

Topic: Basic Elements of a Quantum Computer

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Key Questions:

Which components (or features) does a generic quantum computer have?

Example: Memory to store information.

How are those different from a classical computer?

Your Background:

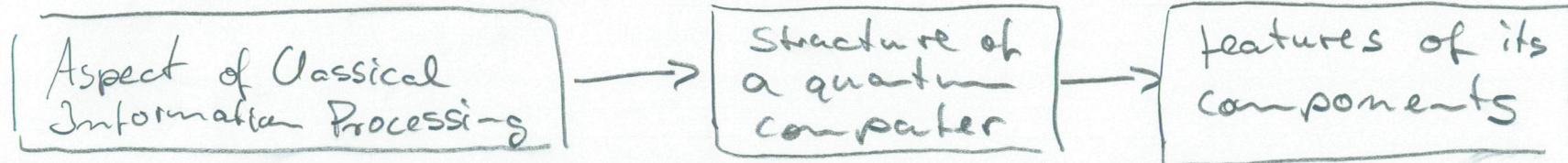
- General knowledge of how computers work
- Knowledge of quantum mechanics

COMBINE

Goal:

- Understand basic structure and operation of a quantum computer.
- Make use of this 'hardware independent' knowledge about quantum computers to evaluate different physical realizations

Outline:



Classical Information Processing

(1)

- Carrier of information in binary representation

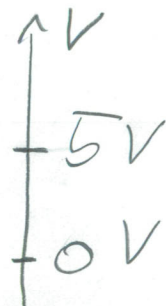
BIT

possible values

1
0

physical representation

example:



voltage level in a circuit

- modification of information in BIT by operating with a physical process on the BIT

- any logical operation on bits can be decomposed into single and two-bit operations

- Why is this useful?
- You may want to think about why this is possible at all!
- Same is true for quantum computers.

How is information physically modified in physical realizations of bits? Give examples.

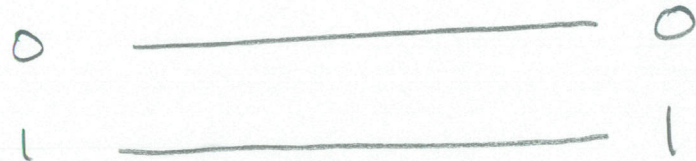
- CD
- hard disk
- RAM

2) The same question will be important in quantum computers!

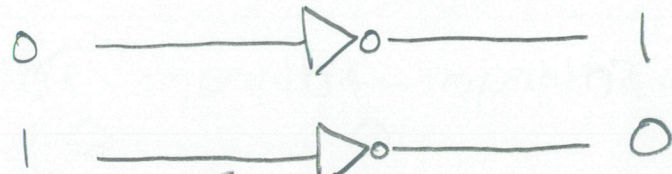
Single Bit Operation

time →

input bit ————— output bit



IDN operation



NOT operation

Would you think single qubit operations are simple to realize in a quantum computer?

What could be potential problems?

- decoherence
- spurious interactions
- ...

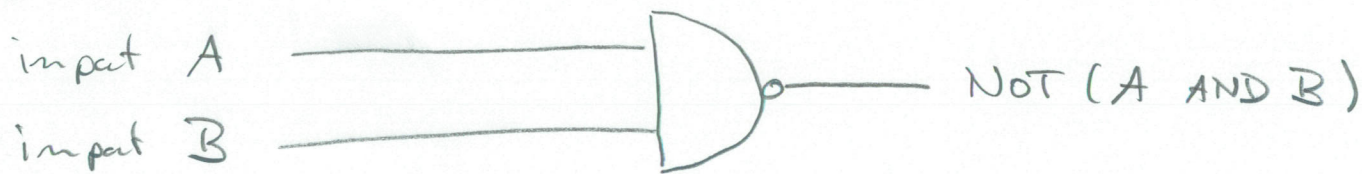
- Wire represents bit
- preserves state

- Symbol represents operation
- changes state

Same representation of information in circuit model for quantum computation

Two Bit Operations

The NAND gate



truth table	A	B	NOT (A AND B)
	0	0	1
	0	1	1
	1	0	1
	1	1	0

Properties:

- Universal logic gate

↳ any function operating on bits can be computed using NAND gates

↳ examples: AND, OR, XOR, NOR

(ancilla bits & possibilities to make copies are required)

Why do you think universality is useful!

