Setup:

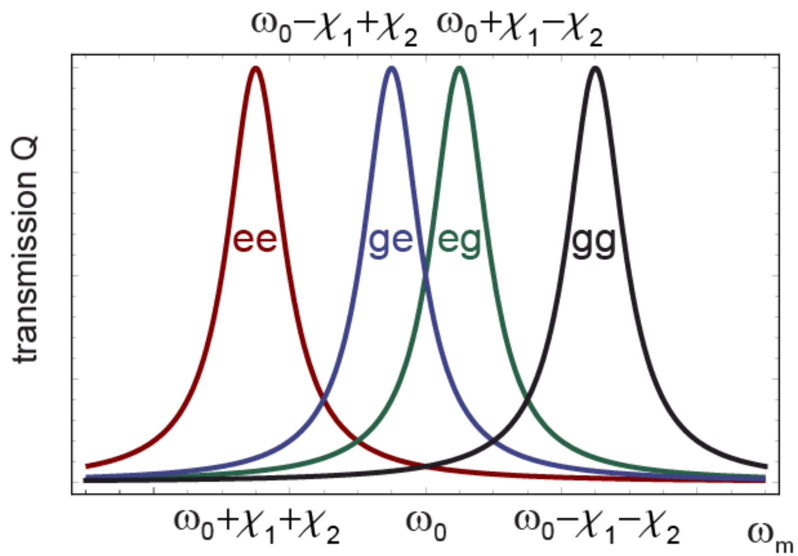
- single qubit operations
- only single detection device



Two qubit setup

joint averaged read-out of two-qubit state:





dispersive two-qubit Hamiltonian:

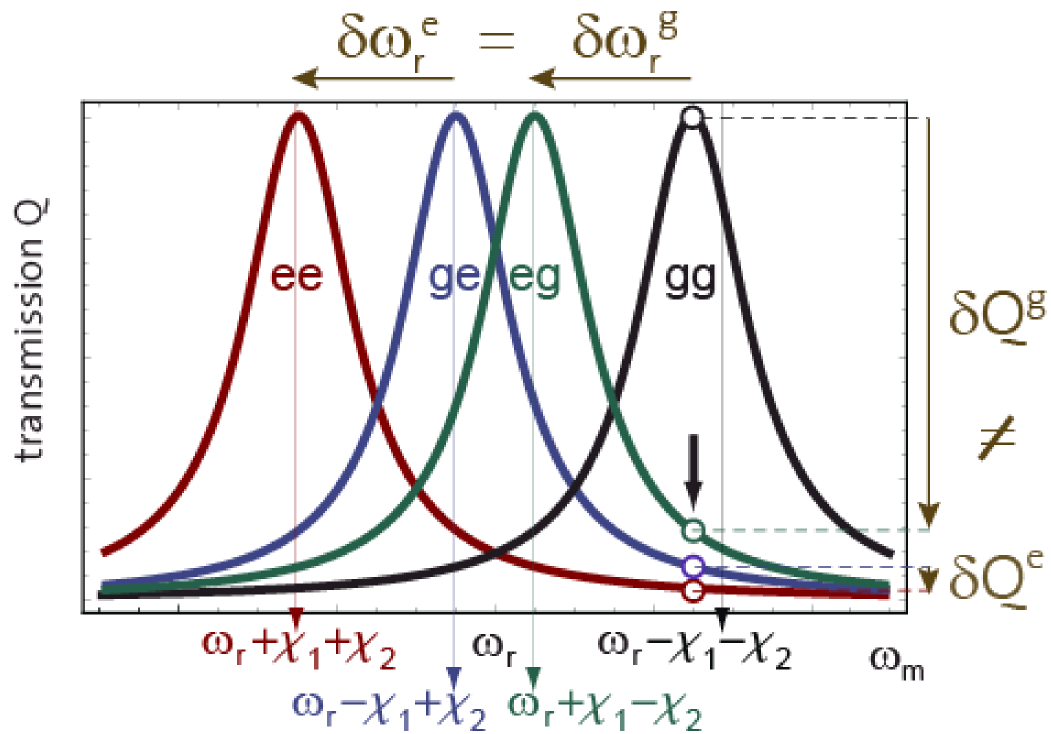
$$(\sigma_{z1} = 1 \otimes \sigma_z; \sigma_{z2} = \sigma_z \otimes 1)$$

$$H_0 = \hbar(\omega_r + \underbrace{\chi_1\hat{\sigma}_{z1} + \chi_2\hat{\sigma}_{z2}}_{\delta\hat{\omega}_r})\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}\sum_{j=1,2}(\omega_{aj} + \chi_j)\hat{\sigma}_{zj}$$

