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Quantum error correction on a hybrid spin system

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Outline

- Error correction: why we need it, how it works
- Experimental realization of an electron-nuclear quantum register in an hybrid spin system
- Implementation of joint initialization, projective read-out and non-local gate operations using the ancillary electron spin of a nitrogen-vacancy defect in diamond
- Experimental demonstration of phase-flip error correction

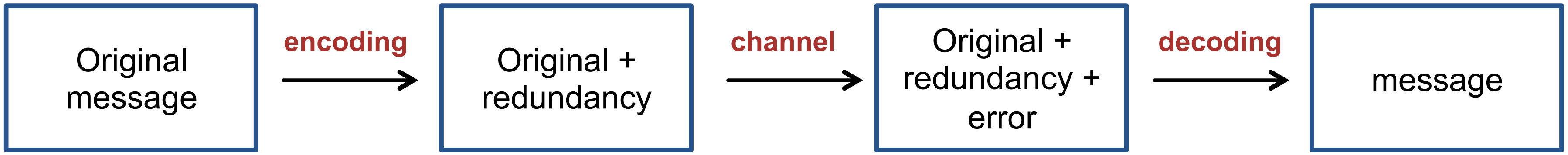
Noise is a major threat to reliable quantum information processing

- Why classical computers don't use error correction?
  - large number of electrons vs. single (or a small number) of particles
  - digital (after each step they correct themselves to the closer 1 or 0) vs. analog (with continuous states errors can sum up)
- Quantum states are intrinsically delicate: looking at one collapses it! And the environment is constantly trying to look at the state (decoherence)

WE NEED ERROR CORRECTION !

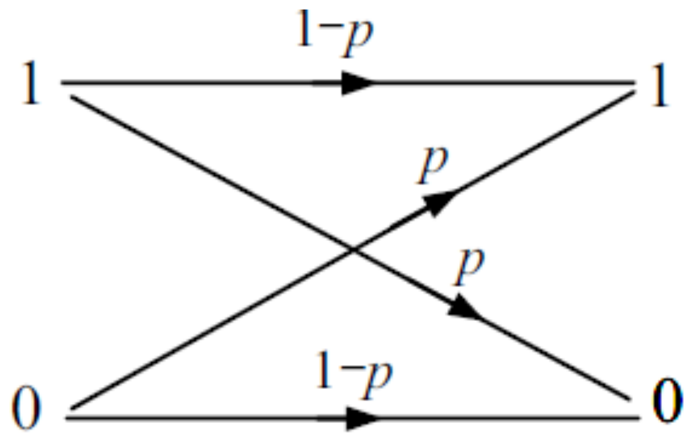
Quantum error correction and the threshold theorem allow fault-tolerant quantum computation

- Quantum error correction codes protect quantum information against noise adding redundant information



- Arbitrarily good quantum computation can be achieved even with faulty gates, provided that the error probabilities per gate is below a certain constant threshold

Error correcting protocols are based on adding redundant information: a classical case study



- sending a bit through a noisy classical communication channel
- Noise modeling: flip the bit with $p > 0$
- Encoding process

$$0 \rightarrow 000 = logical\ 0$$

$$1 \rightarrow 111 = logical\ 1$$

- Send the bits through the channel
- Decode through majority voting
- Reliable if $p < \frac{1}{2}$

Error correcting codes must take into account the differences between classical and quantum case

- If $|\psi\rangle = a|0\rangle + b|1\rangle$

A No cloning theorem

$$U(|\psi\rangle \otimes |\phi\rangle = |\psi\rangle \otimes |\psi\rangle \qquad \forall |\psi\rangle$$

B Errors are continuous

$$a, b \in \mathbb{C}$$

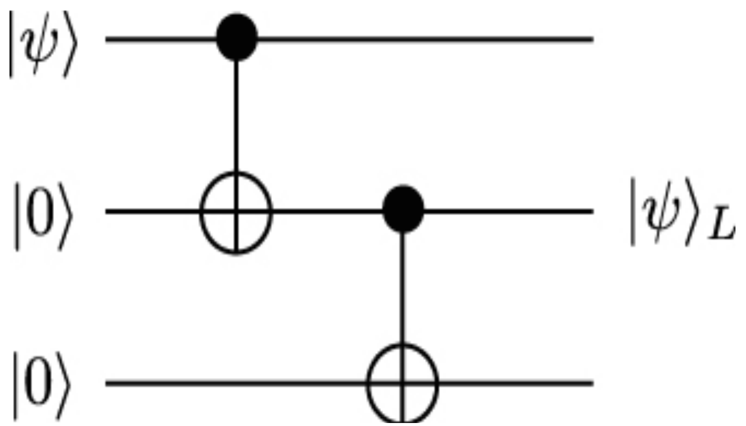
C Measurements destroy quantum information

