Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Quantum Simulation with trapped ions

How to trap ions?

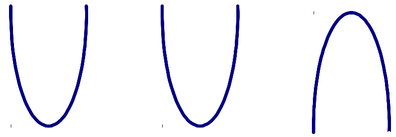
$$\Phi(x,y,z,t) = U\frac{1}{2}(\alpha x^2 + \beta y^2 + \gamma z^2) + \tilde{U}\cos(\omega_{\mathrm{rf}} t)\frac{1}{2}(\alpha' x^2 + \beta' y^2 + \gamma' z^2)$$

with $\Delta\Phi = 0$

meaning:

$$\alpha + \beta + \gamma = 0$$

$$\alpha' + \beta' + \gamma' = 0$$



How to trap ions?

For ex. :

$$\alpha = \beta = \gamma = 0 \qquad , \qquad -(\alpha + \beta) = \gamma > 0$$

$$\alpha' + \beta' = -\gamma' \qquad \qquad \alpha' = -\beta'$$

Linear Paul Trap

Linear Paul Trap

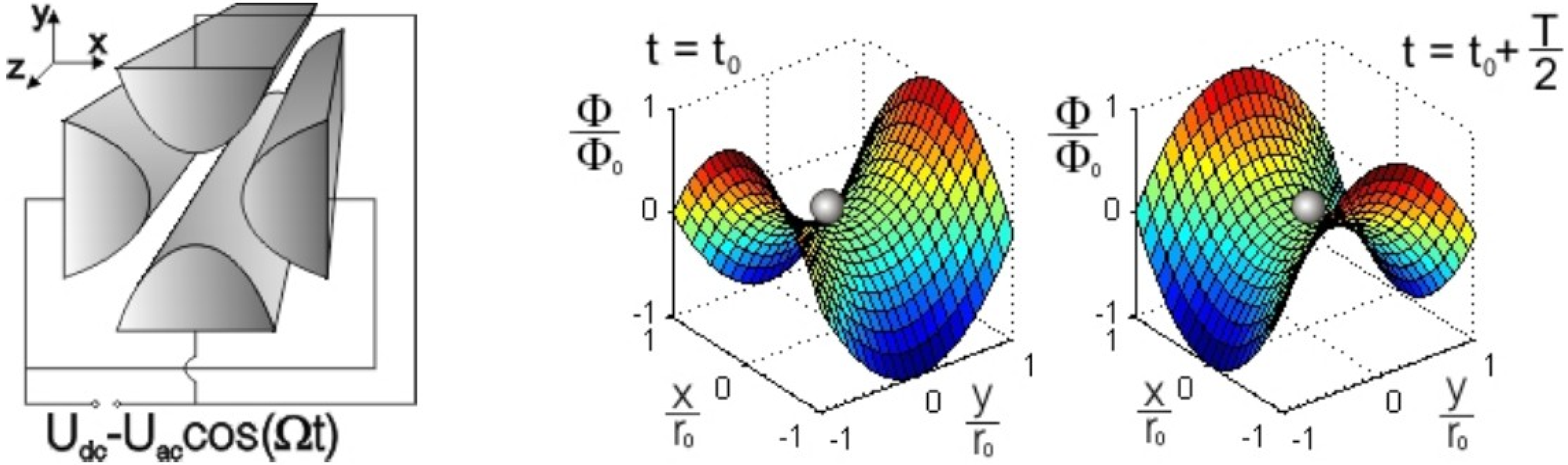
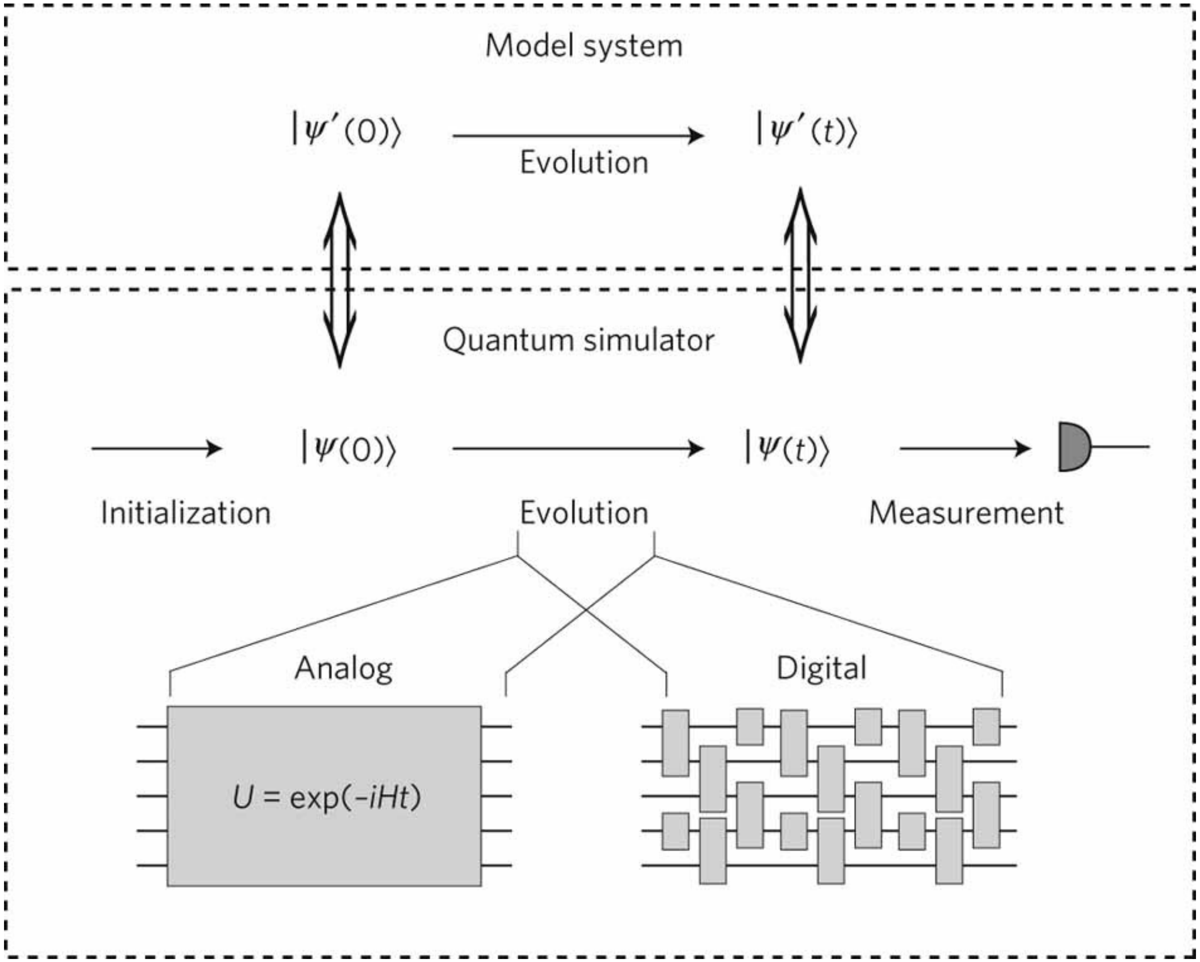


