

Why have the first quantum information processing experiments been performed with photons?

- ↳ Preparation of single photon states by attenuation
- ↳ Detection with high efficiency (single photon detectors)
- ↳ Manipulation of polarization/path (phase shifter, beam splitter, mirror, ...)

→ well developed for photons!

Also: only weak interaction with environment
(long coherence time, long-distance transmission)

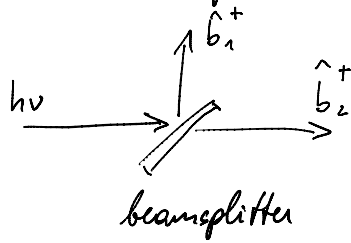
BUT: photon-photon interaction also weak
⇒ impractical for 2-qubit gates

Photon qubits for Quantum Communication:

(e.g. Kok & Lovett - Optical Quantum Inf. Proc.)

e.g. using polarisation + spatial degree of freedom

spatial modes:

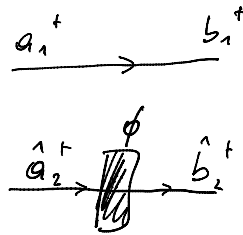


$b_1, b_2 \dots$ spatial modes

$$\begin{aligned}
 |0\rangle &\equiv b_1^\dagger |0\rangle_1 |0\rangle_2 = b_1^\dagger |0,0\rangle = |1,0\rangle \\
 &\quad \uparrow \\
 &\quad \text{photons in mode 1} \\
 |1\rangle &= b_2^\dagger |0,0\rangle = |0,1\rangle
 \end{aligned}
 \left. \vphantom{\begin{aligned} |0\rangle \\ |1\rangle \end{aligned}} \right\} \text{"dual rail representation"}$$

operations:

phase shifter

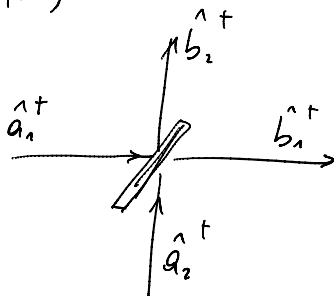


(dielectric with refractive index $n_n \neq 1$)

$$\begin{aligned}
 \hat{a}_1^\dagger &\rightarrow \hat{b}_1^\dagger \\
 \hat{a}_2^\dagger &\rightarrow e^{i\phi} \hat{b}_2^\dagger
 \end{aligned}$$

$$\begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} \hat{b}_1^\dagger \\ \hat{b}_2^\dagger \end{pmatrix}$$

beam splitter (50/50)
(half silvered mirror)



$$\hat{a}_1^\dagger \xrightarrow{BS} \frac{1}{\sqrt{2}} (b_1^\dagger + b_2^\dagger)$$

$$\hat{a}_2^\dagger \xrightarrow{BS} \frac{1}{\sqrt{2}} (-b_1^\dagger + b_2^\dagger)$$

(minus sign:
reflected beam gets
phase shift; unitary
transformation)

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} b_1^\dagger \\ b_2^\dagger \end{pmatrix}$$

Example: photon at input 1:

$$|0\rangle = |1,0\rangle = a_1^\dagger |0,0\rangle \xrightarrow{BS} \frac{1}{\sqrt{2}} (b_1^\dagger + b_2^\dagger) |0,0\rangle$$

$$= \frac{1}{\sqrt{2}} (|1,0\rangle + |0,1\rangle) =$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

→ 50/50 beamsplitter: 50% probability for scattering into either output port

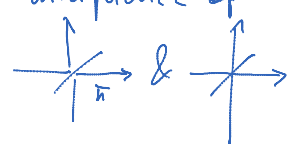
photons at input 1 & 2: (Hong-Ou-Mandel)

$$a_1^\dagger a_2^\dagger |0\rangle_1 |0\rangle_2 \rightarrow \frac{1}{2} (b_1^\dagger + b_2^\dagger) (-b_1^\dagger + b_2^\dagger) |0\rangle_1 |0\rangle_2 =$$