[Filipp *et al.*, PRL **102** (2009)]

Homodyne measurement

Amplitude difference (δQ) depends on state of second qubit:



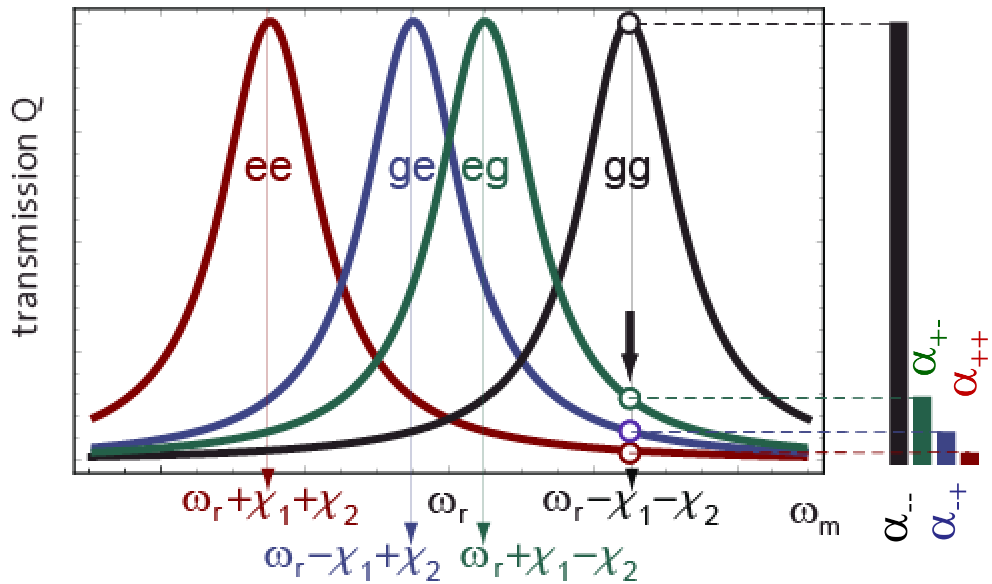
→ **qubit-qubit correlations** can be determined from transmission measurement

Measurement operator

Homodyne voltage measurement: $Q = \langle \hat{M}_Q \rangle = \mathrm{Tr}[\rho \hat{M}_Q]$

