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Swiss Federal Institute of Technology Zurich

Grover algorithm in superconducting circuits

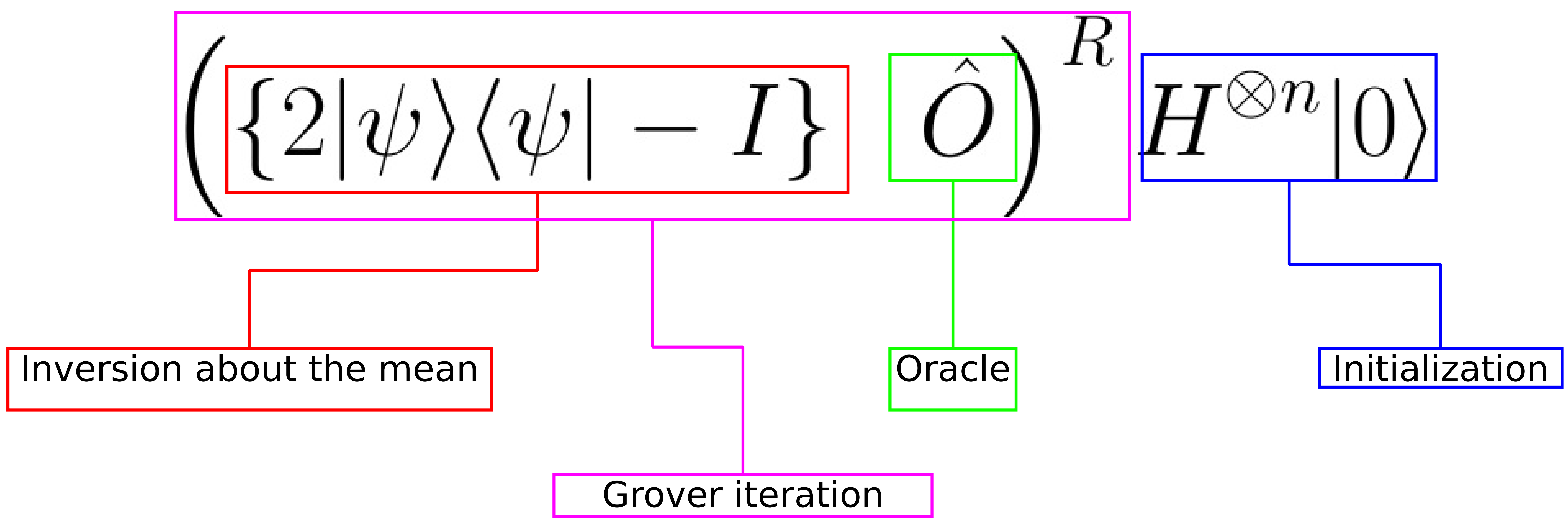
Folkert de Vries & Sergió Solórzano

Grover's search algorithm

Given a name find the phone number -> structured search (easy)

Given a phone number, find the matching name -> unstructured search (hard)

$$\left(\{2|\psi\rangle\langle\psi| - I\}\ \hat{O}\right)^R H^{\otimes n}|0\rangle$$



The oracle

Don't know the solution but is able to recognize it.

By definition:

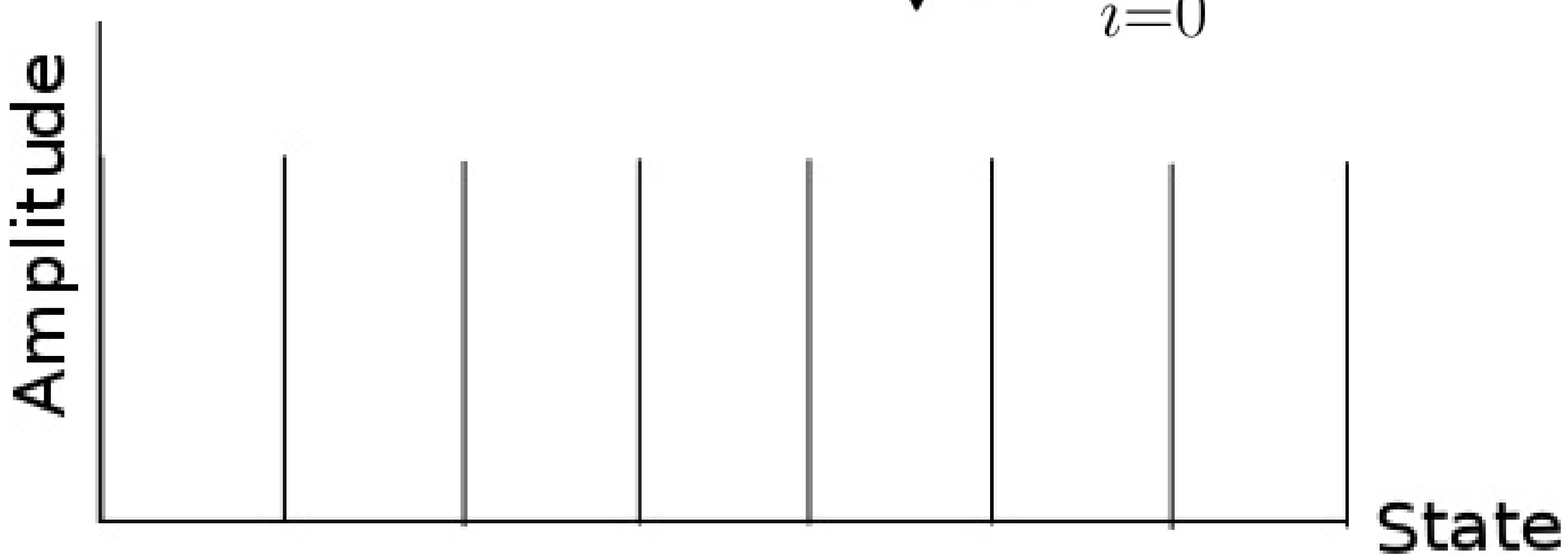
$$\hat{O}|x\rangle = (-1)^{f(x)}|x\rangle$$

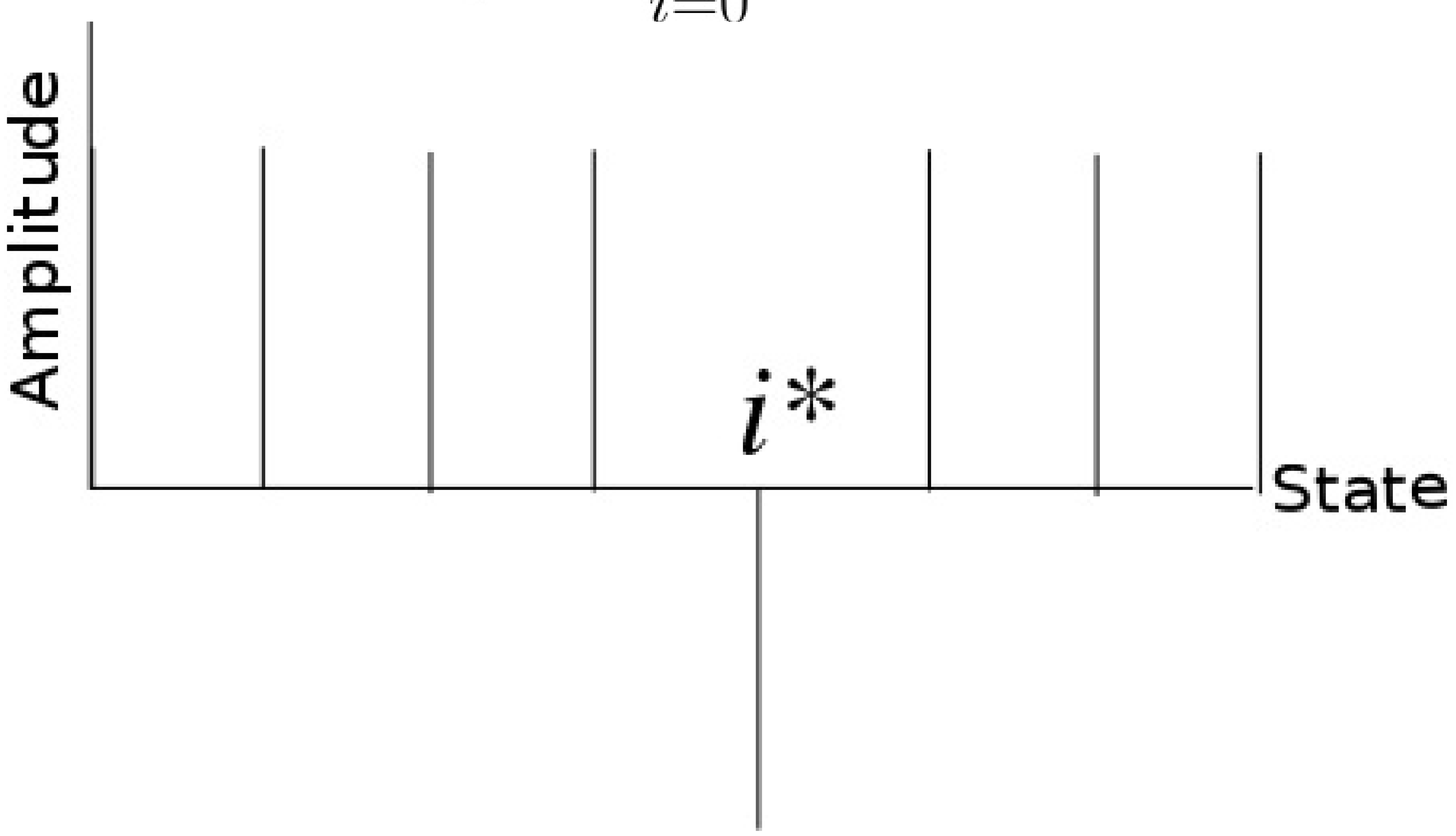
$$f(x) = \begin{cases} 1 & x \text{ Is a solution.} \\ 0 & x \text{ Is not a solution.} \end{cases}$$

