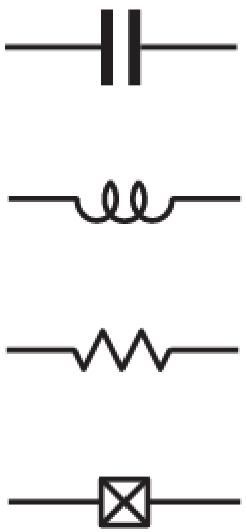
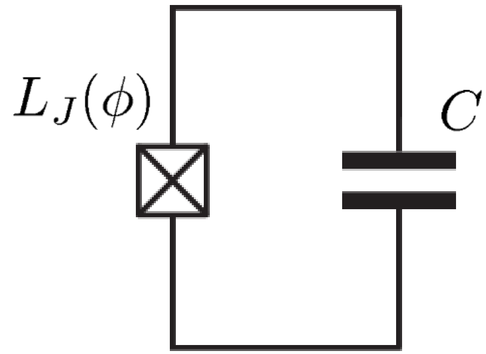
Constructing Non-Linear Quantum Electronic Circuits

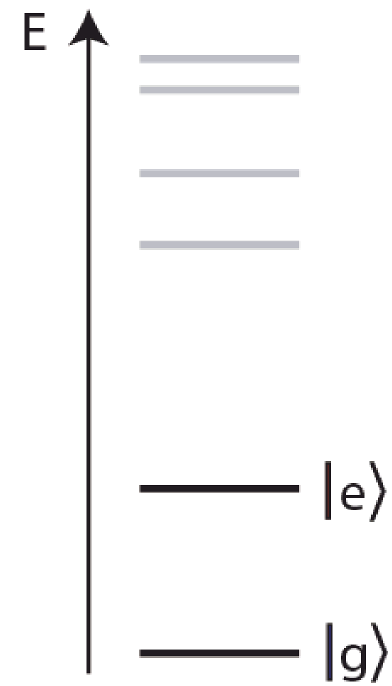
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



Josesphson junction:
a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1}$$

$$= \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic artificial atom

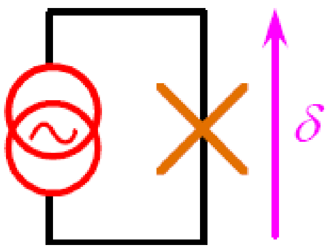
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

A Classification of Josephson Junction Based Qubits

How to make use in of Jospehson junctions in a qubit?

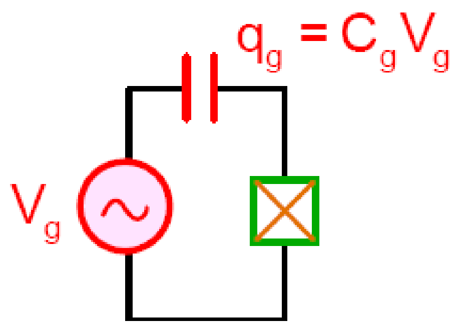
Common options of bias (control) circuits:

phase qubit



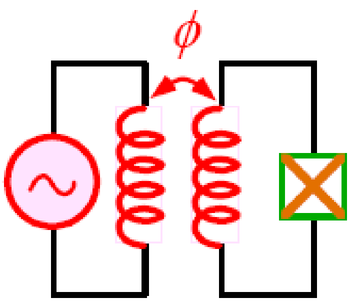
current bias

charge qubit
(Cooper Pair Box, Transmon)



charge bias

flux qubit

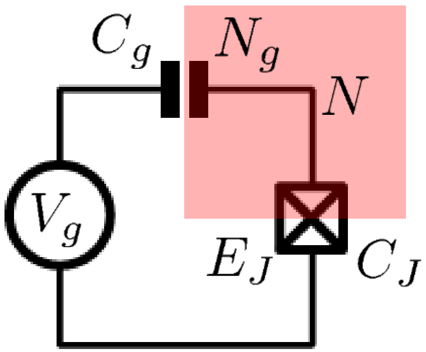


flux bias

How is the control circuit important?

The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $N = \frac{Q}{2e}$

continuous gate charge: $N_g = \frac{C_g V_g}{2e}$

total box capacitance $C_\Sigma = C_g + C_J$

Hamiltonian: $H = H_{\text{el}} + H_{\text{mag}}$

electrostatic part:

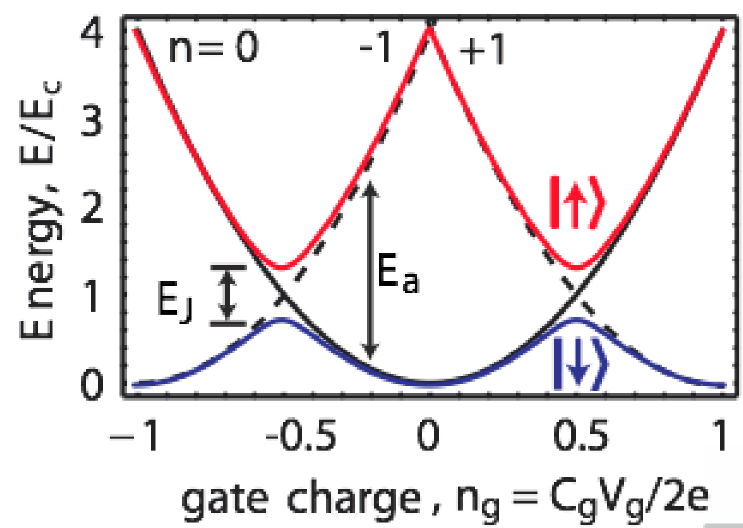
$$H_{\text{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\text{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{mag}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator:

$$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}}\sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the **charge basis** N:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the **phase basis** δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 - E_J\cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

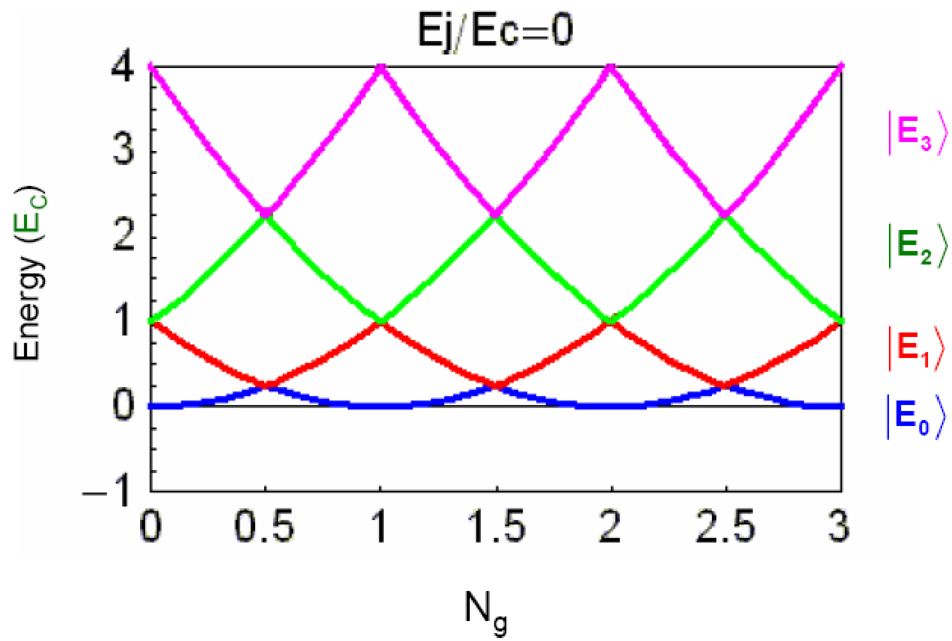
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

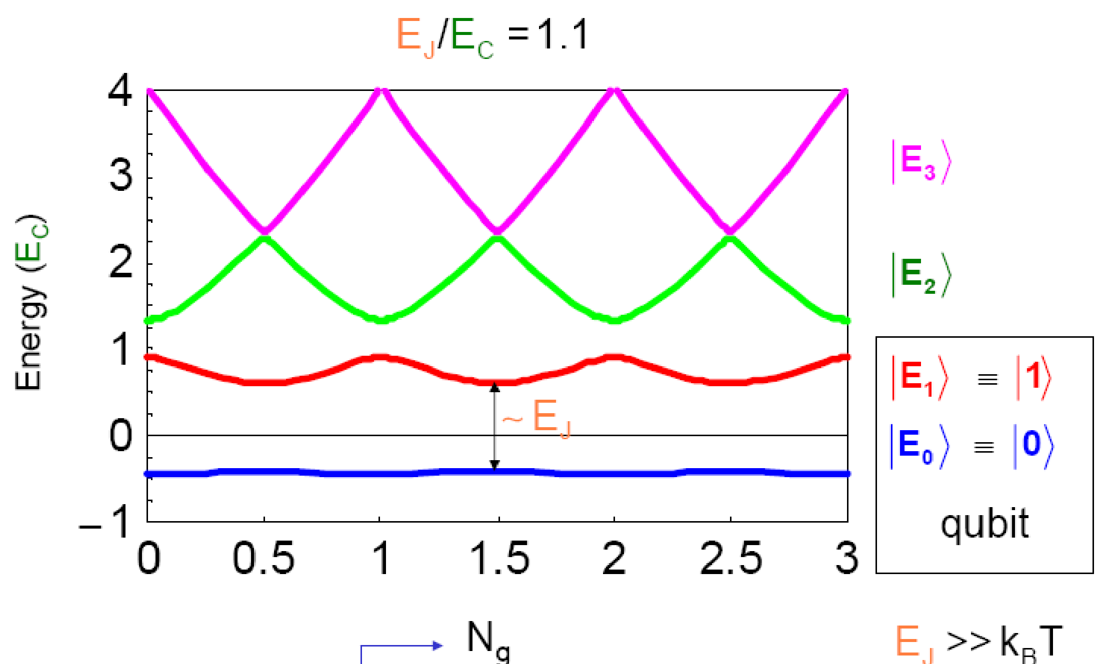
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g



energy bands for finite E_J

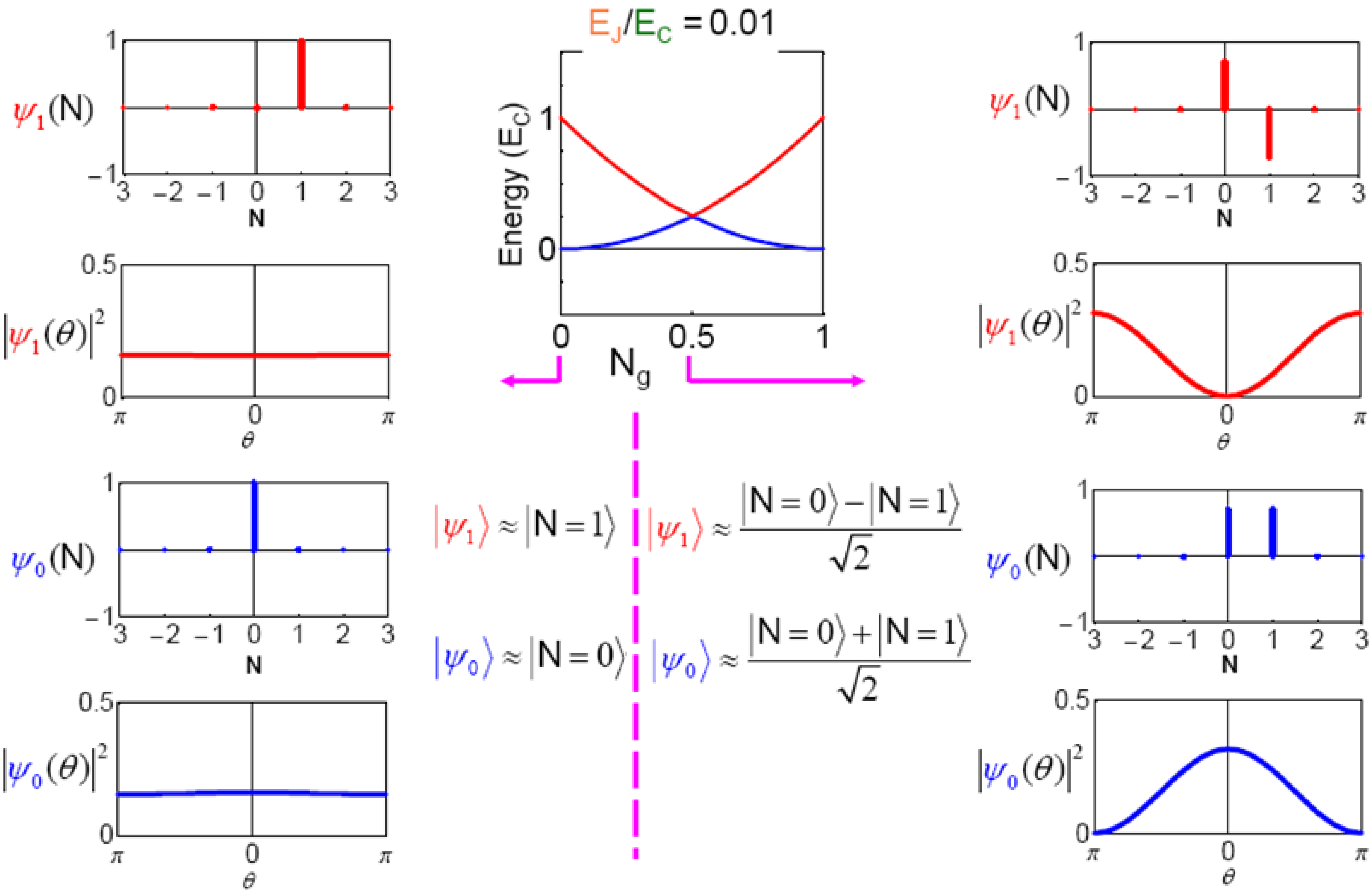
- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



$E_J >> k_B T$

tunable artificial atom

Charge and Phase Wave Functions ($E_J \ll E_C$)