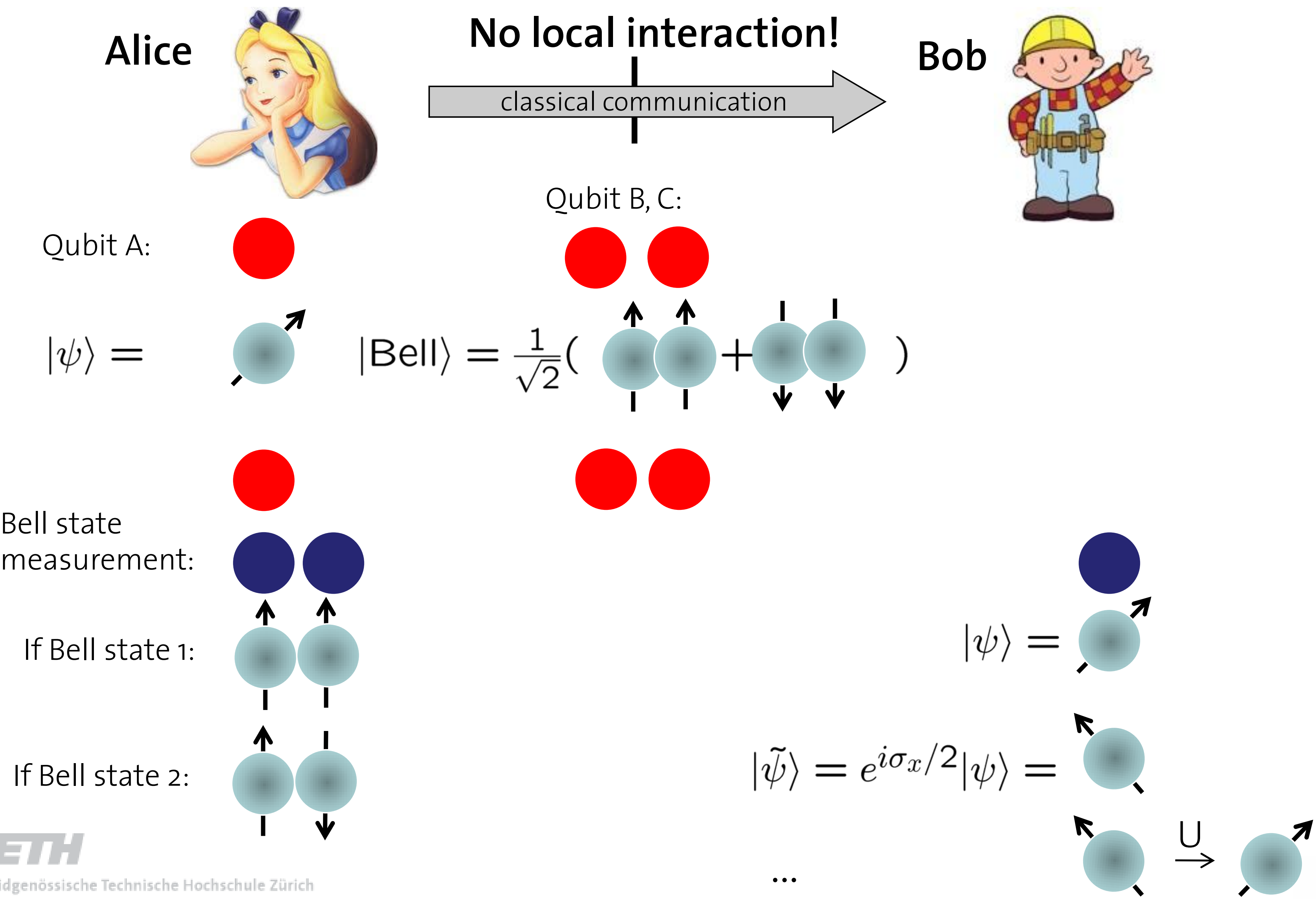
steady state amplitude:

$$\hat{M}_Q = \frac{\kappa}{(\Delta_{rm} + \hat{\delta\omega})_r^2 + (\kappa/2)^2}$$