Initialization

Sets a state that uses the quantum mechanical "built in" paralelization

$$H^{\otimes n}|0\rangle = |\psi\rangle = \frac{1}{\sqrt{N}}\sum_{i=0}^{N}|i\rangle \qquad \hat{O}|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{i=0}^{N}(-1)^{f(i)}|i\rangle$$





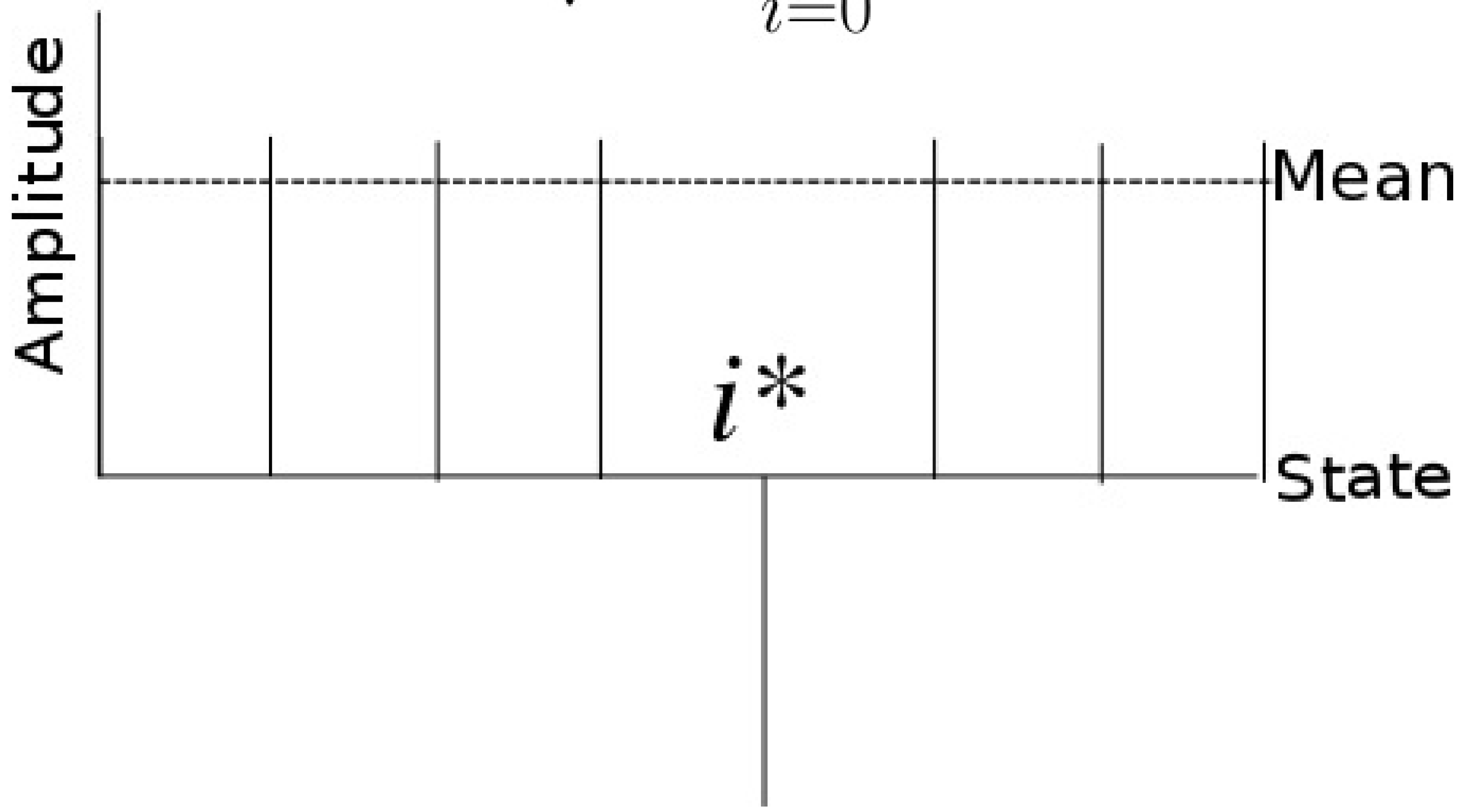
Not useful yet, all states are equally probable.

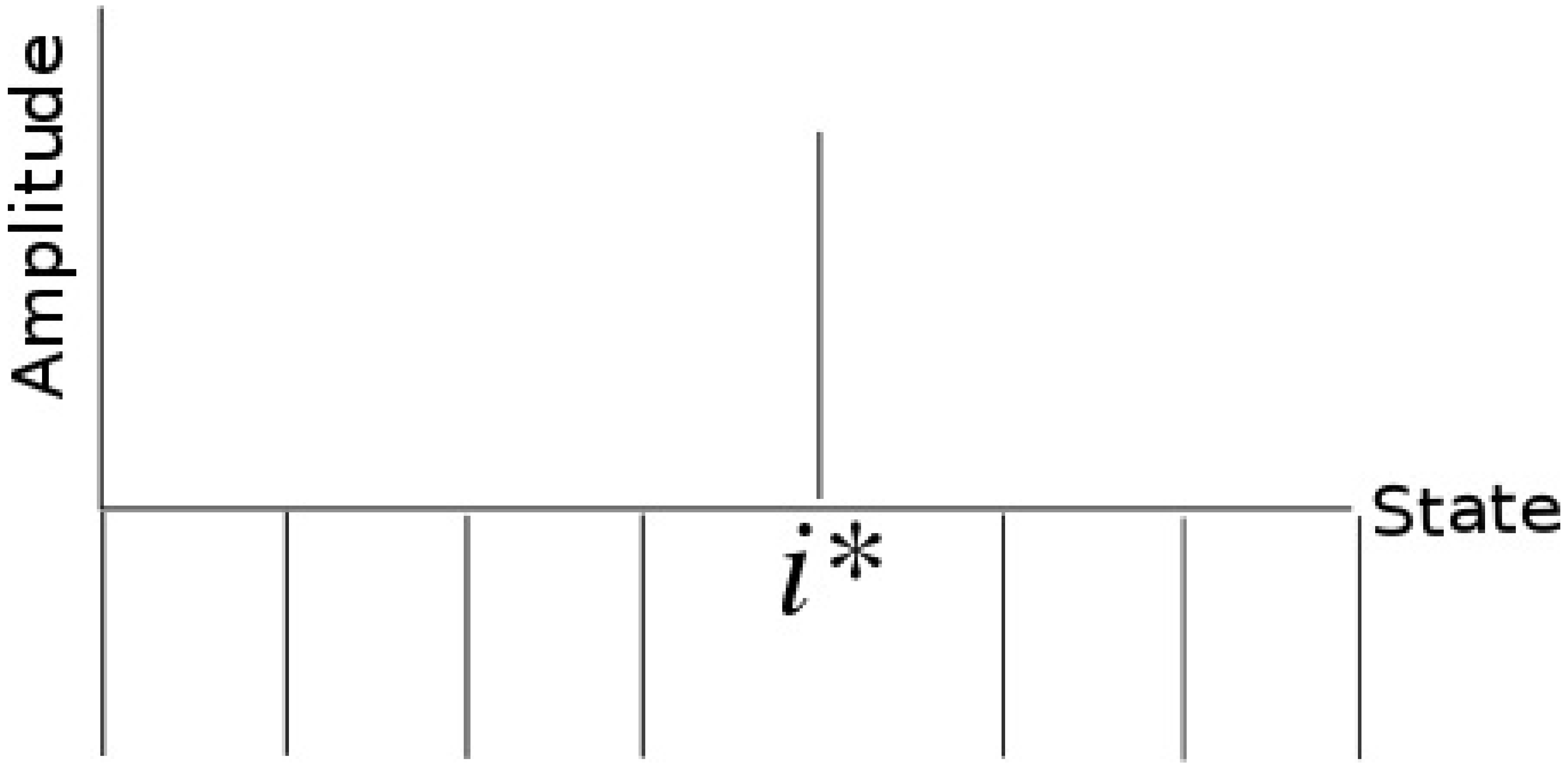
Inversion about the mean

$$\{2|\psi\rangle\langle\psi|-I\}\sum_{i=0}^{N}\alpha_i|i\rangle = \sum_{i=0}^{N}\left(-\alpha_i + 2\langle\alpha\rangle\right)|i\rangle$$