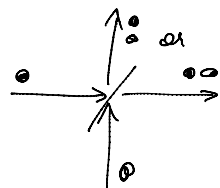
$$= \frac{1}{2} (b_1^\dagger b_1^\dagger + b_2^\dagger b_1^\dagger - b_1^\dagger b_2^\dagger - b_2^\dagger b_2^\dagger) |0\rangle_1 |0\rangle_2 =$$

$$= \frac{\sqrt{2}}{2} (-|2\rangle_1 |0\rangle_2 + |0\rangle_1 |2\rangle_2)$$

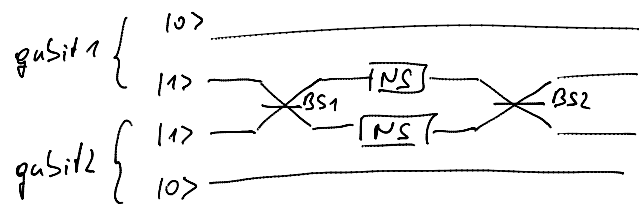
⊙ destructive interference of



→ perfect antibunching



(Non-deterministic) C-Phase gate:



non-linear sign gate (NS) $\propto |0\rangle + 3|1\rangle + 8|2\rangle \rightarrow \alpha|0\rangle + \beta|1\rangle - \gamma|2\rangle$

induced by x) non-linearity

x) non-deterministically (success prob 25%)

operation: $|00\rangle_L = |1001\rangle \xrightarrow{BS+NS+BS} |00\rangle_L$

(no two photons @ NS gates)

$$|01\rangle_L = |1010\rangle \rightarrow |01\rangle_L$$

$$|10\rangle_L = |0101\rangle \rightarrow |10\rangle_L$$

$$|11\rangle_L = |0110\rangle \xrightarrow{BS1} |0\rangle \otimes \frac{1}{\sqrt{2}} (|20\rangle + |02\rangle) \otimes |0\rangle$$

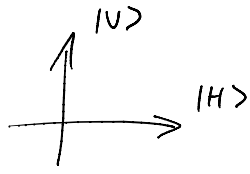
$$\xrightarrow{NS} -|0\rangle \otimes \frac{1}{\sqrt{2}} (|20\rangle + |02\rangle) \otimes |0\rangle$$

$$\xrightarrow{BS2} -|0110\rangle = -|11\rangle_L$$

(bunching)

success probability: $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

polarization basis:

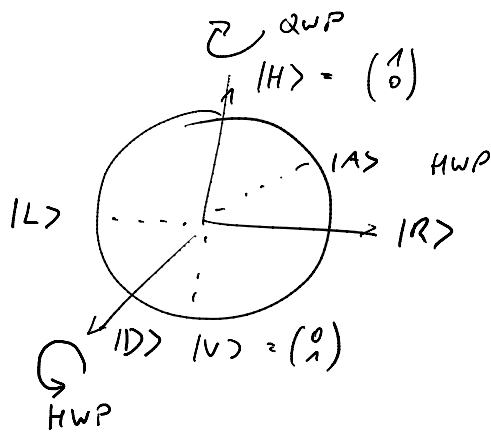


a_H^+ ... creation of photon in mode a with horizontal pol.

a_V^+ ... vertical pol.

$$|0\rangle \equiv a_H^+ |0,0\rangle_{HV} = |1,0\rangle_{HV} = |H\rangle$$

$$|1\rangle \equiv a_V^+ |0,0\rangle_{HV} = |0,1\rangle_{HV} = |V\rangle$$



$$|L\rangle = \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle)$$

$$|R\rangle = \frac{1}{\sqrt{2}} (|H\rangle - i|V\rangle)$$

$$|D\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle)$$

$$|A\rangle = \frac{1}{\sqrt{2}} (|H\rangle - |V\rangle)$$

Operations:

half-wave plate: $|H\rangle \rightarrow \cos 2\theta |H\rangle + i \sin 2\theta |V\rangle$
 $|V\rangle \rightarrow i \sin 2\theta |H\rangle + \cos 2\theta |V\rangle$

$$U_{\text{HWP}}(\theta) = \begin{pmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{pmatrix}$$

\Rightarrow rotation about x-axis

$$\theta = \frac{\pi}{4}: U_{\text{HWP}} = e^{i\frac{\pi}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv X$$

$$|V\rangle \rightarrow |H\rangle; |H\rangle \rightarrow |V\rangle$$

quarter wave plate: $\phi_f - \phi_s = \frac{\pi}{2} \quad \theta = \frac{\pi}{4}$

$$U_{\text{QWP}} = e^{-i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \equiv Z/2$$

\Rightarrow rotation about z-axis

$$|L\rangle = \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle) \hat{=} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$U_{\text{QWP}} |L\rangle \rightarrow e^{-i\frac{\pi}{4}} (|H\rangle - |V\rangle) \propto |A\rangle$$

\rightarrow transforms linear \leftrightarrow circular transformation

$\rightarrow \frac{1}{2} + \frac{1}{4}$ waveplates are sufficient to create arbitrary single-qubit operations!

