# Cavity Quantum Electrodynamics (QED): Coupling a Harmonic Oscillator to a Qubit

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

# What is it good for?

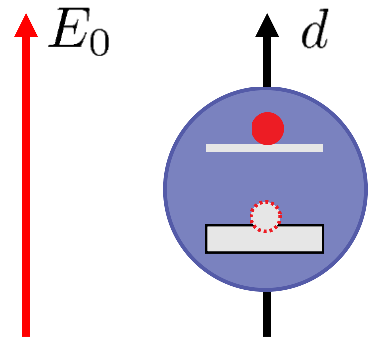
- Isolating a qubit from its environment
- Maintain addressability of a qubit
- Reading out the state of a qubit
- Coupling qubits to each other
- Converting stationary qubits to flying qubits



A. Blais, *et al.* , *PRA* **69,** 062320 (2004)

# Investigating the Interaction of Light and Matter

challenging on the level of single (artificial) atoms and single photons




- mode-matching (controlling the absorption probability)
- single photon fields $E_o$ (small in 3D)
- dipole moment $d$ (usually small ~ $ea_o$)
- photon/dipole interaction $\hbar g \sim dE_0$ (usually small)

What to do?

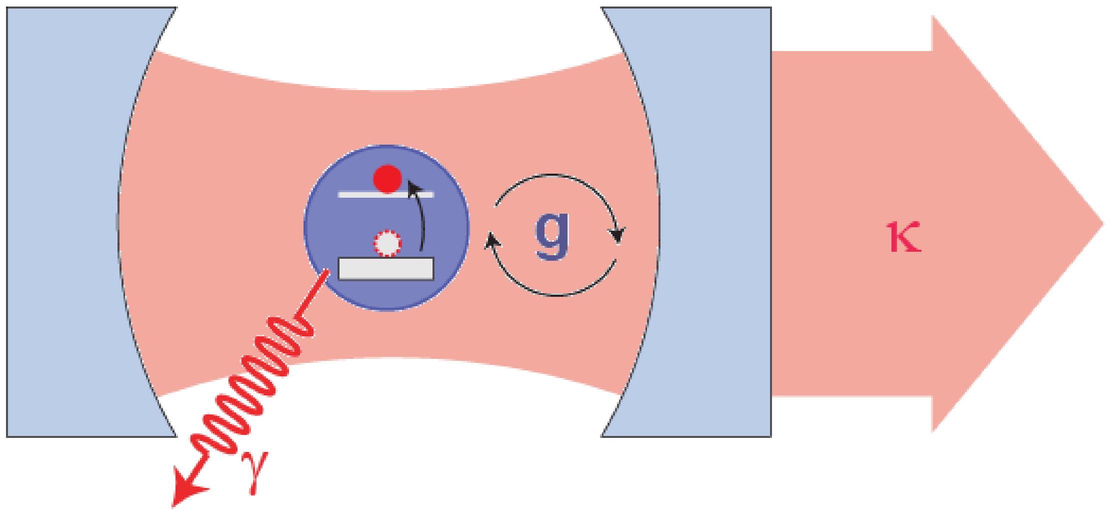
- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits



D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

# Cavity Quantum Electrodynamics

coupling photons to qubits:




Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit $(g = dE_0/\hbar > \gamma,\ \kappa\ ,1/t_{\text{transit}})$



D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

# Dressed States Energy Level Diagram

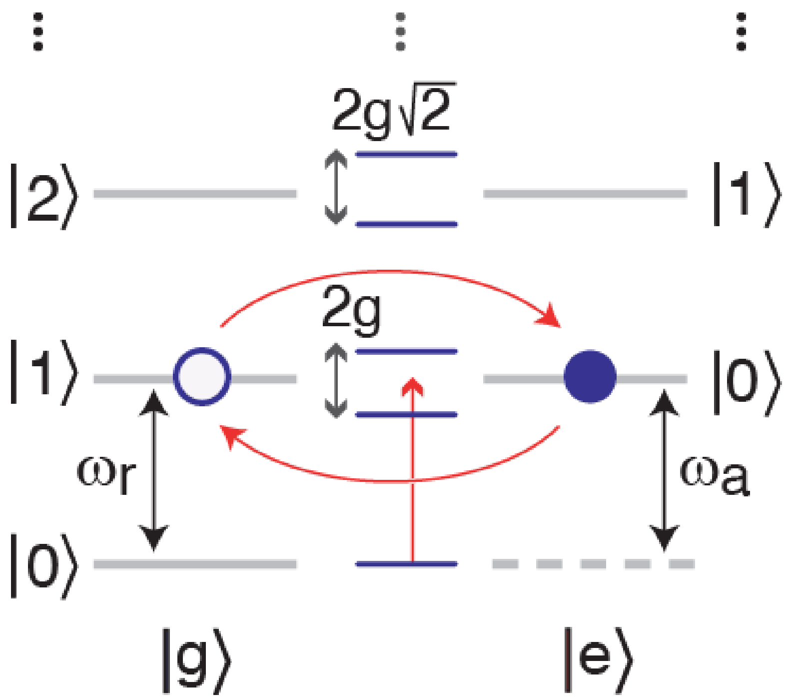
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\ \kappa$$



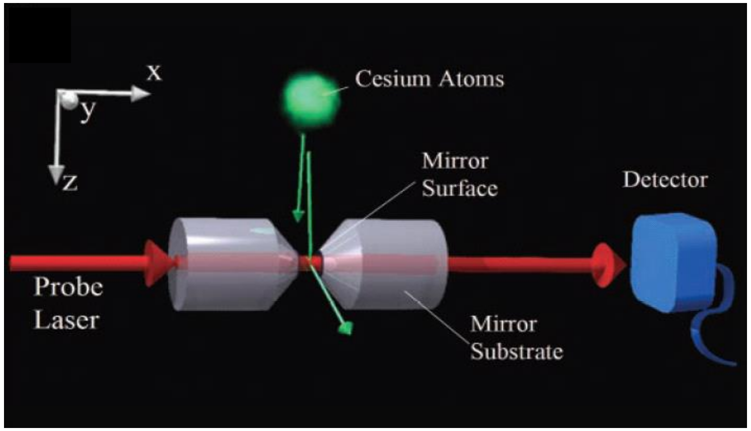

Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

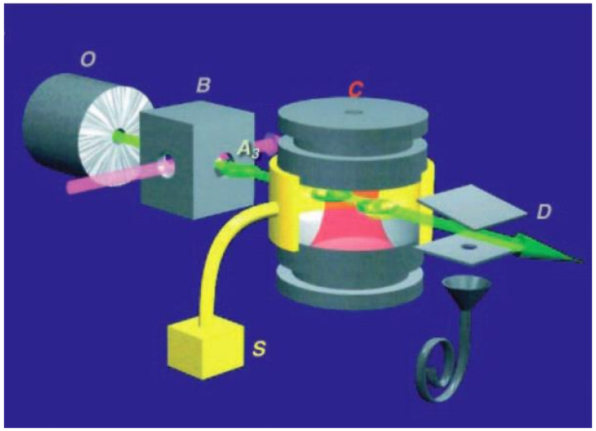
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

