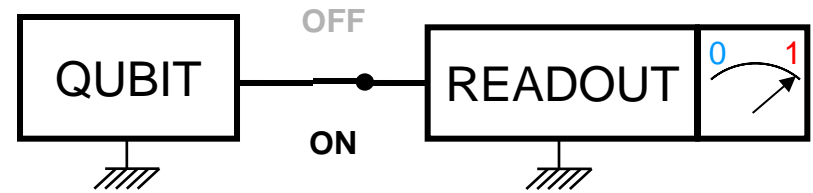
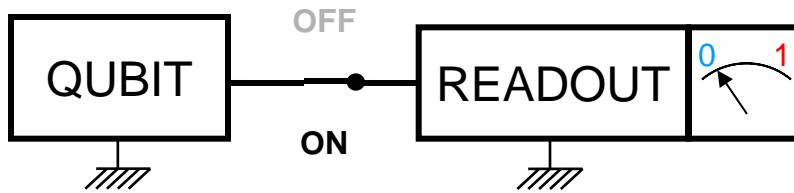
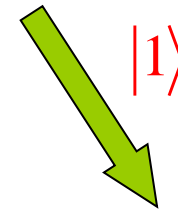
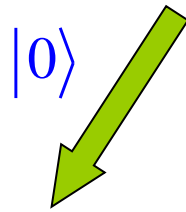
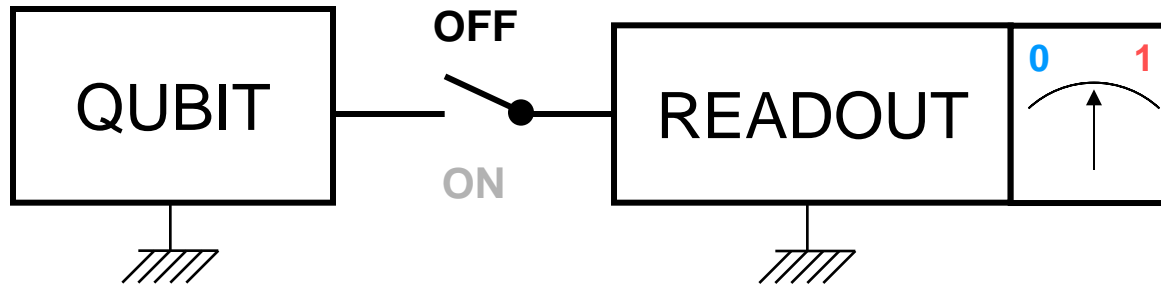


Read-Out ...

... of superconducting qubits

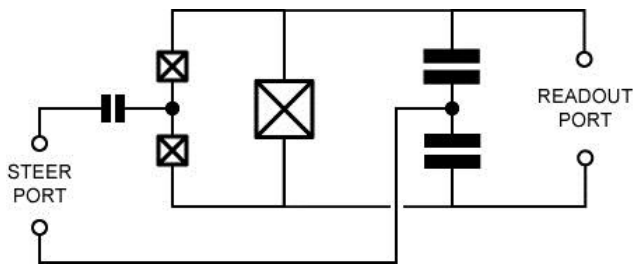
Qubit Read Out



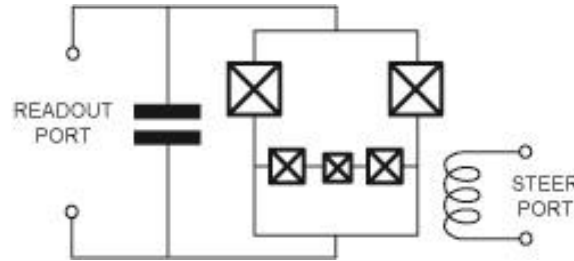
desired: good on/off ratio
no relaxation in on state (QND)