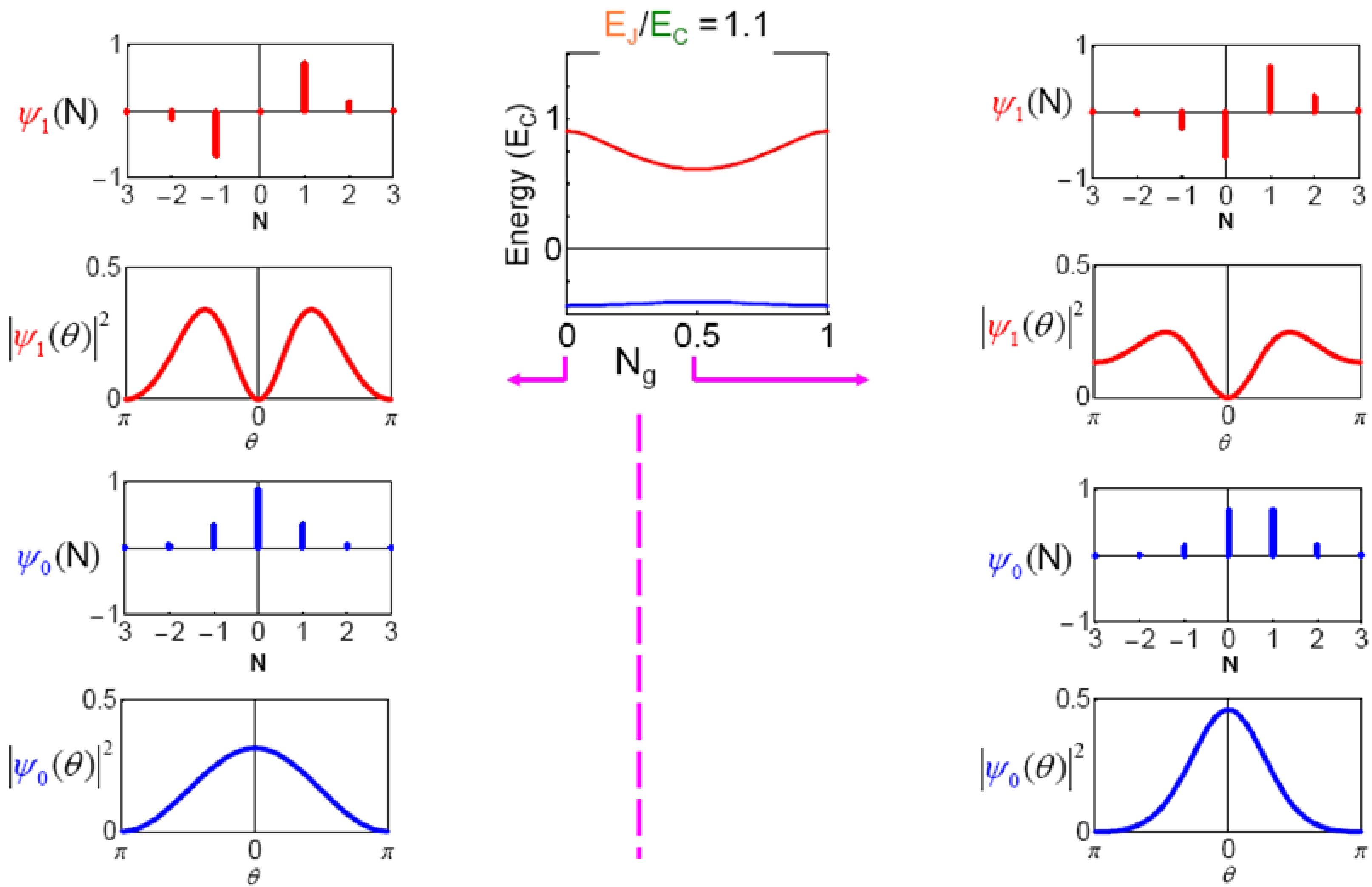


$$|\psi_1\rangle \approx |N=1\rangle \qquad |\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx |N=0\rangle \qquad |\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$

courtesy CEA Saclay

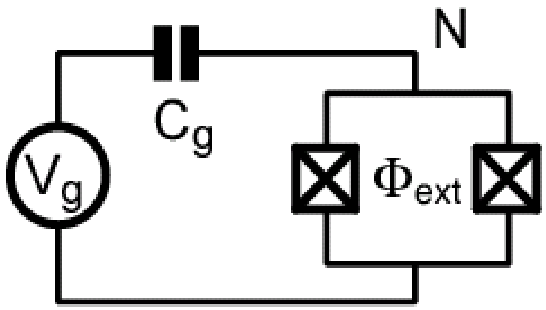
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

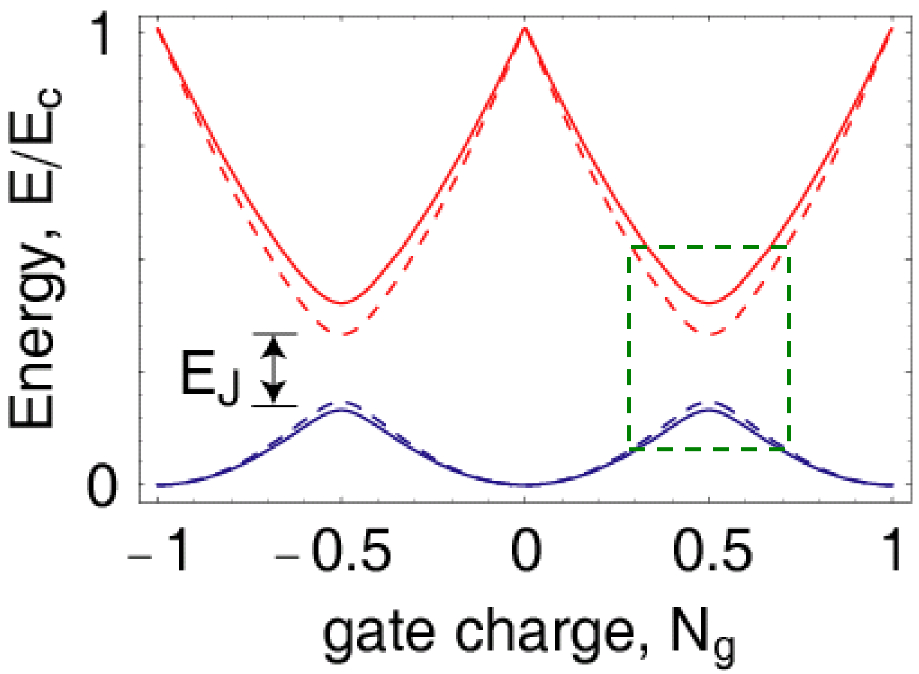
split Cooper pair box in perpendicular field



$$H = E_C\left(N - N_g\right)^2 - E_{J,\max}\cos\left(\pi\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\phi_{\mathrm{ext}}}{\phi_0}\right)$$



consider two state approximation

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\rm CPB} = \hat{H}_{\rm el} + \hat{H}_{\rm J} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

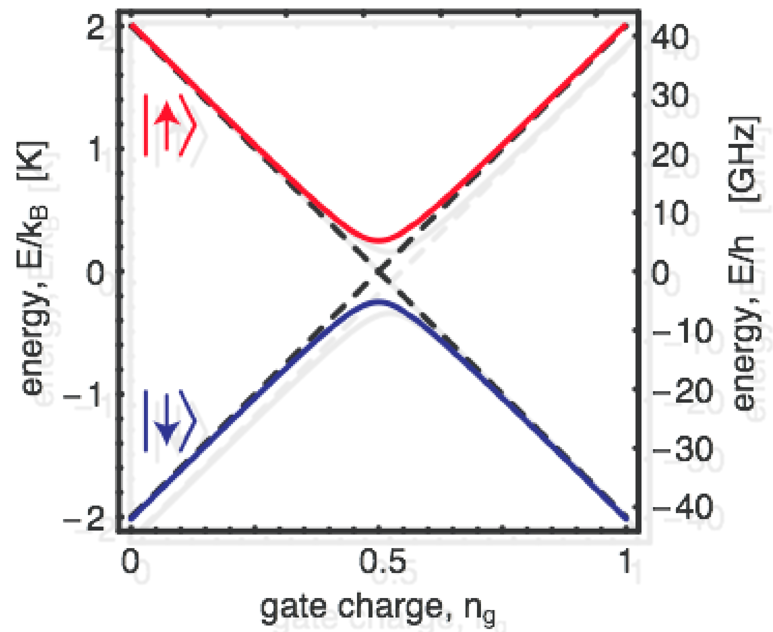
$$\hat{H}_{\rm CPB} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_{\rm J}}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\rm el}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

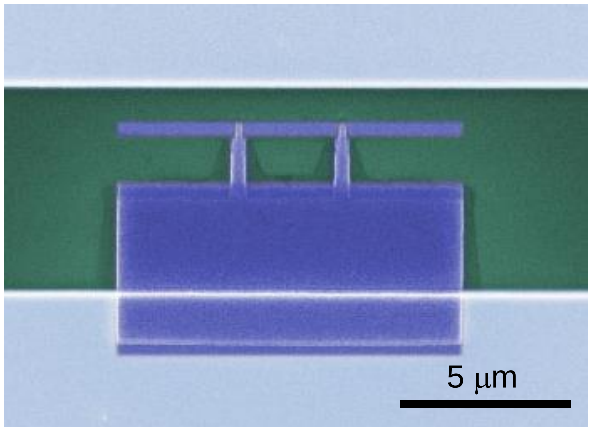


Shnirman *et al.*, *Phys. Rev. Lett.* **79**, 2371 (1997)

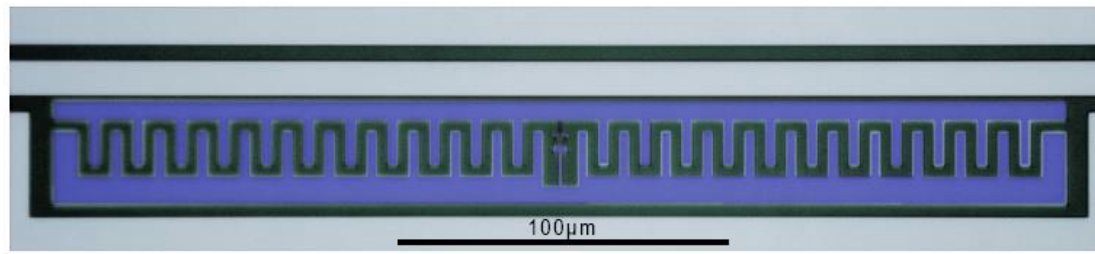
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

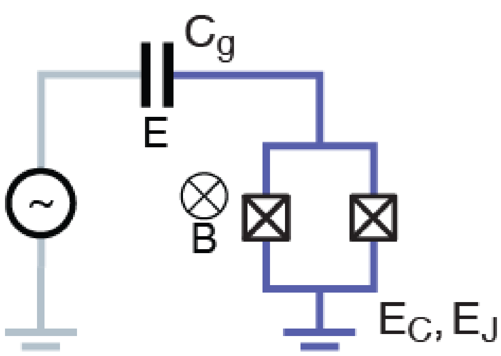
standard CPB:

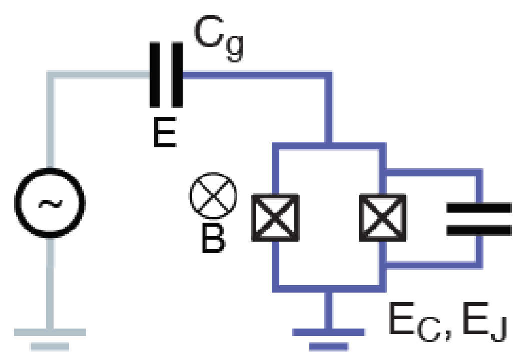


Transmon qubit:



circuit diagram:

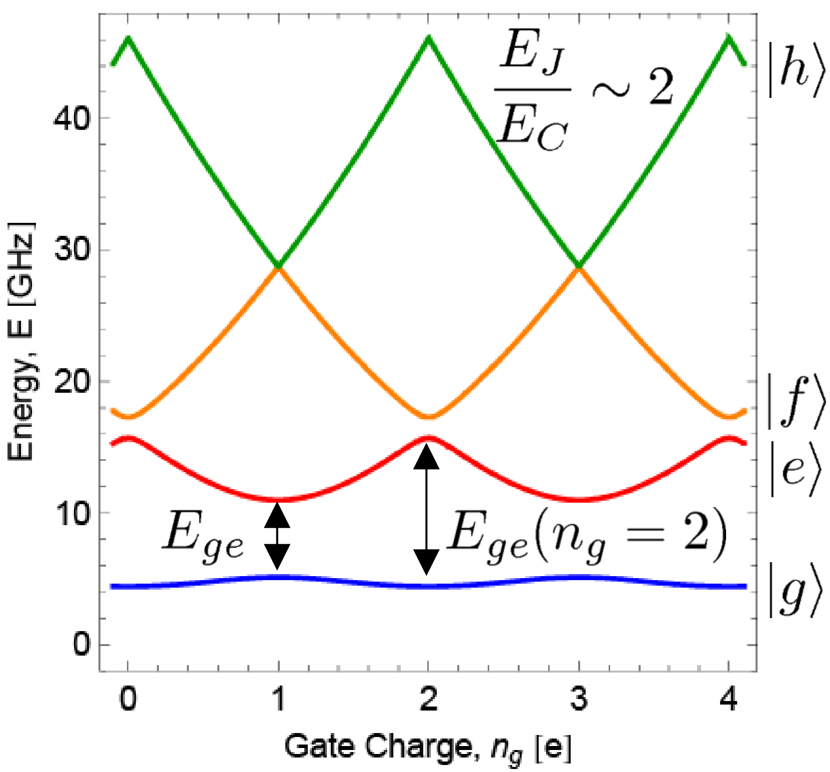




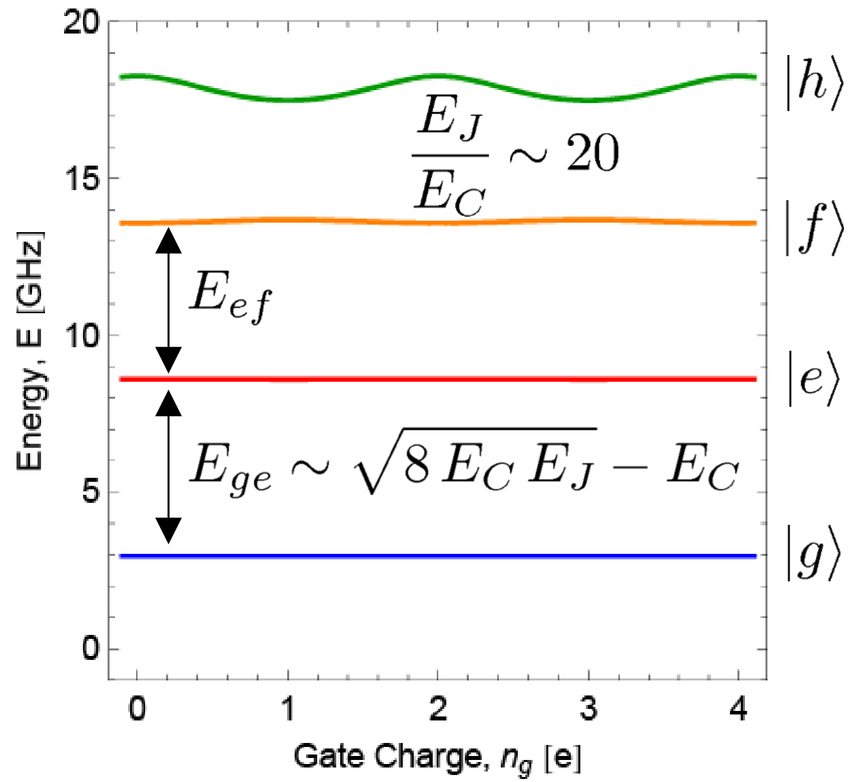
J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B **77**, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

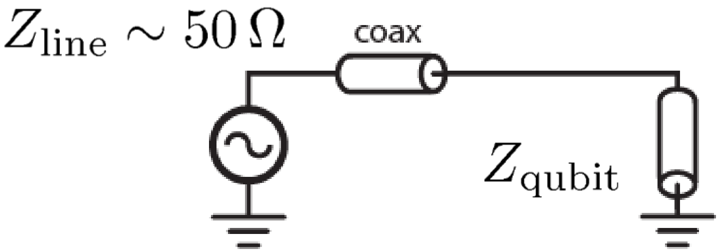
relative anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

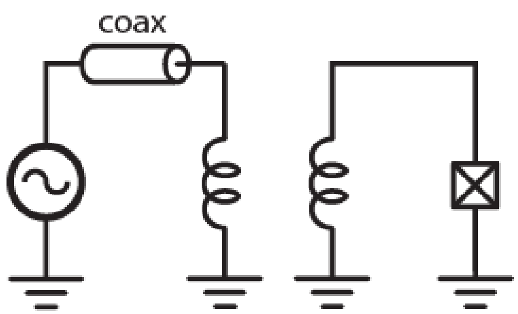
Control of Coupling to Electromagnetic Environment

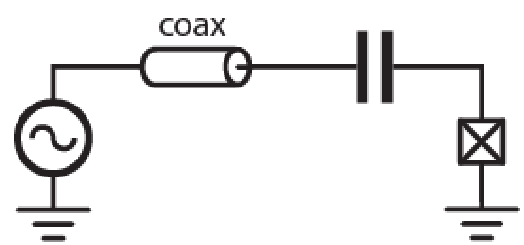
coupling to environment (bias wires):



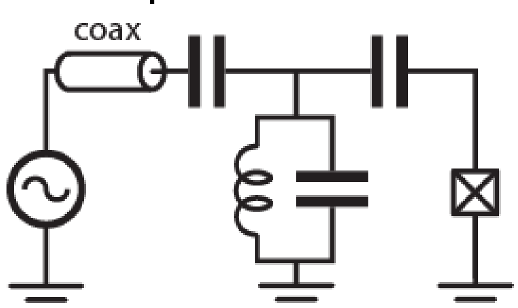
decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...