# Cavity Quantum Electrodynamics (QED)



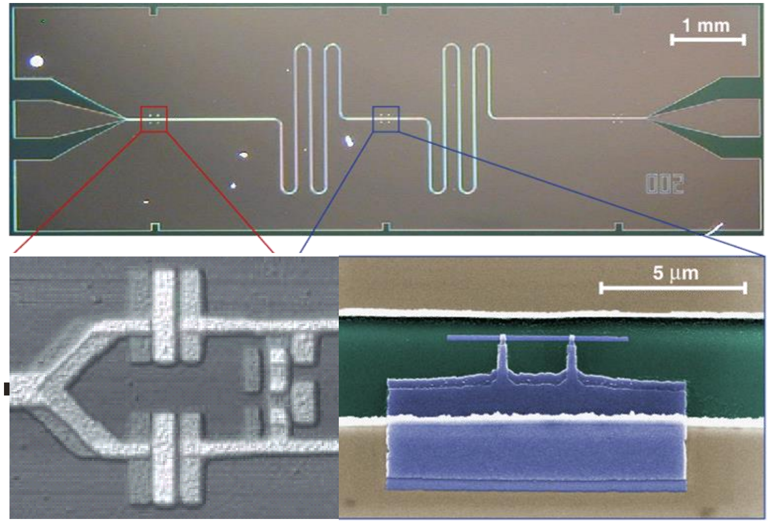

*alkali atoms*

*MPQ, Caltech, ...*



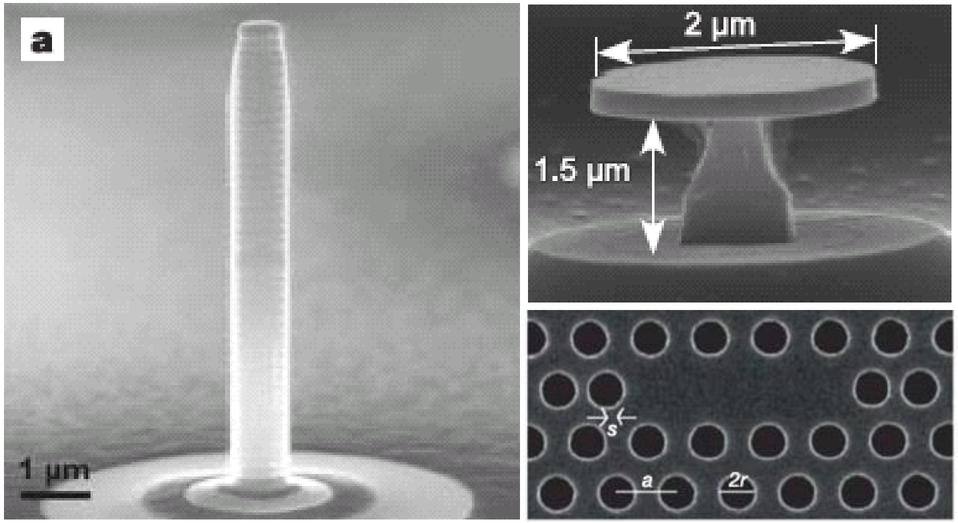

*Rydberg atoms*

*ENS, MPQ, ...*




*superconductor circuits*

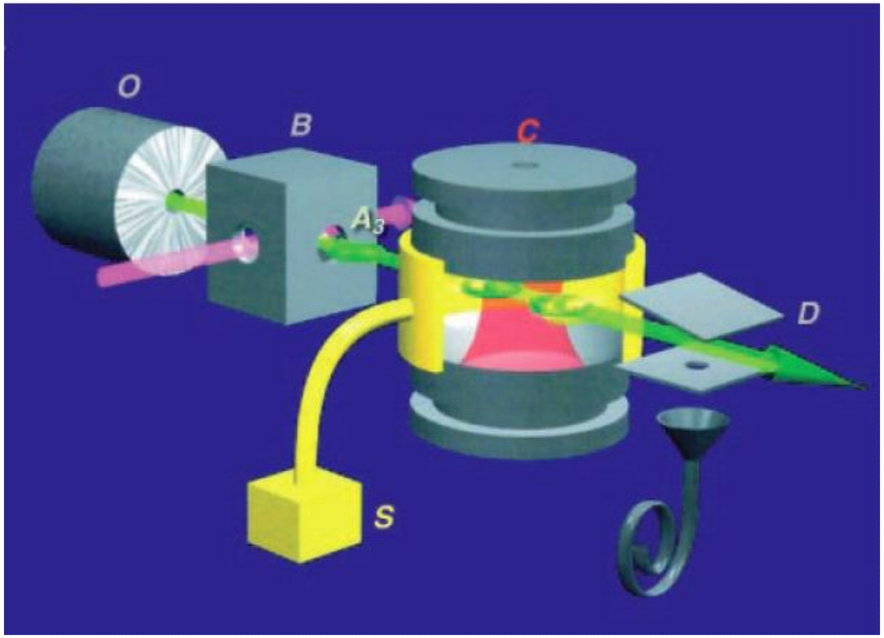
*Yale, Delft, NTT, ETHZ, NIST, …*

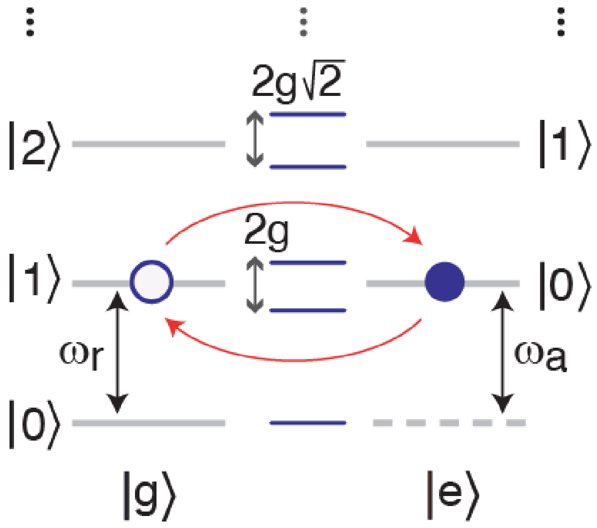



*semiconductor quantum dots*

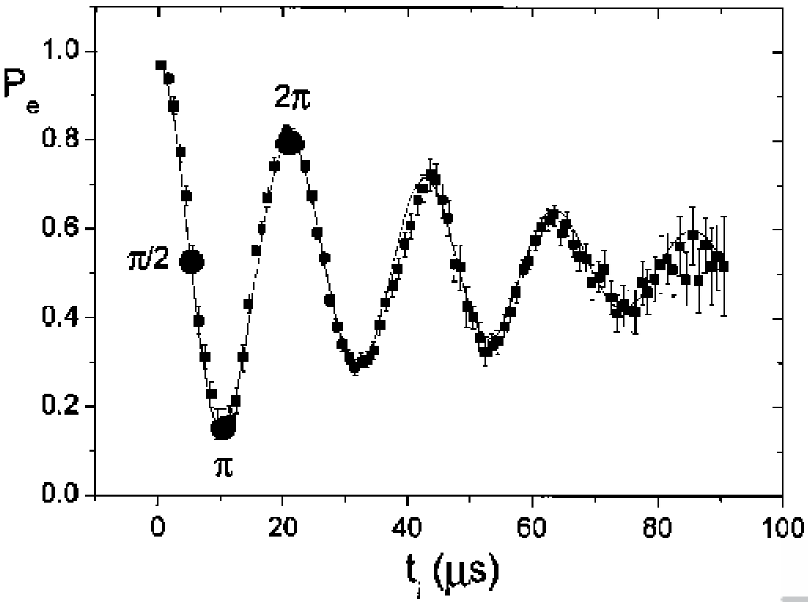
*Wurzburg, ETHZ, Stanford …*

# Vacuum Rabi Oscillations with Rydberg Atoms

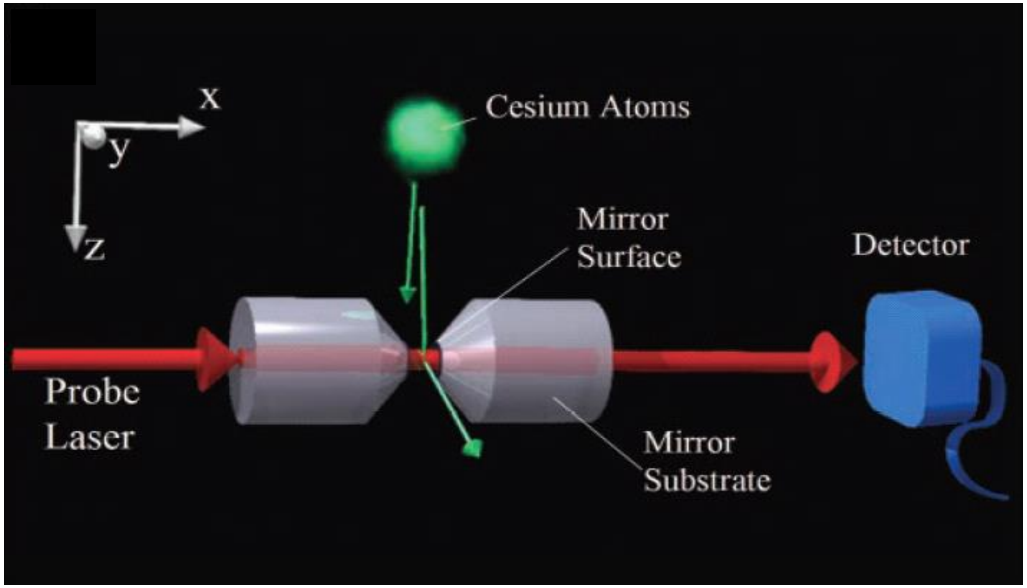


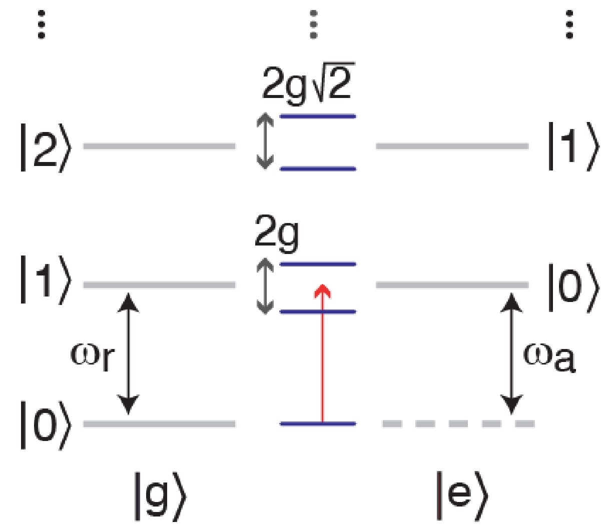




Review: J. M. Raimond, M. Brune, and S. Haroche
*Rev. Mod. Phys.* **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
*Phys. Rev. Lett.* **93**, 103001 (2004)

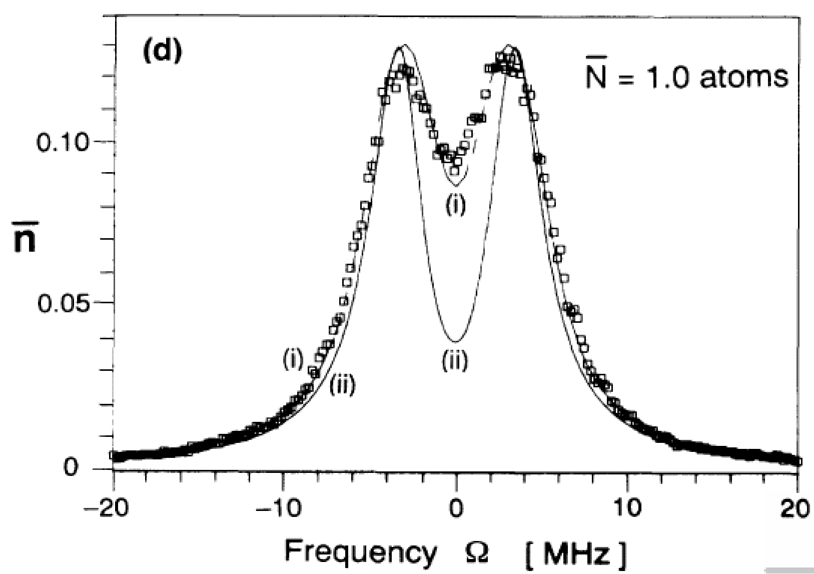


# Vacuum Rabi Mode Splitting with Alkali Atoms

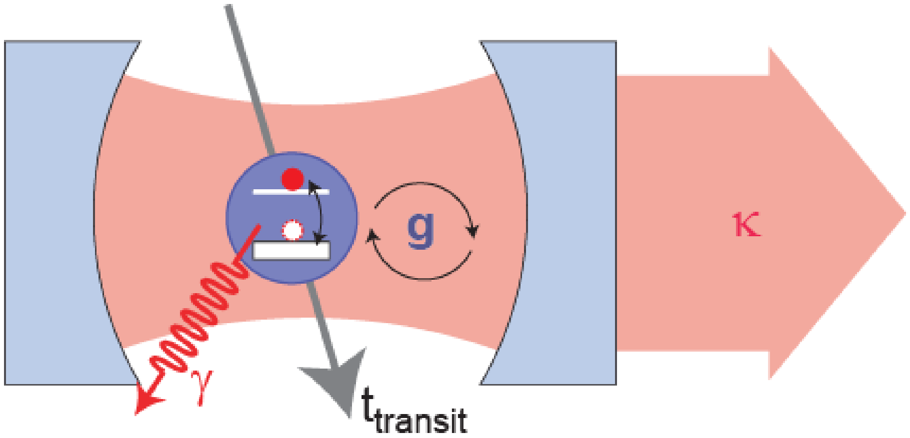



R. J. Thompson, G. Rempe, & H. J. Kimble,
*Phys. Rev. Lett.* **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble,
*Phys. Rev. Lett.* **93**, 233603 (2004)



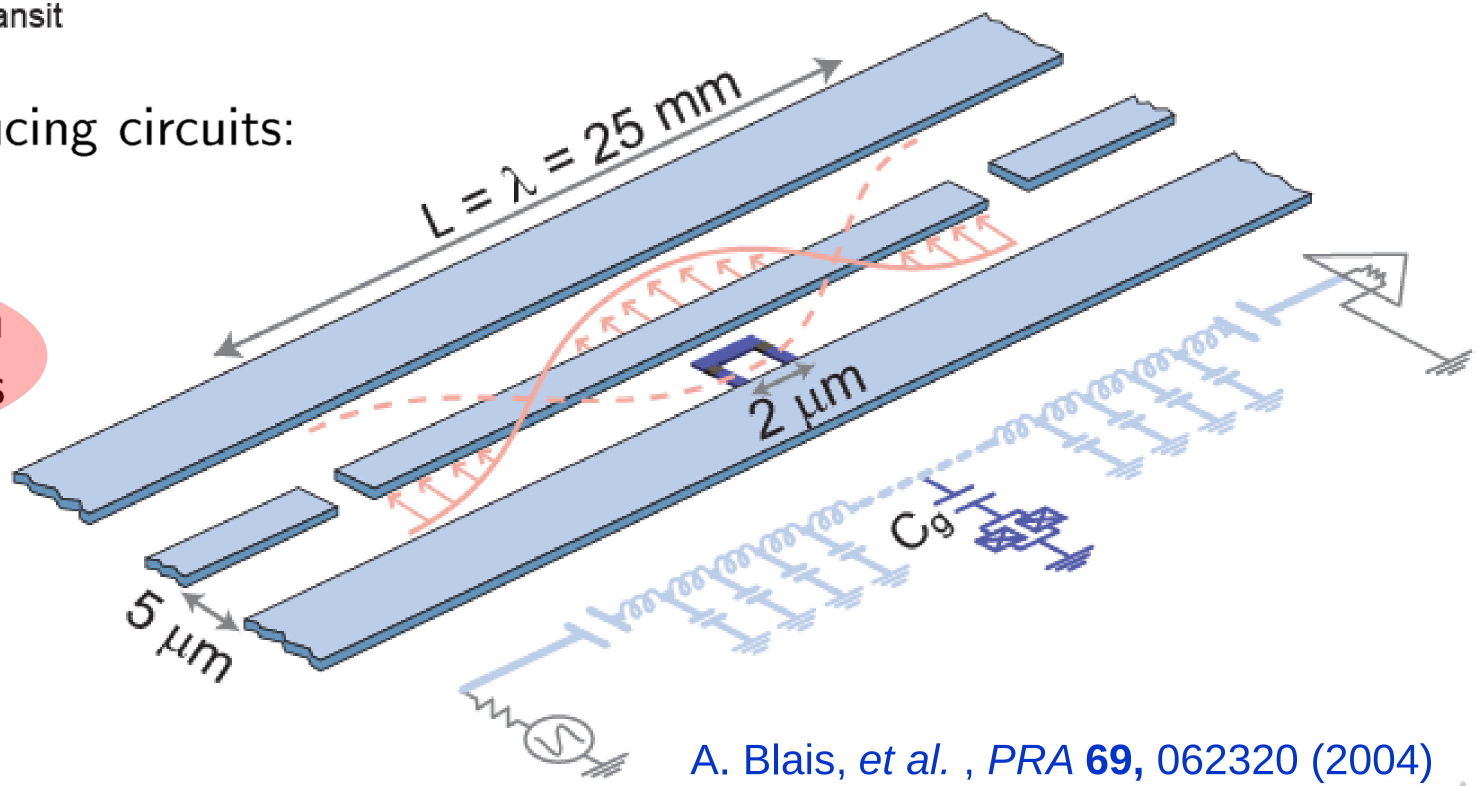
# Cavity QED with Superconducting Circuits




coherent quantum mechanics
with individual photons and qubits ...

