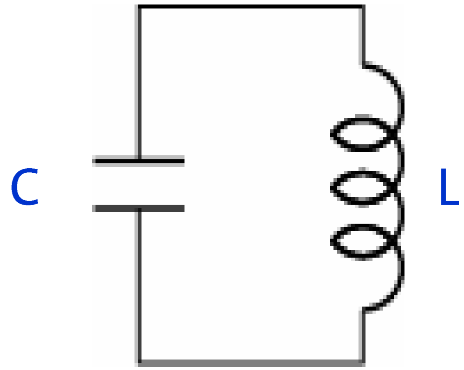
Realizations of Harmonic Oscillators

Superconducting Harmonic Oscillators

a simple electronic circuit:



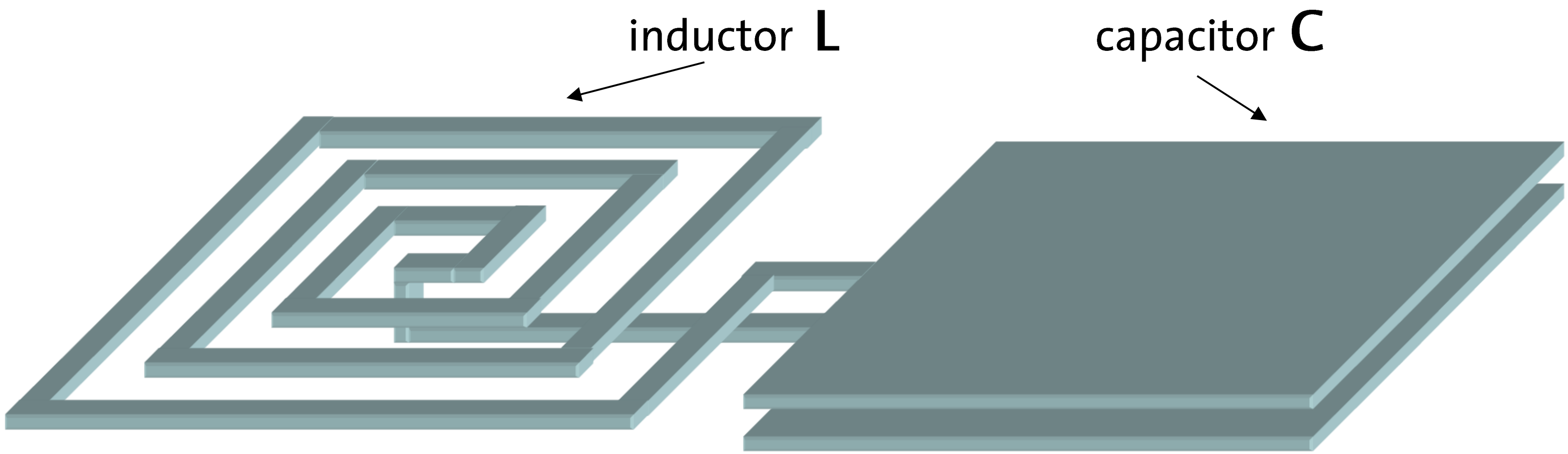
- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm

- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF

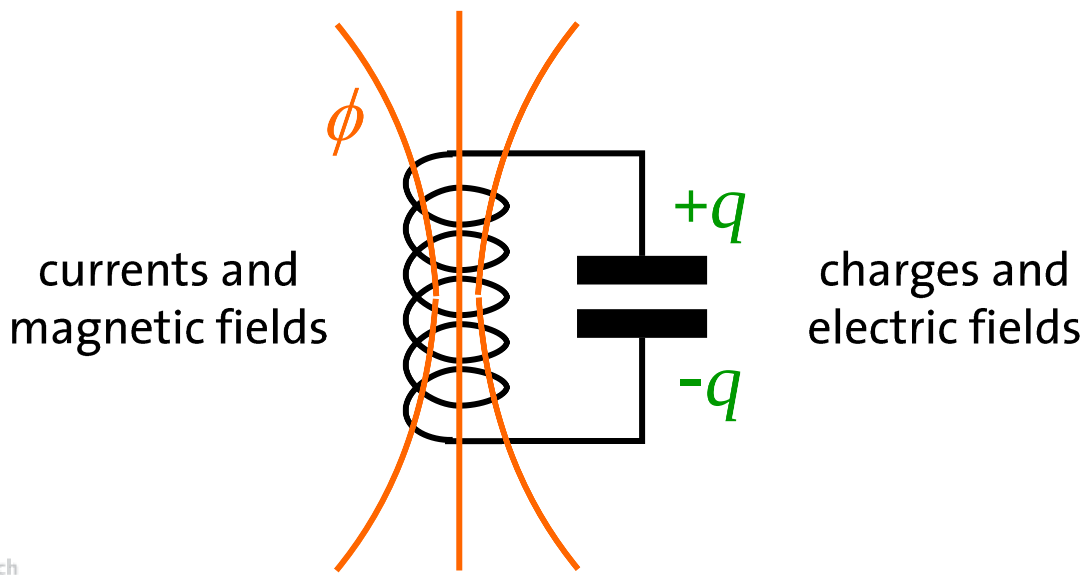
- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

Realization of H.O.: Lumped Element Resonator

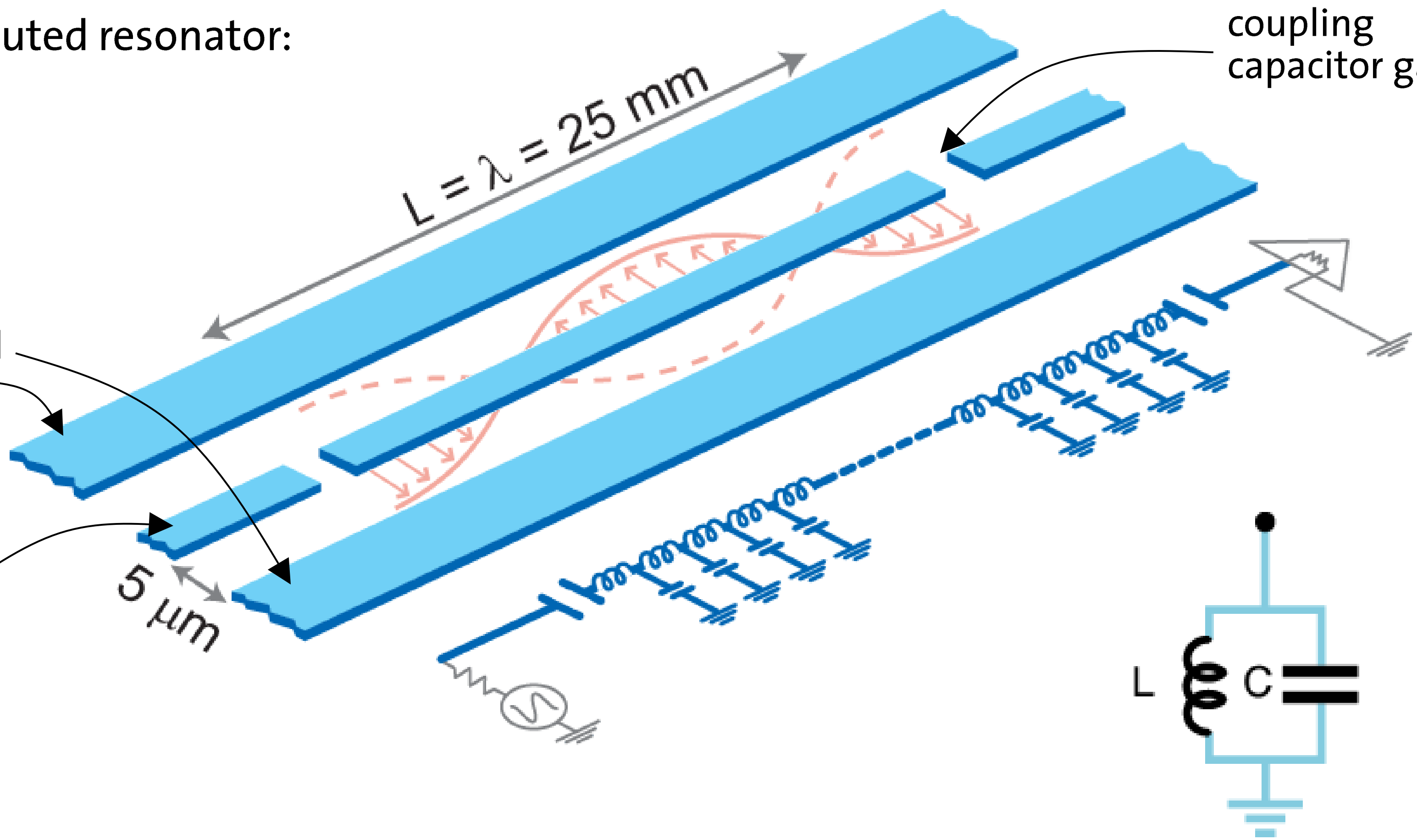


a harmonic oscillator



Realization of H.O.: Transmission Line Resonator

distributed resonator:

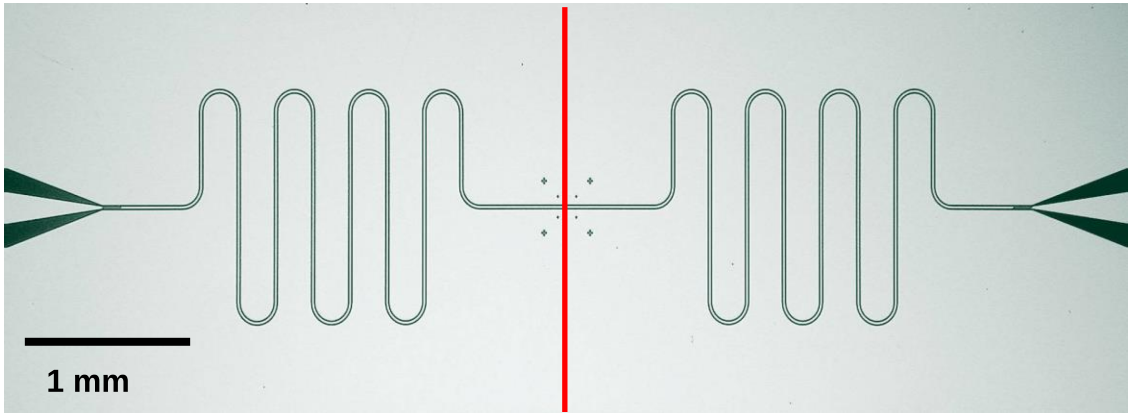


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

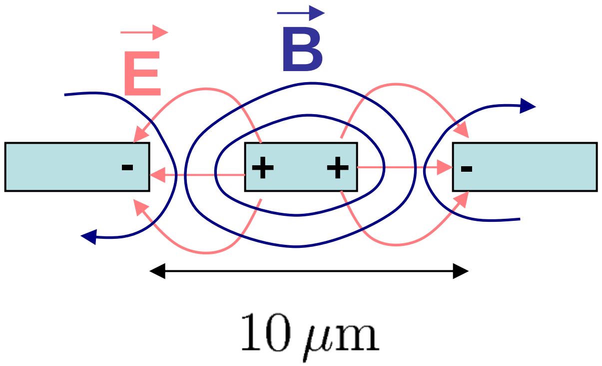
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

