QSIT 2010 - Questions 2

15. March 2013, HIT F 13

1. **State preparation**
   Any single qubit state can be prepared by applying a sequence of unitary operations onto the inital state. Assuming that the system is initially in its ground state, $|\psi_i\rangle = |0\rangle$, determine the unitary matrix (sequence) that results in the following final states:

   (a) $|\psi_f\rangle = |1\rangle$
   (b) $|\psi_f\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$
   (c) $|\psi_f\rangle = \sin\frac{3\pi}{8}|1\rangle - \cos\frac{3\pi}{8}|0\rangle$
   (d) $|\psi_f\rangle = e^{i\pi/4}\sin\frac{3\pi}{8}|1\rangle - \cos\frac{3\pi}{8}|0\rangle$

2. **Quantum State Tomography.**

   To determine the state of a N-level quantum system a specific number of measurements have to be performed on identically prepared systems. From the results of such a complete set of measurements the state can then be fully characterized.

   (a) How many measurements do you need to determine the quantum state of the system?
   (b) Write down explicitly, what measurements can be used and how you can infer the state from the results of these measurements.
   (c) How is the number of required measurements related to the normalization of the state? What does it mean, if the state is found to be not normalized?
   (d) Which measurements are required to characterize a state of two qubits?

3. **Density matrix of a qubit entangled with another one**

The density operator formalism is used to describe a quantum system whose state is not completely known. Suppose a quantum system is in state $|\psi_i\rangle$ with respective probability p_i. The density operator for the system is defined as

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|.$$

Let us consider a system of two qubits, which is described by $|\psi_{\mathrm{AB}}\rangle$and let $\hat{O}$ be an observable of the qubit A. Then its expectation value is described by

$$\langle O \rangle = \mathrm{tr}[\rho_{\mathrm{A}}\hat{\mathrm{O}}],$$

where $\rho_{\mathrm{A}} = \mathrm{tr}_{\mathrm{B}}[\rho_{\mathrm{AB}}]$ is the reduced density operator of qubit A. For maximally entangled states such as the Bell states, ρ_{A} describes a maximally mixed state.

(a) Suppose that the system is in state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}} + |1\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}})$. What is the state of qubit A ignoring the state of qubit B?

(b) What is are the expectation values for σ_x^{A}, σ_y^{A}, σ_z^{A} for the $|\Psi^+\rangle$ state?