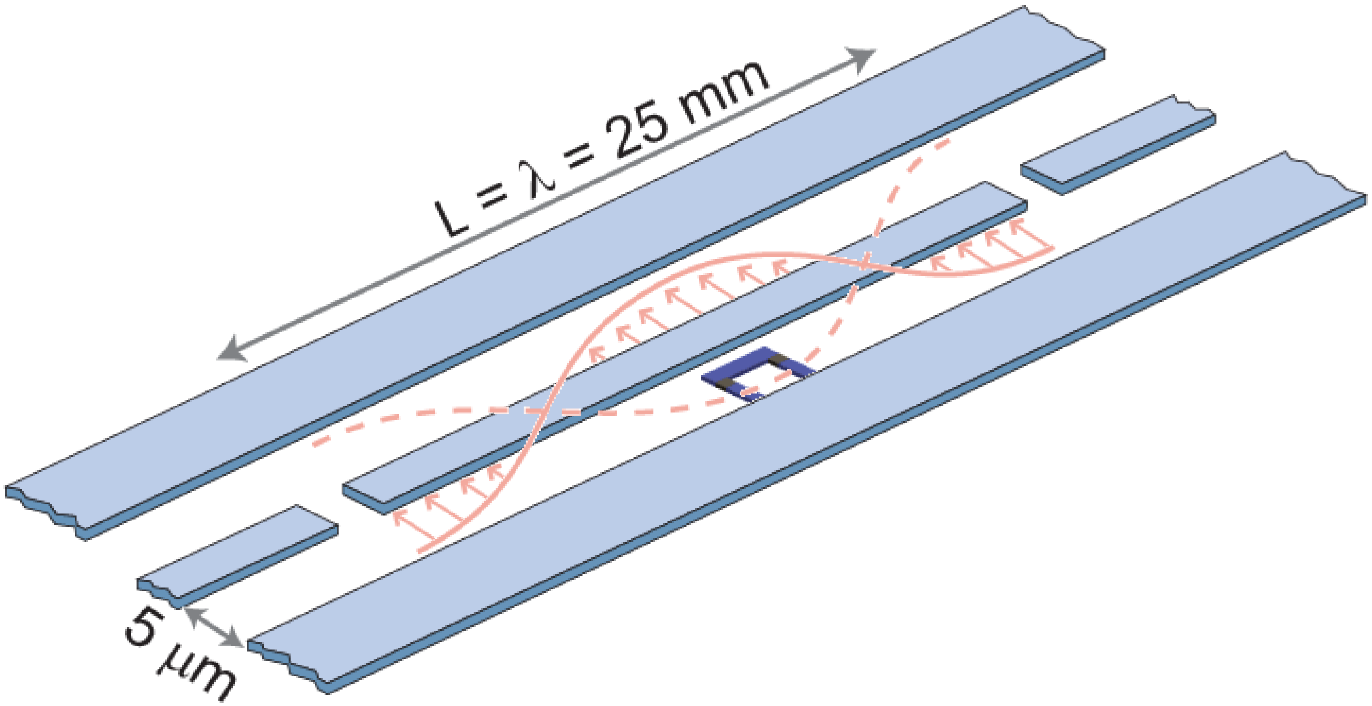
... in superconducing circuits:

circuit quantum electrodynamics




A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

# Circuit Quantum Electrodynamics



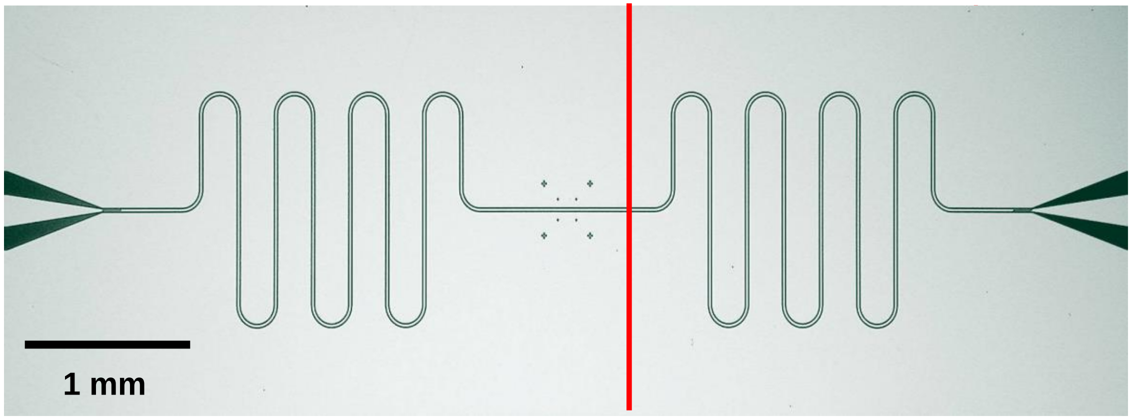

elements

- the cavity: a superconducting 1D transmission line resonator with large vacuum field $E_o$ and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large $E_J/E_C$ with large dipole moment $d$ and long coherence time $1/\gamma$

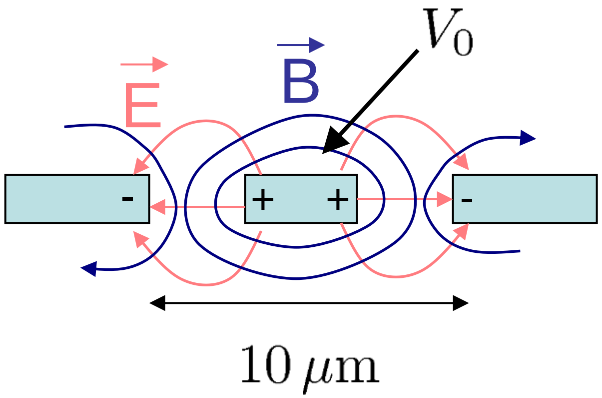


A. Blais *et al.*, *PRA* **69,** 062320 (2004)

# Vacuum Field in 1D Cavity




cross-section
of transm. line (TEM mode):




voltage across resonator in vacuum state $(n = 0)$

$$V_{0,\text{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\text{V}$$

harmonic oscillator

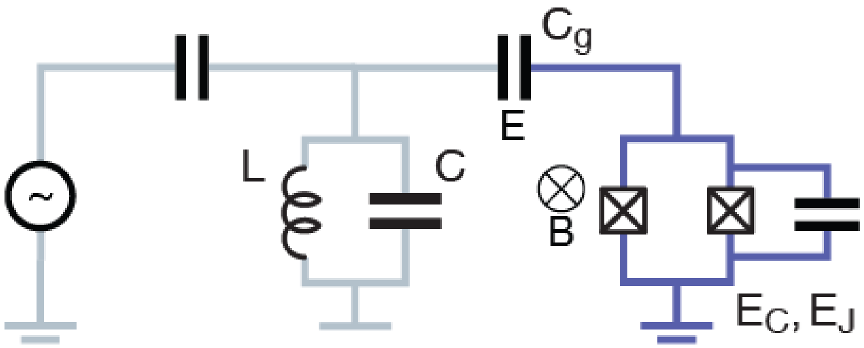
$$H_r = \hbar\omega_r \left( a^\dagger a + \frac{1}{2} \right)$$

$$E_0 = \frac{V_{0,\text{rms}}}{b} \approx 0.2\,\text{V}/\text{m}$$

$\times 10^6$ larger than $E_0$ in 3D microwave cavity

for $\omega_r/2\pi \approx 6\,\text{GHz}$ $(C \sim 1\,\text{pF})$, $b \approx 5\,\mu\text{m}$

# Qubit/Photon Coupling




Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

# Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy $N_g$ = 1/2 and change of qubit basis (*z* to *x*, *x* to *-z*)

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

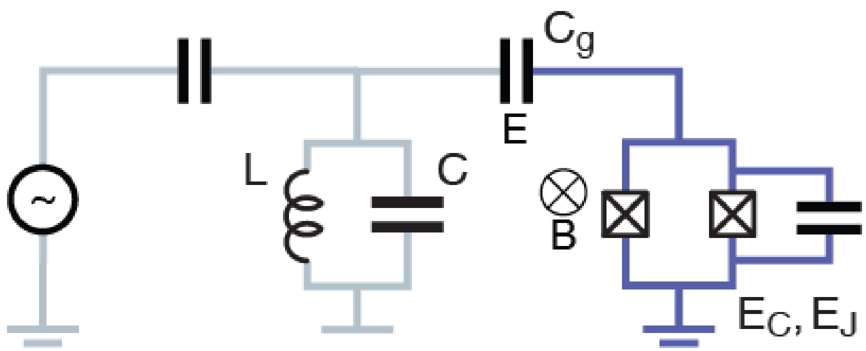
$$\hat{H}_g = \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \cancel{\hat{a}\hat{\sigma}^-} + \cancel{\hat{a}^\dagger\hat{\sigma}^+} + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma}2e\sqrt{\frac{\hbar\omega_r}{2C}}$

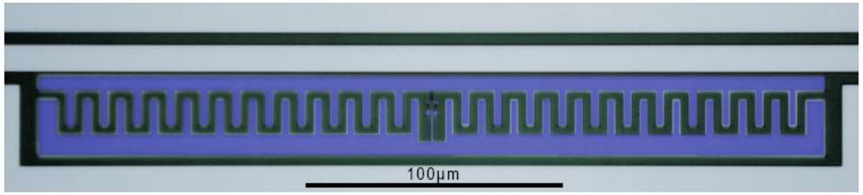
Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1 \ldots 300\ \mathrm{MHz}$

$$g \gg [\kappa, \gamma]\ \text{possible!}$$

# Qubit/Photon Coupling in a Circuit




qubit coupled to resonator




coupling strength:

$$\hbar g = eV_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

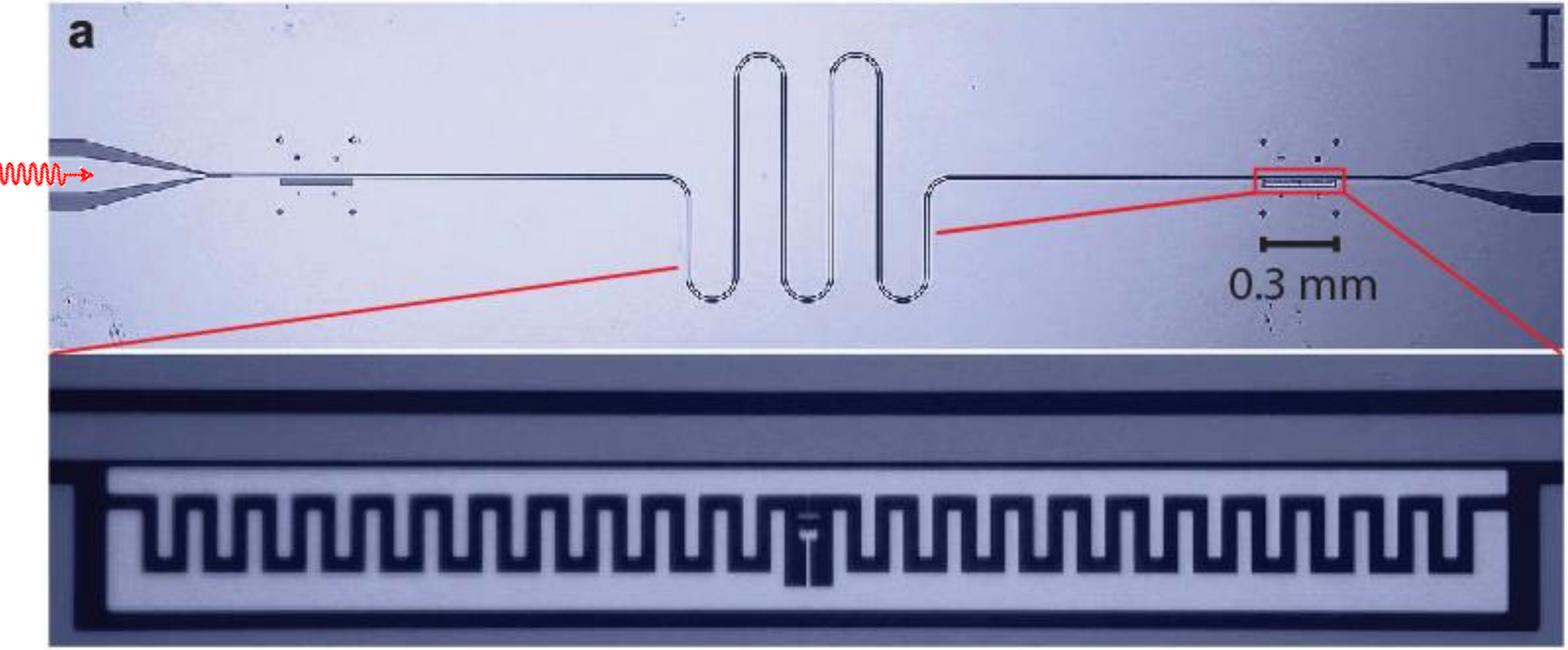
$$\Longrightarrow \nu_{\mathrm{vac}} = \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, e a_0$$