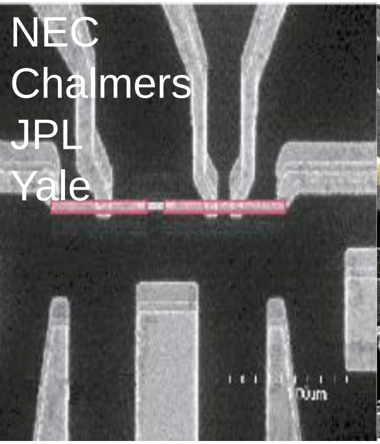


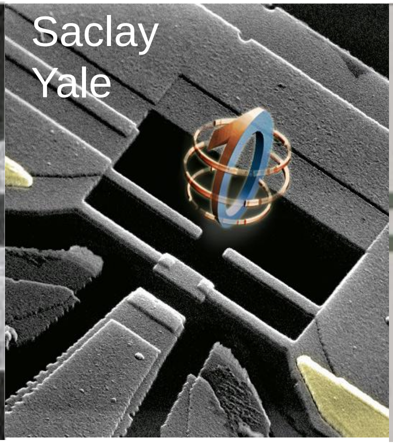
... or resonant impedance transformers

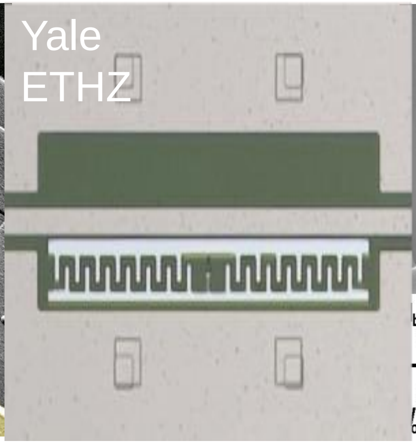


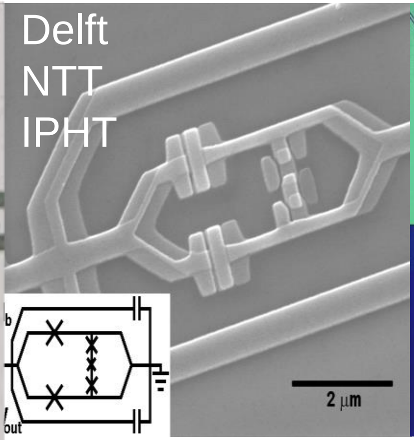
control spontaneous emission by circuit design

Realizations of Superconducting Artificial Atoms





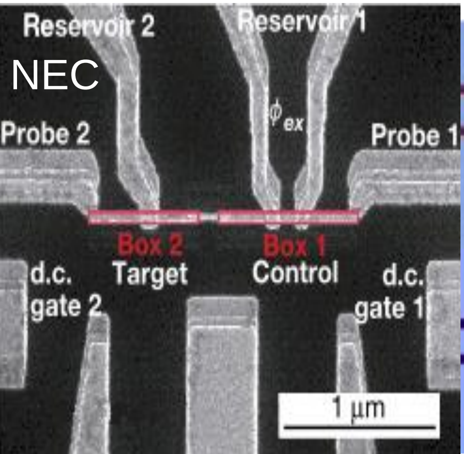


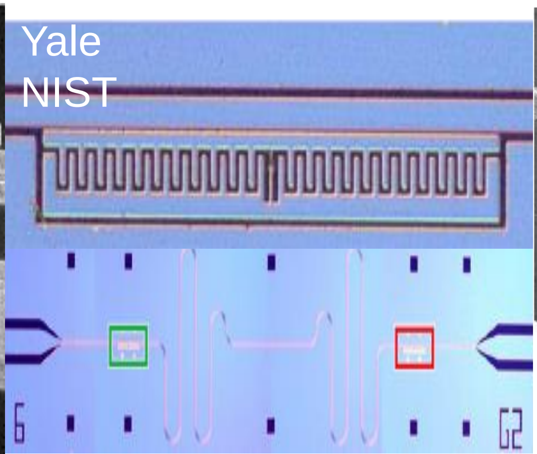


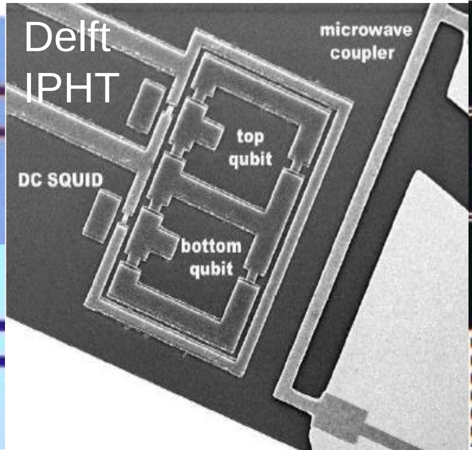
'artificial atoms' -- single superconducting qubits

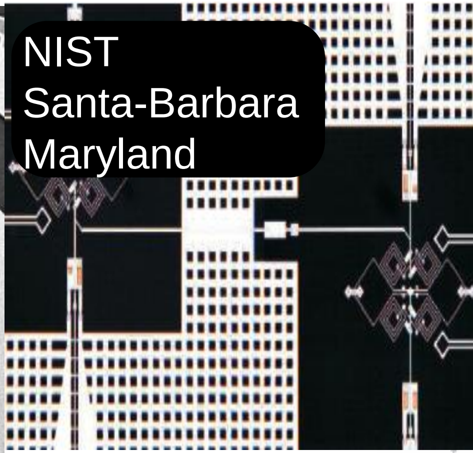
review:
J. Clarke and F. Wilhelm
Nature **453**, 1031 (2008)

'artificial molecules' -- coupled superconducting qubits





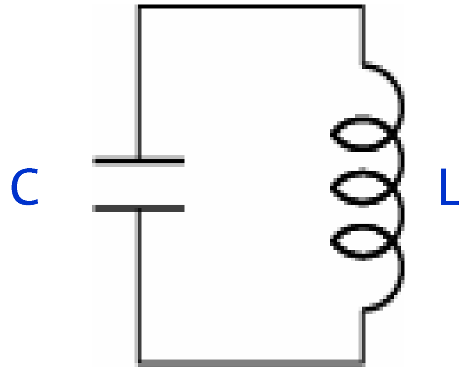




Realizations of Harmonic Oscillators

Superconducting Harmonic Oscillators

a simple electronic circuit:



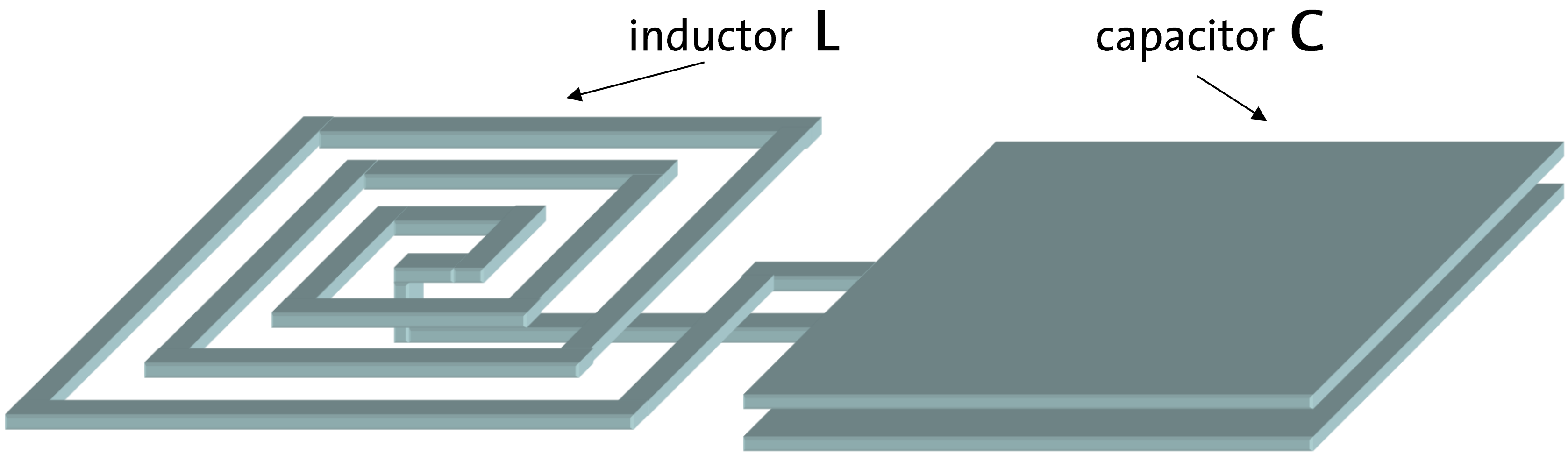
- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm

- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF

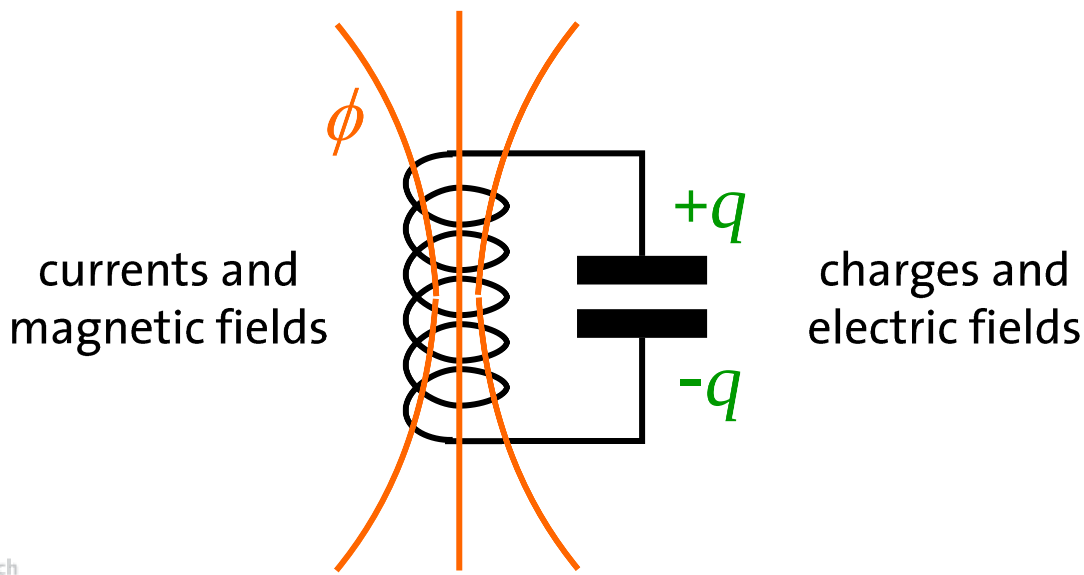
- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

Realization of H.O.: Lumped Element Resonator

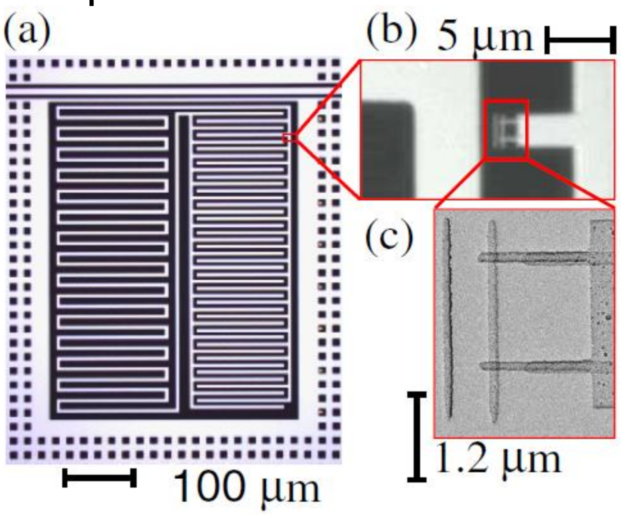


a harmonic oscillator



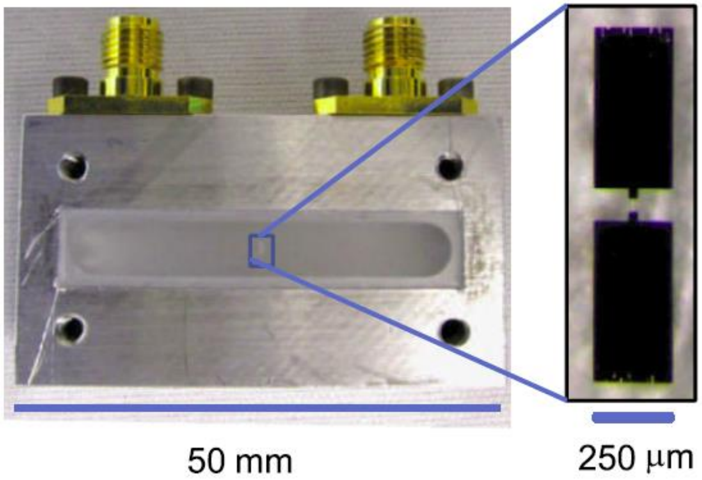
Types of Superconducting Harmonic Oscillators

lumped element resonator:



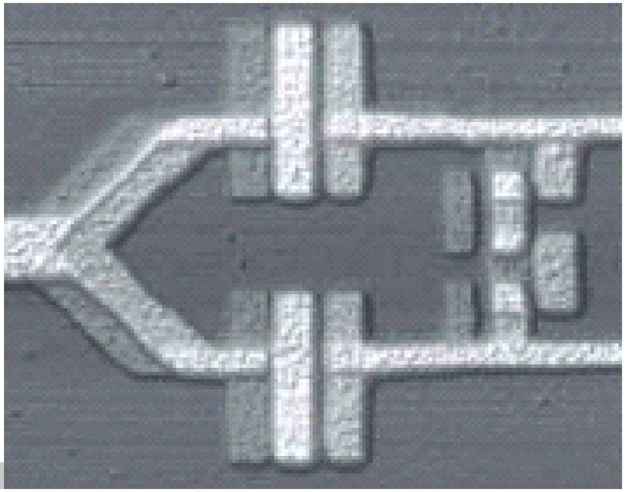
Z. Kim *et al.*, *PRL* **106**, 120501 (2011)

3D cavity:



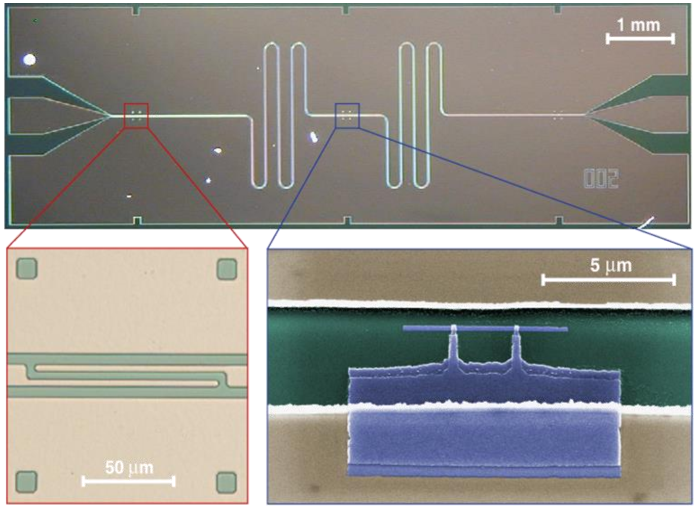
H. Paik *et al.*, *PRL* **107**, 240501 (2011)

weakly nonlinear junction:



I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

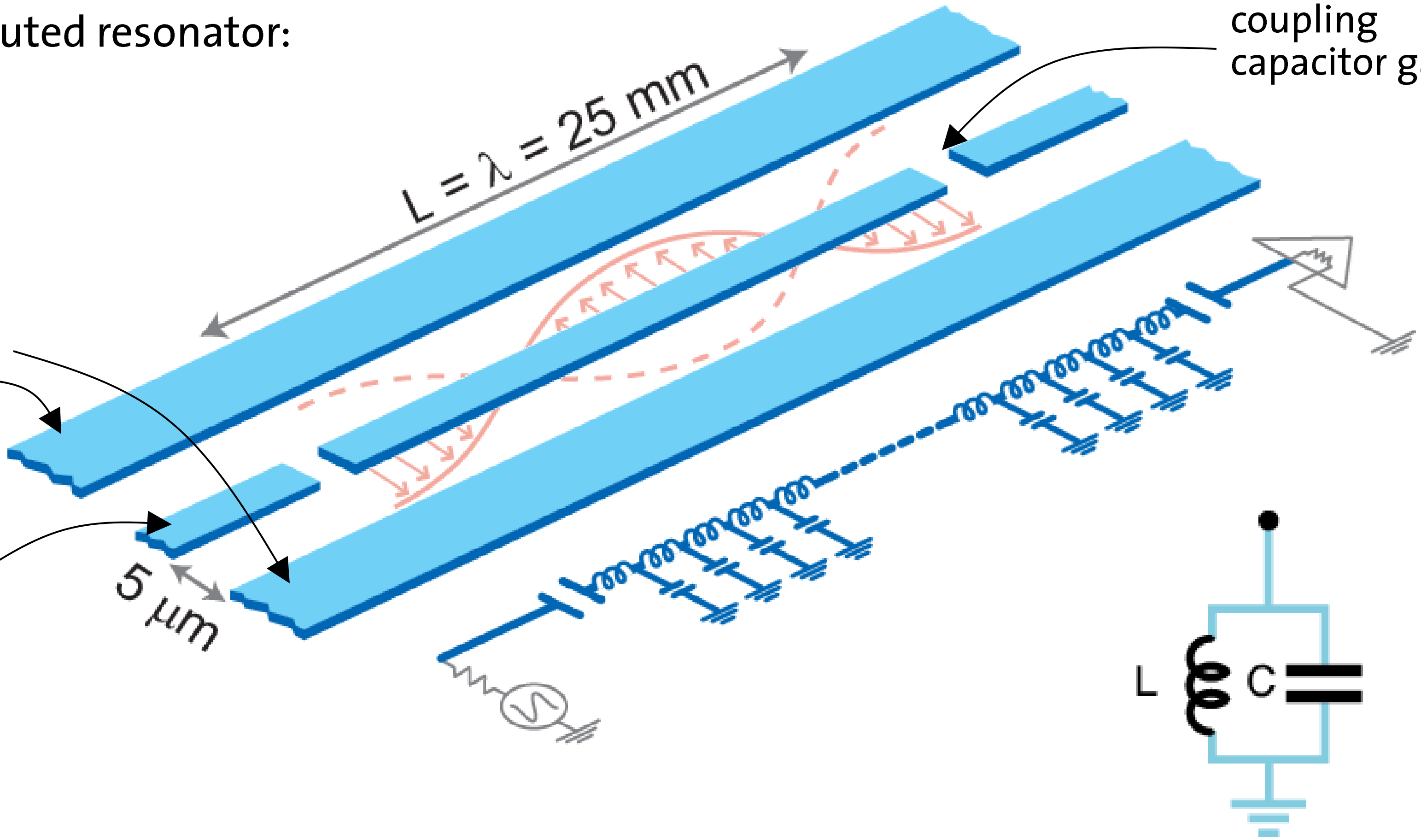
planar transmission line resonator:



A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Realization of H.O.: Transmission Line Resonator

distributed resonator:

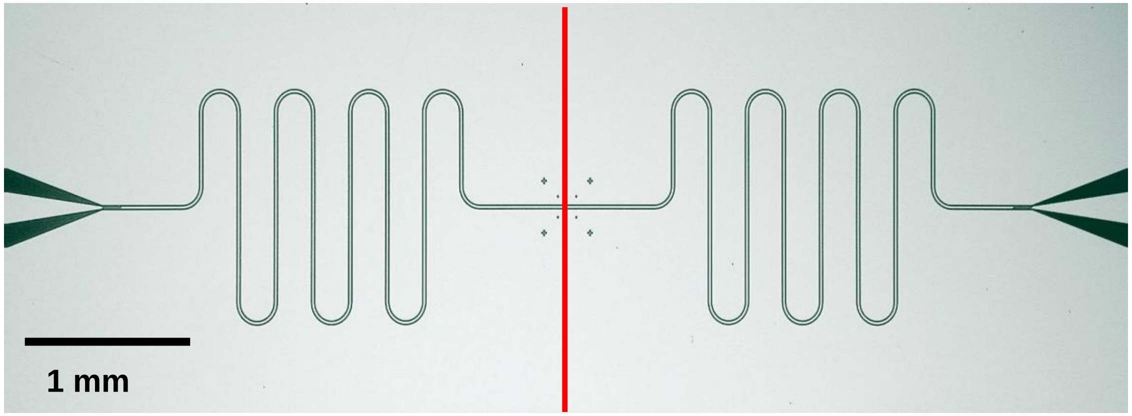


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

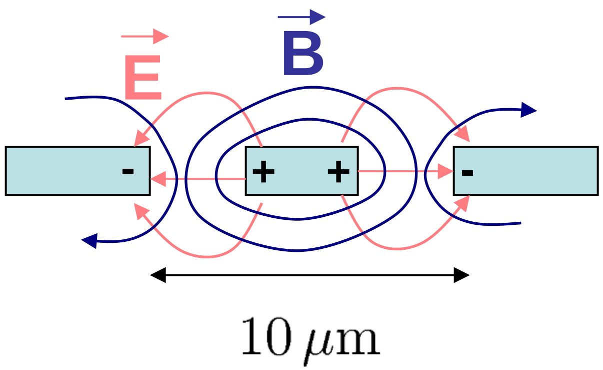
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

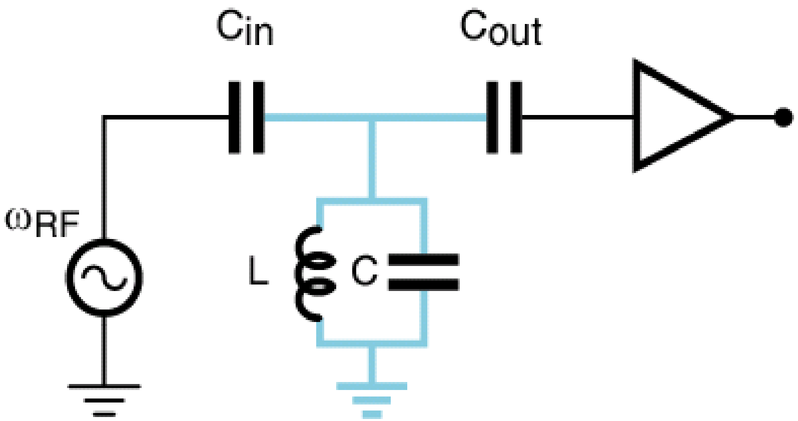
coplanar waveguide:



cross-section of transm. line (TEM mode):

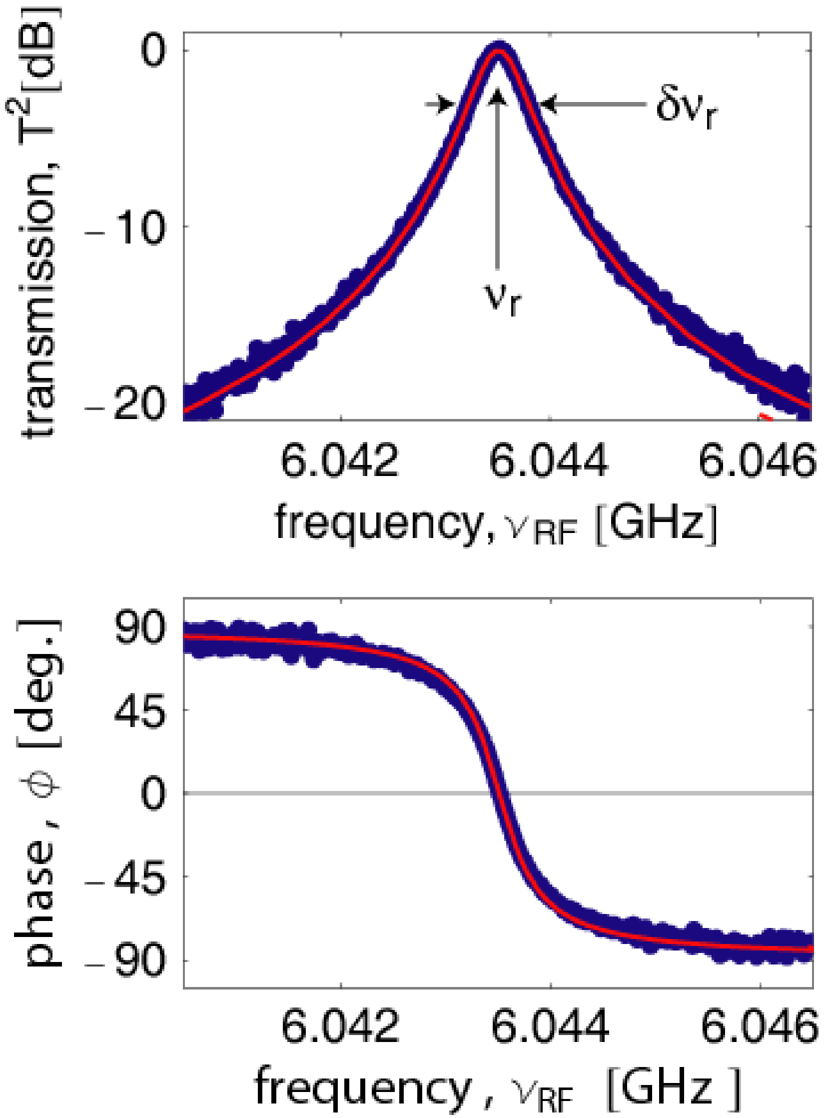


measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

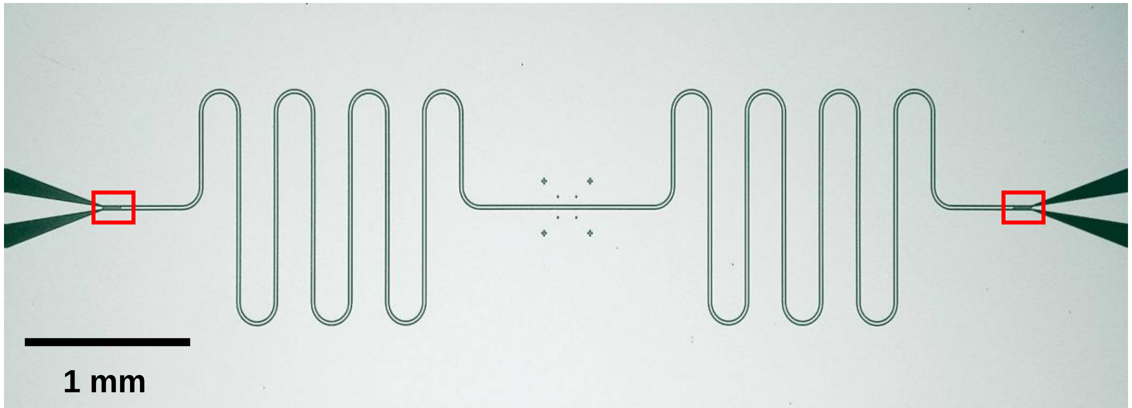
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

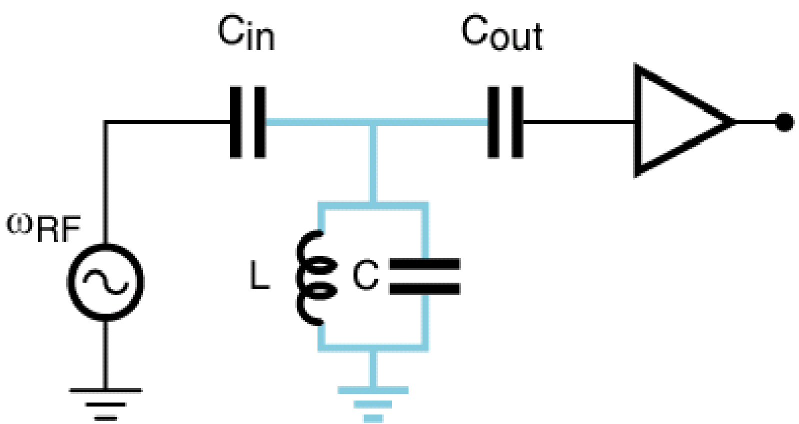
photon lifetime:

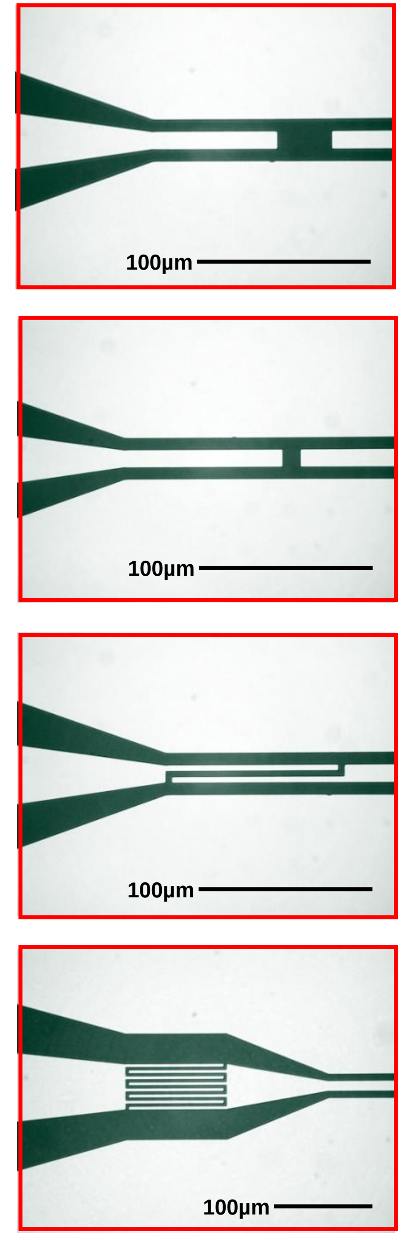
$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Controlling the Photon Life Time