FIG. 1: (left) *Electrode configuration of a linear Paul trap.* ***(right)*** *Electric potential in the Paul trap.*

Picture from HU-Berlin: https://www.physik.hu-berlin.de/nano-en/forschung-en/np

Overview of quantum simulation



Picture from Nature 484, 489–492 (26 April 2012): http://www.nature.com/nature/journal/v484/n7395/full/nature10981.html

Digital quantum simulation

What we want: $e^{-iHt/\hbar} = e^{-i(H_A+H_B)t/\hbar}$

What we have: $e^{-iH_At/\hbar}\,,\; e^{-iH_Bt/\hbar} \rightarrow e^{-iH_At/\hbar}e^{-iH_Bt/\hbar}$

When is $e^{X+Y} = e^X e^Y$?

Reminders

$$e^X = \sum_{k=0}^{\infty} \frac{1}{k!} X^k$$

$$e^{X+Y} = \sum_{k=0}^{\infty} \frac{1}{k!} (X+Y)^k$$

$$[X,Y] = 0 \Longrightarrow e^{X+Y} = e^X e^Y$$

$$([X,Y] = XY - YX)$$

Zaussenhaus formula

$$e^{t(X+Y)} = e^{tX} e^{tY} e^{-\frac{t^2}{2}[X,Y]} e^{\frac{t^3}{6}(2[Y,[X,Y]]+[X,[X,Y]])} + O(t^4)$$

$$e^{t(X+Y)} = e^{tX}\ e^{tY}\ e^{-\frac{t^2}{2}[X,Y]} + O(t^3)$$

$$= e^{(t/2)X}\ e^{(t/2)Y}\ e^{-\frac{(t/2)^2}{2}[X,Y]}\ e^{(t/2)X}\ e^{(t/2)Y}\ e^{-\frac{(t/2)^2}{2}[X,Y]} + O(t^3)$$

$$= (e^{\frac{tX}{2}}\ e^{\frac{tY}{2}})^2 + O(t^2/2)$$

Three times

$$e^{X+Y} = \left(e^{(t/3)X}\ e^{(t/3)Y}\ e^{-\frac{(t/3)^2}{2}[X,Y]}\right)^3 + O(t^3)$$

$$= \left(e^{\frac{tX}{3}}\ e^{\frac{tY}{3}}\right)^3 + O(t^2/3)$$

One last time

$$e^{X+Y} = \left(e^{(t/n)X}\ e^{(t/n)Y}\ e^{-\frac{(t/n)^2}{2}[X,Y]}\right)^n + O(t^3)$$

$$= \left(e^{\frac{tX}{n}}\ e^{\frac{tY}{n}}\right)^n + O(t^2/n)$$

Suzuki-Trotter expansion

$$e^{tZ} = \lim_{n\to\infty}\left(e^{\frac{tX}{n}} \, e^{\frac{tY}{n}} \right)^n$$

$$Z = X + Y$$

$$e^{-\frac{iHt}{\hbar}} = \lim_{n\to\infty}\left(\prod_k e^{-\frac{iH_k t}{n\hbar}}\right)^n$$

Gate operations

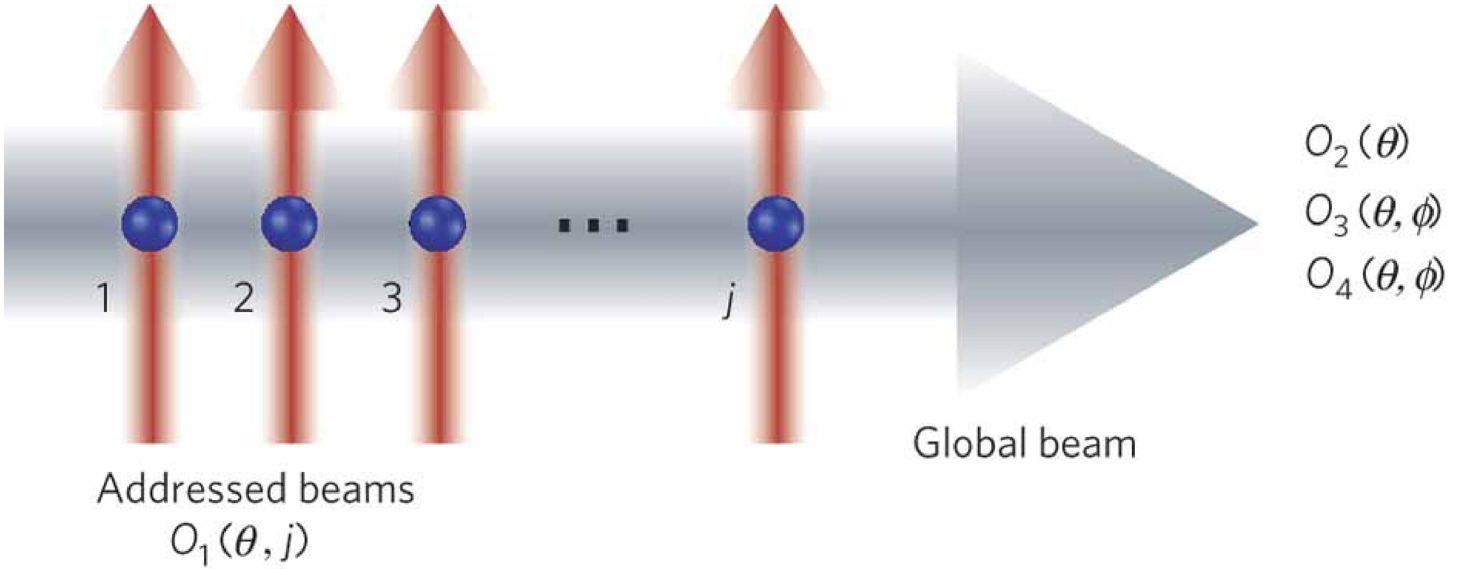
$$O_1(\theta, i) = exp(-i\theta\sigma_z^i)$$

