

Error Correction in Superconducting Circuits

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Motivation

- Decoherence/Environmental influences
- Dephasing
- Gate operations and quantum computing need time

Outline

- Theory
 - Classical error correction
 - Quantum error correction
 - Error suppression factor Λ
- Experiment
 - Setup
 - Measurement procedure
- Results

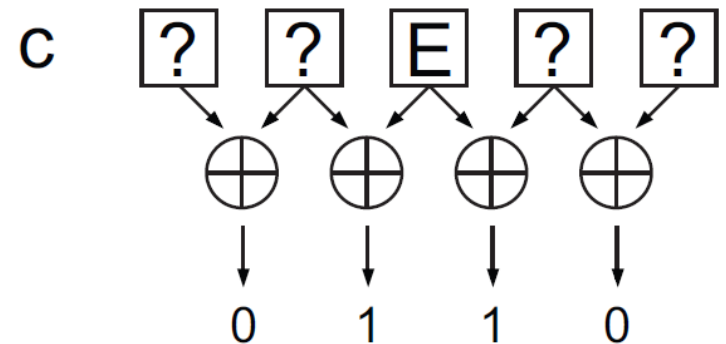
Classical Repetition Code

Improve error tolerance by storing multiple copies of a bit.

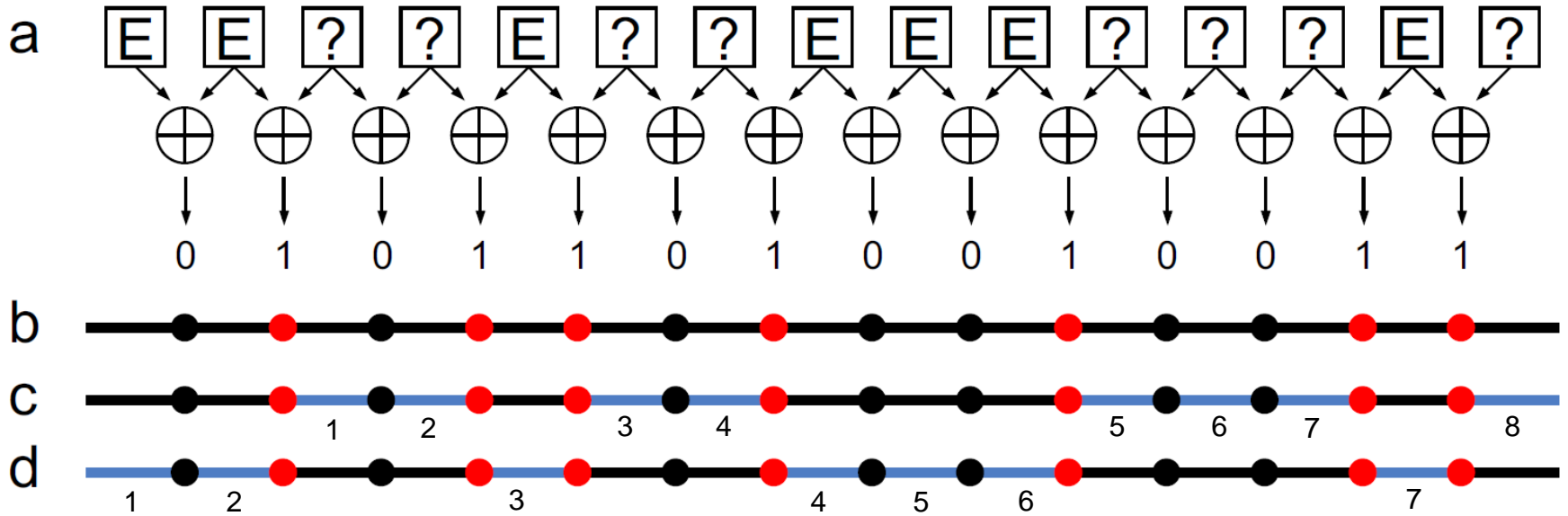
Assumption: independent errors.

Correction method: Majority voting.

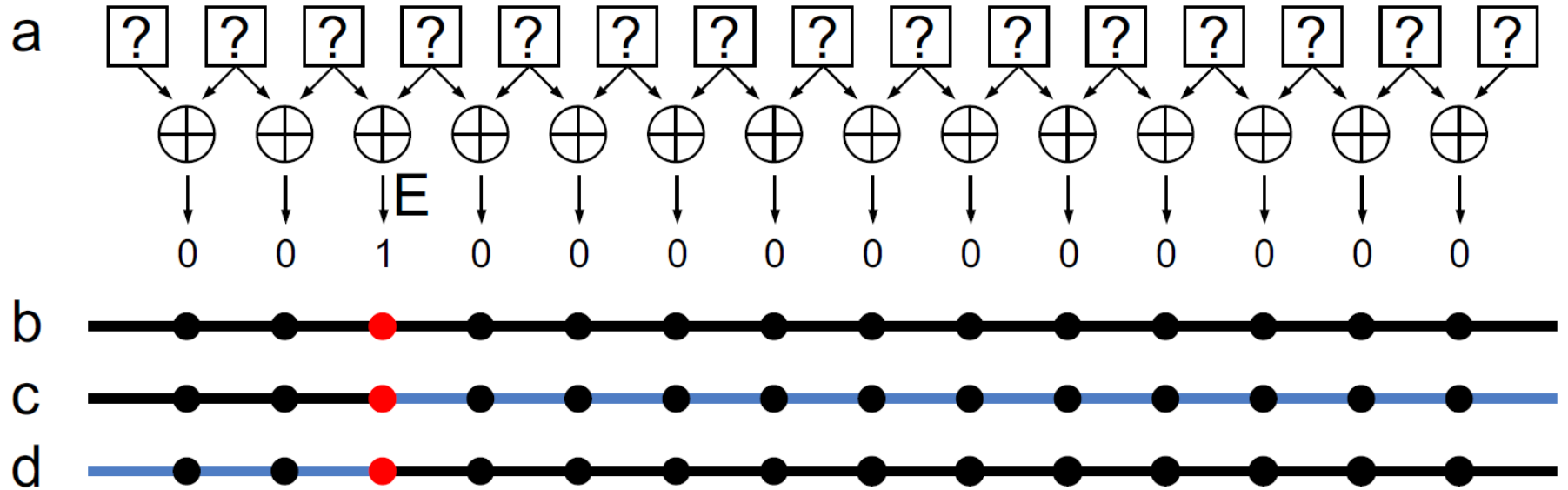
Parity information is sufficient.



An Example



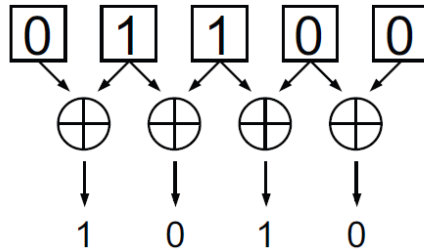
Parity Measurement Errors



Correction method does not work for measurement errors.

