Constructing Non-Linear Quantum Electronic Circuits

circuit elements:

anharmonic oscillator:

non-linear energy level spectrum:

Josesphson junction:

a non-dissipative nonlinear element (inductor)
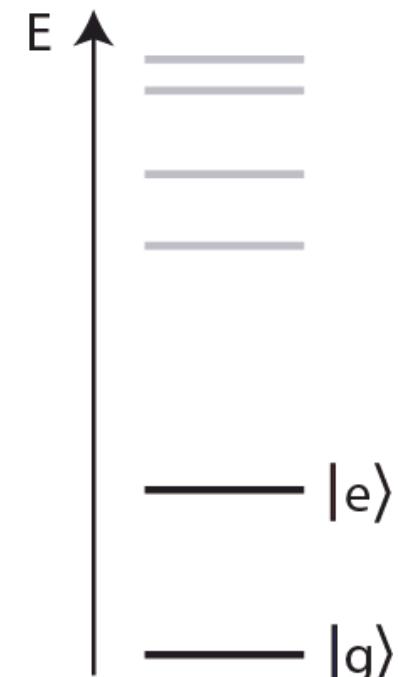
$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1}$$

$$= \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic artificial atom

A Classification of Josephson Junction Based Qubits

How to make use in of Jospehson junctions in a qubit?

Common options of bias (control) circuits:
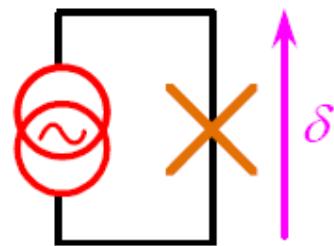
| phase qubit | charge qubit (Cooper Pair Box, Transmon) | flux qubit |

| current bias | charge bias | flux bias |
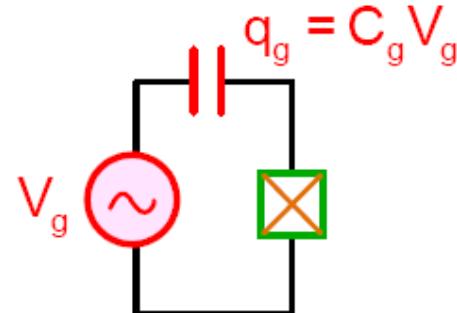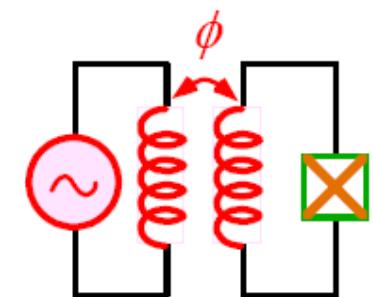
How is the control circuit important?

The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box
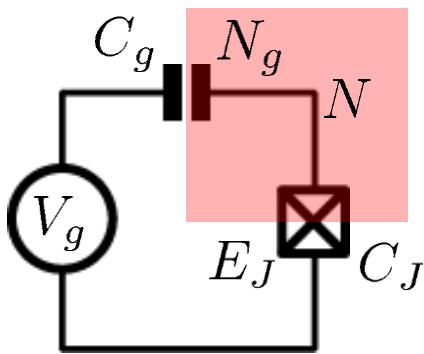
discrete charge on island:
$$N = \frac{Q}{2e}$$

continuous gate charge:
$$N_g = \frac{C_g V_g}{2e}$$

total box capacitance
$$C_\Sigma = C_g + C_J$$

Hamiltonian:
$$H = H_{\text{el}} + H_{\text{mag}}$$

electrostatic part:
$$H_{\text{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:
$$H_{\text{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy
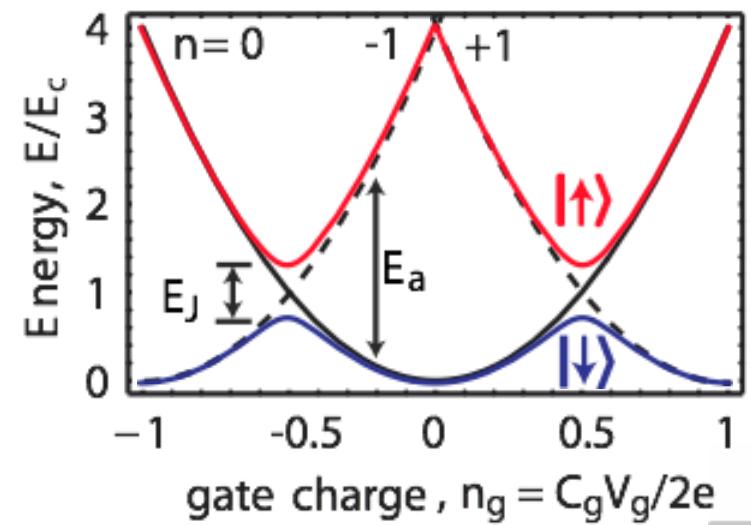
Hamilton Operator of the Cooper Pair Box

Hamiltonian:
$$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

commutation relation:
$$[\hat{\delta}, \hat{N}] = i$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

charge number operator:
$$\hat{N}|N\rangle = N|N\rangle \quad \text{eigenvalues, eigenfunctions}$$

$$\sum_N |N\rangle\langle N| = 1 \quad \text{completeness}$$

$$\langle N|M\rangle = \delta_{NM} \quad \text{orthogonality}$$

phase basis:
$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle \quad \text{basis transformation}$$

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** N:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n |\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 - E_J \cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

energy level diagram for $E_J=0$:

- energy bands are formed

- bands are periodic in N_g
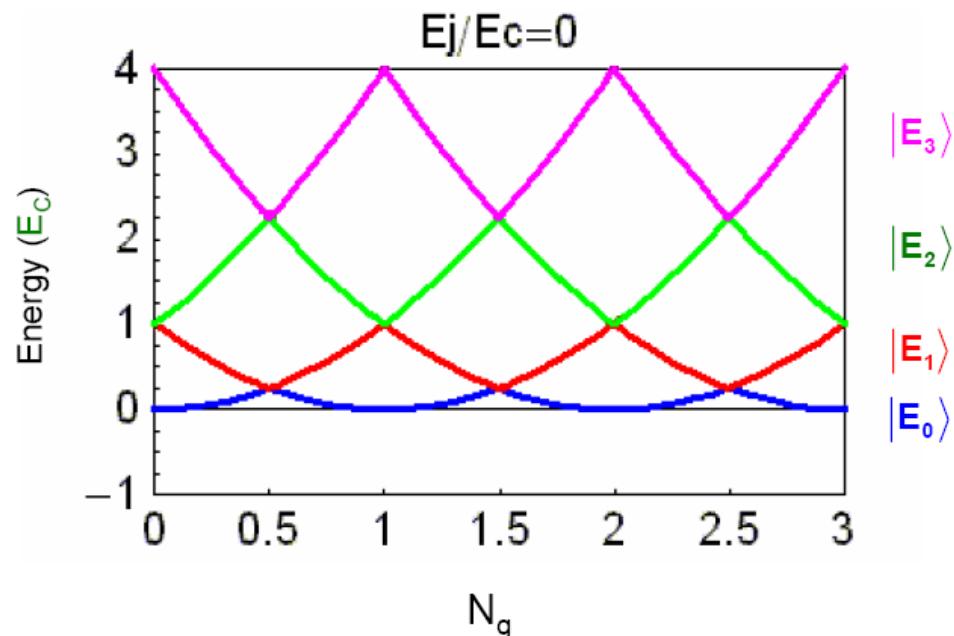
energy bands for finite E_J

- Josephson coupling lifts degeneracy

- E_J scales level separation at charge degeneracy

Charge and Phase Wave Functions ($E_J \ll E_C$)

$E_J/E_C = 0.01$

$\psi_1(N)$

N

$|\psi_1(\theta)|^2$

θ

$\psi_0(N)$

N

$|\psi_0(\theta)|^2$

θ

Energy (E_C)