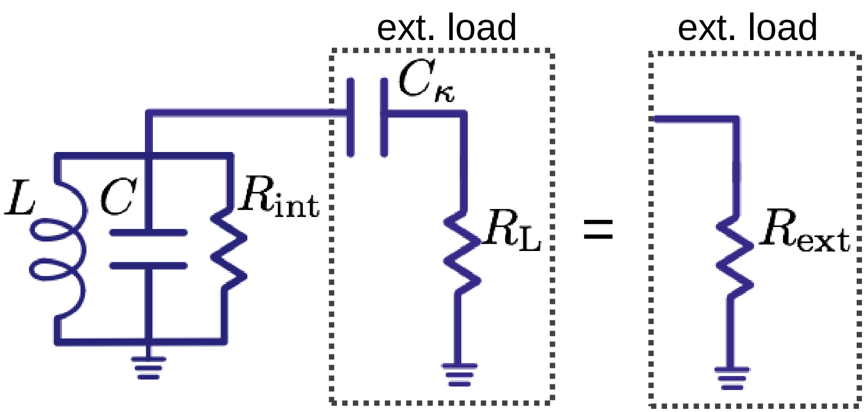


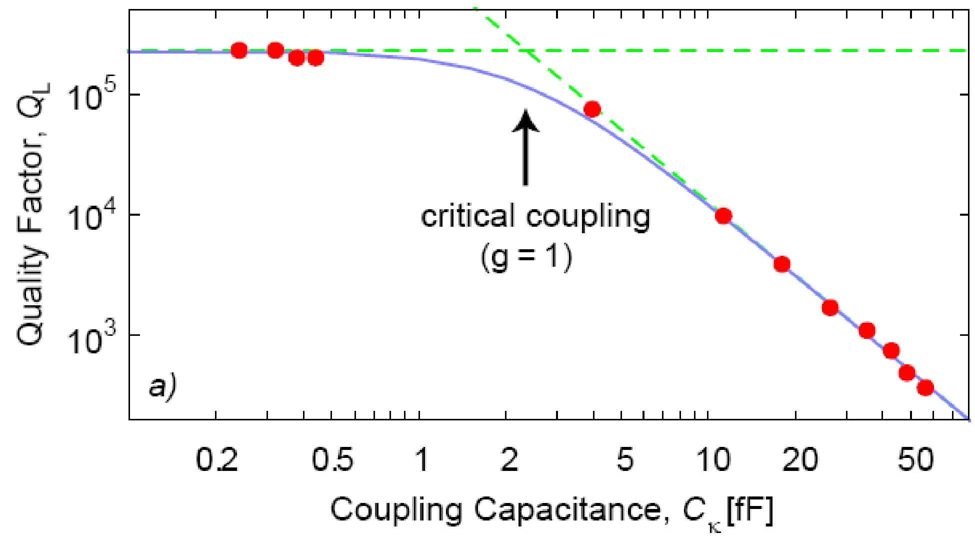


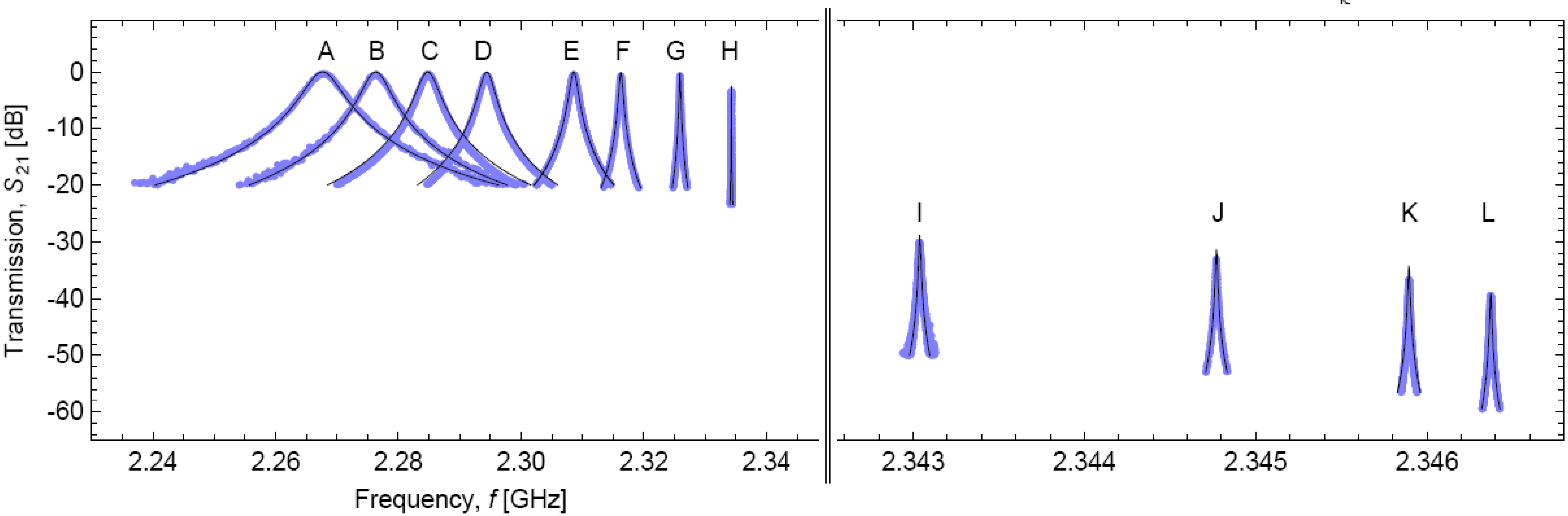


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Quality Factor Measurement





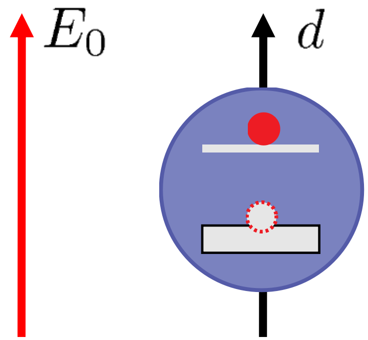


M. Goeppl *et al.*, *J. Appl. Phys.* **104**, 113904 (2008)

Coupling a Harmonic Oscillator to a Qubit

Investigating the Interaction of Light and Matter

challenging on the level of single (artificial) atoms and single photons



- mode-matching (controlling the absorption probability)
- single photon fields E_o (small in 3D)
- dipole moment d (usually small ~ ea_o)
- photon/dipole interaction $\hbar g \sim dE_0$ (usually small)

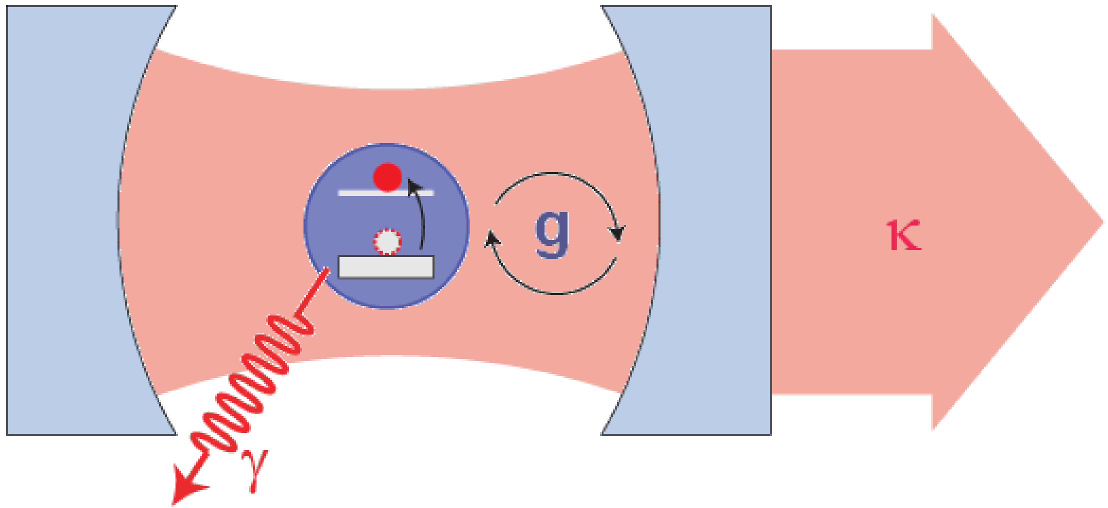
What to do?

- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit $(g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}})$

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

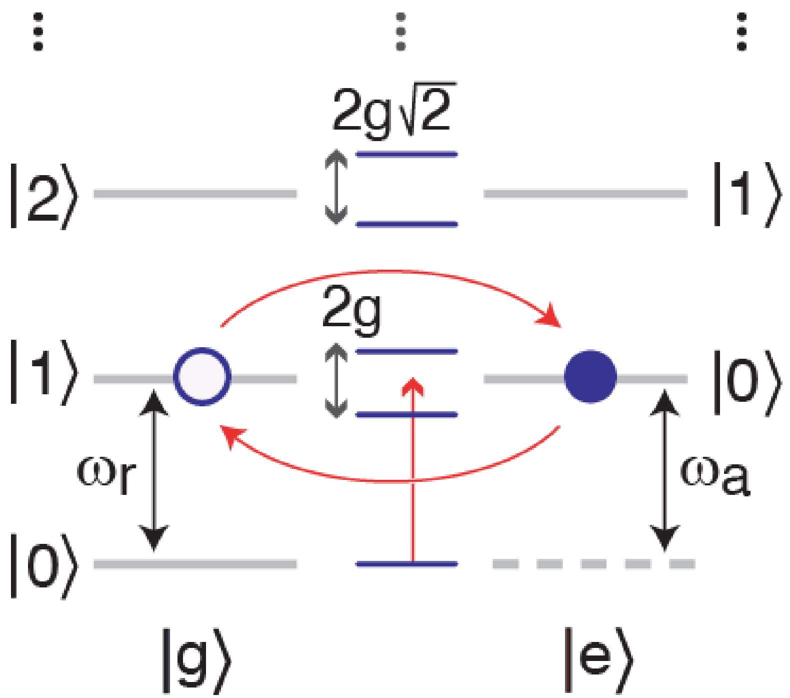
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

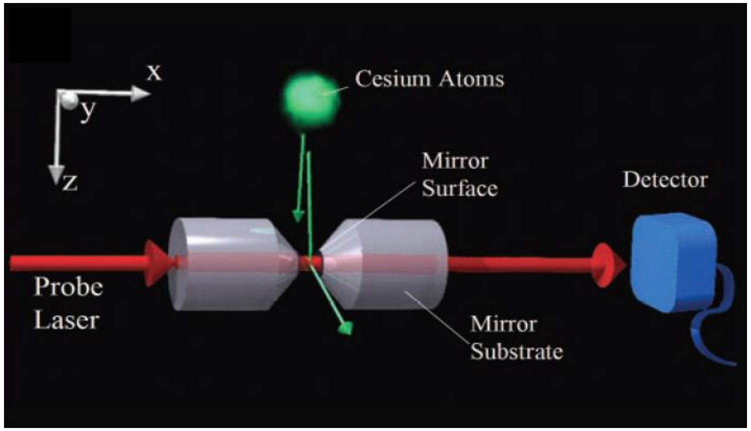
$$g = \frac{dE_0}{\hbar} > \gamma,\ \kappa$$



Jaynes-Cummings Ladder

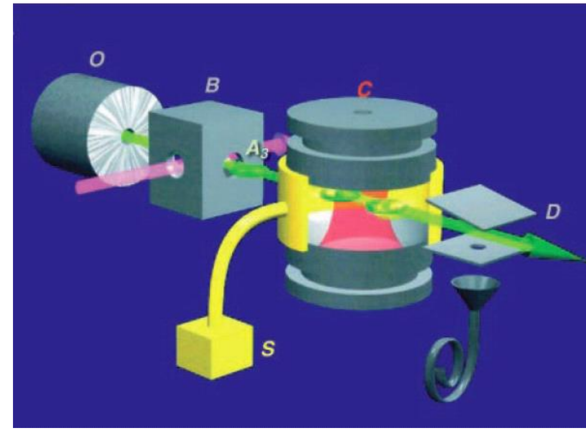
Atomic cavity quantum electrodynamics reviews:
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Cavity Quantum Electrodynamics (QED)



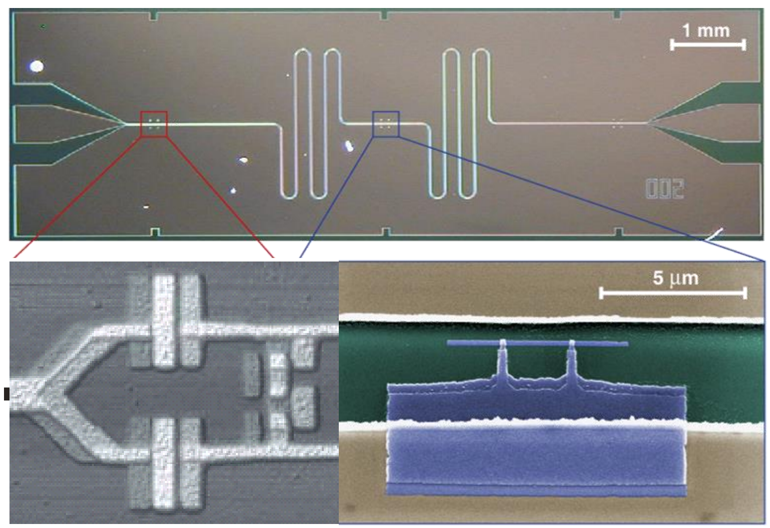
alkali atoms

MPQ, Caltech, ...



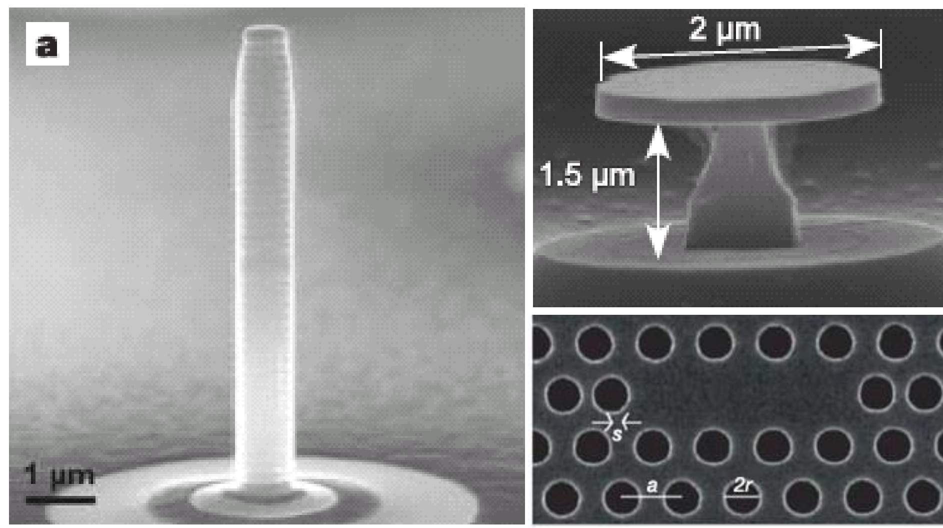
Rydberg atoms

ENS, MPQ, ...