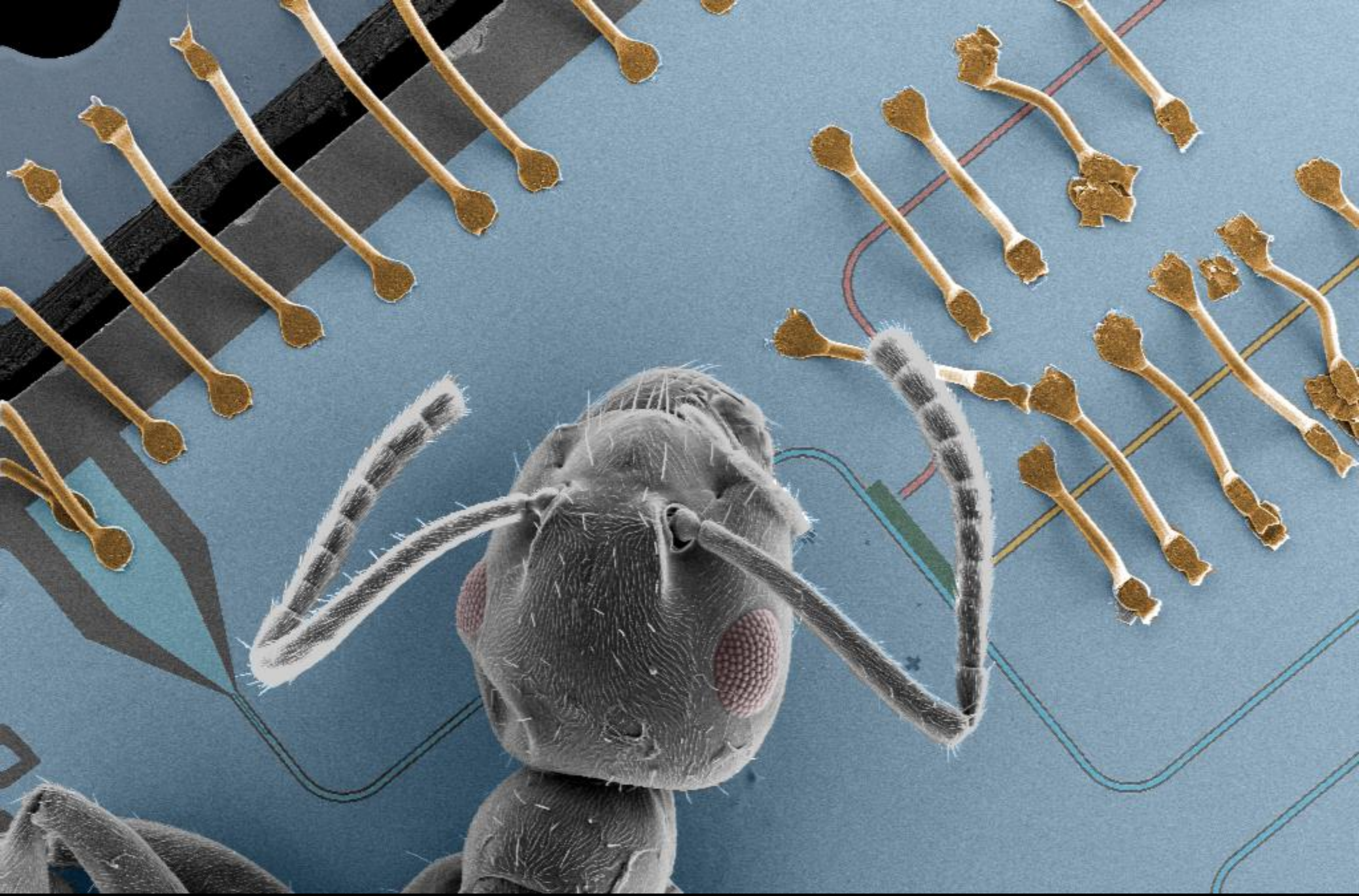
# Circuit QED with One Photon




superconducting cavity QED circuit



A. Wallraff, …, R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)



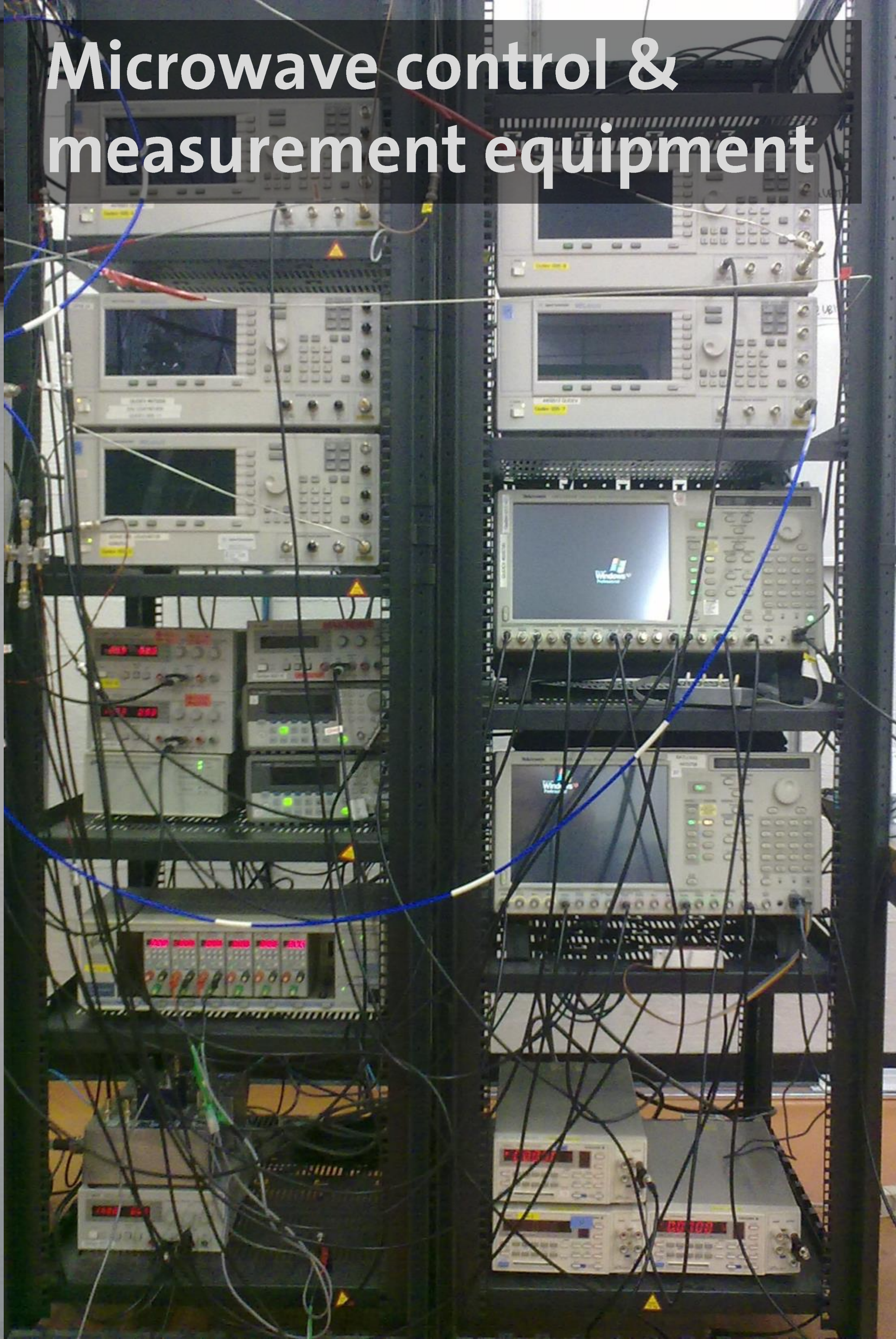


J. Mlynek *et al.*, Quantum Device Lab, ETH Zurich (2012)

# Sample Mount

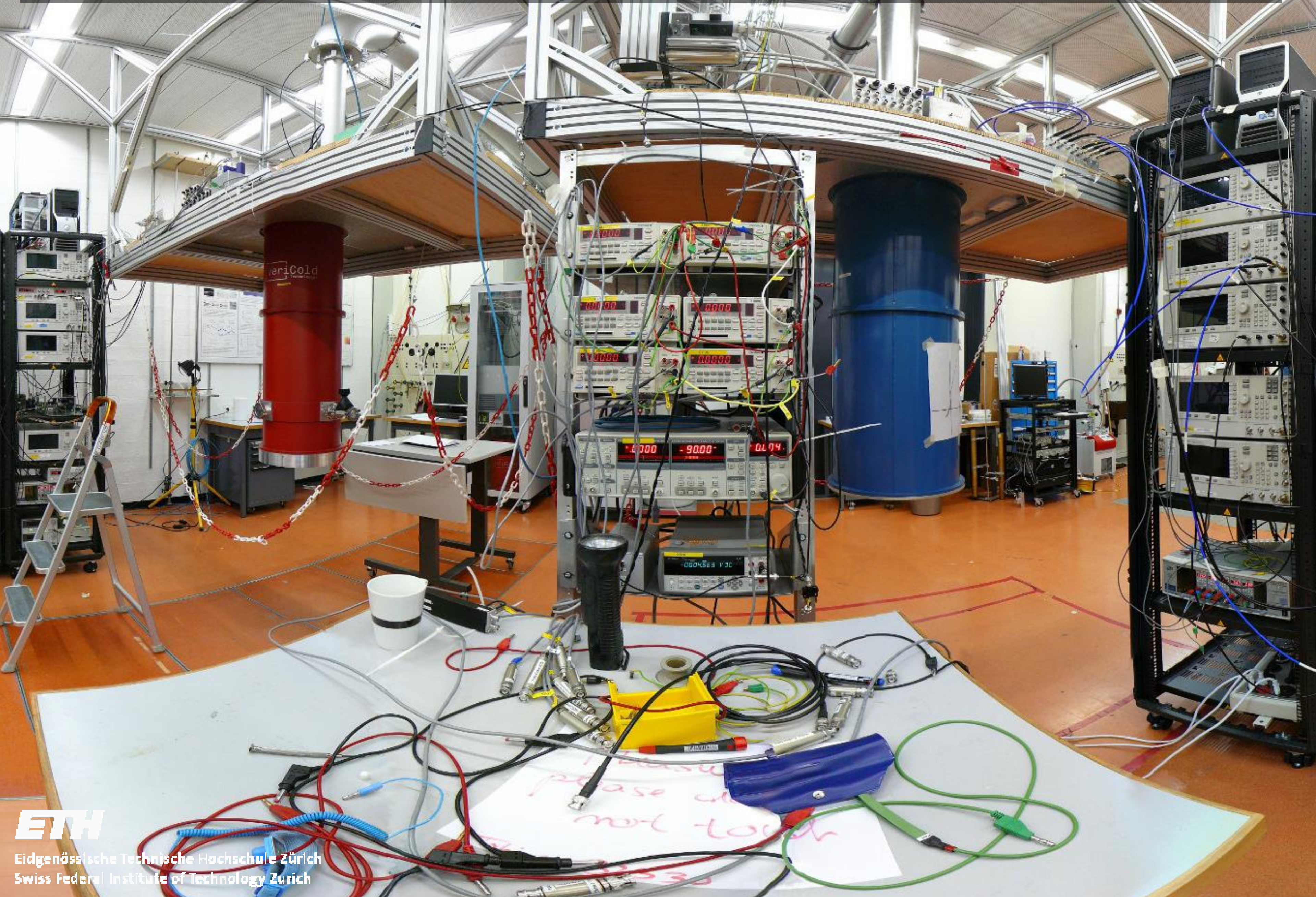



M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)


ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

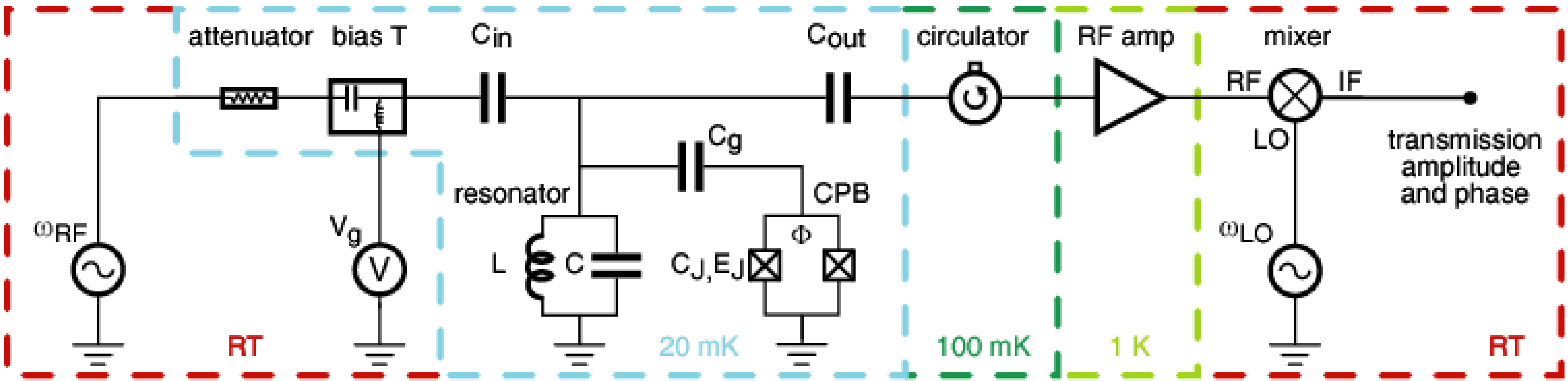
# A Circuit QED Lab at ETH Zurich



# How to Measure Single Microwave Photons

- average power to be detected

  $\rightarrow \langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$

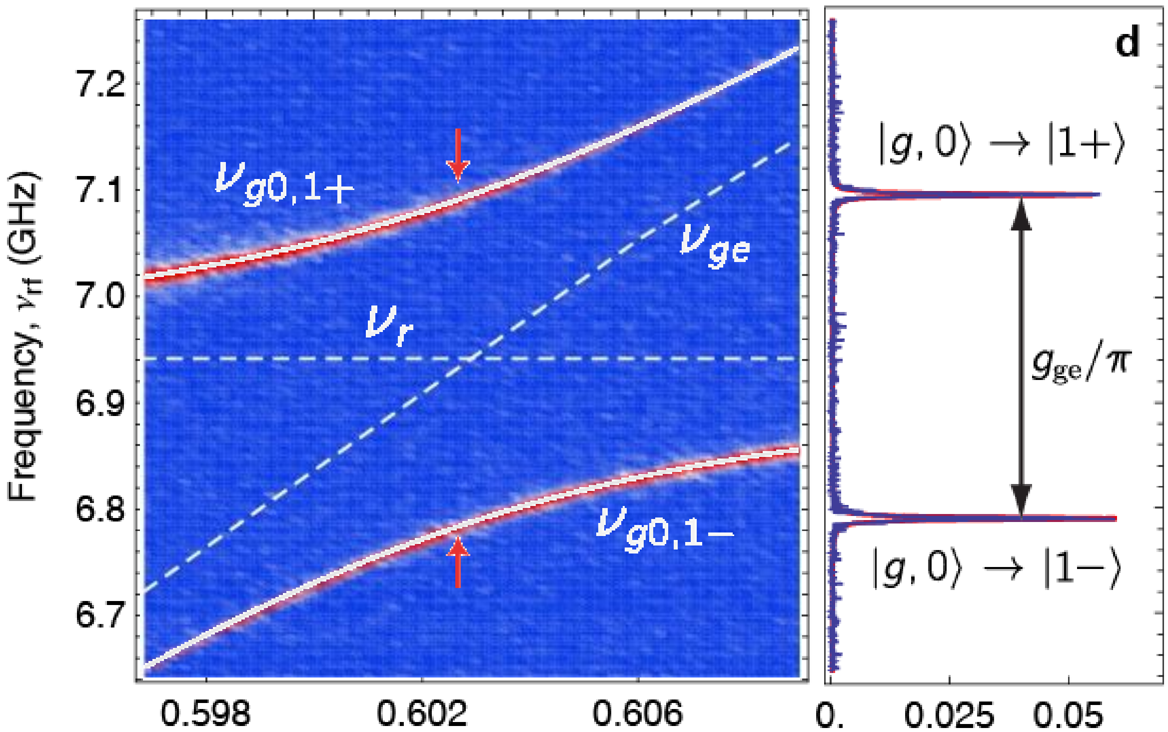



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)