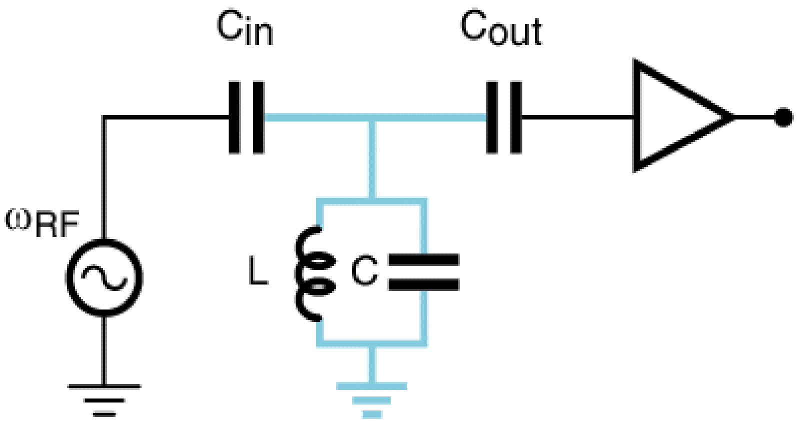
coplanar waveguide:



cross-section of transm. line (TEM mode):

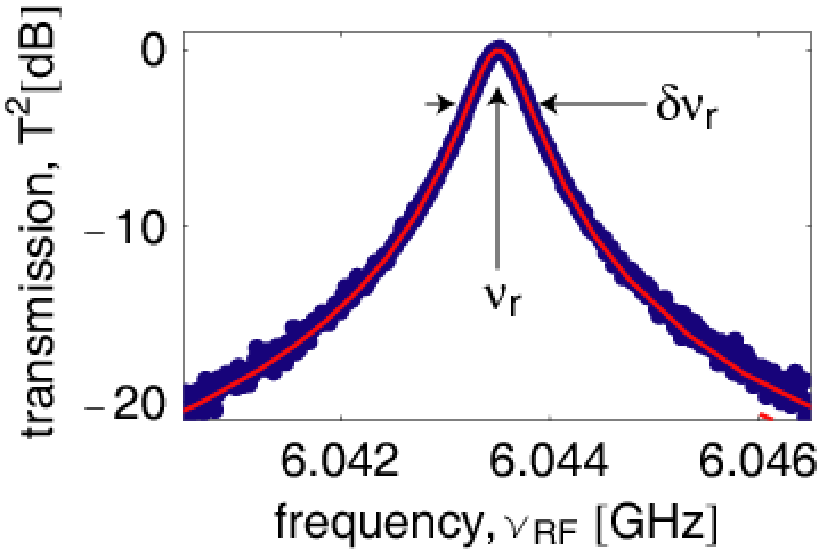


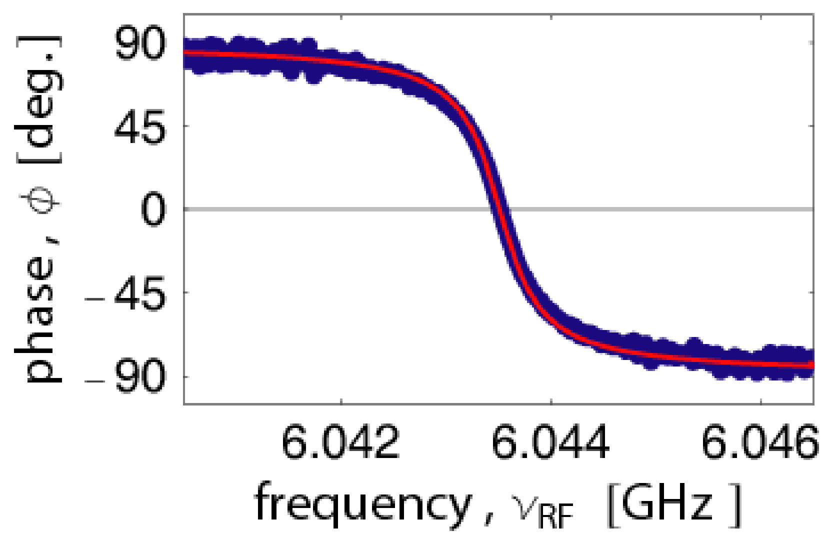
measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

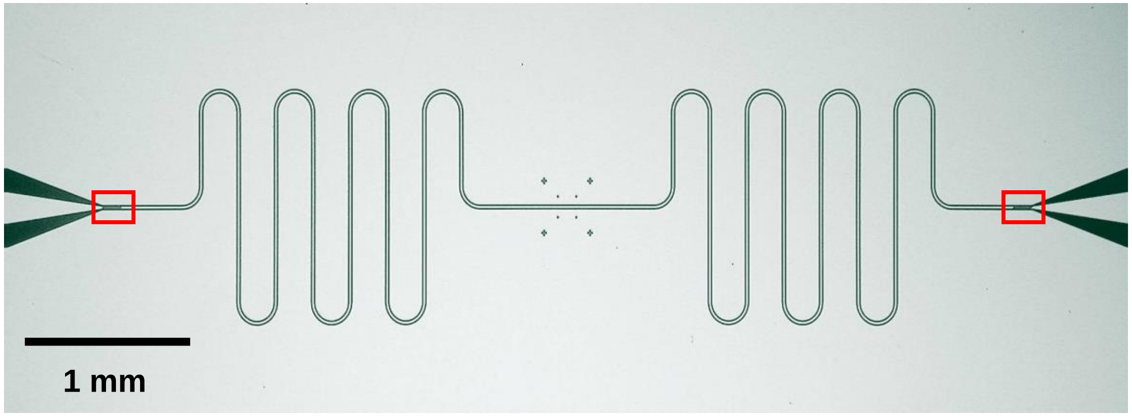
photon decay rate:

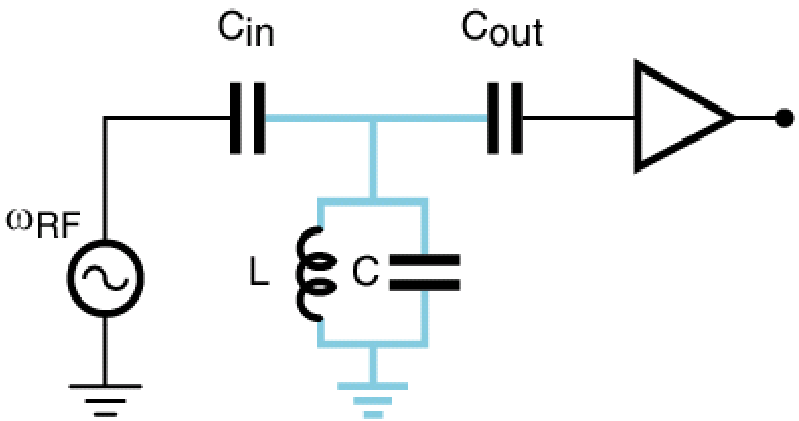
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

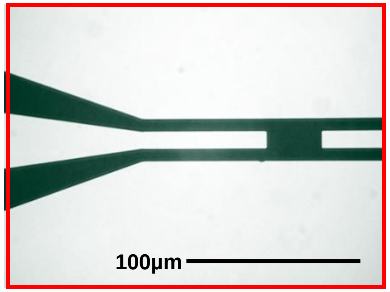
$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

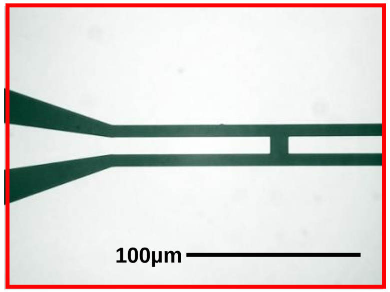
Controlling the Photon Life Time

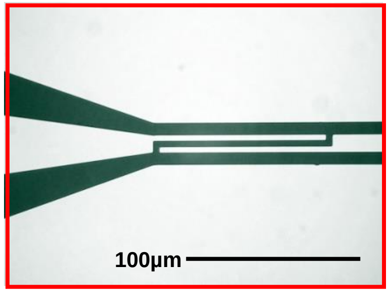


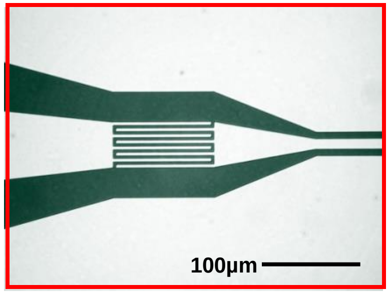


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

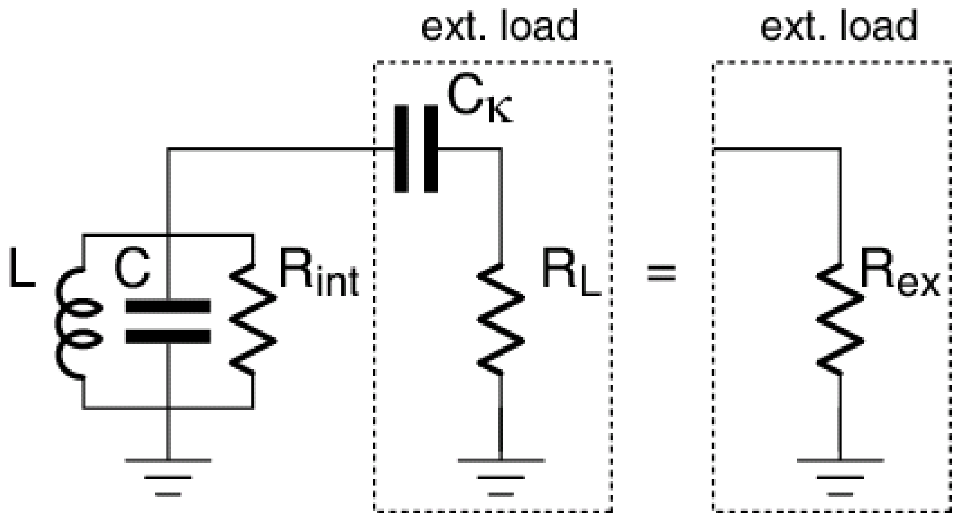








Internal and External Dissipation in an LC Oscillator



internal losses: $R_{\rm int}$
conductor, dielectric

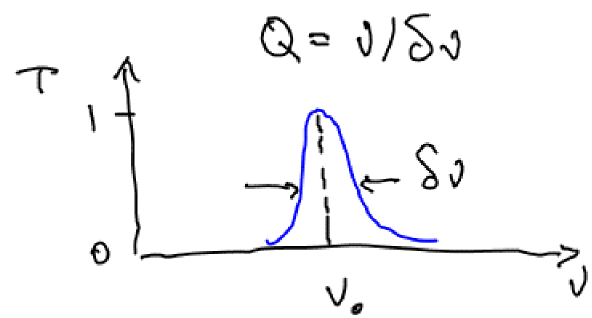
external losses: $R_{\rm ext}$
radiation, coupling