$$O_3(\theta, \phi) = exp(-i\theta \sum_i \sigma_\phi^i)$$

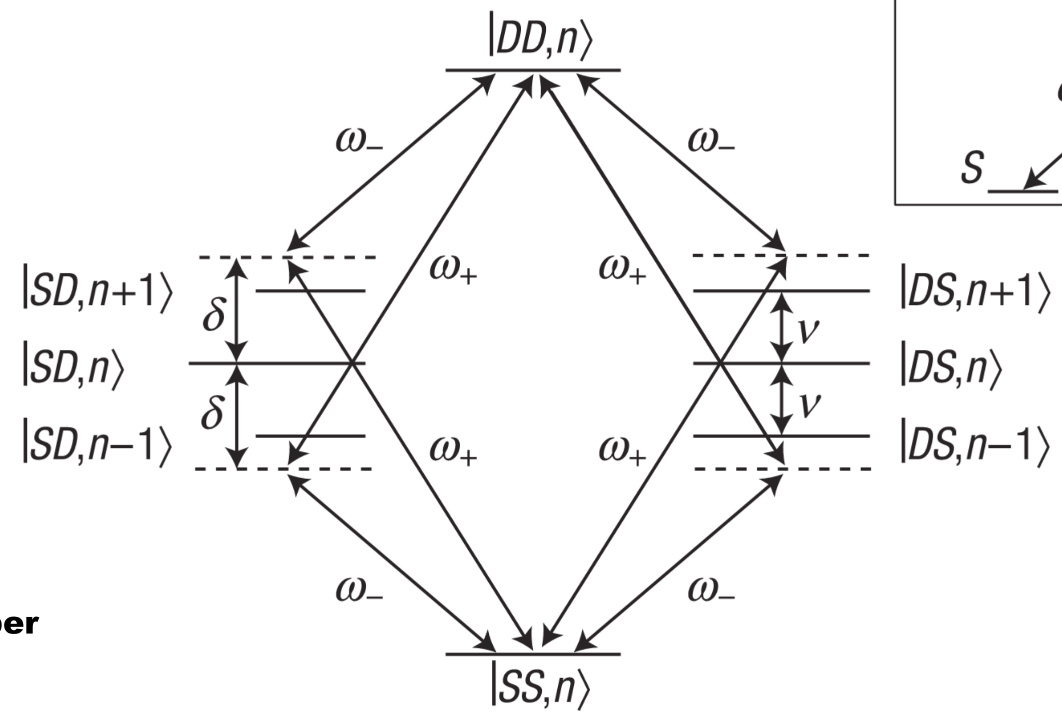
$$O_2(\theta) = exp(-i\theta \sum_i \sigma_z^i)$$

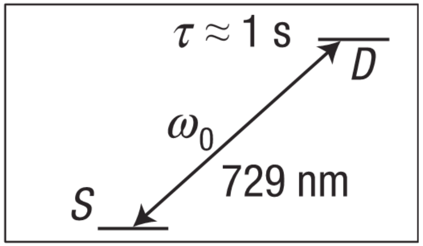
$$O_4(\theta, \phi) = exp(-i\theta \sum_{i<j} \sigma_\phi^i \sigma_\phi^j)$$