superconductor circuits

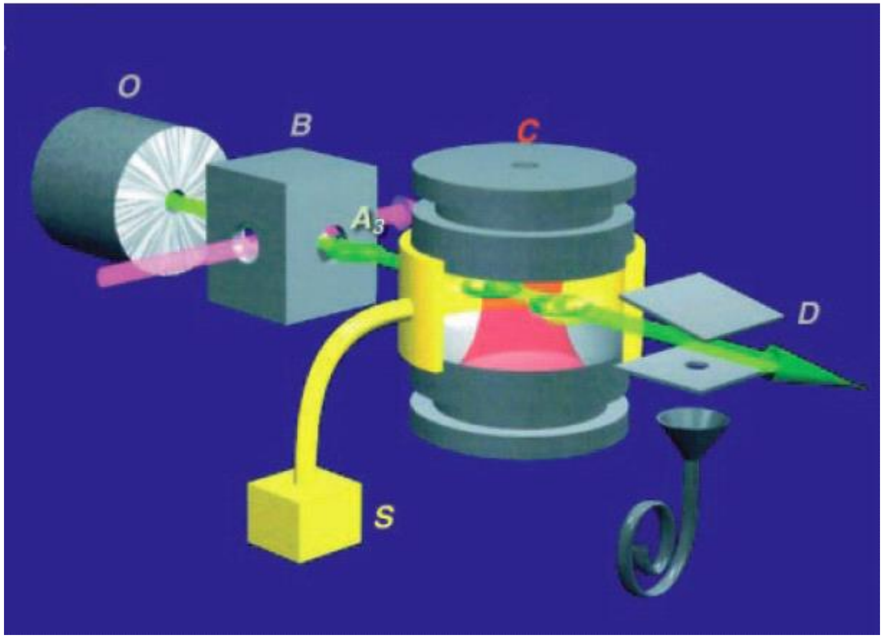
Yale, Delft, NTT, ETHZ, NIST, …

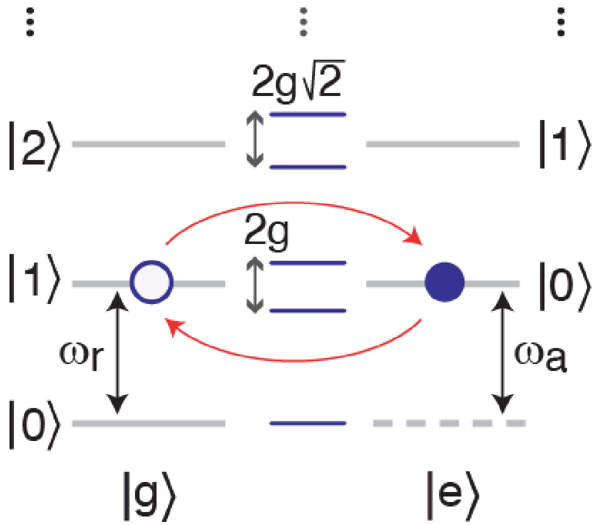


semiconductor quantum dots

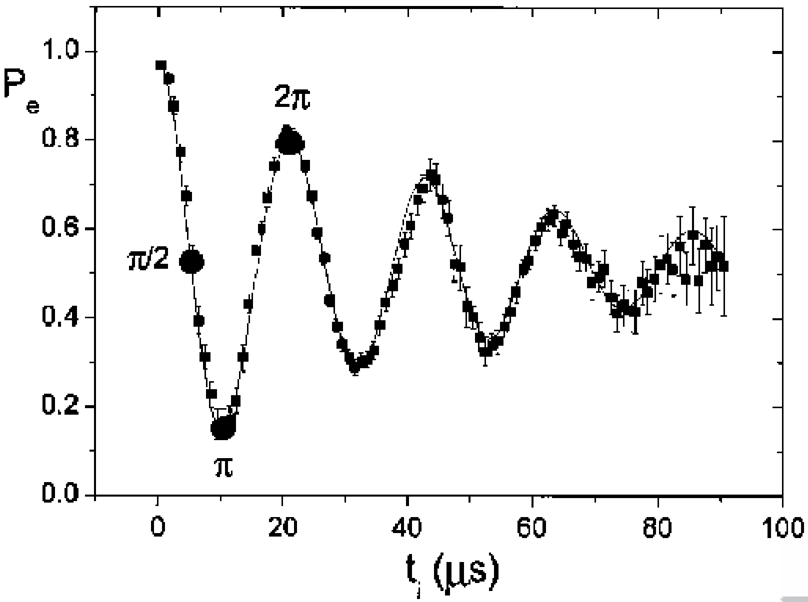
Wurzburg, ETHZ, Stanford …

Vacuum Rabi Oscillations with Rydberg Atoms





Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)



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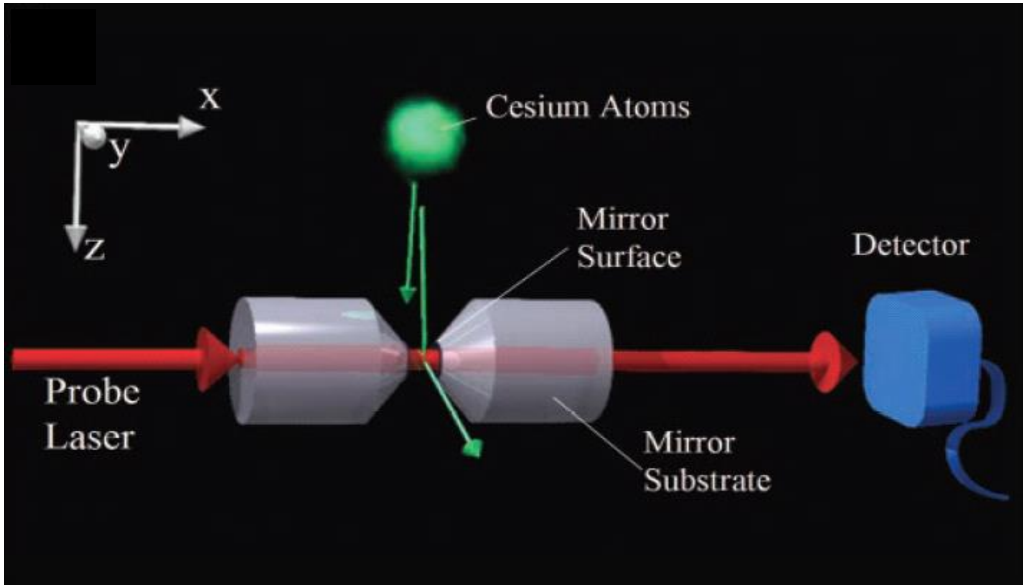
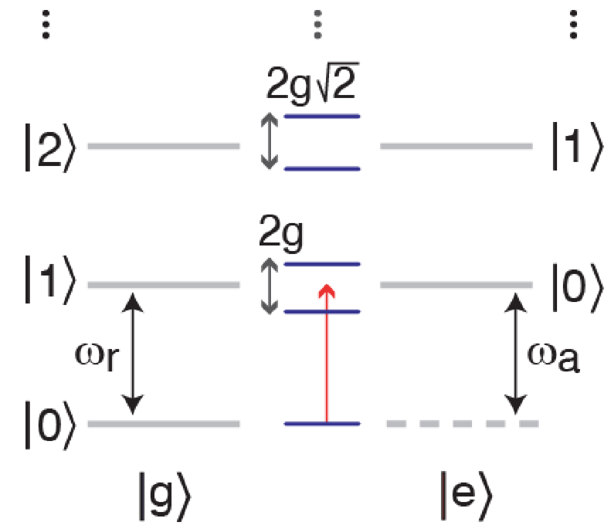
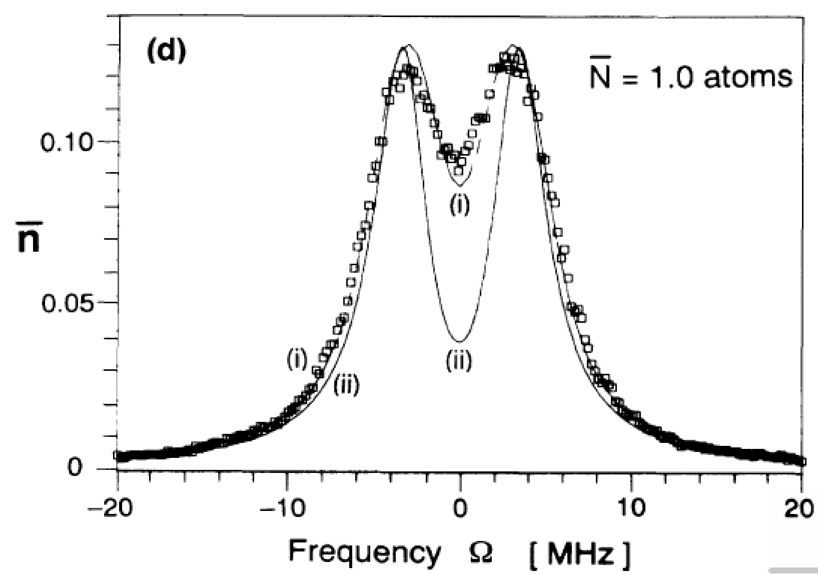
Vacuum Rabi Mode Splitting with Alkali Atoms

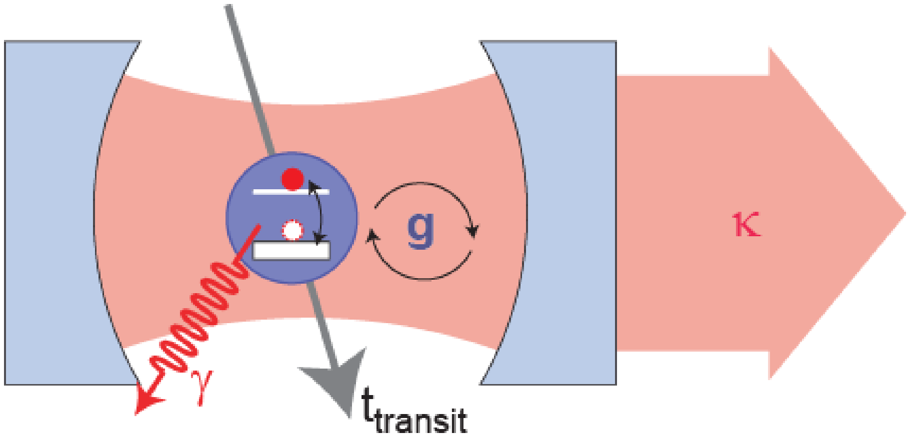




R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble
Phys. Rev. Lett. **93**, 233603 (2004)



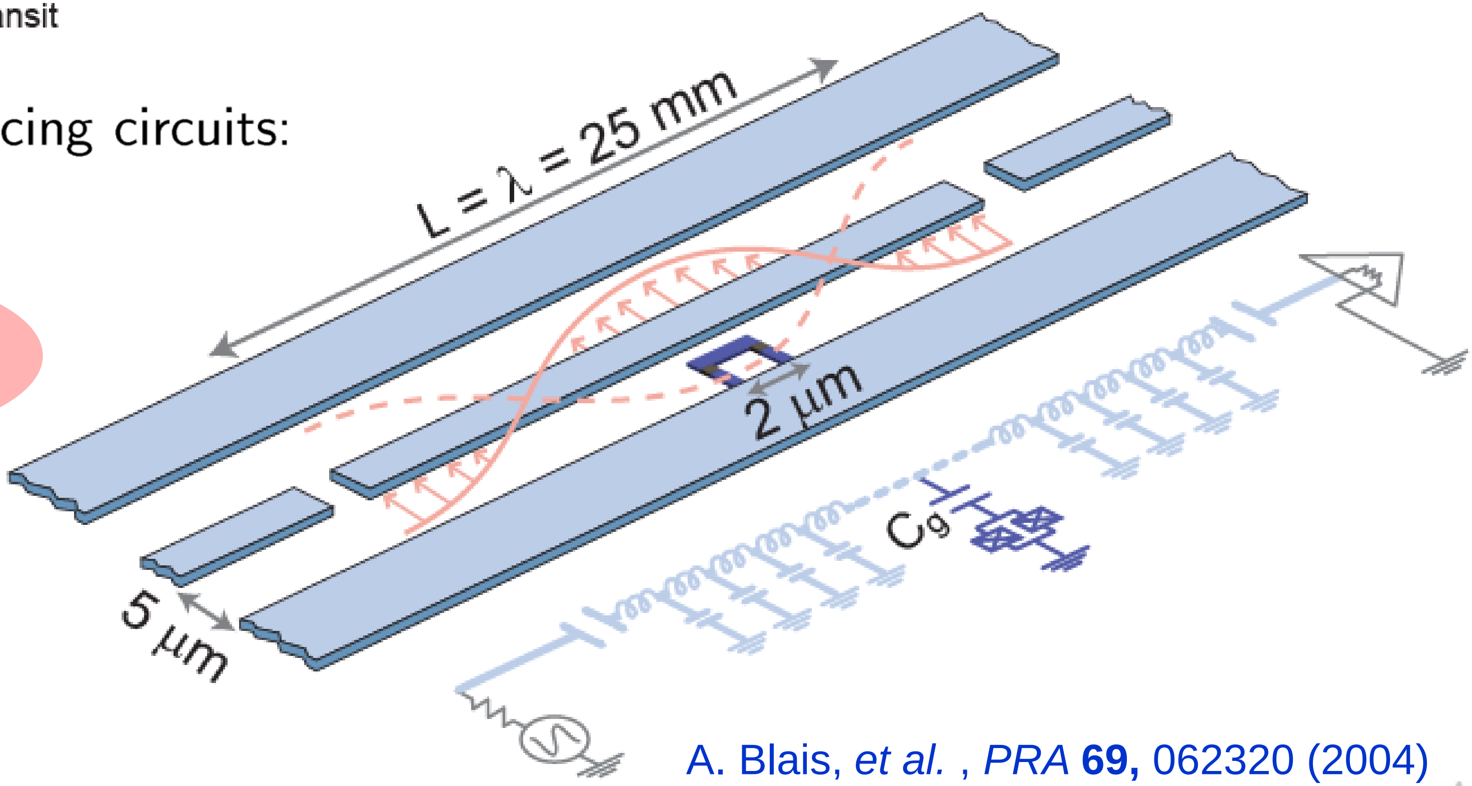
Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