$$\hat{O}|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{i=0}^{N}(-1)^{f(i)}|i\rangle$$

$$\{2|\psi\rangle\langle\psi|-I\}\hat{O}|\psi\rangle$$





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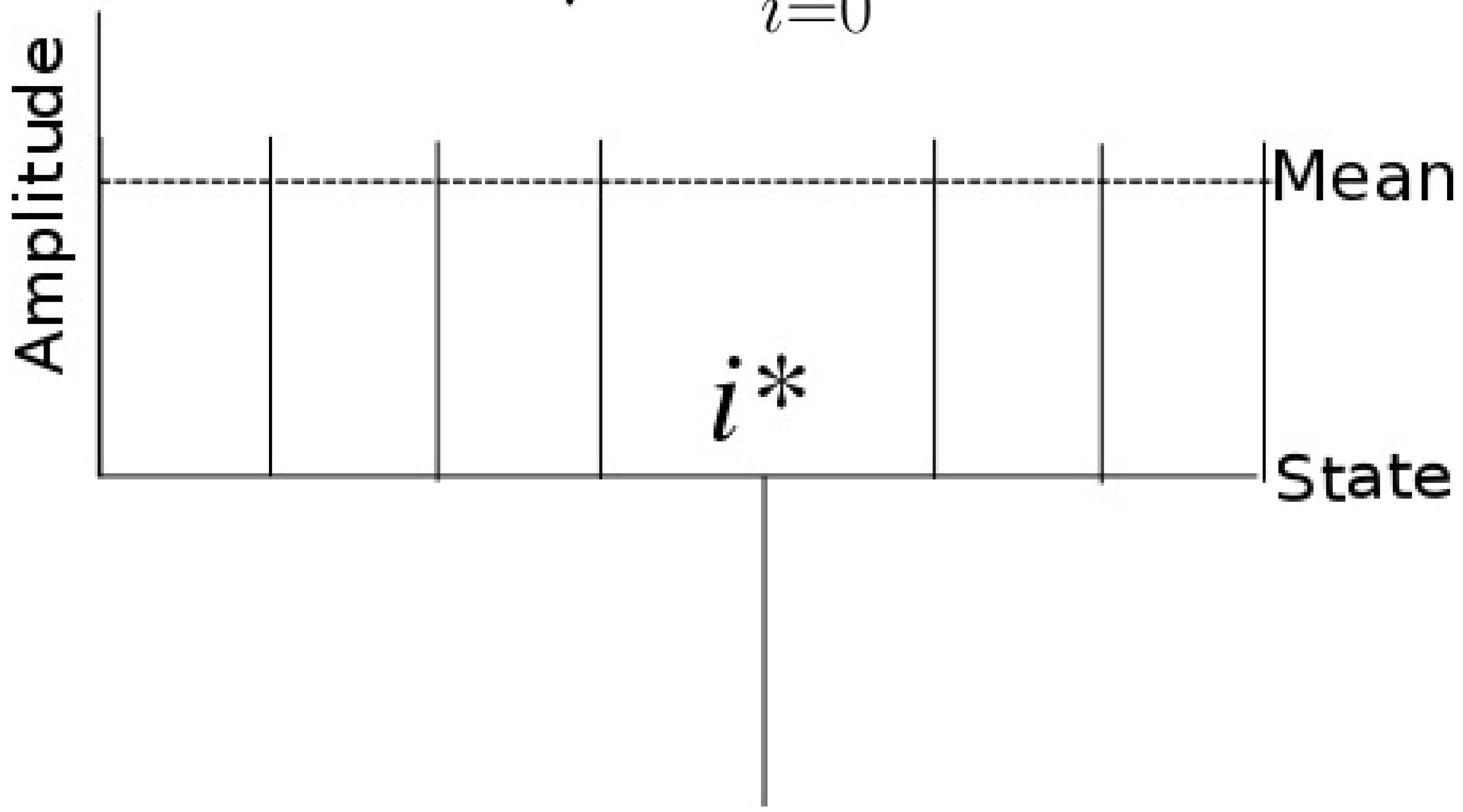
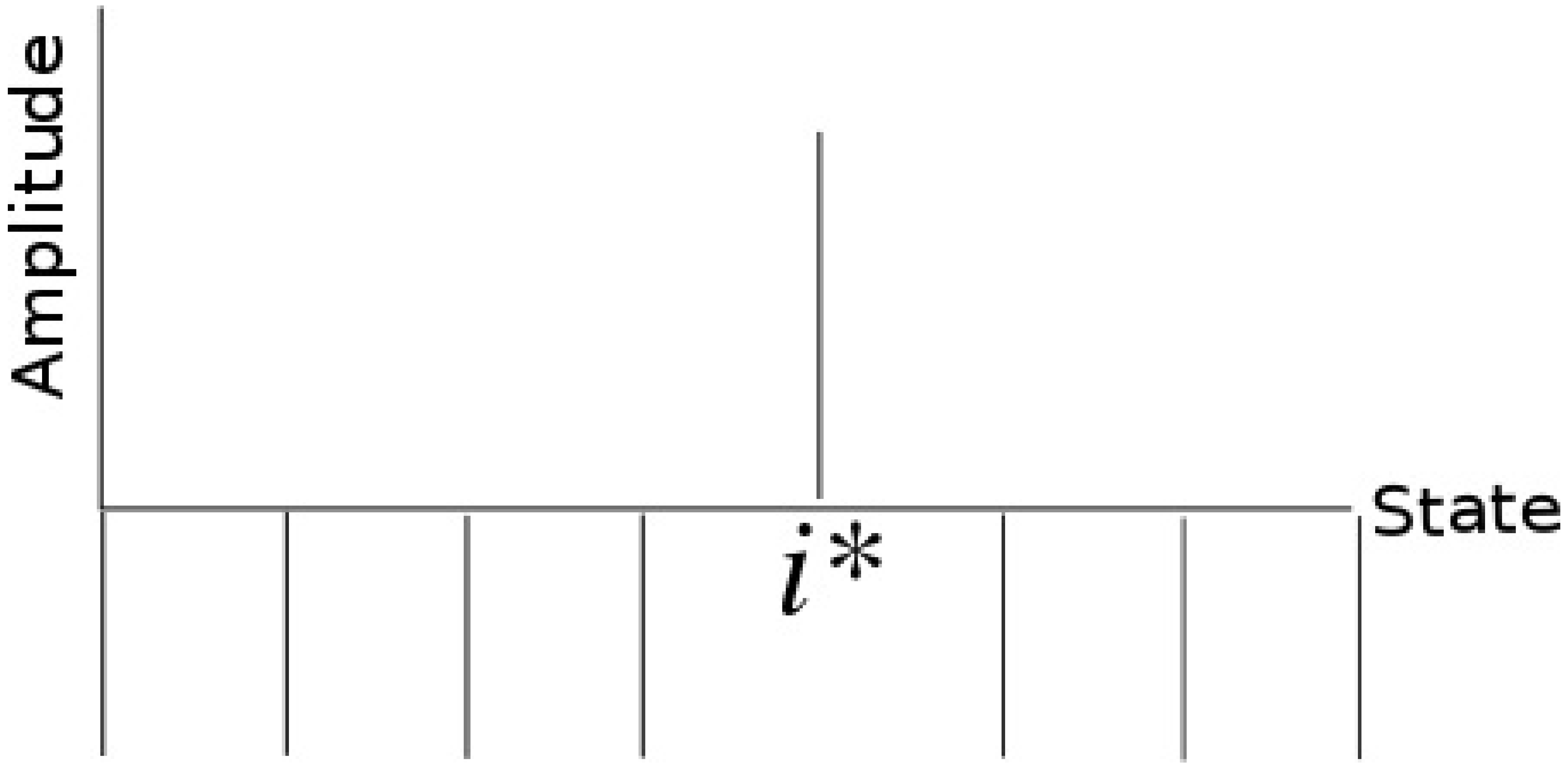
Inversion about the mean

$$\{2|\psi\rangle\langle\psi| - I\} \sum_{i=0}^{N} \alpha_i |i\rangle = \sum_{i=0}^{N} \left(-\alpha_i + 2\langle\alpha\rangle\right) |i\rangle$$

$$\hat{O}|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N} (-1)^{f(i)} |i\rangle$$

$$\{2|\psi\rangle\langle\psi| - I\} \hat{O}|\psi\rangle$$





The amplitude of the target state grows while all other aplitudes shrink.