N_g

$\psi_1(N)$

N

$|\psi_1(\theta)|^2$

θ

$\psi_0(N)$

N

$|\psi_0(\theta)|^2$

θ
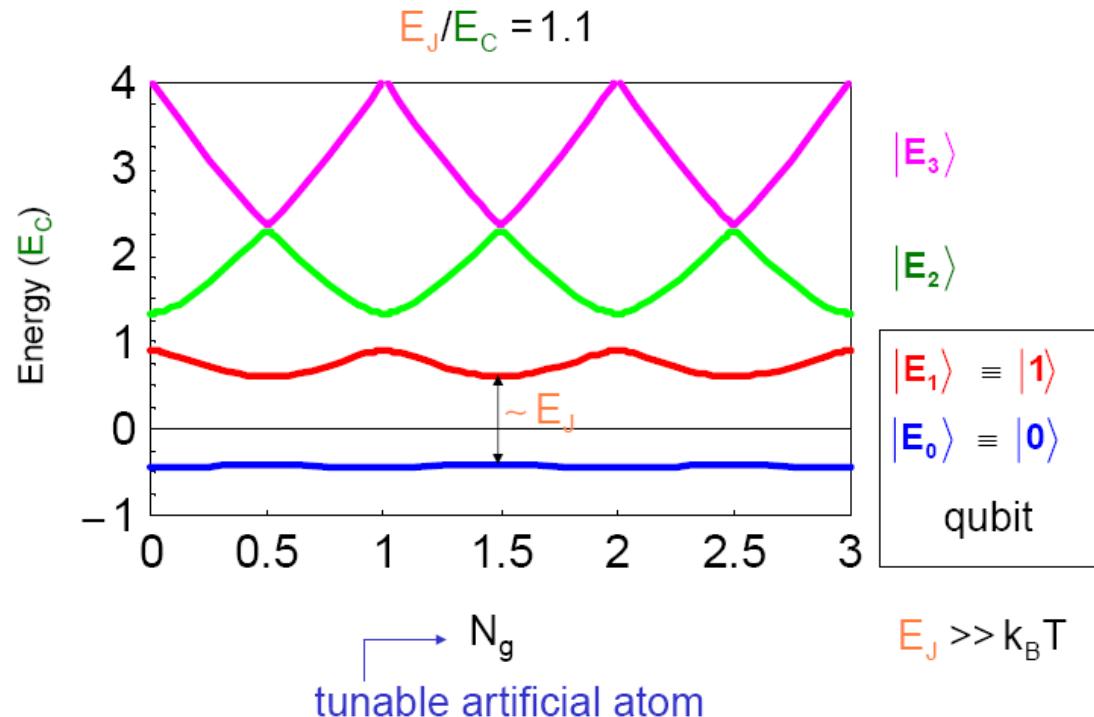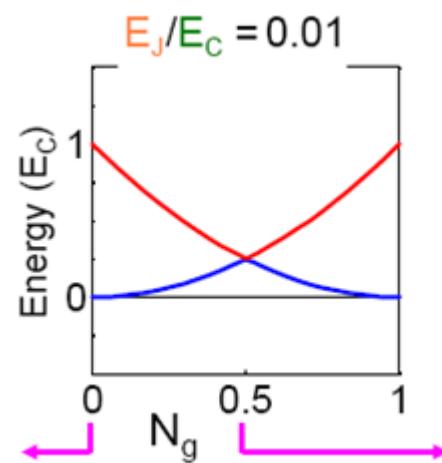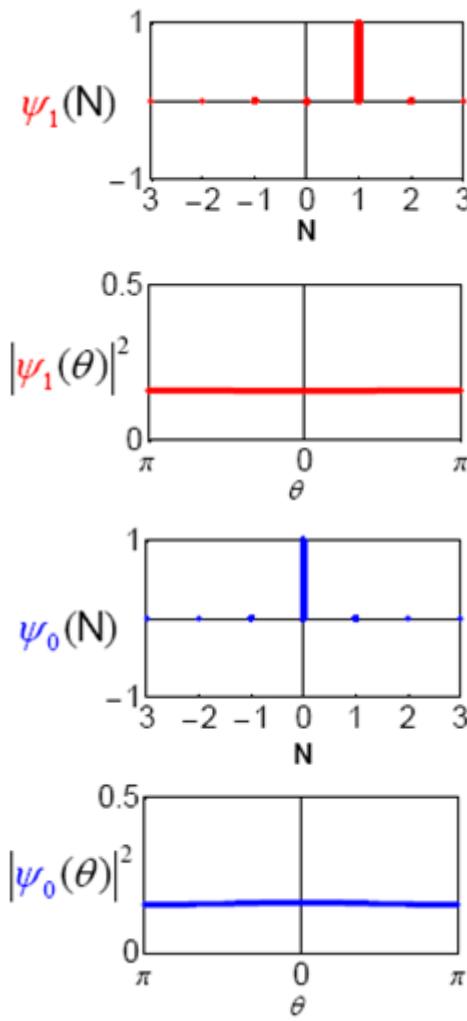
$$\left|\psi_1\right\rangle \approx \left|N=1\right\rangle$$

$$\left|\psi_1\right\rangle \approx \frac{\left|N=0\right\rangle - \left|N=1\right\rangle}{\sqrt{2}}$$

$$\left|\psi_0\right\rangle \approx \left|N=0\right\rangle$$

$$\left|\psi_0\right\rangle \approx \frac{\left|N=0\right\rangle + \left|N=1\right\rangle}{\sqrt{2}}$$

courtesy CEA Saclay
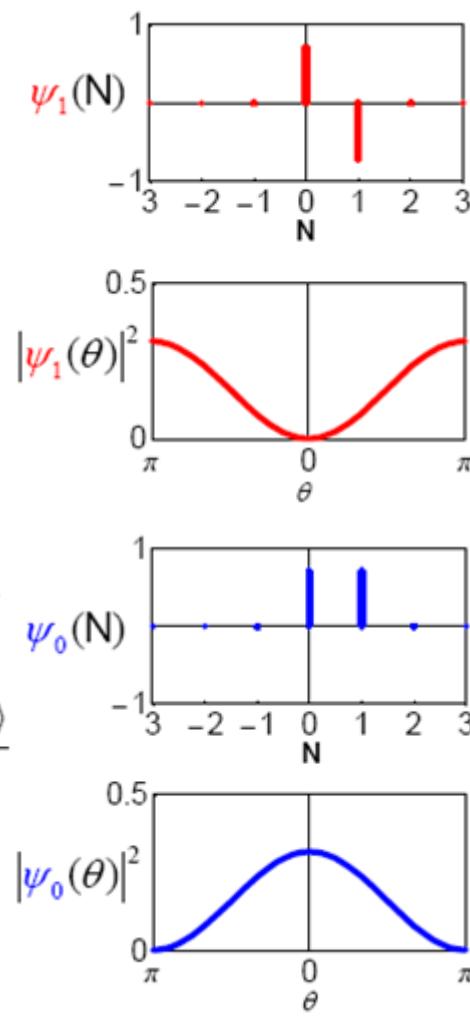
Charge and Phase Wave Functions ($E_J \sim E_C$)
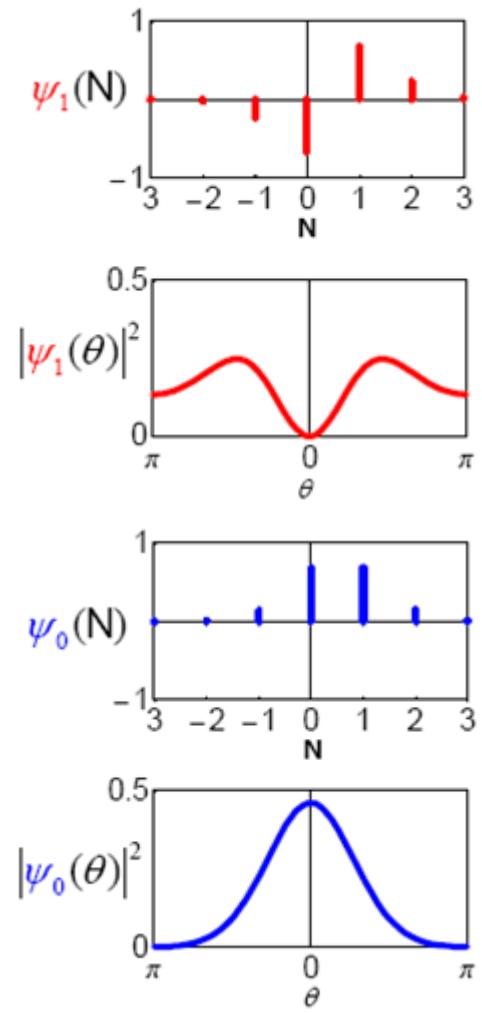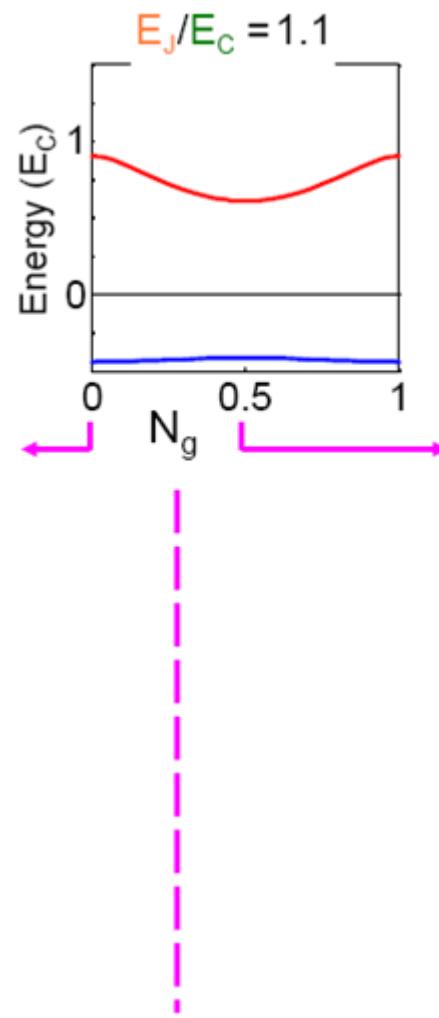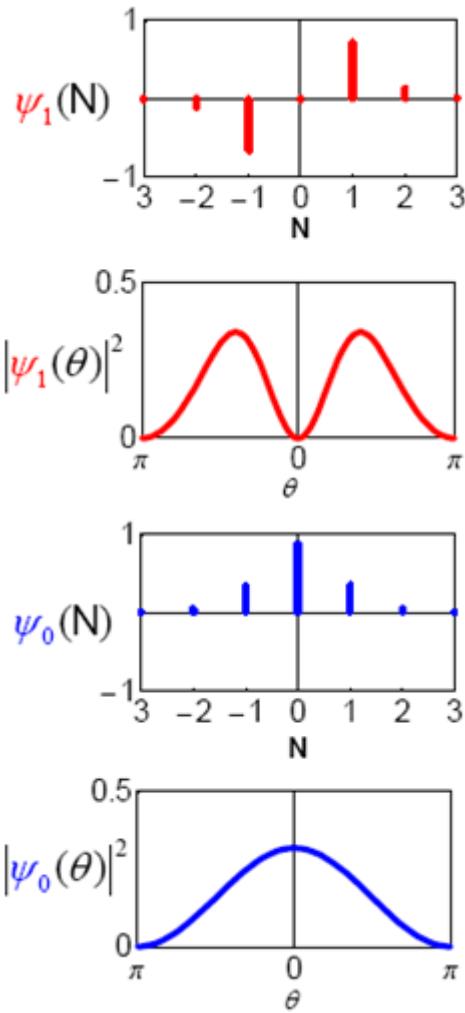
Tuning the Josephson Energy