$$\sigma_\phi = cos(\phi)\,\sigma_x + sin(\phi)\,\sigma_y$$



O4 Gate



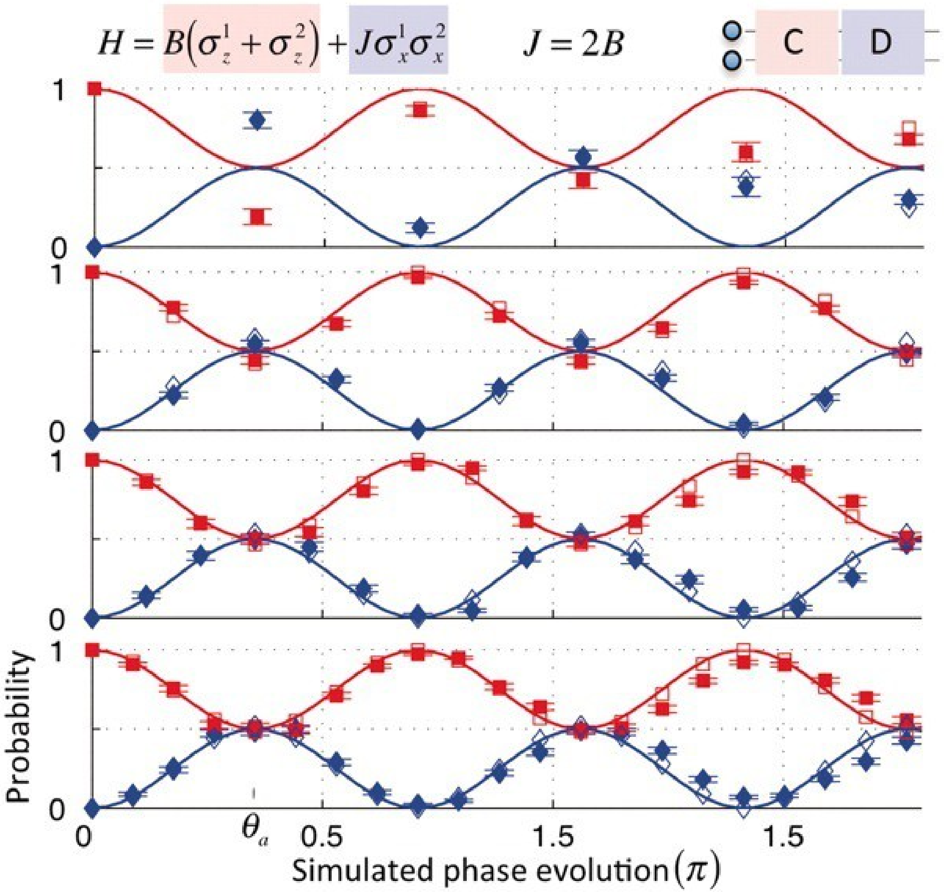


n: vibrational quantum number of the axial COM (center of mass) mode

$$|SS\rangle \xrightarrow{\tau_{\text{gate}}} |SS\rangle + i|DD\rangle \xrightarrow{\tau_{\text{gate}}} |DD\rangle \xrightarrow{\tau_{\text{gate}}} |DD\rangle + i|SS\rangle \xrightarrow{\tau_{\text{gate}}} |SS\rangle \xrightarrow{\tau_{\text{gate}}} \cdots$$

Real simulation

n=
1
2
3
4



C = O2(θa/2n)

D = O4(θa/n,0)

Lines: exact dynamics

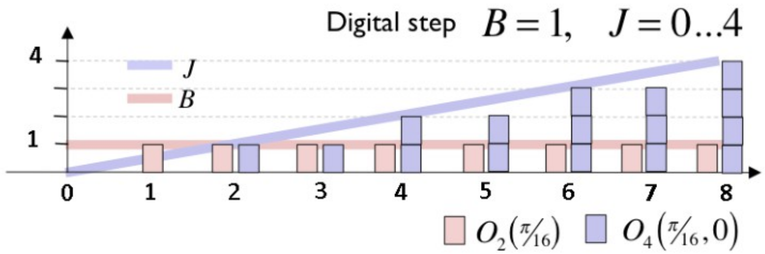
Unfilled shapes: ideal digitized

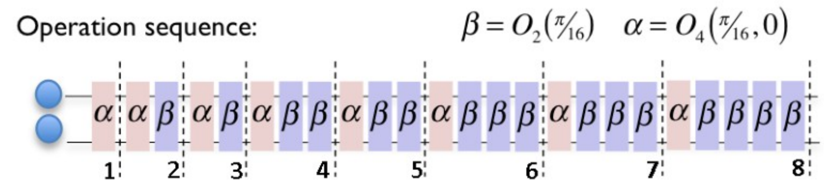
Filled shapes: data (■↑↑ ♦↓↓)