The bit flip code: encoding

- Error modeling: bit flipped with p probability

$$a|0\rangle + b|1\rangle \rightarrow a|1\rangle + b|0\rangle$$

- Encoding of a single qubit state in three entangled qubits



Encoded state

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow |\psi_L\rangle = a|000\rangle + b|111\rangle$$

The bit flip code: error-detection and recovery

- Measuring the error syndromes (projection operators) we gain information about which error has occurred

$$
\begin{aligned}
P_0 &\equiv |000\rangle\langle 000| + |111\rangle\langle 111| && \text{no error} \\
P_1 &\equiv |100\rangle\langle 100| + |011\rangle\langle 011| && \text{bit flip on qubit one} \\
P_2 &\equiv |010\rangle\langle 010| + |101\rangle\langle 101| && \text{bit flip on qubit two} \\
P_3 &\equiv |001\rangle\langle 001| + |110\rangle\langle 110| && \text{bit flip on qubit three}
\end{aligned}
$$

- Syndromes contain no information about the state being protected
- The value of the error syndrome tells us what procedure to use to recover the initial state

The phase flip code: overview

- Error modeling: phase flipped with p probability

$$a|0\rangle + b|1\rangle \rightarrow a|0\rangle - b|1\rangle$$

- Phase flips error rate is usually much higher than the bit flip error rate
- Working in the $|+\rangle\,,\ |-\rangle$ basis we can turn the phase flip channel into a bit flip channel

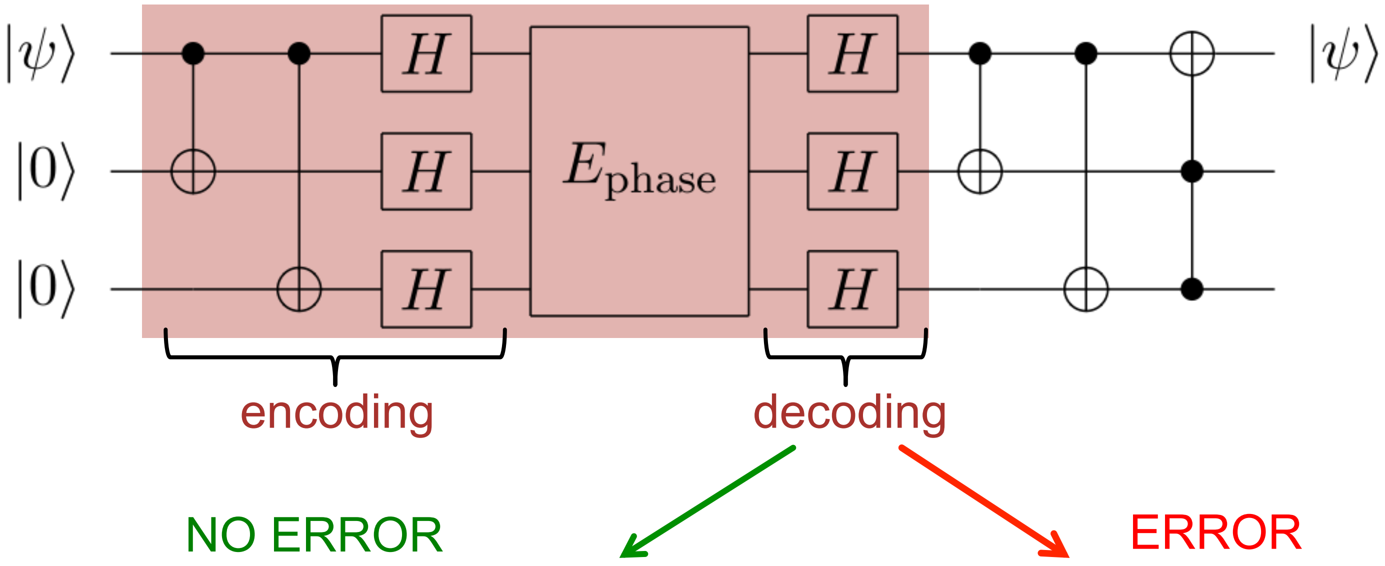
$$|+\rangle \equiv \frac{(|0\rangle+|1\rangle)}{\sqrt{2}}, \quad |-\rangle \equiv \frac{(|0\rangle-|1\rangle)}{\sqrt{2}}$$