split Cooper pair box in perpendicular field
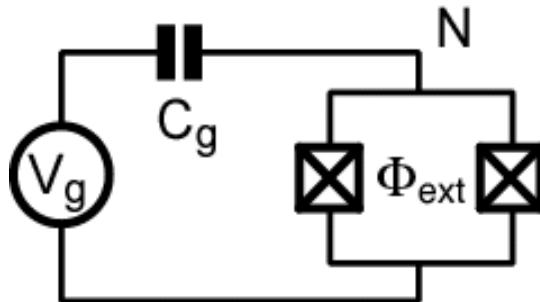
$$H = E_C \left(N - N_g \right)^2 - E_{J,\mathrm{max}} \cos\left(\pi \frac{\phi_{\mathrm{ext}}}{\phi_0} \right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\mathrm{max}} \cos\left(\pi \frac{\phi_{\mathrm{ext}}}{\phi_0} \right)$$
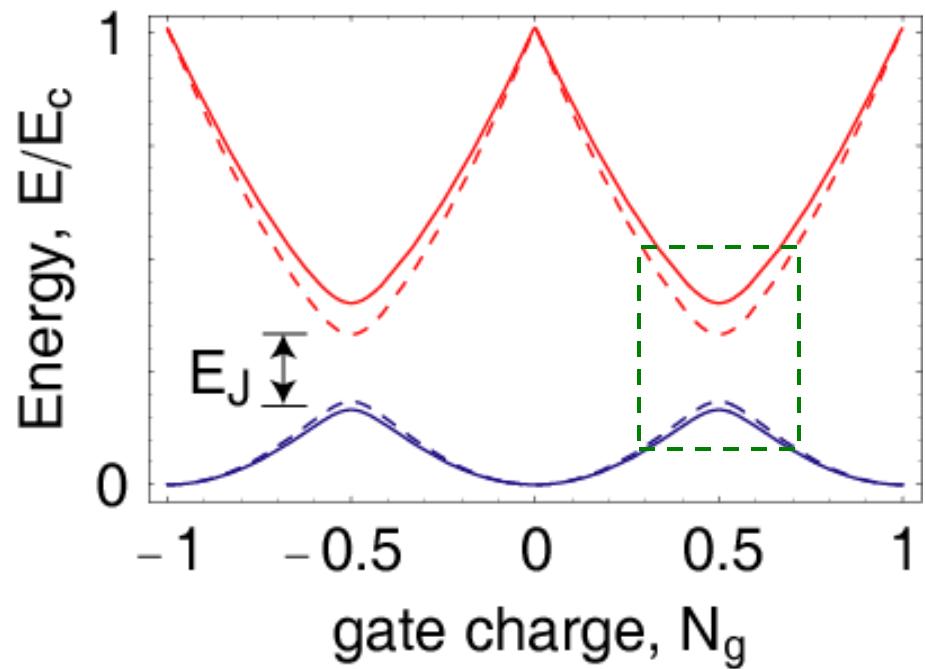
consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

$$\hat{H}_{\mathrm{CPB}} = \sum_N \left[E_C(N - N_g)^2 \, |N\rangle\langle N| - \frac{E_{\mathrm{J}}}{2} \left(|N\rangle\langle N+1| + |N+1\rangle\langle N| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\hat{H} \;=\; -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

$$\;=\; -\frac{1}{2}(E_{\mathrm{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x)$$
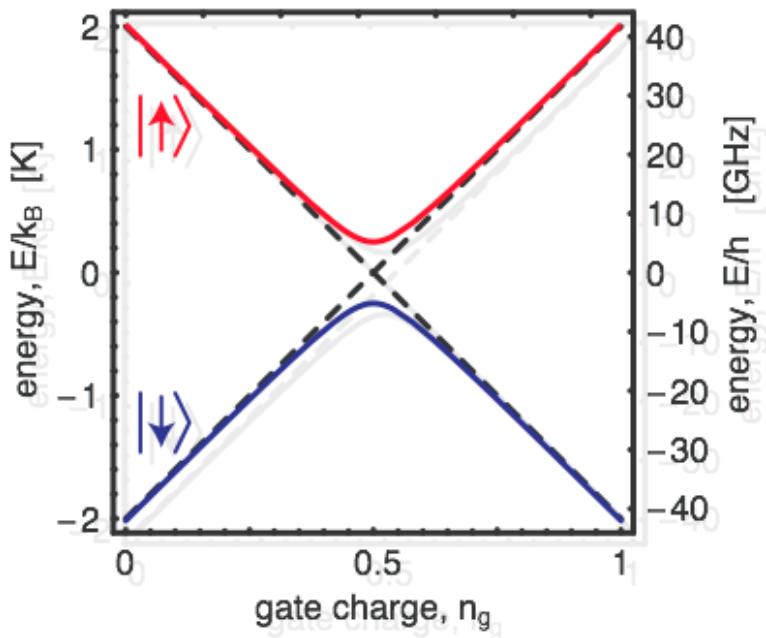
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

standard CPB:
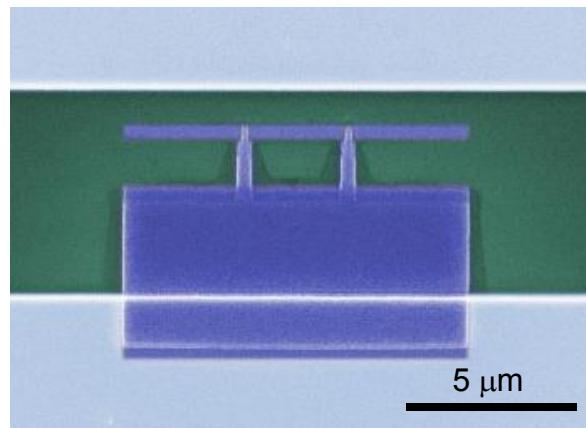
Transmon qubit:
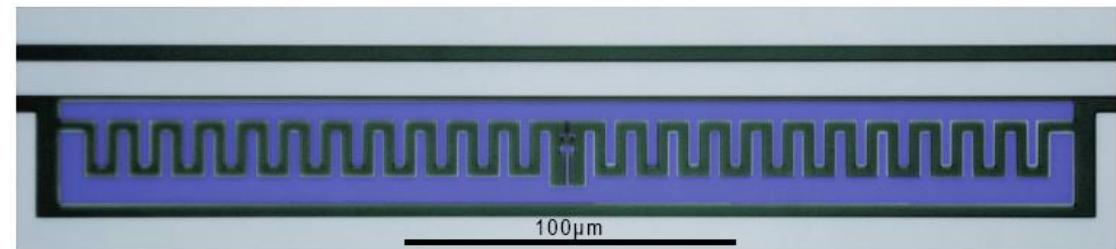
circuit diagram:
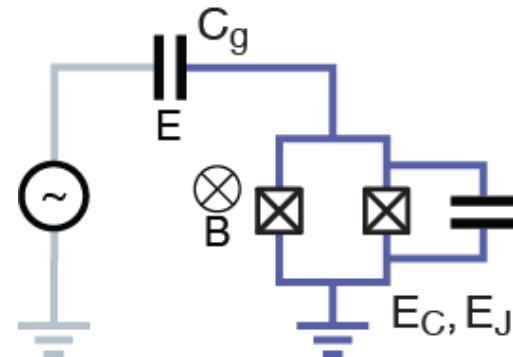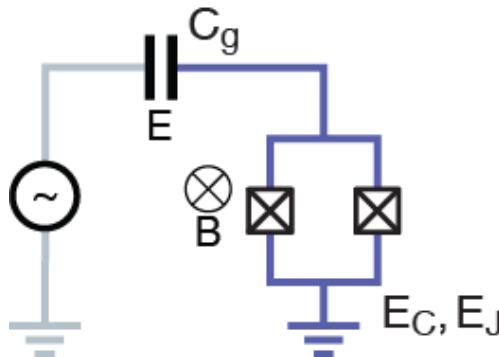
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:

$$\frac{E_J}{E_C} \sim 2$$

$|h\rangle$, $|f\rangle$, $|e\rangle$, $|g\rangle$

E_{ge}
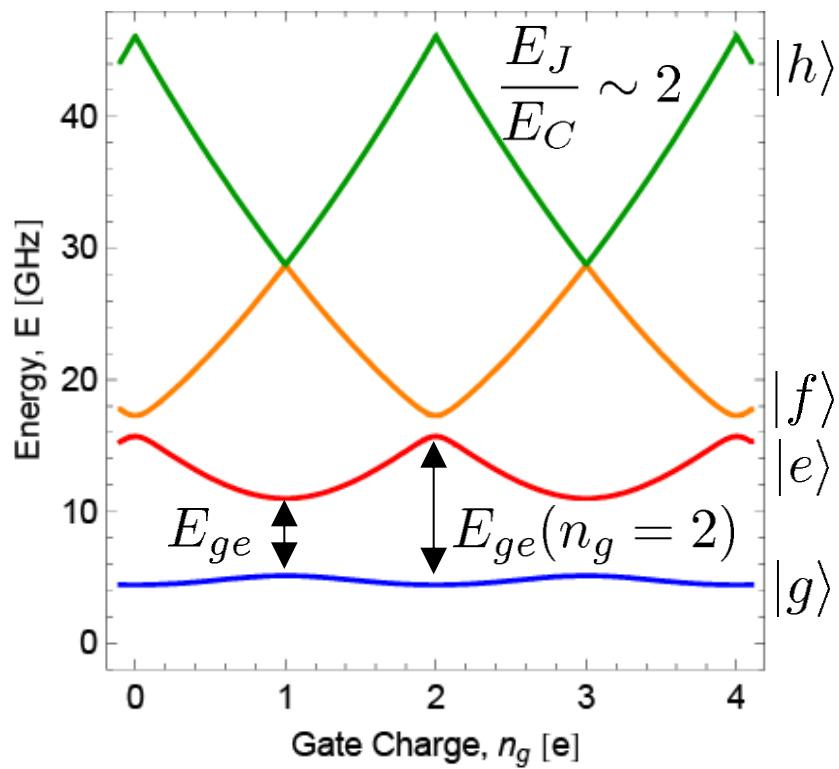
$E_{ge}(n_g = 2)$

Transmon energy levels:

$$\frac{E_J}{E_C} \sim 20$$

$|h\rangle$, $|f\rangle$, $|e\rangle$, $|g\rangle$

E_{ef}

$$E_{ge} \sim \sqrt{8\, E_C\, E_J} - E_C$$
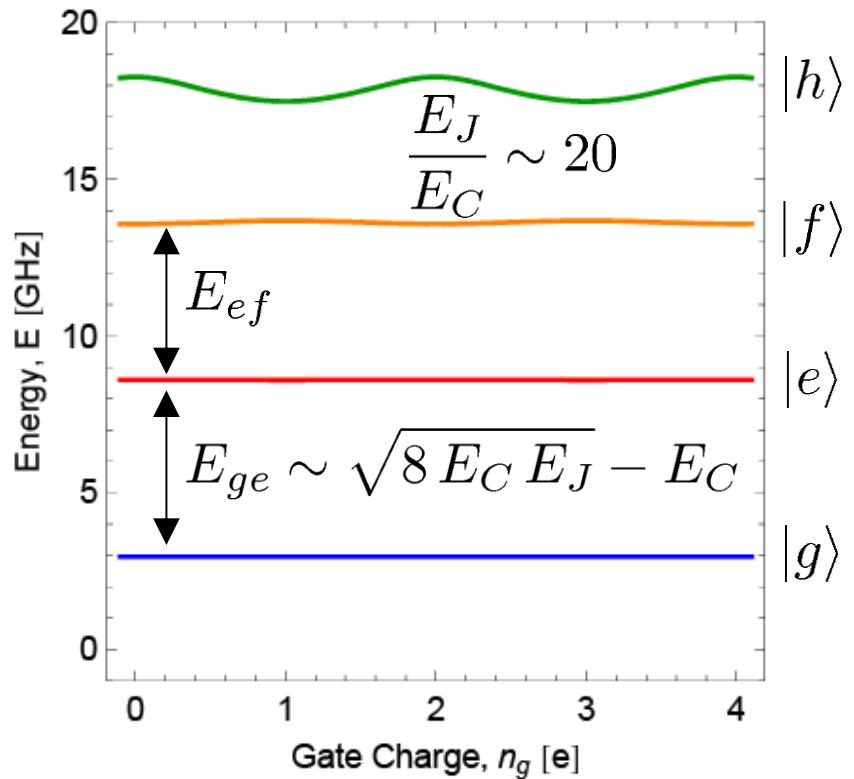
dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

relative anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

How to Operate Circuits Quantum Mechanically?

$T \sim 0.01 \, \mathrm{K}$
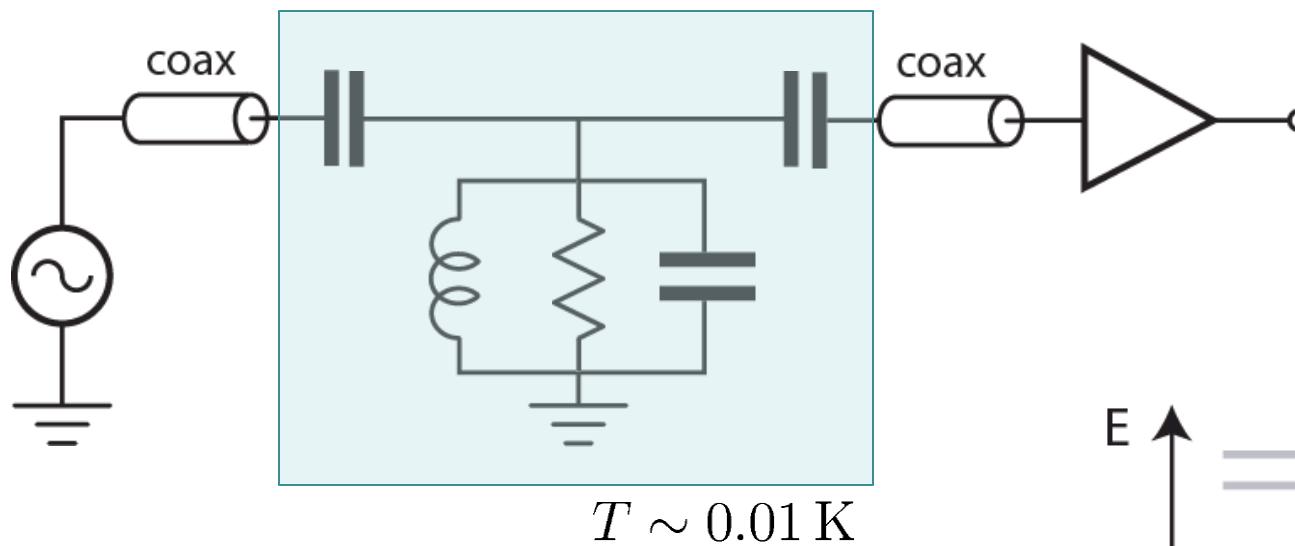
recipe:

- avoid dissipation

- work at low temperatures

- isolate quantum circuit from environment

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)
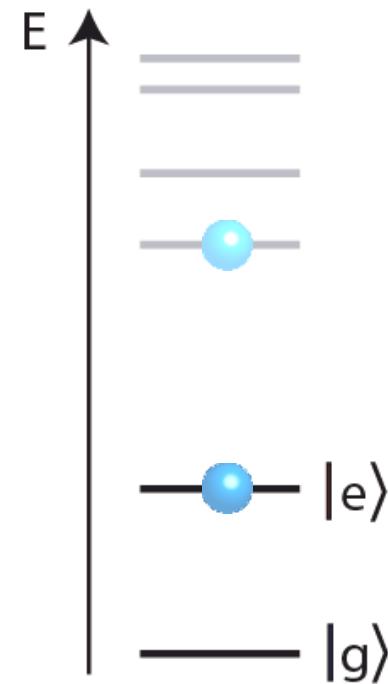
Control of Coupling to Electromagnetic Environment

coupling to environment (bias wires):

$$Z_{\mathrm{line}} \sim 50\,\Omega$$

coax

Z_{qubit}
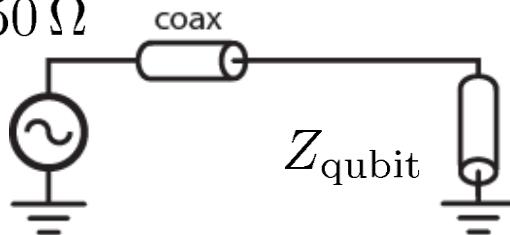
decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...

coax

coax
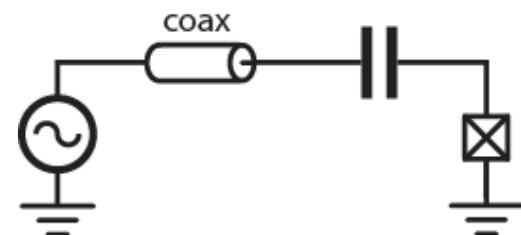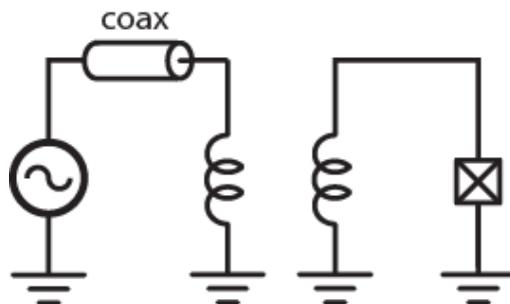
... or resonant impedance transformers

coax
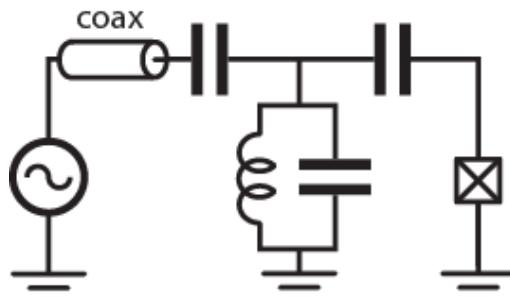
control spontaneous emission by circuit design

Realizations of Superconducting Artificial Atoms
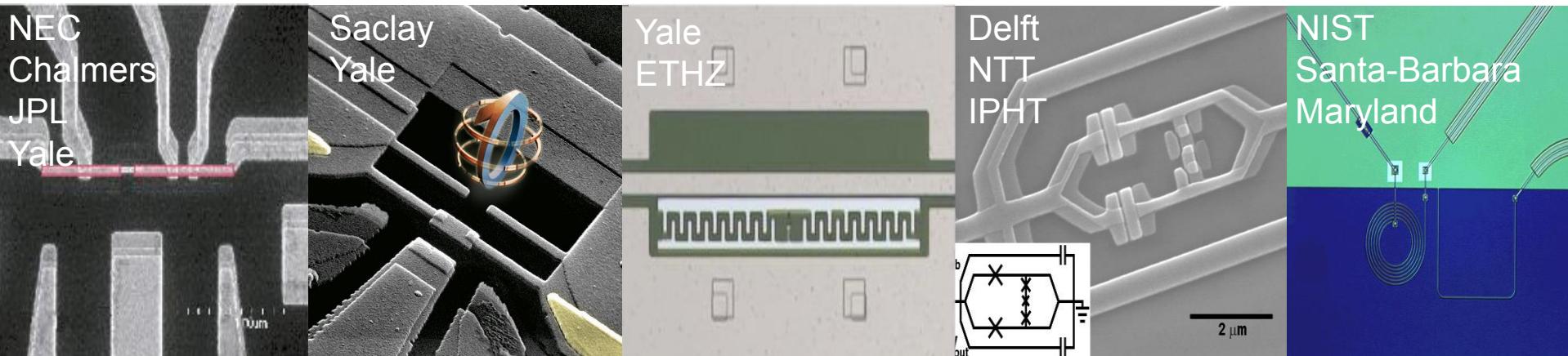
'artificial atoms' -- single superconducting qubits

review:
J. Clarke and F. Wilhelm
Nature 453, 1031 (2008)