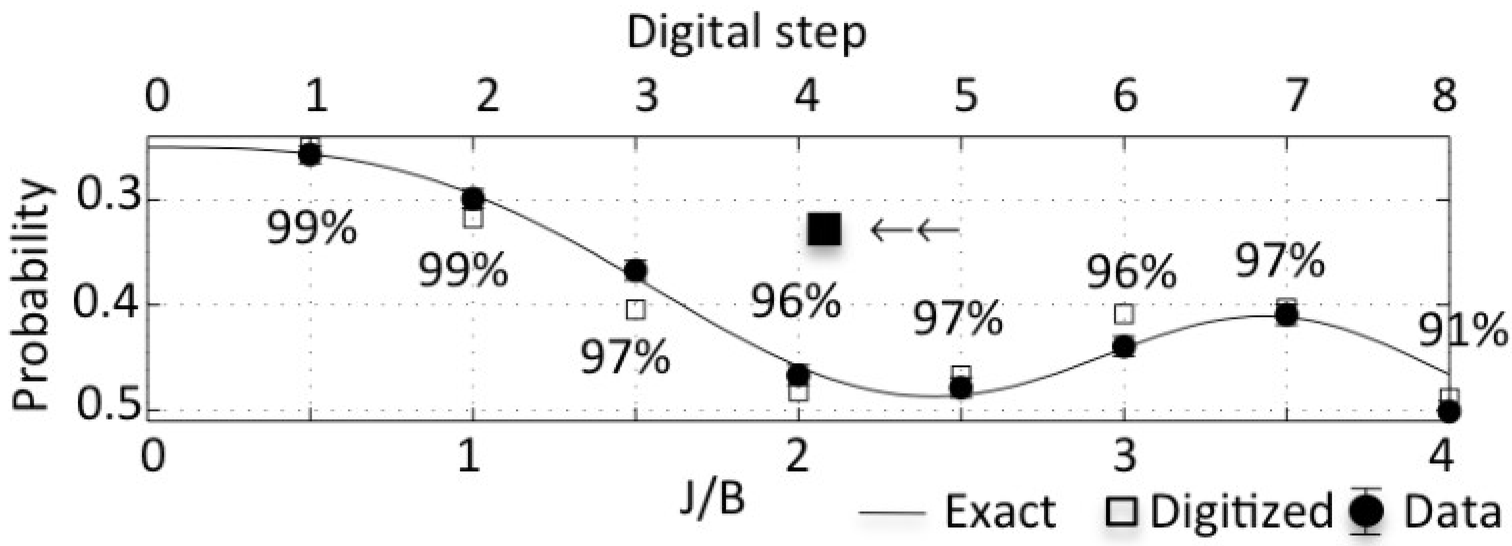
$$\theta_a = \frac{\pi}{2\sqrt{2}}$$

Linear ramp

$$H = B\left(\sigma_z^1 + \sigma_z^2\right) + J\sigma_x^1\sigma_x^2$$







Percentages: measured fidelities between the measured and ideal digitised states

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More results

C = O2(π/16)

D = O4(π/16,0)