Read Out Strategies

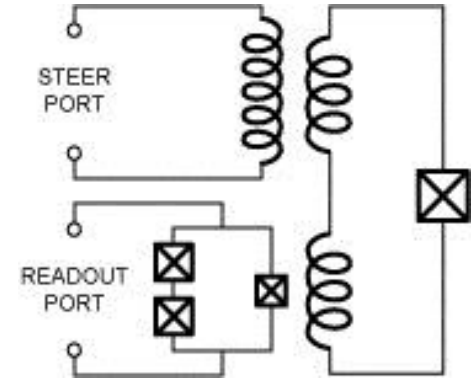
demolition measurements (switching/latching measurements)



Quantonium (Saclay, Yale)

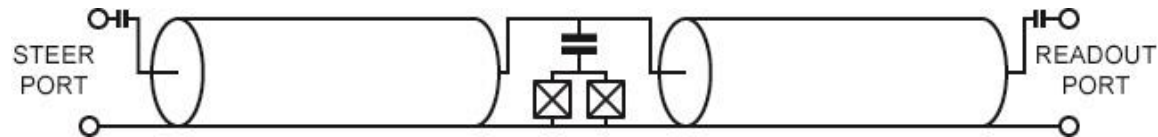


Flux Qubit (TU Delft, NEC)



Phase Qubit (NIST, UCSB)

quantum non-demolition (QND) measurements



Yale (circuit QED)

also: Chalmers, Delft, Yale (JBA)

Dispersive Approximation of the J-C Hamiltonian

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+)$$

Unitary transformation

$$\begin{aligned} \tilde{H} &= U H U^\dagger & \text{with} & & U &= \exp \frac{g}{\Delta} (a \sigma^+ - a^\dagger \sigma^-) \\ & & \text{and} & & \Delta &= \omega_a - \omega_r \end{aligned}$$

Results in dispersive approximation up to 2nd order in g

$$\tilde{H} \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

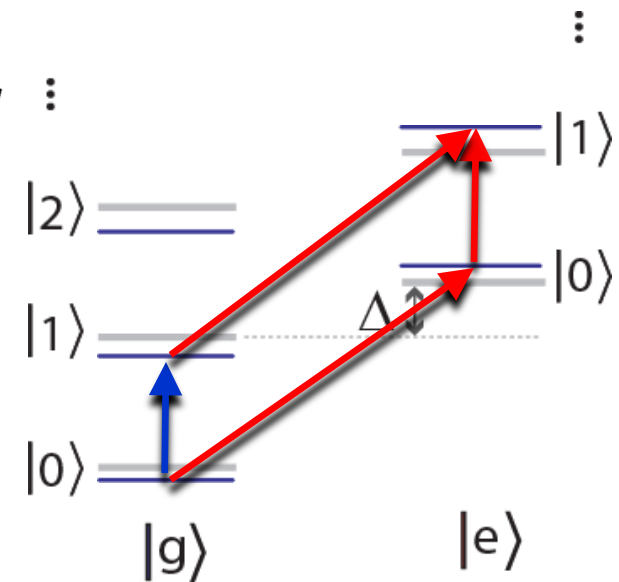
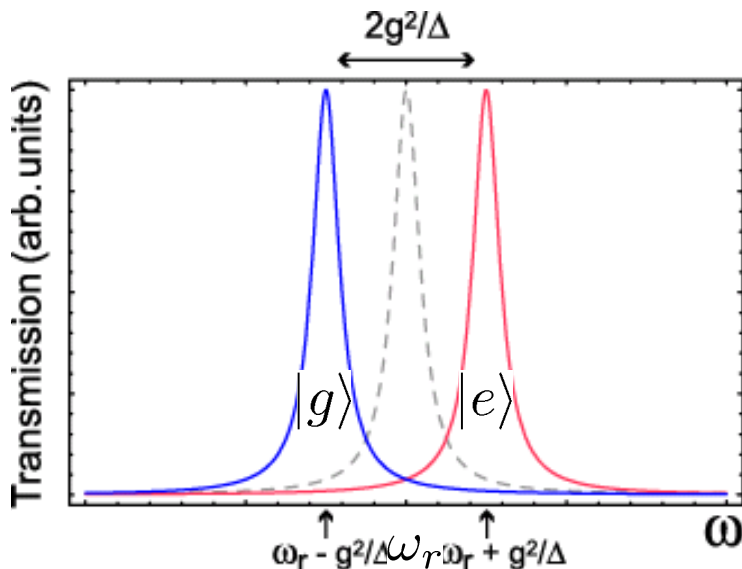
Non-Resonant (Dispersive) Interaction

approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$:

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

//

cavity frequency shift



qubit detuned by Δ
from resonator

A. Blais *et al.*, *PRA* **69**, 062320 (2004)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

D. I. Schuster *et al.*, *Phys. Rev. Lett.* **94**, 123062 (2005)

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

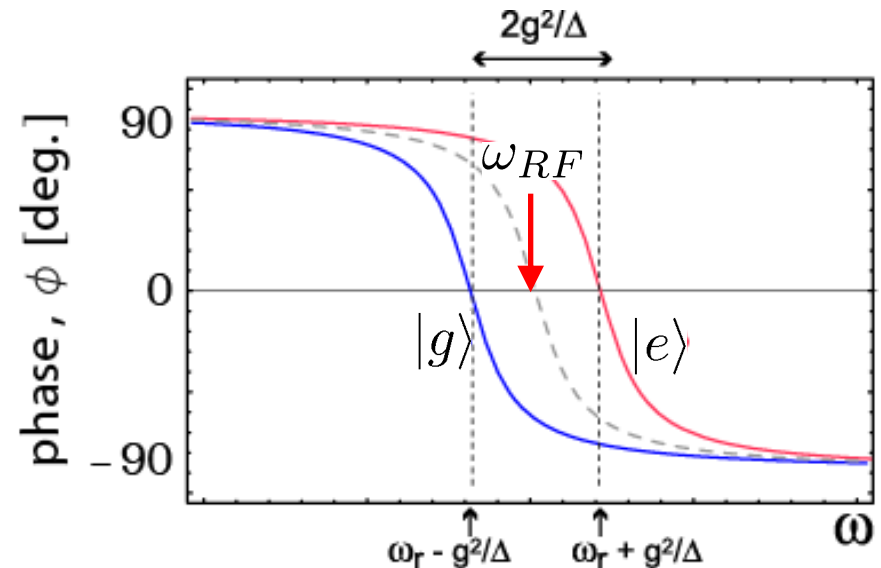
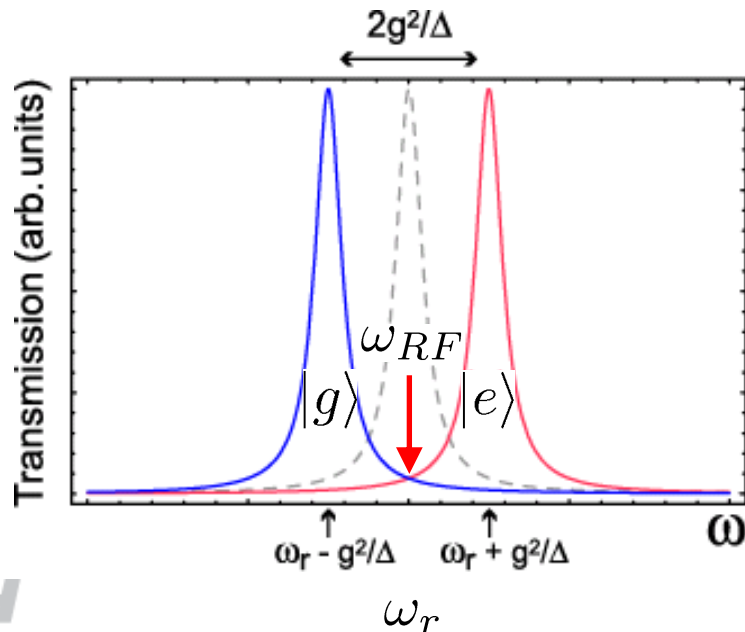
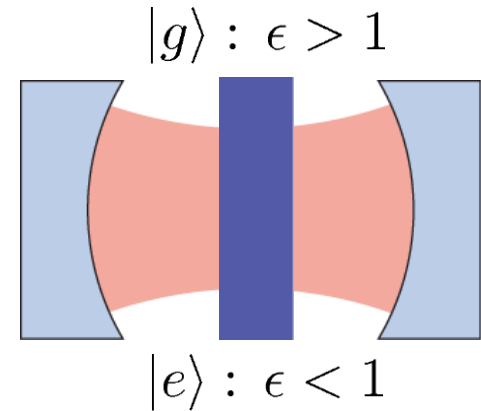
Dispersive Read-Out

approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

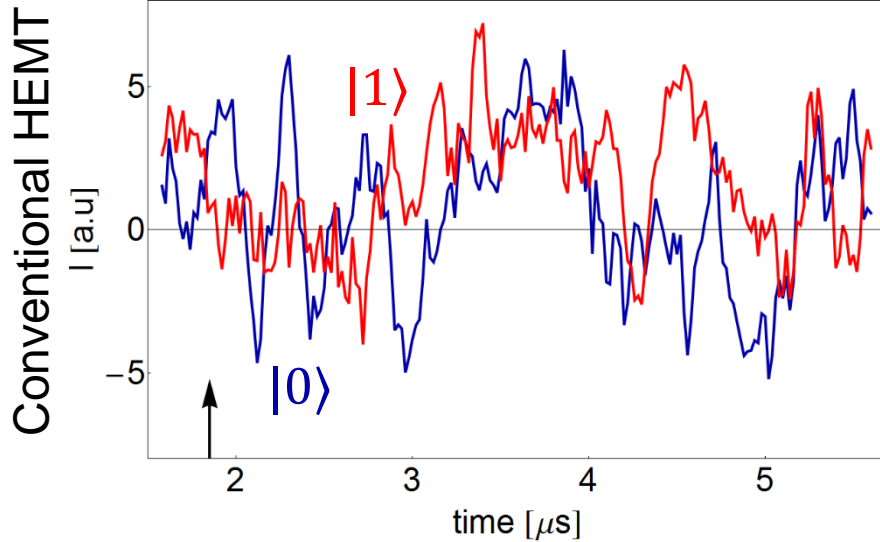
//

cavity frequency shift

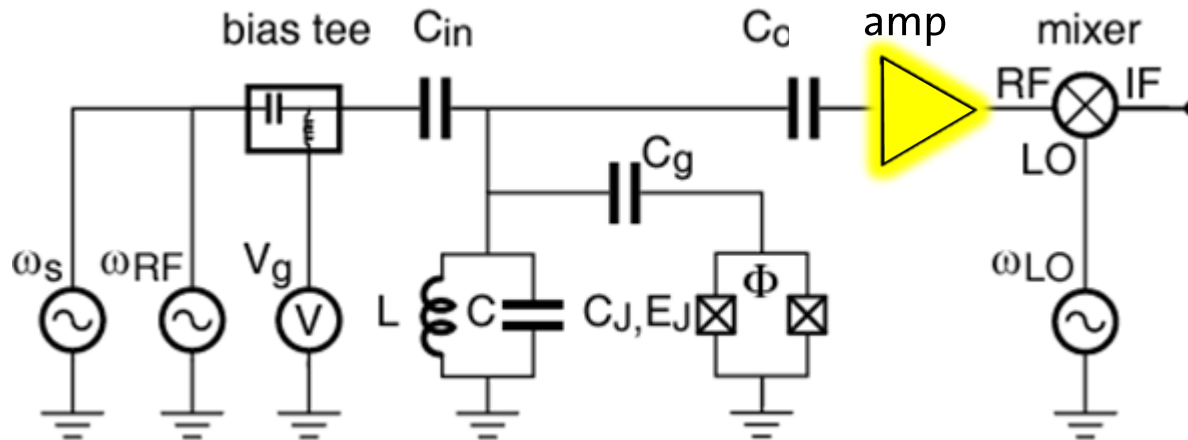
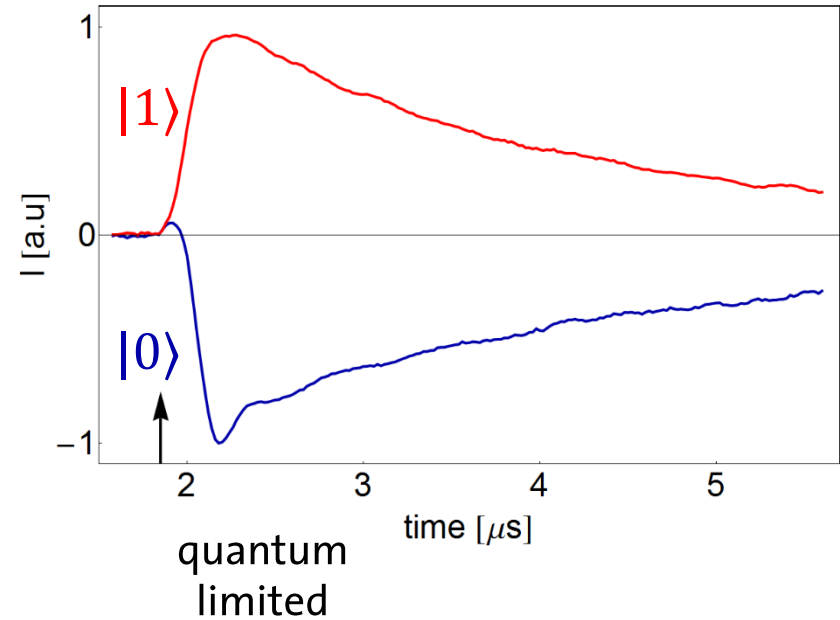