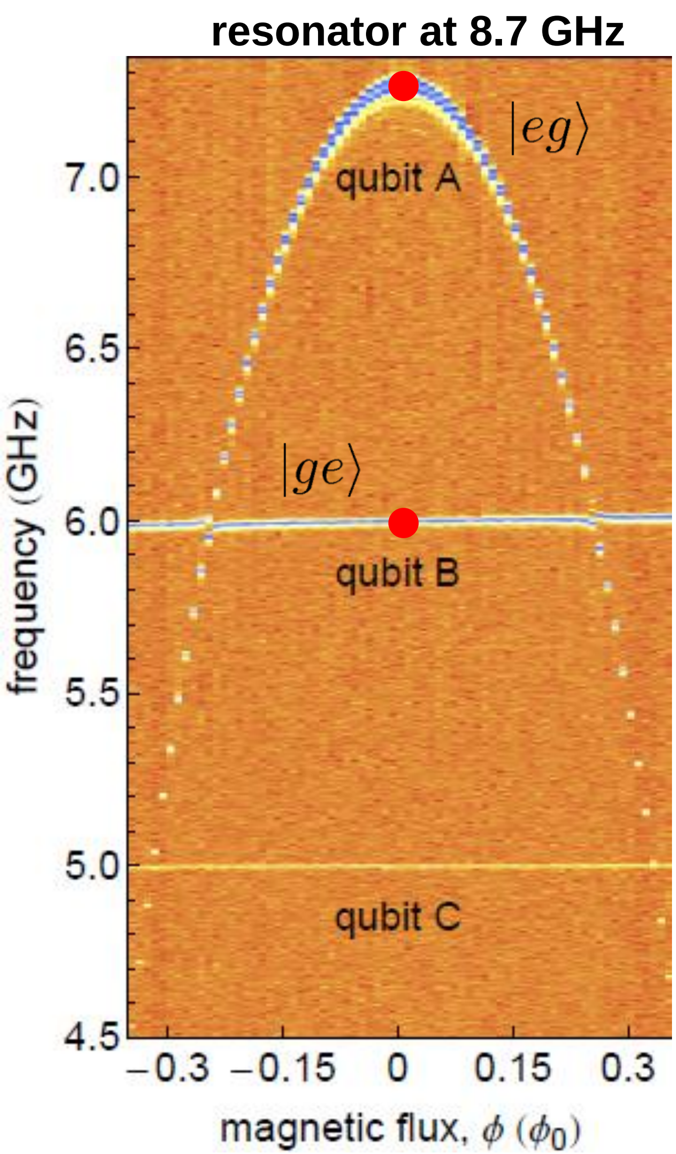


$$\hat{M}_Q = \beta_{00}\,\mathrm{id}\otimes\mathrm{id} + \beta_{10}\,\hat{\sigma}_{z1}\otimes\mathrm{id} + \beta_{01}\,\mathrm{id}\otimes\hat{\sigma}_{z2} + \beta_{11}\,\hat{\sigma}_{z1}\otimes\hat{\sigma}_{z2}$$
$$= \alpha_{++}|ee\rangle\langle ee| + \alpha_{+-}|eg\rangle\langle eg| + \alpha_{-+}|ge\rangle\langle ge| + \alpha_{--}|gg\rangle\langle gg|$$