- The change of basis is performed by the Hadamard gate

The phase flip code: encoding

- The initial state $|\psi\rangle = a|0\rangle + b|1\rangle$ is encoded as

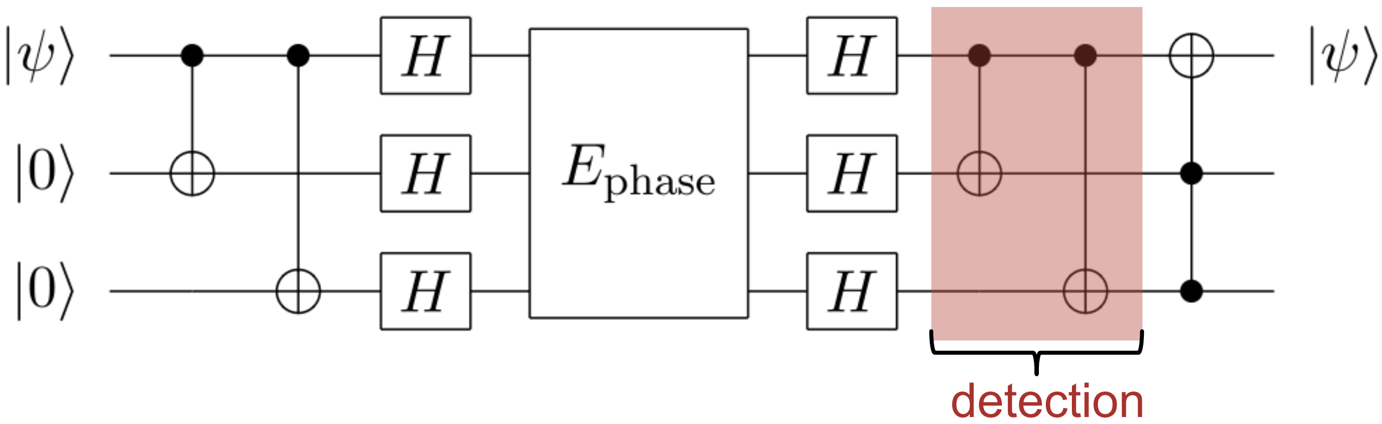
$$|\psi\rangle = a|+++\rangle + b|---\rangle$$



$$|\psi\rangle = a|000\rangle + b|111\rangle \qquad |\psi\rangle = a|100\rangle + b|011\rangle$$