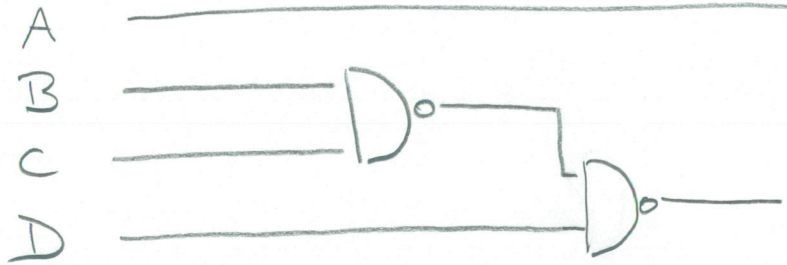
NOTE:

Universal logic gates also do exist for quantum computers!

Circuit Representation

(4)

INPUT



OUTPUT

$$E = A$$

$$F = (B \text{ NAND } C) \text{ NAND } D$$

Any computable function can be represented as a circuit composed of logic universal gates acting on a set of input bits generating a set of output bits

Circuit properties :

- bits can be copied (FAN OUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than number of input bits
- loops are not allowed

In your opinion, which of these properties might be less / hard to realize in a quantum computer?

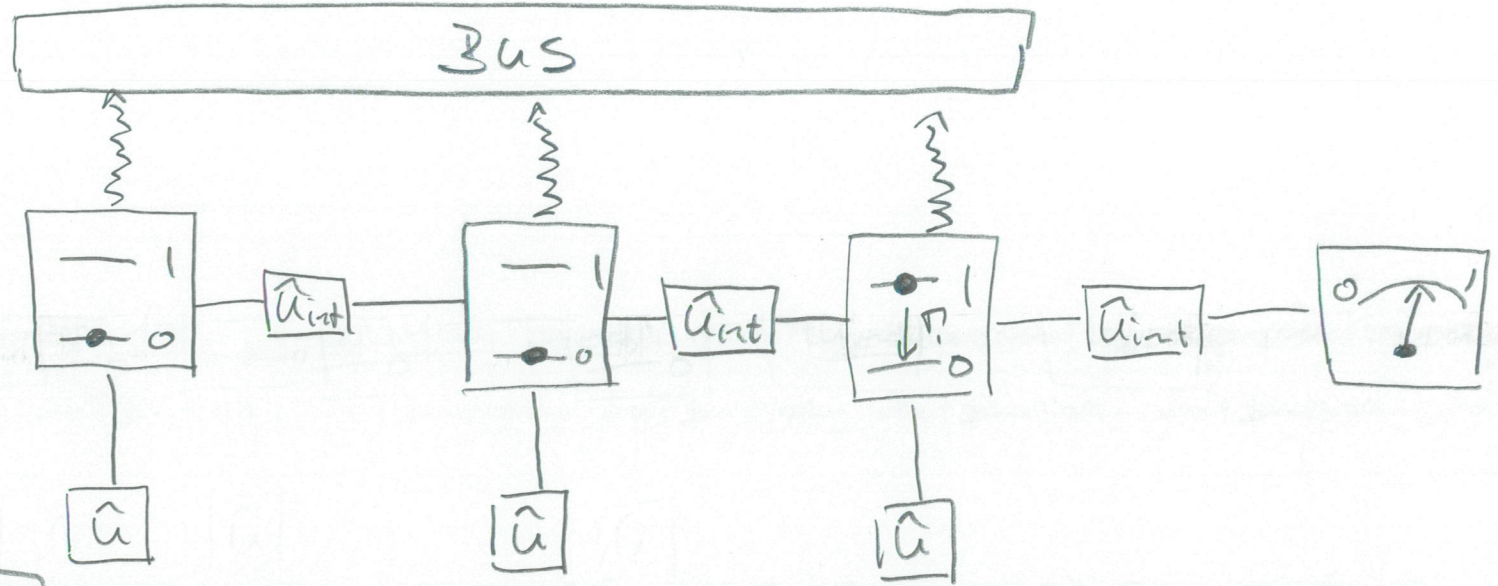
- reversibility?
- preservation of state?
- copying of information
- universality?

A Generic Quantum Processor

(5)

features:

- (1) quantum bits
- (2) initialization
- (3) coherence
- (4) universal gates
- (5) read-out



Di Vincenzo Criteria

- (6) conversion
- (7) transfer

In your opinion, what kind of components are absolutely essential to realize any quantum computer?

