QSIT 2013 - Questions 3

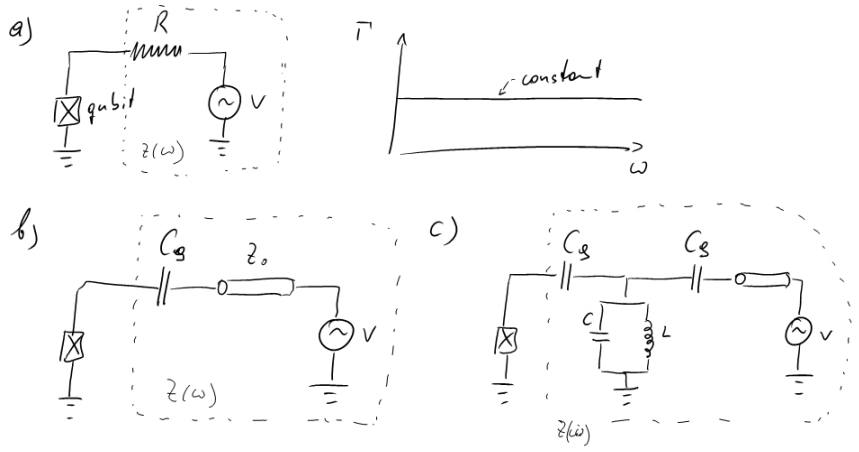
22. March 2013, HIT F 13

1. **Energy relaxation of a qubit**
   In analogy to the harmonic oscillator, the energy decay time of a qubit is given by $T_1 = RC$, where C is the intrinsic capacitance of the qubit. R denotes the effective resistance $R = 1/\mathrm{Re}[Y(\omega)]$ obtained from the impedance of the environment $Z(\omega) = 1/Y(\omega)$ as seen from the position of the qubit.

   If the impedance of the environment is purely resistive, e.g. $Z(\omega) = 50\ \Omega$, the decay rate $\Gamma = 1/T_1$ is frequency independent (see Figure a).

   (a) Derive the impedance of a Cooper-pair box qubit that is capacitively coupled to a transmission line ($Z_0 = 50\ \Omega$) via a gate capacitance C_g (Figure b). Sketch the decay rate Γ as a function of frequency.

   (b) What is the spectral shape of Γ for a coupling to an LC oscillator (Figure c)?



2. **Superconducting loop**
SQUID stands for Superconducting Quantum Interference Device and consists of a superconducting loop intersected by two Josephson junctions as shown in the figure below. In general the junction is described the its phase ϕ which depends on the supercurrent flowing through it according to

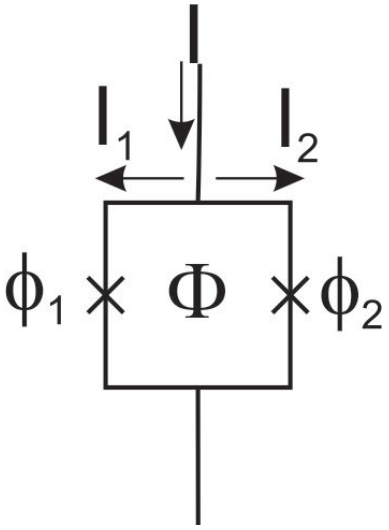
$$I = I_c \sin \phi, \tag{1}$$

where I_c is the critical current of the junction. In addition to Eq. (1) we can relate magnetic flux Φ through the loop and all currents by Kirchhoff rules:

$$I_2 + I_1 = I, \tag{2}$$

$$\Phi + \frac{\Phi_0}{2\pi}(\phi_1 - \phi_2) = 0 \tag{3}$$

(a) Show that the total current I is related to the average phase $\phi_p = (\phi_1 + \phi_2)/2$ by the current-phase relationship similar to (1) with the critical current modulated by the magnetic flux through the loop $I_c(\Phi)$. Assume both junctions to be identical.



3. **Quantum search algorithm (Grover's algorithm)**

You are given a search space with $N = 16$ elements, and the element you are looking is encoded in the state $|5\rangle$.

(a) How many qubits do you need to encode the whole search space?

(b) Construct the needed oracle.

(c) How many Grover iterations are needed to find the needed element?