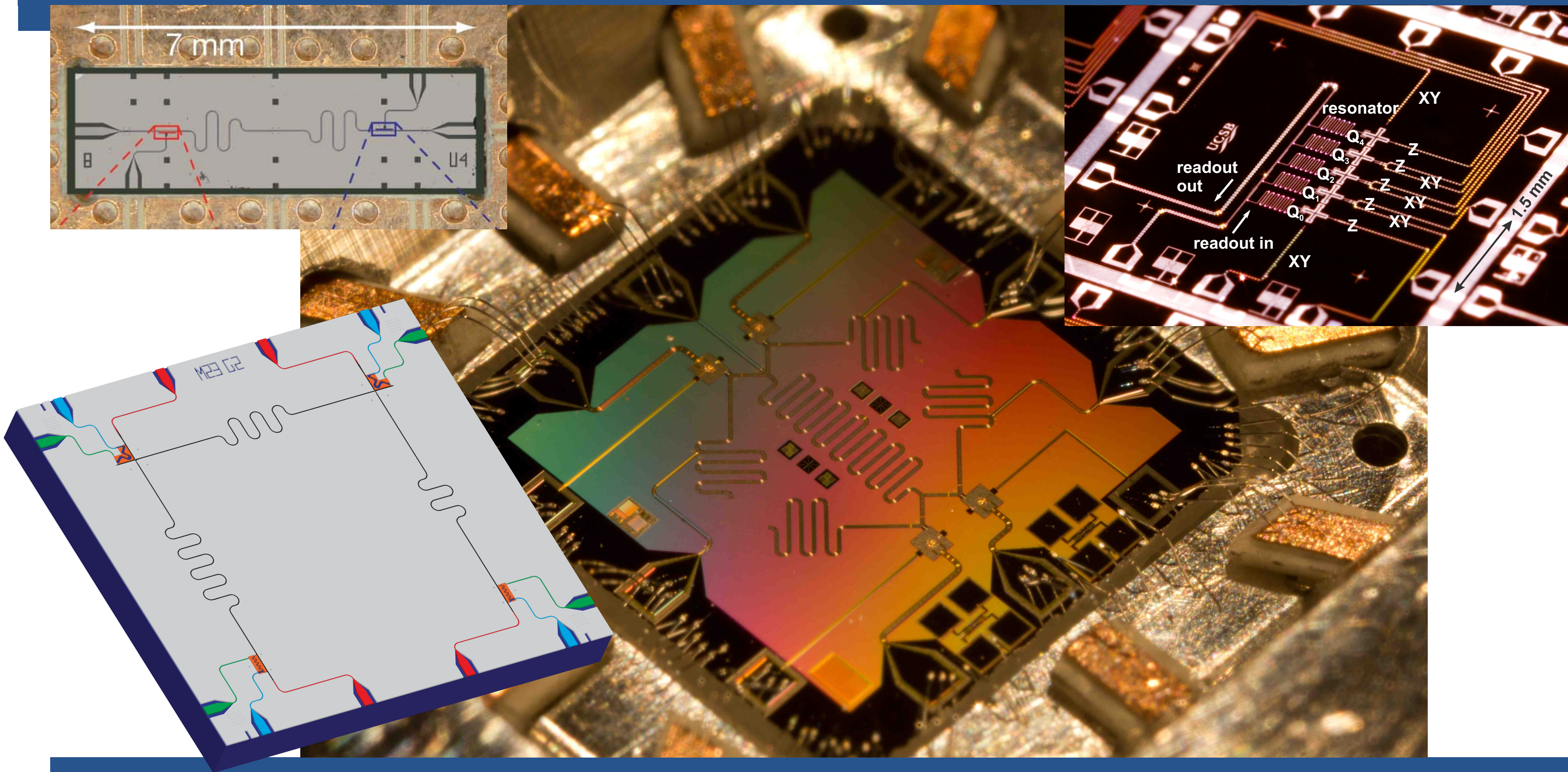


Quantum Computing with Superconducting Qubits

Joannis Koepsell, Tim Menke

11.04.2014

Outline

- Grover algorithm: theory
- Implementing the C-Phase gate on a 2-qubit processor
- Experimental demonstration of the Grover algorithm
- High-fidelity gates and multi-qubit entanglement on a 5-qubit processor

Motivation: Searching a Database

- Structured Database: Given a name, find the phone number
  - Easy to do!
- Unstructured Database: Given a phone number, find the name
  - Classically: N-1 operations

→ Quantum Grover Algorithm: $\mathcal{O}(\sqrt{N})$

QSIT Student Presentation 2013

Grover Algorithm: Ingredients

- Store indices of $N = 2^n$ elements in n qubits
  - Example: for N = 4 the elements are represented by the states

$$\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$$

- Oracle *O*: recognizes the solution

$$O|x\rangle = (-1)^{f(x)}|x\rangle$$

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a solution to the search problem} \\ 0, & \text{otherwise} \end{cases}$$

➡ In the general case, oracle is a "black box"

Nielsen & Chuang 2010

Grover Algorithm: Initialization

- Start with state $|\mathbf{0}\rangle = |0, 0, ...0\rangle$

- Put computer in equal superposition state:

$$|\psi\rangle = H^{\otimes n}|\mathbf{0}\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle$$

Apply Hadamard gate to each qubit

→ Exploit quantum parallelism

Nielsen & Chuang 2010

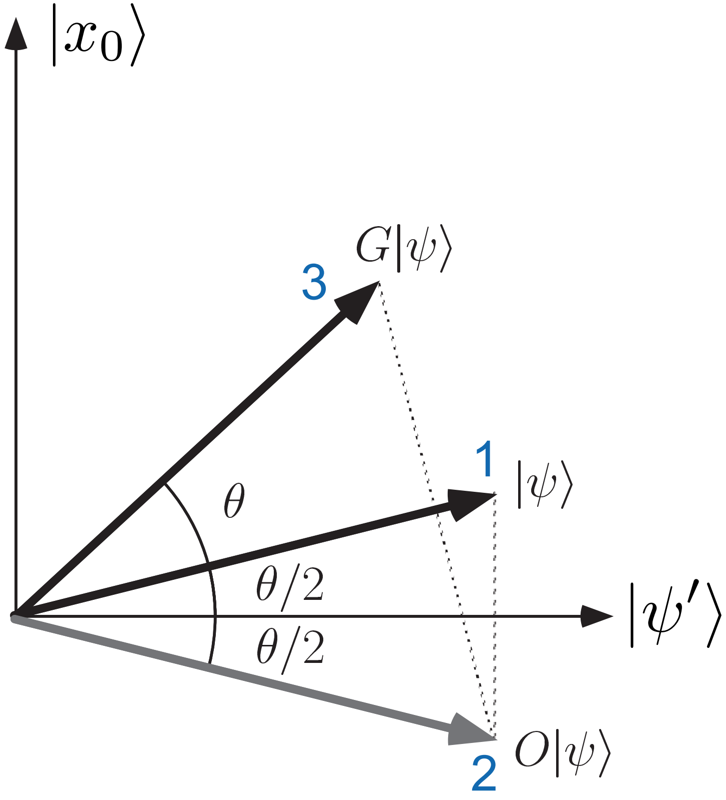
Grover Algorithm: Iteration

- In each iteration step
  1. Apply Oracle *O*
  2. Apply "inversion about mean"

$$2\,|\psi\rangle\langle\psi| - I$$

Grover operator

$$G = (2\,|\psi\rangle\langle\psi| - I)\,O$$



$$|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{x\neq x_0}|x\rangle + \frac{1}{\sqrt{N}}|x_0\rangle$$

$=: |\psi'\rangle$ — Subspace that is not a solution: $O\,|\psi'\rangle = |\psi'\rangle$

Solution subspace $O\,|x_0\rangle = -\,|x_0\rangle$

Nielsen & Chuang 2010

Grover Algorithm: Result

- Each iteration step corresponds to a rotation

$$G^k |\Psi\rangle = \cos\left(\frac{2k+1}{2}\theta\right) |\Psi'\rangle + \sin\left(\frac{2k+1}{2}\theta\right) |x_0\rangle$$

- Iterations needed to get solution:

$$G^R |\psi\rangle = |x_0\rangle \quad \text{for} \quad \boxed{R \approx \frac{\pi}{4}\sqrt{N}}$$

- Special case: for N = 4 (two qubits), only a single iteration is needed!

Nielsen & Chuang 2010

Decomposition of Inversion about the Mean

- We want to implement inversion about mean on computer

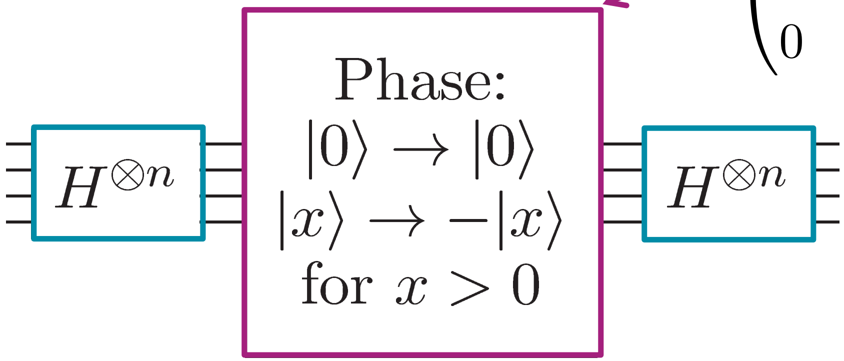
→ Decompose into single- and multi-qubit gates

$$2\left|\psi\right\rangle\left\langle\psi\right| - I = \underline{H^{\otimes n}}\,\underline{\left(2\left|0\right\rangle\left\langle 0\right| - I\right)}\,\underline{H^{\otimes n}}$$

For two qubits

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

1) Hadamard gate
2) Conditional phase shift
3) Hadamard gate



Nielsen & Chuang 2010

Decomposition of Inversion about the Mean

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For two qubits

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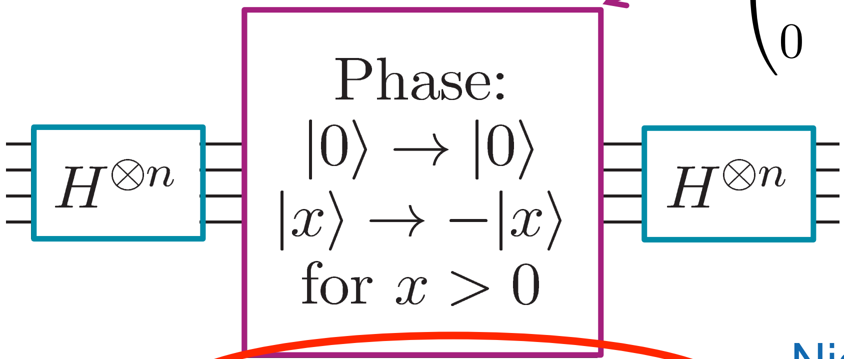


Need C-Phase gate to implement Grover algorithm!

Multi-qubit operation!

