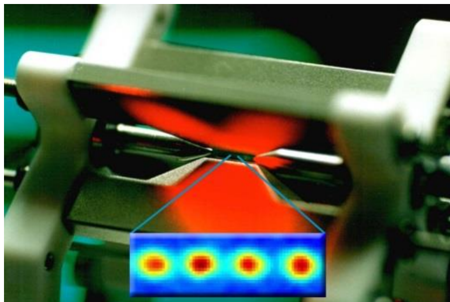
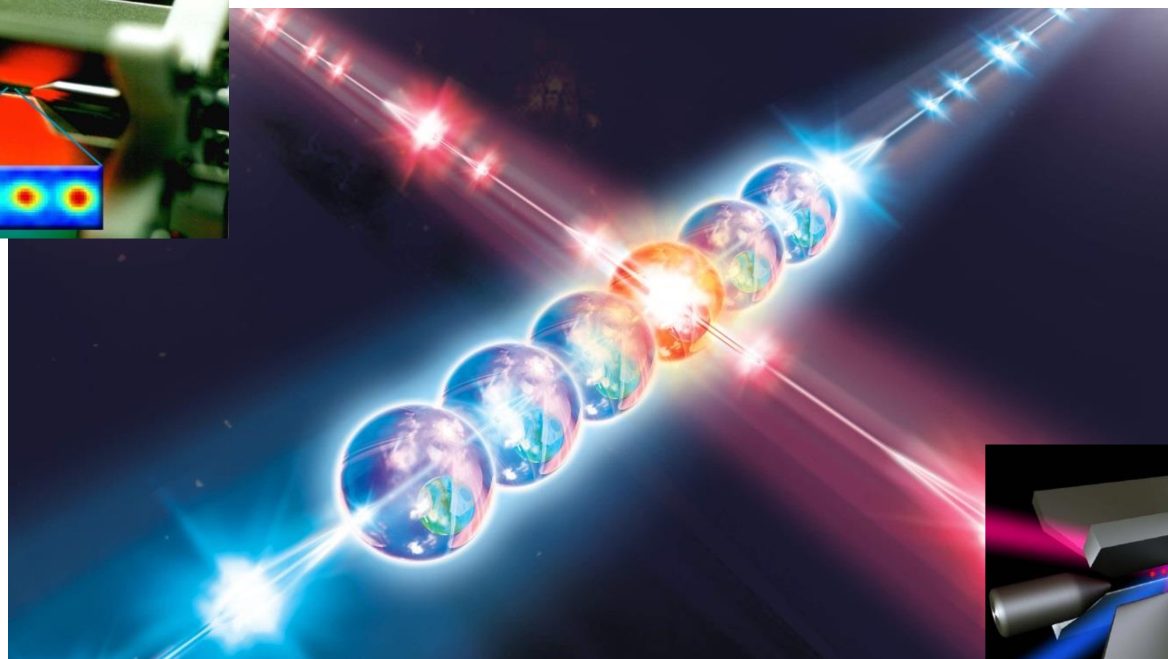
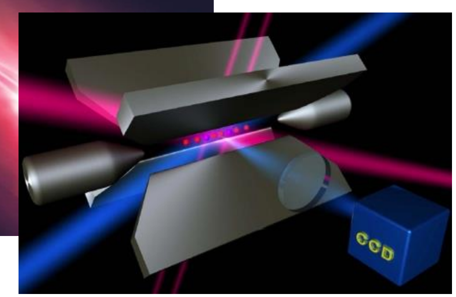
Quantum Simulation with Trapped Ions







Valentin Goblot

Antonio Rubio Abadal

Outline

- Quantum Simulation: Analog and Digital approaches
- Trapped ions for Digital Quantum Simulation
- Simulation of various spin systems
- Conclusion and References

Motivation for Quantum Simulation

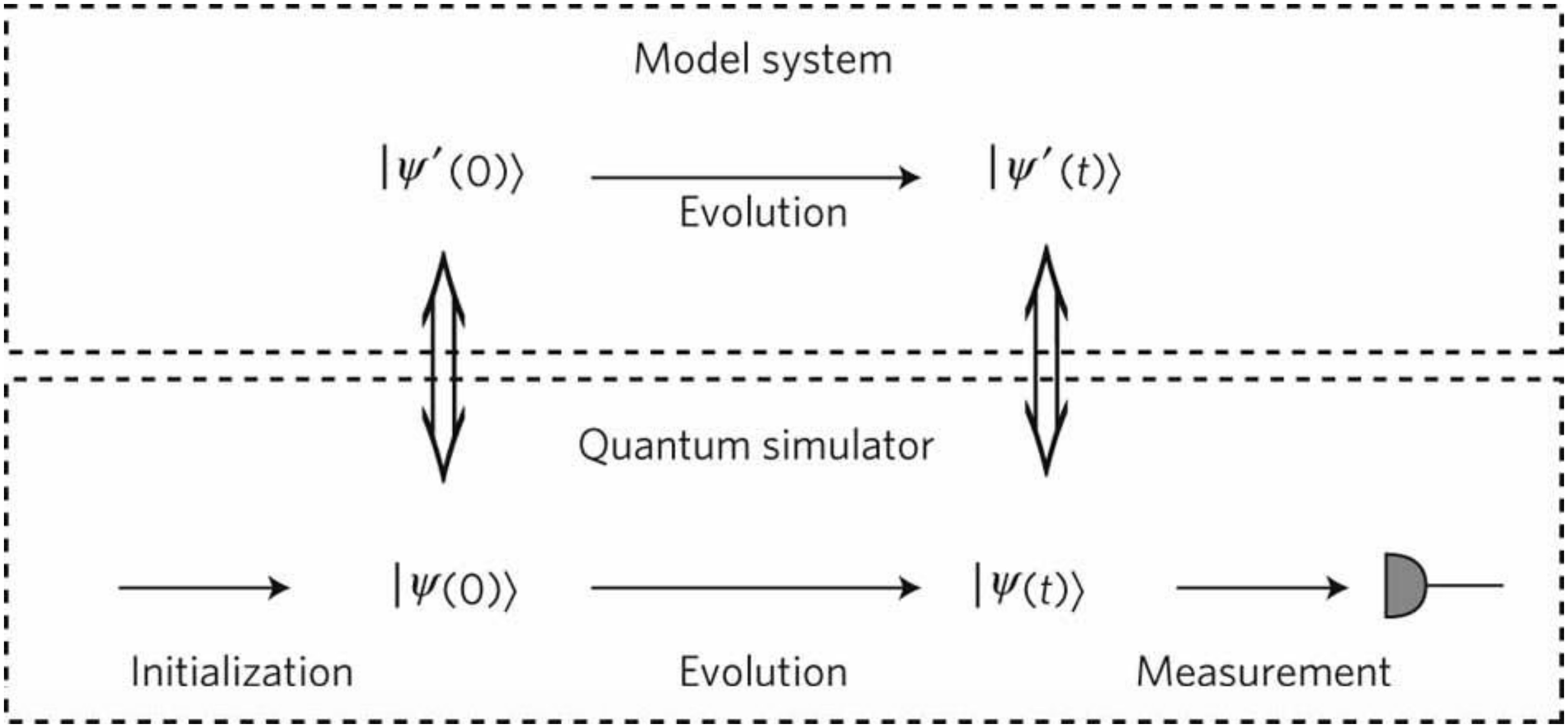
- Simulate quantum systems: difficult for the exponential growth of the Hilbert space
- R. Feynman (1982), Idea of a hypothetical universal quantum simulator: “quantum mechanical device for the efficient simulation of quantum systems”.
- S. Lloyd (1996), “A standard quantum computer can be programmed to be a Universal Quantum Simulator“.