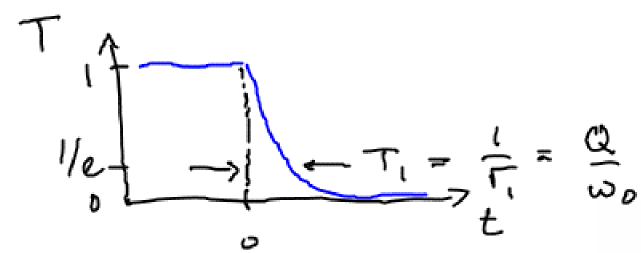
total losses $$\frac{1}{R} = \frac{1}{R_{\rm int}} + \frac{1}{R_{\rm ext}}$$

impedance $$Z = \sqrt{\frac{L}{C}}$$

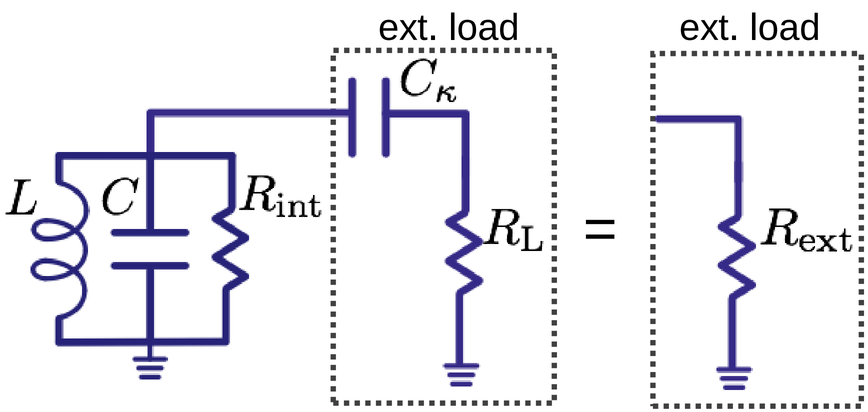
quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

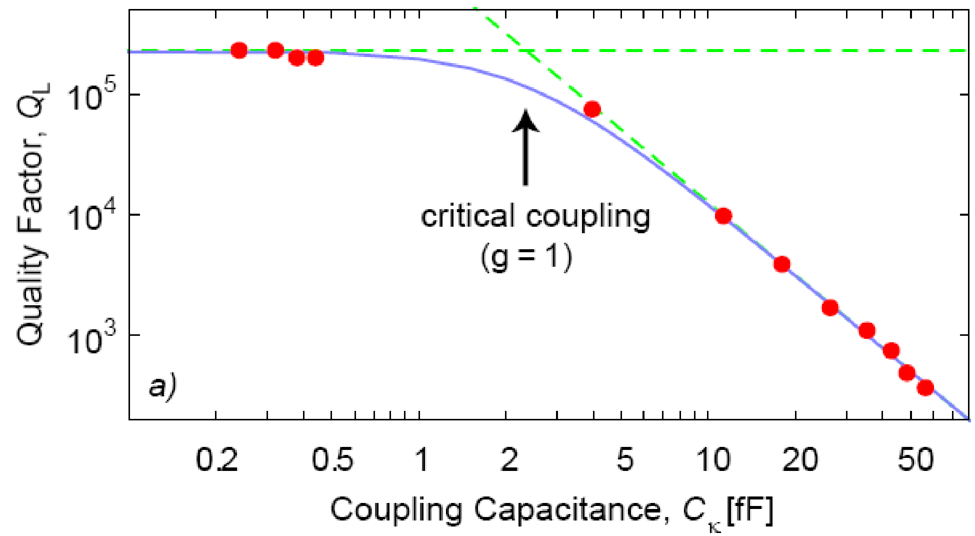
excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$

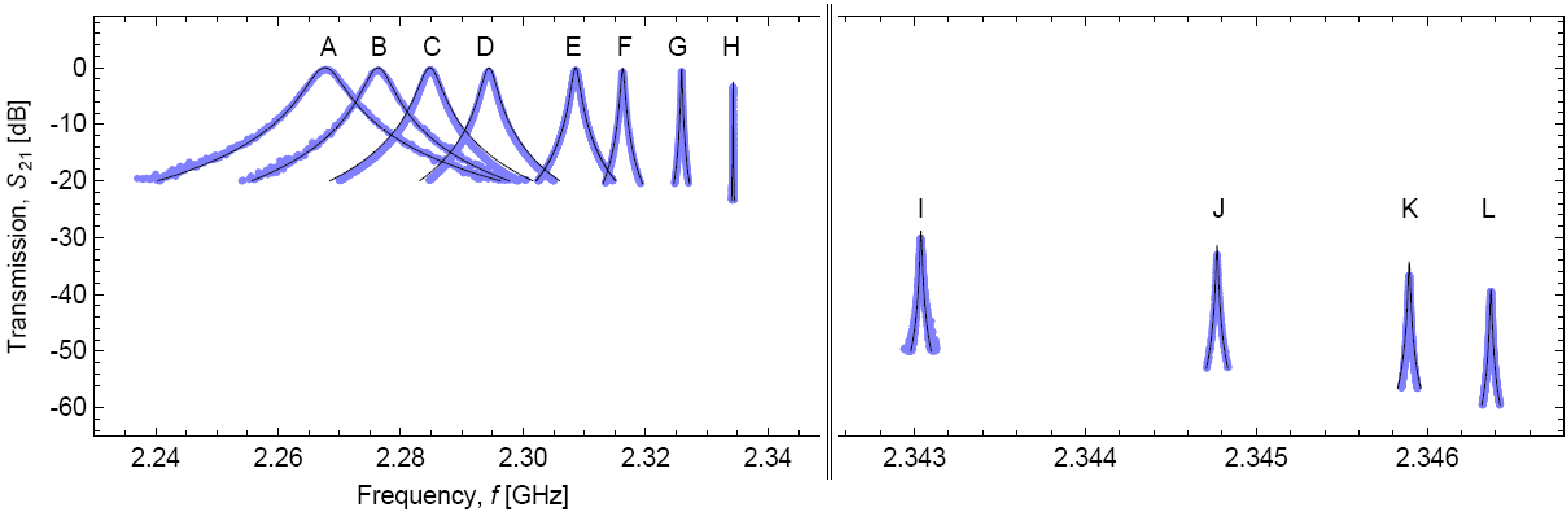




Quality Factor Measurement

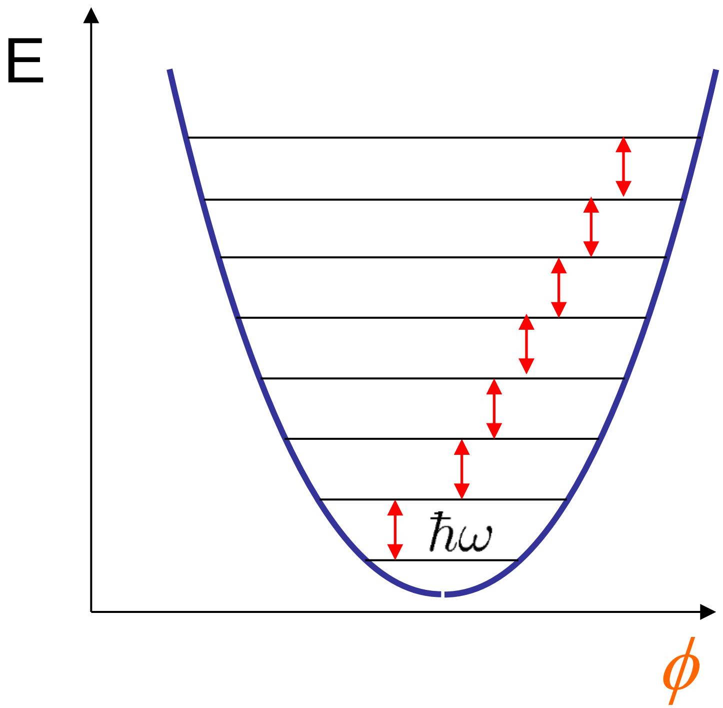






M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\text{th}} \rangle = \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$

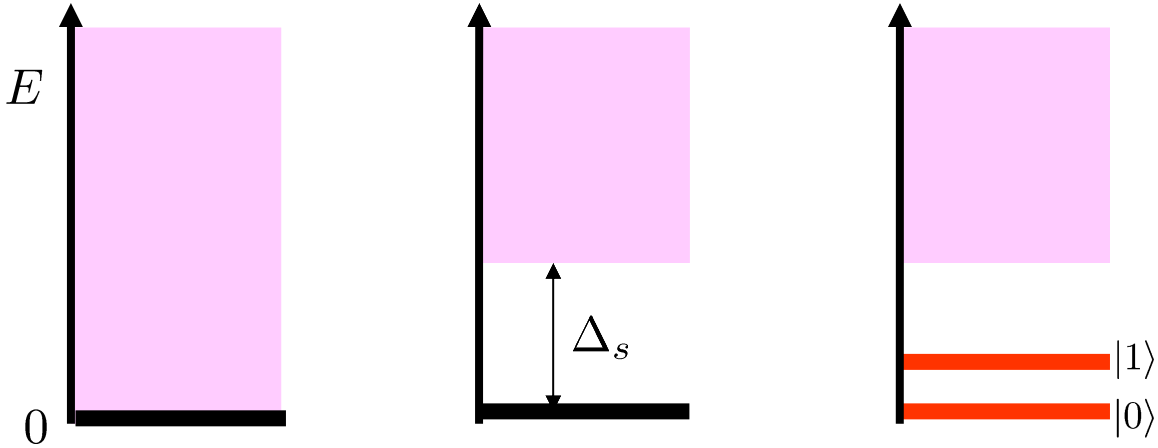
low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{\text{th}} \rangle \sim 10^{-11}$$

Why Superconductors?



normal metal superconductor How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

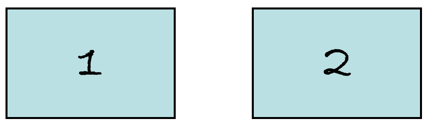
Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta/h$ = 725 GHz, T_c = 9.2 K
- Aluminum (Al): $2\Delta/h$ = 100 GHz, T_c = 1.2 K

Cooper pairs:
bound electron pairs

Bosons (S=0, L=0)

2 chunks of superconductors

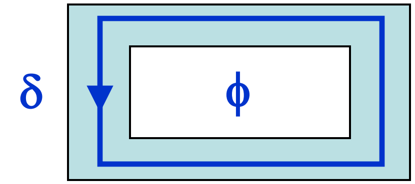


macroscopic wave function

$$\psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

Cooper pair density n_i
and global phase δ_i

phase quantization: $\delta = n\, 2\pi$
flux quantization: $\phi = n\, \phi_0$



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

How to Prove that a Harmonic Oscillator is Quantum?

measure:

- resonance frequency
- average charge (momentum)
- average flux (position)

all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

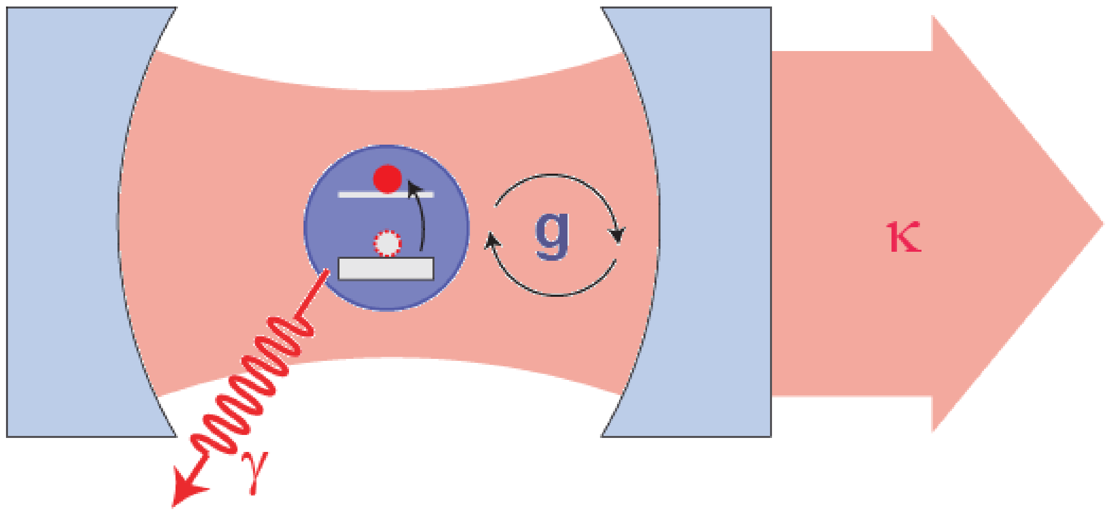
solution:

- make oscillator non-linear in a controllable way

Coupling a Harmonic Oscillator to a Qubit

Cavity Quantum Electrodynamics

coupling photons to qubits:



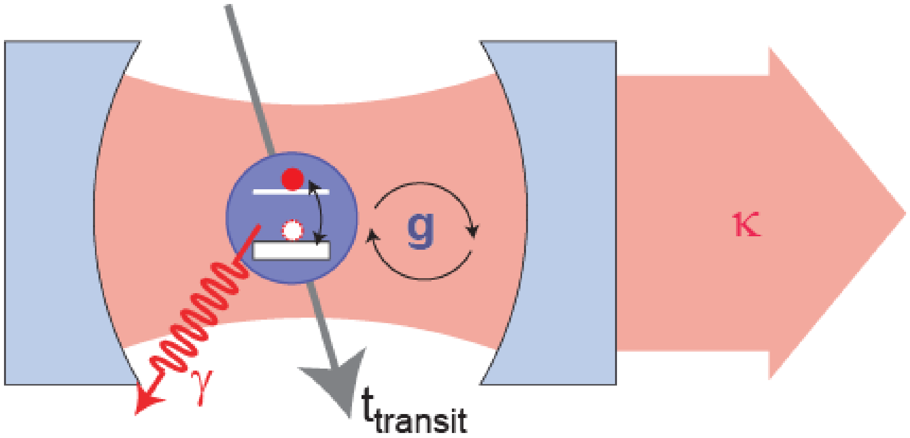
Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit $(g = dE_0/\hbar > \gamma,\ \kappa\ ,1/t_{\mathrm{transit}})$

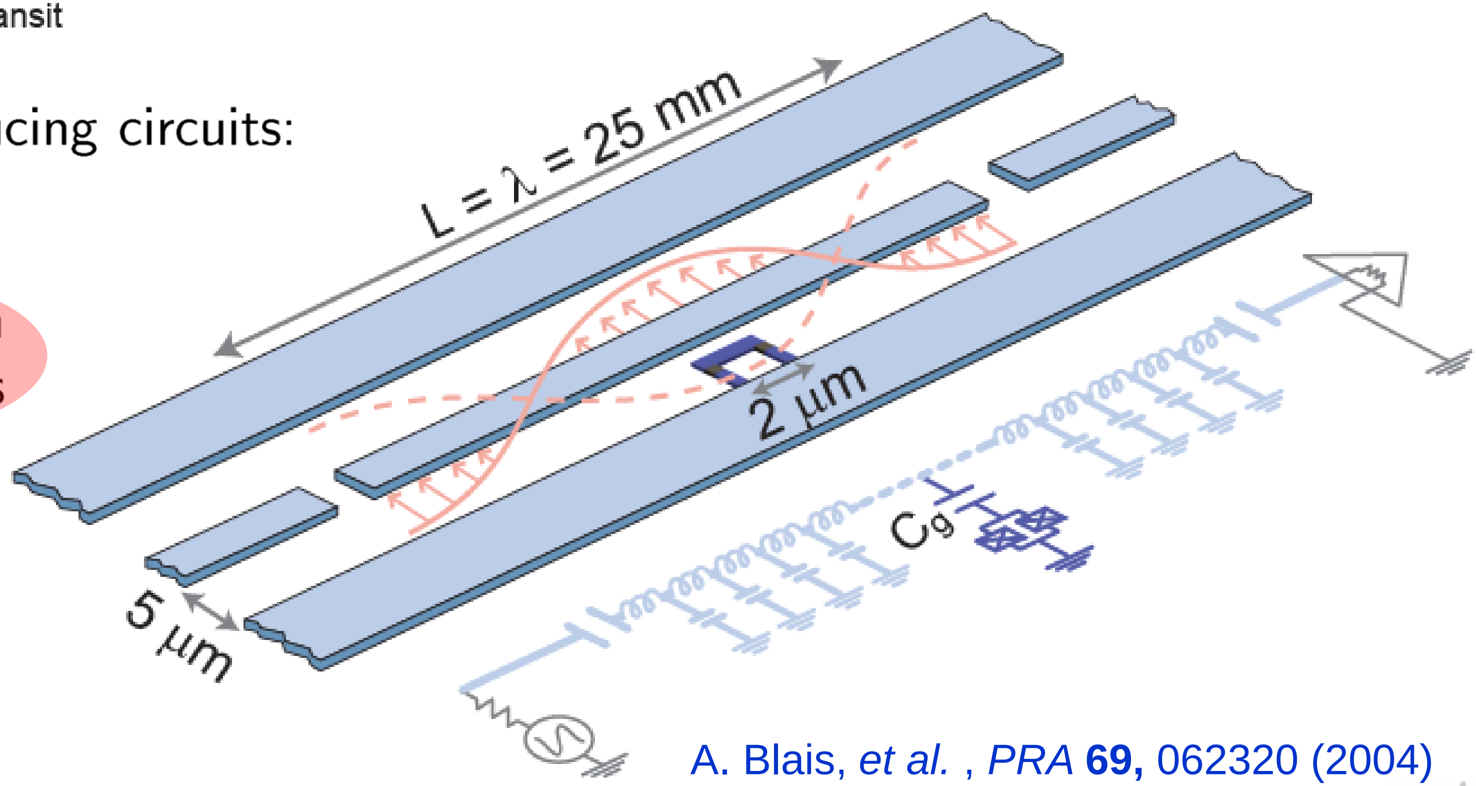
D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

... in superconducing circuits:

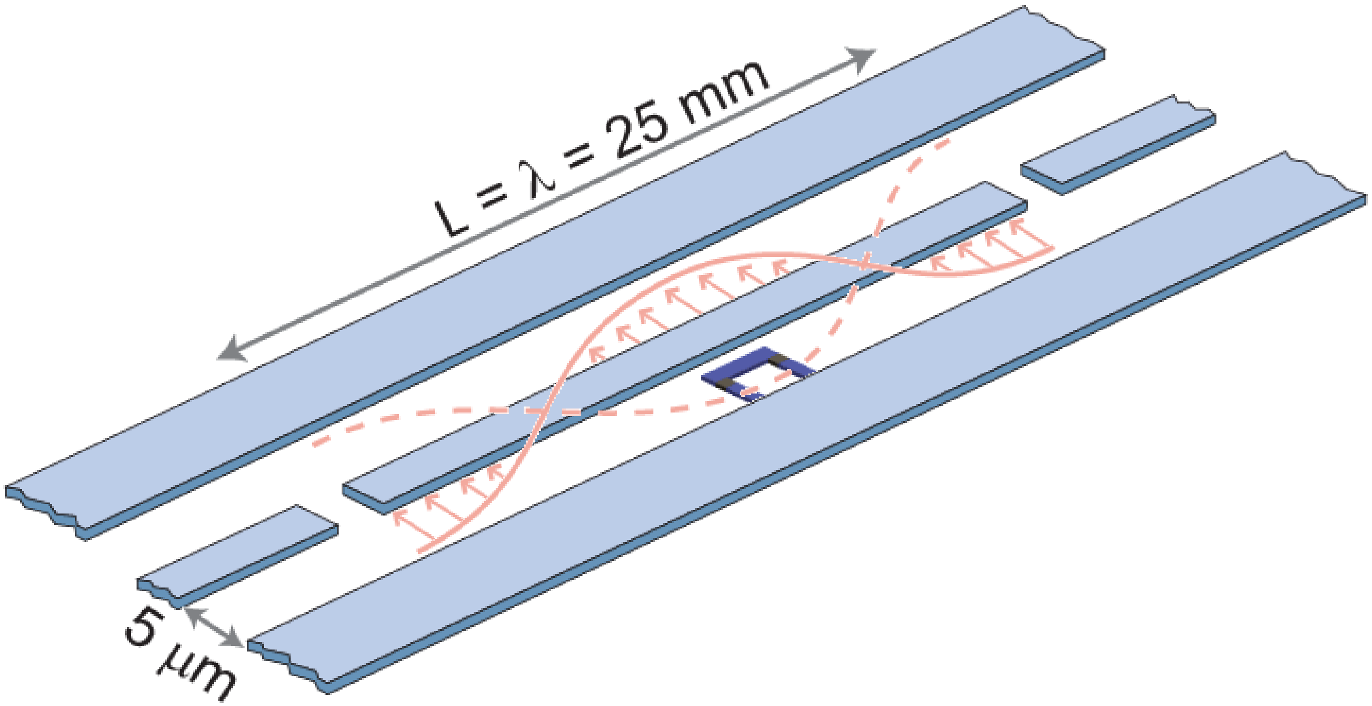


circuit quantum electrodynamics

A. Blais, *et al.* , *PRA* **69,** 062320 (2004)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics

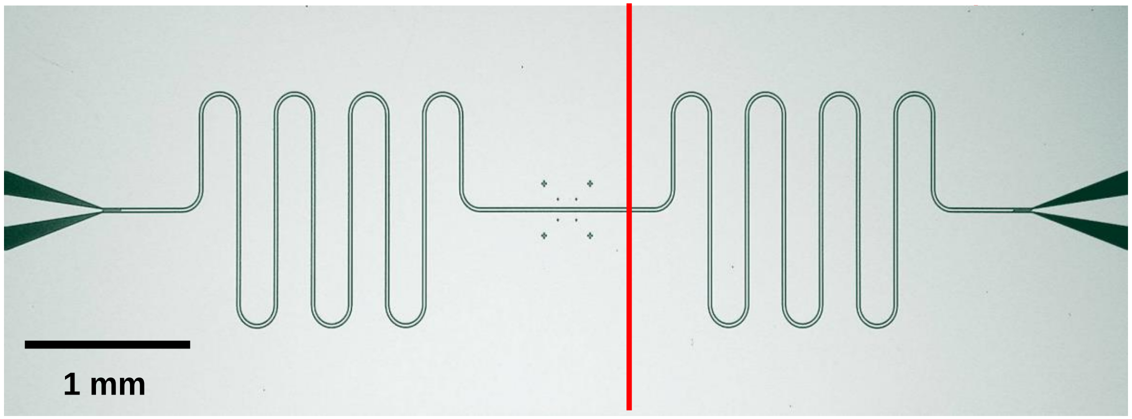


elements

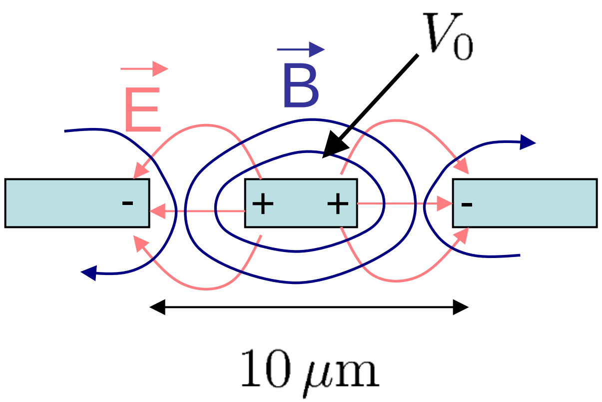
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment d and long coherence time $1/\gamma$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Vacuum Field in 1D Cavity