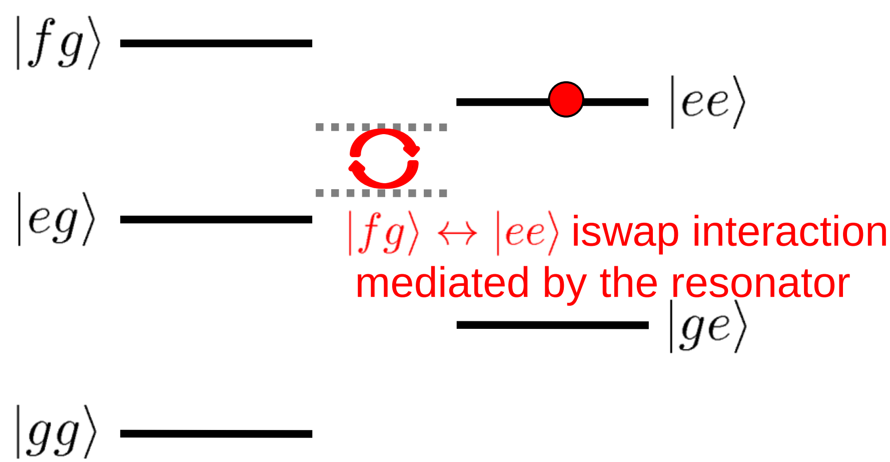
Quantum Teleportation



C-Phase gate



Tune frequency of qubit A with fast magnetic flux pulses



$$|ee\rangle \longrightarrow i|fg\rangle \longrightarrow -|ee\rangle$$

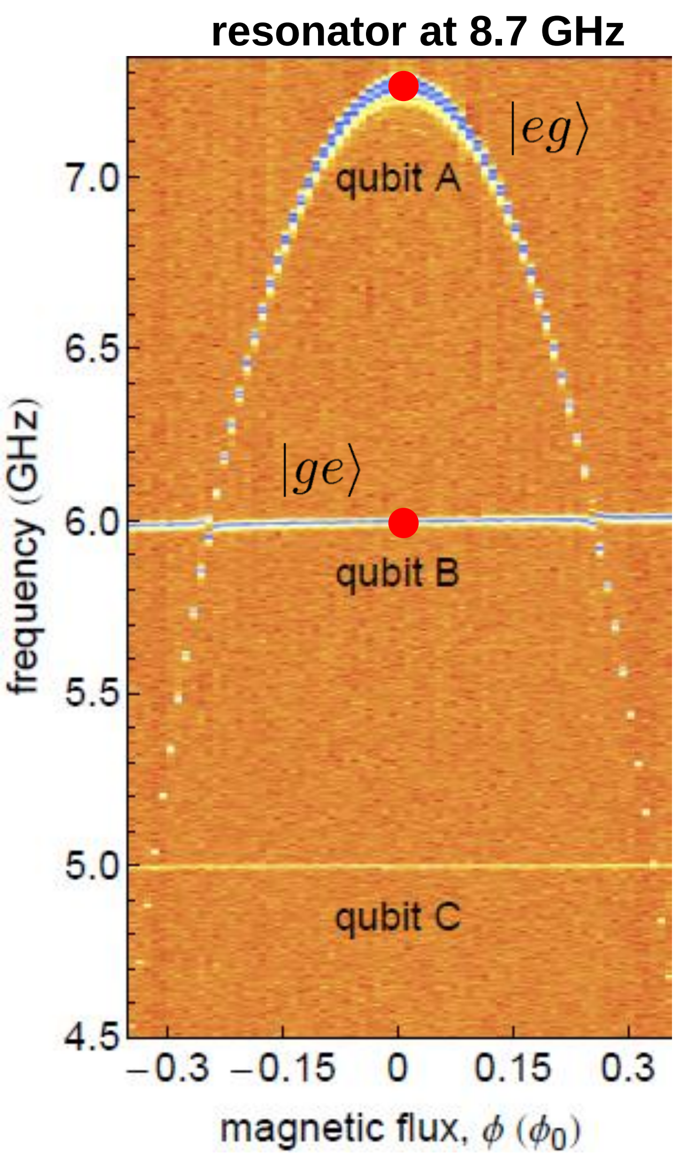
$$|eg\rangle \longrightarrow |eg\rangle \longrightarrow |eg\rangle$$

$$|ge\rangle \longrightarrow |ge\rangle \longrightarrow |ge\rangle$$

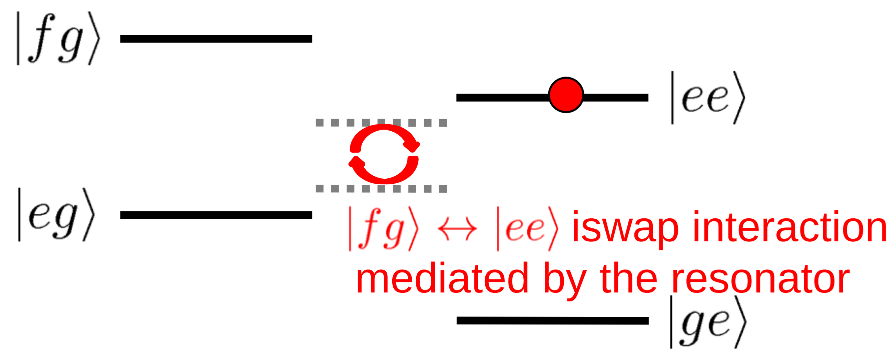
$$|gg\rangle \longrightarrow |gg\rangle \longrightarrow |gg\rangle$$

Proposal: F. W. Strauch, *et al.*, PRL 91 (2003) First realization*:* L. DiCarlo, *et al.*, Nature 460 (2009)