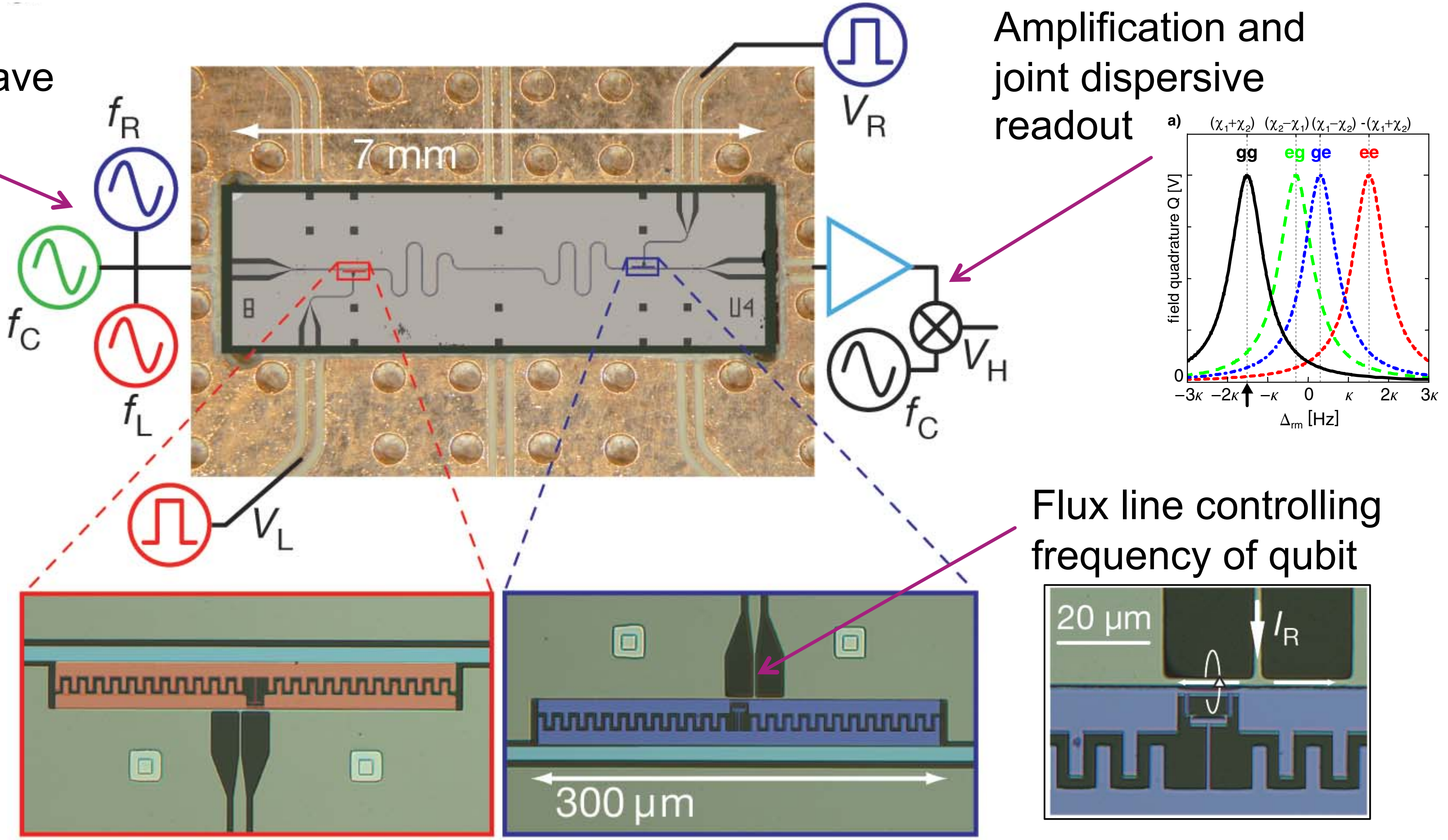
Nielsen & Chuang 2010

Implementing Grover Alg. on 2-Qubit Processor

DiCarlo *et al.*, Nature **460**, 240 (2009)