cross-section
of transm. line (TEM mode):



voltage across resonator in vacuum state $(n = 0)$

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

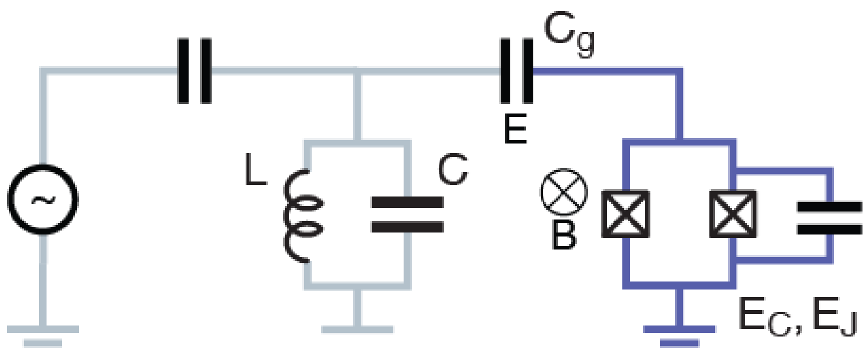
$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

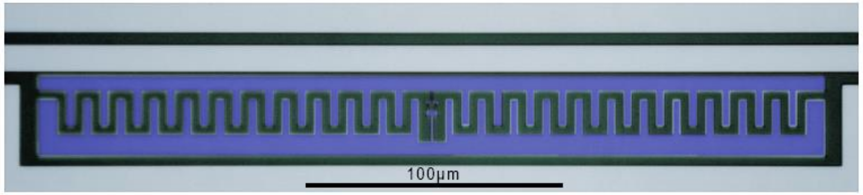
$\times 10^6$ larger than E_0
in 3D microwave cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ $(C \sim 1\,\mathrm{pF})$, $b \approx 5\,\mu\mathrm{m}$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = eV_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \dots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \dots 10^4\, ea_0$$

Dressed States Energy Level Diagram

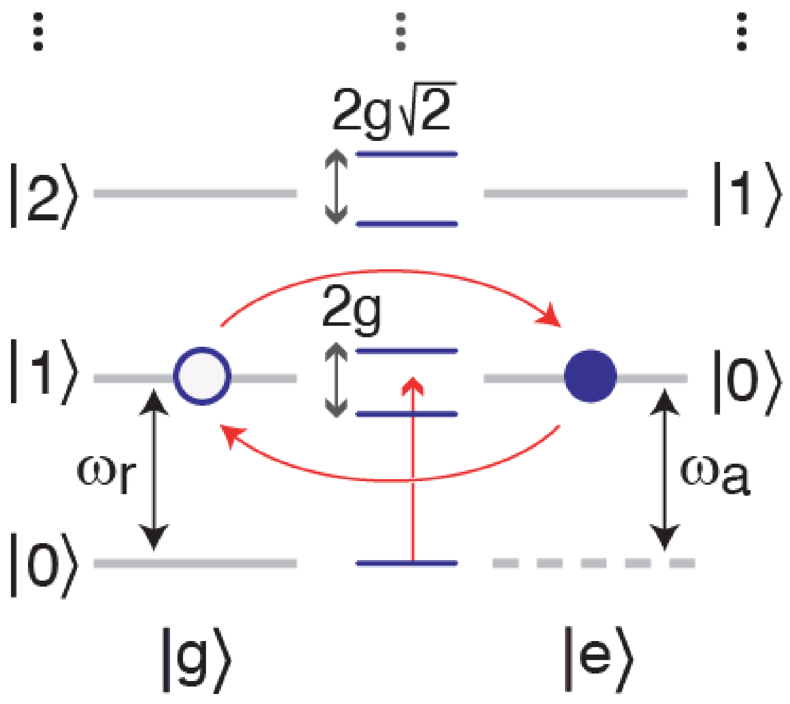
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\ \kappa$$



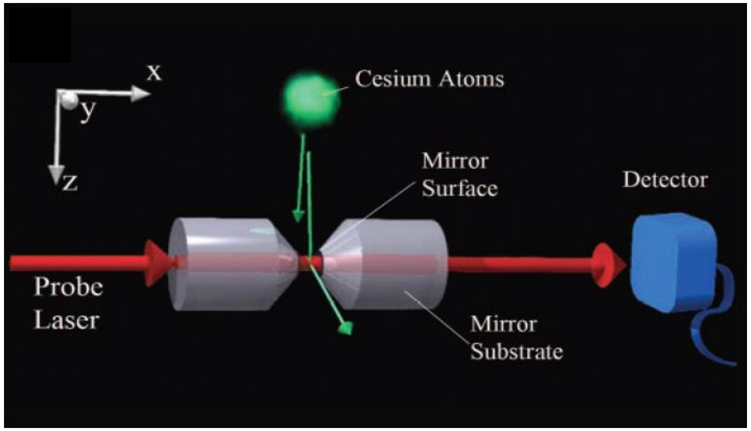
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

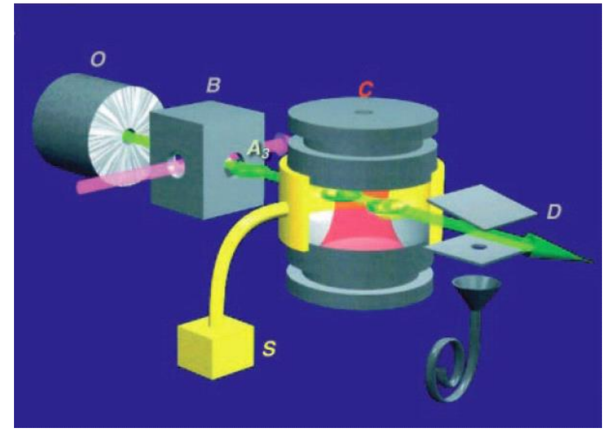
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Cavity Quantum Electrodynamics (QED)



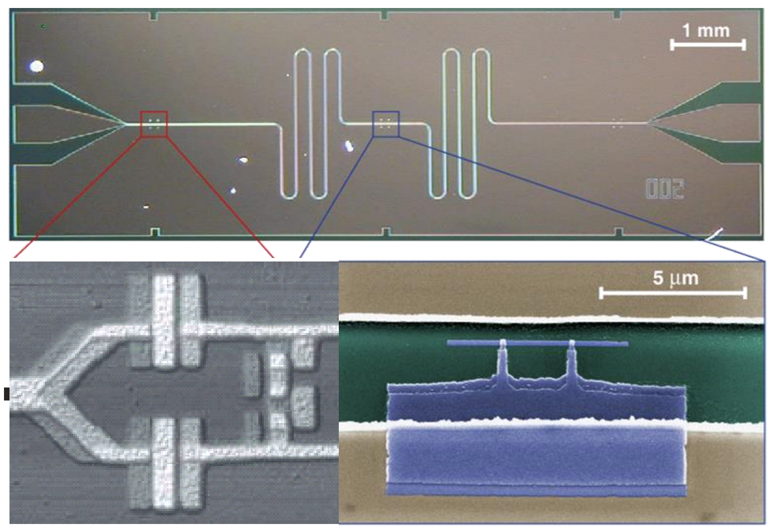
alkali atoms

MPQ, Caltech, ...



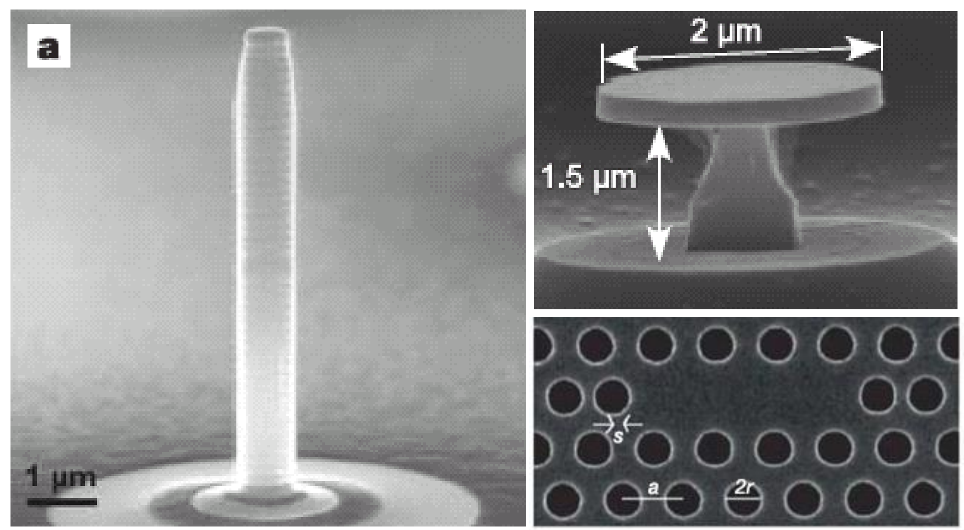
Rydberg atoms

ENS, MPQ, ...