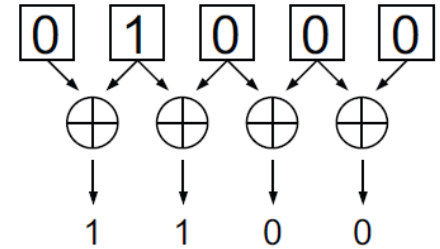
Another Example

t=0



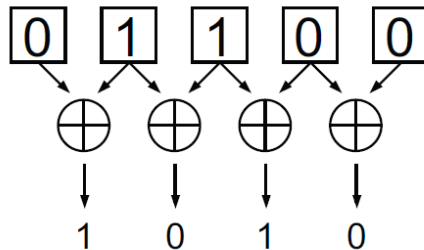
t=3

All parities identical,
no detection events.



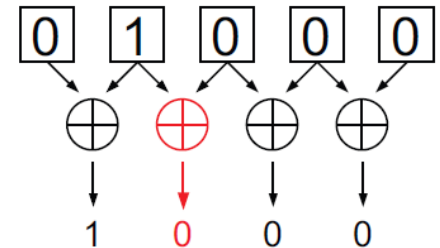
t=1

All parities identical,
no detection events.



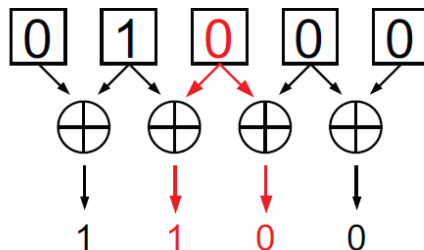
t=4

One parity differs,
one detection event.



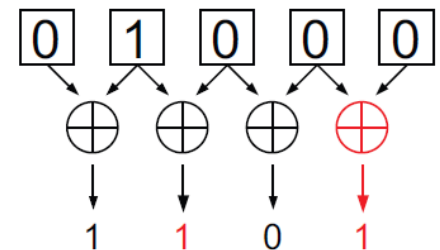
t=2

Two parities differ,
two detection events.

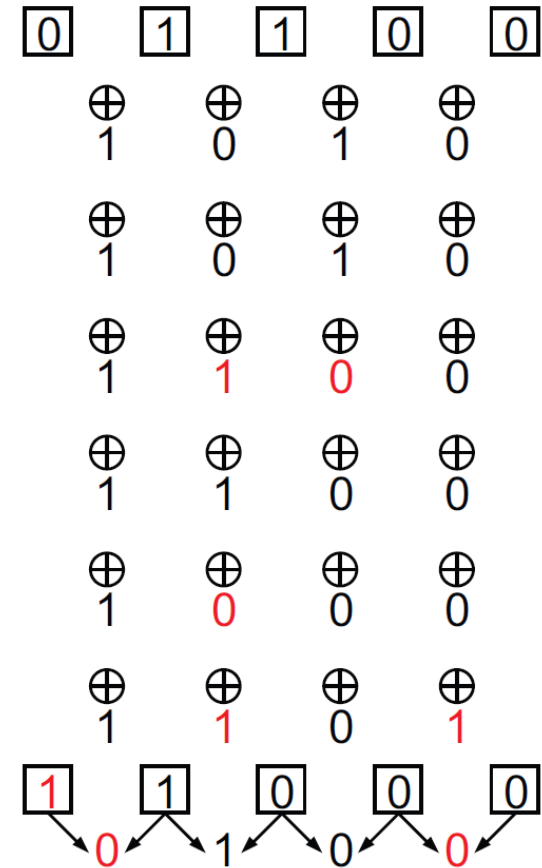
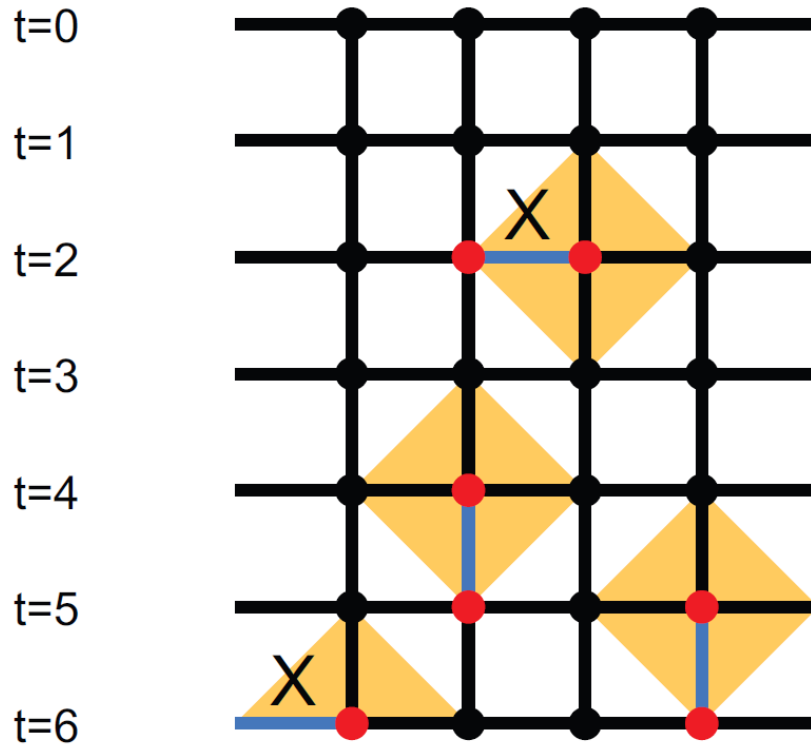


t=5

Two parities differ,
two detection events.



Error Connectivity Graph



Quantum Repetition Code

Goal: Make qubits less susceptible to errors by using multiple qubits to store the state.

Bit-flip (X) and phase-flip (Z) errors

Logical states for repetition code with m qubits.

$$|0_L\rangle = |0..0\rangle \quad |1_L\rangle = |1..1\rangle$$

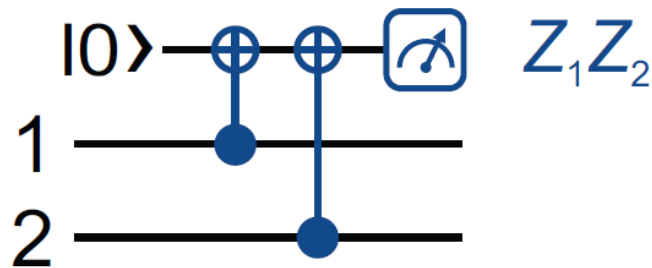
n th-order fault tolerance is defined to mean that any combination of n errors is tolerable. ($n < \lfloor m/2 \rfloor$)

How to measure the parity of qubits?

The $\hat{Z}\hat{Z}$ operator measures the parity of two qubits because

$$\begin{aligned} \hat{Z}\hat{Z}|00\rangle &= +|00\rangle & \hat{Z}\hat{Z}|01\rangle &= -|01\rangle \\ \hat{Z}\hat{Z}|11\rangle &= +|11\rangle & \hat{Z}\hat{Z}|10\rangle &= -|10\rangle \end{aligned}$$

Implement this by introducing an ancilla measurement qubit to CNOT gates.



$|00\rangle$ and $|11\rangle$ give $|0\rangle$ for the ancilla qubit state

$|01\rangle$ and $|10\rangle$ give $|1\rangle$ for the ancilla qubit state

An example.

