Classical information processing

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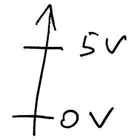
*) classical bit: possible values 0, 1 physical realization



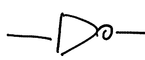
voltage level in a circuit

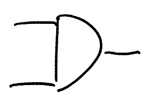
*) manipulation of bits with a physical process:

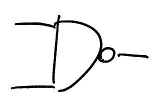
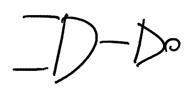
e.g. magnetization on harddisc, flip flop circuit for RAM

*) any logical operation can be decomposed into single & two-bit operations (gate)

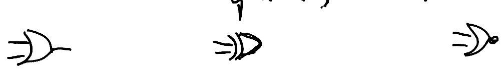
examples: identity-gate a ——— a 'do nothing'

NOT-gate a  NOT a

AND-gate a, b  a AND b

NAND-gate a, b  a NAND b = 

input qubit(s) operation output qubit(s)

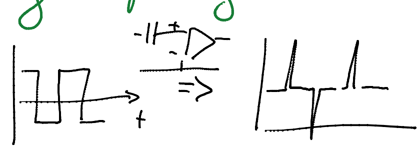


OR-gate, XOR gate, NOR gate

Why is digital computing much better than analog computing?

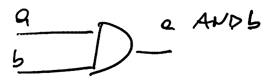
*) resilience to errors

*) more applications (universality)



Truth tables:

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

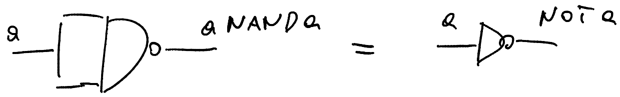


Universal logic gates: NOR, NAND

Any other logical gate can be built from using only NOR (or only NAND) gates.

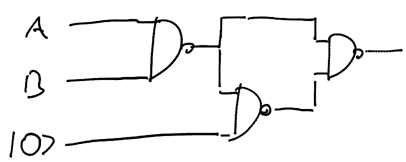
a	a NAND a
0	1
1	0

e.g.



⇒ universal gates do also exist for quantum computers!

Circuit representation:



((A NAND B) NAND 0) NAND A $\hat{=}$ A AND B

properties:
- bits can be copied (FANOUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than input bits

Which of these properties are easy / hard to realize in a quantum computer?
- copying qubits is not possible (no-cloning theorem)

*) ancillas ✓

*) crossover ⟺ swap operation ✓

*) # outputs = # inputs (reversibility)

↳ e.g. AND-gate is not reversible, inputs (a, b) cannot be reconstructed from output

Generic Quantum Computing Architecture

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features: (Di Vincenzo Criteria)

1. quantum bits
2. initialization
3. coherence
4. universal gates
5. read out

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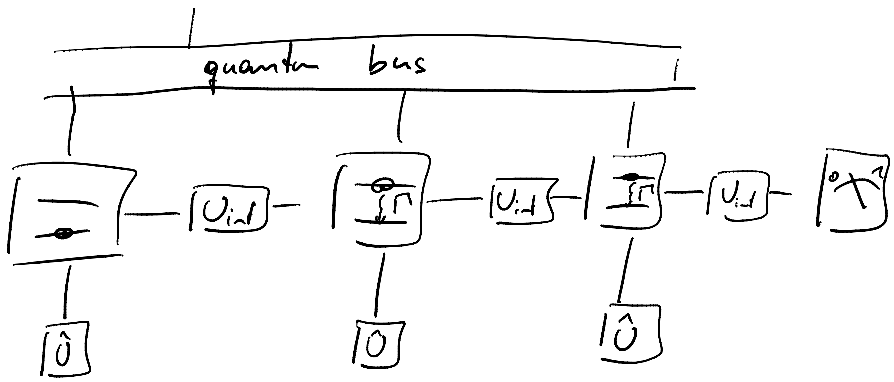
6. conversion stationary/mobile
7. transmission of qubits



The quantum bit

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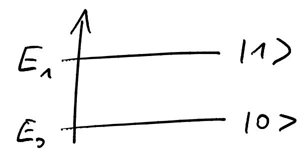
*) quantum mechanical system with two distinct states



examples: neutron / proton / electron spin,
atomic two levels
⋮

Under which conditions can you consider a quantum mechanical system as an effective two-level system (equivalent to a spin-$\frac{1}{2}$ particle)?

$\Rightarrow$ frequency separation

*) representation of qubit states: vectors in Hilbert space (2-dim, complex)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrel{\hat{=}} |\downarrow\rangle \mathrel{\hat{=}} |H\rangle$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathrel{\hat{=}} |\uparrow\rangle \mathrel{\hat{=}} |V\rangle$$

$|\rangle$ .. Dirac notation
$()$ .. vector notation

*) most general qubit state

$$\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C}}$$

superposition state of $|0\rangle$ & $|1\rangle$

probability to find system in 0 or 1:

$$P_0 = |\alpha|^2$$

$$P_1 = |\beta|^2$$

law of probabilities: $P_0 + P_1 = 1$

$\Rightarrow$ Normalization: $|\alpha|^2 + |\beta|^2 = 1$

What is the main difference to classical bit?