'artificial molecules' -- coupled superconducting qubits
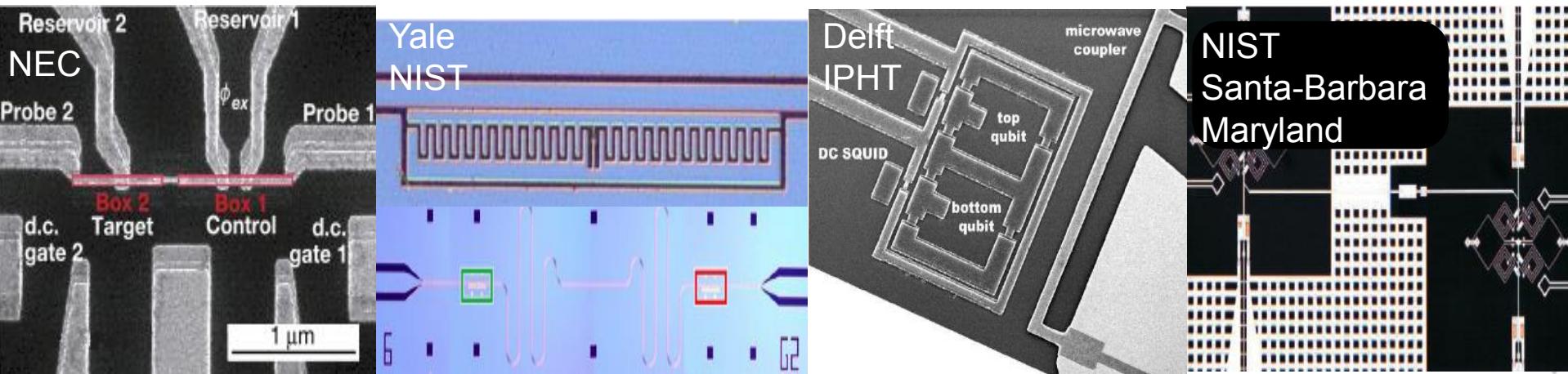
Realizations of Harmonic Oscillators

Superconducting Harmonic Oscillators

a simple electronic circuit:
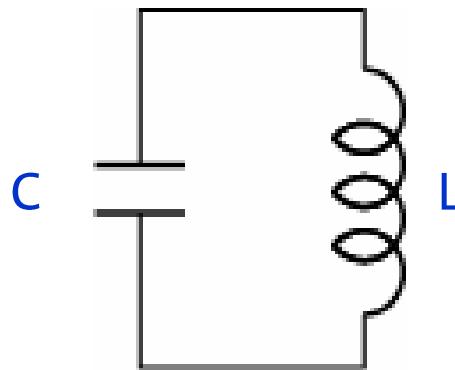
- typical inductor: $L = 1\,nH$

- a wire in vacuum has inductance $\sim 1\,nH/mm$

- typical capacitor: $C = 1\,pF$

- a capacitor with plate size $10\,\mu m \times 10\,\mu m$ and dielectric AlOx ($\varepsilon = 10$) of thickness $10\,nm$ has a capacitance $C \sim 1\,pF$

- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,GHz$$

Realization of H.O.: Lumped Element Resonator

inductor **L**

capacitor **C**
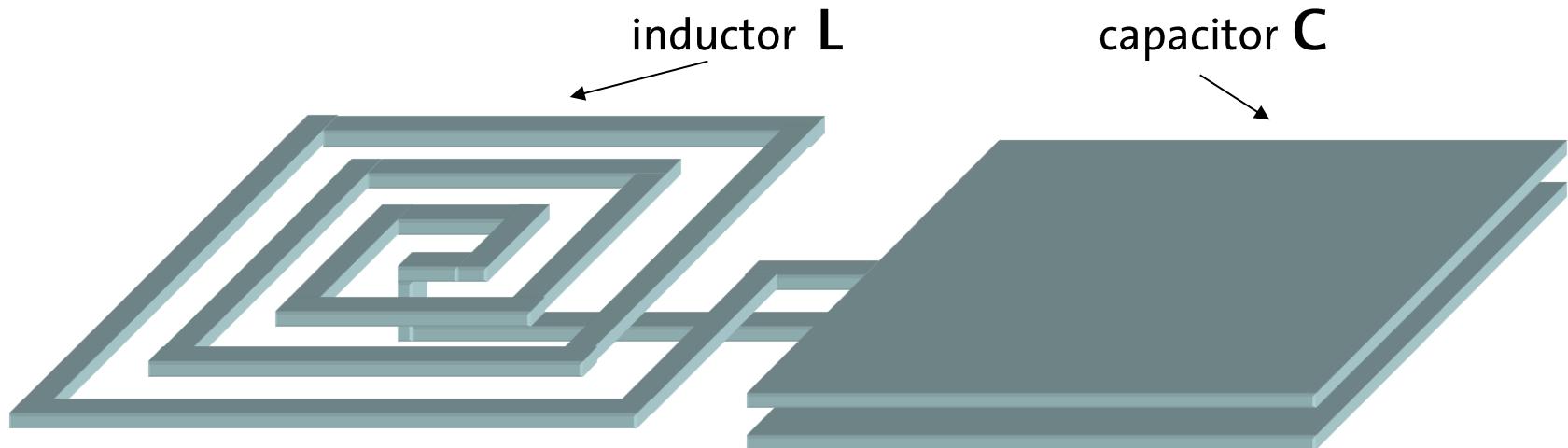
a harmonic oscillator

ϕ

currents and
magnetic fields

$+q$

$-q$

charges and
electric fields
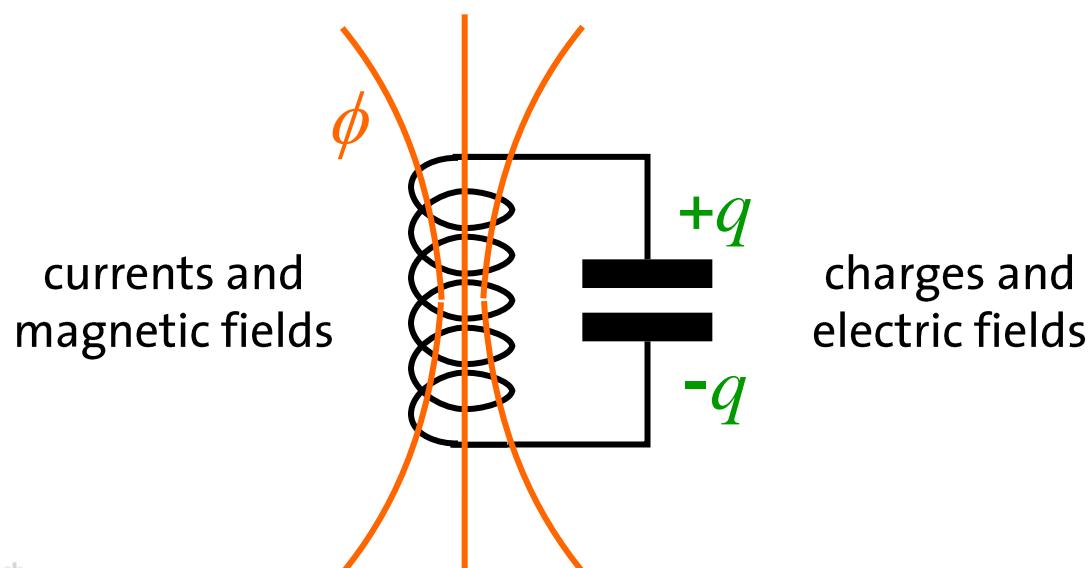
Types of Superconducting Harmonic Oscillators

lumped element resonator:
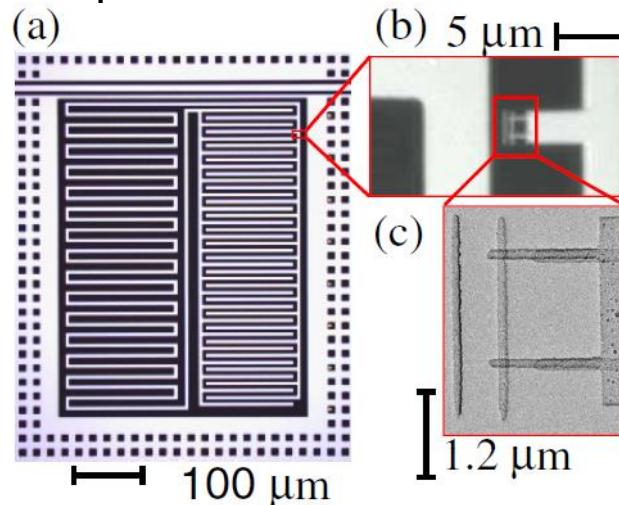
Z. Kim *et al., PRL* **106**, 120501 (2011)

3D cavity:

H. Paik *et al., PRL* **107**, 240501 (2011)

weakly nonlinear junction:


I. Chiorescu *et al., Nature* **431**, 159 (2004)

planar transmission line resonator:
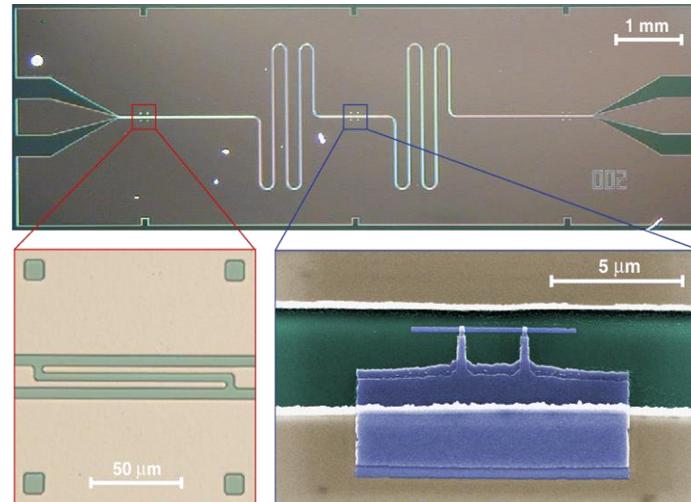
A. Wallraff *et al., Nature* **431**, 162 (2004)