The phase flip code: detection

- The error is detected via the other two qubits

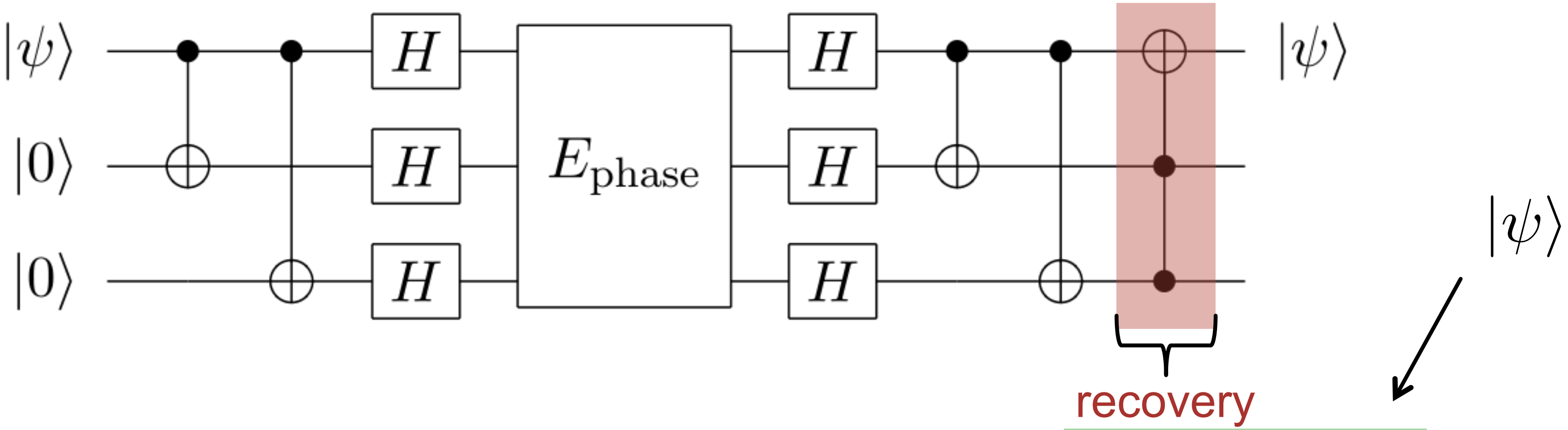


NO ERROR $|\psi\rangle = a\,|000\rangle + b\,|111\rangle \longrightarrow (a\,|0\rangle + b\,|1\rangle) \otimes |00\rangle$

ERROR $|\psi\rangle = a\,|100\rangle + b\,|011\rangle \longrightarrow (a\,|1\rangle + b\,|0\rangle) \otimes |11\rangle$

The phase flip code: recovery

- The first qubit is flipped conditional on the state of the two others transferring any errors onto the two other qubits



NO ERROR $(a|0\rangle + b|1\rangle) \otimes |00\rangle \longrightarrow (a|0\rangle + b|1\rangle) \otimes |00\rangle$

ERROR $(a|1\rangle + b|0\rangle) \otimes |11\rangle \longrightarrow (a|0\rangle + b|1\rangle) \otimes |11\rangle$

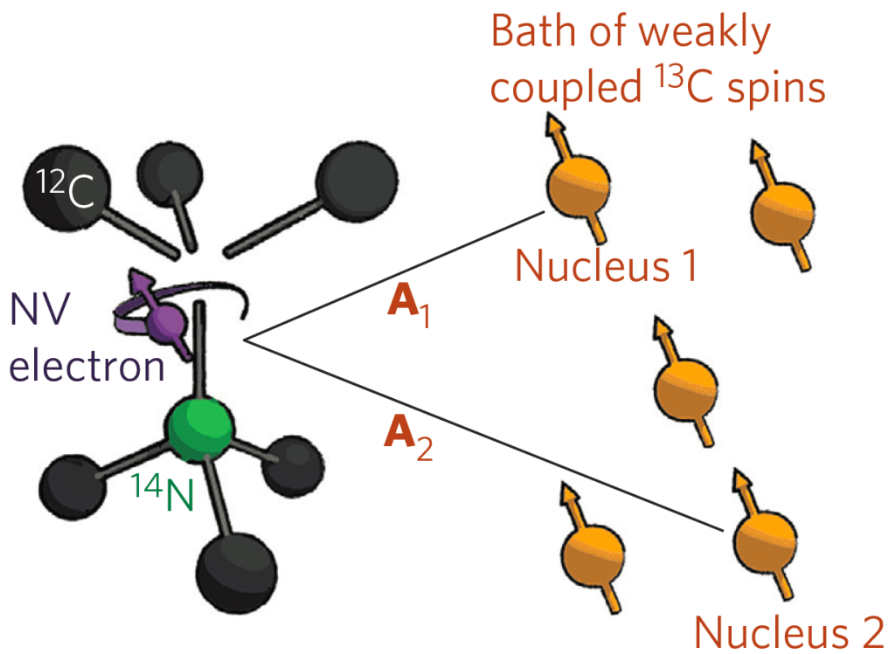
The phase flip correction protocol is specifically tailored for quantum processes

A Encoding does not violate the no cloning theorem (redundancy, not repetition)

B Error detection projects continuous state rotations onto the two possible cases of not having an error or having an error on the qubit (no need of knowing the error)

C The code preserves quantum information (non demolition detection)

