Why have the first quantum information processing experiments been performed with photons?

*) Preparation of single photon states by attenuation

*) Detection with high efficiency (single photon detectors)

*) Manipulation of polarization/path (phase shifter, beam splitter, mirror, ...)

→ well developed for photons!

Also: only weak interaction with environment
(long coherence time, long-distance transmission)

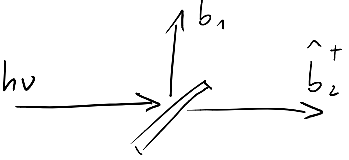
BUT: photon-photon interaction also weak
⇒ impractical for 2-qubit gates

Photon qubits for Quantum Communication:

(e.g. Koh & Lovett – Optical Quantum Inf. Proc.)

e.g. using polarisation + spatial degree of freedom

spatial modes:



beamsplitter

b_1, b_2 .. spatial modes

$|0\rangle = b_1^+ |0\rangle_1 |0\rangle_2 = b_1^+ |0,0\rangle = |1,0\rangle$

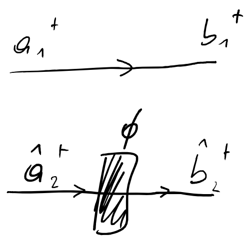
photons in mode 1

$|1\rangle = b_2^+ |0,0\rangle = |0,1\rangle$

"dual rail representation"

operations:

phase shifter



(dielectric with refractive index $n_1 \neq 1$)

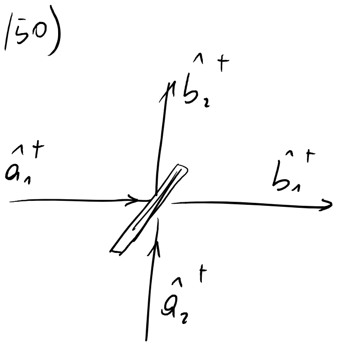
$\hat{a}_1^+ \longrightarrow \hat{b}_1^+$

$a_2^+ \longrightarrow e^{i\phi} b_2^+$

$$\begin{pmatrix} a_1^+ \\ a_2^+ \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} b_1^+ \\ b_2^+ \end{pmatrix}$$

beam splitter (50/50)

(half silvered mirror)



$$\hat{a}_1^\dagger \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(b_1^\dagger + b_2^\dagger\right)$$

$$\hat{a}_2^\dagger \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(-b_1^\dagger + b_2^\dagger\right)$$

(minus sign: reflected beam gets phase shift; unitary transformation)

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} b_1^\dagger \\ b_2^\dagger \end{pmatrix}$$

<u>Example</u>: <u>photon at input 1</u>:

$$|0\rangle = |1,0\rangle = a_1^\dagger |0,0\rangle \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(b_1^\dagger + b_2^\dagger\right)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,1\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

-> 50/50 beamsplitter: 50% probability for scattering into either output port

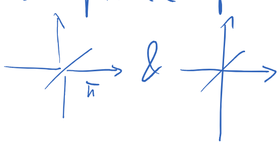
<u>photons at input 1 & 2</u>: (Hong-Ou-Mandel)

$$a_1^\dagger a_2^\dagger |0\rangle_1 |0_2\rangle \longrightarrow \frac{1}{2}\left(b_1^\dagger + b_2^\dagger\right)\left(-b_1^\dagger + b_2^\dagger\right)|0\rangle_1|0\rangle_2 =$$

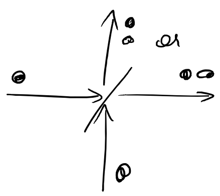
$$= \frac{1}{2}\left(b_1^\dagger b_1^\dagger + \cancel{b_2^\dagger b_1^\dagger} - \cancel{b_1^\dagger b_2^\dagger} - b_2^\dagger b_2^\dagger\right)|0\rangle_1|0\rangle_2 =$$

$$= \frac{\sqrt{2}}{2}\left(-|2\rangle_1|0\rangle_2 + |0\rangle_1|2\rangle_2\right)$$