Outline

- Quantum Simulation: Analog and Digital approaches
- Trapped ions for Digital Quantum Simulation
- Simulation of various spin systems

Quantum Simulation

- Quantum simulator: A controllable quantum system that can be used to simulate other quantum systems



Potential applications for QS

- Quantum Magnetism (Ising, XY)
- Quantum Phase Transitions (Superfluid/Mott Insulator)
- High-Tc Superconductors (Hubbard model)
- High energy physics (Dirac equation)

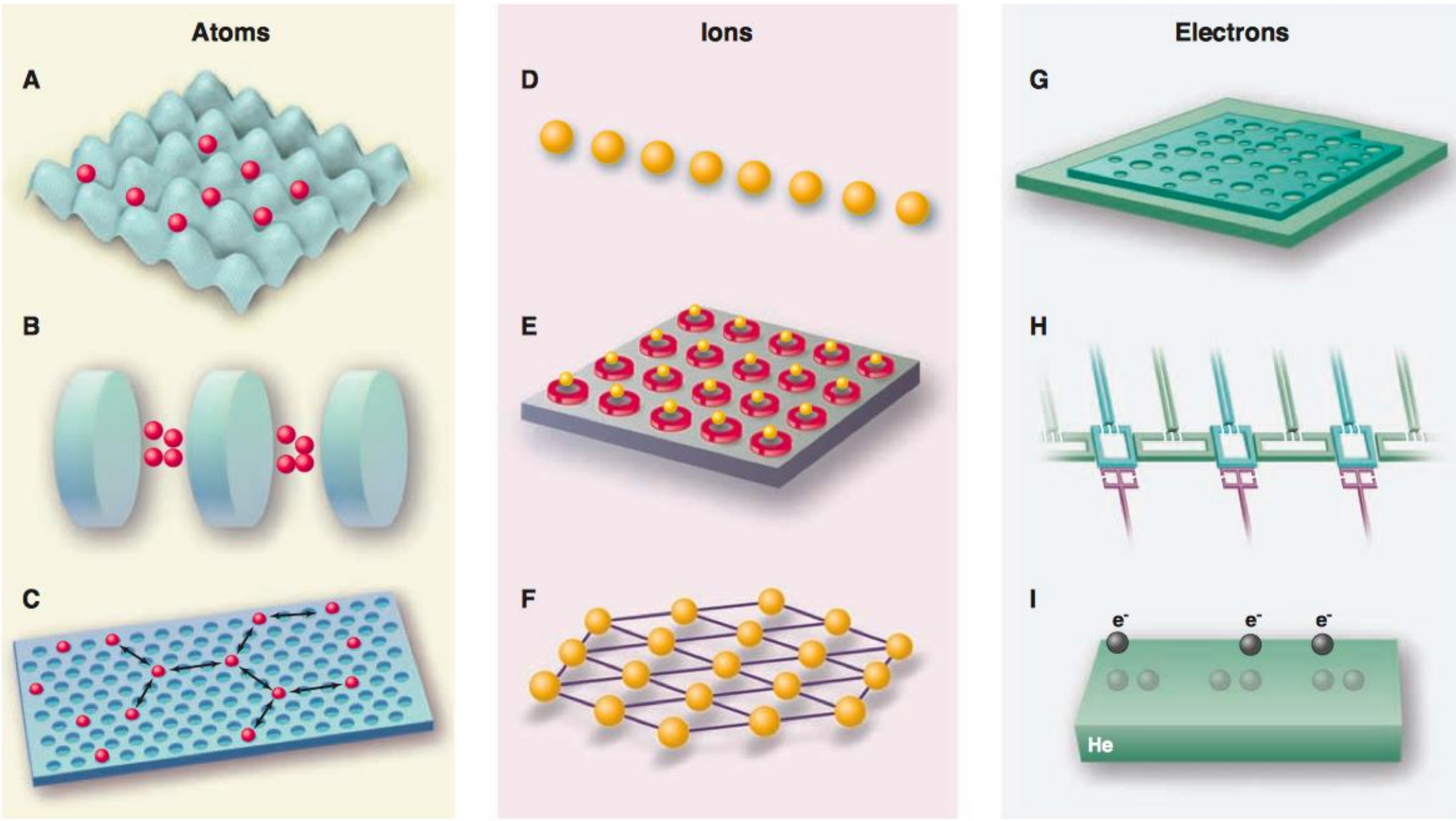
Potential applications for QS

High-energy physics:	
	Lattice gauge theories
	Dirac particles
Cosmology:	
	Hawking radiation
	Unruh effect
	Universe expansion
Atomic physics:	
	Cavity QED
	Cooling
Open systems:	
Chemistry:	
	Thermal rate calculations
	Molecular energies
	Chemical reactions
Quantum chaos:	
Other:	
	Schrödinger equation
	Quantum thermodynamics
	Nonlinear interferometers