- prevent leakage of thermal photons (cold attenuators and circulators)

# Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

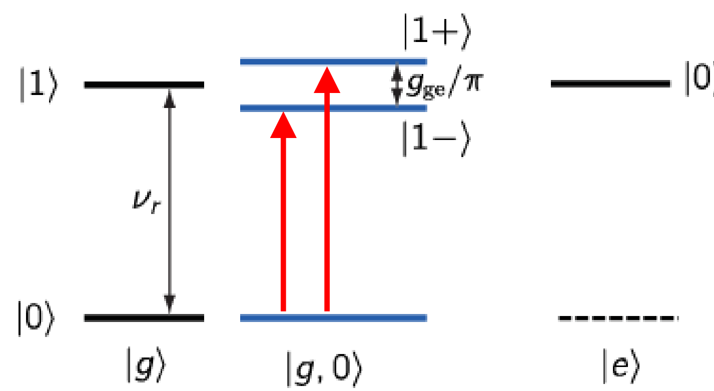



very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$




forming a 'molecule' of a qubit and a photon

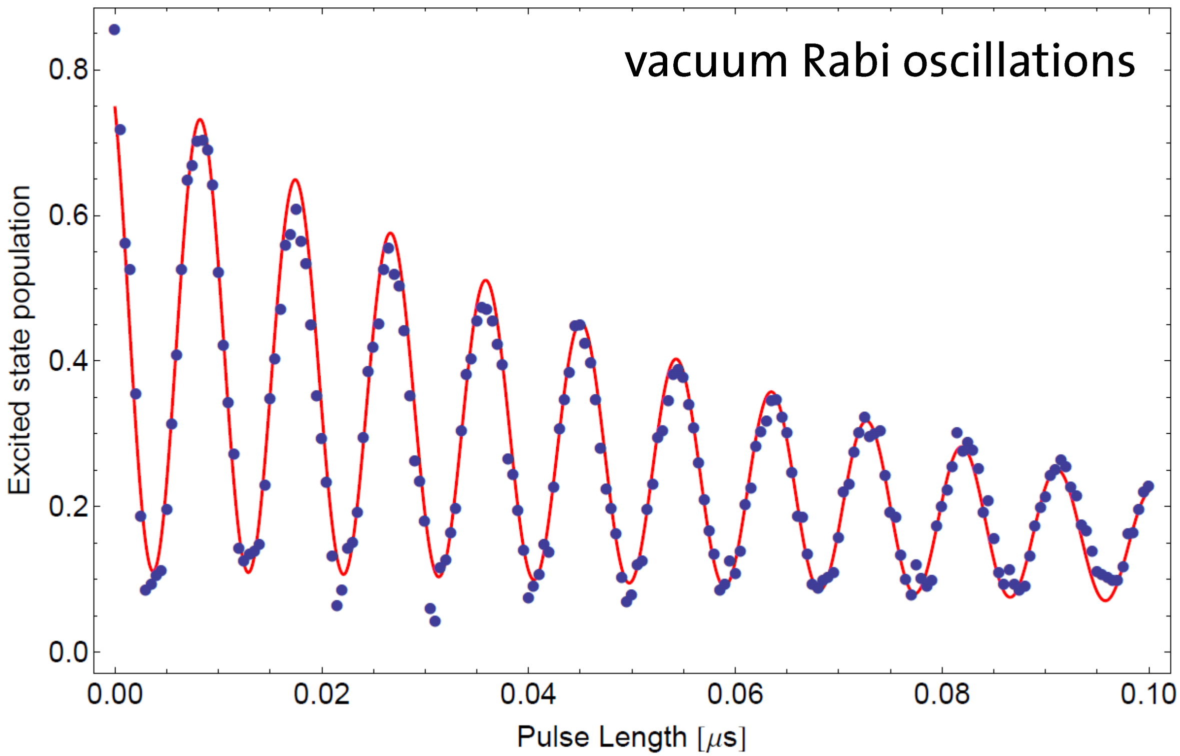
first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

# Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

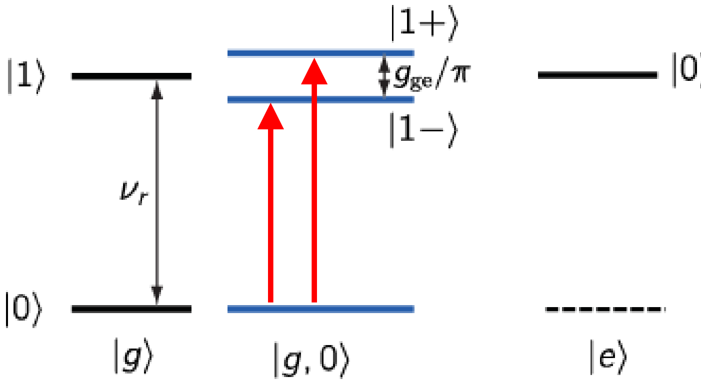



very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{\mathrm{ge}} \gg \kappa, \gamma$$




forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

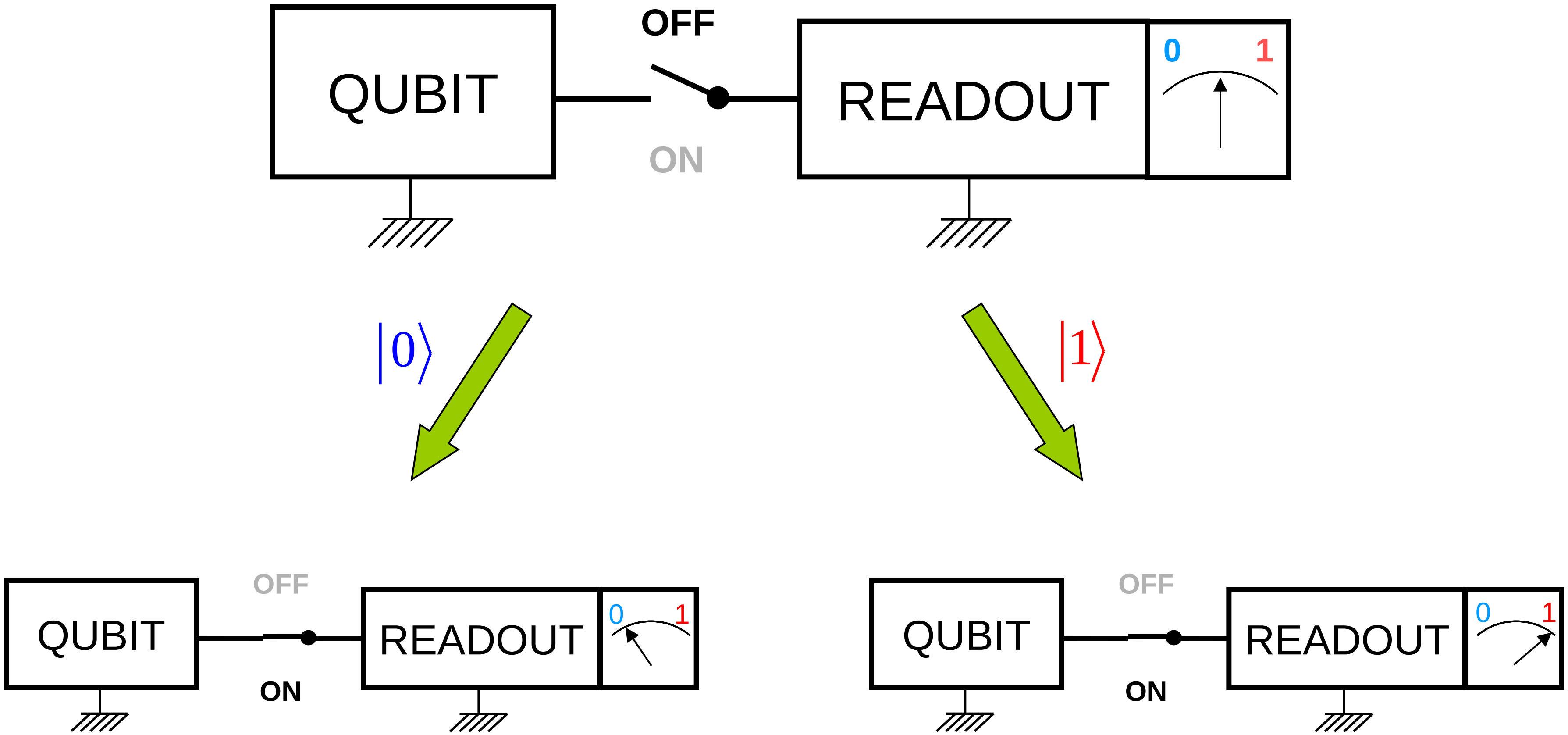
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

# Read-Out ...

## ... of superconducting qubits

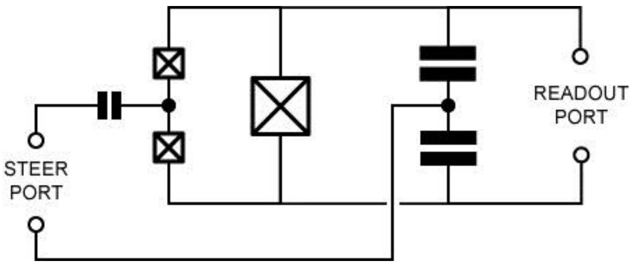
# Qubit Read Out



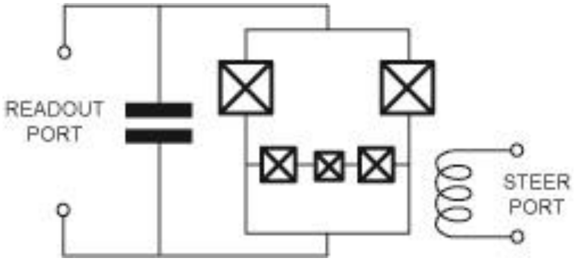

desired: good on/off ratio
no relaxation in on state (QND)

# Read Out Strategies

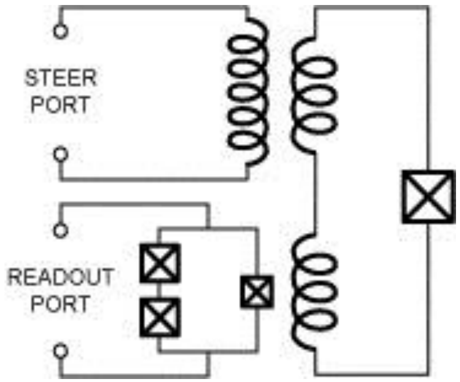
demolition measurements (switching/latching measurements)




Quantronium (Saclay, Yale)

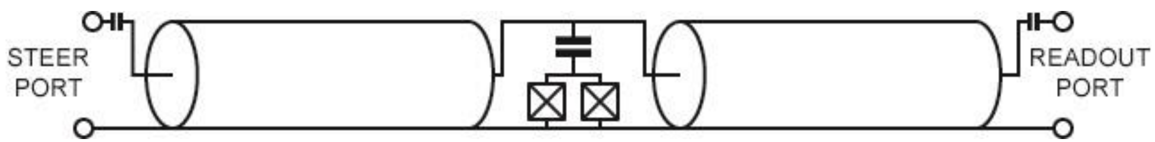



Flux Qubit (TU Delft, NEC)




Phase Qubit (NIST, UCSB)

quantum non-demolition (QND) measurements




Yale (circuit QED)

also: Chalmers, Delft, Yale (JBA)