Initial state:

$$|\Psi\rangle = (\alpha |00\rangle_D + \beta |11\rangle_D) \otimes |0\rangle_A$$

Rotate qubit 1 around the x-axis:

$$|\Psi'\rangle = [\gamma(\alpha |00\rangle_D + \beta |11\rangle_D) + \delta(\alpha |10\rangle_D + \beta |01\rangle_D)] \otimes |0\rangle_A$$

Apply the CNOT gates:

$$|\Psi''\rangle = \gamma(\alpha |00\rangle_D + \beta |11\rangle_D) \otimes |0\rangle_A + \delta(\alpha |10\rangle_D + \beta |01\rangle_D) \otimes |1\rangle_A$$

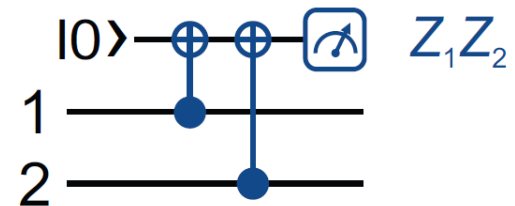
After measurement:

$$|\Psi\rangle = (\alpha |00\rangle_D + \beta |11\rangle_D) \otimes |0\rangle_A$$

$$|\Phi\rangle = (\alpha |10\rangle_D + \beta |01\rangle_D) \otimes |1\rangle_A$$

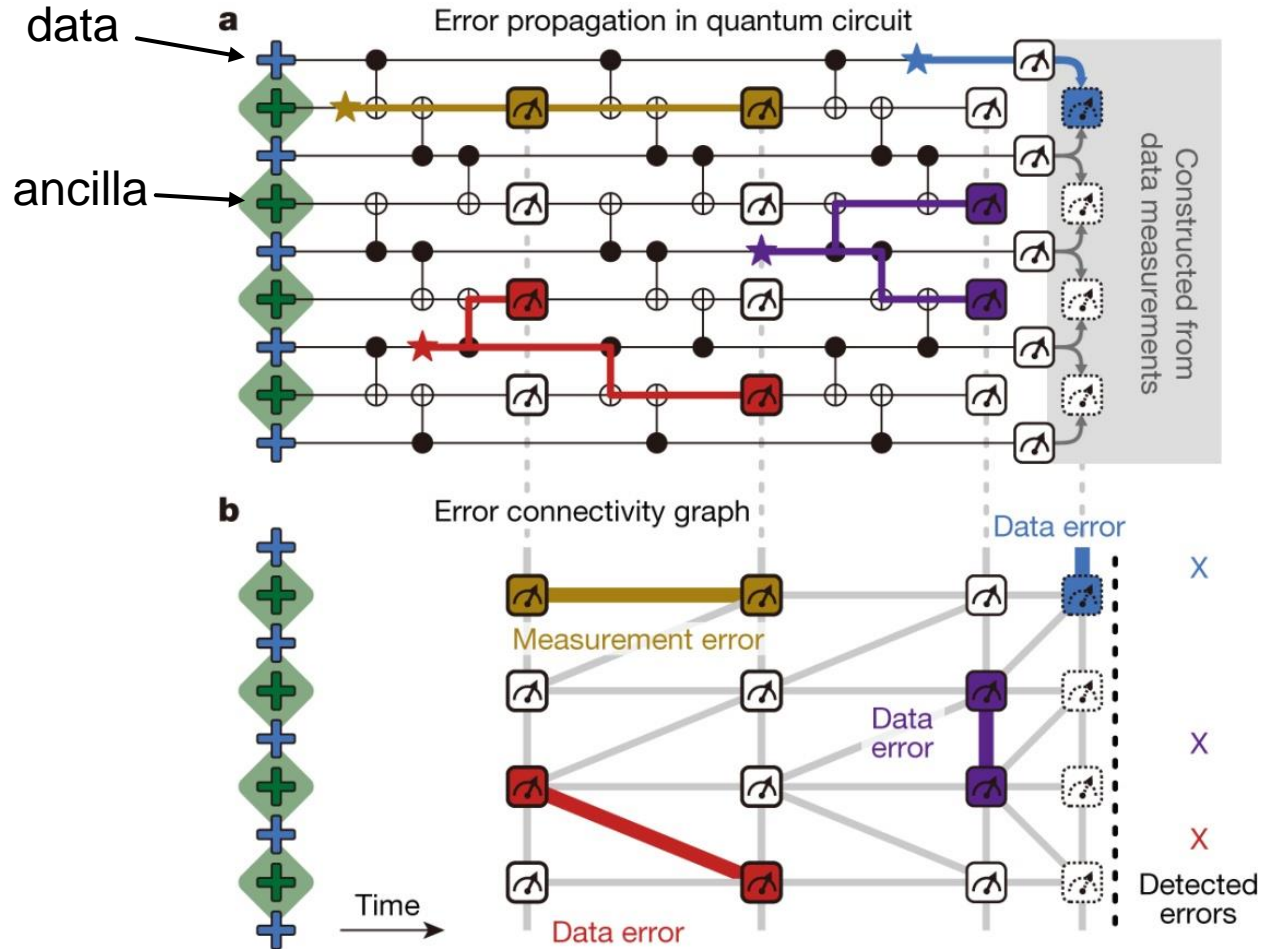
Correction:

$$|\Psi\rangle = X_1 |\Phi\rangle$$



Parity measurement does not change the state of the data qubits!

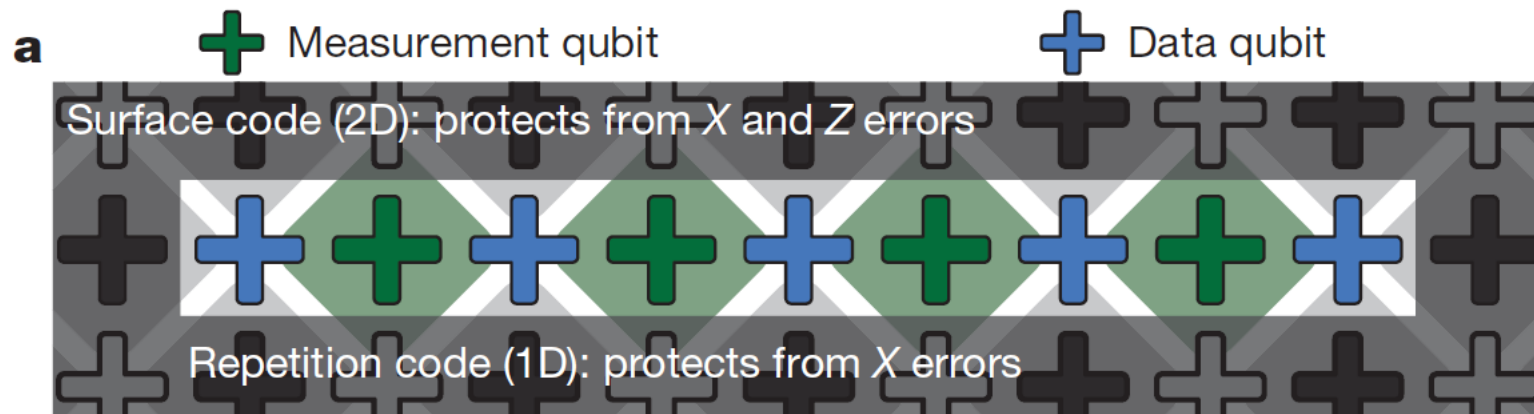
Error propagation and identification.