Improved using a Quantum Limited Amplifier

single-shot measurements:



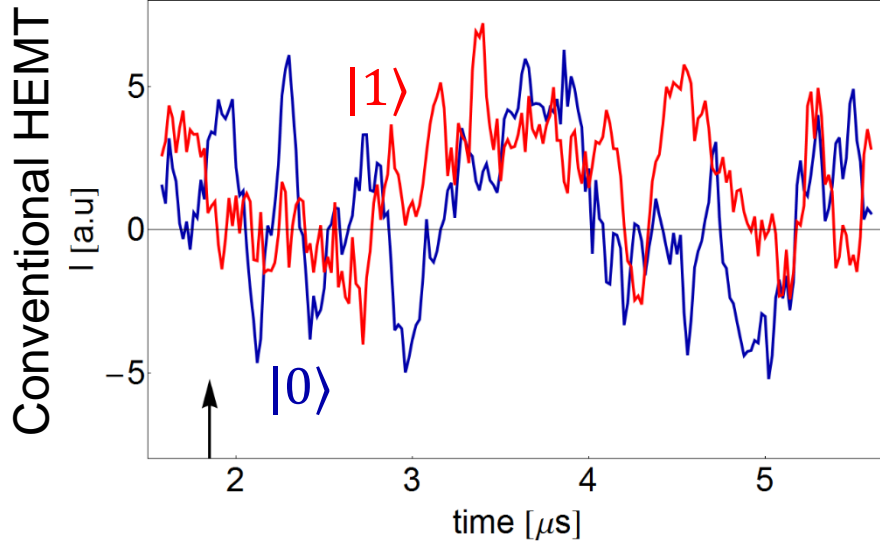
averaged measurements ($8 \cdot 10^4$):