C-Phase gate



Tune frequency of qubit A with fast magnetic flux pulses

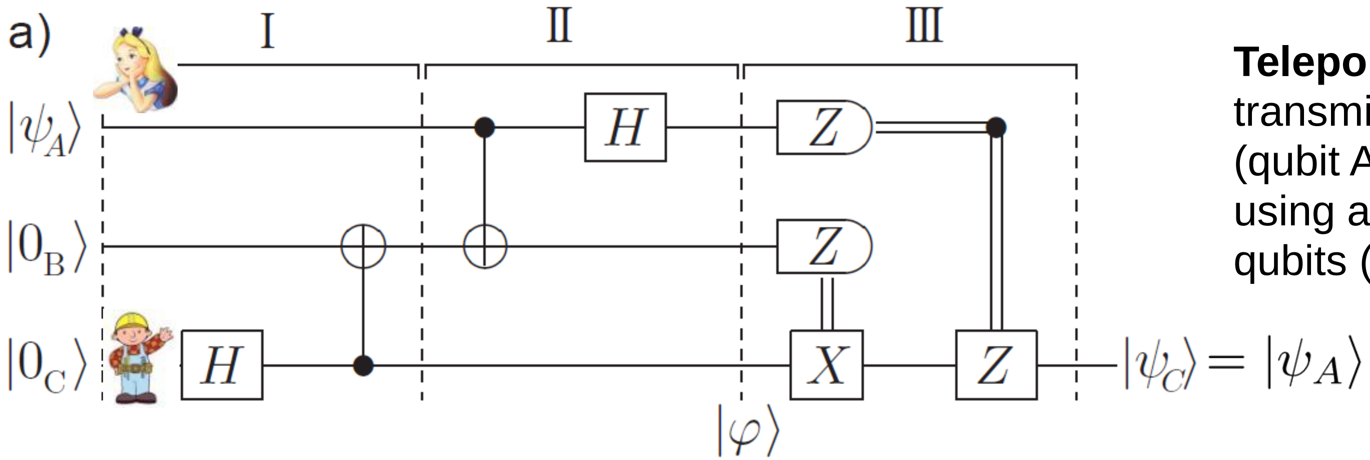


controlled-phase gate:
adds -1 to state $|ee\rangle$

$$U_{cZ_{11}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{matrix} |ee\rangle \\ |eg\rangle \\ |ge\rangle \\ |gg\rangle \end{matrix}$$

Proposal: F. W. Strauch, *et al.*, PRL 91 (2003) First realization: L. DiCarlo, *et al.*, Nature 460 (2009)

Teleportation Circuit



Teleportation:
transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

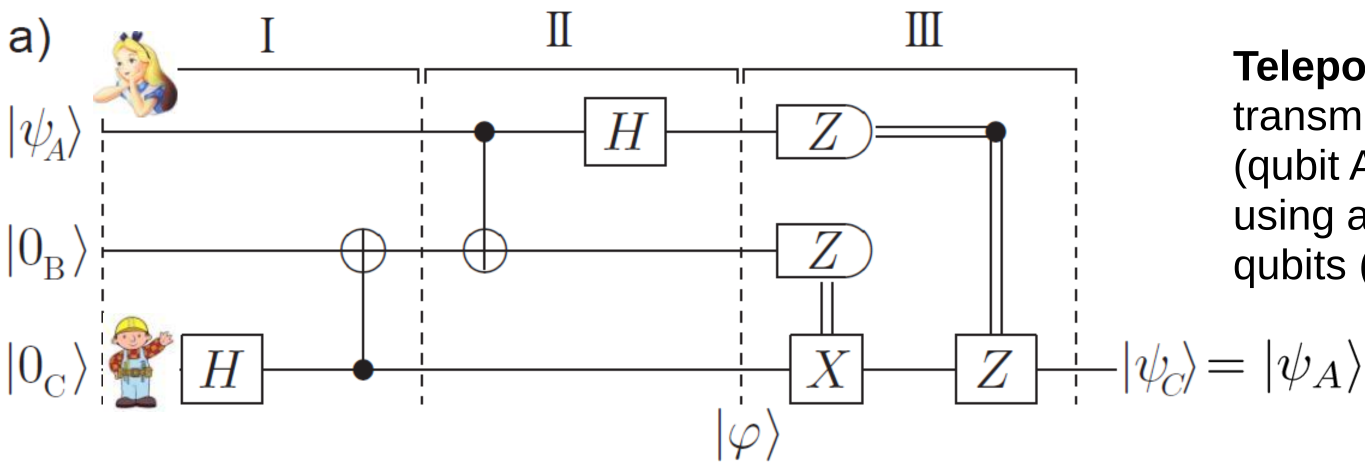
Preparation of Bell state

Measurement + classical communication

Bell Measurement

Teleportation Circuit

a)