NV Centre represents a promising approach for quantum computation



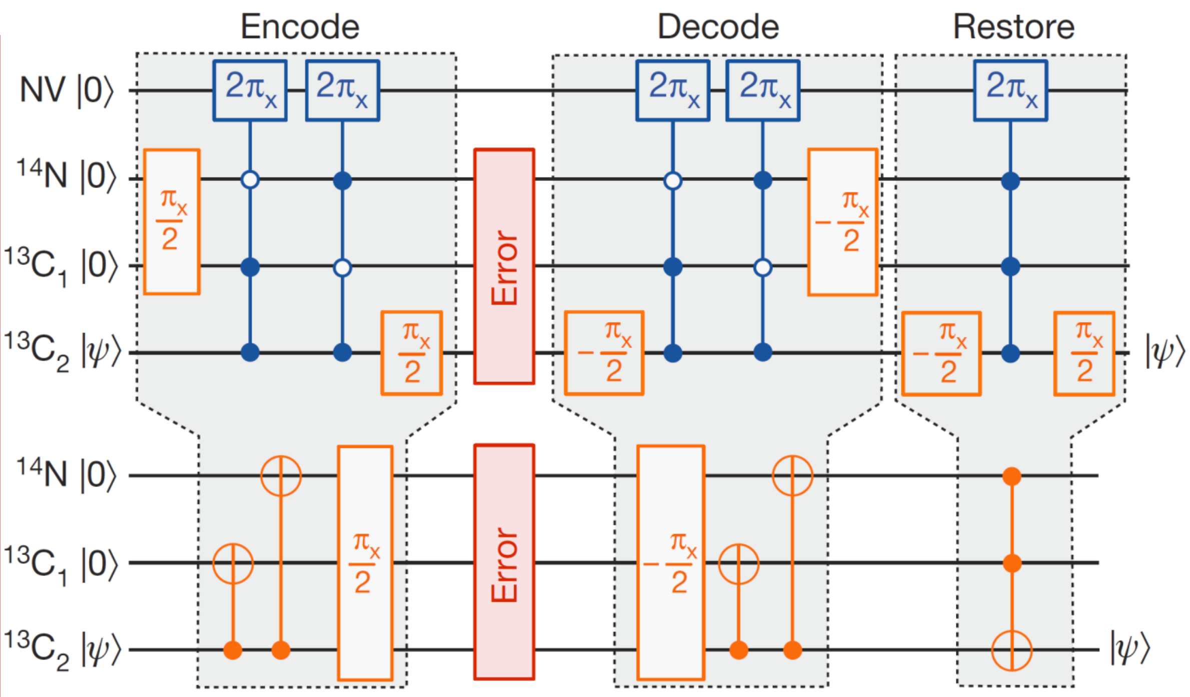
T. H. Taminiau et al., Nat. Nanotechnol. **9**, 3 (2014)

- Use defect electron and surrounding ^{13}C / ^{14}N as qubits
- Long coherence time of nuclear spins: qubits
- Electron spin: fast readout
- Number of nuclear spins suitable for quantum information processing depends on the strength of the coupling between nuclear and electron spin
- State preparation (waveguides, SIL) is required

Implementation of the phase flip error correction protocol in an NV center diamond

- The physical implementation of the code requires the following ingredients:

- Rotations of nuclear spin
  - Conditional
  - Unconditional
- CNOT gates
- Initialization
- Readout
- Controlled Error



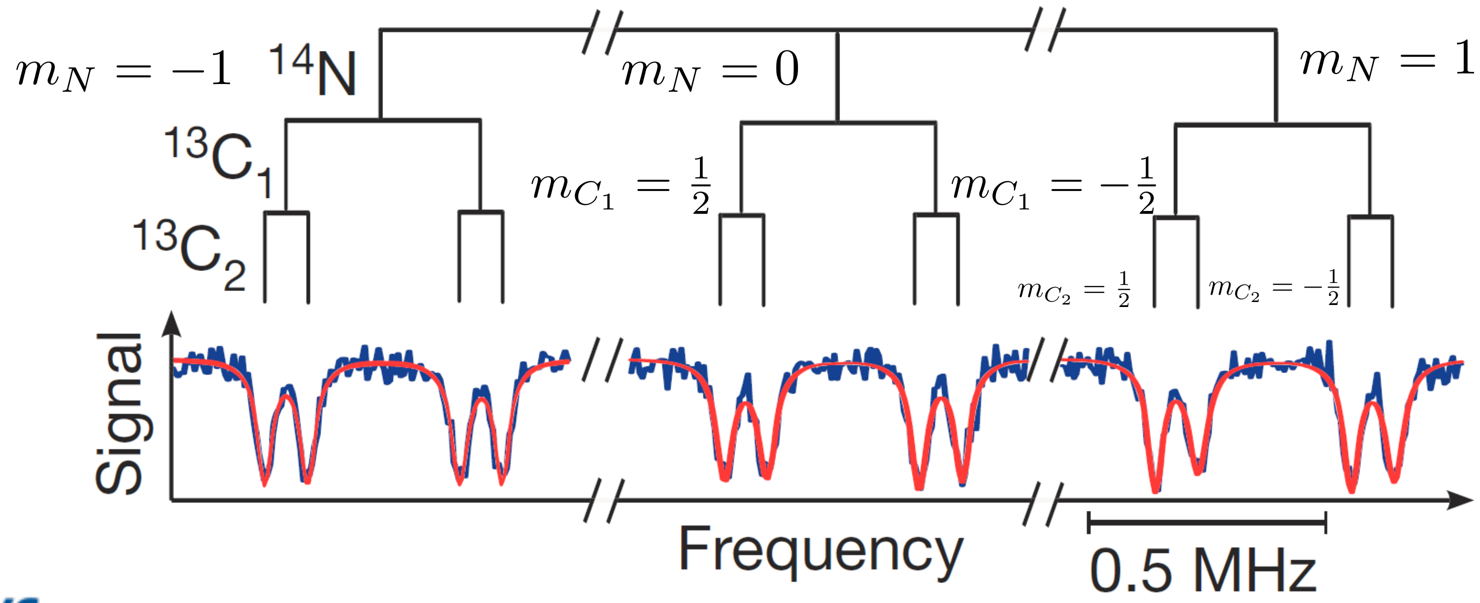
G. Waldherr et al., Nature, 2014/01/29/online