Realization of H.O.: Transmission Line Resonator

distributed resonator:

coupling capacitor gap

$L = \lambda = 25$ mm

ground

signal

5 μm

L   C
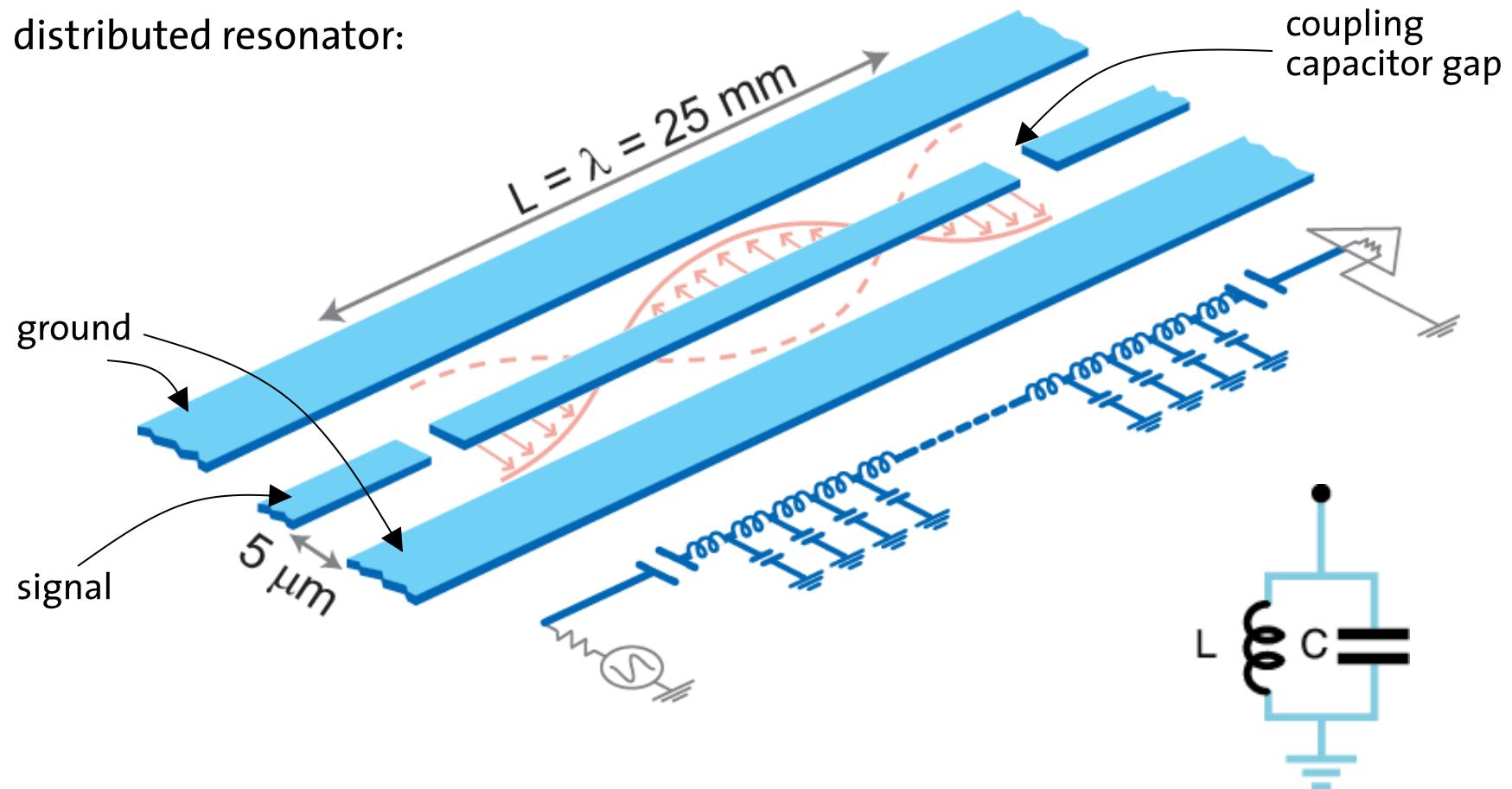
- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* 104, 113904 (2008)

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Realization of Transmission Line Resonator

coplanar waveguide:

1 mm
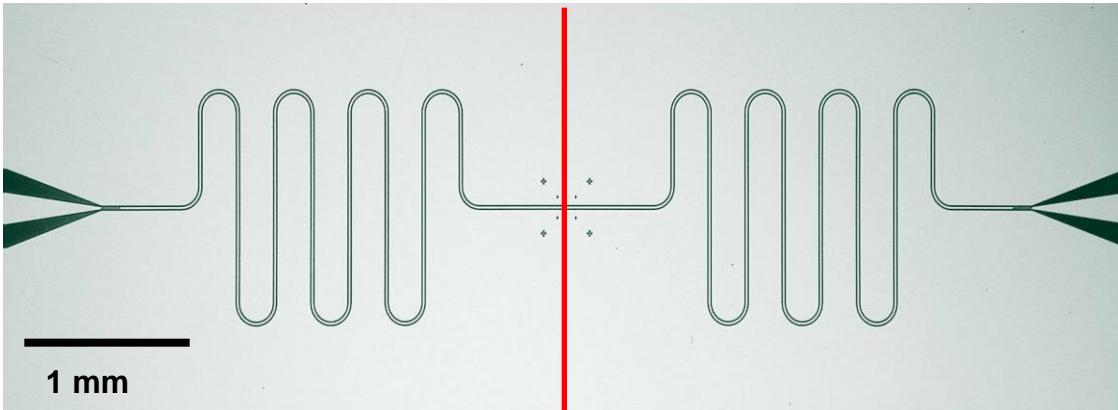
cross-section of transm. line (TEM mode):

$\vec{E}$ $\vec{B}$

$10\,\mu\mathrm{m}$
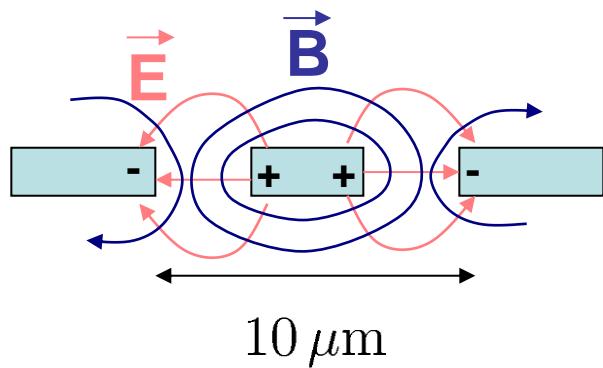
measuring the resonator:

C_{in} C_{out}

ω_{RF}

L   C
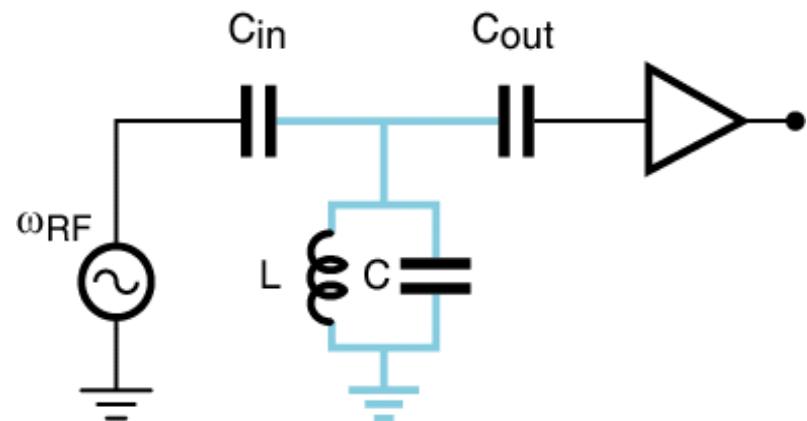
photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime
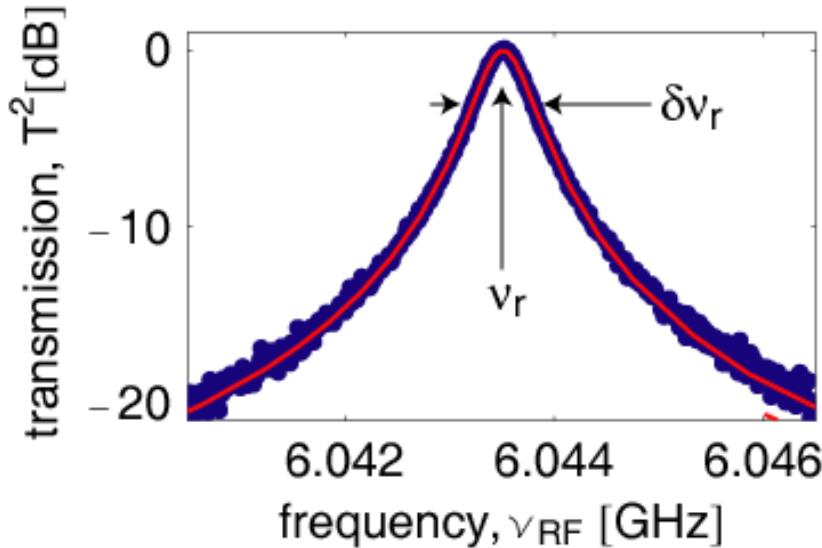
resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$
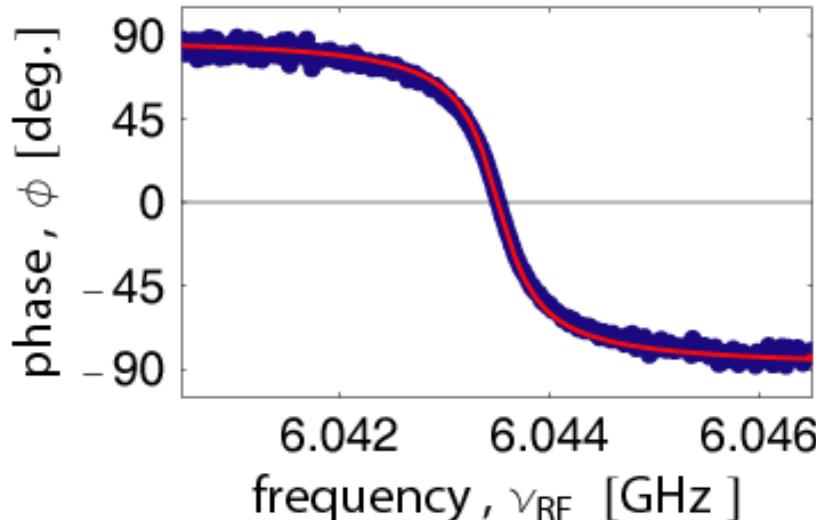
quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Controlling the Photon Life Time