Inversion about the mean

$$2|\psi\rangle\langle\psi| - I = H^{\otimes n}\left(2|0\rangle\langle 0| - I\right) H^{\otimes n}$$

$$= H^{\otimes n} C_{00} H^{\otimes n}$$

$$C_{00}|x\rangle = -(-1)^{\delta_{x,0}}|x\rangle$$

The inversion about the mean operation can be decomposed into one-qubit and two-qubit operations

Grover's search algorithm

$$\left(\{2|\psi\rangle\langle\psi| - I\}\;\hat{O}\right)^R H^{\otimes n}|0\rangle$$

- $\{2|\psi\rangle\langle\psi| - I\}$: Inversion about the mean
- $\hat{O}$: Oracle
- $\left(\{2|\psi\rangle\langle\psi| - I\}\;\hat{O}\right)$: Grover iteration
- $H^{\otimes n}|0\rangle$: Initialization

$R \approx O(\sqrt{N})$ Steps

$O(\log(N))$ Qubits

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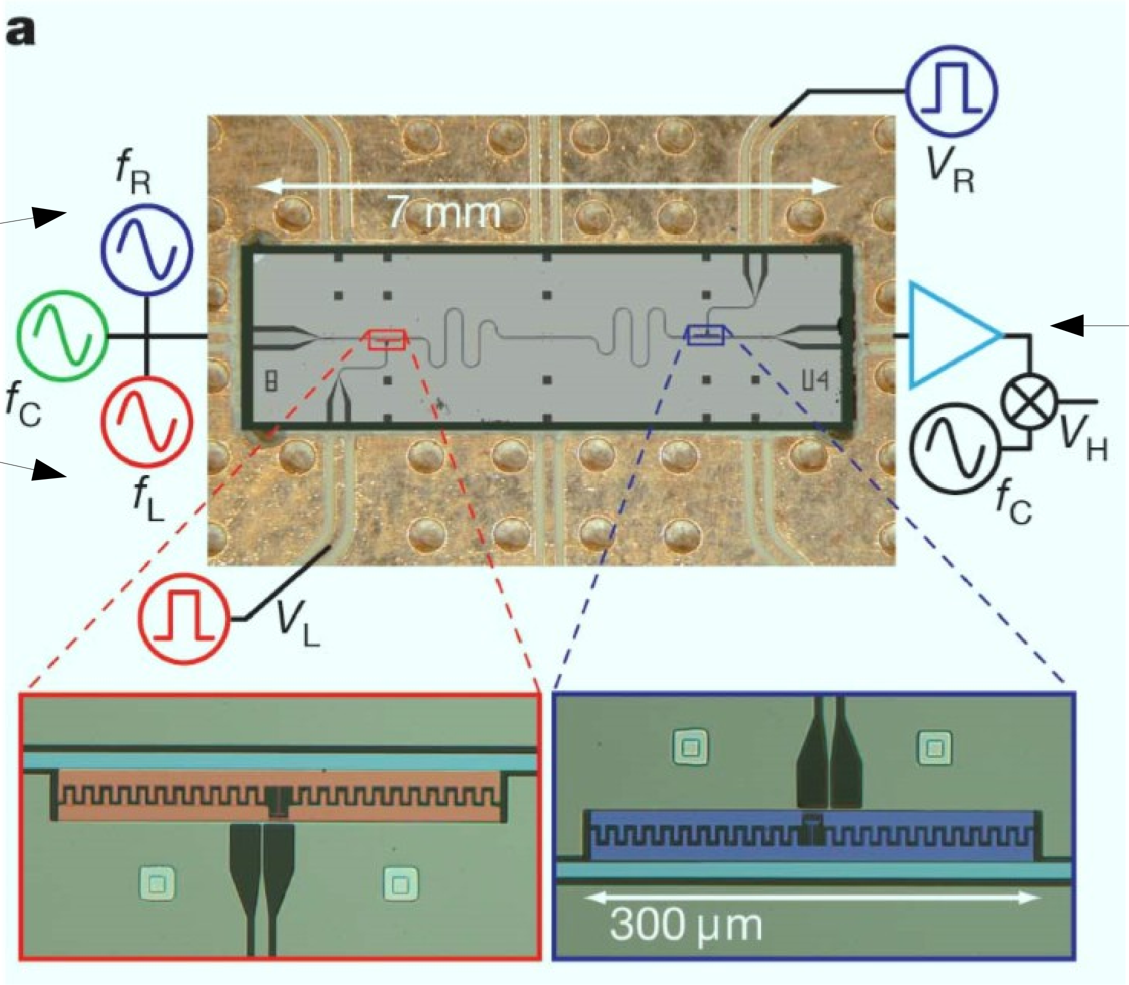
Experiment 1

Single qubit rotations



Two qubit readout

DiCarlo et al. Nature 460, 240 (2009)

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) - \sum_{i=1,2}\frac{\hbar\omega_i}{2}\sigma_{i,z} + \hbar\sum_{i=1,2} g_i(a^\dagger\sigma_i^- + a\sigma_i^+).$$

↓ Dispersive regime

$$H = \hbar\left(\omega_\mathrm{r} + \chi_1\sigma_{1,z} + \chi_2\sigma_{2,z}\right)a^\dagger a +$$

$$\sum_{i=1,2}\hbar\frac{\chi_i - \omega_i}{2}\sigma_{i,z} + \hbar J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+).$$

Jaynes-Cummings Hamiltonian

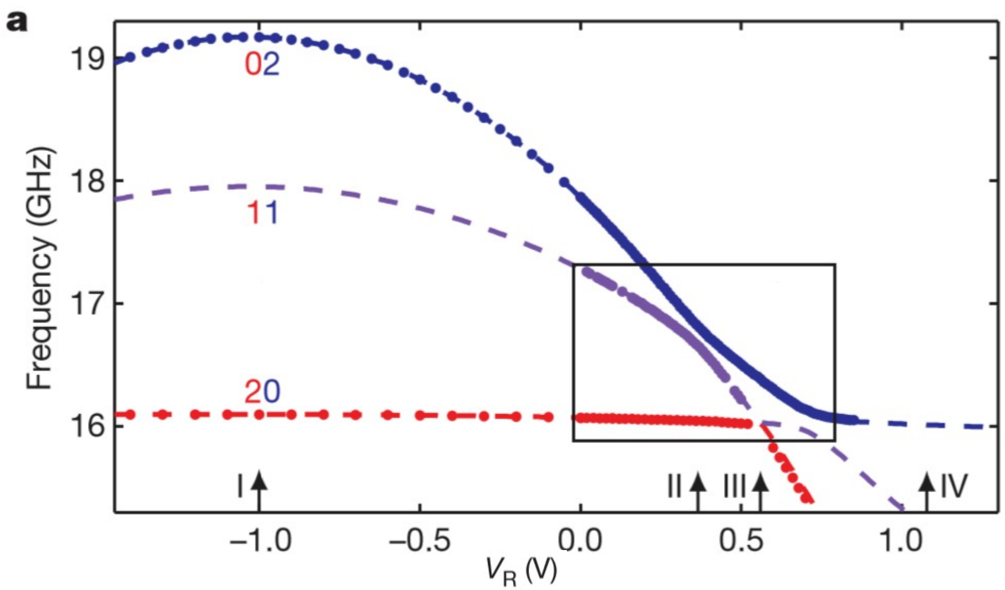
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) - \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_{i,z} + \hbar \sum_{i=1,2} g_i (a^\dagger \sigma_i^- + a\sigma_i^+).$$

↓ Dispersive regime

$$H = \hbar\left(\omega_\mathrm{r} + \chi_1\sigma_{1,z} + \chi_2\sigma_{2,z}\right) a^\dagger a + \sum_{i=1,2} \hbar \frac{\chi_i - \omega_i}{2}\sigma_{i,z} + \hbar J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+).$$

$$J = \tfrac{1}{2} g_1 g_2 \left(\tfrac{1}{\omega_1 - \omega_\mathrm{r}} + \tfrac{1}{\omega_2 - \omega_\mathrm{r}}\right)$$