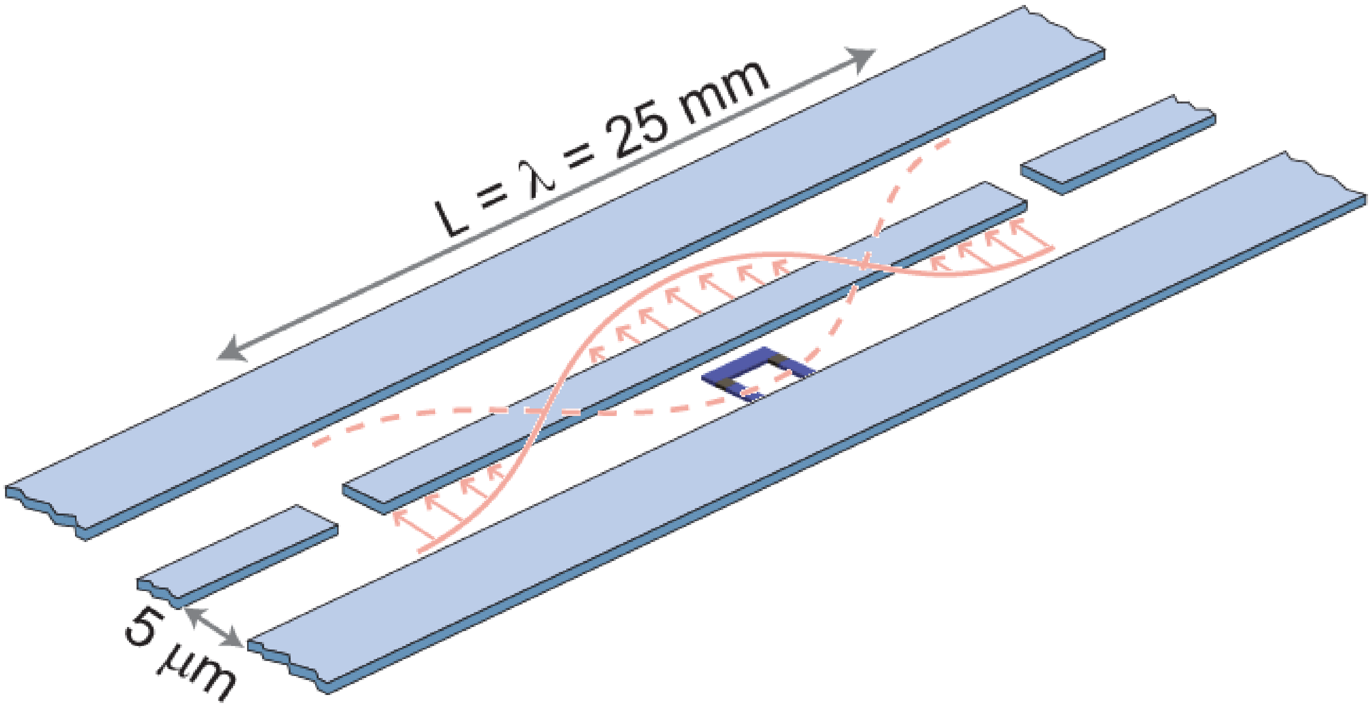
... in superconducing circuits:

circuit quantum electrodynamics



A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics

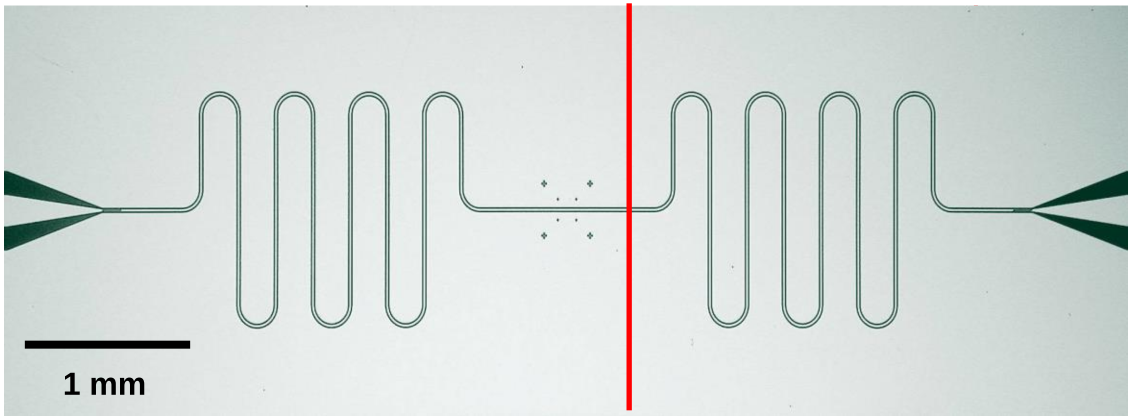


elements

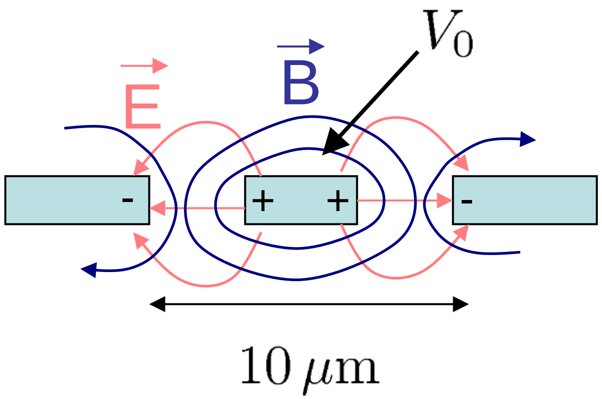
- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_o and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment d and long coherence time $1/\gamma$

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Vacuum Field in 1D Cavity



cross-section
of transm. line (TEM mode):



voltage across resonator in vacuum state $(n = 0)$

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

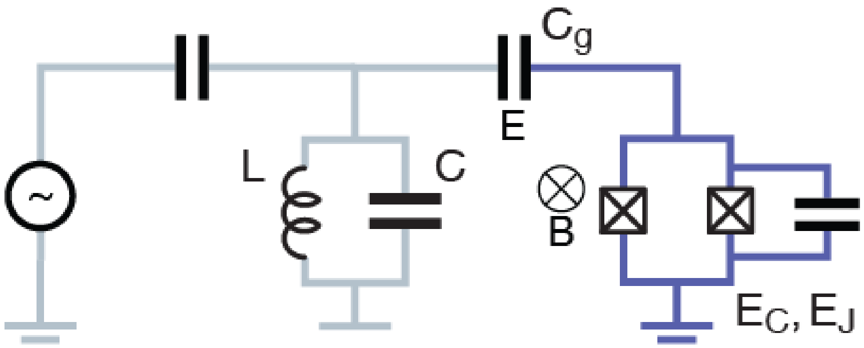
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

$\times 10^6$ larger than E_0
in 3D microwave cavity

for $\omega_r / 2\pi \approx 6\,\mathrm{GHz}$ $(C \sim 1\,\mathrm{pF})$, $b \approx 5\,\mu\mathrm{m}$

Qubit/Photon Coupling



Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy N_g = 1/2 and change of qubit basis (*z* to *x*, *x* to *-z*)

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

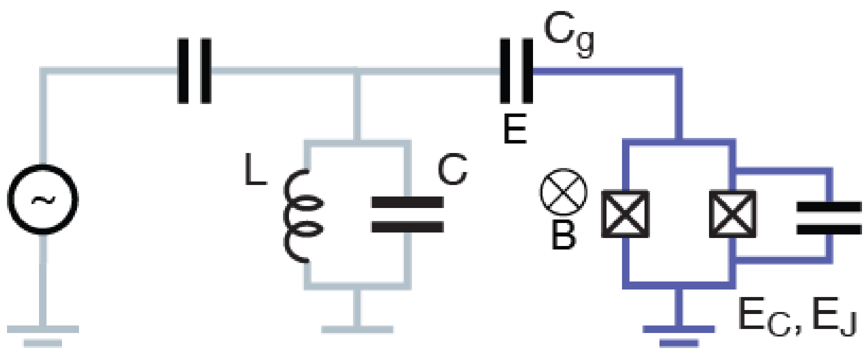
$$\hat{H}_g = \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \cancel{\hat{a}\hat{\sigma}^-} + \cancel{\hat{a}^\dagger\hat{\sigma}^+} + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma}2e\sqrt{\frac{\hbar\omega_r}{2C}}$

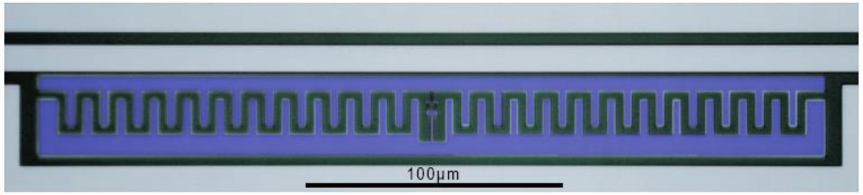
Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1 \ldots 300\,\mathrm{MHz}$

$$g \gg [\kappa, \gamma] \text{ possible!}$$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = eV_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

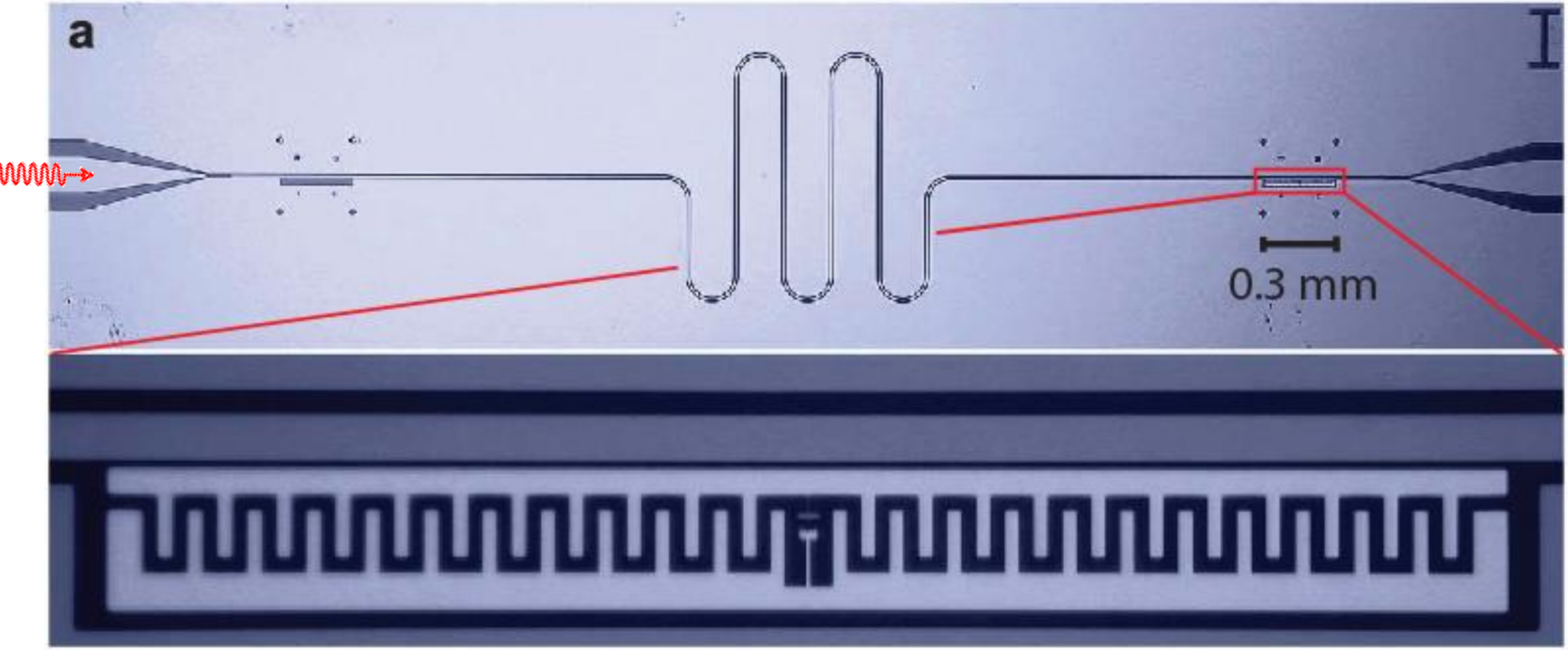
$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

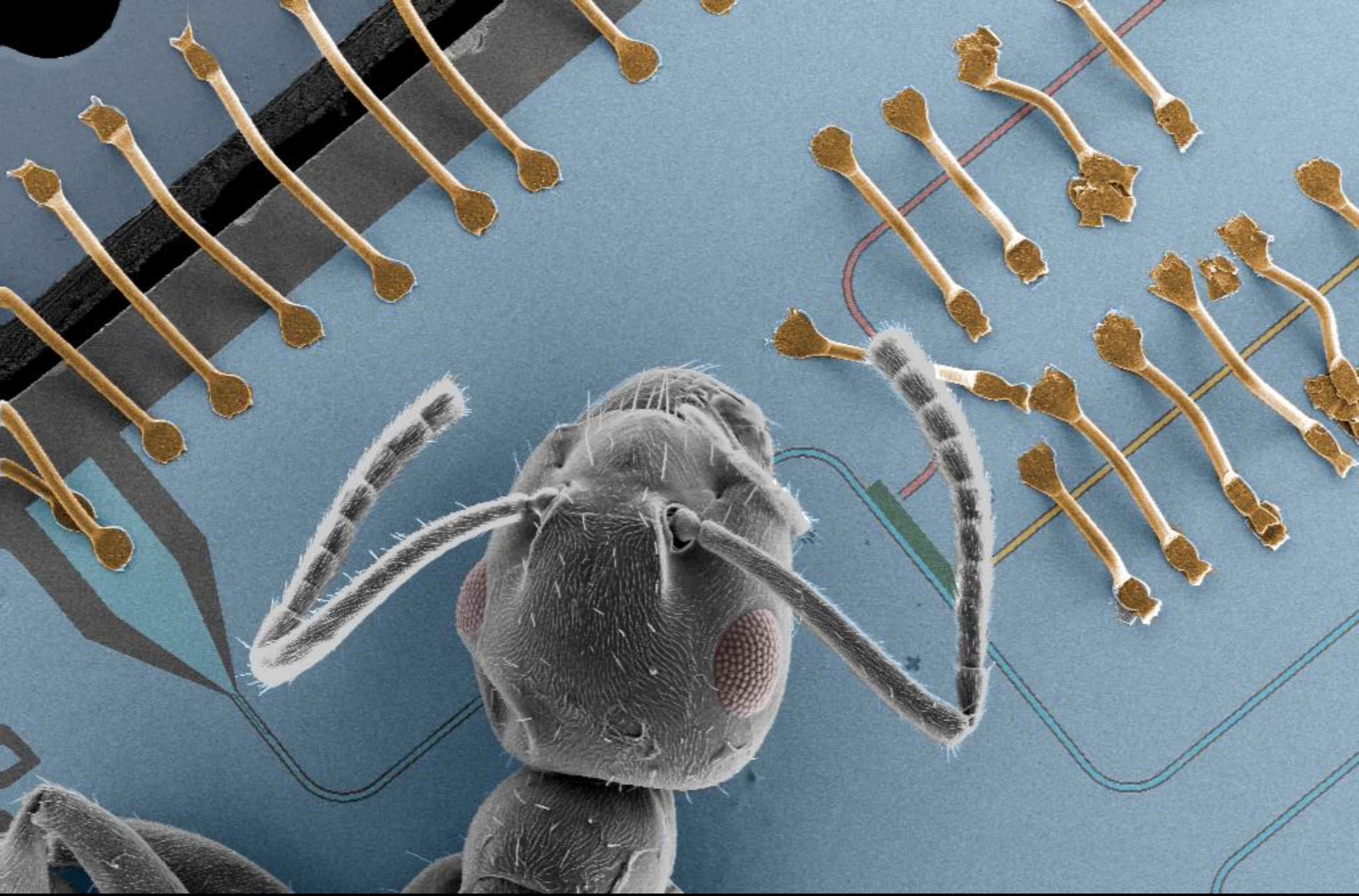
$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, ea_0$$

Circuit QED with One Photon



superconducting cavity QED circuit

A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)