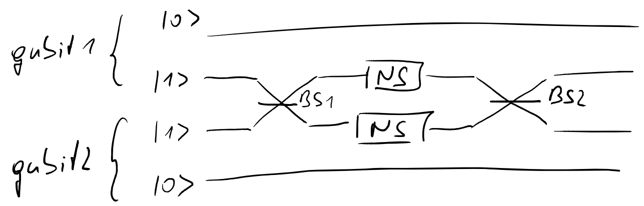
⊛ destructive interference of



-> perfect bunching



(Non-deterministic) C-Phase gate:



non-linear sign gate (NS) $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \longrightarrow \alpha|0\rangle + \beta|1\rangle - \gamma|2\rangle$

induced by
- non-linearity
- non-deterministically (success prob 25%)

operation: $|00\rangle_L = |1001\rangle \xrightarrow{BS+NS+BS2} |00\rangle_L$

(no two photons @ NS gates)

$|01\rangle_L = |1010\rangle \longrightarrow |01\rangle_L$

$|10\rangle_L = |0101\rangle \longrightarrow |10\rangle_L$

$|11\rangle_L = |0110\rangle \xrightarrow{BS1} |0\rangle \otimes \frac{1}{\sqrt{2}}\left(-|20\rangle + |02\rangle\right) \otimes |0\rangle$

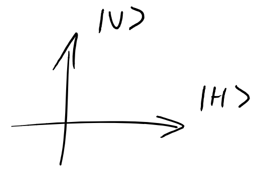
$\xrightarrow{NS} -|0\rangle \otimes \frac{1}{\sqrt{2}}\left(-|20\rangle + |20\rangle\right) \otimes |0\rangle$

$\xrightarrow{BS2} -|0110\rangle = -|11\rangle_L$

(bunching)

success probability: $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

polarization basis:

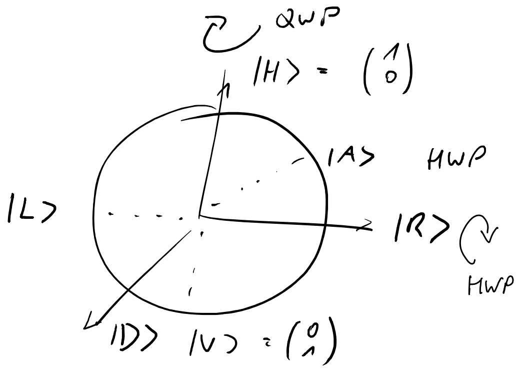


$a_H^\dagger$ ... creation of photon in mode a with horizontal pol.

$a_V^\dagger$ ... ——— " ——— vertical pol.

$$|0\rangle = a_H^\dagger |0,0\rangle_{HV} = |1,0\rangle_{HV} = |H\rangle$$

$$|1\rangle = a_V^\dagger |0,0\rangle_{HV} = |0,1\rangle_{HV} = |V\rangle$$



$$|L\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i|V\rangle\right)$$

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right)$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

Operations:

half-wave plate: $|H\rangle \to \cos 2\theta\, |H\rangle - \sin 2\theta\, |V\rangle$
$|V\rangle \to \sin 2\theta\, |H\rangle + \cos 2\theta\, |V\rangle$

$$U_{HWP}(\theta) = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$\Rightarrow$ rotation about y-axis

$$\theta = \frac{\pi}{4}: \quad U_{HWP} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv$$

$$|V\rangle \to |H\rangle \; ; \quad |H\rangle \to |V\rangle$$

quarter wave plate: $\phi_f - \phi_s = \frac{\pi}{2}$ $\qquad \theta = \frac{\pi}{4}$

$$U_{QWP} = e^{-i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \equiv Z/2$$

$\Rightarrow$ rotation about z-axis

$$|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle) \hat{=} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$U_{QWP}|L\rangle \to e^{-i\frac{\pi}{4}}(|H\rangle - |V\rangle) \propto |A\rangle$$

$\to$ transforms linear $\leftrightarrow$ circular transformation

$\Rightarrow$ $\frac{\lambda}{2} + \frac{\lambda}{4}$ waveplates are sufficient to create arbitrary single-qubit operations!