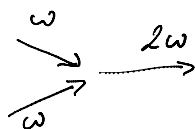
Parametric Down Conversion

for creation of entangled photon pairs

due to non-linear response of medium to electric field

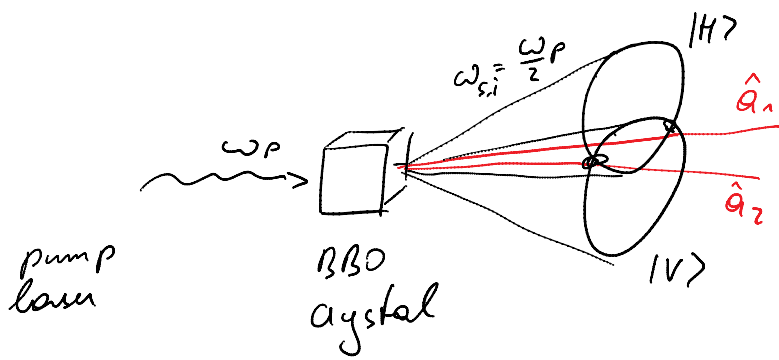
polarization $P = \epsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots)$

for $E(x,t) = E_0 e^{i(kx - \omega t)} \Rightarrow \chi^{(2)} E^2 \propto e^{-2i\omega t}$



up-conversion

reverse process: down-conversion:

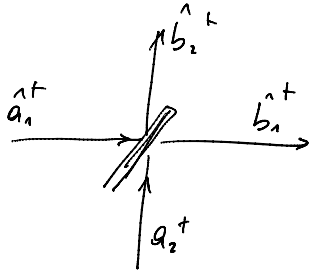


- conservation of
- energy ($\omega_p = \omega_s + \omega_i$)
 - momentum ($\vec{k}_p = \vec{k}_s + \vec{k}_i$)

at crossing points: $|\Psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2)$

anti-symmetric Bell state

Bell State measurement using 50/50 beamsplitter

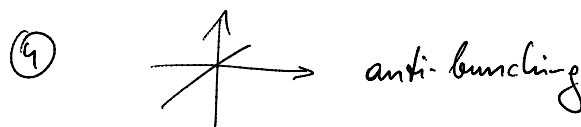
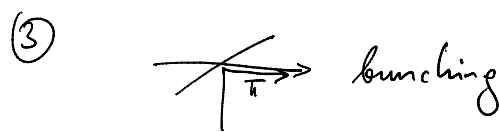
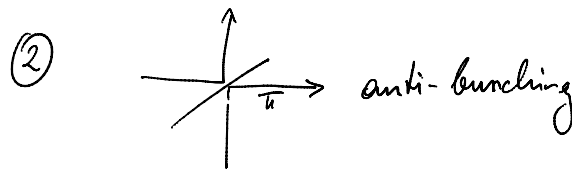
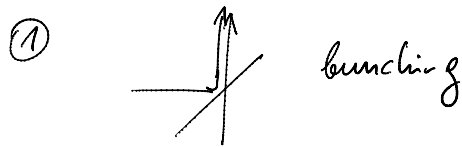


$$a_1^{(\dagger)} \xrightarrow{BS} \frac{1}{\sqrt{2}} (\hat{a}_1^{(\dagger)} + \hat{a}_2^{(\dagger)})$$

$$a_2^{(\dagger)} \xrightarrow{BS} \frac{1}{\sqrt{2}} (-\hat{a}_1^{(\dagger)} + \hat{a}_2^{(\dagger)})$$

two photons incident on the beamsplitter:

4 possibilities:



Hong-Ou Mandel effect: for identical polarization ($\Phi_{\pm}^{\dagger} = \frac{1}{\sqrt{2}} (|HH\rangle \pm |VV\rangle)$)

→ destructive interference
due to phase shift of π

cannot be distinguished

⇒ bunching

BUT: anti-bunching for anti-symmetric state
 $|\psi^-\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle)!$


input state: $(\hat{v}_i = \hat{a}_{iV}, \hat{h}_i = \hat{a}_{iH})$ $\frac{1}{\sqrt{2}} (h_1^+ v_2^+ \mp v_1^+ h_2^+)$ $\xrightarrow{\text{BS}}$ "apply beam splitter transformation to each mode"

\swarrow anti-sym
 \nwarrow symm.