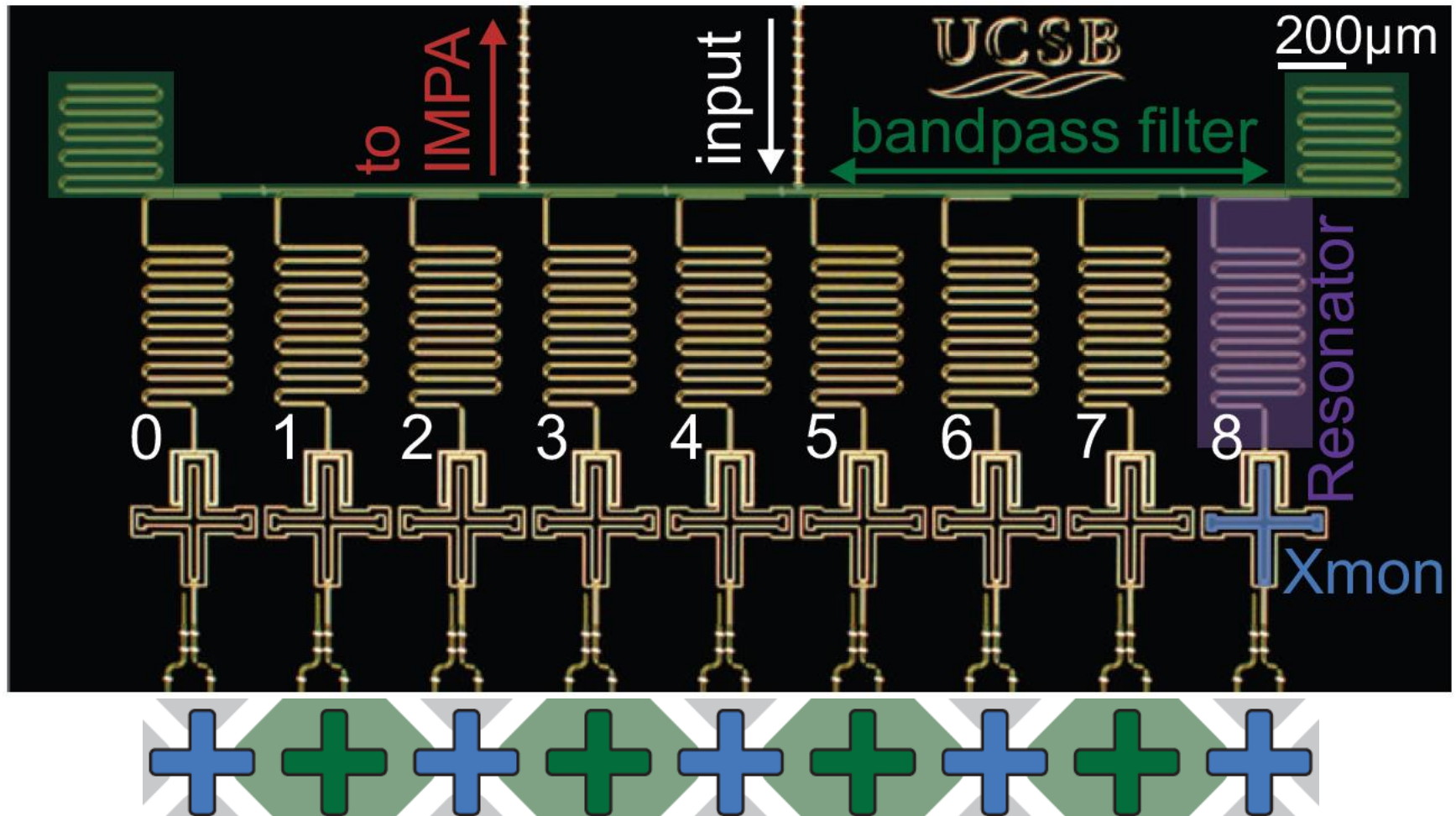
Error suppression factor Λ and Fidelity

- A measure of how far below the threshold error rate a system is
- Logical error rate $\varepsilon \propto \frac{1}{\Lambda^{n+1}}$ $\Lambda > 1$
with n -th order fault tolerance n
- The “Fidelity” is a measure of closeness of two quantum states