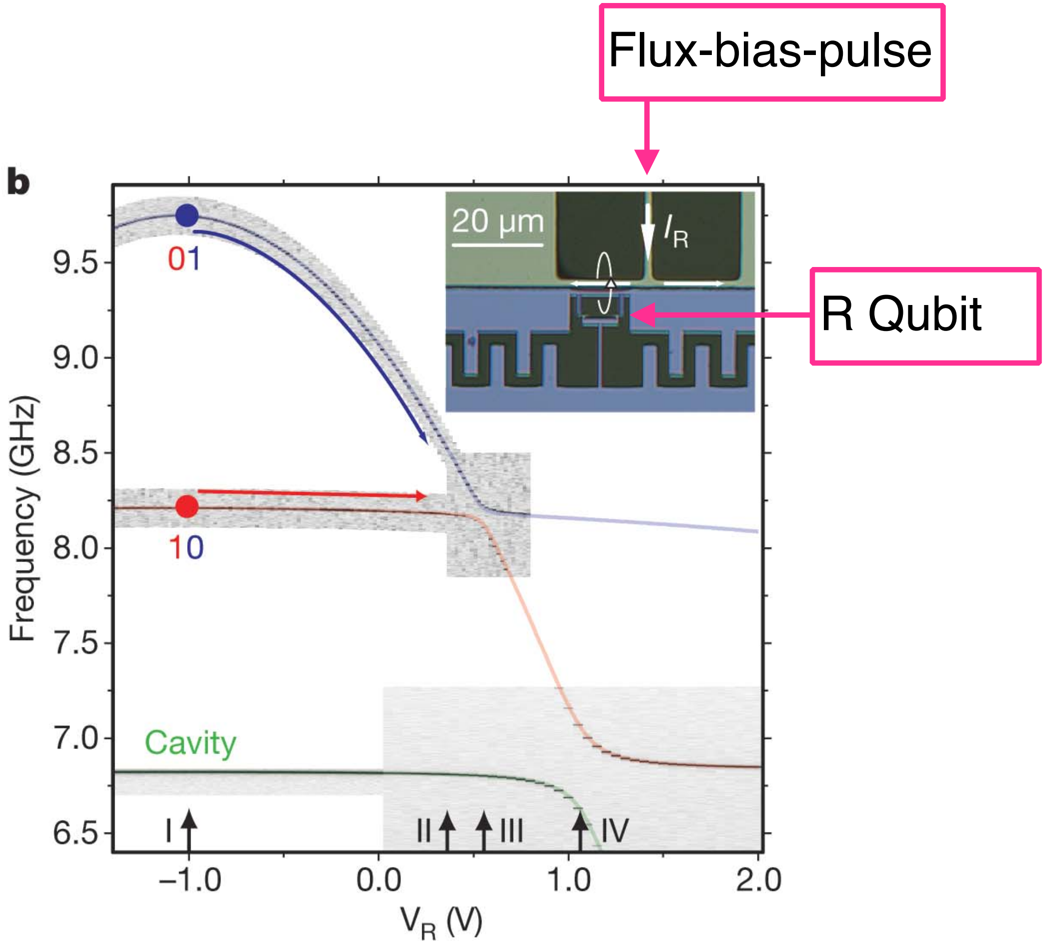
Microwave source

Amplification and joint dispersive readout



Flux line controlling frequency of qubit

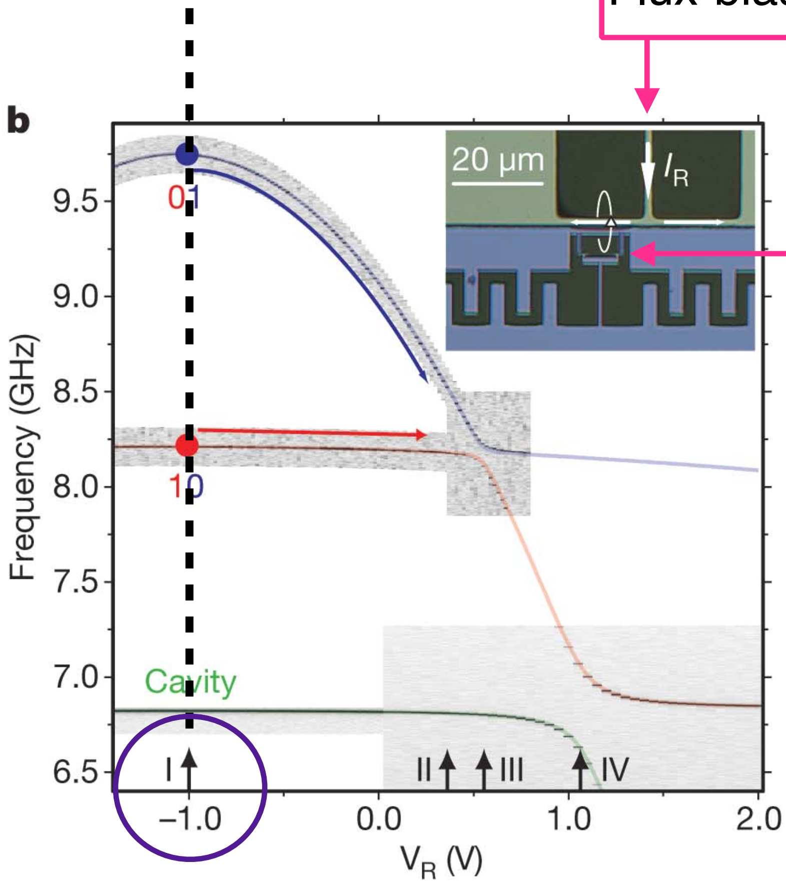
Implementation of C-Phase gate



DiCarlo 2009

Implementation of C-Phase gate

Flux-bias-pulse

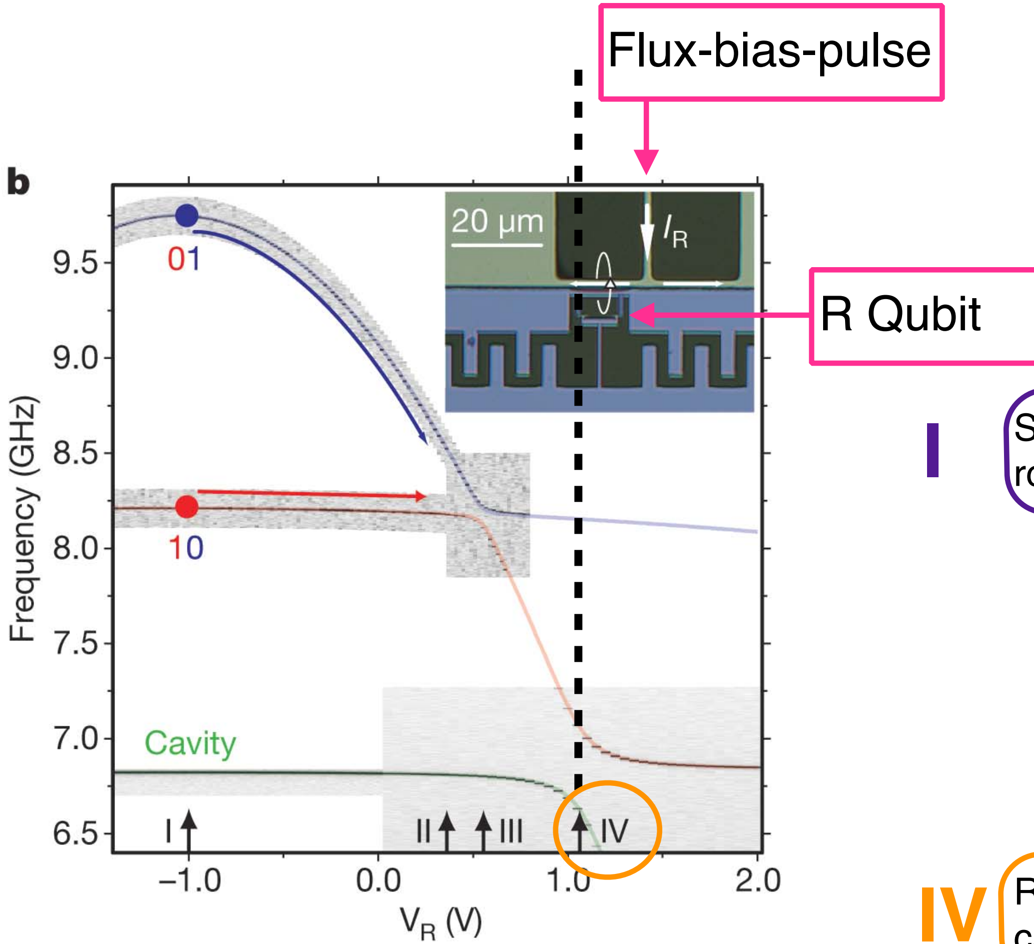


R Qubit

State preparation, single-qubit rotations + measurement

DiCarlo 2009

Implementation of C-Phase gate

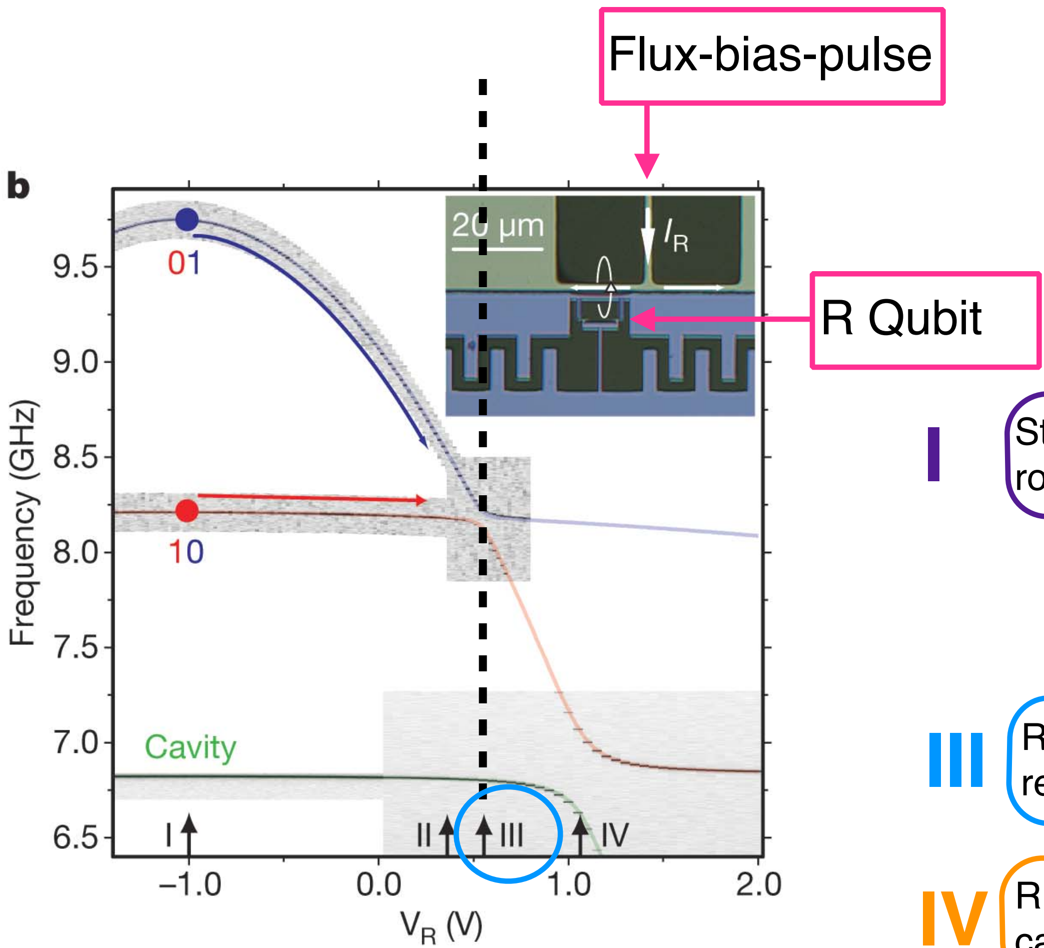


I State preparation, single-qubit rotations + measurement

IV Right qubit in resonance with cavity; vacuum Rabi splitting

DiCarlo 2009

Implementation of C-Phase gate



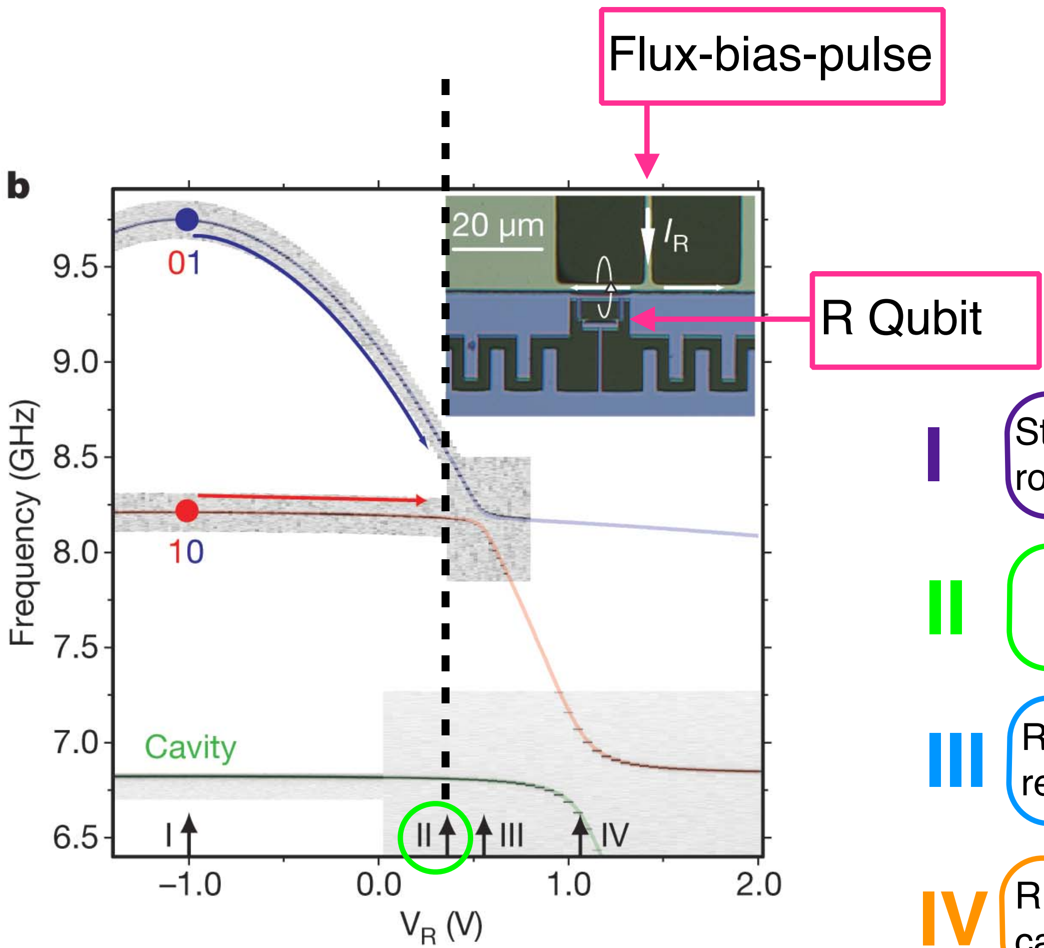
I State preparation, single-qubit rotations + measurement

III R and L qubit in mutual resonance; (cavity-mediated)

IV Right qubit in resonance with cavity; vacuum Rabi splitting

DiCarlo 2009

Implementation of C-Phase gate



I State preparation, single-qubit rotations + measurement

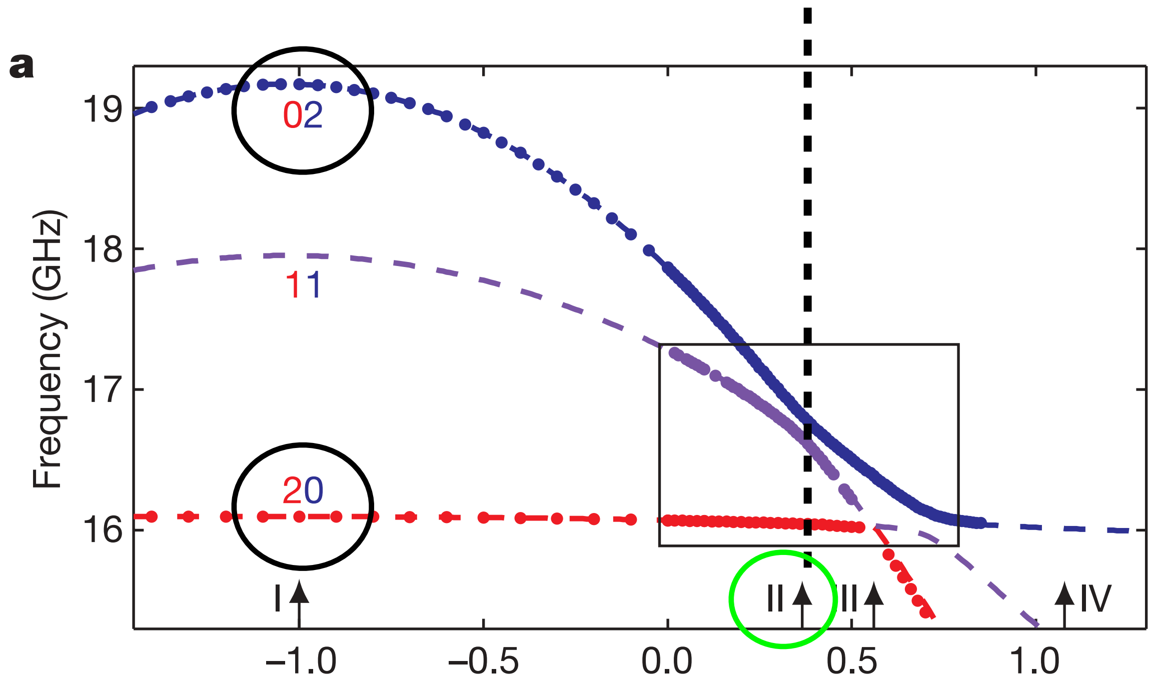
II …no interaction apparent… why do we look at this point?