- phase matters, that's why decoherence is an issue
- parallelism: operations act on both qubit in $|0\rangle$ AND in $|1\rangle$

[extension to many qubits: 2 qubits 00, 01, 10, 11

3 qubits $\underbrace{000, 001, \ldots}_{\text{8 possible states}}$

→ parallel operation on 2^n states!]

Bloch Sphere

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$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$ $\Rightarrow$ 3 parameters

$\Rightarrow$ parameterization? $\alpha = \cos\frac{\theta}{2} e^{i\varphi_\alpha}$

$\beta = \sin\frac{\theta}{2} e^{i\varphi_\beta}$

$$|\psi\rangle = \cos\frac{\theta}{2} e^{i\varphi_\alpha}|0\rangle + \sin\frac{\theta}{2} e^{i\varphi_\beta}|1\rangle =$$

$$= e^{i\varphi_\alpha}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

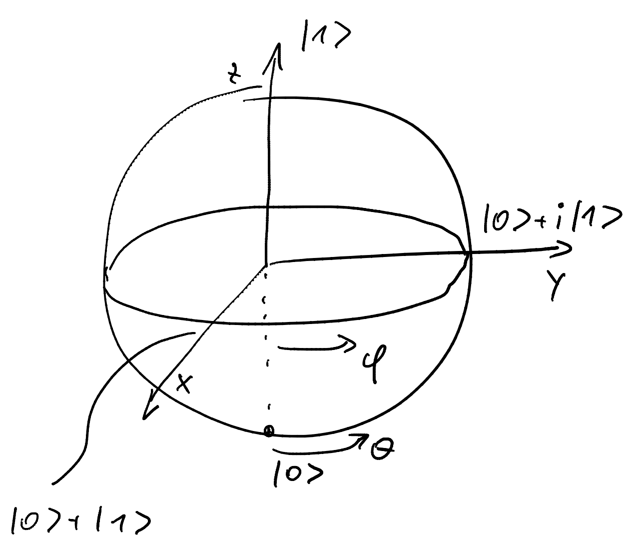
Does the global phase factor $e^{i\varphi_\alpha}$ matter?

Not for a single qubit! All measurements are insensitive to a global phase:

$$\langle\psi|\hat{O}|\psi\rangle = \langle\psi'|e^{-i\varphi}\hat{O}\,e^{i\varphi}|\psi'\rangle = \langle\psi'|O|\psi'\rangle$$

$\Rightarrow$ 2 parameters: $\theta \in [0,\pi]$ ... polar angle

$\varphi \in [0, 2\pi]$ ... azimuthal angle



equal superposition states lie on the equatorial plane with $\theta = \frac{\pi}{2}$ $\left(\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\varphi}|1\rangle\right)$$

Why '$\cos\frac{\theta}{2}$' and not '$\cos\theta$'?

spin-$\frac{1}{2}$ / 2-level system shows 4π-symmetry, i.e. state returns to itself only for $\theta = 4\pi$, otherwise it obtains additional $\ominus$ sign: (observable in interference experiment)

$$|\psi\rangle_{\theta=0} = -|\psi\rangle_{\theta=2\pi} = |\psi\rangle_{\theta=4\pi}$$

(Representation of group SU(2) instead of SO(3))

Single Qubit Gates

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Circuit representation:

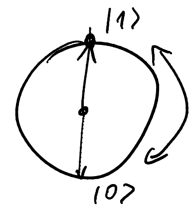
$$|\psi\rangle_{IN} \;—\; \boxed{\hat{U}} \;—\; |\psi\rangle_{OUT}$$

Work out how specific operations are represented on the Bloch sphere:

*) identity: $\hat{U} = \hat{I} = \mathbb{1}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

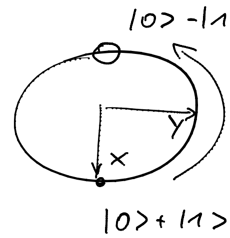
*) bit flip: $\hat{U} = \hat{X} = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{=}$ NOT operation

(rotation by π about x-axis)



*) phase flip: $\hat{U} = \hat{Z} = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



(top view)

*) conjugate bit flip: $\hat{U} = \hat{Y} = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
(bit-phase flip)

$$|0\rangle \to i\,|1\rangle$$
$$|1\rangle \to -i\,|0\rangle$$

$\sigma_x, \sigma_y, \sigma_z$ .. Pauli matrices

$\{\mathbb{1}, \sigma_x, \sigma_y, \sigma_z\}$ basis set for operators on a single qubit

Hadamard gate: generation of superposition from $|0\rangle$ or $|1\rangle$

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left(\hat{X} + \hat{Z} \right)$$

$\hat{=}$ rotation by π about $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$-axis

$$|0\rangle \; —\boxed{\hat{H}}— \; \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \; —\boxed{\hat{H}}— \; \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$\Rightarrow$ used in many quantum algorithms to prepare (initial) superposition state