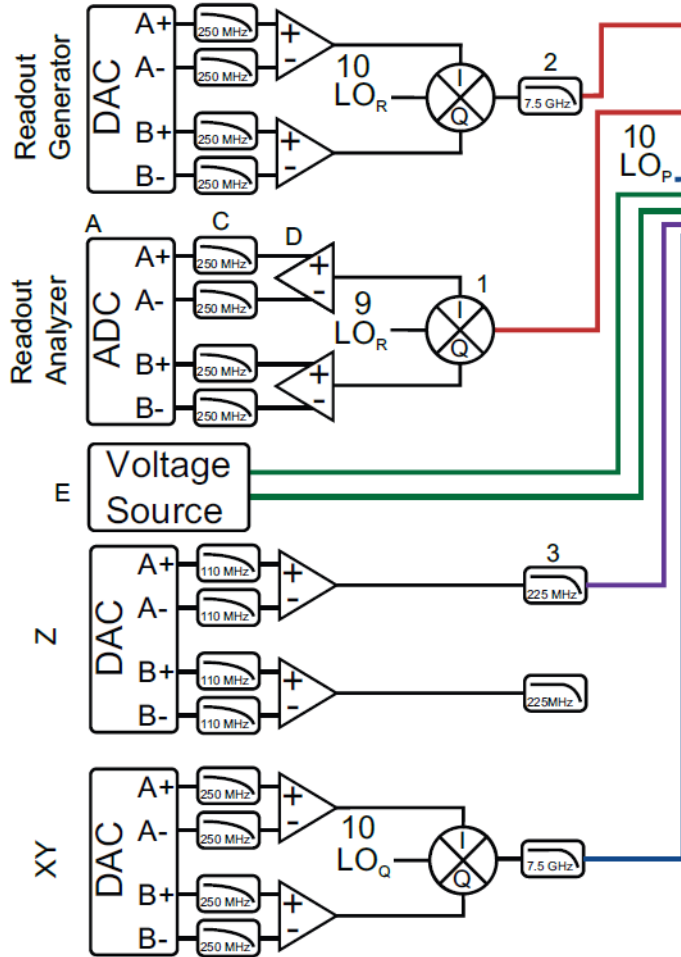
Setup



Chip architecture - components

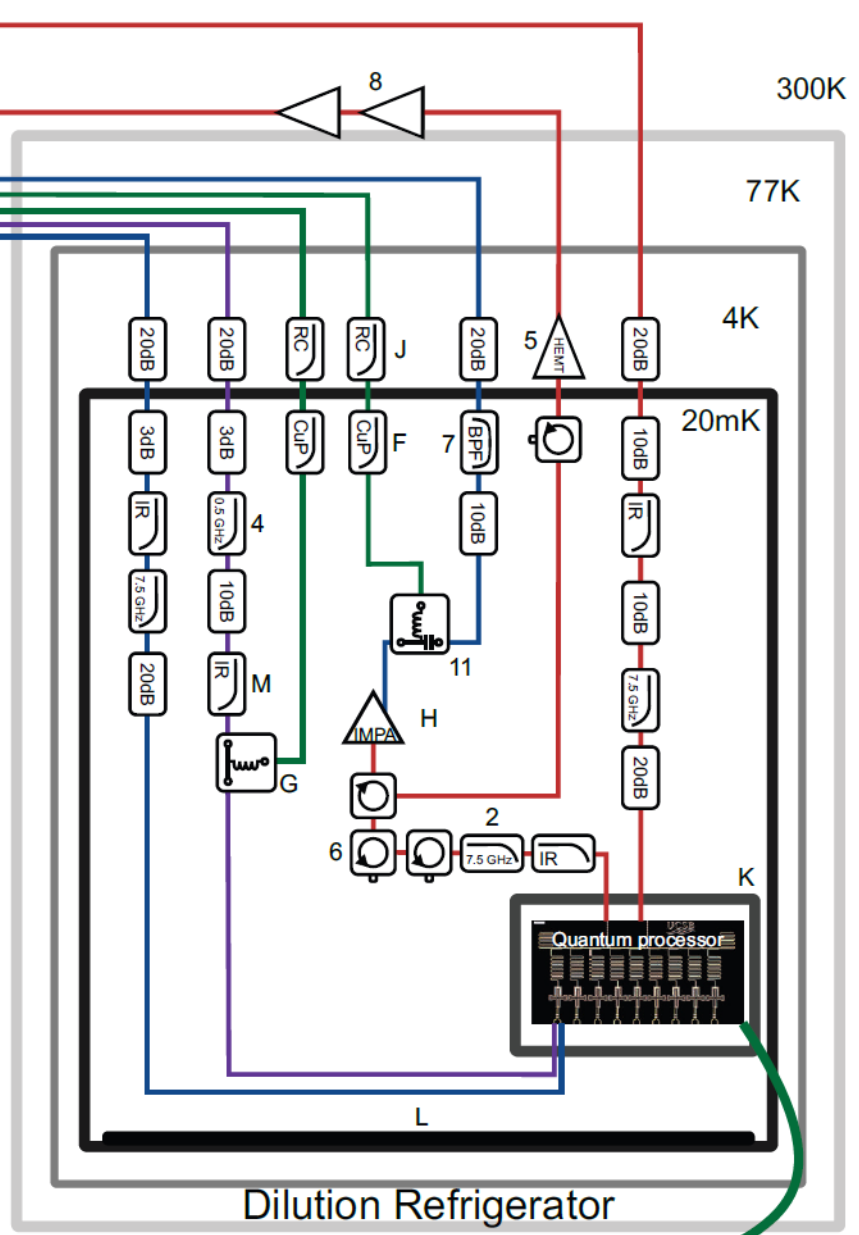


Readout