Rotations (1/4)

- Consider a single nuclear spin: $\{|\uparrow\rangle\,,\ |\downarrow\rangle\}$
- Electron spin $m_s \in \{0,\ \pm 1\}$
- If $m_s = 0$
  - No splitting of nuclear spin
  - Larmor precession ω_L of nuclear spin in ext. B-field
- If $m_s = -1$
  - Hyperfine splitting of $|1\uparrow\rangle$ and $|1\downarrow\rangle$ by ω_1
  - No Larmor precession due to splitting
- Depending on electron spin: precession of nuclear spin
  → $C_e NOT_n$

Rotations (2/4)

- Transition frequency of electron $m_s = 0 \rightarrow m_s = -1$
- Splittings: $^{14}N: \ 2.16\,\mathrm{MHz}, \ ^{13}C_1: \ 413\,\mathrm{KHz}, \ ^{13}C_2: 89\,\mathrm{KHz}$
- Splitting between $m_s = \pm 1: \sim 800\,\mathrm{MHz}$ [Dutt]



G. Waldherr et al., Nature, 2014/01/29/online

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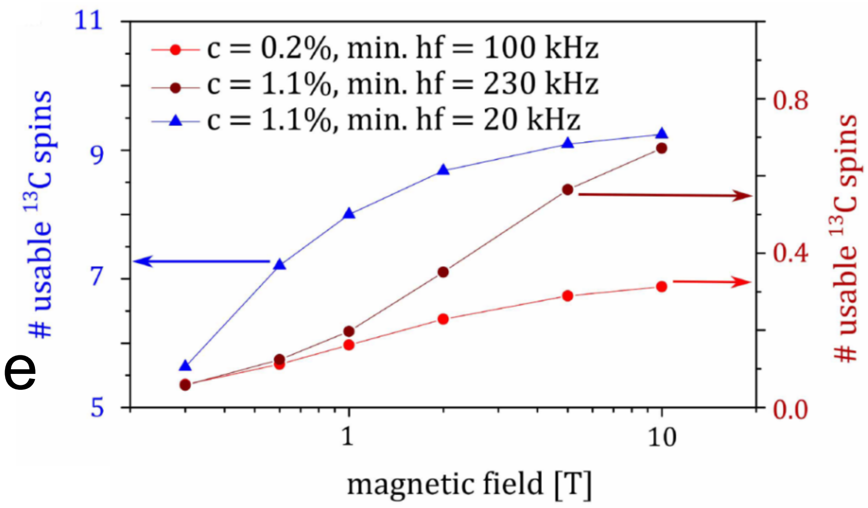
Rotations (3/4)

- Microwave transition frequency of electron spin depends on state of register
  → $C_n NOT_e$
- Exact splitting given by distances between nuclei
- Maximum number of accessible spins
  - Hyperfine resolution of lines
  - $2^{10} = 1024$, hence at most 10 coupled nuclear spins