III R and L qubit in mutual resonance; (cavity-mediated)

IV Right qubit in resonance with cavity; vacuum Rabi splitting

DiCarlo 2009

Implementation of C-Phase gate

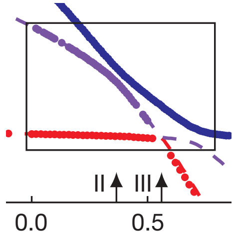


Behaviour of non-computational states

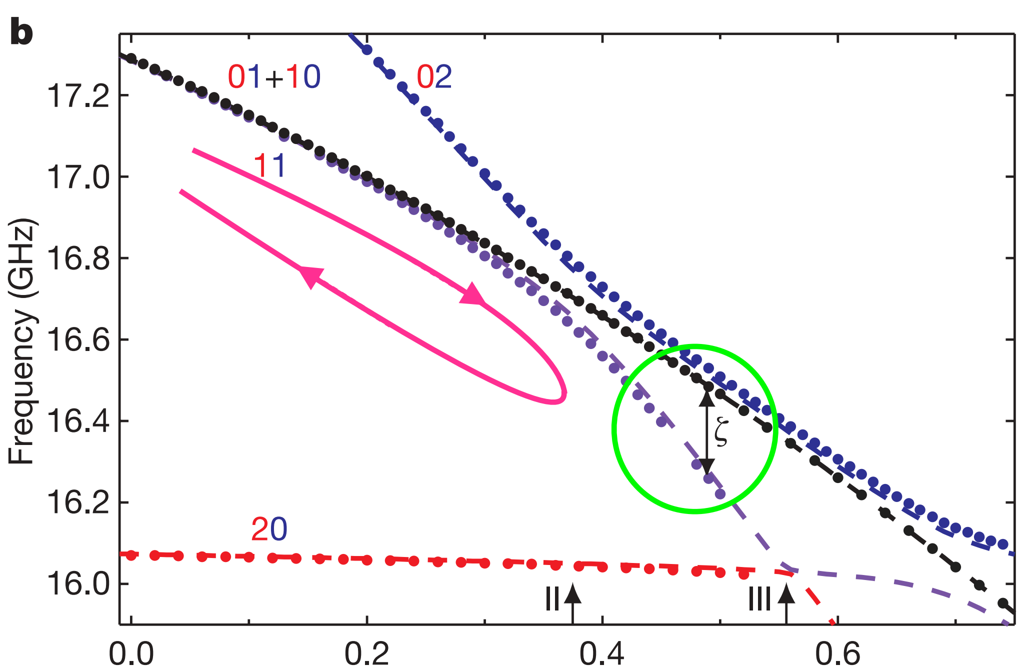
Avoided crossing

$$|1, 1\rangle \leftrightarrow |0, 2\rangle$$

DiCarlo 2009



Idea: A pulse **I** ⇄ **II** will induce a phase shift!



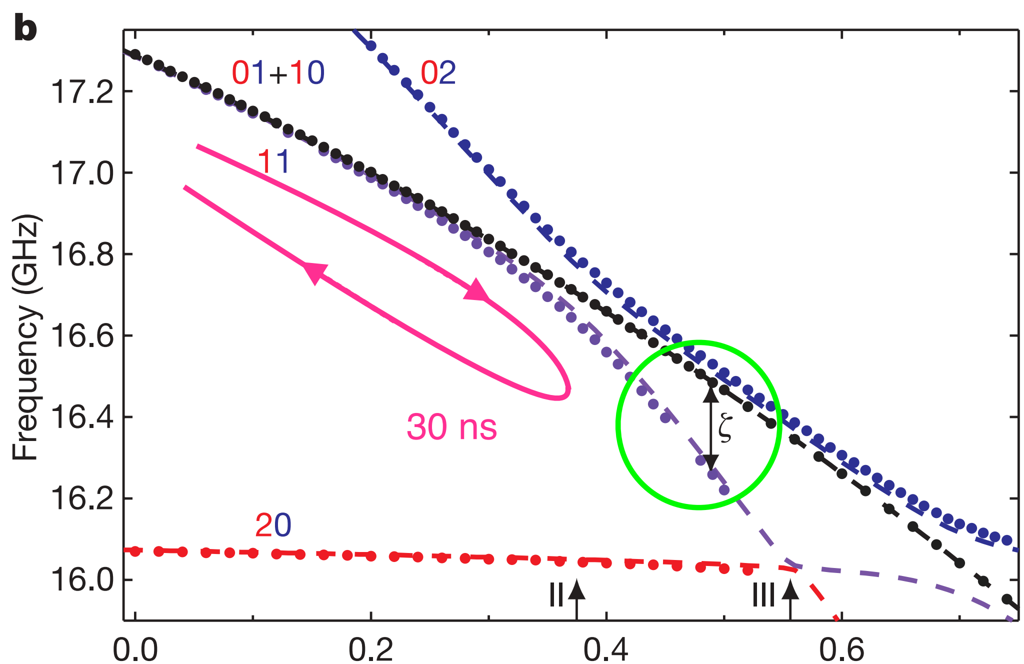
DiCarlo 2009

Idea: A pulse **I** ⇄ **II** will induce a phase shift!

What is the phase each state picks up during this pulse?

$$\phi_{lr} = 2\pi \int \delta f_{lr}(t)\,\mathrm{d}t$$

deviation from frequency at I



DiCarlo 2009

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{11}} \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

cU_{11} **C-Phase!**

$$\phi_{11} = \phi_{01} + \phi_{10} + \int \zeta(t)dt$$

$$\phi_{10} = 2\pi k$$

$$\phi_{01} = 2\pi m$$

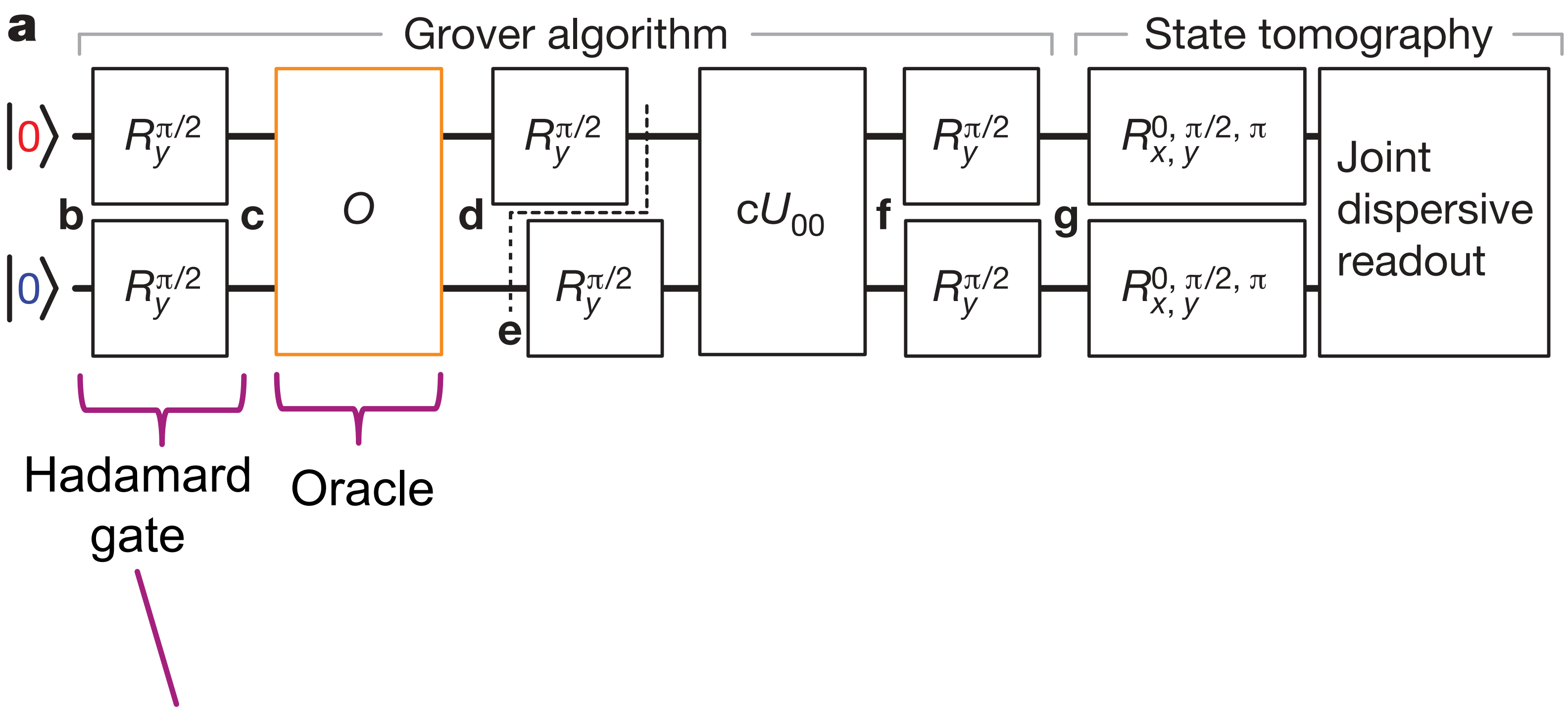
$$\int \zeta(t)dt = (2n+1)\pi$$

Adjusting:

- Amplitude of simult. weak pulse on L-qubit
- Rising & falling edges of pulse
- Control by two orders of magnitude (residual 1.2MHz at I)

cU_{ij} : All four C-Phase Gates possible

Implementing the Grover Algorithm on a 2-Qubit Processor



Single qubit rotations via microwave pulse resonant with qubit

DiCarlo 2009

Oracle for Grover Algorithm

- Desired action of oracle:

$$O\,|x\rangle = (-1)^{f(x)}\,|x\rangle$$

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a solution to the search problem} \\ 0, & \text{otherwise} \end{cases}$$