1 mm

C_{in} C_{out}

ωRF L   C
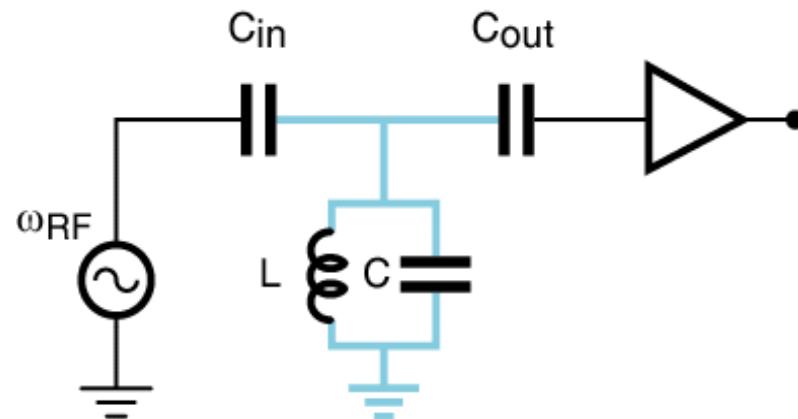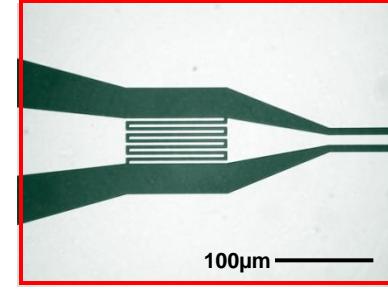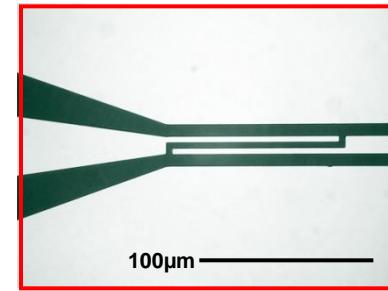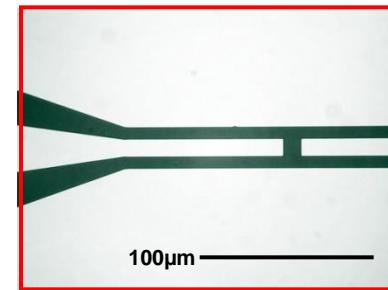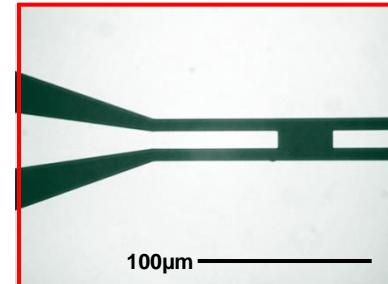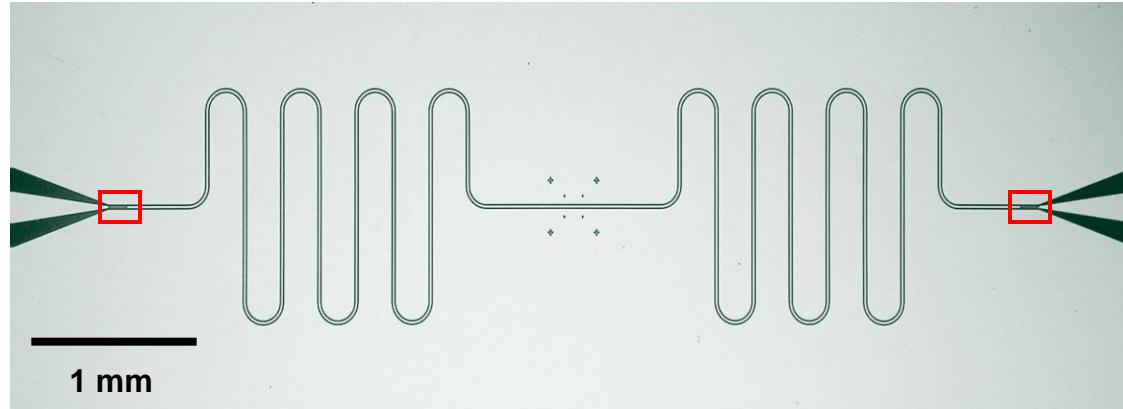
photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

100µm

100µm

100µm

100µm

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Swiss Federal Institute of Technology Zurich

Internal and External Dissipation in an LC Oscillator
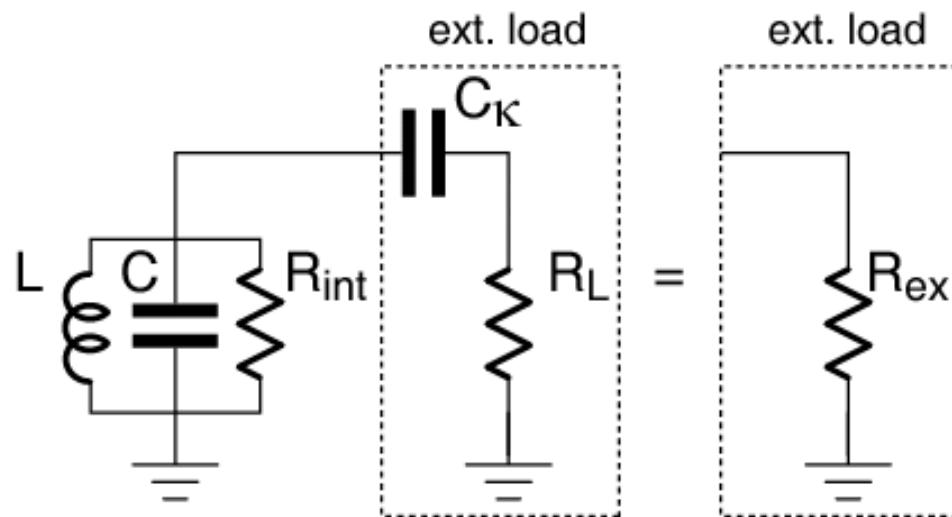
internal losses: R_{int}
conductor, dielectric

external losses: R_{ext}
radiation, coupling

total losses $\dfrac{1}{R} = \dfrac{1}{R_{\text{int}}} + \dfrac{1}{R_{\text{ext}}}$