superconductor circuits

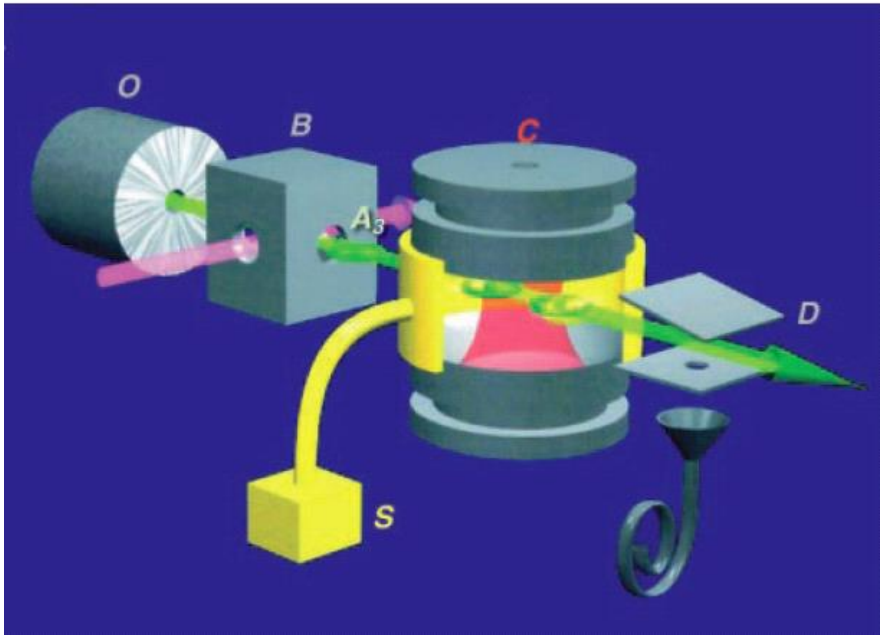
Yale, Delft, NTT, ETHZ, NIST, …

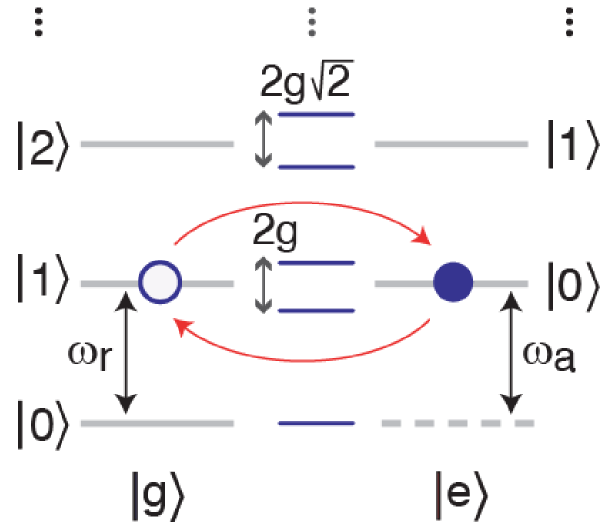


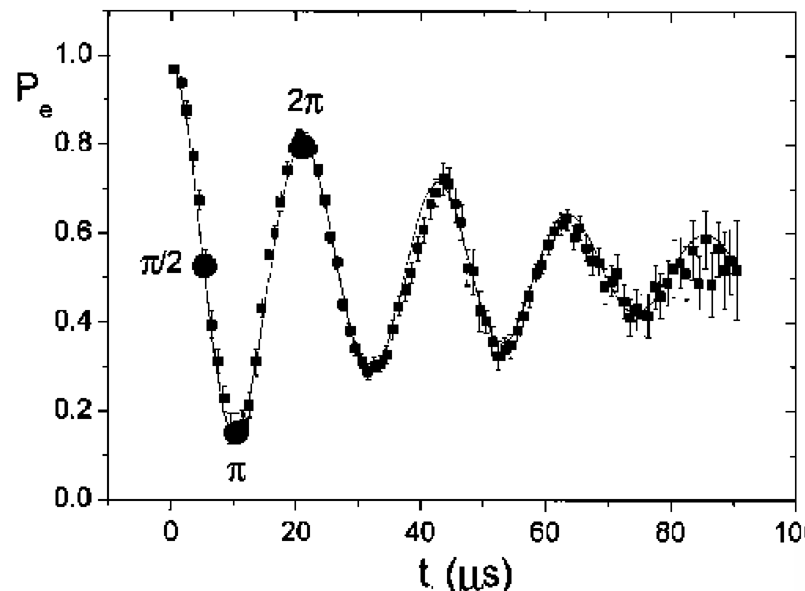
semiconductor quantum dots

Wurzburg, ETHZ, Stanford …

Vacuum Rabi Oscillations with Rydberg Atoms

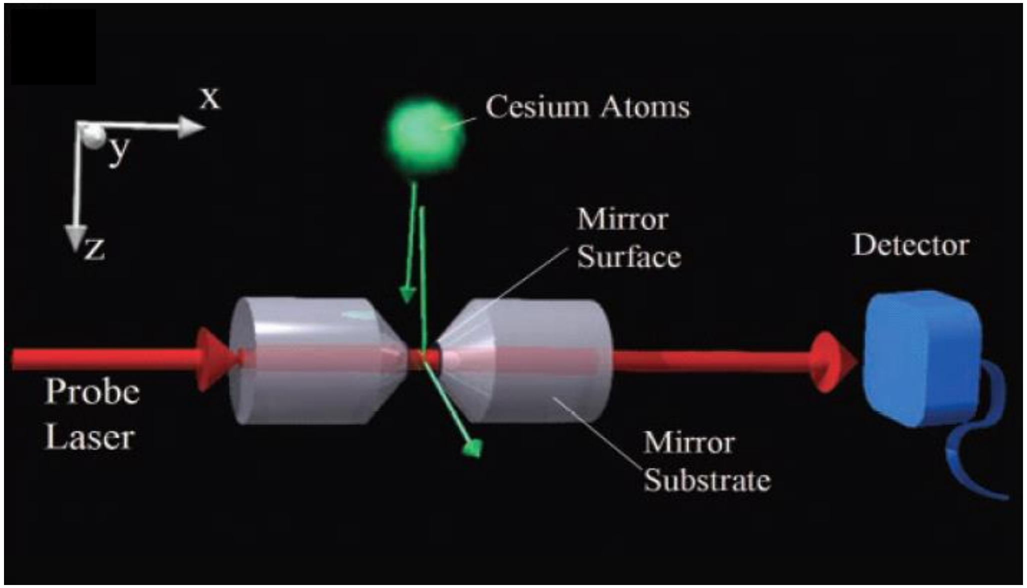


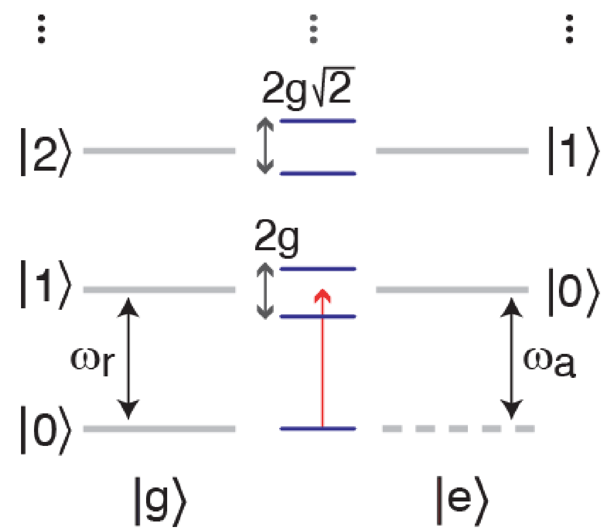




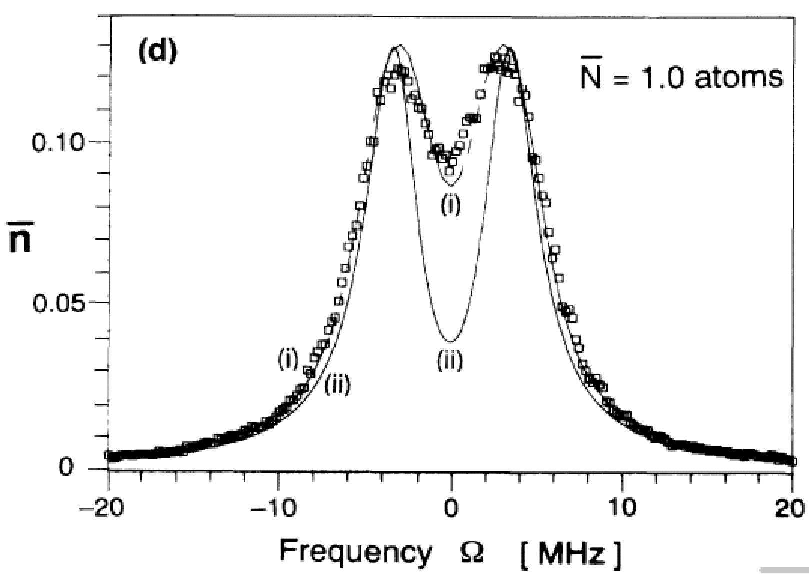
Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)

Vacuum Rabi Mode Splitting with Alkali Atoms

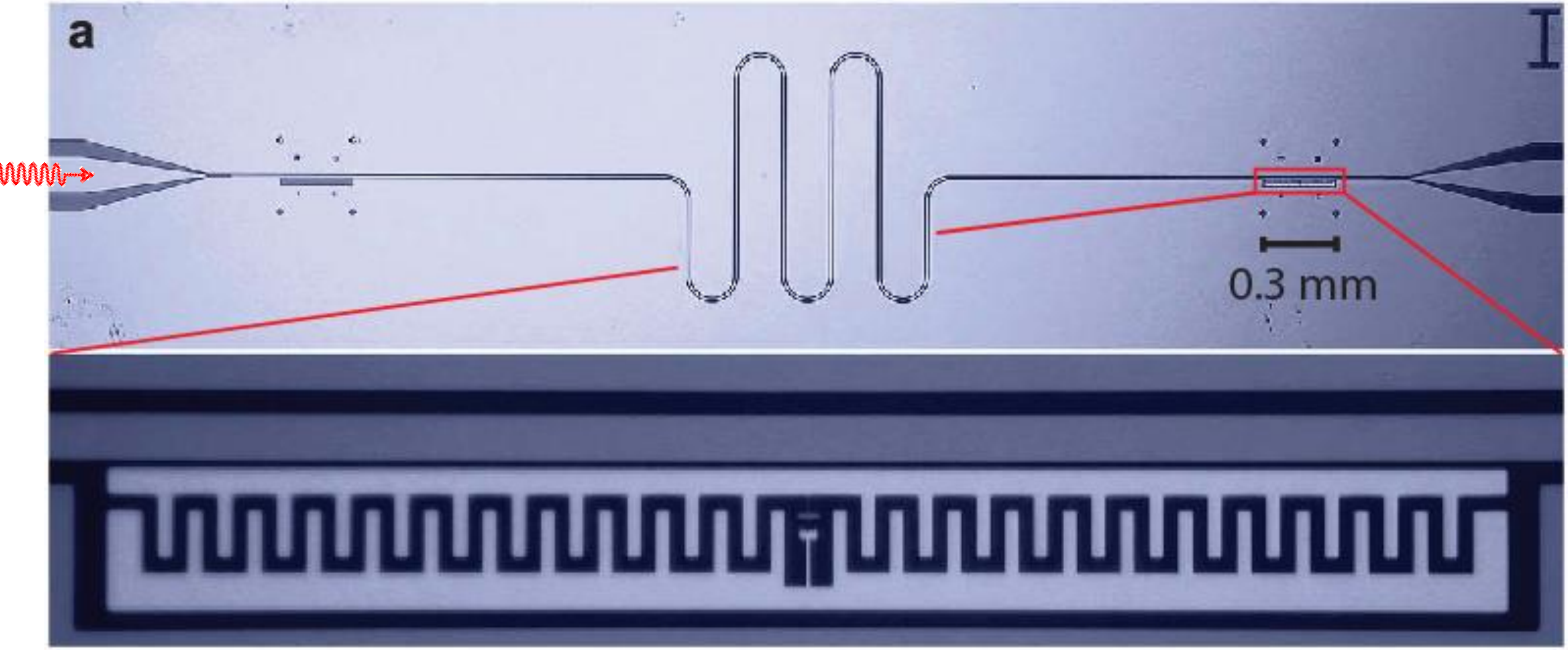




R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble
Phys. Rev. Lett. **93**, 233603 (2004)