Condensed matter physics:	
	Quantum phase transitions
	Hubbard models
	Spin models
	Spin glasses
	High Tc superconductivity
	BCS pairing
	BCS-BEC crossover
	Metamaterials
	Disordered systems
	Frustrated systems
	Tonks-Girardeau gas
	Anyons

I. Buluta, F. Nori, Science 326, 108 (2009).

Implementations of Quantum Simulation



I. Buluta, F. Nori, Science 326, 108 (2009).

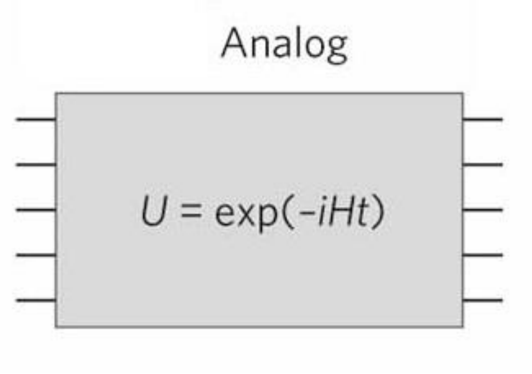
Quantum Simulation: Analog and digital

Quantum Simulation

→ Analog Quantum Simulation (AQS)

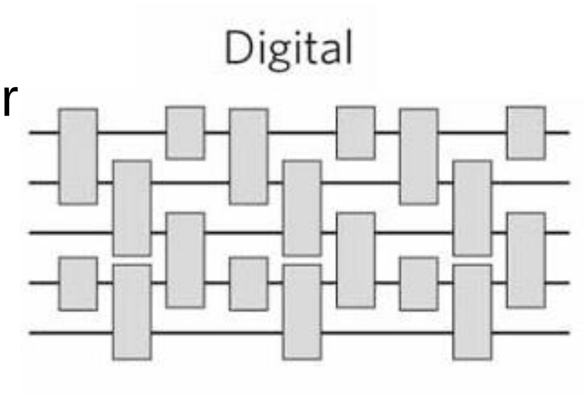
→ Digital Quantum Simulation (DQS)

Analog Quantum Simulation



- Mapping of the evolution of the simulated system onto the controlled evolution of the quantum simulator.
- The simulated system and the simulator can be physically quite different systems but mathematically equivalent.
- The Analog Quantum Simulator is a Quantum Emulator, a dedicated device to the particular problem.
- The simulator performs an exact computation

Digital Quantum Simulation



- The simulated state is encoded in a register of quantum information carriers. (programmable quantum computer)

- The interaction is written as the sum of many local interactions. (approximation by a stroboscopic sequence)

- Requires a set of universal quantum operations.

- A Digital Quantum Simulator is capable of simulating any other local system. (Universal Quantum Simulator)

DQS: The Trotter Approximation

Time-independent Hamiltonian: $$H_{sys} = \sum_{j=1}^{l} H_j$$

$$U = e^{-iH_{sys}t/\hbar} \neq \prod_{j=1}^{l} e^{-iH_j t/\hbar}$$ as in general $$\left[H_j, H_k\right] \neq 0$$

Trotter formula: $$e^{-iH_{sys}t/\hbar} = \lim_{n\to\infty}\left(\prod_{j=1}^{l} e^{-iH_j t/n\hbar}\right)^n$$

