Adiabatic Quantum Computation
An alternative approach to a quantum computer

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Literature:

E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, D. Preda
A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem.
e-print quant-ph/0104129; Science 292, 472 2001.

Jèrèmie Roland and Nicolas J. Cerf
Quantum search by local adiabatic evolution
Physical Review A, Volume 65, 042308

Idea

Alternative approach to quantum computation.

- Encode problem in a constructed Hamiltonian.

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- Encode problem in a constructed Hamiltonian.
- Encode solution in ground state of this Hamiltonian.

Spin glass

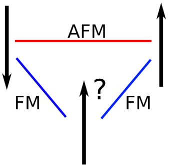
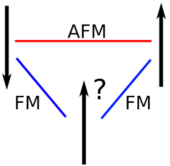


Figure: Three frustrated spins with ferromagnetic and anti-ferromagnetic coupling.

Spin glass



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- Frustrations lead to many local minima.

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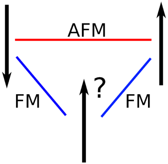

Figure: Three frustrated spins with ferromagnetic and anti-ferromagnetic coupling.

- Frustrations lead to many local minima.
- Minima are separated by large potential walls.

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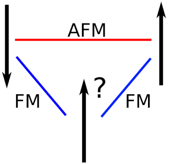

Figure: Three frustrated spins with ferromagnetic and anti-ferromagnetic coupling.

- Frustrations lead to many local minima.
- Minima are separated by large potential walls.
- Ground state not reachable by cooling.

Adiabatic Theorem

Theorem

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- A system in the ground state remains in the ground state.
- The perturbation does not have to be small.
- Can switch Hamiltonian: $H(t) = (1 - \frac{t}{T})H_0 + \frac{t}{T}H_P$

Runtime of an adiabatic algorithm

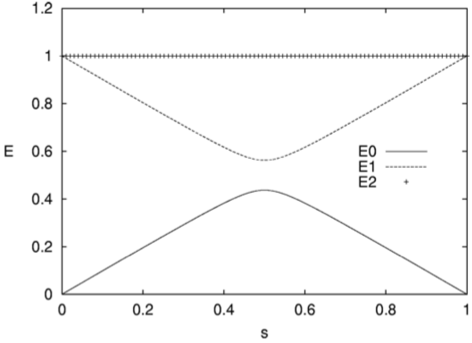

Figure: Eigenvalues of the time-dependent Hamiltonian. E_0 and E_1 avoid crossing each other.

Runtime of an adiabatic algorithm

For probability $1 - \epsilon^2$ of remaining in the ground state:

$$g_{\min} = \min_{0 \le t \le T} [E_1(t) - E_0(t)] \tag{1}$$

$$D_{\max} = \max_{0 \le t \le T} |\langle E_1; t| \frac{dH}{dt} |E_0; t\rangle| \tag{2}$$

Condition for the adiabatic Theorem:

$$\frac{D_{\max}}{g_{\min}^2} \le \epsilon \tag{3}$$

Exact Cover

String of n bits $z_1, z_2 ... z_n$ satisfying a set of clauses of the form $z_i + z_j + z_k = 1$.
Determining a string satisfying all clauses involves checking all 2^n assignments, and is a NP-complete problem.

Define energy for a clause:

$$h_C(z_{i_C}, z_{j_C}, z_{k_C}) = \begin{cases} 0 & \text{if } z_{i_C} + z_{j_C} + z_{k_C} = 1 \\ 1 & \text{if } z_{i_C} + z_{j_C} + z_{k_C} \neq 1 \end{cases} \tag{4}$$

Define total energy:

$$h = \sum_C H_C \tag{5}$$

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The energy $h \geq 0$ and $h(z_1, z_2, ...z_n) = 0$ only if the string satisfies all clauses.

Problem Hamiltonian

Use spin-$\frac{1}{2}$ qubits labeled by $|z_1\rangle$ where $z_1 = 0, 1$.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

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Define operator corresponding to clause C

$$H_{P,C}(|z_1\rangle ... |z_n\rangle) = h_C(z_{i_C}, z_{j_C}, z_{k_C})|z_1\rangle ... |z_n\rangle \tag{7}$$

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Problem Hamiltonian is given by

$$H_P = \sum_C H_{P,C} \tag{8}$$

Initial Hamiltonian

Use x basis for initial state

$$|x_i = 0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \text{ and } |x_i = 1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \tag{9}$$

Define operator

$$H_B^{(i)}|x_i = x\rangle = \frac{1}{2}(1 - \sigma_x^{(i)})|x_i = x\rangle = x|x_i = x\rangle \tag{10}$$

Initial Hamiltonian is given by

$$H_P = \sum_{i=1}^{n} d_i H_B^{(i)} \tag{11}$$

where d_i is the number of clauses involving bit i.
The ground state is given by

$$|x_1 = 0\rangle ... |x_n = 0\rangle = \frac{1}{2^{n/2}}(|z_1 = 0\rangle + |z_1 = 1\rangle)...(|z_n = 0\rangle + |z_n = 1\rangle) \tag{12}$$

Runtime for probability 1/8

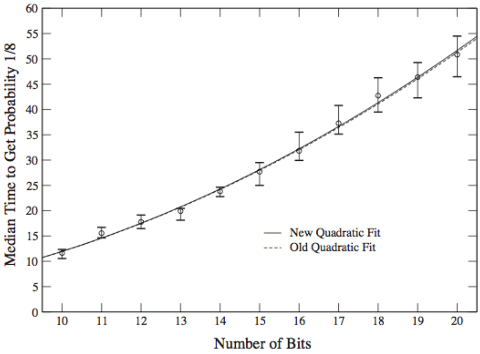

Figure: Median time to achieve success probability of 1/8 for different sized problems.

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- Encode solution in ground state of a Hamiltonian.
- Adiabatic theorem provides way to reach the ground state.