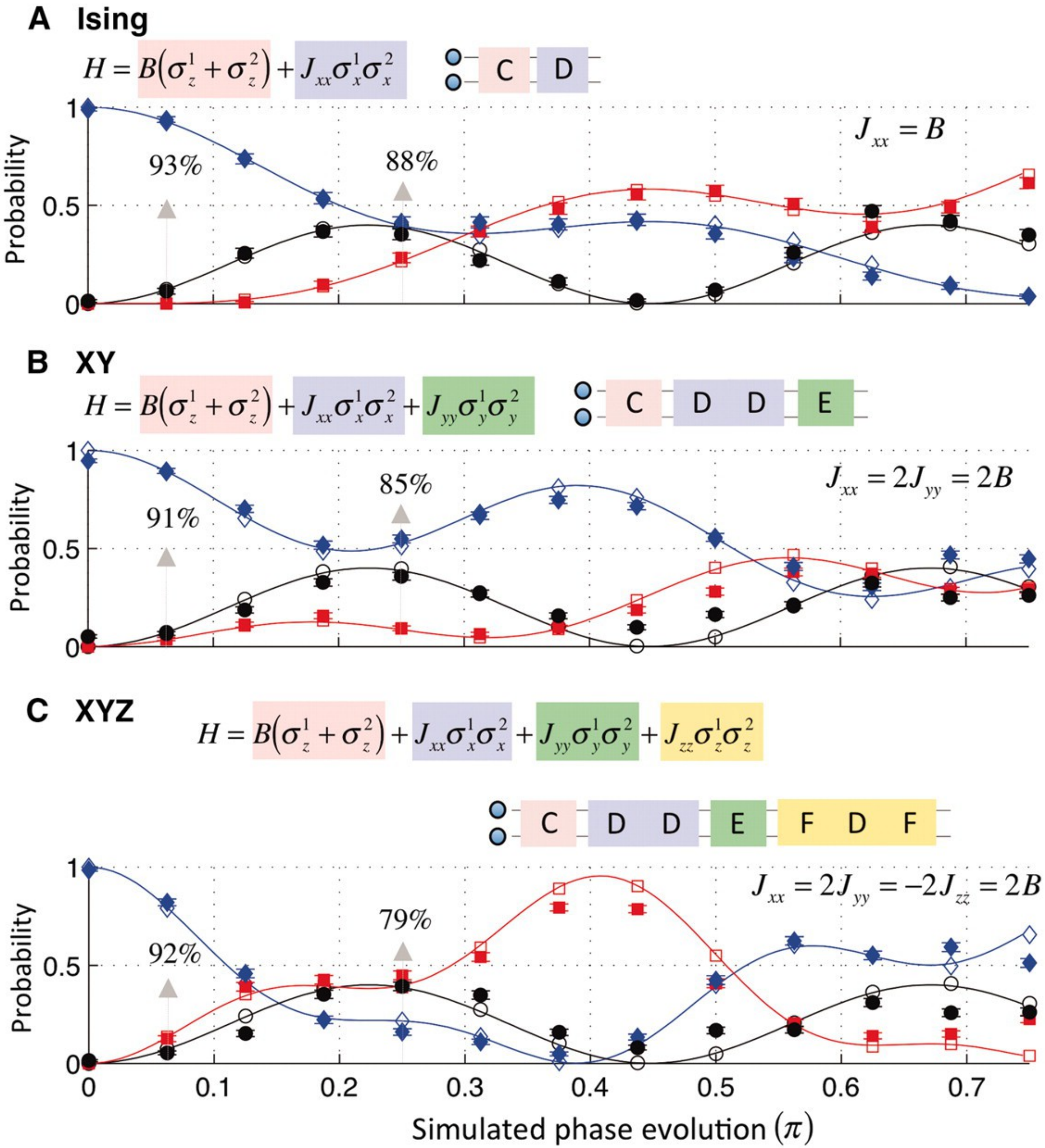
E = O4(π/16,π)

F = O3(π/4,0)