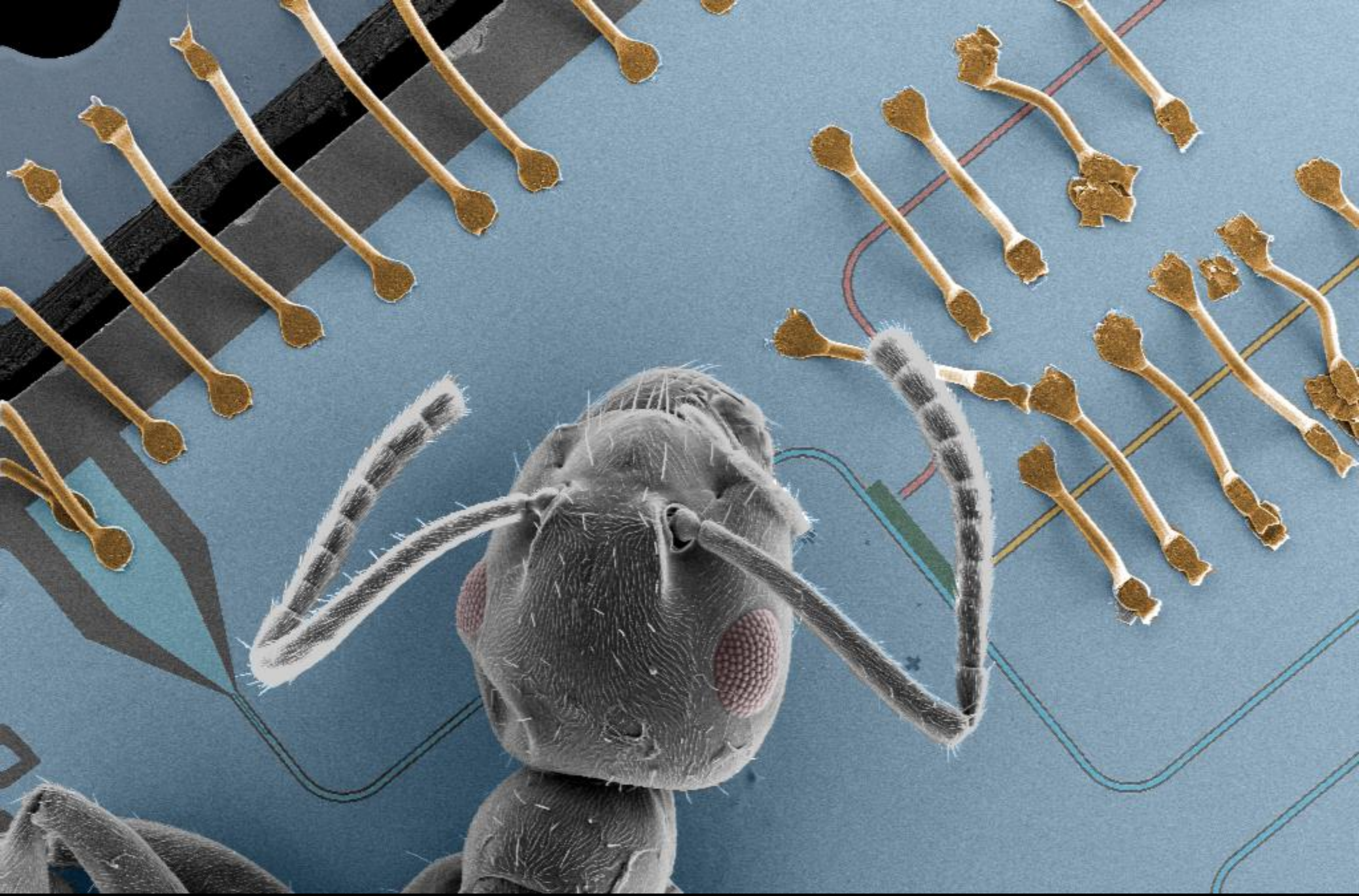


Circuit QED with One Photon



superconducting cavity QED circuit

A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)



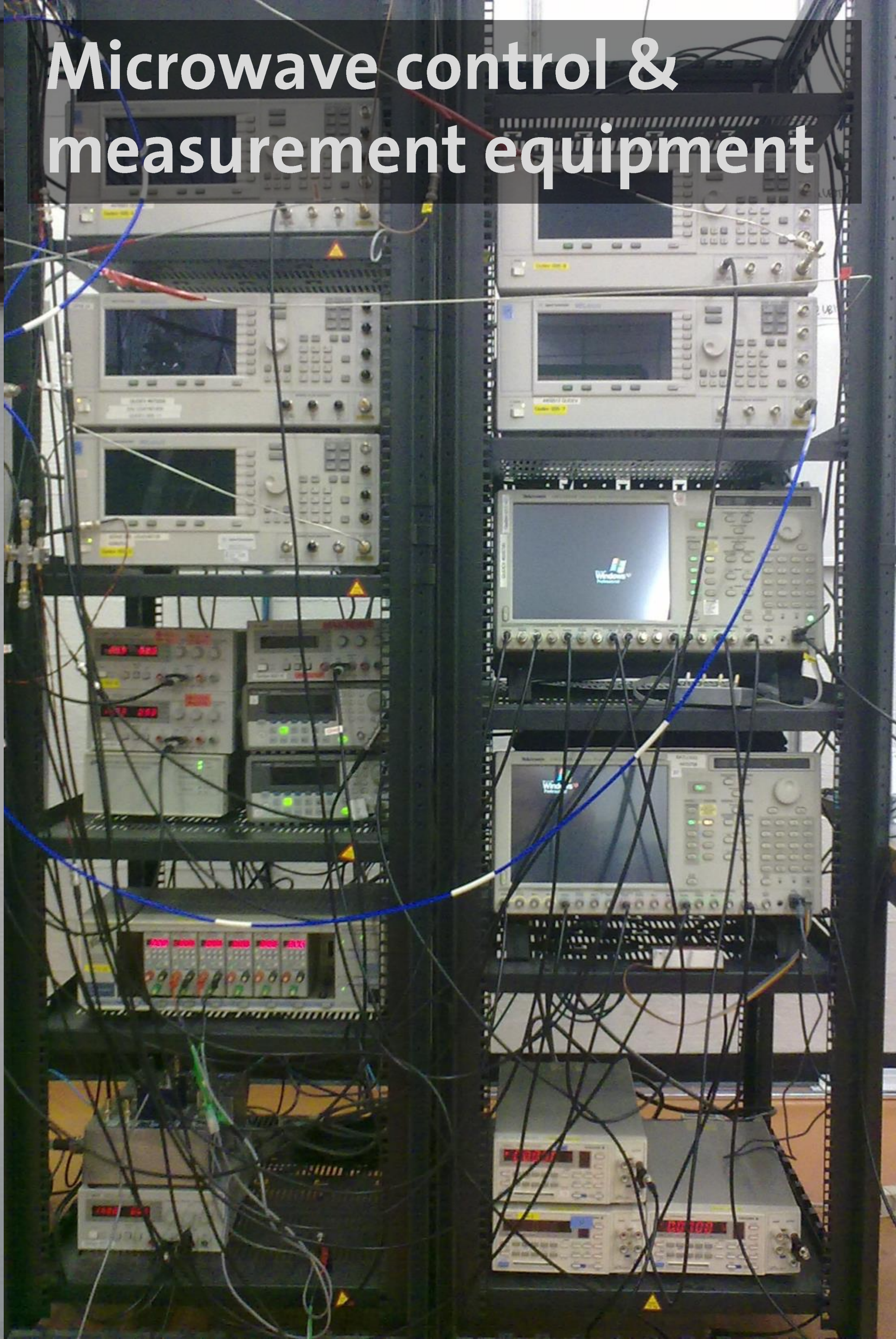
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

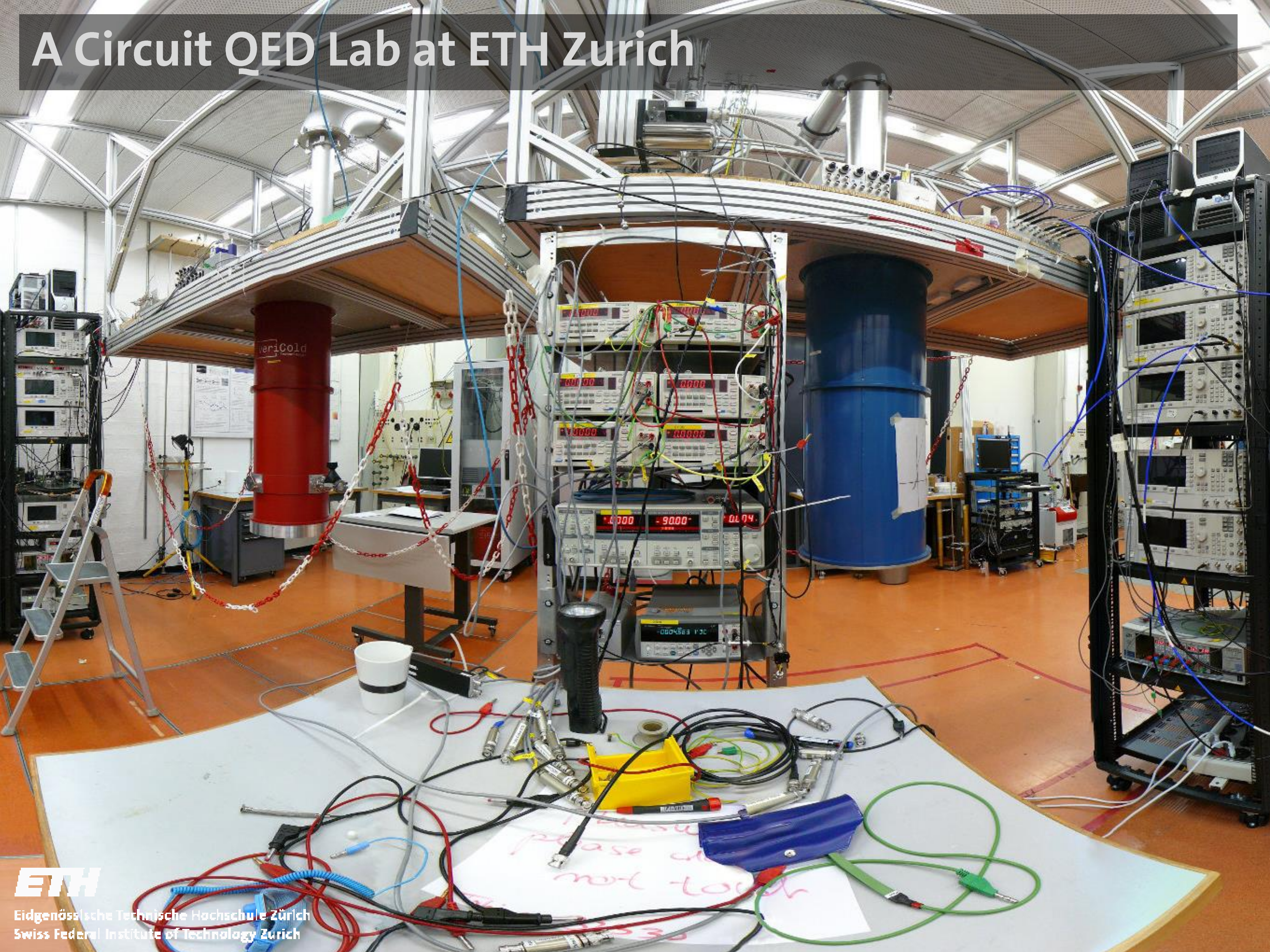
Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)


