QSIT 2013 - Questions 1

1. March 2013, HIT F 13

1. **State Space in Quantum Mechanics** What is the relevant Hilbert space that represents the dynamics of the following systems. Write down their basis states.

   (a) A neutron in a static magetic field.

   (b) A neutron in a magnetic gradient field.

   (c) A small mirror attached to a spring.

   (d) A small mirror attached to a spring exposed to laser radiation .

   (e) A ground-state hydrogen atom at room temperature.

   (f) A ground-state hydrogen atom exposed to laser radiation at a wavelength of 121 nm.

   (g) An ensemble of N hydrogen atoms in the ground state at room temperature.

2. **Bloch Sphere** Any quantum state of a spin-1/2 (or two-level system) can be represented on the *Bloch sphere*. Calculate the polar and azimuthal angles of the following states and draw the states on the Bloch sphere.

   (a) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}|1\rangle\right)$

   (b) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle - i\sqrt{2}|1\rangle\right)$

   (c) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(\sqrt{2}|0\rangle - i2|1\rangle\right)$

   (d) $|\psi\rangle = \frac{e^{i\pi/4}}{\sqrt{3}}\left(|0\rangle - i\sqrt{2}|1\rangle\right)$

3. **The no-cloning theorem**

   Prove that exact copying of an arbitrary state is not possible.
   Hint: Suppose that using a unitary operator U you can copy state $|\psi\rangle$ to $|\phi\rangle$, i.e. $U(|\psi\rangle \otimes |\phi\rangle) = |\psi\rangle \otimes |\psi\rangle$. Using unitary properties of operator U, show that copying of an arbitrary state is not possible.

4. **Rabi Oscillations**

   A spin-1/2 particle is placed in a magetic field of magnitude B_z pointing in the z-direction . At time t_0 an additional field B_x is applied in the x-direction. Calculate the expected excited state population as a function of time and draw a diagram. Assume that $B_x \gg B_z$ and that the particle is initially in its ground state. What changes, if the additional magnetic field points in the y-direction instead?