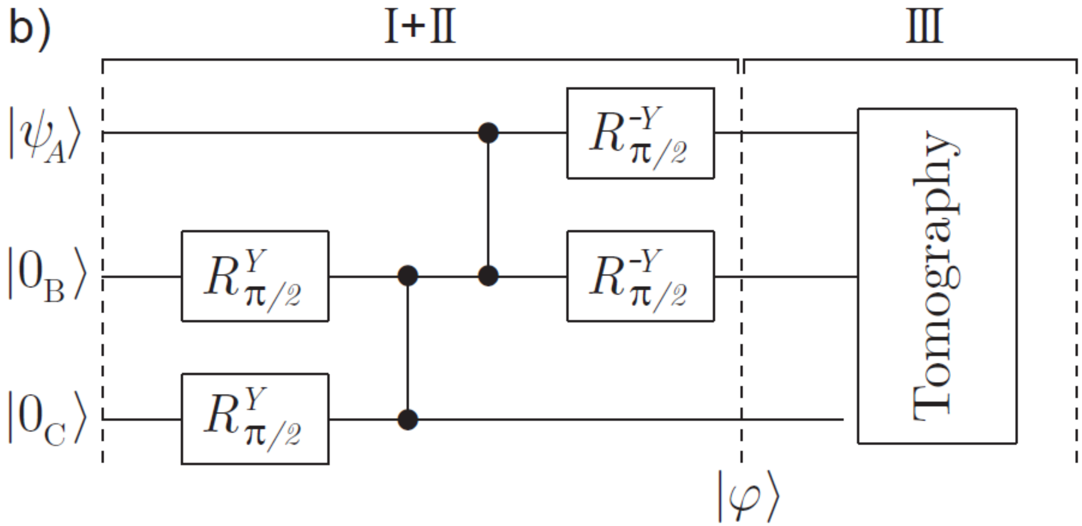
Teleportation:
transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

Preparation of Bell state

Measurement + classical communication

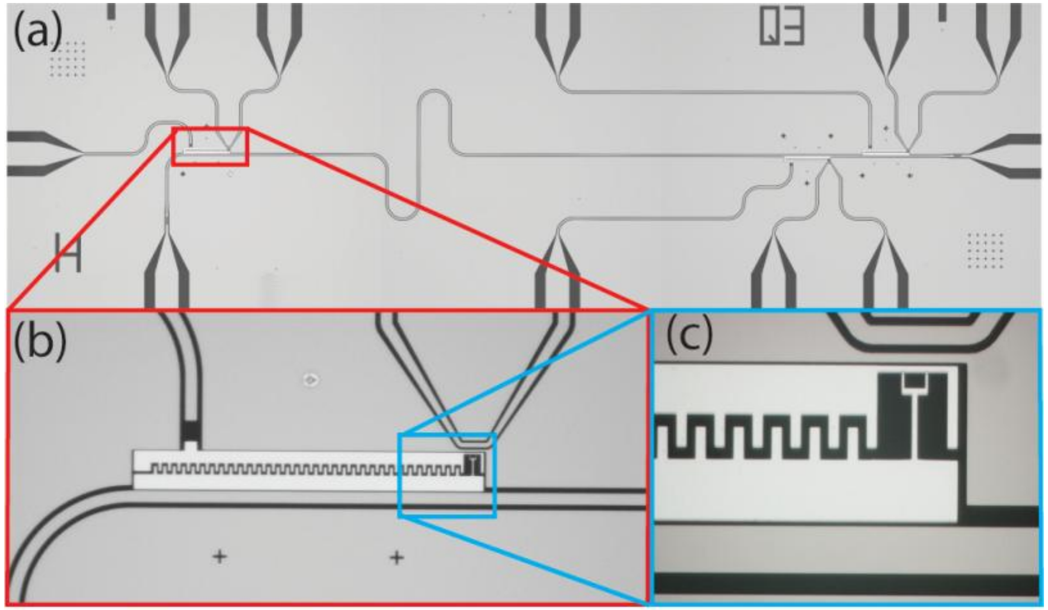
Bell Measurement

b)