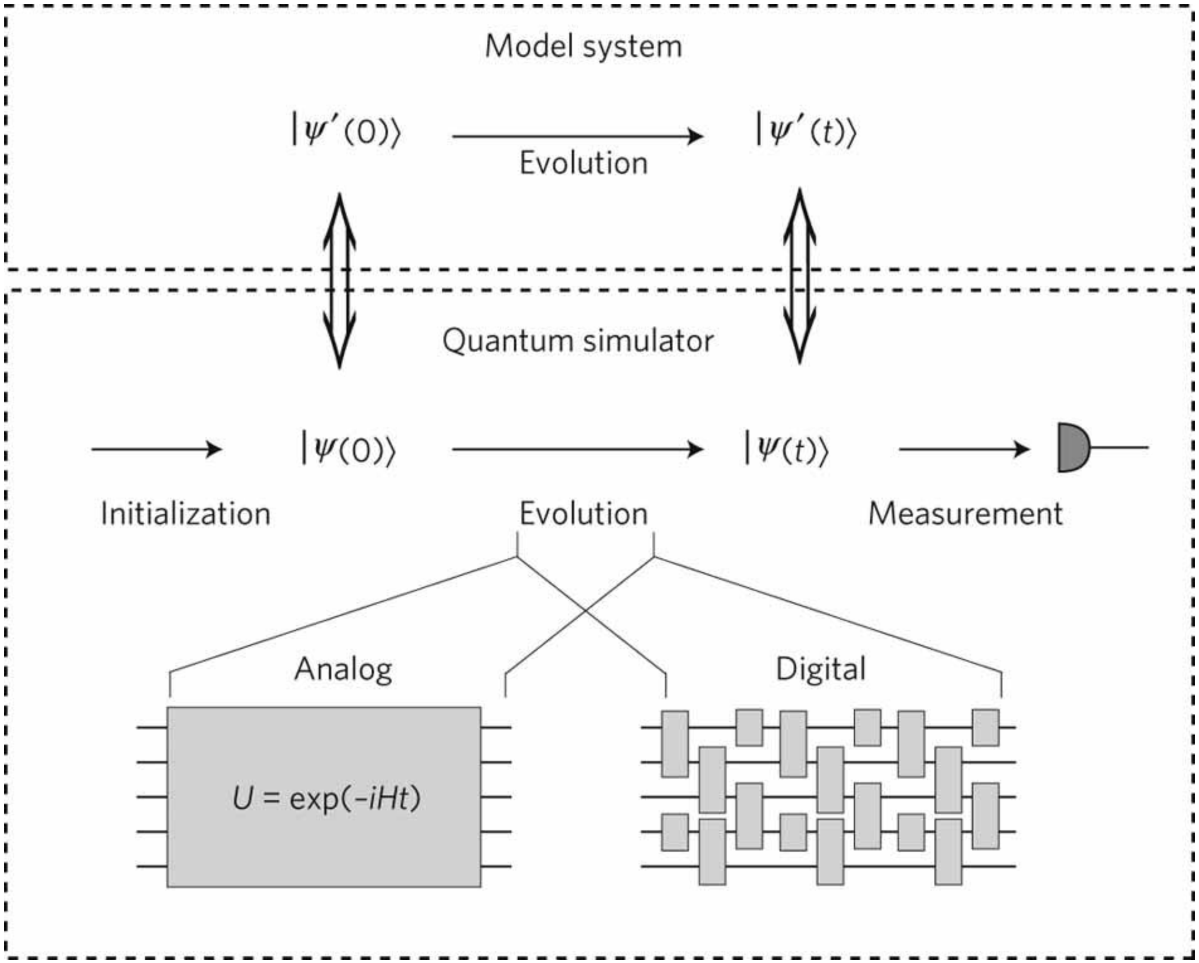
DQS: The Trotter Approximation

$$H_{sys} = A + B$$

For finite n: $$e^{-iH_{sys}t/\hbar} = \left(e^{-iAt/n\hbar} e^{-iBt/n\hbar}\right)^n + O(t^2/n)$$

Trotter formula: $$e^{-iH_{sys}t/\hbar} = \lim_{n\to\infty}\left(\prod_{j=1}^{l} e^{-iH_j t/n\hbar}\right)^n$$

Analog and Digital QS

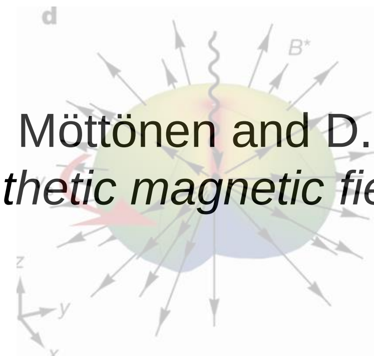


Analog and Digital QS

- A Digital Quantum Simulator (DQS) requires quantum error correction, and therefore is a scalable quantum computer.
- An Analog Quantum Simulator (AQS) doesn‘t need to be fault-tolerant.
- The DQS is a programmable quantum simulator, while the AQS is designed to behave analogously to a specific quantum system.

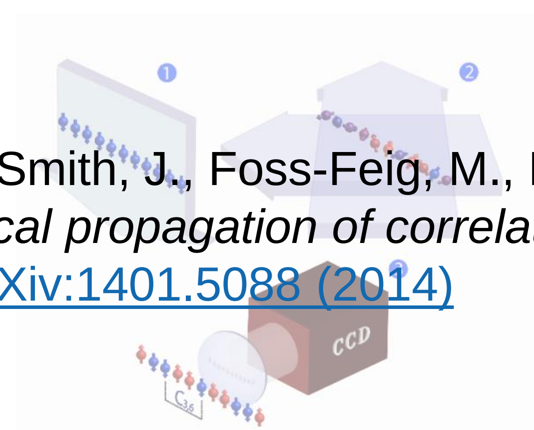
Examples of analog quantum simulation

Neutral atoms:



M. W. Ray, E. Ruokokoski, S. Kandel, M. Möttönen and D. S. Hall.
Observation of Dirac monopoles in a synthetic magnetic field
Nature **505**, 657–660 (2014)

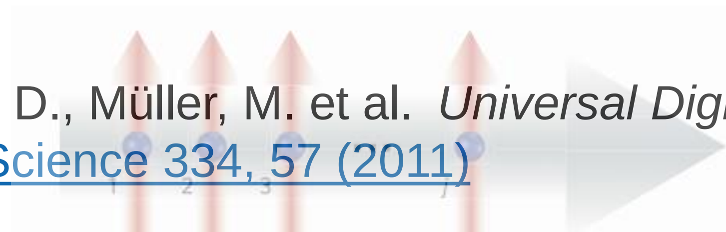
Trapped ions:



Richerme, P., Gong, Z., Lee, A., Senko, C., Smith, J., Foss-Feig, M., Michalakis, S., V. Gorshkov, A., and Monroe, C. *Non-local propagation of correlations in long-range interacting quantum systems* [ArXiv:1401.5088 (2014)]

Example of digital quantum simulation

<u>Trapped ions:</u>



Lanyon, B. P., Hempel, C., Nigg, D., Müller, M. et al. *Universal Digital Quantum Simulation with Trapped Ions.* [Science 334, 57 (2011)]

Outline

- Quantum Simulation: Analog and Digital approaches
- Trapped ions for Digital Quantum Simulation
- Simulation of various spin systems

Trapped ions for digital quantum simulation

Universal Digital Quantum Simulation with Trapped Ions
B. P. Lanyon *et al.*
Science **334**, 57 (2011);
DOI: 10.1126/science.1208001

Model:
Ising, XY and XYZ Hamiltonian

DQS ⟷

Simulator:
String of trapped ions

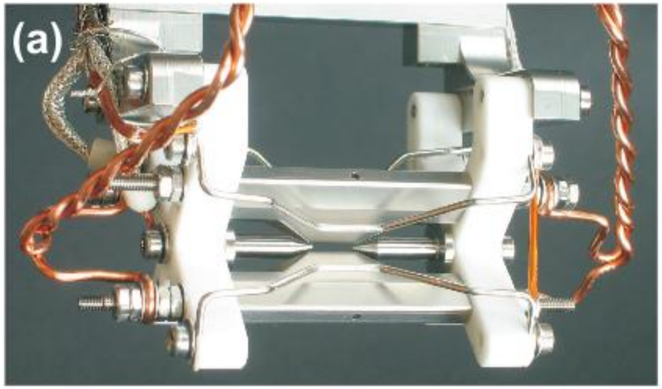
Error correction demonstrated:

P. Schindler et al., **Experimental Repetitive Quantum Error Correction.** Science 332, 1059 (2011)

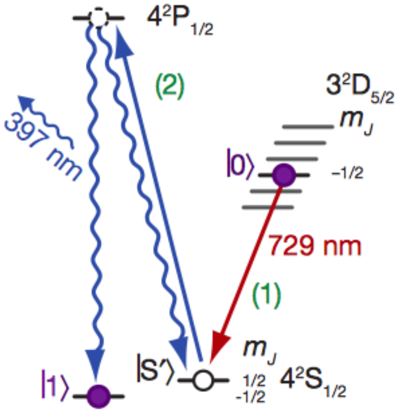
Description of the system

String of $^{40}Ca^{+}$ ions

Linear Paul trap



- $|\uparrow\rangle = |D_{5/2}\rangle$,
- $|\downarrow\rangle = |S_{1/2}\rangle$



Operation set

- First goal: simulate two-spin Ising model

$$H = B({\sigma_z}^1 + {\sigma_z}^2) + J{\sigma_x}^1{\sigma_x}^2$$

- Decomposition of the Hamiltonian:

$$H = B(\sigma_z^1 + \sigma_z^2) + J\sigma_x^1\sigma_x^2 \quad = H_1 + H_2$$

- Decomposition of evolution in many small time steps:

$$e^{-iHt/\hbar} = \lim_{n\to\infty}\left(\prod_k e^{-iH_k t/n\hbar}\right)^n$$

- We need gates to simulate evolution operators of the form:

$$U_1 = e^{-i\theta({\sigma_z}^1 + {\sigma_z}^2)} \qquad U_2 = e^{-i\theta{\sigma_x}^1{\sigma_x}^2} \qquad \theta = E\frac{t}{\hbar} \qquad H = E\tilde{H}$$

Operation set

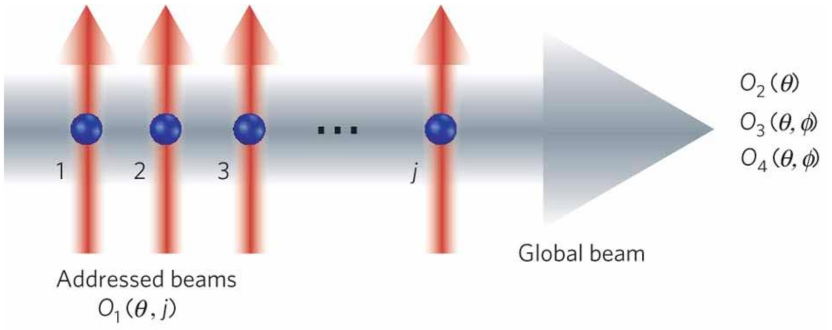
- The following toolbox is chosen:

$$\begin{cases} O_1(\theta, j) = e^{-i\theta\sigma_z{}^j} \\ O_2(\theta) = e^{-i\theta\ \sum\sigma_z{}^i} \\ O_3(\theta, \varphi) = e^{-i\theta\ \sum\sigma_\varphi{}^i} \\ O_4(\theta, \varphi) = e^{-i\sum_{i<j}\sigma_\varphi{}^i\sigma_\varphi{}^j} \end{cases}$$

$$\sigma_\varphi{}^i = \cos\varphi\,\sigma_x{}^i + \sin\varphi\,\sigma_y{}^i$$

Mølmer–Sørensen gate

- Gates implementation via laser pulses:



Outline

- Quantum Simulation: Analog and Digital approaches
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- Simulation of various spin systems

Simulation: two-spin Ising model

$$H = B\left(\sigma_z^1 + \sigma_z^2\right) + J\sigma_x^1\sigma_x^2$$

- Single digital step represented as gates:

- Here,

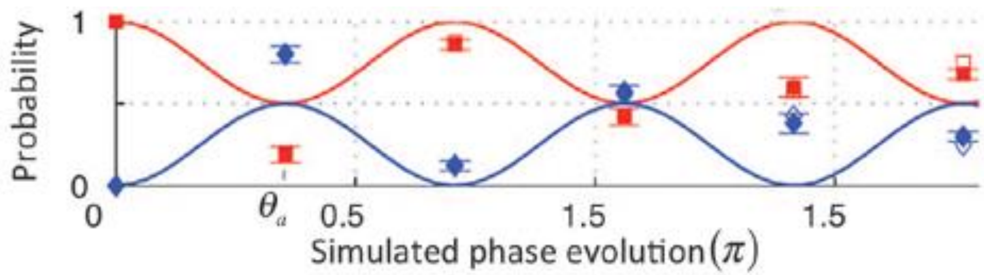
$$C = O_2(\theta_c) = e^{-i\theta_c\left({\sigma_z}^1 + {\sigma_z}^2\right)} \qquad D = O_4(\theta_d, 0) = e^{-i\theta_d {\sigma_x}^1 {\sigma_x}^2}$$

- For simulation, following parameters are used: $J = 2B$

$$C = O_2\left(\frac{\theta_a}{2n}\right) \qquad D = O_4\left(\frac{\theta_a}{n}, 0\right) \qquad \theta_a = \frac{\pi}{2\sqrt{2}}$$