Lines: exact dynamics

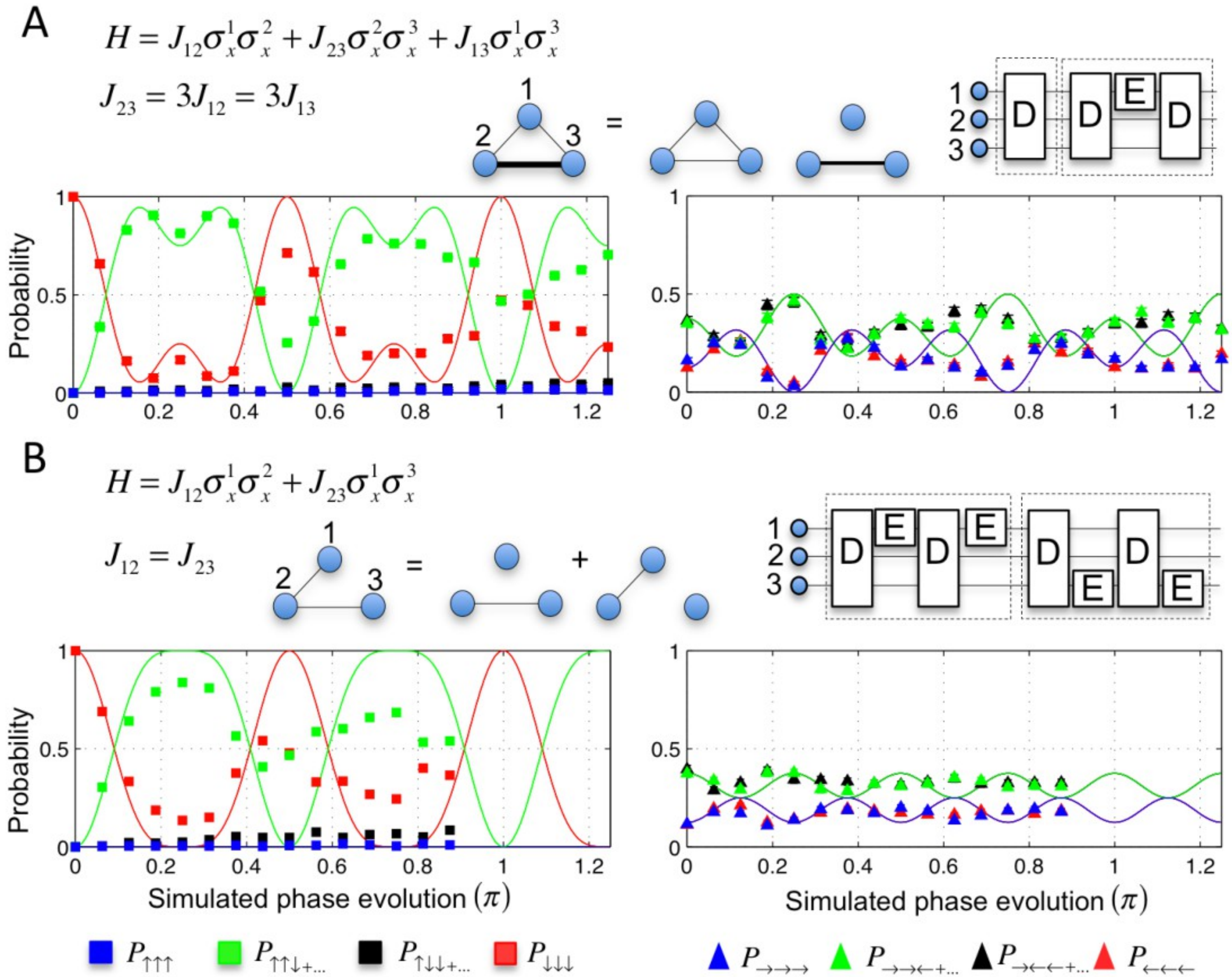
Unfilled shapes: ideal digitized

Filled shapes: data (♦→← ■←→ ●←← or →→)



Questions?

Backup


A
$H = J_{12}\sigma_x^1\sigma_x^2 + J_{23}\sigma_x^2\sigma_x^3 + J_{13}\sigma_x^1\sigma_x^3$
$J_{23} = 3J_{12} = 3J_{13}$
Probability
B
$H = J_{12}\sigma_x^1\sigma_x^2 + J_{23}\sigma_x^1\sigma_x^3$
$J_{12} = J_{23}$
Probability
Simulated phase evolution (π)
Simulated phase evolution (π)
$P_{\uparrow\uparrow\uparrow}$
$P_{\uparrow\uparrow\downarrow+...}$
$P_{\uparrow\downarrow\downarrow+...}$
$P_{\downarrow\downarrow\downarrow}$
$P_{\rightarrow\rightarrow\rightarrow}$
$P_{\rightarrow\rightarrow\leftarrow+...}$
$P_{\rightarrow\leftarrow\leftarrow+...}$
$P_{\leftarrow\leftarrow\leftarrow}$