Oracle: Controlled-phase gate

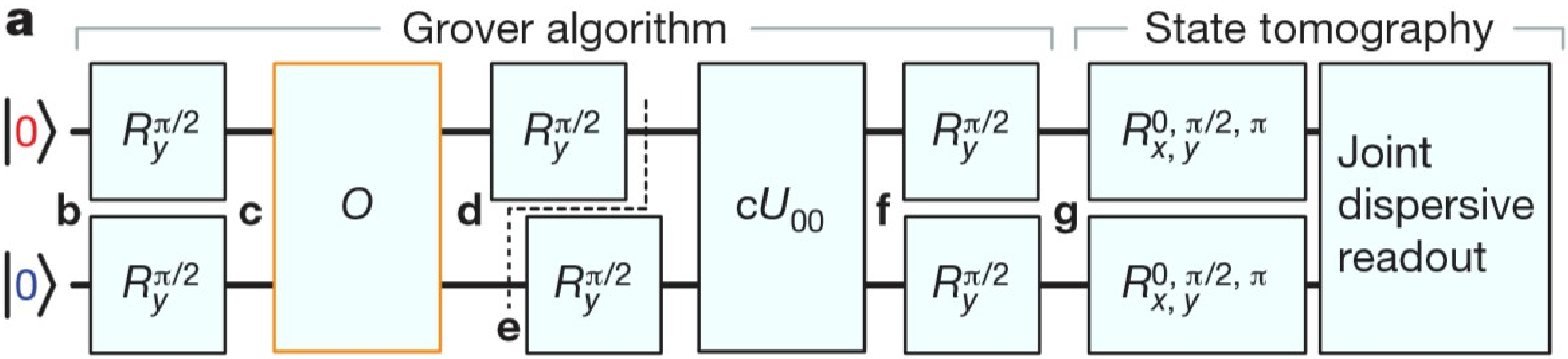


$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |10\rangle$$
$$|11\rangle \rightarrow -|11\rangle$$