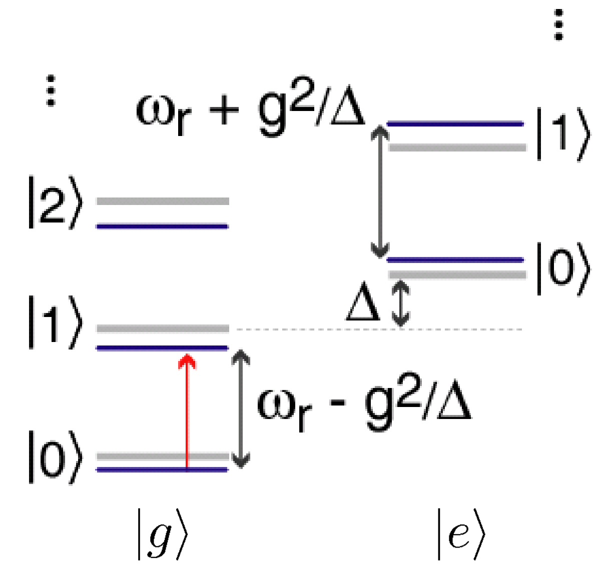
# Non-Resonant Qubit-Photon Interaction

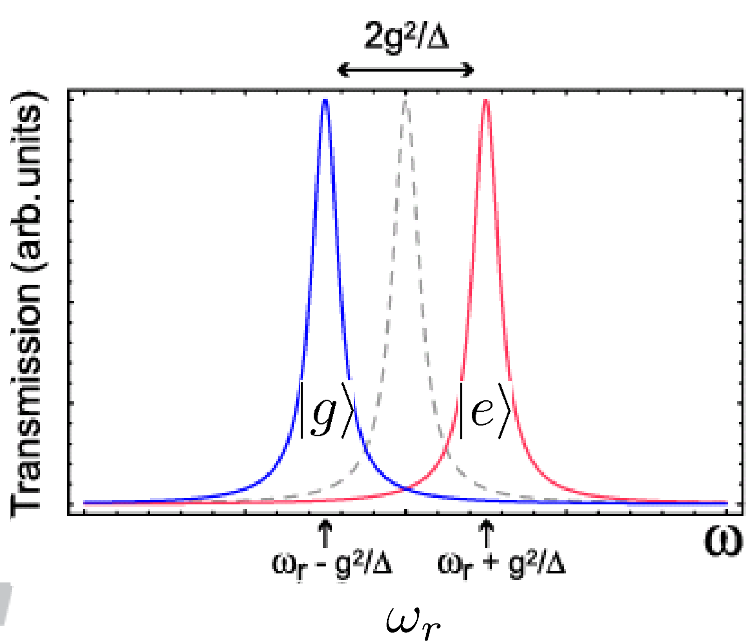
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

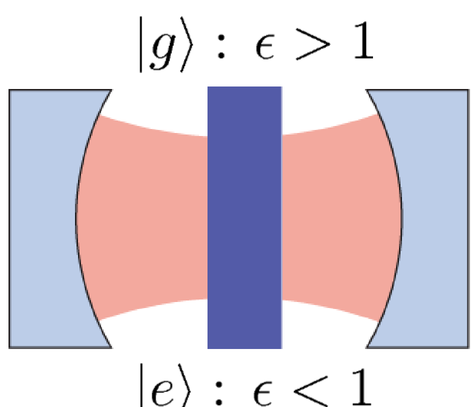
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//
cavity frequency shift
*and* qubit ac-Stark shift

//
Lamb shift




ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

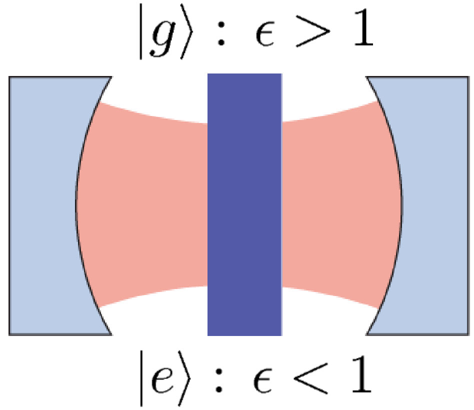
# Dispersive Read-Out

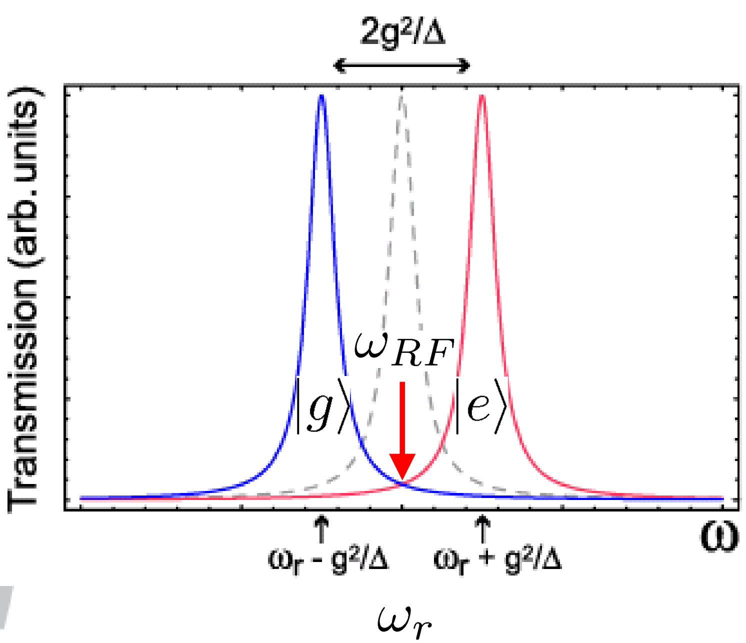
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

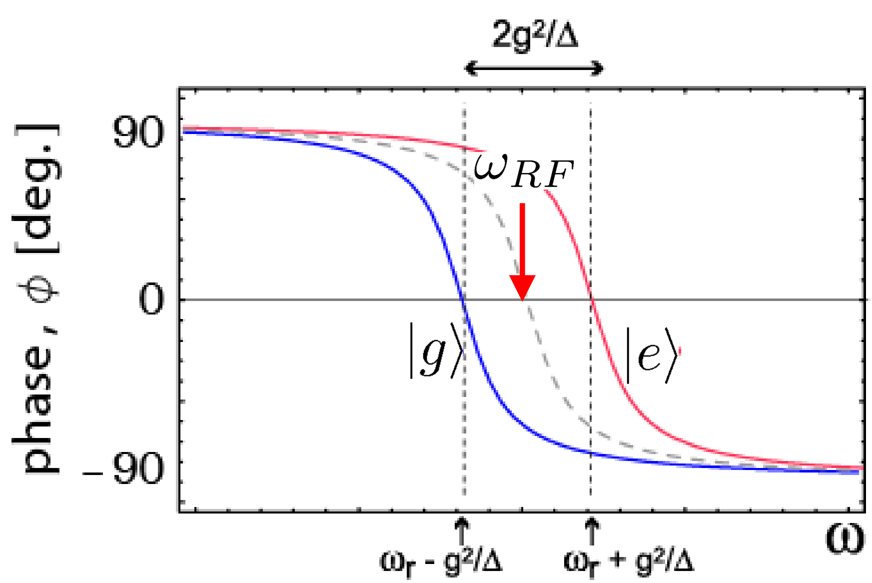
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

// cavity frequency shift *and* qubit ac-Stark shift

// Lamb shift



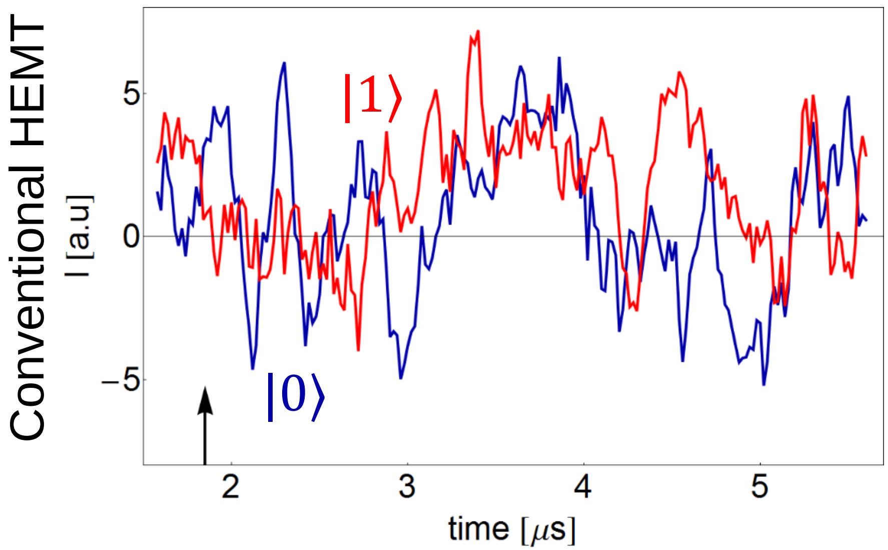





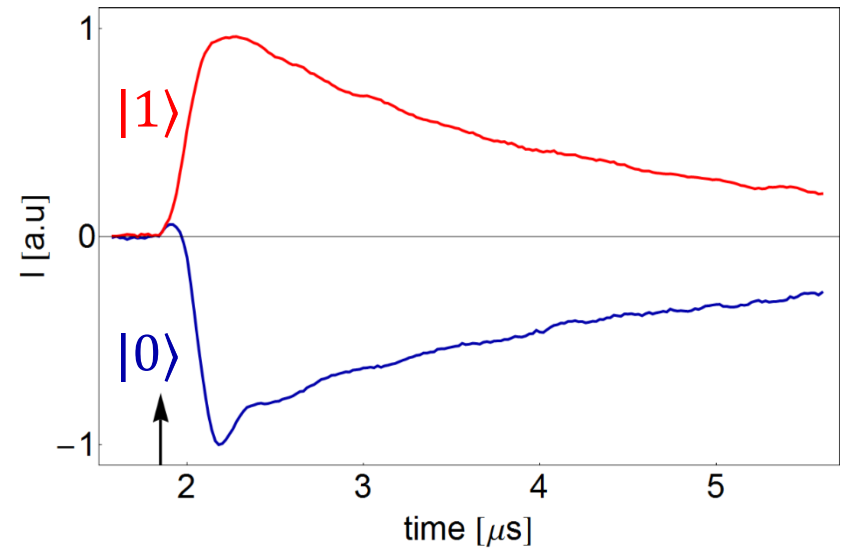

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

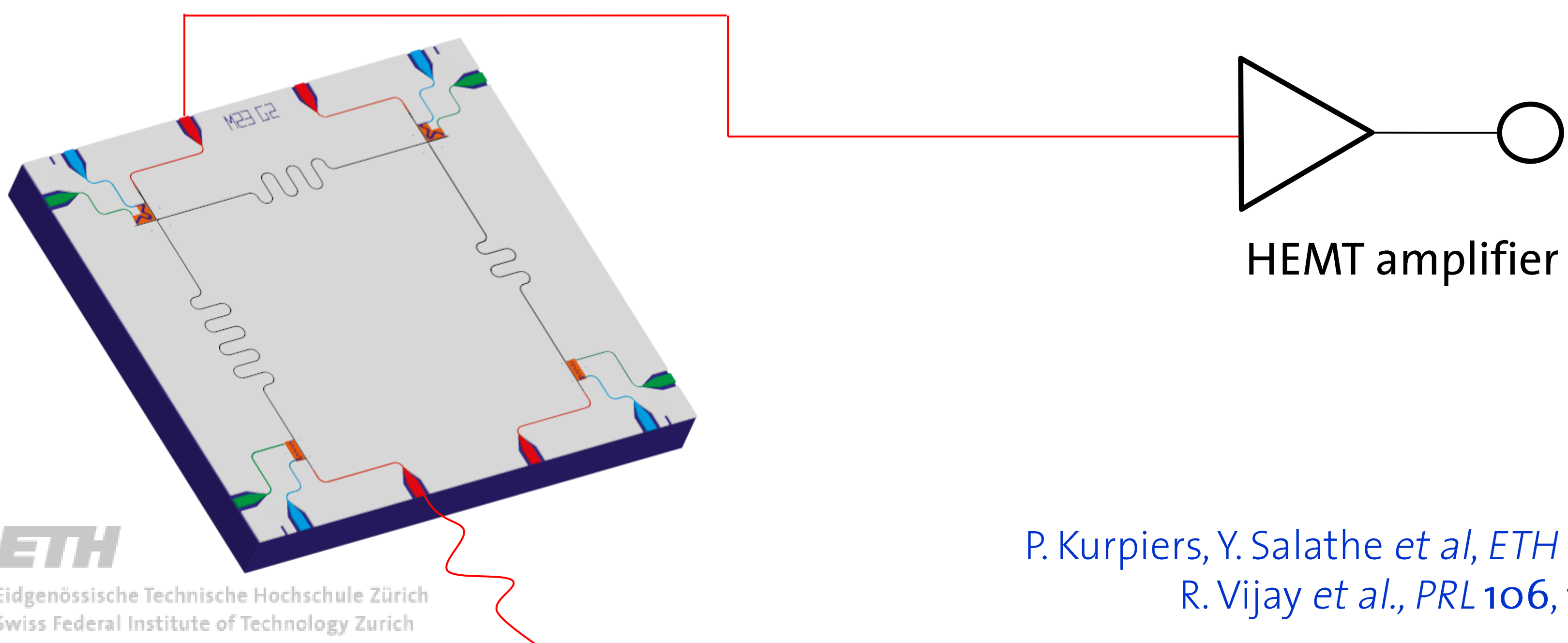
# Averaged Qubit-Readout

single-shot measurements:




averaged measurements (8 10⁴):