implemented three qubit tomography at step III

ETH Quantum processor platform with 3-Qubits

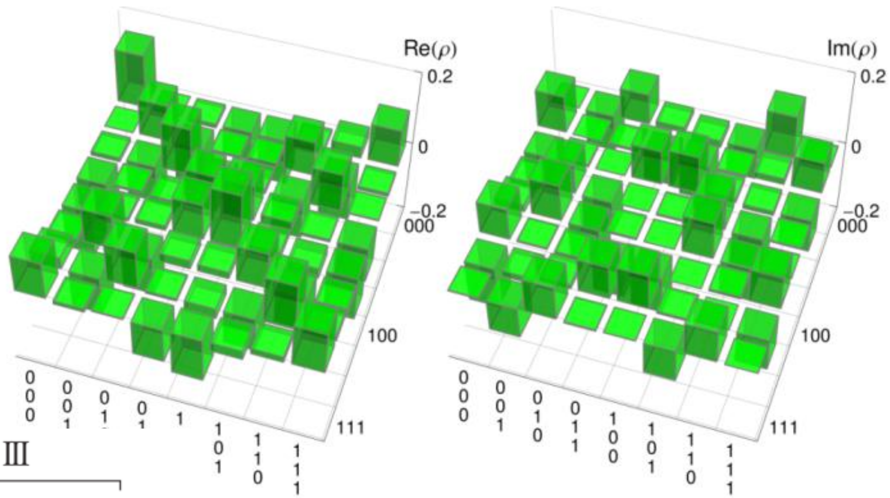


- Full individual coherent qubit control via local charge and flux lines
- Large coupling strength to resonator $g \sim 300 - 350$ MHz
- Transmon coherences times: $T_1 \sim 0.8 - 1.2\ \mu s$, $T_2 \sim 0.4 - 0.7 \mu s$.

[Baur, Fedorov, Steffen, SF, da Silva, Wallraff. arXiv:1107.4774]

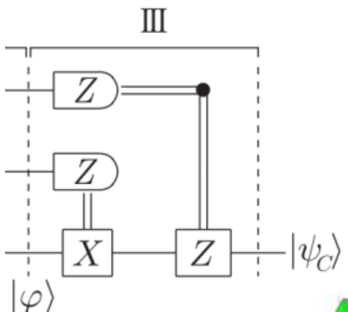
State tomography of the entangled 3-qubit state

Example: State to be teleported on qubit A is $|\Psi\rangle = \frac{1}{\sqrt{2}}(|g\rangle + i|e\rangle)$

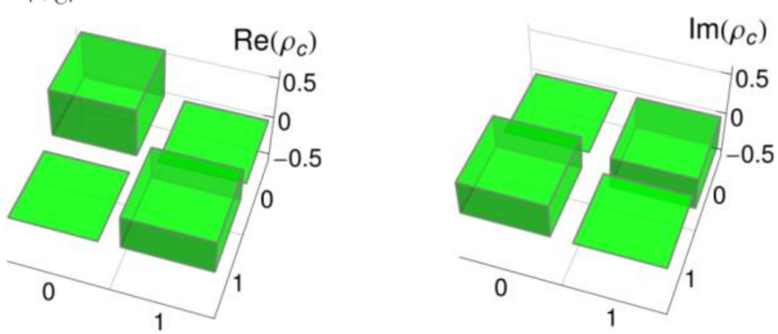


$$
\begin{aligned}
|\varphi\rangle = \{ & |g_A g_B\rangle \otimes |\Psi\rangle_C \\
& + |g_A e_B\rangle \otimes \sigma_x |\Psi\rangle_C \\
& + |e_A g_B\rangle \otimes \sigma_z |\Psi\rangle_C \\
& + |e_A e_B\rangle \otimes (-\sigma_z \sigma_x)|\Psi\rangle_C \}
\end{aligned}
$$

$$\rho = |\varphi\rangle\langle\varphi|$$



Simulating measurement of qubit A and B with projection on $|g_A g_B\rangle$:



$$
\begin{aligned}
\rho_C &= \langle g_A g_B|\rho|g_A g_B\rangle \\
&= |\Psi\rangle\langle\Psi|
\end{aligned}
$$

fidelity 88%

[Baur, Fedorov, Steffen, Filipp, da Silva, Wallraff. PRL 108, 040502 (2012)]

DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A scalable physical system with well-characterized qubits. ✓
#2. The ability to initialize the state of the qubits. ✓
#3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
#4. A universal set of quantum gates. ✓
#5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
#7. The ability to faithfully transmit flying qubits between specified locations. ✓