impedance $Z = \sqrt{\dfrac{L}{C}}$

quality factor $Q = \dfrac{R}{Z} = \omega_0 R C$

excited state decay rate $\Gamma_1 = \dfrac{\omega_0}{Q} = \dfrac{1}{RC}$
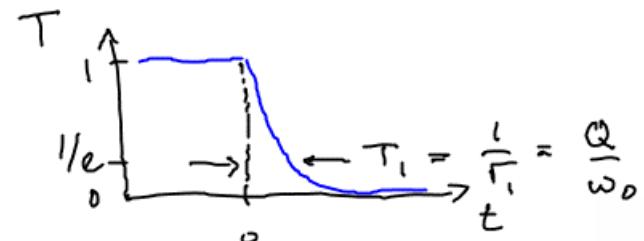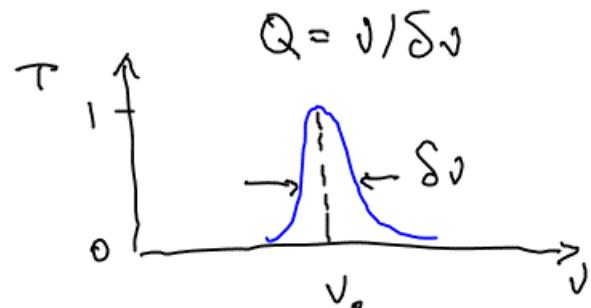
Quality Factor Measurement
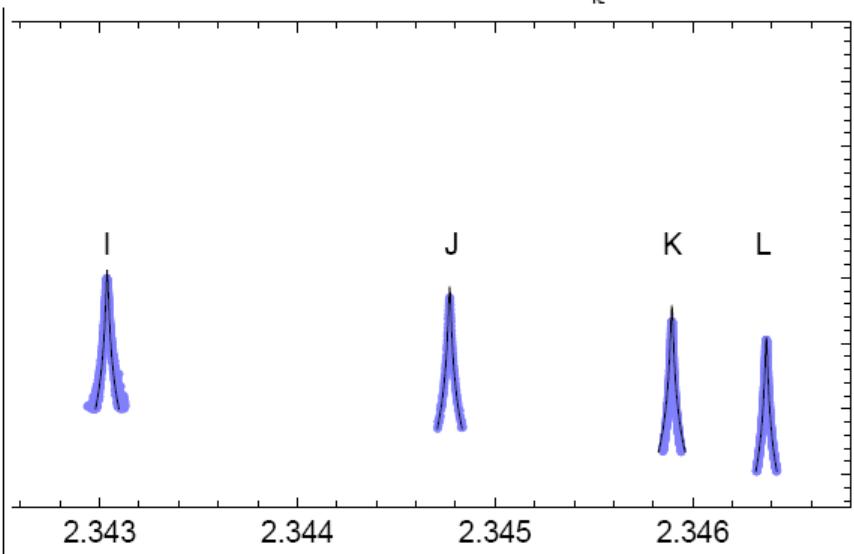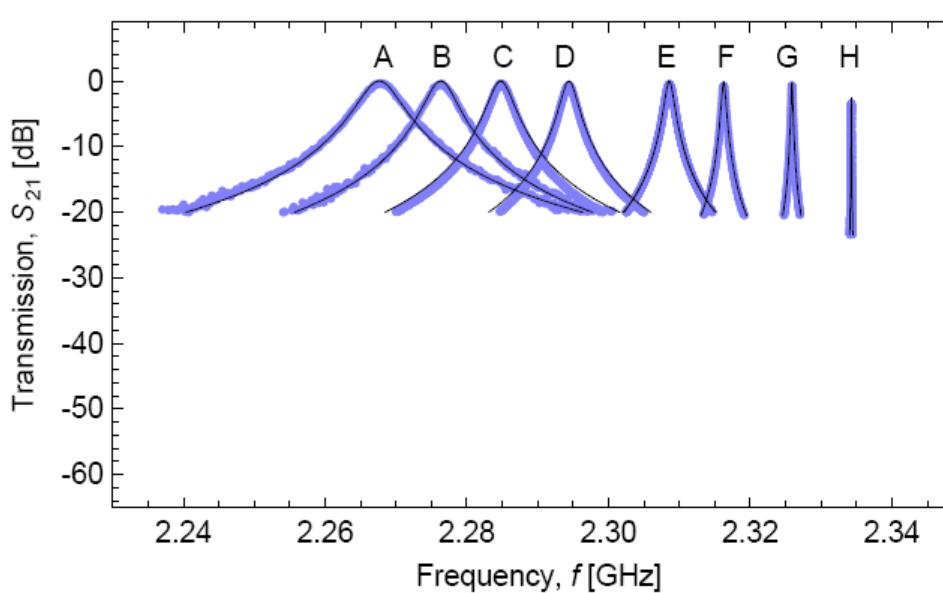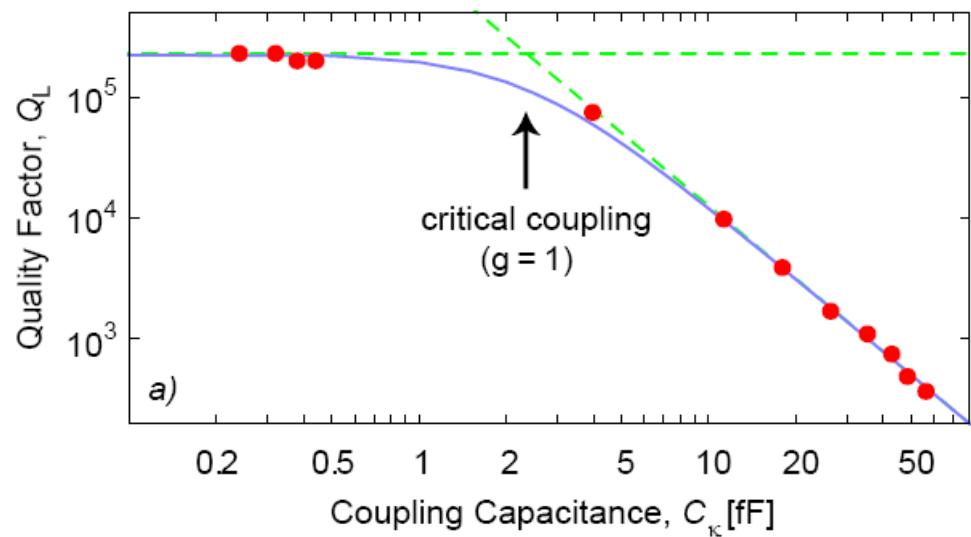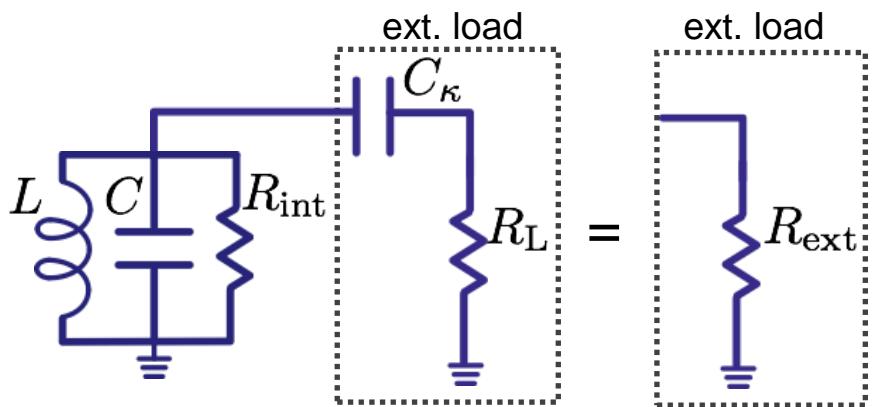
M. Goeppl *et al., J. Appl. Phys.* **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature

E

ϕ

$\hbar\omega$
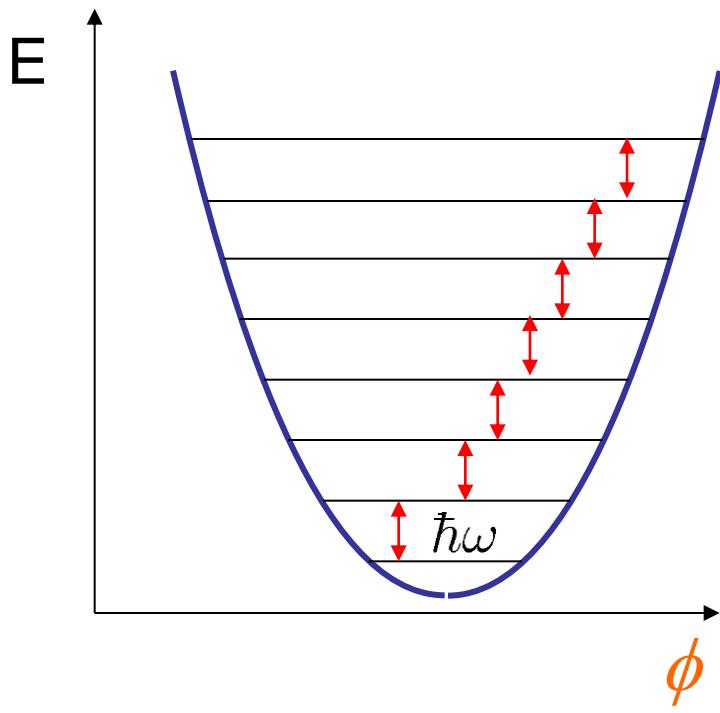
thermal occupation:

$$\langle n_{\text{th}} \rangle = \frac{1}{\exp\left(h\nu/k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK   20 mK

$$\langle n_{\text{th}} \rangle \sim 10^{-11}$$

Why Superconductors?

E

0
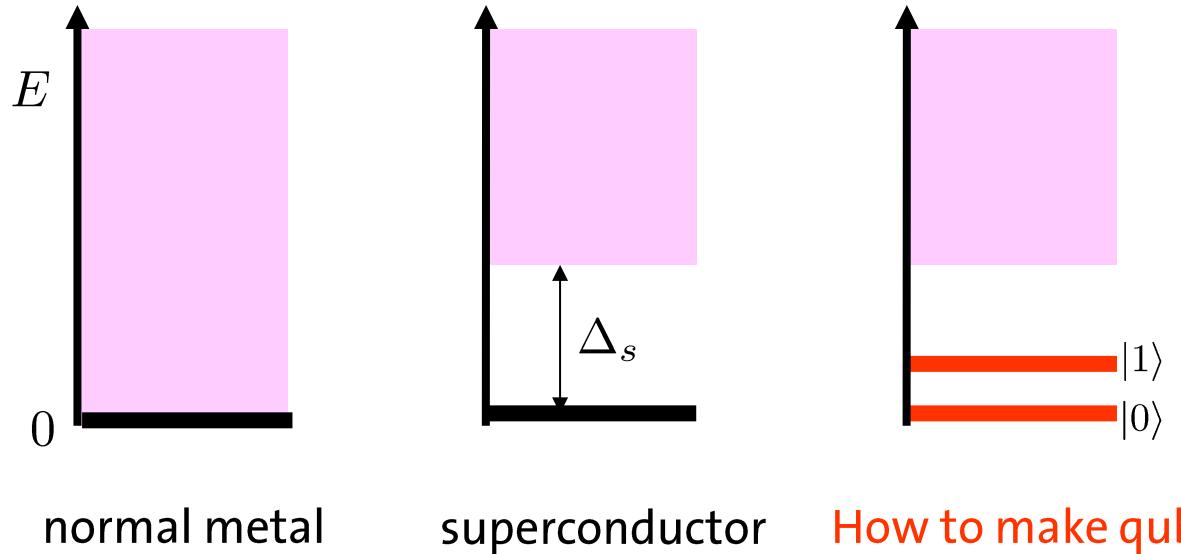
normal metal   superconductor   How to make qubit?

Δ_s

$|1\rangle$
$|0\rangle$

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):

- Niobium (Nb): $2\Delta/h$ = 725 GHz, T_c = 9.2 K

- Aluminum (Al): $2\Delta/h$ = 100 GHz, T_c = 1.2 K

Cooper pairs:
bound electron pairs

Δ
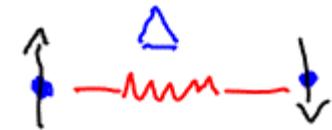
Bosons (S=0, L=0)

2 chunks of superconductors

| 1 | 2 |

macroscopic wave function

$$\psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

Cooper pair density n_i
and global phase δ_i

phase quantization: $\delta = n\,2\pi$
flux quantization: $\phi = n\,\phi_0$

δ ϕ