Simulation: two-spin Ising model

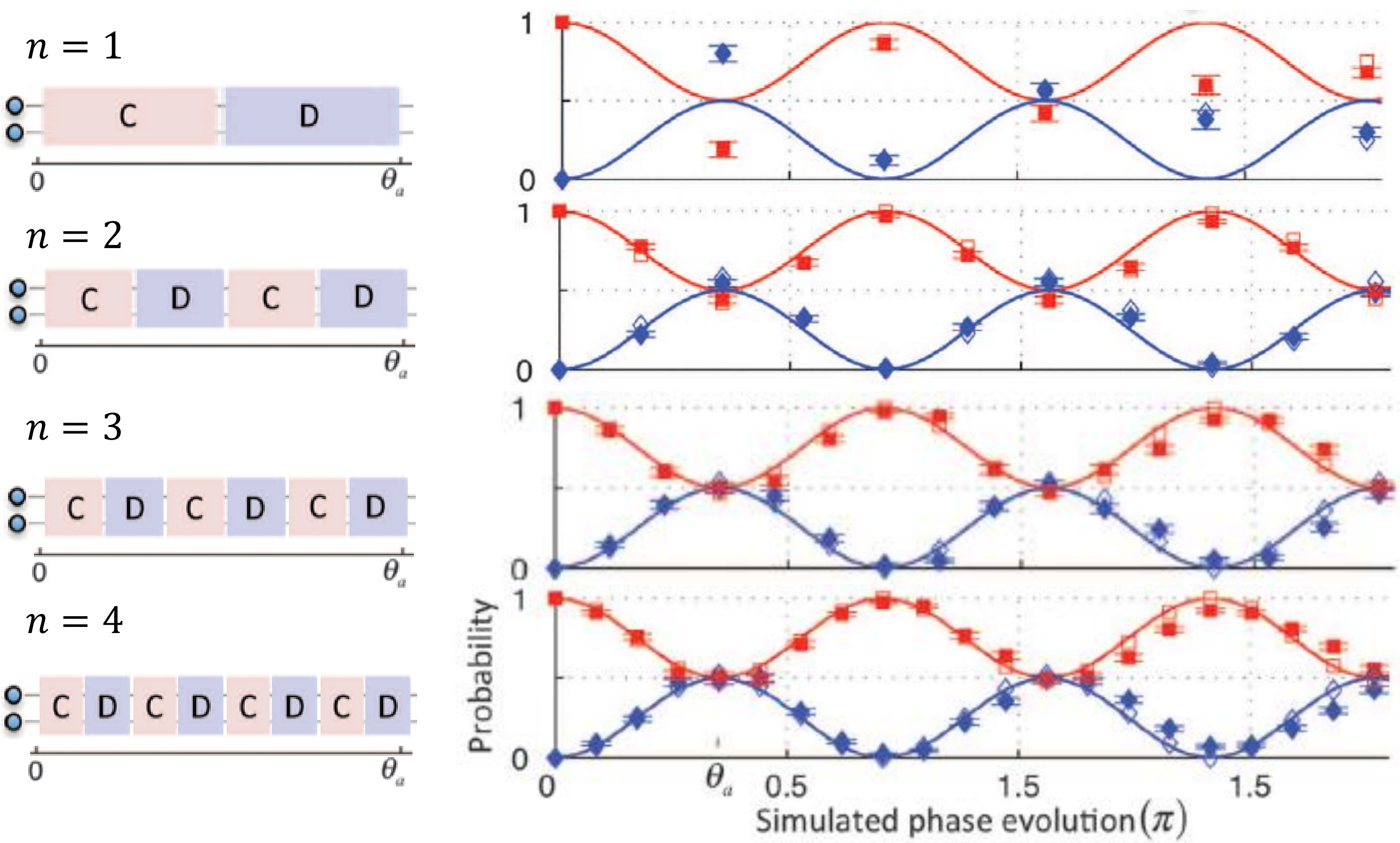
$n = 1$



Fidelity: 61%

- $|\downarrow\downarrow\rangle$
- $|\uparrow\uparrow\rangle$

Simulation: two-spin Ising model



Fidelity: 61%

◆ $|\downarrow\downarrow\rangle$

■ $|\uparrow\uparrow\rangle$

Fidelity: 91%

Simulations: Ising, XY and XYZ models

- Ising model: $H = B(\sigma_z^1 + \sigma_z^2) + J_{xx}\sigma_x^1\sigma_x^2$ $\quad J_{xx} = B$

$$C = O_2\left(\frac{\pi}{16}\right) = e^{-i\frac{\pi}{16}(\sigma_z{}^1 + \sigma_z{}^2)} \qquad D = O_4\left(\frac{\pi}{16}, 0\right) = e^{-i\frac{\pi}{16}\sigma_x{}^1\sigma_x{}^2}$$

- XY: $H = B(\sigma_z^1 + \sigma_z^2) + J_{xx}\sigma_x^1\sigma_x^2 + J_{yy}\sigma_y^1\sigma_y^2$ $\quad J_{xx} = 2J_{yy} = 2B$