C-Phase gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Schematic setup



Initialization

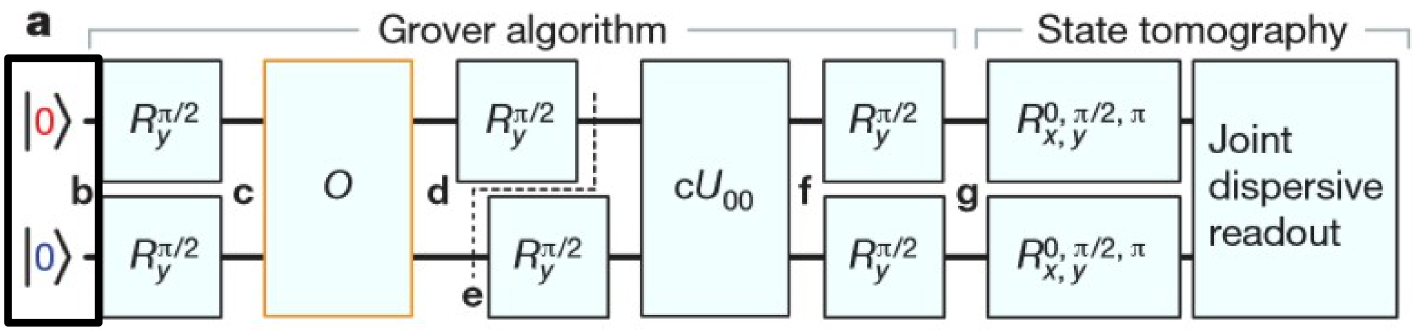
C-Phase

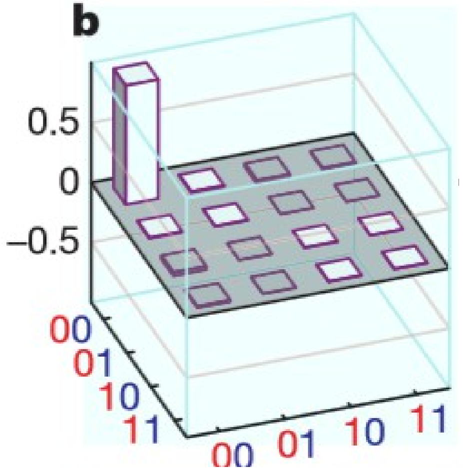
Replacing Hadamards

C_{00}

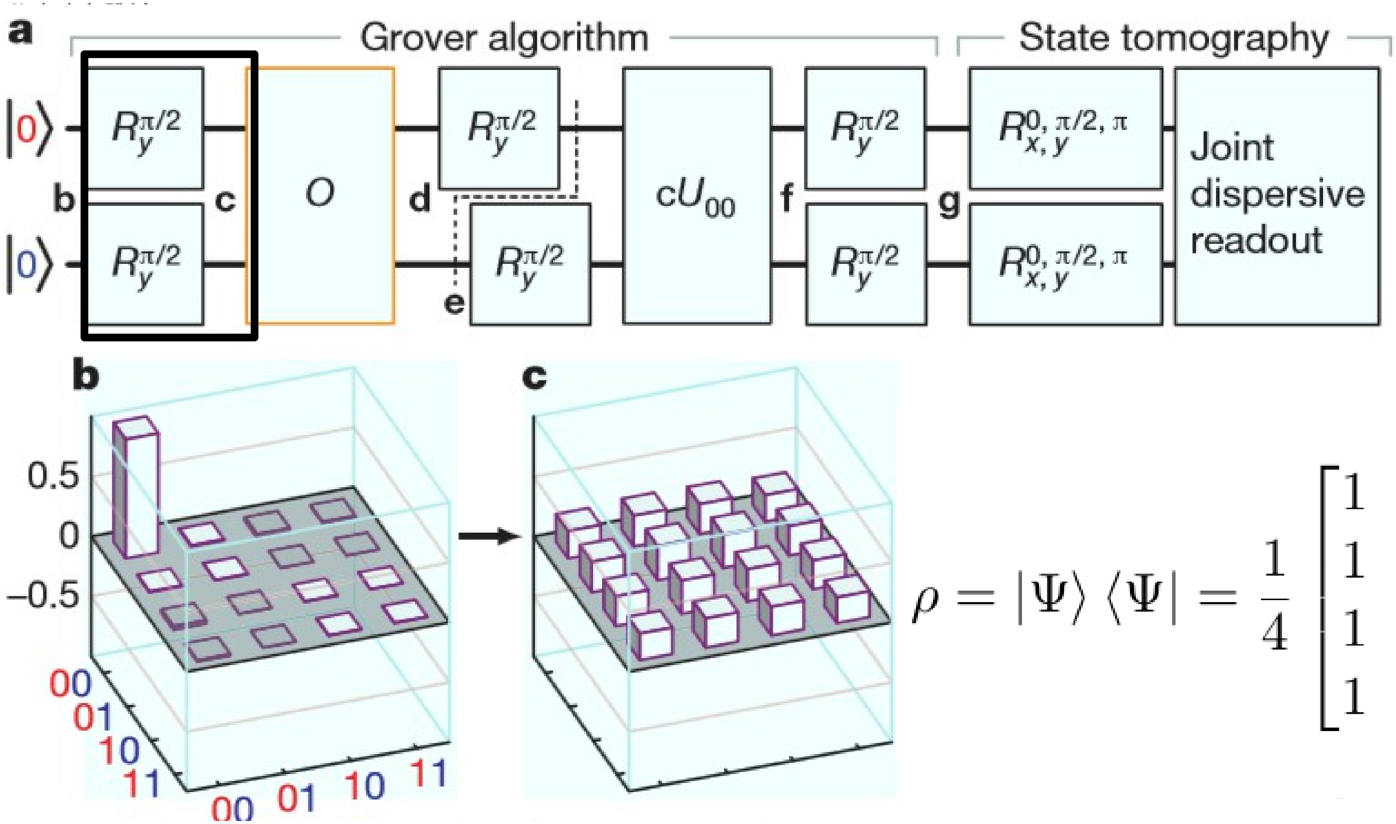
Replacing Hadamards

Inversion about the mean



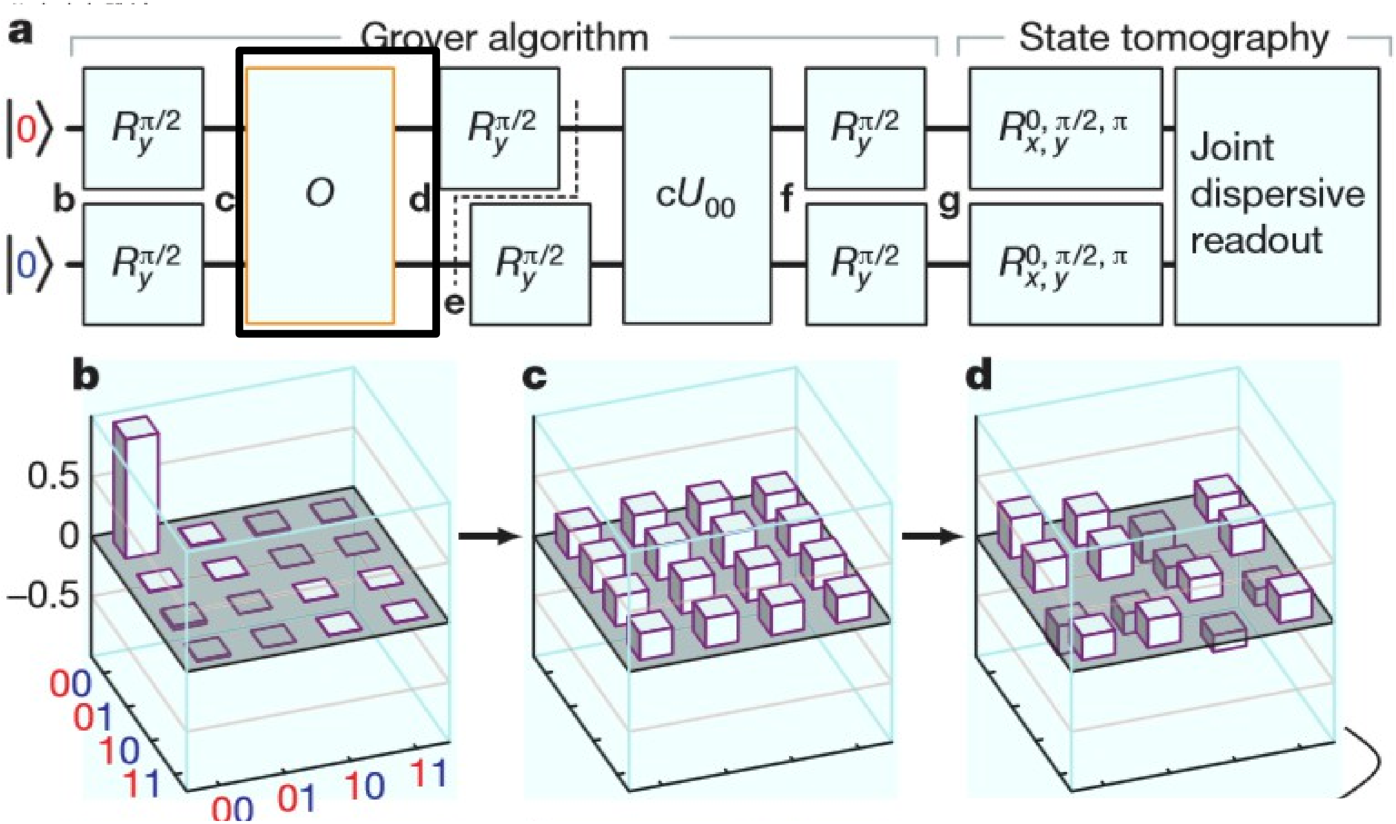


$$|\Psi\rangle = |00\rangle = |0\rangle_1 \otimes |0\rangle_2$$

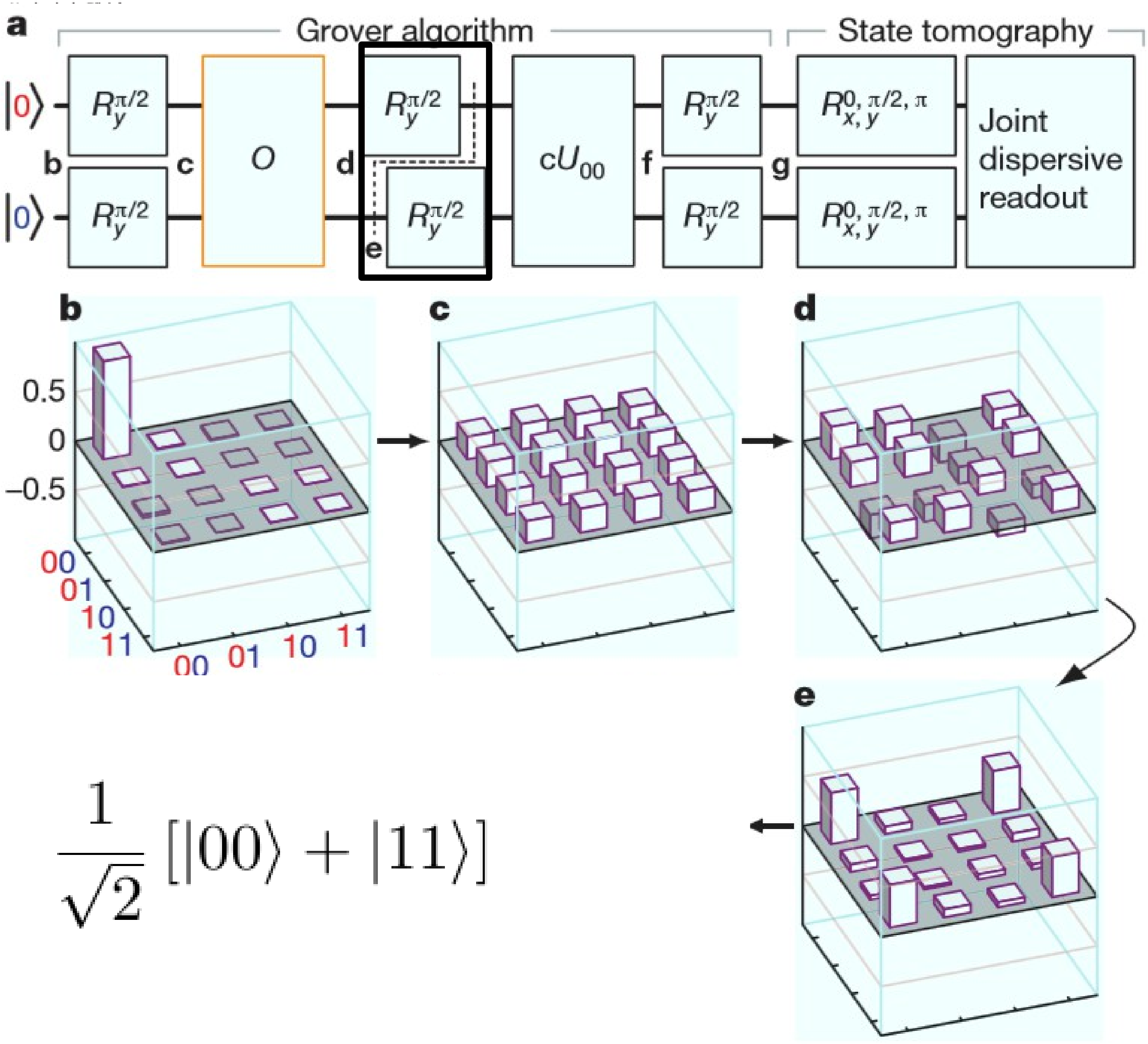


$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

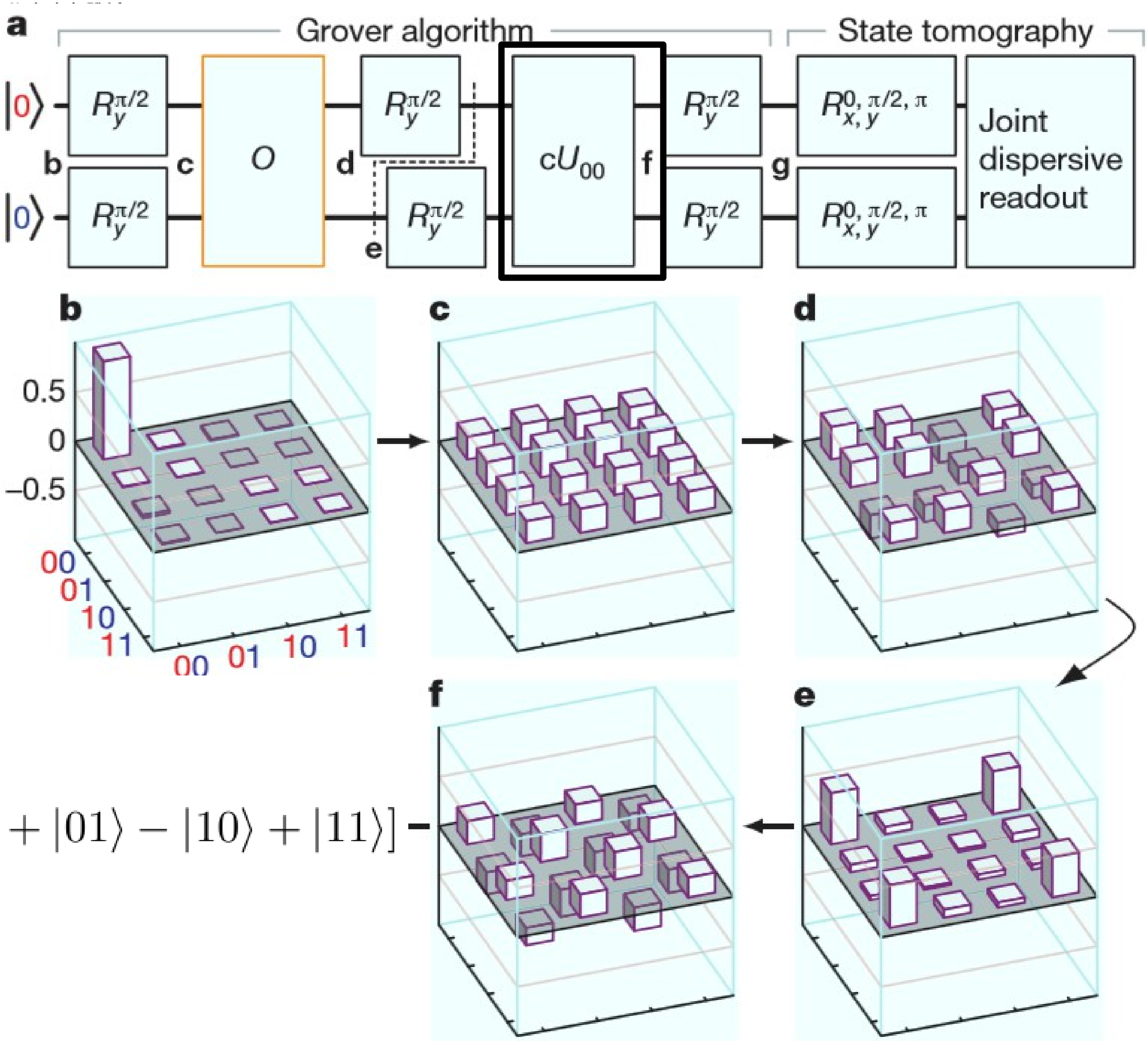
$$R_y^{\pi/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \longrightarrow \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow \frac{1}{2}\left[|00\rangle + |01\rangle - |10\rangle + |11\rangle\right]$$