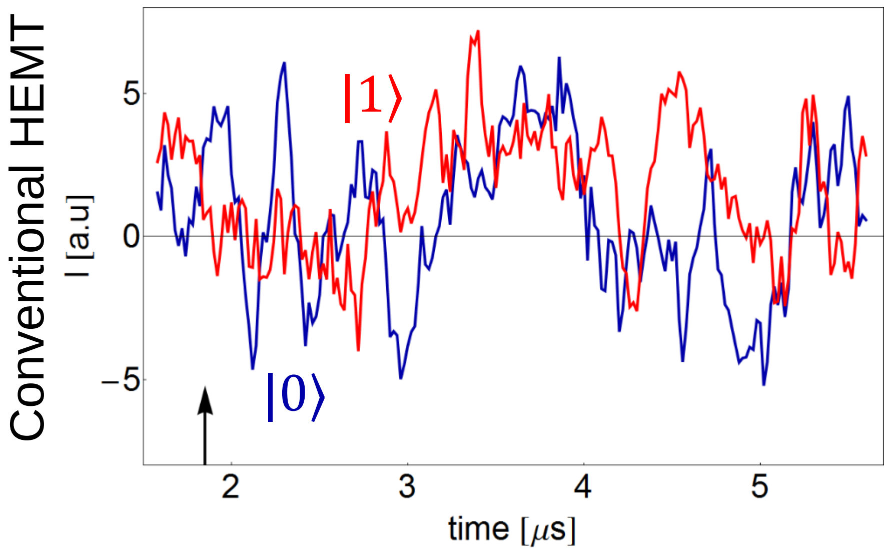



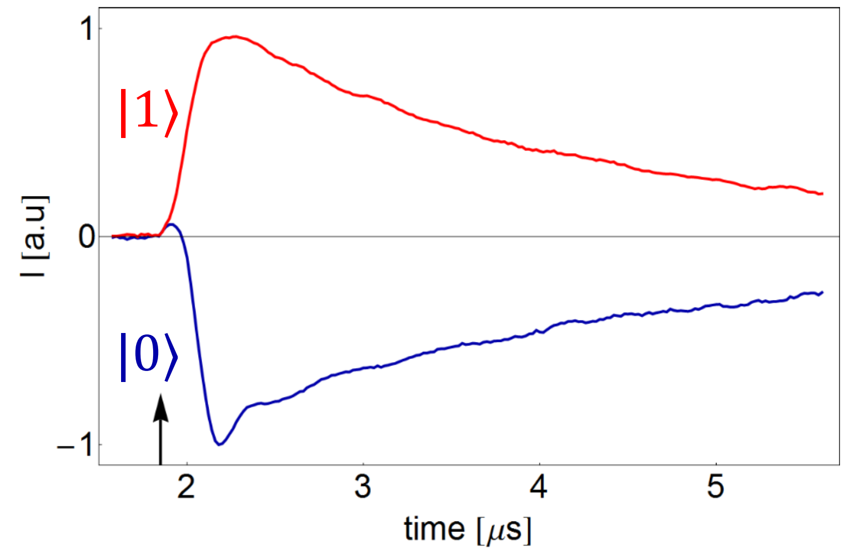

P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* **106**, 110502 (2011)

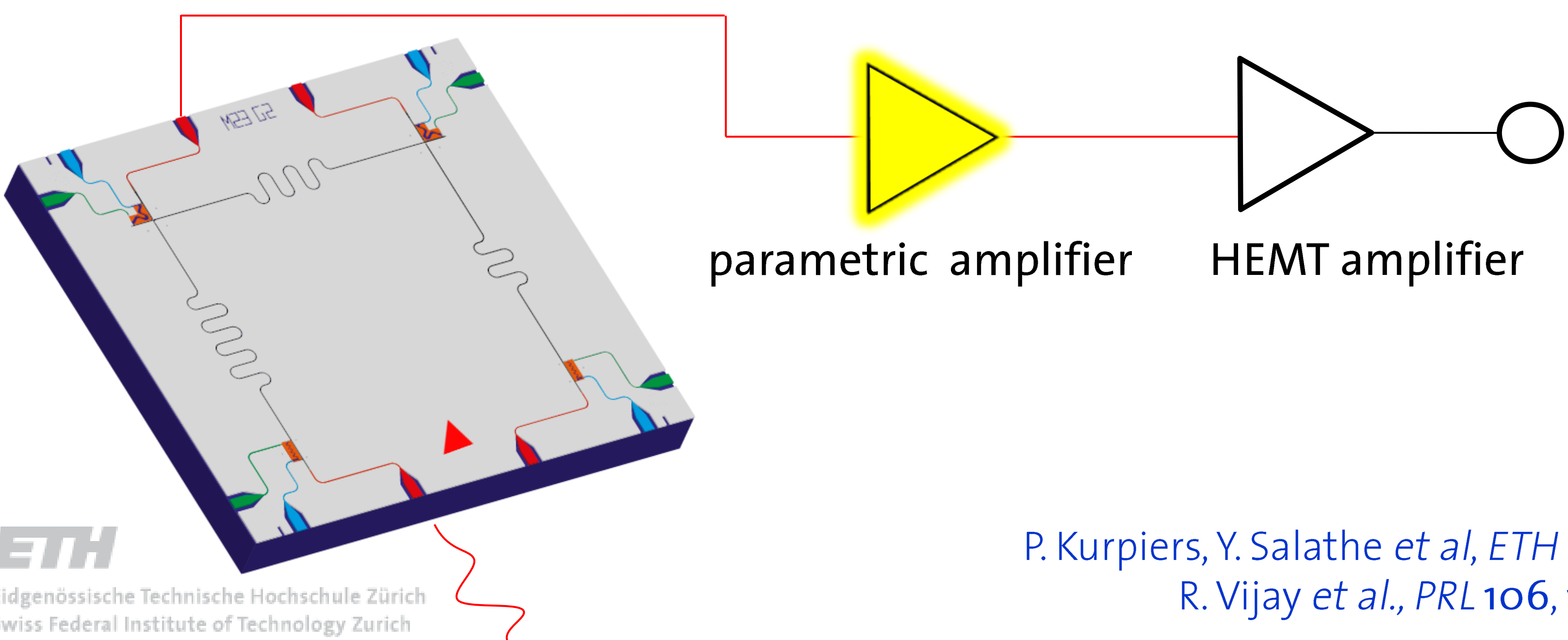
# Improved using a Quantum Limited Amplifier

single-shot measurements:

averaged measurements (8 10⁴):



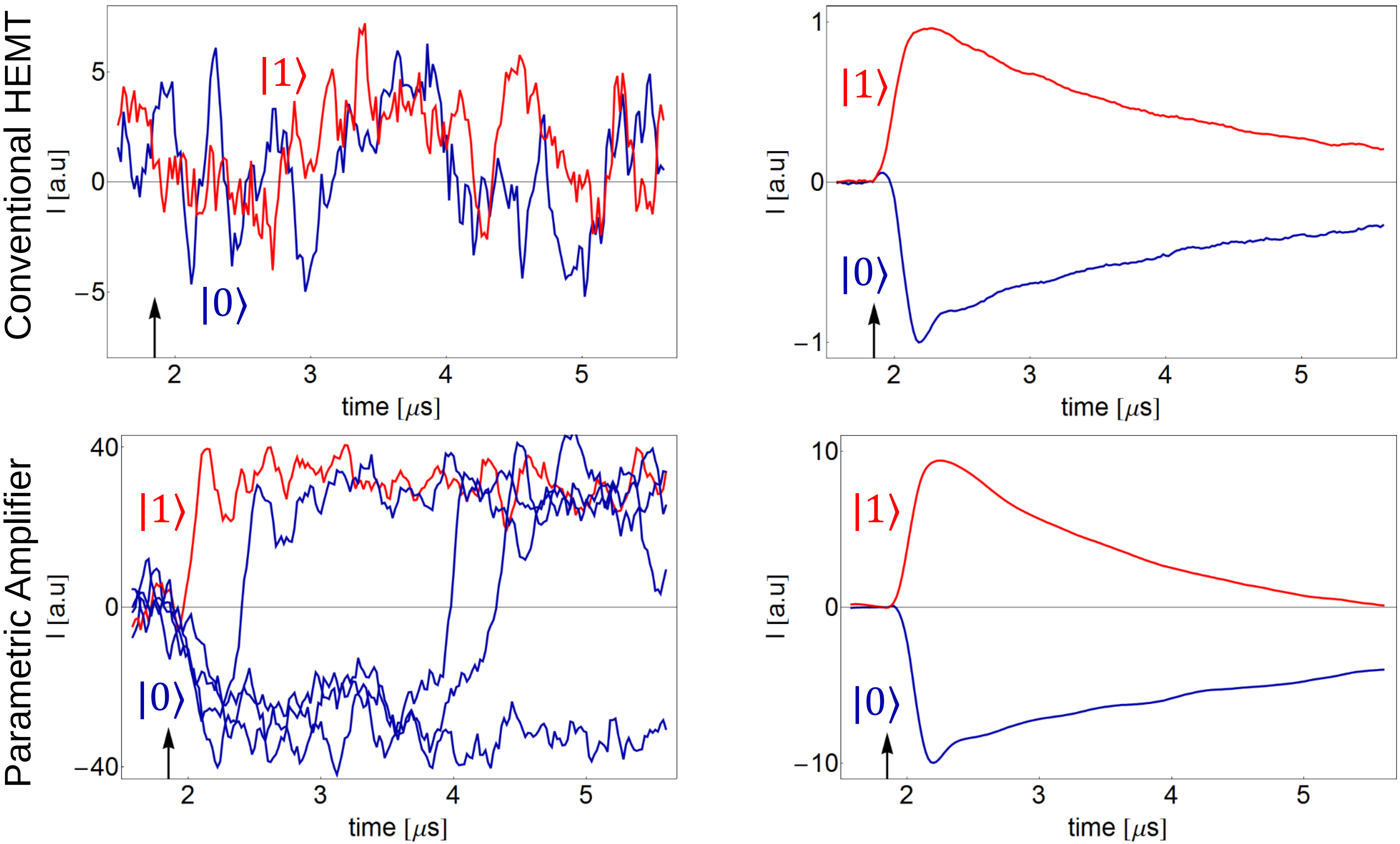



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)

R. Vijay *et al.*, *PRL* **106**, 110502 (2011)

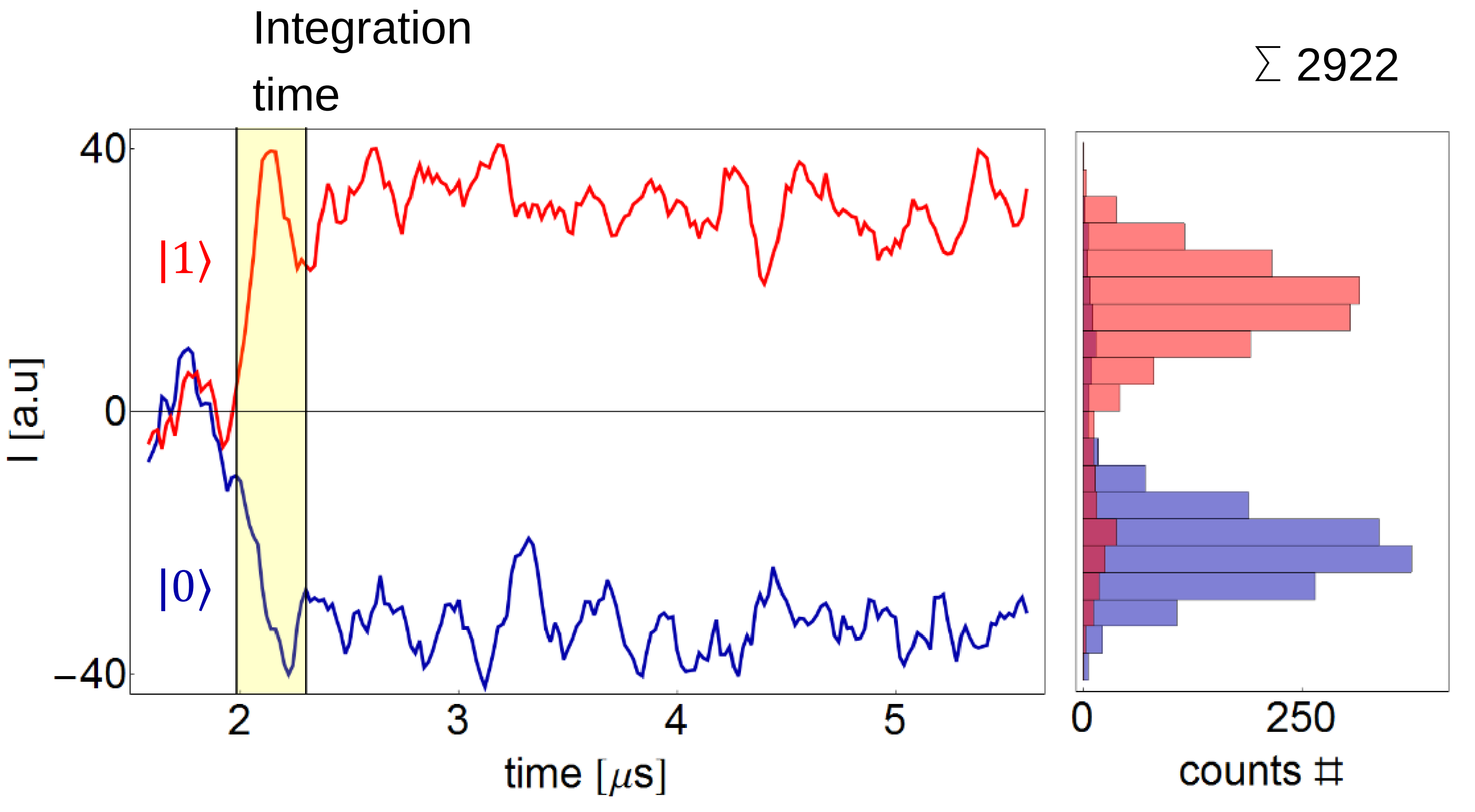
# Single-Shot Single-Qubit Readout

single-shot measurements: averaged measurements (8 $10^4$):