G. Waldherr et al., Nature, 2014/01/29/online

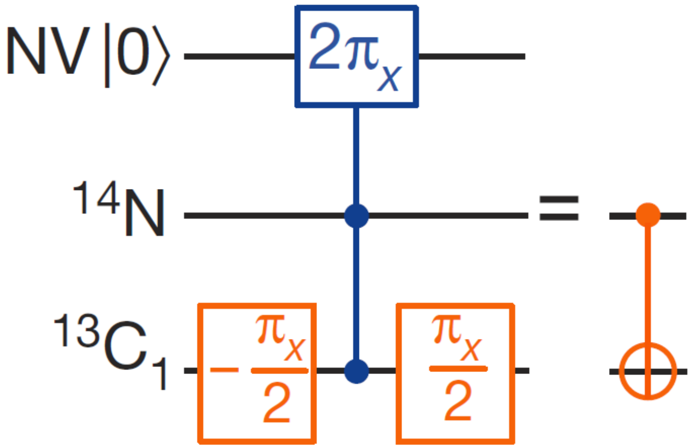
Rotations (4/4)

- Conditional rotation of nuclear spin:
  electron spin in $|0\rangle$ and wait for $\frac{\omega_L}{2} \equiv \pi - pulses$
- Conditional rotation of electron spin:
  apply the correct microwave $\pi - pulses$
- Gate Fidelity $\sim 96\%$ for $\frac{\pi}{2} - pulses$ [Taminiau 2014]

CNOT Gate

- CPhase on electron spin: $|0\rangle \otimes |11\rangle \rightarrow -|0\rangle \otimes |11\rangle$ (rotation by)

$$|00\rangle \rightarrow |0\rangle \otimes (|0\rangle - i\,|1\rangle) \rightarrow |0\rangle \otimes (|0\rangle - i\,|1\rangle) \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |0\rangle \otimes (|0\rangle + i\,|1\rangle) \rightarrow |0\rangle \otimes (|0\rangle - i\,|1\rangle) \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |1\rangle \otimes (|0\rangle - i\,|1\rangle) \rightarrow |1\rangle \otimes (|0\rangle + i\,|1\rangle) \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |1\rangle \otimes (|0\rangle + i\,|1\rangle) \rightarrow |1\rangle \otimes (|0\rangle + i\,|1\rangle) \rightarrow |10\rangle$$



G. Waldherr et al., Nature, 2014/01/29/online

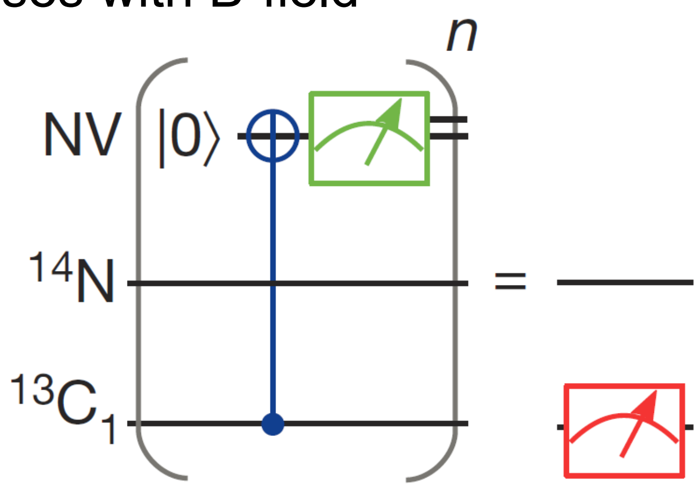
SWAP Gate

- Swap electron spin to nuclear spin

1. Initialization of electron spin with laser pulse to
   $|0\rangle_e \otimes (a|0\rangle + b|1\rangle)_n$
2. $C_nNOT_e : \ |00\rangle \rightarrow |10\rangle$
   $a|10\rangle + b|01\rangle$
3. $C_eNOT_n : \ |01\rangle \rightarrow |00\rangle$
   $a|10\rangle + b|00\rangle = (b|0\rangle + a|1\rangle)_e \otimes |0\rangle_n$