$$\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$



$$\frac{1}{\sqrt{2}}\left[-|00\rangle + |01\rangle - |10\rangle + |11\rangle\right]$$

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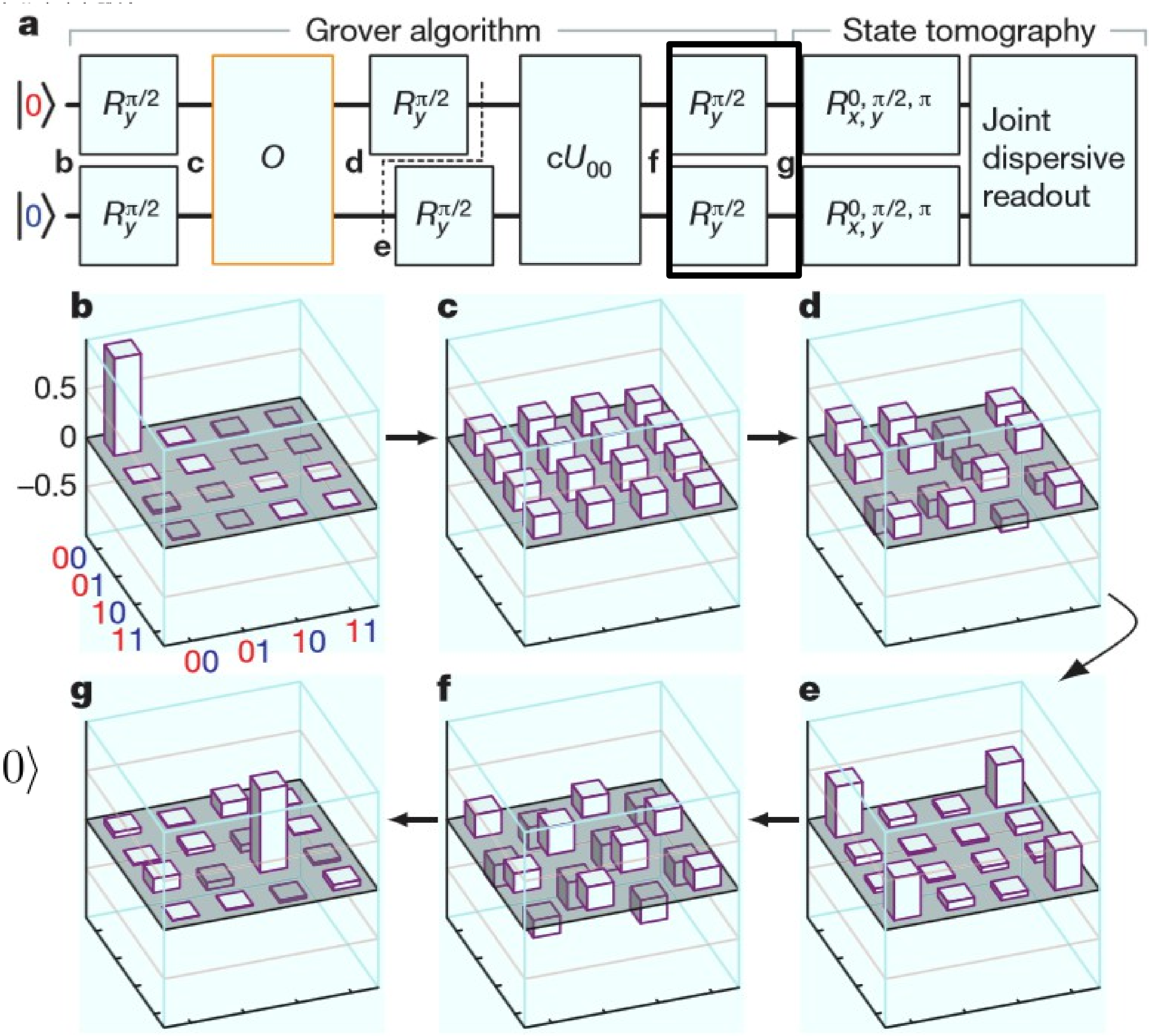
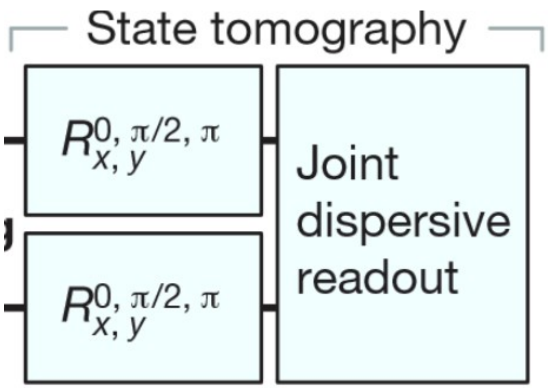


$$|\Psi\rangle = |10\rangle$$

Tomography: Joint dispersive read out

$$M = \beta_1 \sigma_z^L + \beta_2 \sigma_z^R + \beta_{12} \sigma_z^L \otimes \sigma_z^R$$



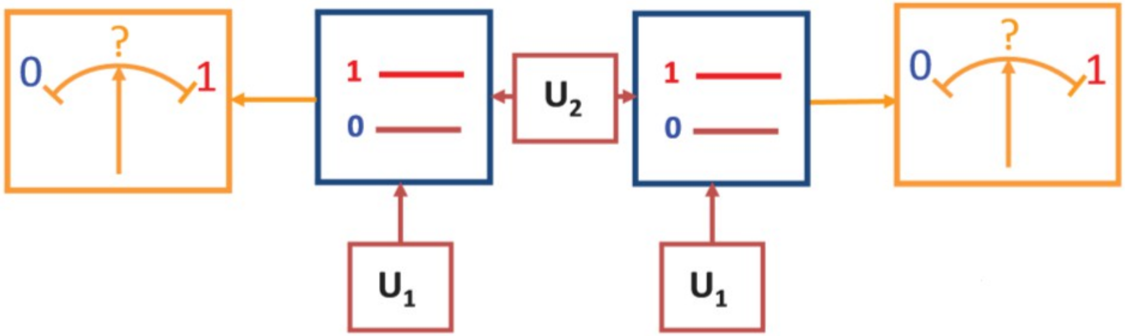
- Dispersive regime, all β's are approximately equal.
- Single qubit rotations: $0 \; R_x^{\pi/2} \; R_y^{\pi/2} \; R_x^{\pi}$
- 15 observables (16 -1) by single qubit rotations before measuring M.
- Likelyhood estimator for the averages over 450.000 runs.