P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* **106**, 110502 (2011)

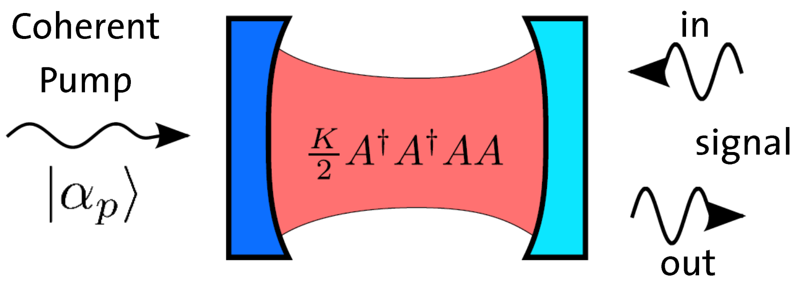
# Statistics of Integrated Single-Shot Readout



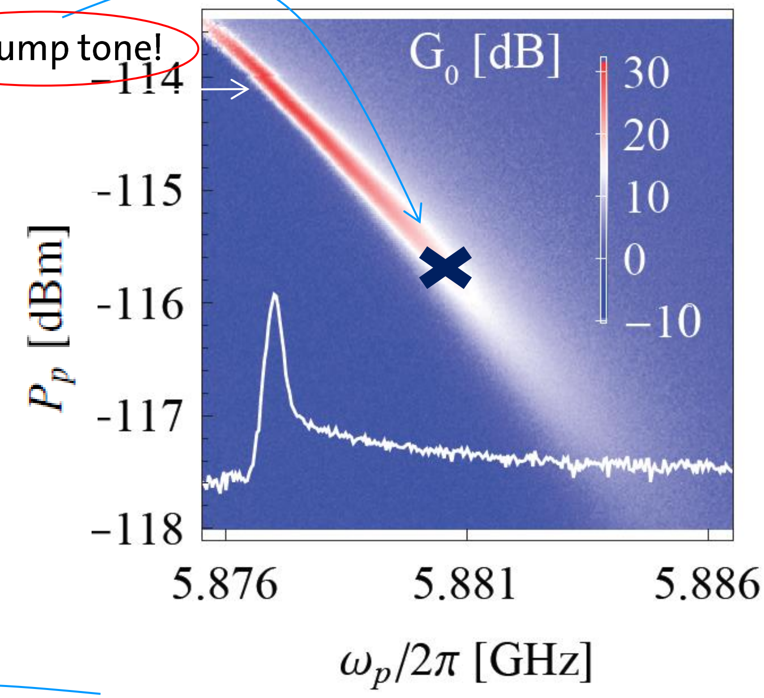



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)

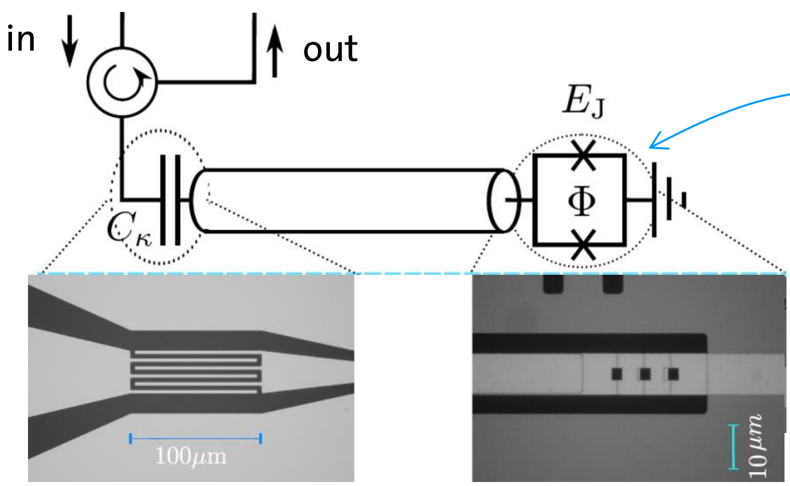
# Near Quantum-Limited Parametric Amplifier




Measured Gain




Circuit QED implementation:



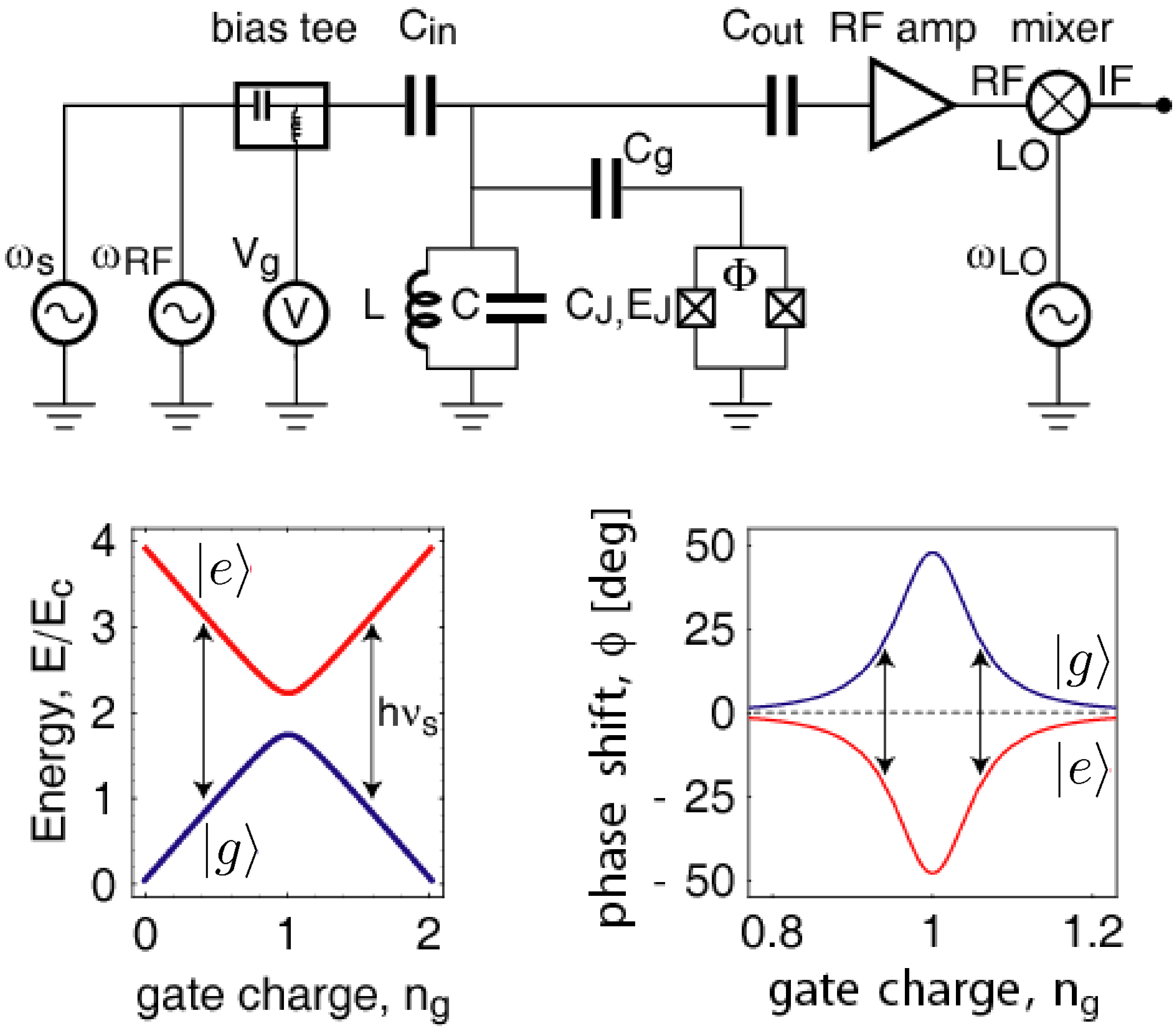

SQUID(-array) provides required nonlinearity

Eichler *et al.*, EPJ Quantum Technology **1**, 2 (2014)
Eichler *et al.*, *Phys. Rev. Lett.* **107**, 113601 (2011)

Caves, *Phys. Rev. D* **26**, 1817 (1982)
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 929 (2008)

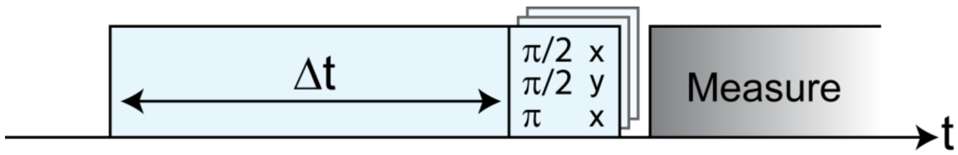
# Qubit Control

# Qubit Spectroscopy with Dispersive Read-Out

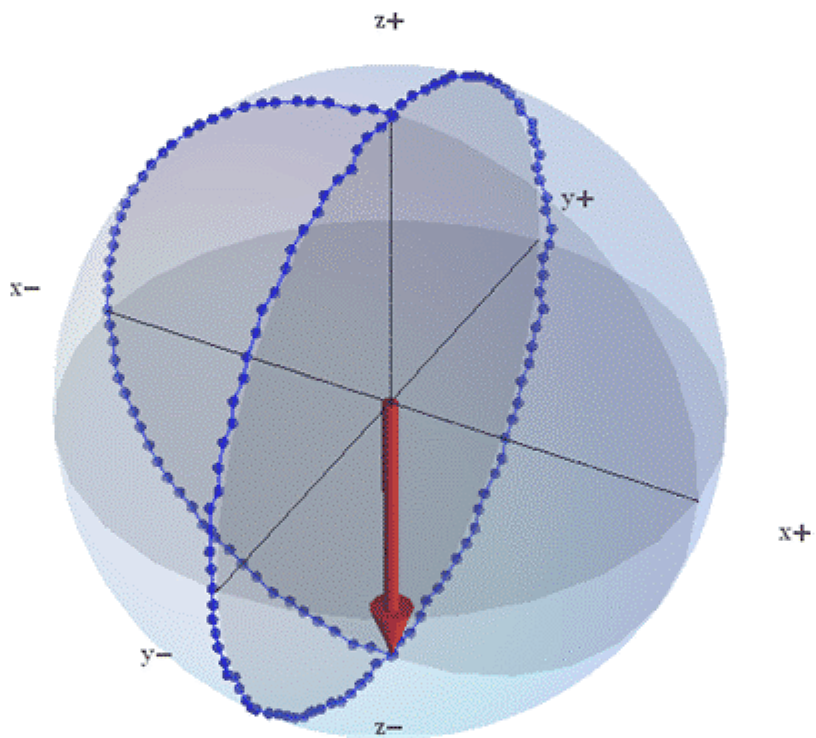


# Single Qubit Gates

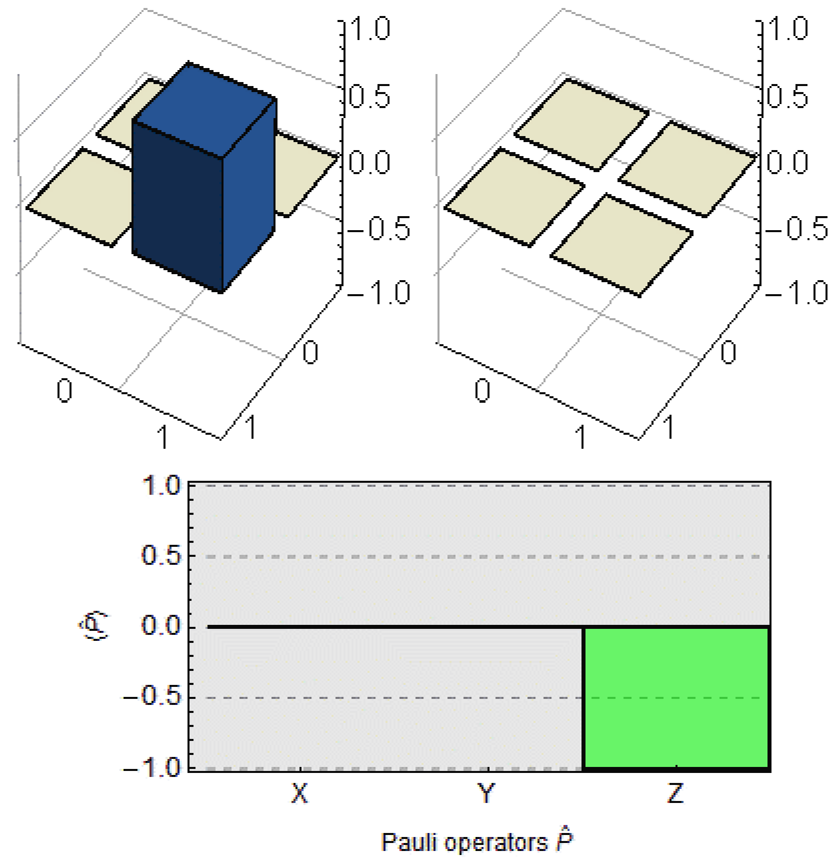
Pulse sequence for qubit rotation and readout:




experimental Bloch vector:




experimental density matrix and Pauli set:




L. Steffen *et al.*, Quantum Device Lab, ETH Zurich (2008)