→ Oracle for x_0 = ij is the C-Phase gate cU_{ij}

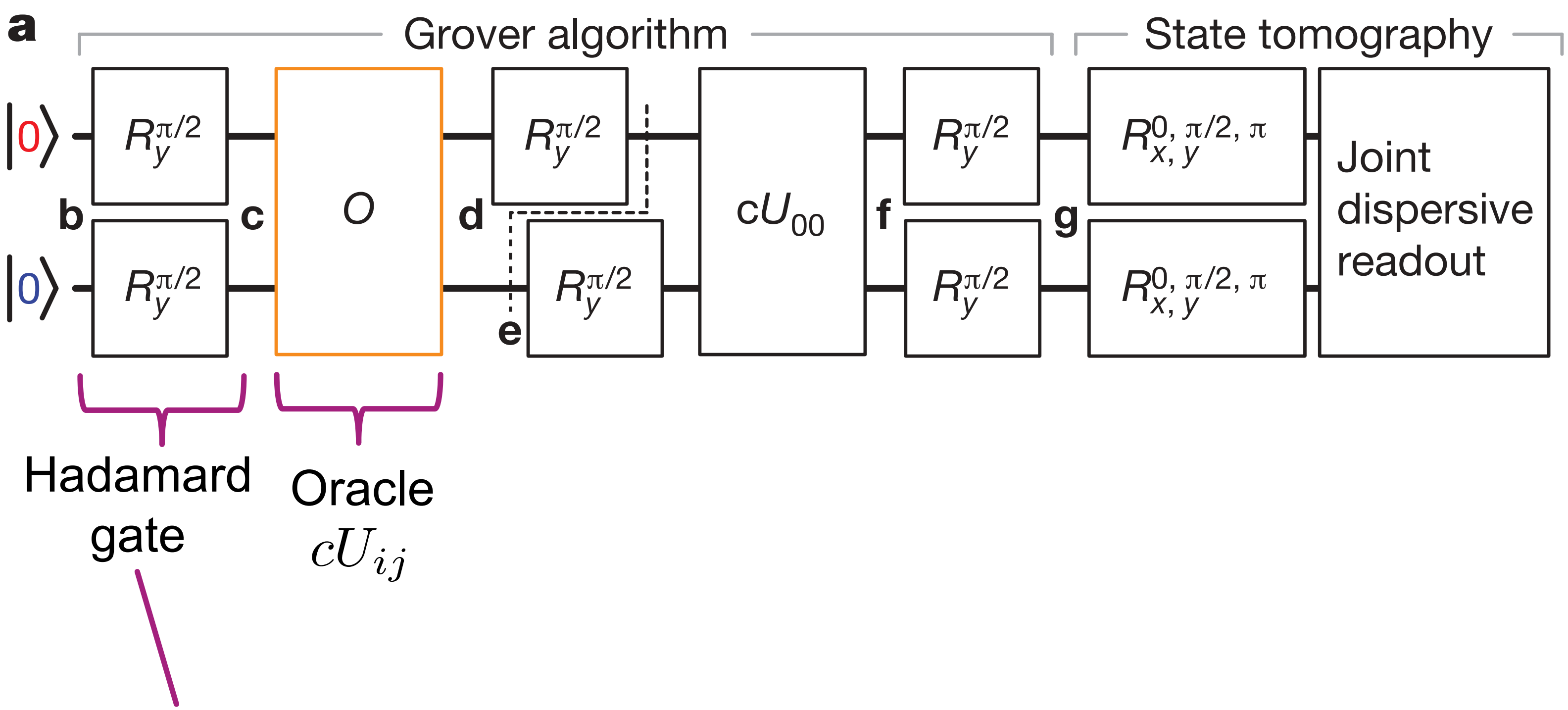
- Example:

  x_0 = 10

$$cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{array}{ccc} |00\rangle & \rightarrow & |00\rangle \\ |10\rangle & \rightarrow & -\,|10\rangle \\ |01\rangle & \rightarrow & |01\rangle \\ |11\rangle & \rightarrow & |11\rangle \end{array}$$

solution marked

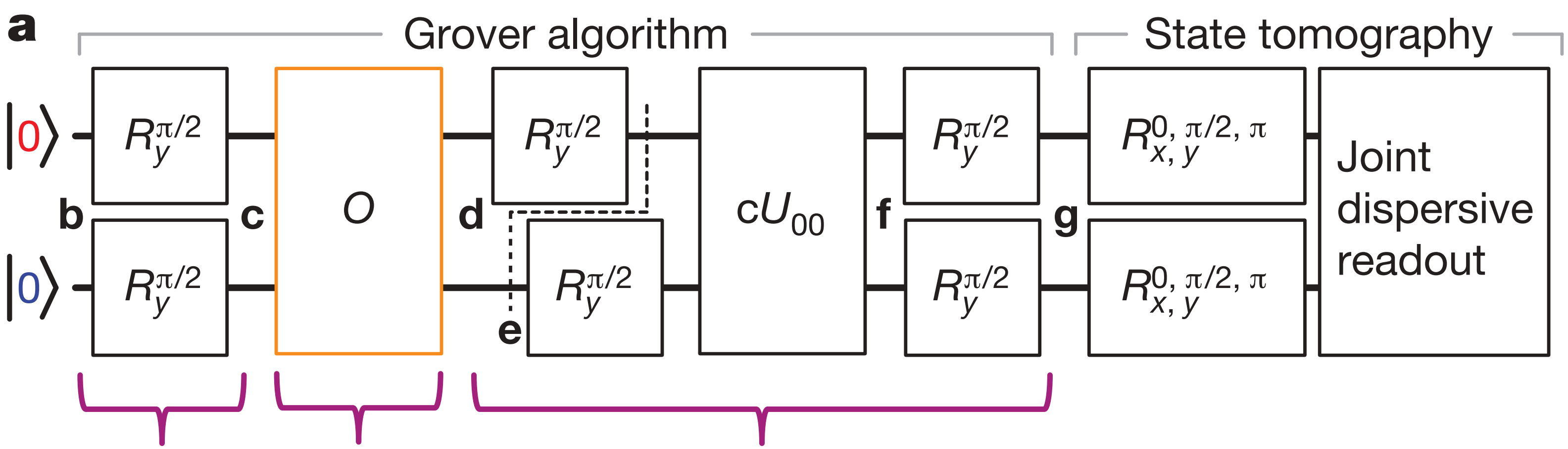
Implementing the Grover Algorithm on a 2-Qubit Processor



Single qubit rotations via microwave pulse resonant with qubit

DiCarlo 2009

Implementing the Grover Algorithm on a 2-Qubit Processor



Hadamard gate

Oracle cU_{ij}

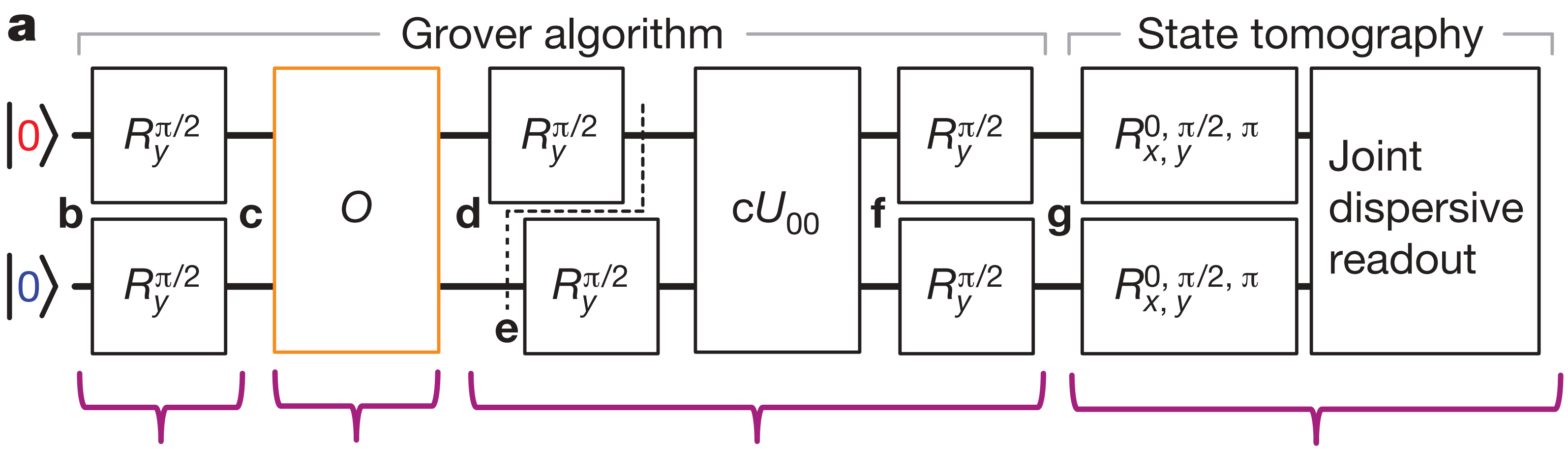
$$2\left|\psi\right\rangle\left\langle\psi\right| - I$$
$$= H^{\otimes n}\left(2\left|0\right\rangle\left\langle 0\right| - I\right) H^{\otimes n}$$

Single qubit rotations via microwave pulse resonant with qubit

$$cU_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DiCarlo 2009

Implementing the Grover Algorithm on a 2-Qubit Processor



Hadamard gate

Oracle cU_{ij}

$$2\left|\psi\right\rangle\left\langle\psi\right| - I$$
$$= H^{\otimes n}\left(2\left|0\right\rangle\left\langle 0\right| - I\right)H^{\otimes n}$$

Apply one of 15 single qubit rotations before measurement

↓

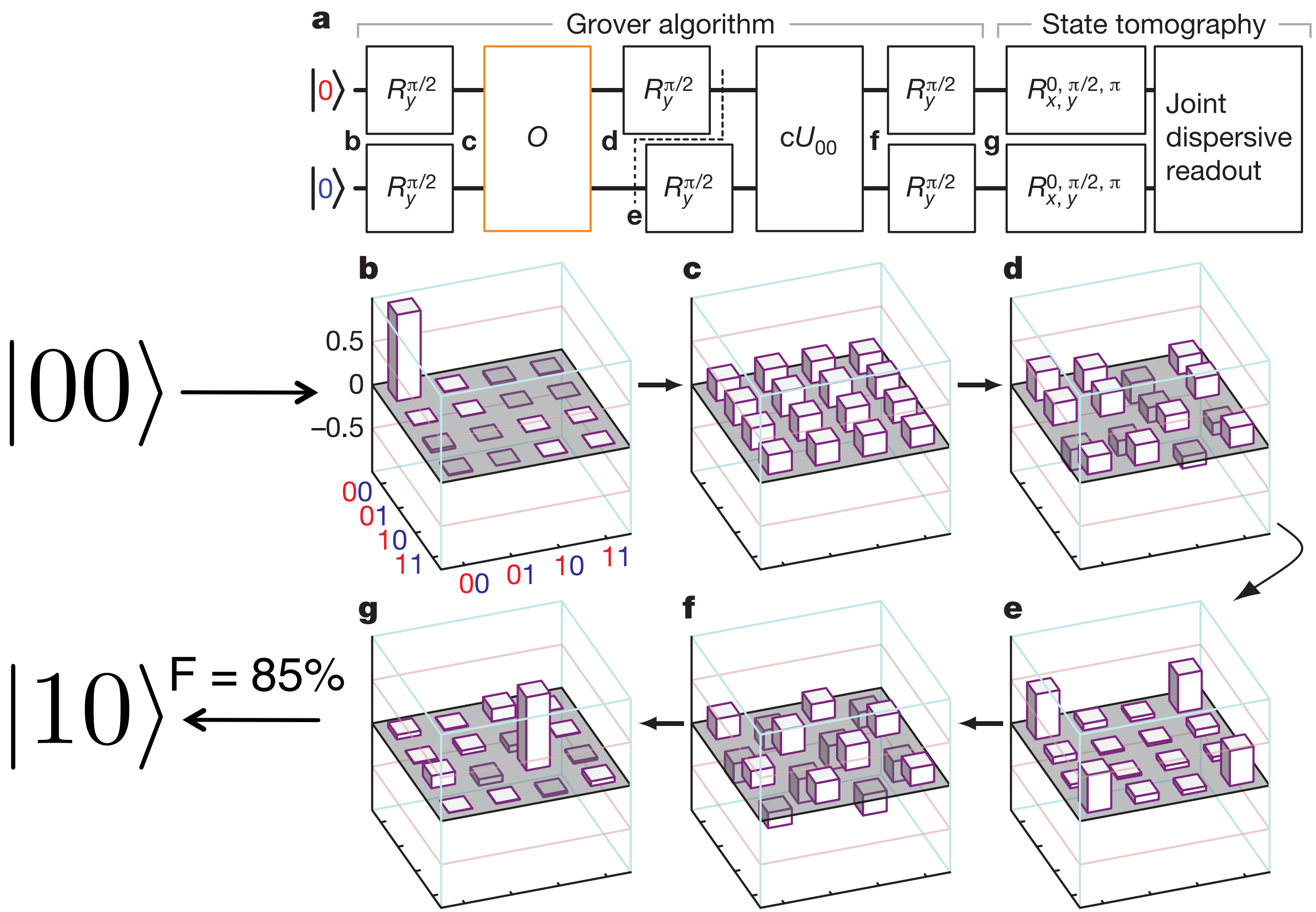
Reconstruct density matrix

Single qubit rotations via microwave pulse resonant with qubit

$$cU_{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DiCarlo 2009

Grover Algorithm on 2-Qubit Processor: Experimental Results



- Main error source: decoherence of qubits during gate sequence
- Ratio of total duration of gate sequence to qubit coherence time:

$$\sim \frac{100\,\mathrm{ns}}{1\,\mu\mathrm{s}} = 0.1$$

DiCarlo 2009

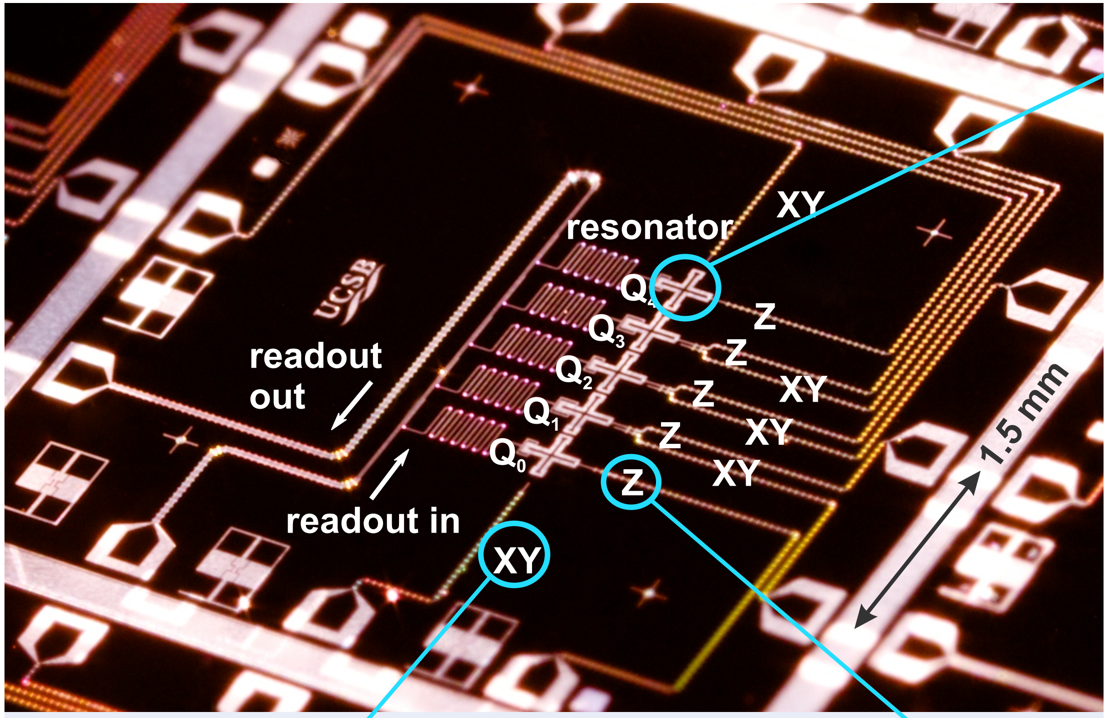
Next Steps on the Road to Quantum Computation

- We showed that quantum algorithms (Grover) can in principle be implemented using superconducting qubits
- Now we want to get closer to real computer
  - Larger number of qubits
    - We only need single-qubit and C-Phase gates because they are universal
  - High fidelity of qubit operations
    - For surface code at least 99% ("surface code threshold")
  - First step of error correction: entanglement of multiple qubits
    - Generate maximally entangled Greenberger-Horne-Zeilinger (GHZ) state

$$|\mathrm{GHZ}\rangle = \frac{|00...0\rangle + |11...1\rangle}{\sqrt{2}}$$

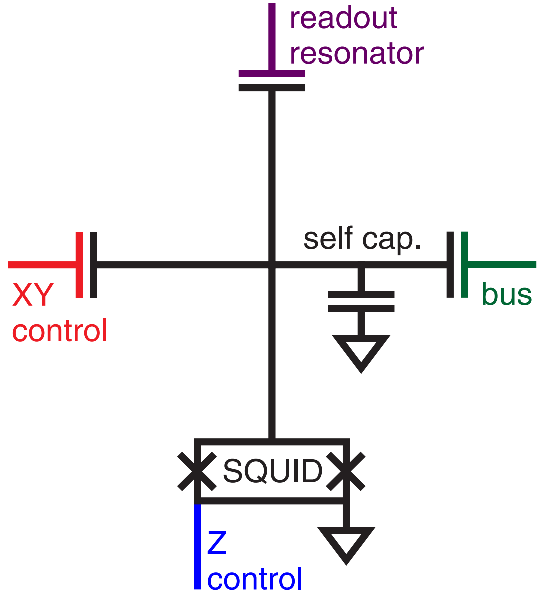
Five-Qubit Architecture

R. Barends *et al.*, arXiv:1402.4848 (2014)



Xmon: cross-shaped Transmon qubit

- Long coherence time $\sim 40\,\mu s$
- Fast control



Single-qubit rotations around x and y

Control of qubit frequency

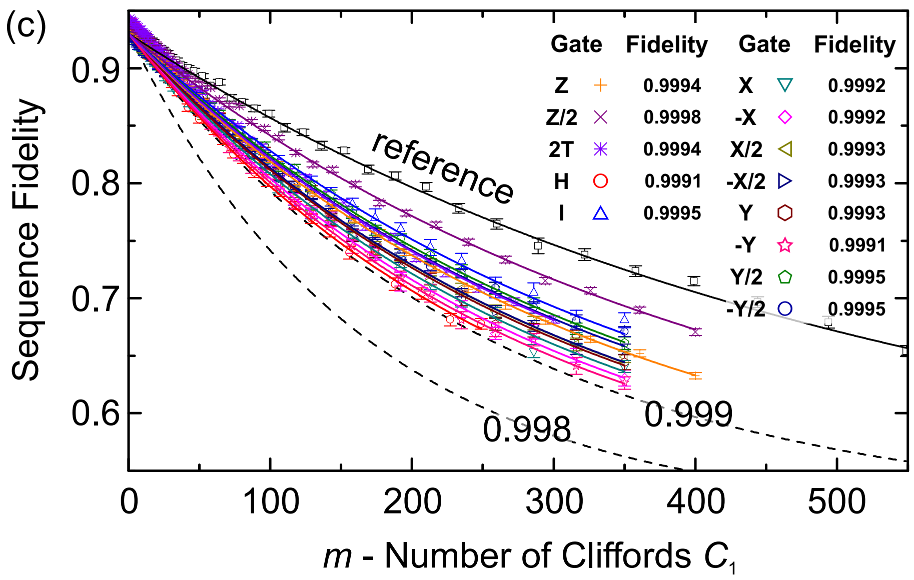
↓

C-Phase implemented same as for 2-qubit architecture

First Result: High-Fidelity Gates

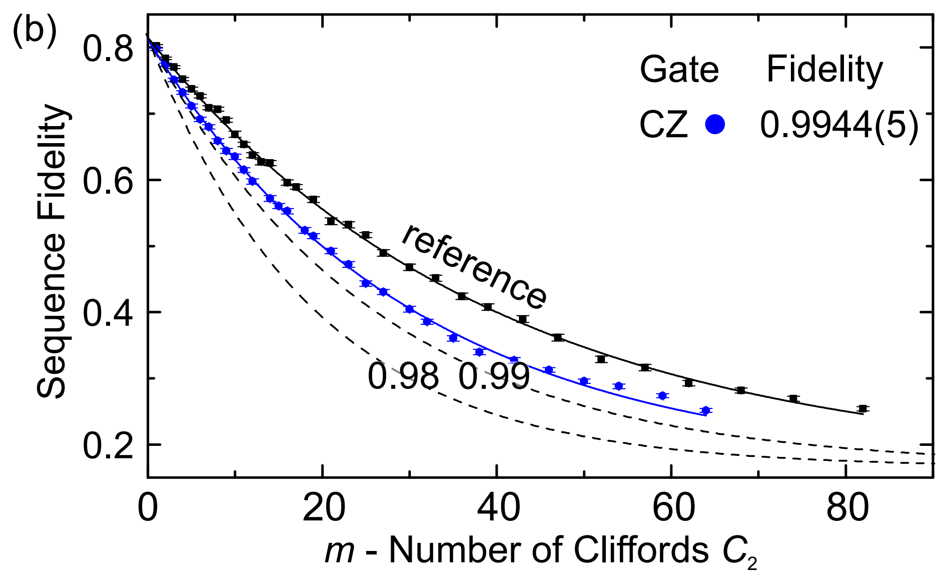
- Characterize fidelity of a gate independent of input state
- Interleave gate with random sequence of qubit operations