Results DiCarlo et al.

$$F(\rho, \psi) = \langle \psi | \rho | \psi \rangle$$

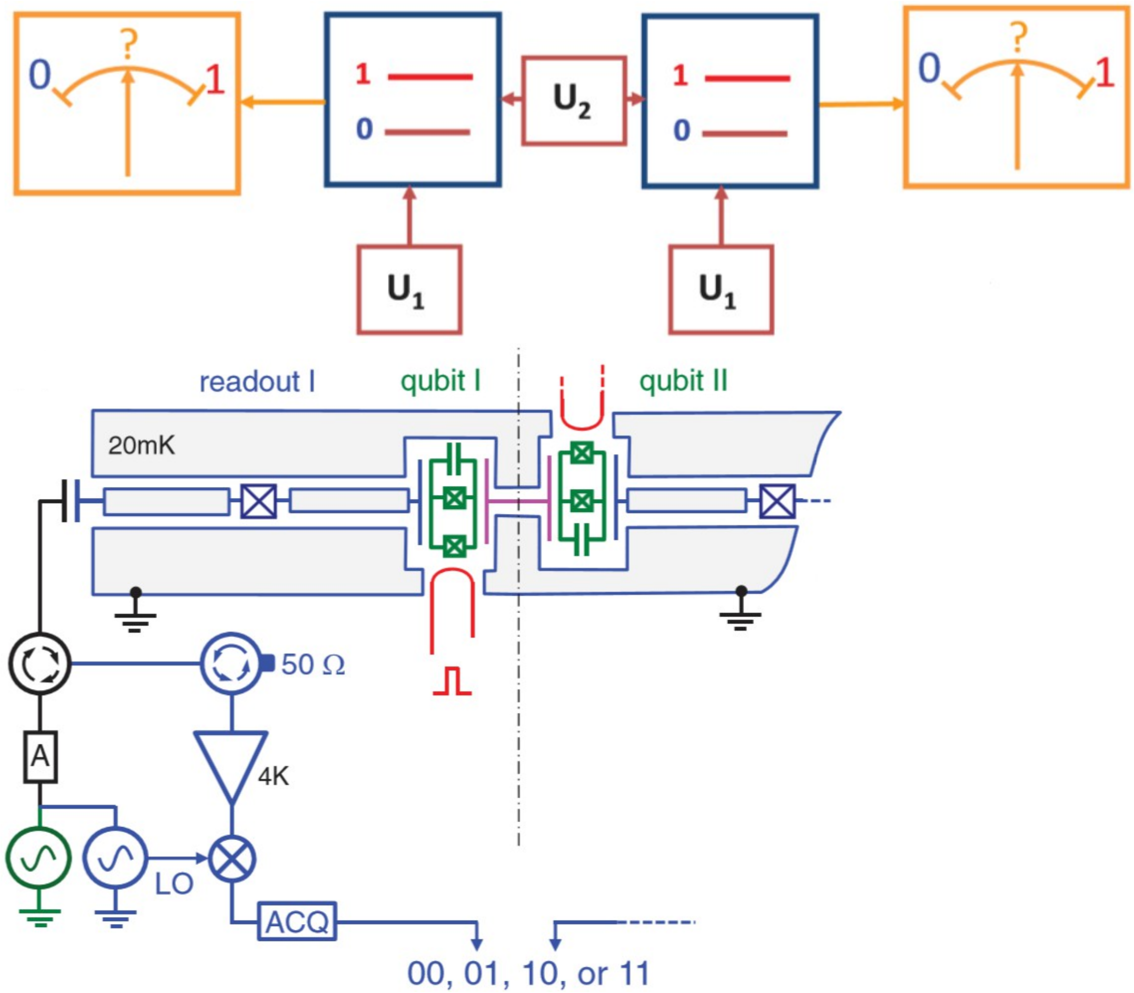
Element		Grover search oracle*			
		f_{00}	f_{01}	f_{10}	f_{11}
$\langle 0,0 \| \rho \| 0,0 \rangle$	Ideal	1	0	0	0
	Measured	0.81(1)	0.08(1)	0.07(2)	0.065(7)
$\langle 0,1 \| \rho \| 0,1 \rangle$	Ideal	0	1	0	0
	Measured	0.066(7)	0.802(9)	0.05(1)	0.054(8)
$\langle 1,0 \| \rho \| 1,0 \rangle$	Ideal	0	0	1	0
	Measured	0.08(1)	0.05(1)	0.82(2)	0.07(1)
$\langle 1,1 \| \rho \| 1,1 \rangle$	Ideal	0	0	0	1
	Measured	0.05(2)	0.07(1)	0.06(1)	0.81(1)

Experiment 2 : single qubit read out



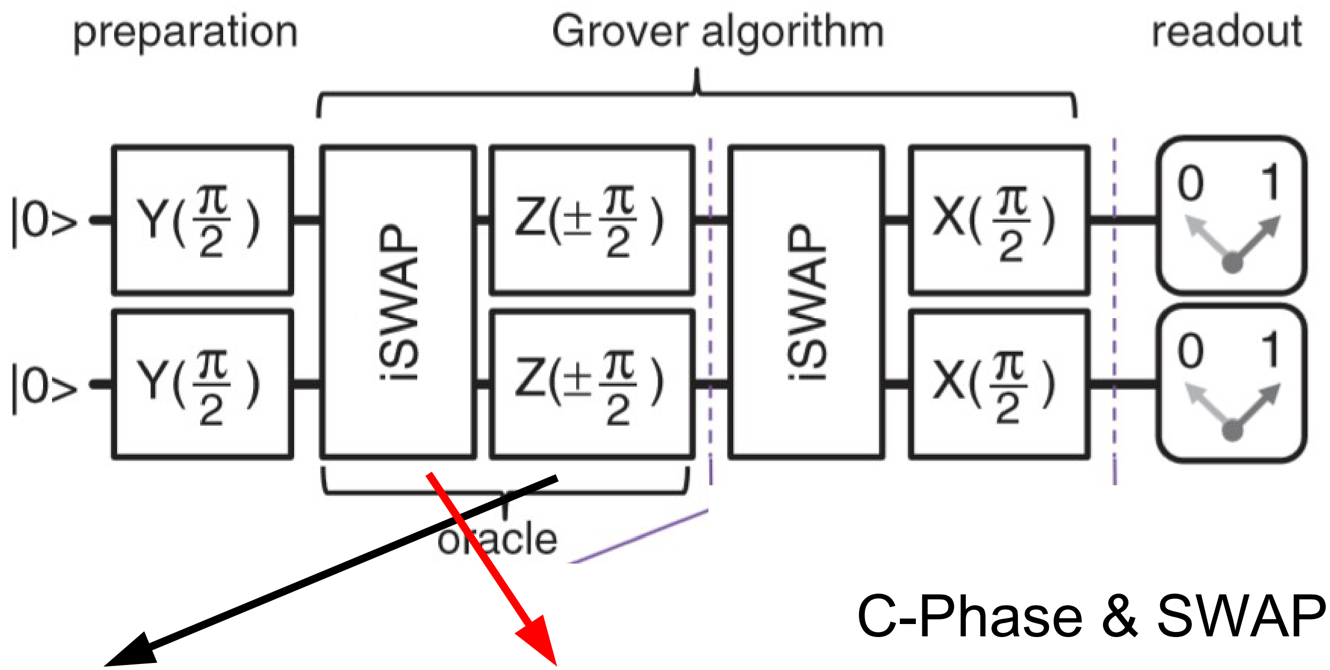
Dewes et al. Phys. Rev. B 85, 140503(R)

Experiment 2 : single qubit read out



Dewes et al. Phys. Rev. B 85, 140503(R)

Schematic setup



C-Phase & SWAP

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & i & 0 & 0\\ 0 & 0 & -i & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & i & 0\\ 0 & i & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Results Dewes et al.

State	Tomography (F)	Single shot (P)
00	0.70	0.67
01	0.62	0.55
10	0.67	0.62
11	0.66	0.52

Single shot measurement ~200 ns

Conclusion

- Grover search algorithm

$$\left(\{2|\psi\rangle\langle\psi| - I\} \quad \hat{O}\right)^R H^{\otimes n}|0\rangle$$

- Experimentally realizable in superconducting circuits
  - Oracle ~ C-Phase
  - average about mean ~ H + C-Phase + H

- Tomography fidelity > 0.8, qubit-resonator-qubit
- Single shot probability > 0.5, qubit-capacitor-qubit

References

- *Quantum speeding-up of computation demonstrated in a superconducting two-qubit processor.* A. Dewes et al. Phys. Rev. B 85, 140503(R) (2012), arxiv:1110.5170
- *Demonstration of two-qubit algorithms using a superconducting quantum processor*, L. DiCarlo et al. Nature 460, 240 (2009).
- *Quantum Computation and Quantum Information*, Nielsen and Chuang.