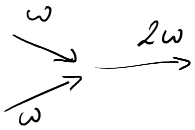
Parametric Down Conversion

for creation of entangled photon pairs

due to non-linear response of medium to electric field

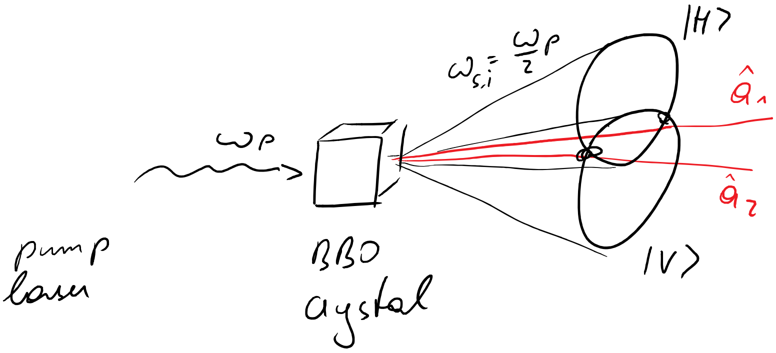
polarization $P = \varepsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots)$

for $E(x,t) = E_0 e^{i(kx - \omega t)}$ $\Rightarrow$ $\chi^{(2)} E^2 \propto e^{-2i\omega t}$



up-conversion

reverse process: down-conversion:

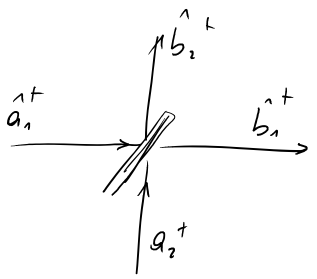


conservation of
- energy $(\omega_p = \omega_s + \omega_i)$
- momentum $(\vec{k}_p = \vec{k}_s + \vec{k}_i)$

at crossing points: $|\psi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2 \right)$

anti-symmetric Bell state

Bell State measurement using 50/50 beamsplitter

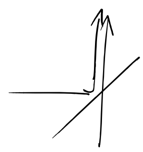


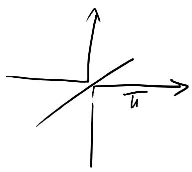
$$\hat a_1^{(\dagger)} \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(\hat b_1^{(\dagger)} + \hat b_2^{(\dagger)}\right)$$

$$\hat a_2^{(\dagger)} \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(-b_1^{(\dagger)} + b_2^{(\dagger)}\right)$$

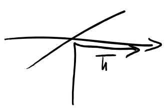
two photons incident on the beamsplitter:

4 possibilities:

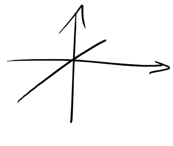
①  bunching

② 

anti-bunching

③ 

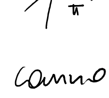
bunching

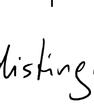
④  anti-bunching

Hong-Ou Mandel effect: for identical polarization $\left(\Phi^{\pm} = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)\right)$

$\Rightarrow$ destructive interference due to phase shift 0 or π

$\Rightarrow$ bunching



&


cannot be distinguished

BUT: anti-bunching for anti-symmetric state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$!

input state: $\frac{1}{\sqrt{2}}\left(h_1^\dagger v_2^\dagger \mp v_1^\dagger h_2^\dagger\right)$ $\xrightarrow{BS}$ "apply beamsplitter transformation to each mode"

$(\hat{v}_i := \hat{a}_{iV},\ \hat{h}_i := \hat{a}_{iH})$

anti-sym

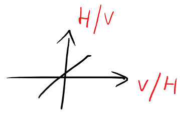
symm.