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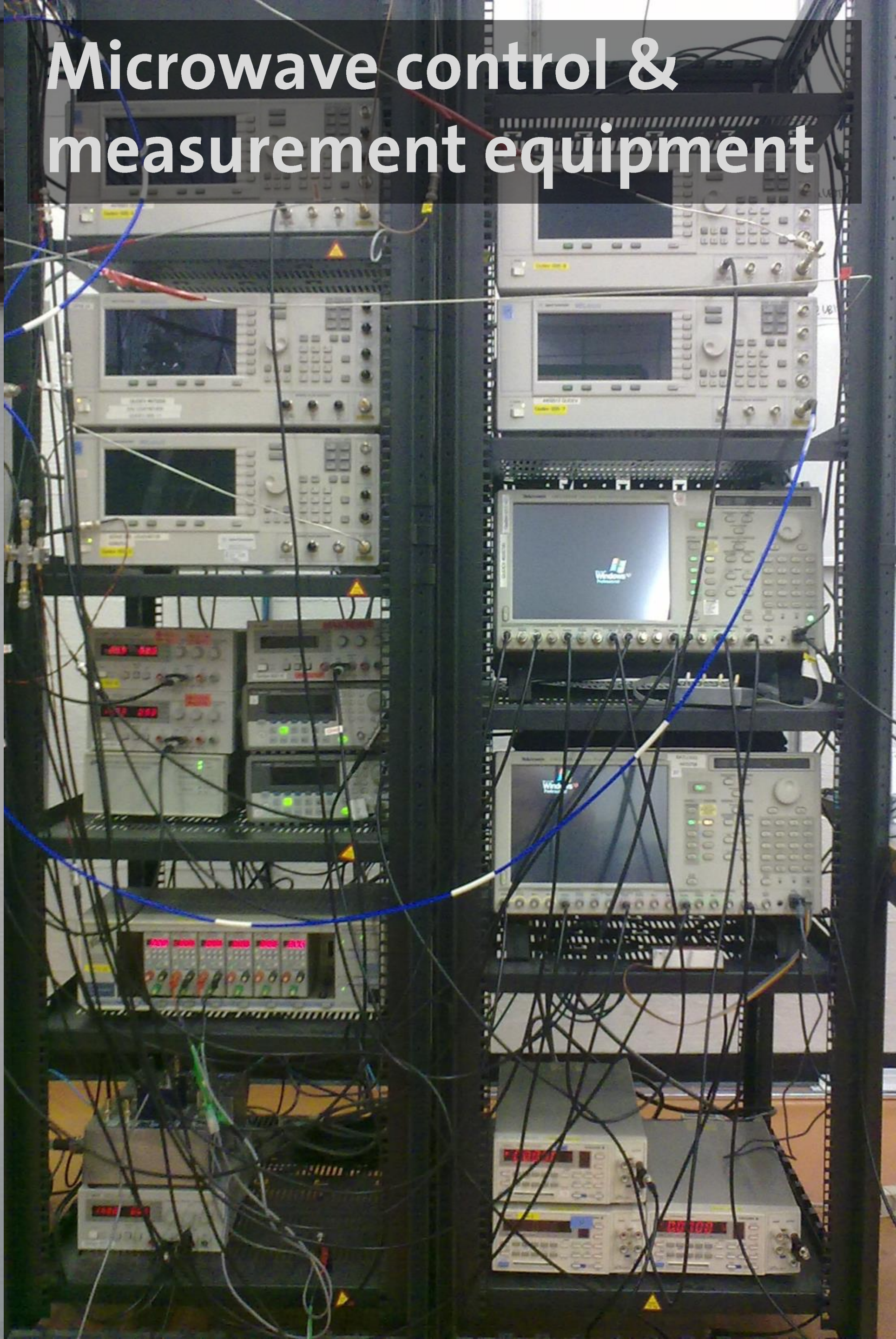
J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

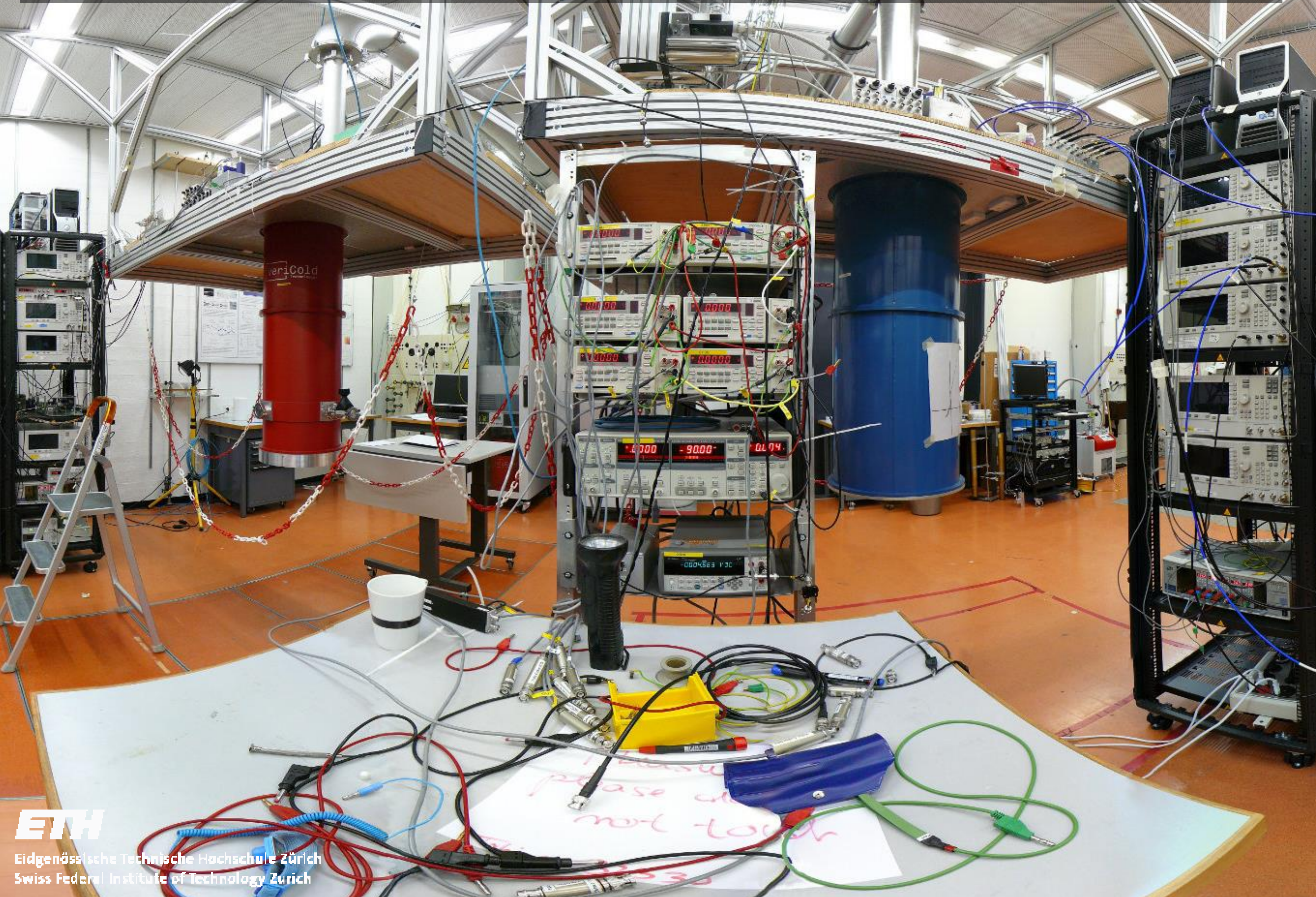
Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)


Cryostate for temperatures
down to 0.02 K
VeriCold
Technologies
~ 20 cm
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich


Microwave control &
measurement equipment

A Circuit QED Lab at ETH Zurich



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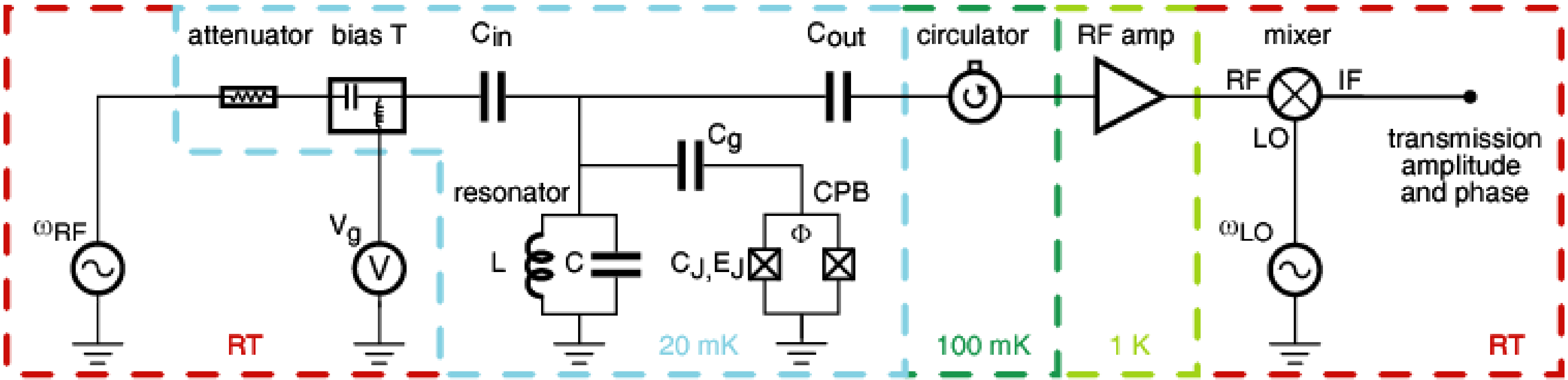
How to Measure Single Microwave Photons

- average power to be detected

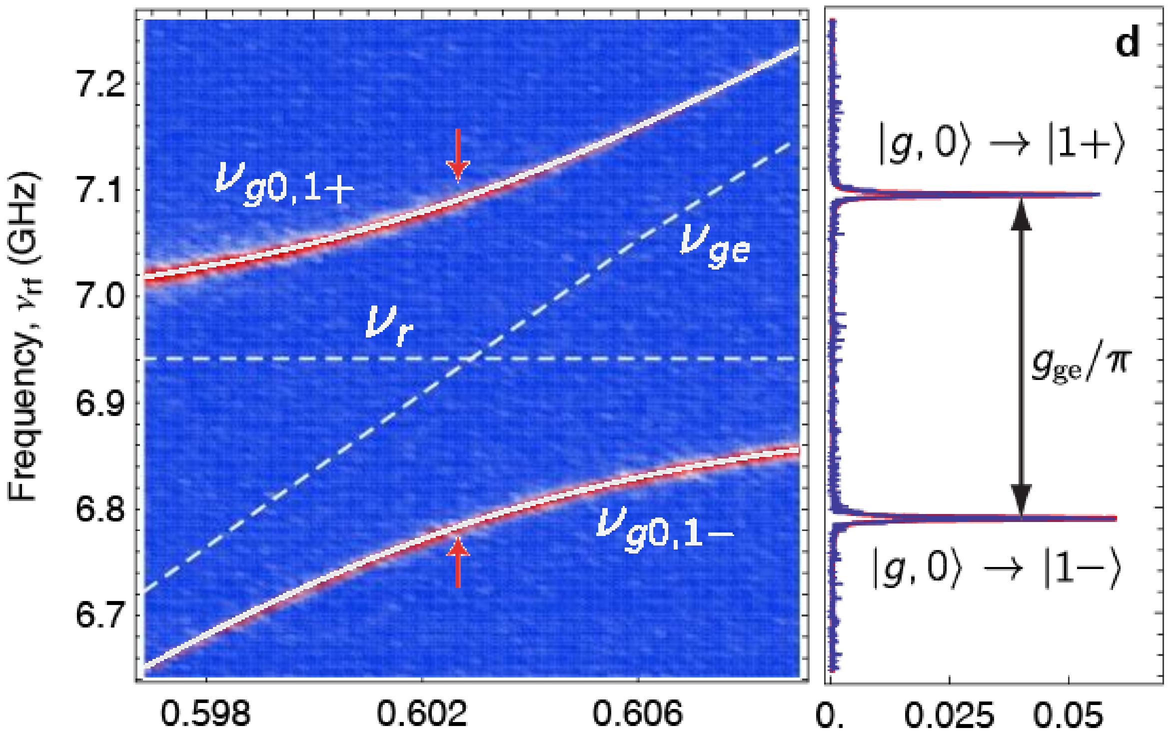
$$\rightarrow \langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

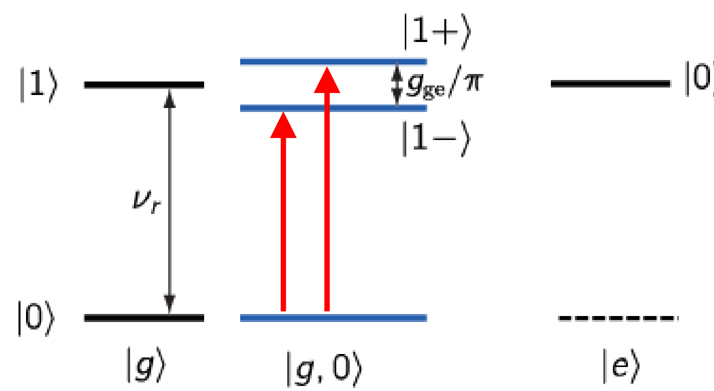


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

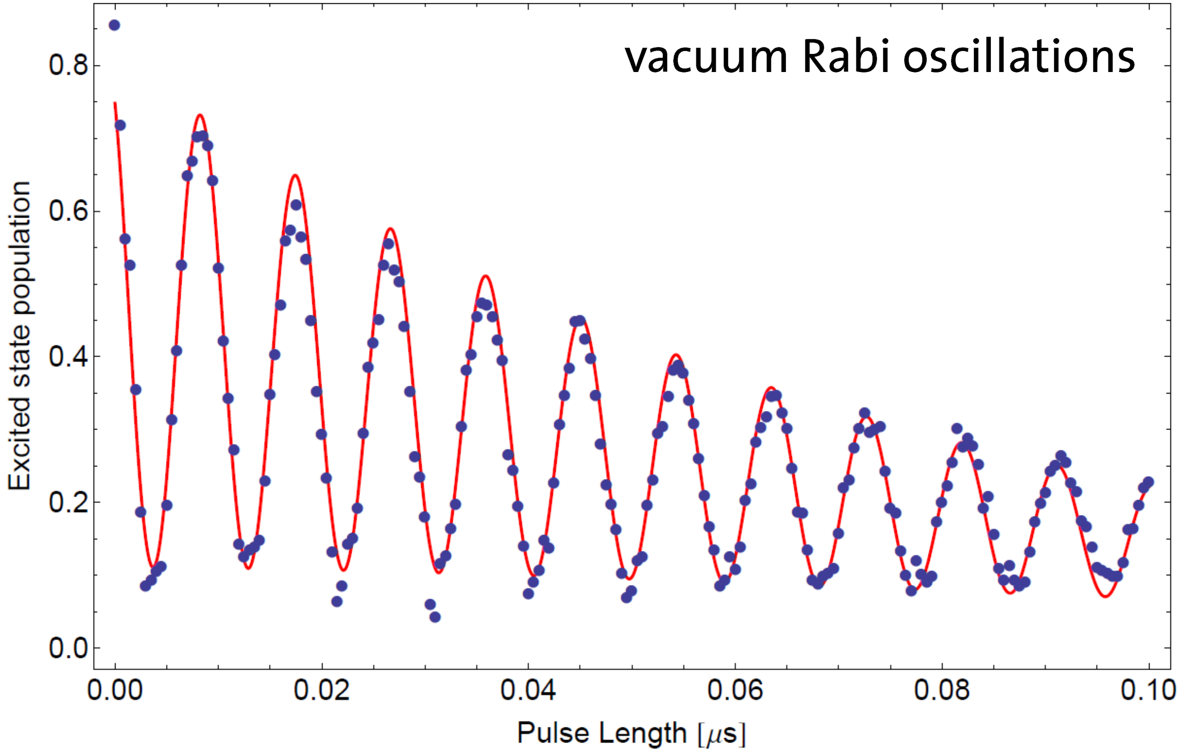
first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):



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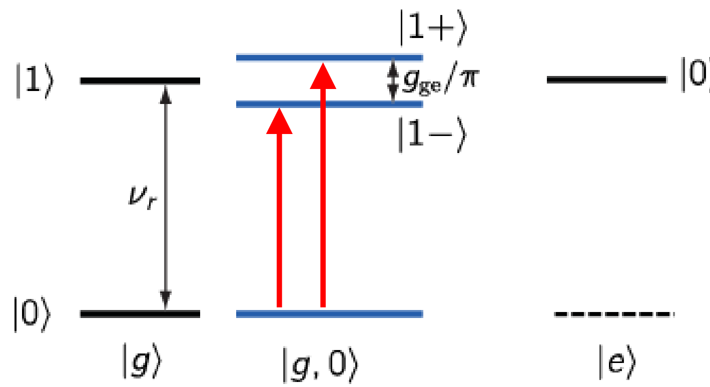

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