The Quantum Bit

- Quantum mechanical system with two distinct states



Give examples of systems with this energy level spectrum. Under which conditions would you be allowed to approximate a given quantum system as a two-level system? (6)

- Representation of qubit basis states $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

vectors in 2D Hilbert space
(1st QM postulate)

- general qubit state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \text{with } \alpha, \beta \in \mathbb{C}$$

properties: - qubit can be in superposition of states $|0\rangle$ and $|1\rangle$

- probability of states $P_0 = |\alpha|^2$

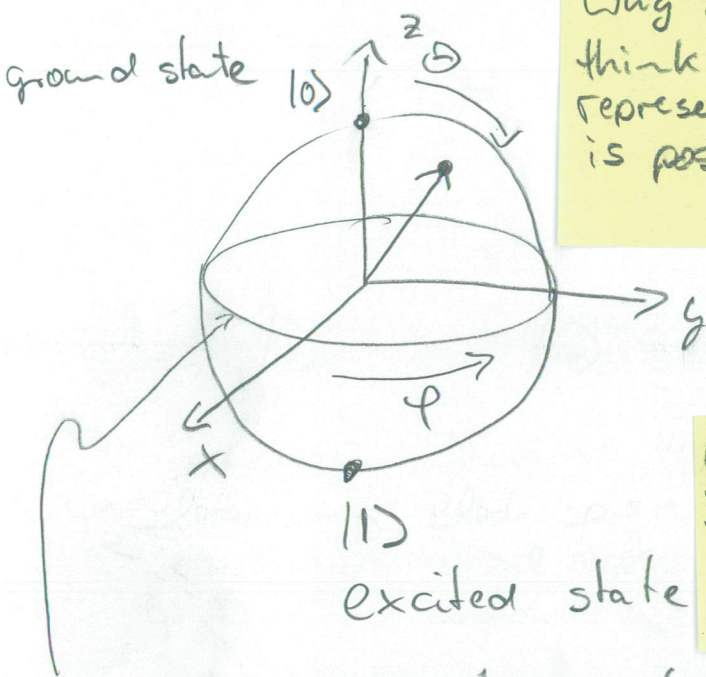
$$P_1 = |\beta|^2$$

$$\text{with } P_1 + P_2 = 1$$

What is the main difference in comp. to a classical bit?

The Bloch Sphere

Representation of pure single qubit state as vector to the surface of a sphere



Why do you think this representation is possible?

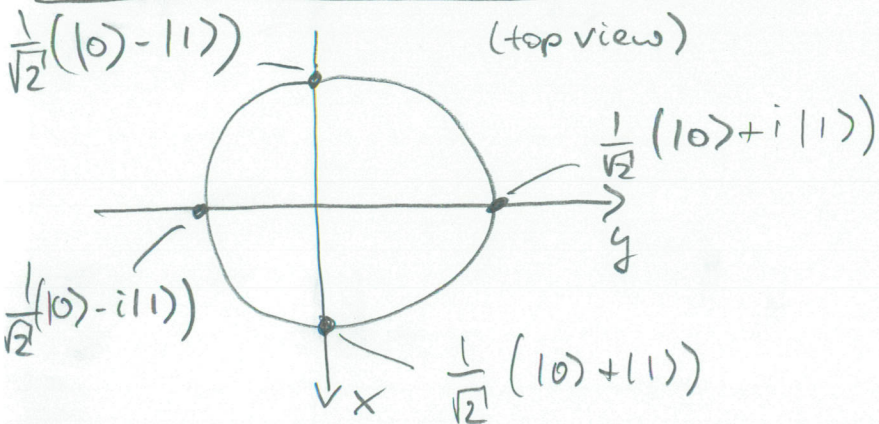
Where on the Bloch sphere are equal superposition states?

- general state $|ψ\rangle = α|0\rangle + β|1\rangle$
- 4 parameters $Re\ α, Im\ α, Re\ β, Im\ β$
- + 1 normalization constraint $|α|^2 + |β|^2 = 1$

• $|ψ\rangle = e^{iγ} \left[\cos\frac{\Theta}{2} |0\rangle + e^{iφ} \sin\frac{\Theta}{2} |1\rangle \right]$

with $γ$: global phase factor
 Θ : polar angle
 $φ$: azimuthal angle

equal superposition states:



equivalent representation

• $|ψ\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{iφ} |1\rangle)$

$\Rightarrow \Theta = \frac{\pi}{2}$: defines equator

$φ = 0, \pi/2, \pi, 3/2\pi$ defines phase angle

Single Qubit Gates

circuit representation



• $\hat{I} = \hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity

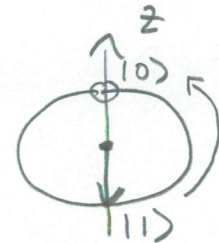
• $\hat{X} = \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Bit flip

• $\hat{Y} = \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ Conjugate bit flip

• $\hat{Z} = \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Phase flip

↑
Pauli matrices

Work out the effect of the single qubit operations on some simple state on the Bloch sphere!



$$|10\rangle \rightarrow -i|11\rangle$$

$$|11\rangle \rightarrow i|10\rangle$$