$$\frac{1}{2}\frac{1}{\sqrt{2}}\left[(h_1^\dagger + h_2^\dagger)(v_2^\dagger - v_1^\dagger) \mp (v_1^\dagger + v_2^\dagger)(h_2^\dagger - h_1^\dagger)\right] =$$

$$= \frac{1}{2\sqrt{2}}\left[h_1^\dagger v_2^\dagger + h_2^\dagger v_2^\dagger - h_1^\dagger v_1^\dagger - h_2^\dagger v_1^\dagger \mp v_1^\dagger h_2^\dagger \mp v_2^\dagger h_2^\dagger \pm v_1^\dagger h_1^\dagger \pm v_2^\dagger h_1^\dagger\right]$$

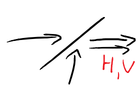
for anti-symmetric spatial wavefunction (-):

$$= \frac{1}{\sqrt{2}}\left(h_1^\dagger v_2^\dagger - v_1^\dagger h_2^\dagger\right) \Rightarrow \text{anti-bunching}$$



for symmetric spatial wavefunction (+):

$$= \frac{1}{\sqrt{2}}\left(-h_1^\dagger v_1^\dagger + h_2^\dagger v_2^\dagger\right) \Rightarrow \text{bunching}$$



oder

bunching also for other symmetric spatial wavefunctions $\left[\frac{1}{\sqrt{2}}\left(h_1^\dagger h_2^\dagger \pm v_1^\dagger v_2^\dagger\right)\right]$

$$\left[\frac{1}{\sqrt{2}}\left(h_1^\dagger h_2^\dagger \pm v_1^\dagger v_2^\dagger\right)\right] \Rightarrow \text{bunching}$$

Super Dense Coding:

1) Preparation of initial entangled state (PDC)

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

2) Generation of 4 maximally entangled 2 photon polarization states

$$|\psi^+\rangle \xrightarrow{\mathbb{1}_2} |\psi^+\rangle$$

$$|\psi^+\rangle \xrightarrow{Y_2} \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) = |\phi^+\rangle \quad (HWP)$$

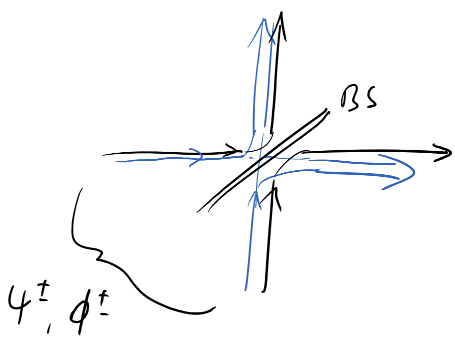
(i) ... omitted

$$|\psi^+\rangle \xrightarrow{Z_2} \frac{1}{\sqrt{2}}\left(|HV\rangle -\,^{(i)}\,|VH\rangle\right) = |\psi^-\rangle \quad (QWP)$$

$$|\psi^+\rangle \xrightarrow{Z_2 Y_2} \frac{1}{\sqrt{2}}\left(|HH\rangle -\,^{(i)}\,|VV\rangle\right) = |\phi^-\rangle \quad (HWP + QWP)$$

3) Bell State measurement using beam splitter

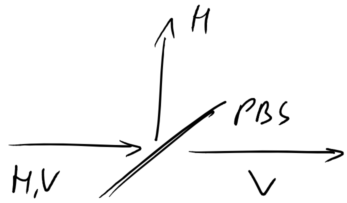
A) distinguish symmetric $(|\psi^+\rangle, |\phi^+\rangle, |\phi^-\rangle)$ from antisymmetric $(|\psi^-\rangle)$ state using a beamsplitter (BS)



anti-bunching for ψ^-

bunching for symm. states

B) distinguish polarization states using polarizing beam splitter



outcomes: $|\psi^+\rangle$: coincidence D_H & D_V or $D_{H'}$ & $D_{V'}$

$|\psi^-\rangle$: coincidence $D_{H'}$ & D_V or D_H & $D_{V'}$

$|\phi^+\rangle, |\phi^-\rangle$: 2 photons in $D_H, D_V, D_{H'}, D_{V'}$

(cannot be distinguished)