Single-qubit gates



Average single-qubit gate fidelity: 99.92%

C-Phase gate



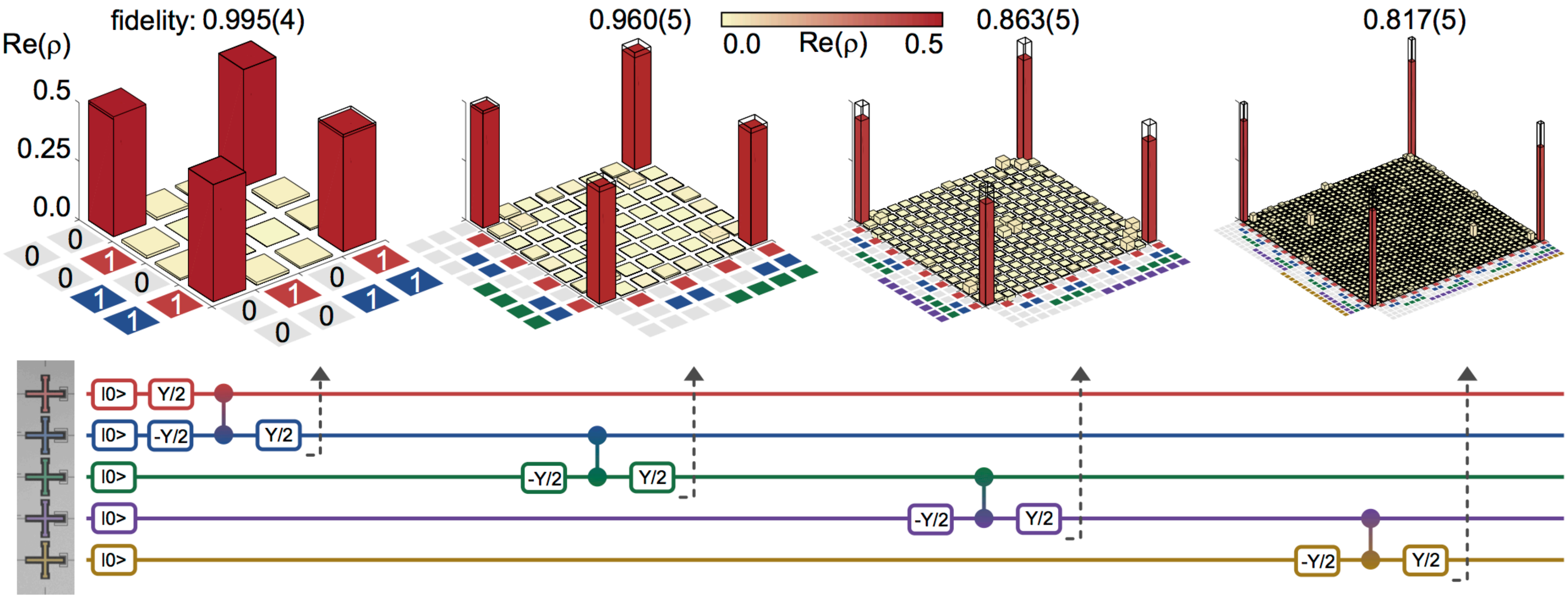
Average C-Phase gate fidelity: 99.4%

Barends 2014

Second Result: Generation of 5-Qubit GHZ State

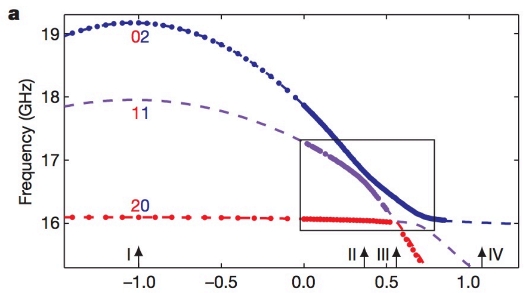
- Successively contruct 5-qubit GHZ state using single- and two-qubit gates

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \longrightarrow \frac{|000\rangle + |111\rangle}{\sqrt{2}} \longrightarrow \frac{|0000\rangle + |1111\rangle}{\sqrt{2}} \longrightarrow \frac{|00000\rangle + |11111\rangle}{\sqrt{2}}$$

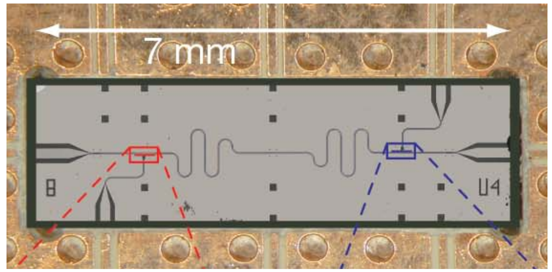


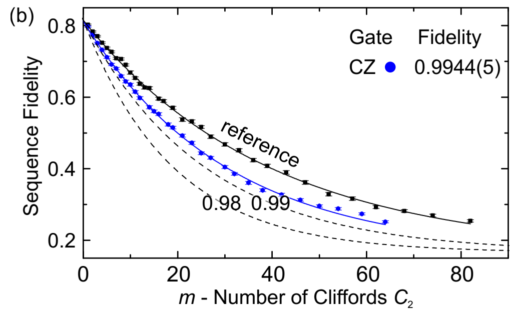
Barends 2014

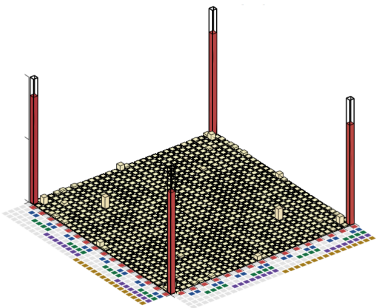
Conclusion

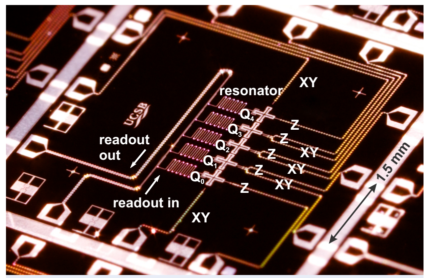


- Implementation of C-Phase two-qubit gate using avoided crossing in frequency spectrum
- Demonstration of Grover algorithm on 2-qubit processor
- High-fidelity single- and two-qubit gates reach error correction threshold
- Construction of maximally entangled GHZ states on 5-qubit processor







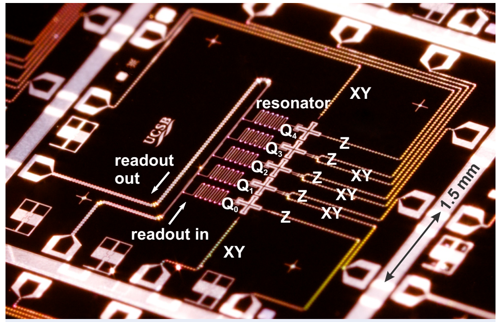


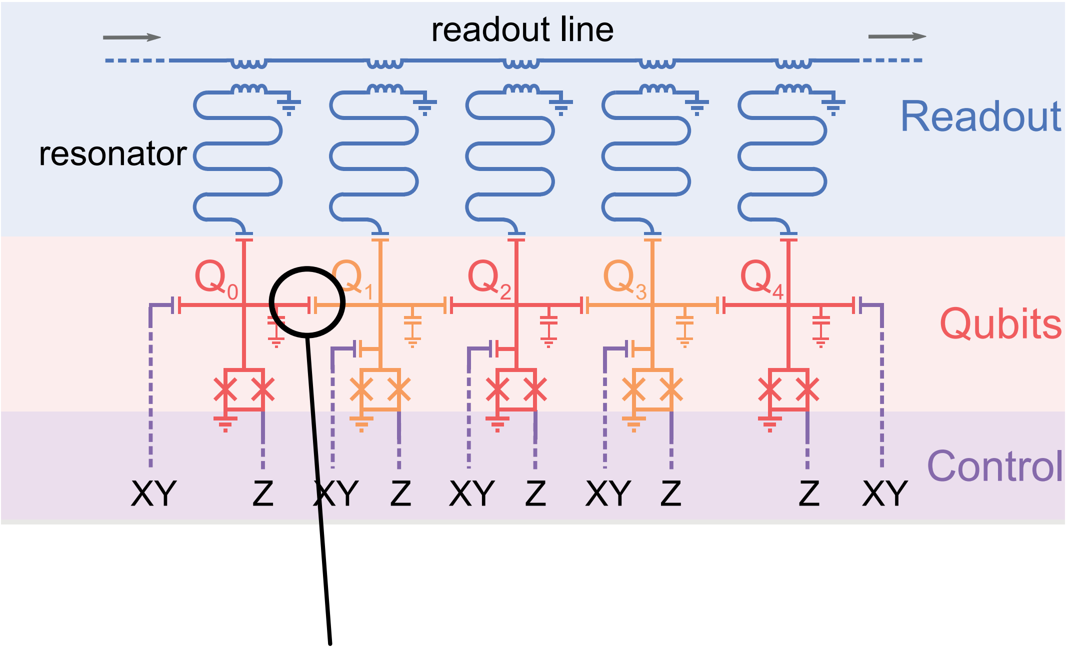
References

- M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press 2010
- DiCarlo *et al.*, Demonstration of two-qubit algorithms with a superconducting quantum processor, Nature **460**, 240 (2009)
- R. Barends *et al.*, Logic gates at the surface code threshold: Superconducting qubits poised for fault-tolerant quantum computing, arXiv:1402.4848 (2014)

Backup Slides

Five-Qubit Architecture

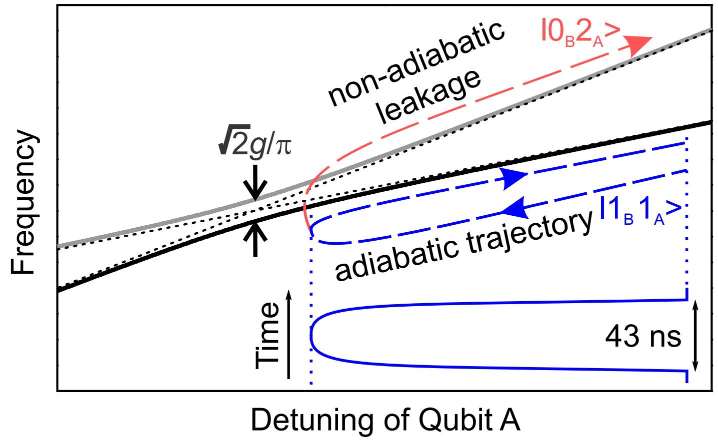




Transmission line decoupled from resonator to avoid cavity-induced relaxation of qubit

Capacitative coupling of nearest neighbors allows entangling

➡ C-Phase gate



Barends 2014

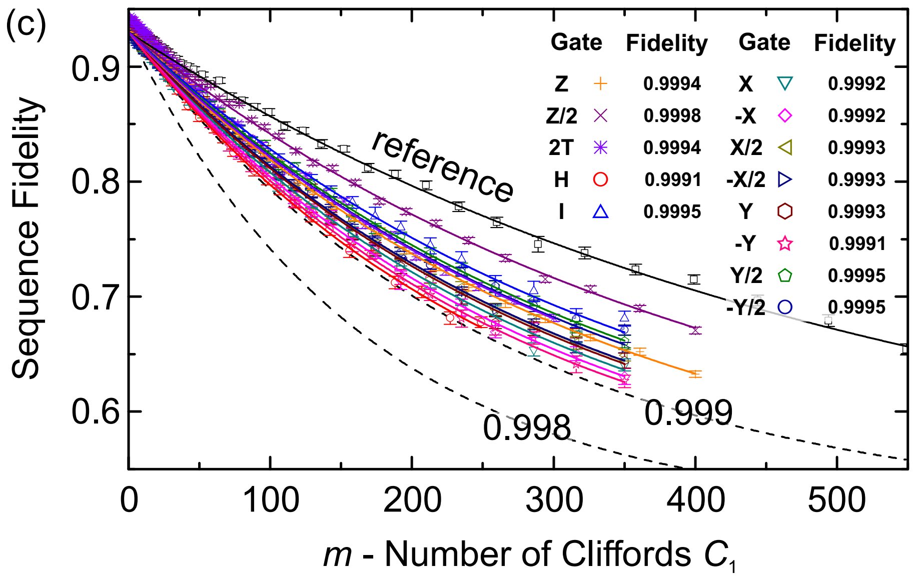
Randomised Benchmarking of Qubit Operations