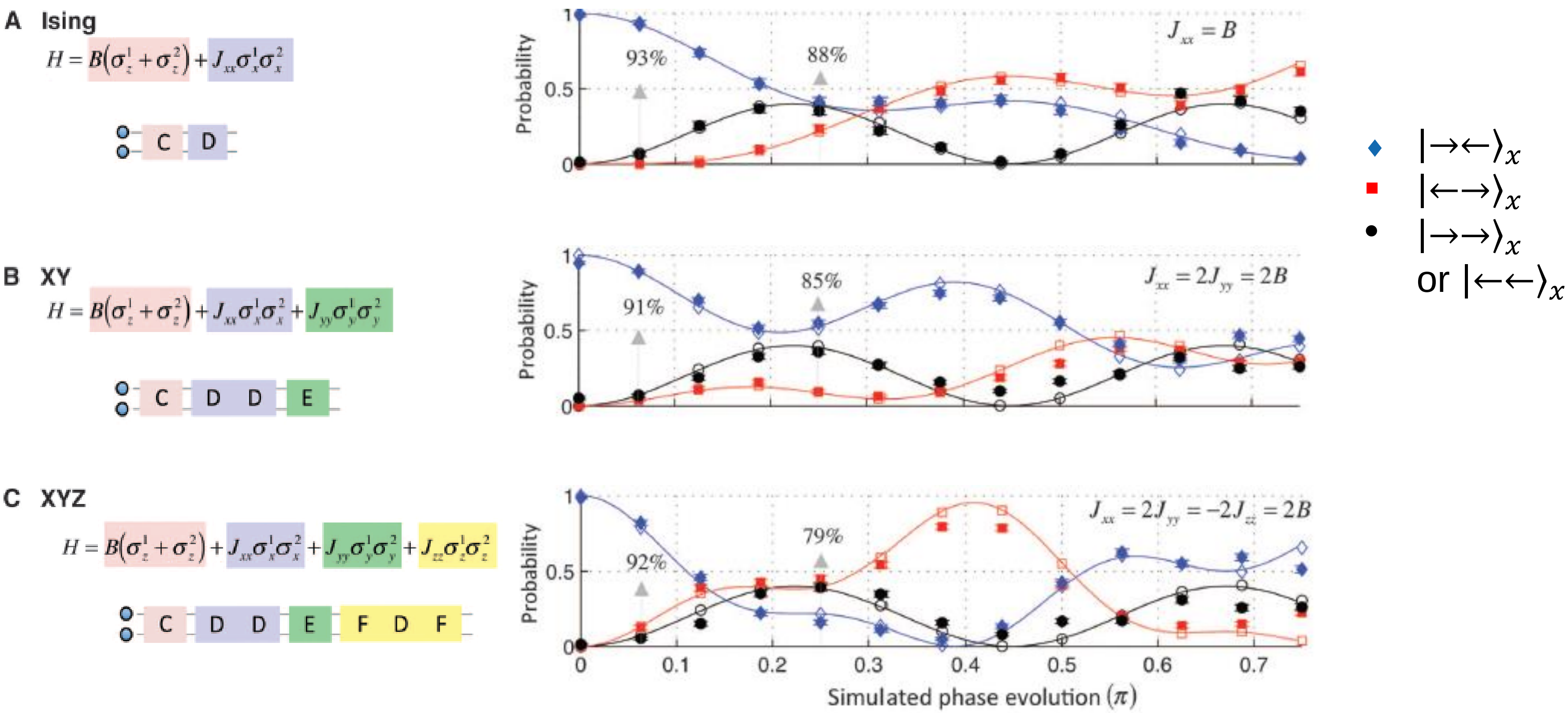
$$E = O_4\left(\frac{\pi}{16}, \frac{\pi}{2}\right) = e^{-i\frac{\pi}{16}\sigma_y{}^1\sigma_y{}^2}$$

- XYZ: $H = B(\sigma_z^1 + \sigma_z^2) + J_{xx}\sigma_x^1\sigma_x^2 + J_{yy}\sigma_y^1\sigma_y^2 + J_{zz}\sigma_z^1\sigma_z^2$ $\quad J_{xx} = 2J_{yy} = 2J_{zz} = 2B$

$$F = O_3\left(\frac{\pi}{4}, 0\right) = e^{-i\frac{\pi}{4}(\sigma_x{}^1 + \sigma_x{}^2)}$$

Simulations: Ising, XY and XYZ models

- Simulations with 12 time steps: respectively 24, 48 and 84 gates

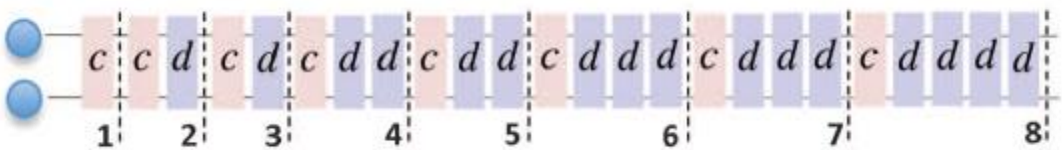


Simulation: time-dependant Ising model

- Linear evolution of coupling from *0* to *4B* :

$$H = B(\sigma_z{}^1 + \sigma_z{}^2) + J(t)\sigma_x{}^1\sigma_x{}^2$$

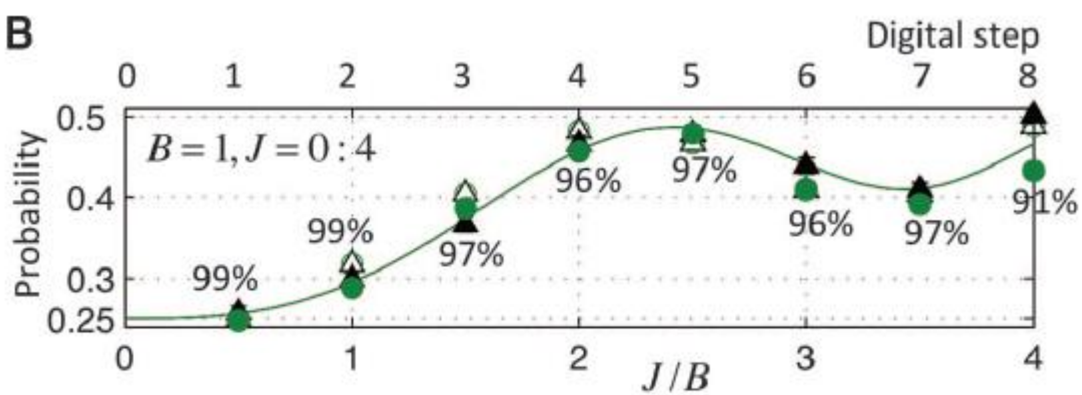
- Sequence of gates different at each time step