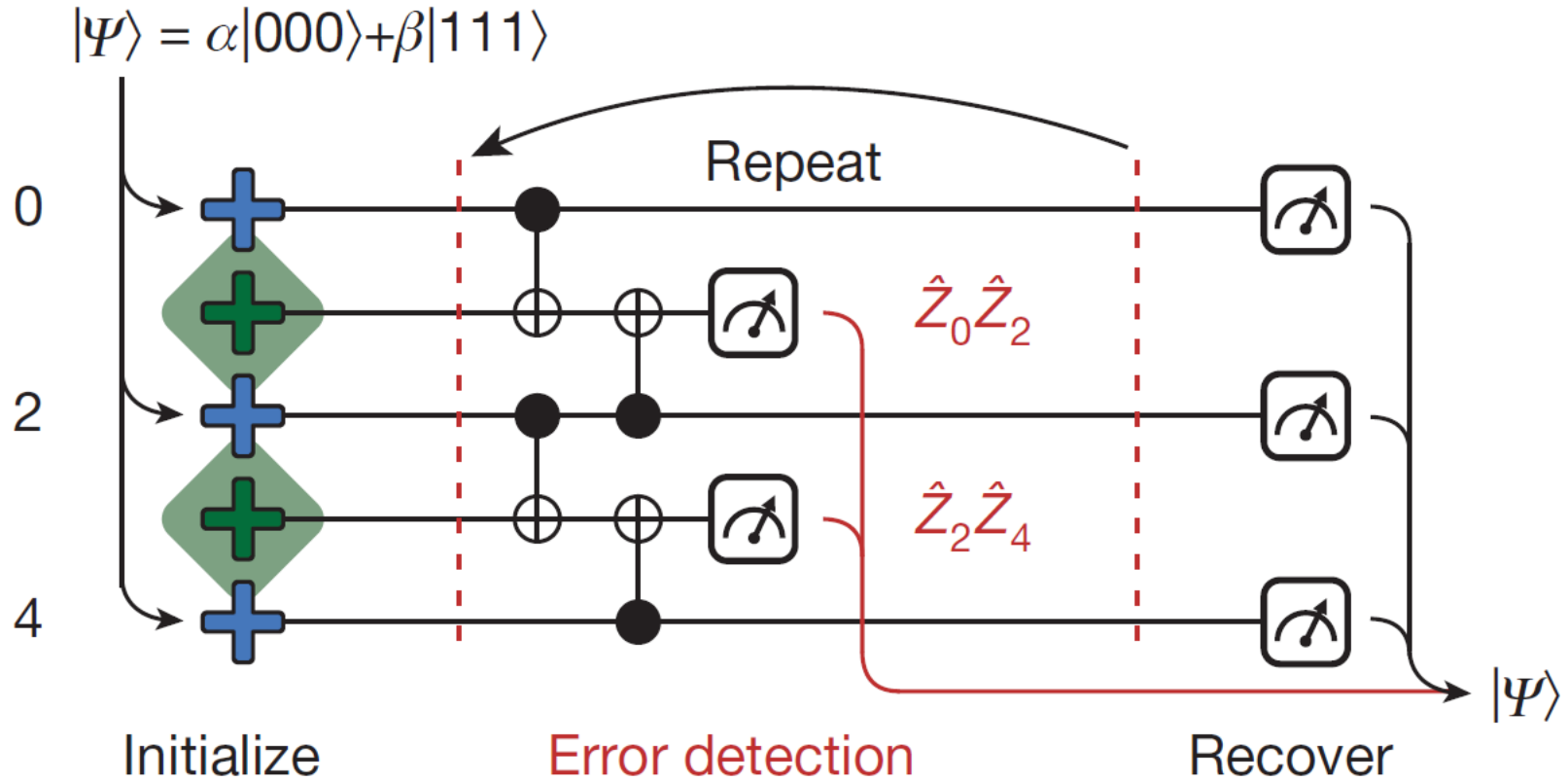


Repeat for other eight qubits

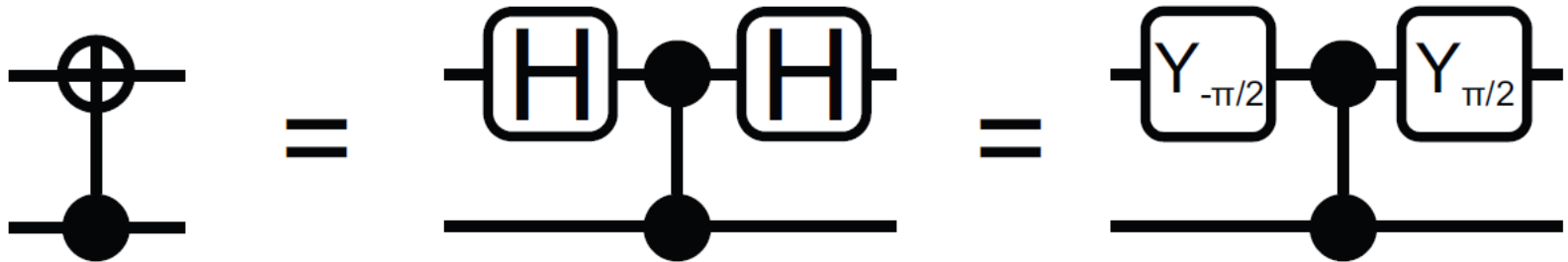
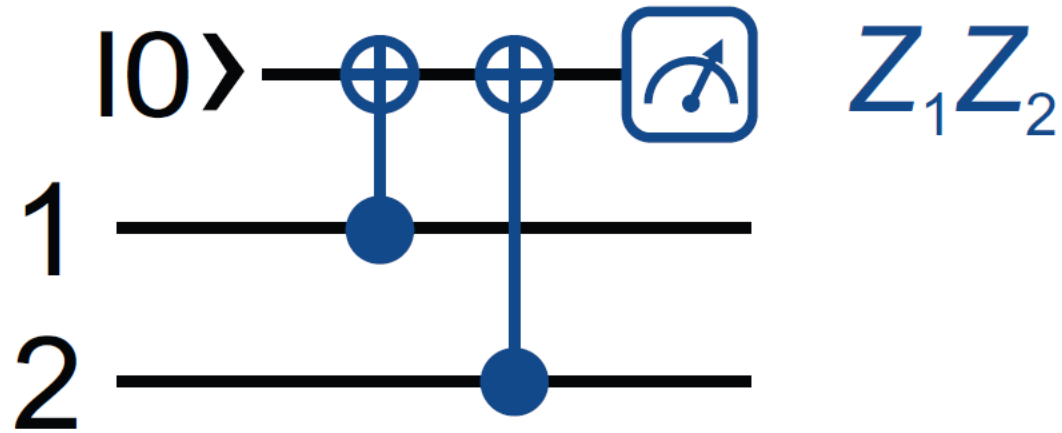
Qubit Control