- Characterize fidelity of a gate independent of input state
- "Sequence of Cliffords": Random sequence of qubit operations
  - Example: for a single qubit, the sequence consists of randomly chosen gates from $\{\hat{I}, \pm\hat{X}/2, \pm\hat{Y}/2, \pm\hat{X}, \pm\hat{Y}\}$
- Experimental procedure:
  1) Apply sequence of Cliffords
  2) Apply qubit operation we want to characterize
  3) Apply recovery sequence that makes the first sequence the identity
- "Reference" obtained by leaving out step 2

Kelly 2014

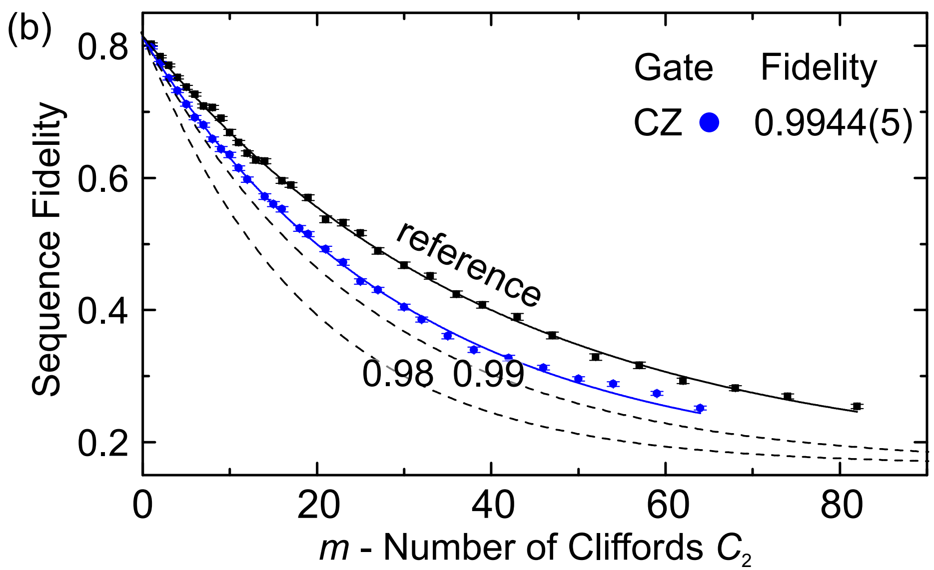
First Result: High-Fidelity Gates

Single-qubit gates



Average single-qubit gate fidelity: 99.92%

C-Phase gate

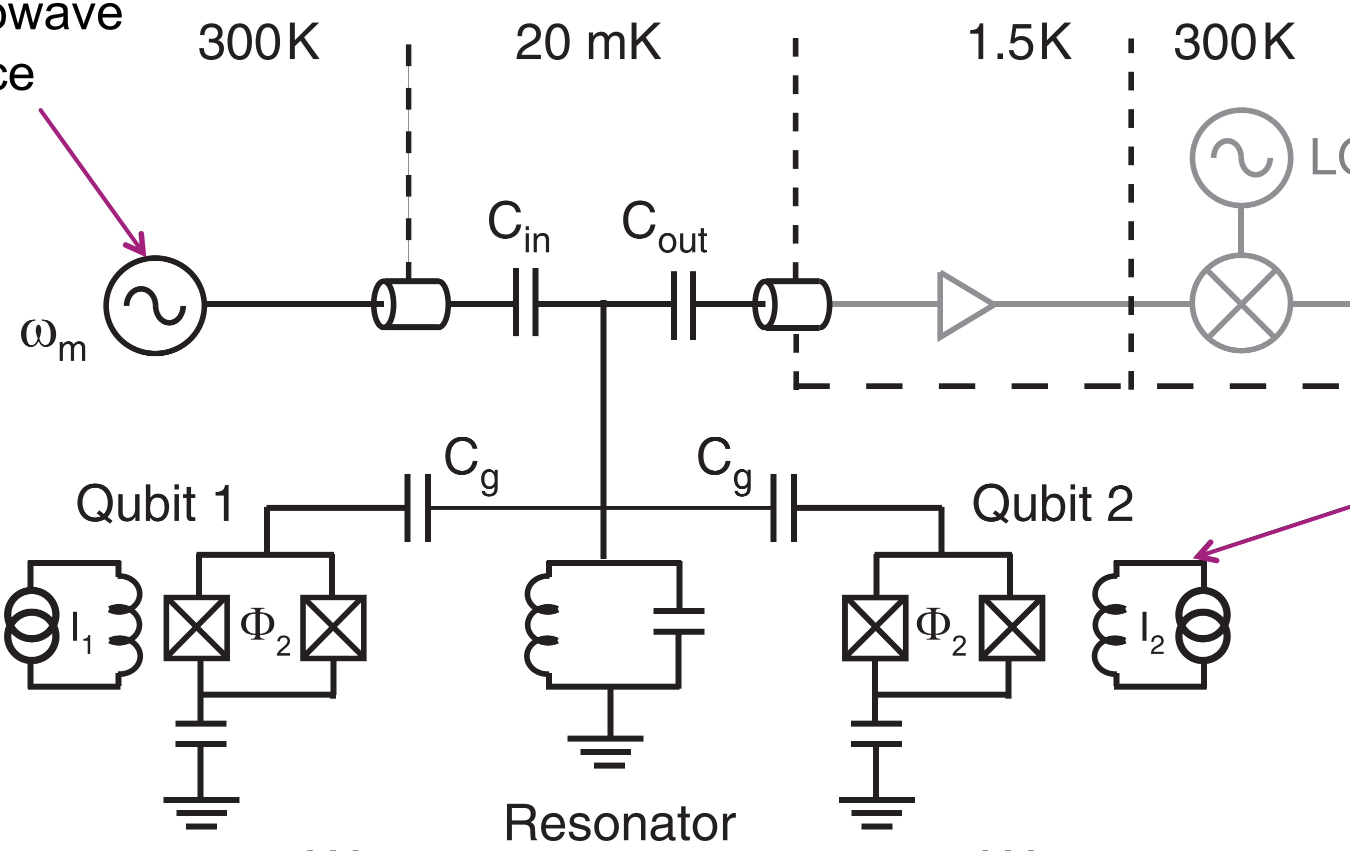


Average C-Phase gate fidelity: 99.4%

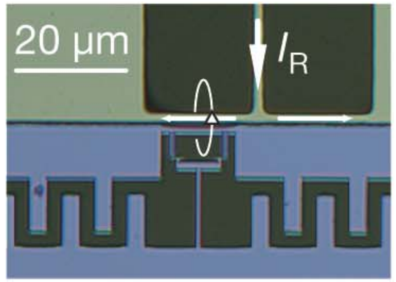
Barends 2014

The Superconducting Quantum Processor

Microwave source



Flux line or flux bias coils???



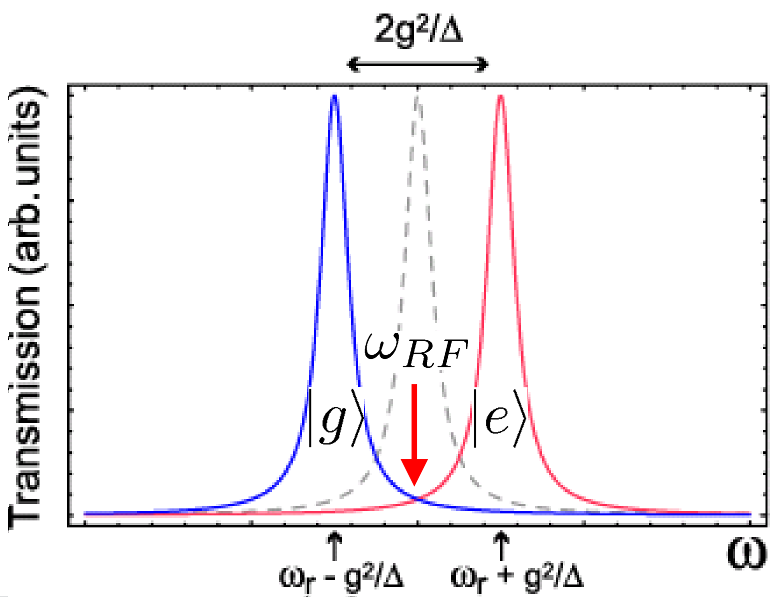
Filipp 2009
DiCarlo 2009

Reminder: Single-Qubit Readout

Hamiltonian of the system in dispersive limit

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) \left(a^\dagger a + \frac{1}{2} \right) - \hbar \frac{\omega_q}{2} \sigma_z$$

Shift of cavity frequency depending on qubit state



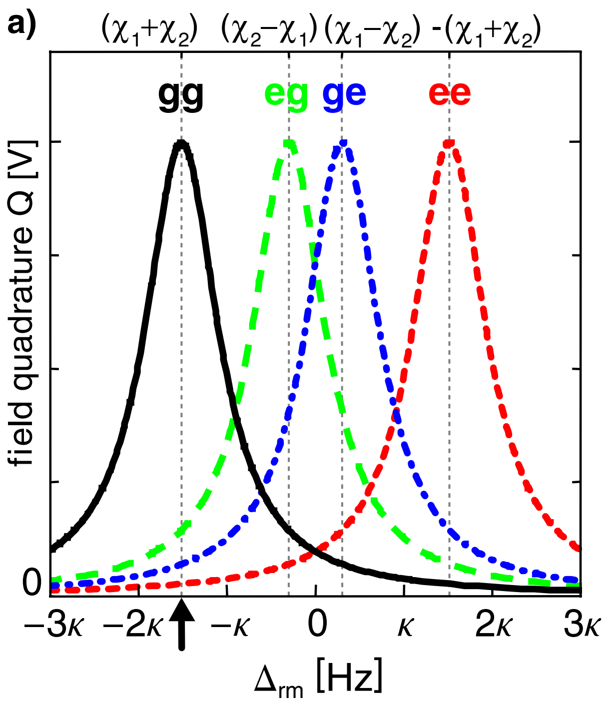
- State-dependent pull of cavity frequency by the qubit
- Apply measurement pulse to resonator
  - If we measure at omega_r +- shift, qubit state can be extracted from number of transmitted photons
  - If we measure at omega_r (bare resonator freq.), state encoded in phase of transmitted pulse (-> single-shot readout possible)

QSIT lecture slides 2014, Blais 2004

Joint Dispersive Readout of Qubits

➜ Detect correlations between qubit states

$$H \approx \hbar \boxed{\left(\Delta_{rm} + \chi_1 \sigma_{z1} + \chi_2 \sigma_{z2}\right)} a^\dagger a + \frac{\hbar}{2} \sum_{j=1,2} \left(\omega_{qj} + \chi_j\right) \sigma_{zj} + \hbar \epsilon(t) \left(a^\dagger + a\right)$$



- Each qubit induces a state-dependent dispersive shift

Filipp 2009