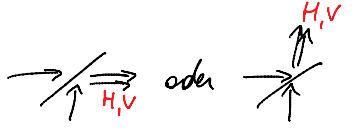
$$\frac{1}{2} \frac{1}{\sqrt{2}} (h_1^+ + h_2^+) (v_2^+ - v_1^+) \mp (v_1^+ + v_2^+) (h_2^+ - h_1^+) =$$

$$= \frac{1}{2\sqrt{2}} \left[\underbrace{h_1^+ v_2^+}_{\text{red}} + \underbrace{h_2^+ v_2^+}_{\text{blue}} - \underbrace{h_1^+ v_1^+}_{\text{blue}} - \underbrace{h_2^+ v_1^+}_{\text{red}} \mp \underbrace{v_1^+ h_2^+}_{\text{red}} \mp \underbrace{v_2^+ h_2^+}_{\text{blue}} \pm \underbrace{v_1^+ h_1^+}_{\text{blue}} \pm \underbrace{v_2^+ h_1^+}_{\text{red}} \right]$$

for anti-symmetric spatial wavefunction (-): "red"

$$= \frac{1}{\sqrt{2}} (h_1^+ v_2^+ - v_1^+ h_2^+) \Rightarrow \text{anti-bunching}$$


for symmetric spatial wavefunction (+): "blue"

$$= \frac{1}{\sqrt{2}} (-h_1^+ v_1^+ + h_2^+ v_2^+) \Rightarrow \text{bunching}$$


bunching also for other symmetric spatial wavefunctions $\left[\frac{1}{\sqrt{2}} (h_1^+ h_2^+ \pm v_1^+ v_2^+) \right]$

$$\left[\frac{1}{\sqrt{2}} (h_1^+ h_2^+ \pm v_1^+ v_2^+) \right] \Rightarrow \text{bunching}$$

Super Dense Coding:

1) Preparation of initial entangled state (PDC)

$$|\psi^+\rangle = \frac{1}{\sqrt{2}} (|HV\rangle + |VH\rangle)$$

2) Generation of 4 maximally entangled 2-photon polarization states

$$|\psi^+\rangle \xrightarrow{I_2} |\psi^+\rangle$$

$$|\psi^+\rangle \xrightarrow{X_2} \frac{1}{\sqrt{2}} (|HH\rangle + |VV\rangle) = |\phi^+\rangle \quad (\text{HWP})$$

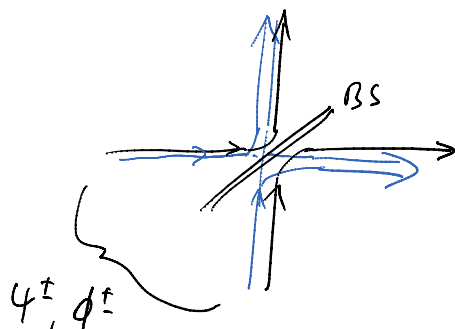
(i) ... omitted

$$|\psi^+\rangle \xrightarrow{Z_2} \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle) = |\psi^-\rangle \quad (\text{QWP})$$

$$|\psi^+\rangle \xrightarrow{Z_2 X_2} \frac{1}{\sqrt{2}} (|HH\rangle - |VV\rangle) = |\phi^-\rangle \quad (\text{HWP} + \text{QWP})$$

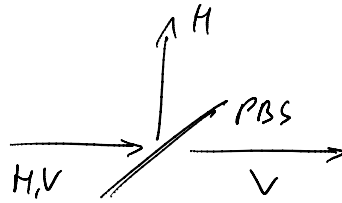
3) Bell State measurement using beam splitter

A) distinguish symmetric ($|\psi^+\rangle, |\phi^+\rangle, |\phi^-\rangle$) from antisymmetric ($|\psi^-\rangle$) state using a beam splitter (BS)



anti-bunching for ψ^-
bunching for symm. states

B) distinguish polarization states using polarizing beam splitter



outcomes: $|\psi^+\rangle$: coincidence D_H & D_V or $D_{H'}$ & $D_{V'}$

$|\psi^-\rangle$: coincidence $D_{H'}$ & D_V or D_H & $D_{V'}$

$|\phi^+\rangle, |\phi^-\rangle$: 2 photons in $D_H, D_V, D_{H'}, D_{V'}$
(cannot be distinguished)