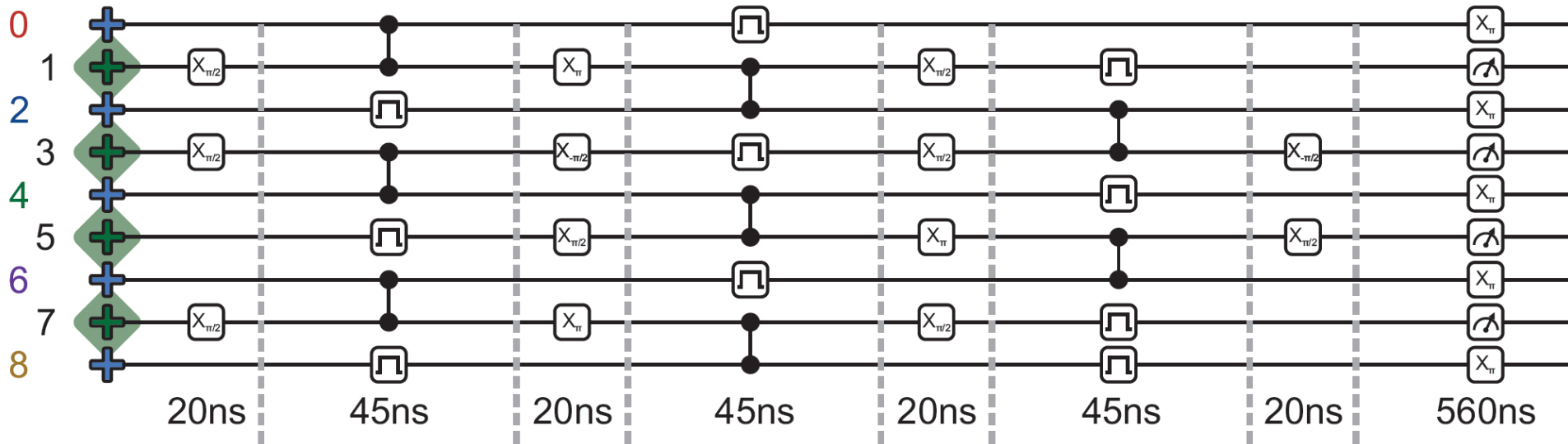
Measurement procedure



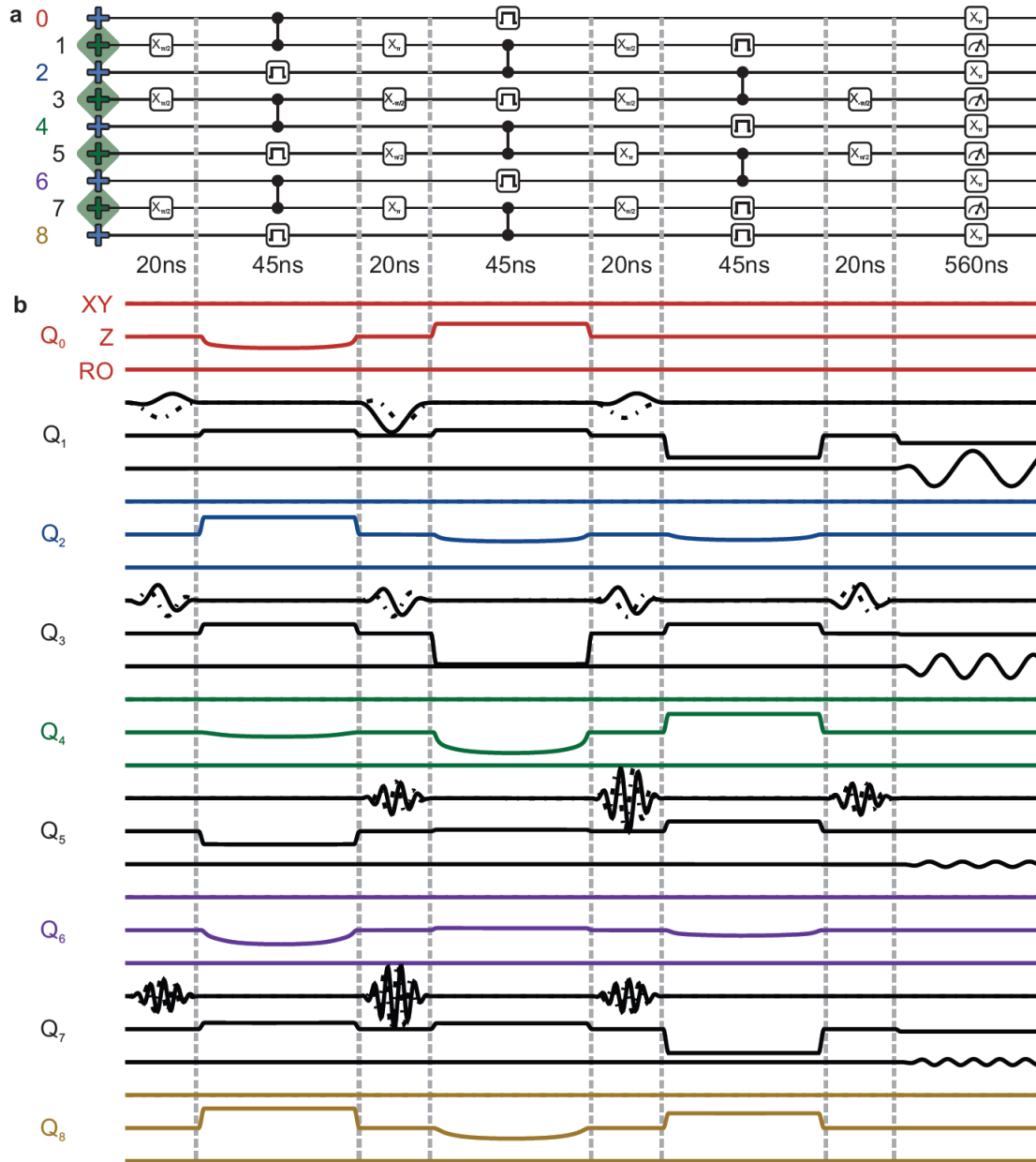
Decomposition of CNOT gate



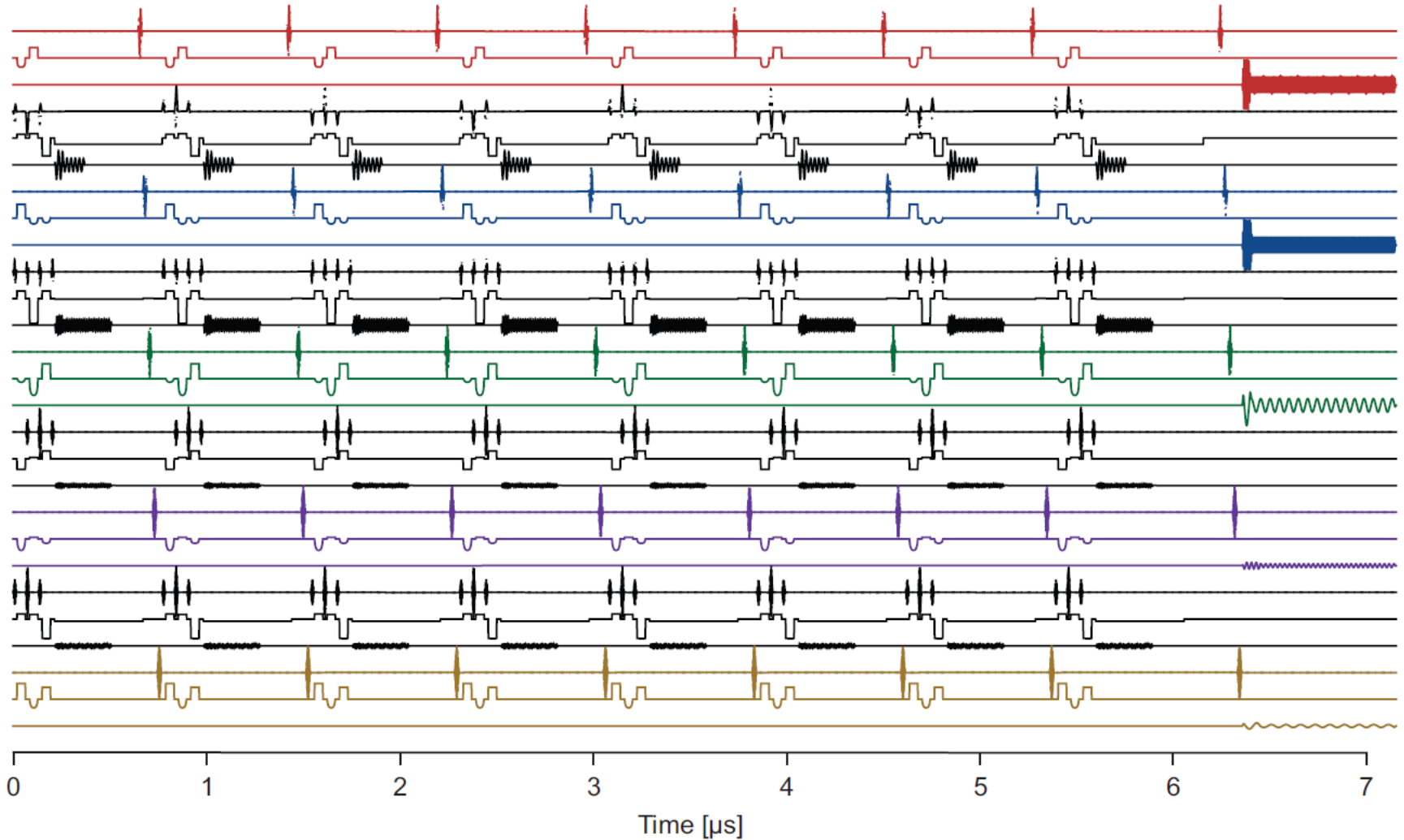
Single repetition code implementation



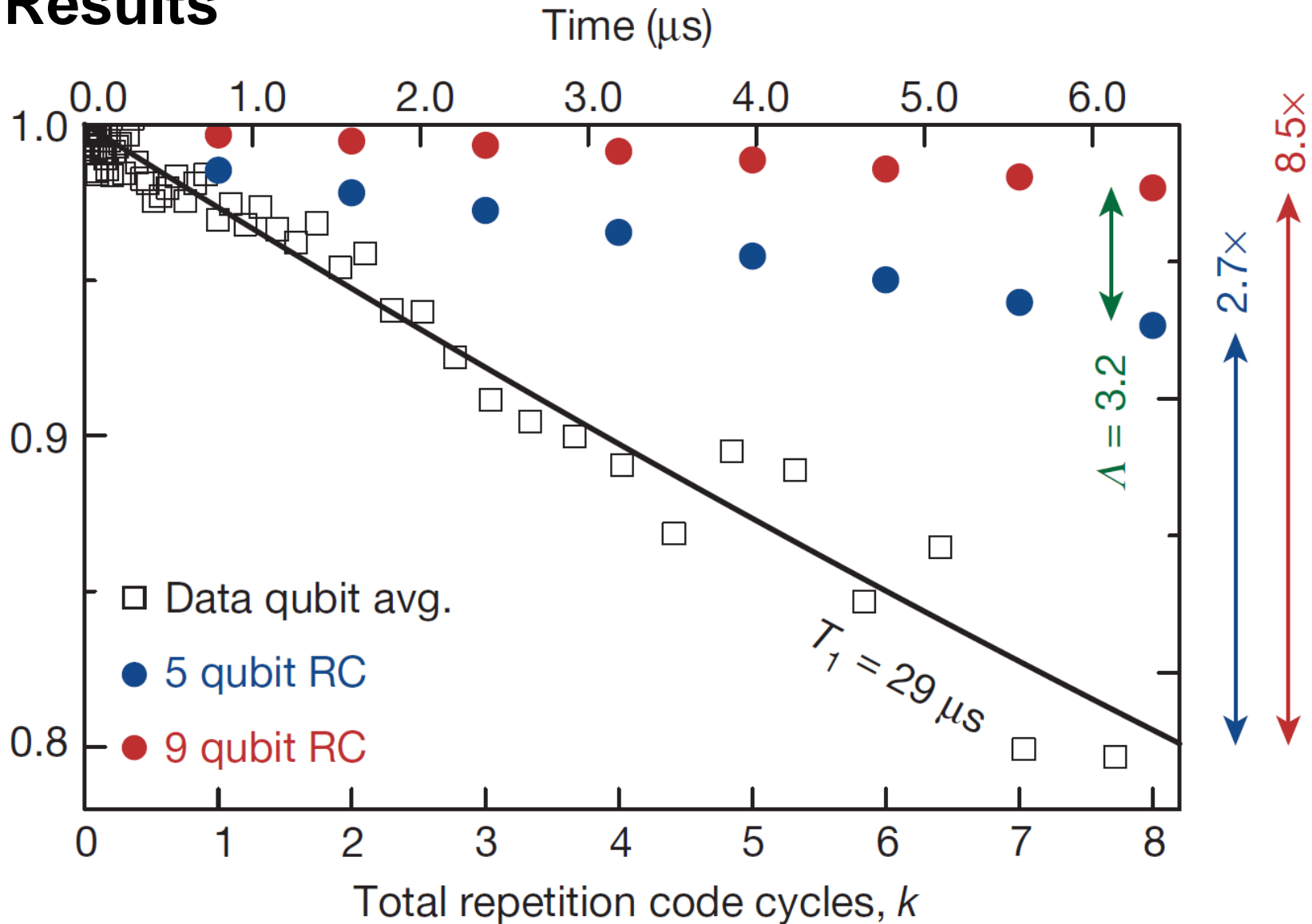
waveform data for 1 cycle



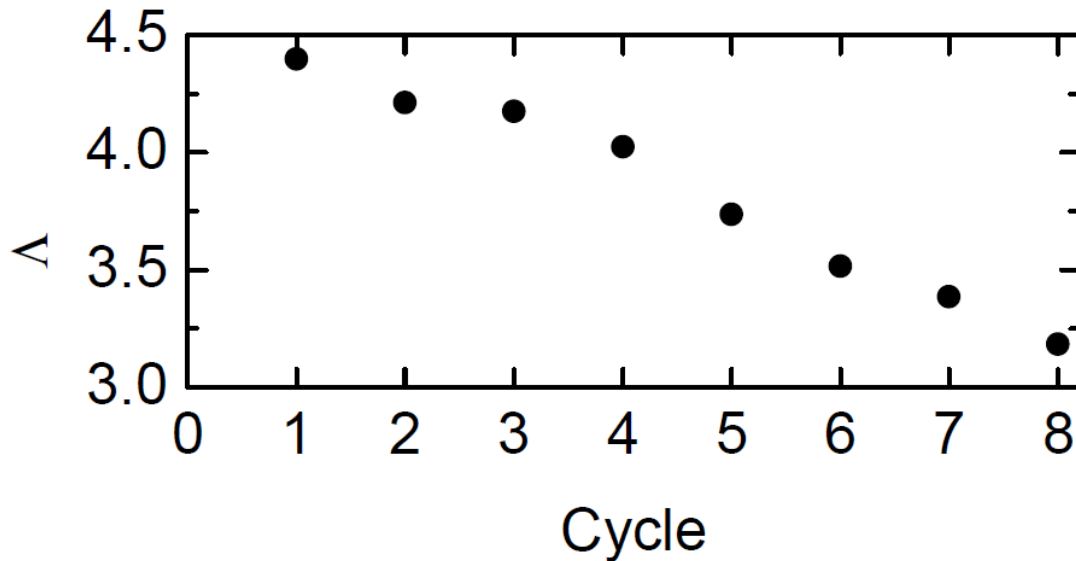
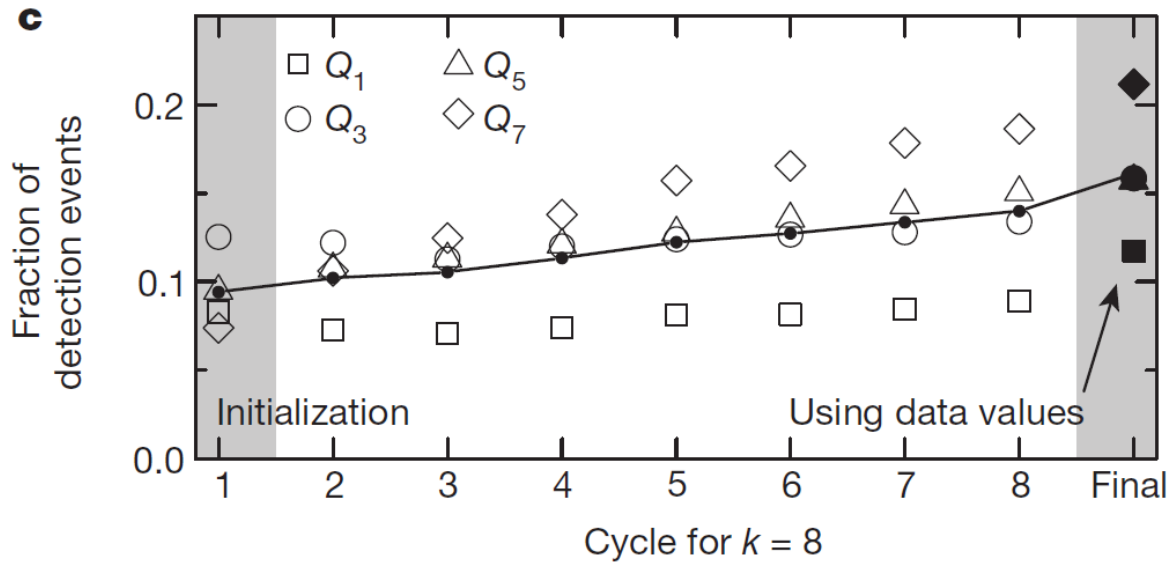
Waveform data for 8 cycles



Results



Logical error rate and Λ



$$\varepsilon \propto \frac{1}{\Lambda^{n+1}}$$

Acknowledgements & References

Acknowledgements

to Dr. Abdulfarrukh Abdumalikov
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Reference

J. Kelly et al., State preservation by repetitive error detection in a superconducting quantum circuit, *Nature* **519**, 66-69 (2015)

Final Slide – Questions?