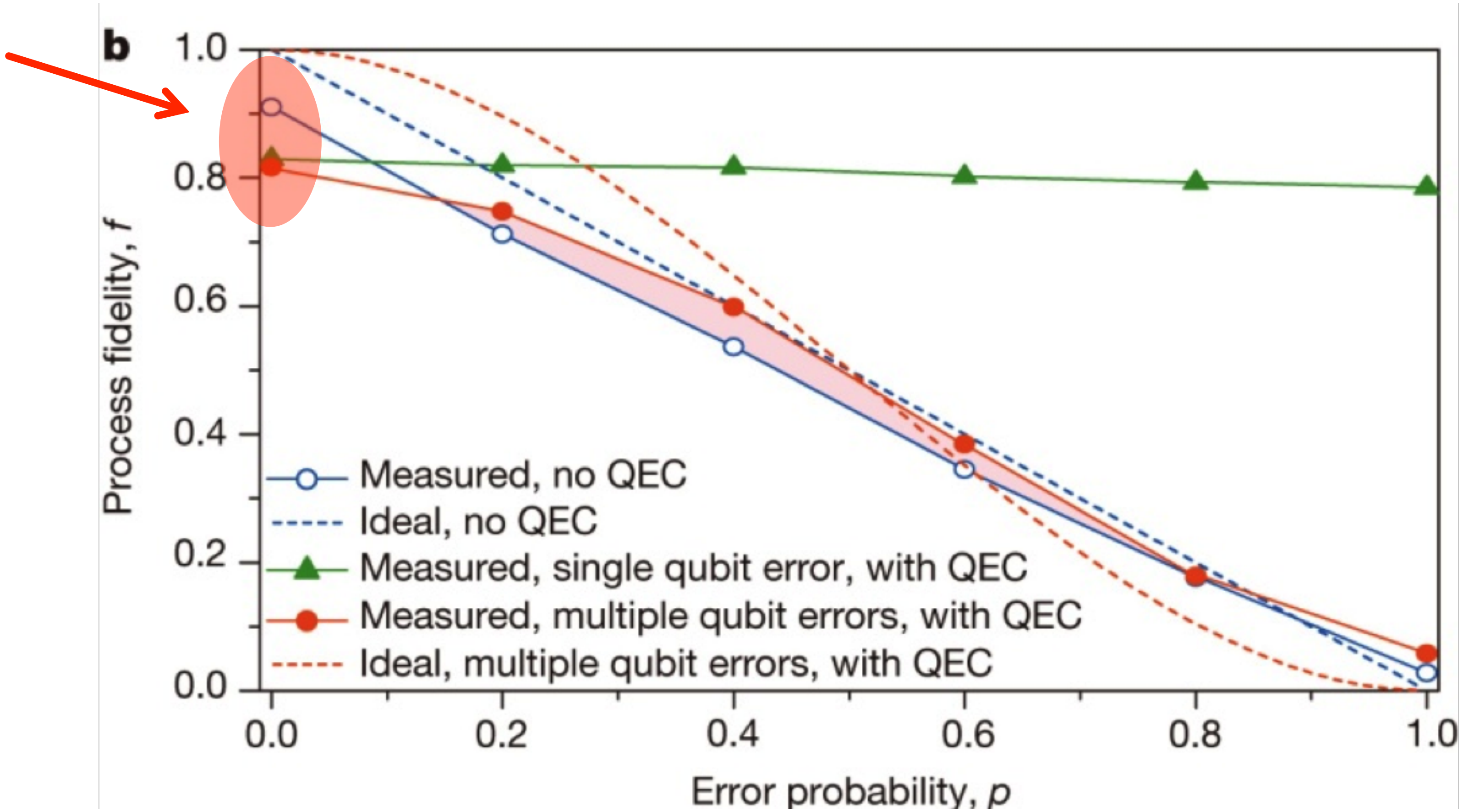
Readout and State Preparation

- Single-Shot Readout (QND)
  - Decoupling of the electron and the nuclear spin (B-field)
  - Test each microwave transition and use fluorescent readout
  - Even ~ 1000 photons do not affect the nuclear spin (scaling)
  - Fidelity $> 95\%$ [Waldherr 2014], increases with B-field
  - Projection into one nuclear spin state
- Preparation
  - Readout
  - SWAP Gate with re-initialization
    $(b\,|0\rangle + a\,|1\rangle)_e \otimes |0\rangle_n \rightarrow |00\rangle$
  - Fidelity $\sim 99\%$ [Waldherr 2014]



G. Waldherr et al., Nature, 2014/01/29/online

Results: controlled single qubit errors are corrected by the algorithm



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