- Result: adiabatic evolution to the antiferromagnetic ground state

Initial state

$|\uparrow\uparrow\rangle$



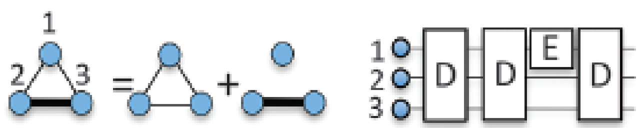
Final state

$|\rightarrow\leftarrow\rangle_x + |\leftarrow\rightarrow\rangle_x$

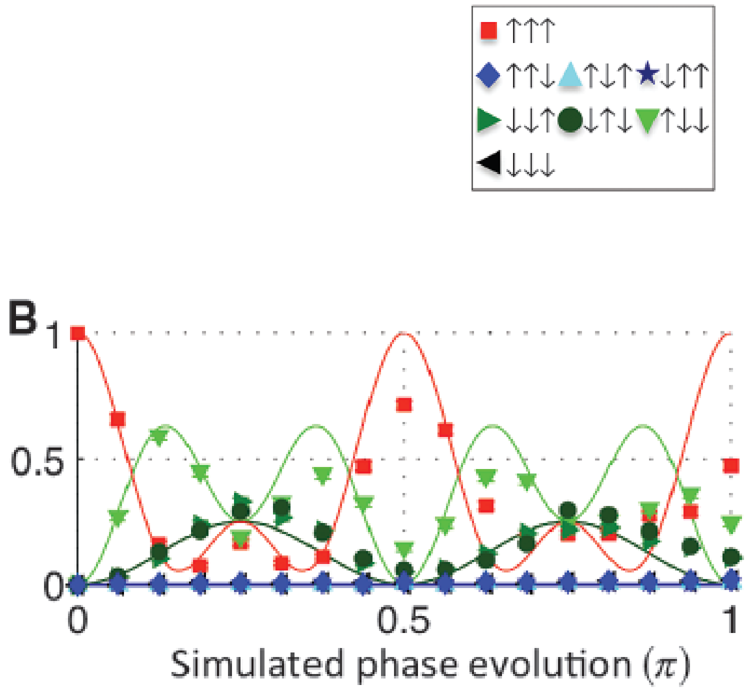
Simulations: three-spin Ising model

- Long range Ising model successfully simulated
- Inhomogeneous spin-spin couplings

$$H = \sum_{i<j} J_{ij}\sigma_x^i\sigma_x^j \qquad J_{23} = 3J_{12} = 3J_{13}$$

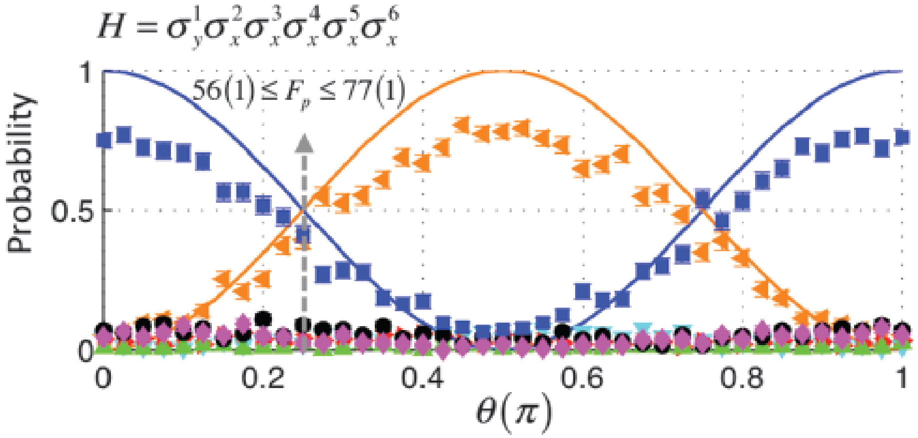


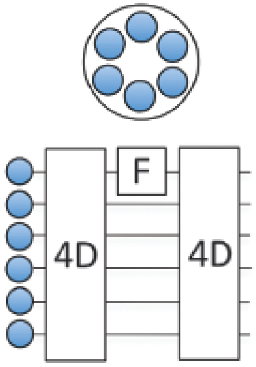
$$E = O_1\left(\frac{\pi}{2}, 1\right)$$ π-pulse



Simulations: many-body interactions

- Biggest simulation: six-body interaction





Conclusion

- Trapped ions constitute a system well suited to Digital Quantum Simulation
- Various types of interactions, for two spins and more, were accurately reproduced
- With current ion trap development and including error correction, a full-scale device seems within reach
- At present times, experimental realization of Analog Quantum Simulation is still easier for many systems

Literature

- I. Buluta, F. Nori, *Science* **326**, 108 (2009)
  Quantum Simulators

- R. Blatt, C. F. Roos, *Nature Physics* **8**, 277 (2012)
  Quantum simulations with trapped ions

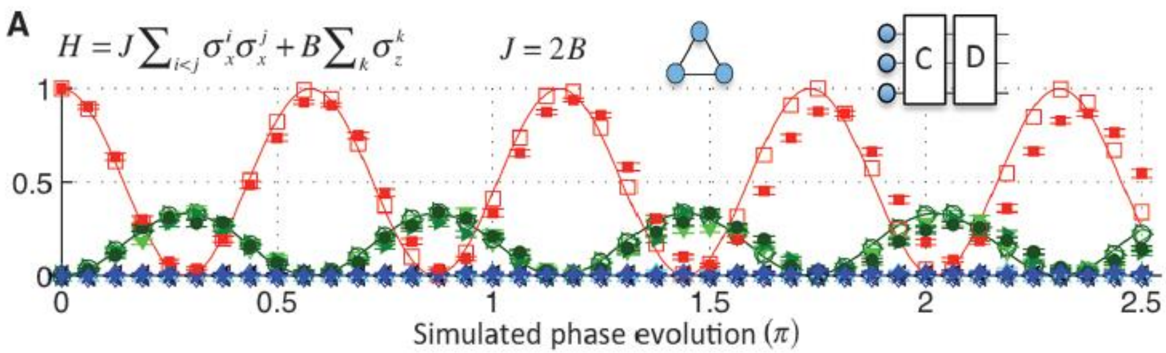
- G. Brumfiel, *Nature* **491**, 322 (2012)
  Simulation: Quantum leaps

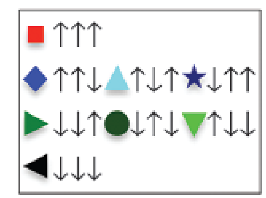
- J. Benhelm, G. Kirchmair, C. F. Roos, R. Blatt, *Nature Physics* **4**, 463 (2008)
  Towards fault-tolerant quantum computing with trapped ions

Thank you!

Simulations: more three-spin Hamiltonians

- Long range Ising model





- Three-body interaction