Cryostate for temperatures down to 0.02 K
VeriCold
Technologies
~ 20 cm
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich


Microwave control & measurement equipment

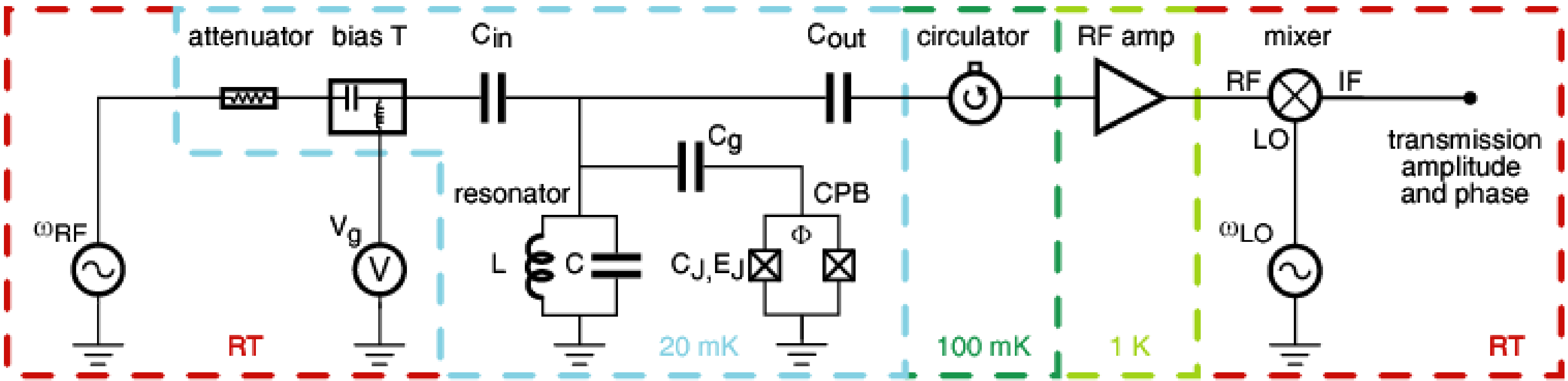
A Circuit QED Lab at ETH Zurich



How to Measure Single Microwave Photons

- average power to be detected

  $\rightarrow \langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$

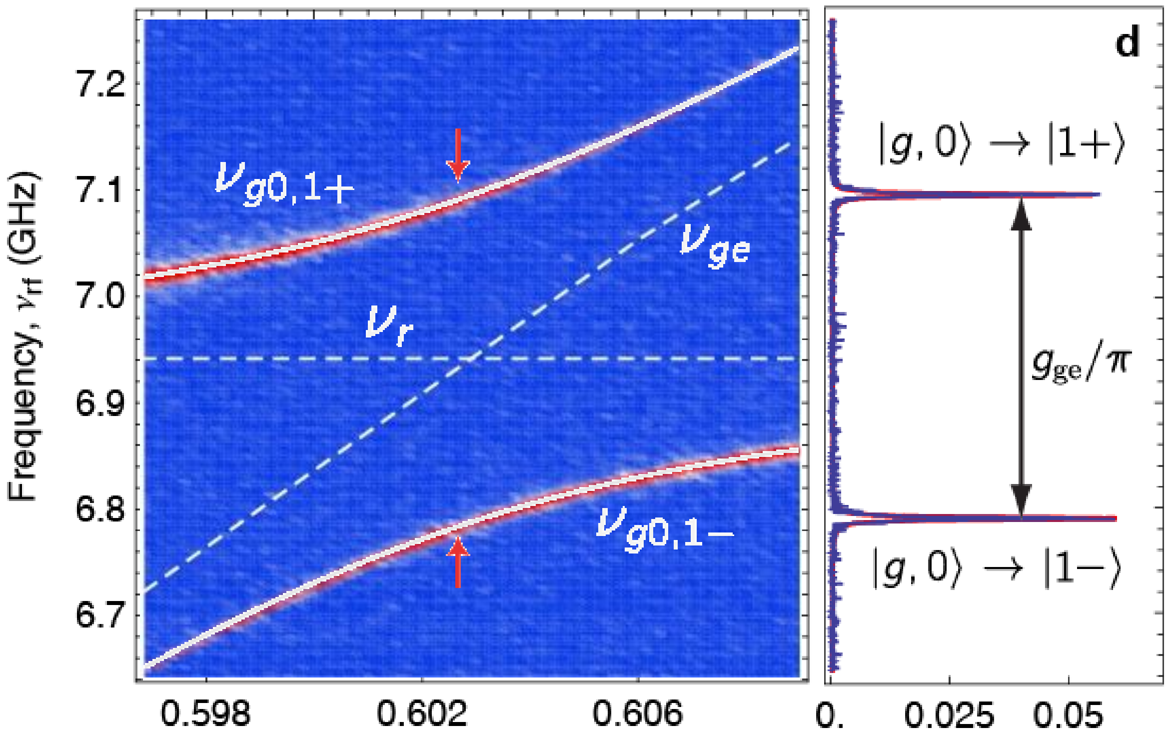


- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)

- prevent leakage of thermal photons (cold attenuators and circulators)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

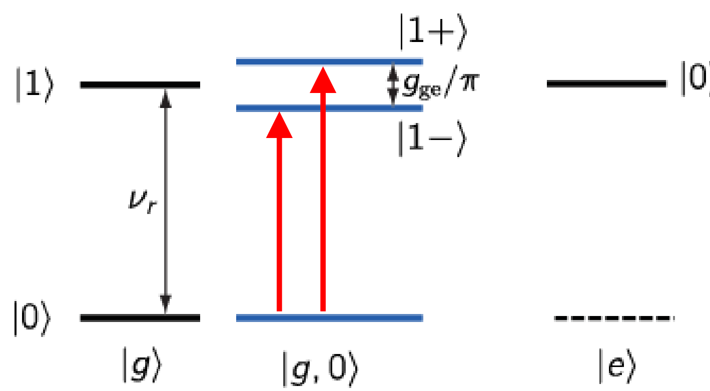


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

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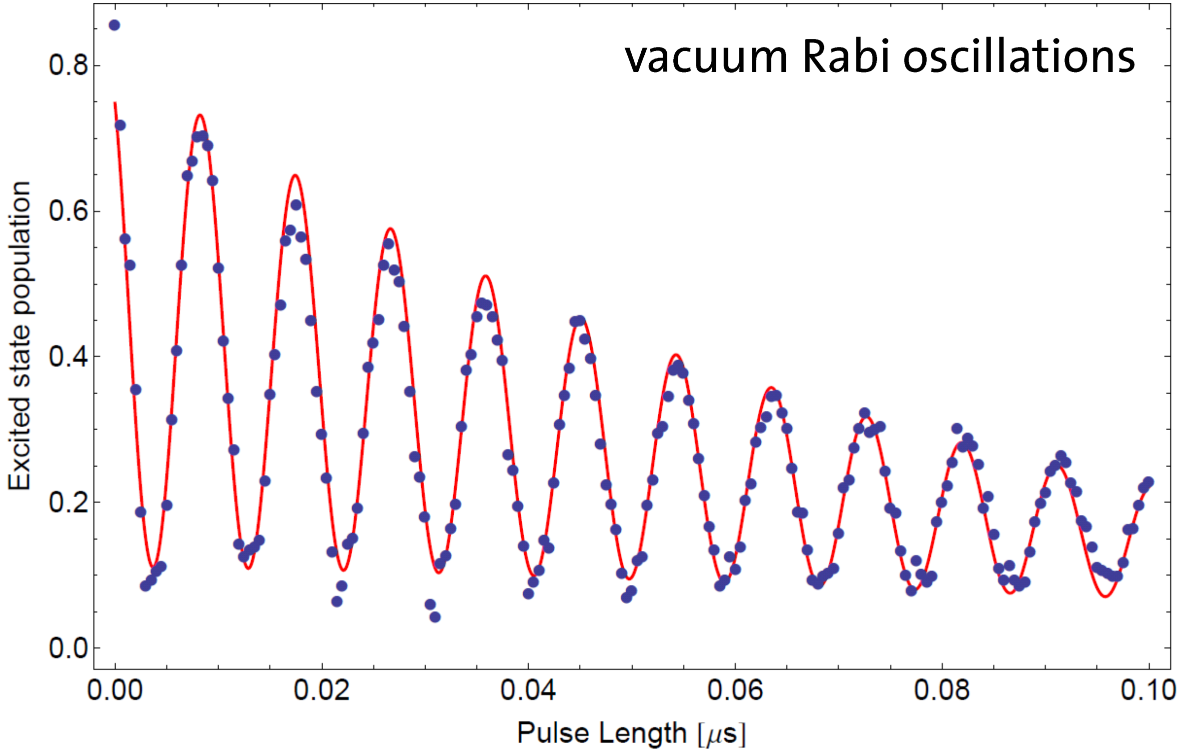

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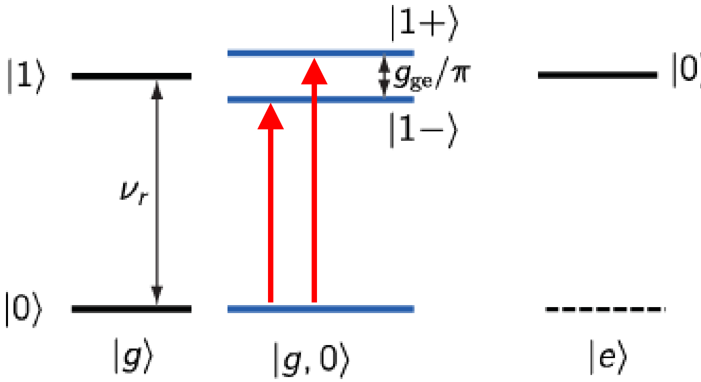
$$\kappa, \gamma < 1\ \text{MHz}$$

$$g_{\text{ge}} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)