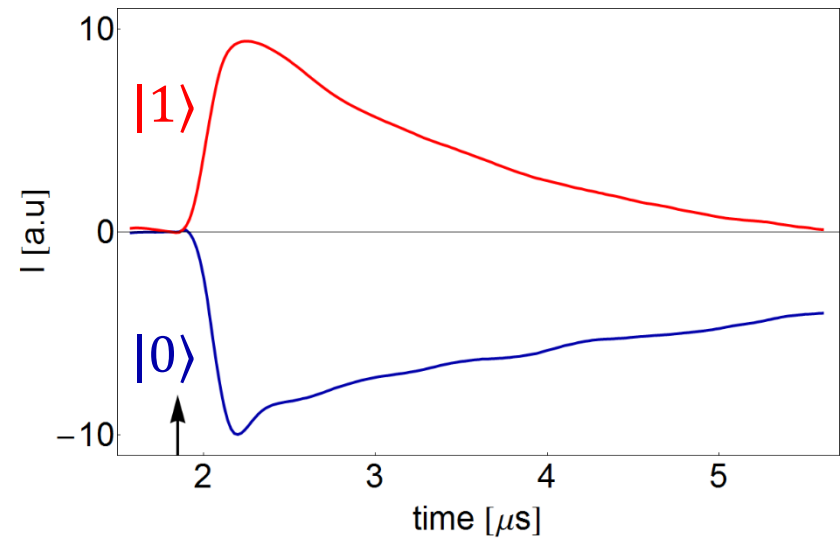
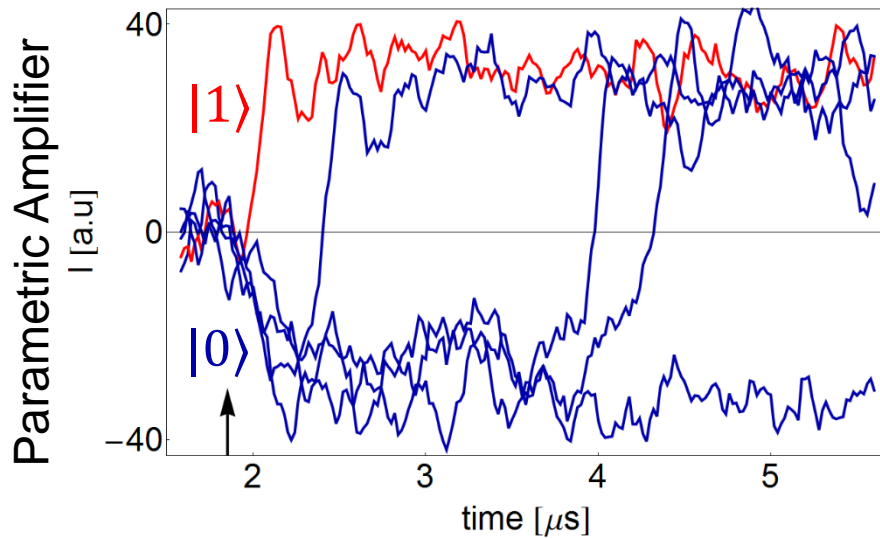
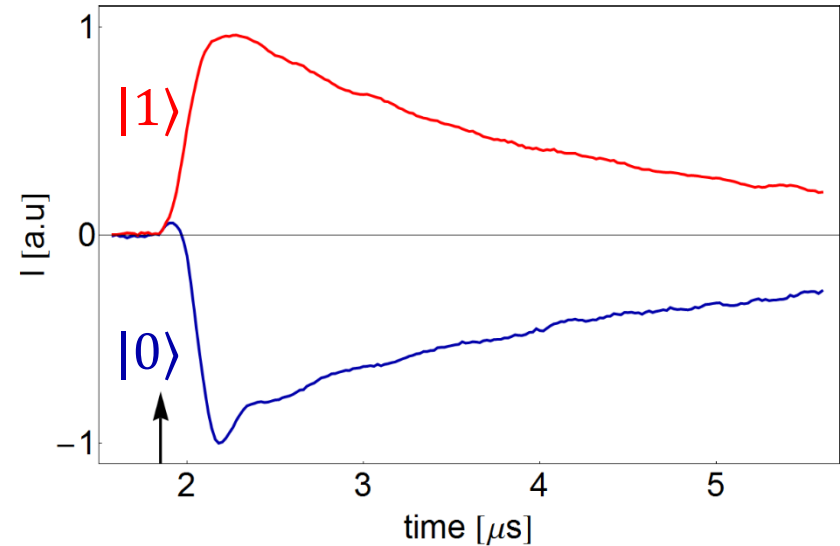
P. Kurpiers, Y. Salathe et al, *ETH Zurich* (2013)
 R. Vijay et al., *PRL* 106, 110502 (2011)

Single-Shot Single-Qubit Readout

single-shot measurements:

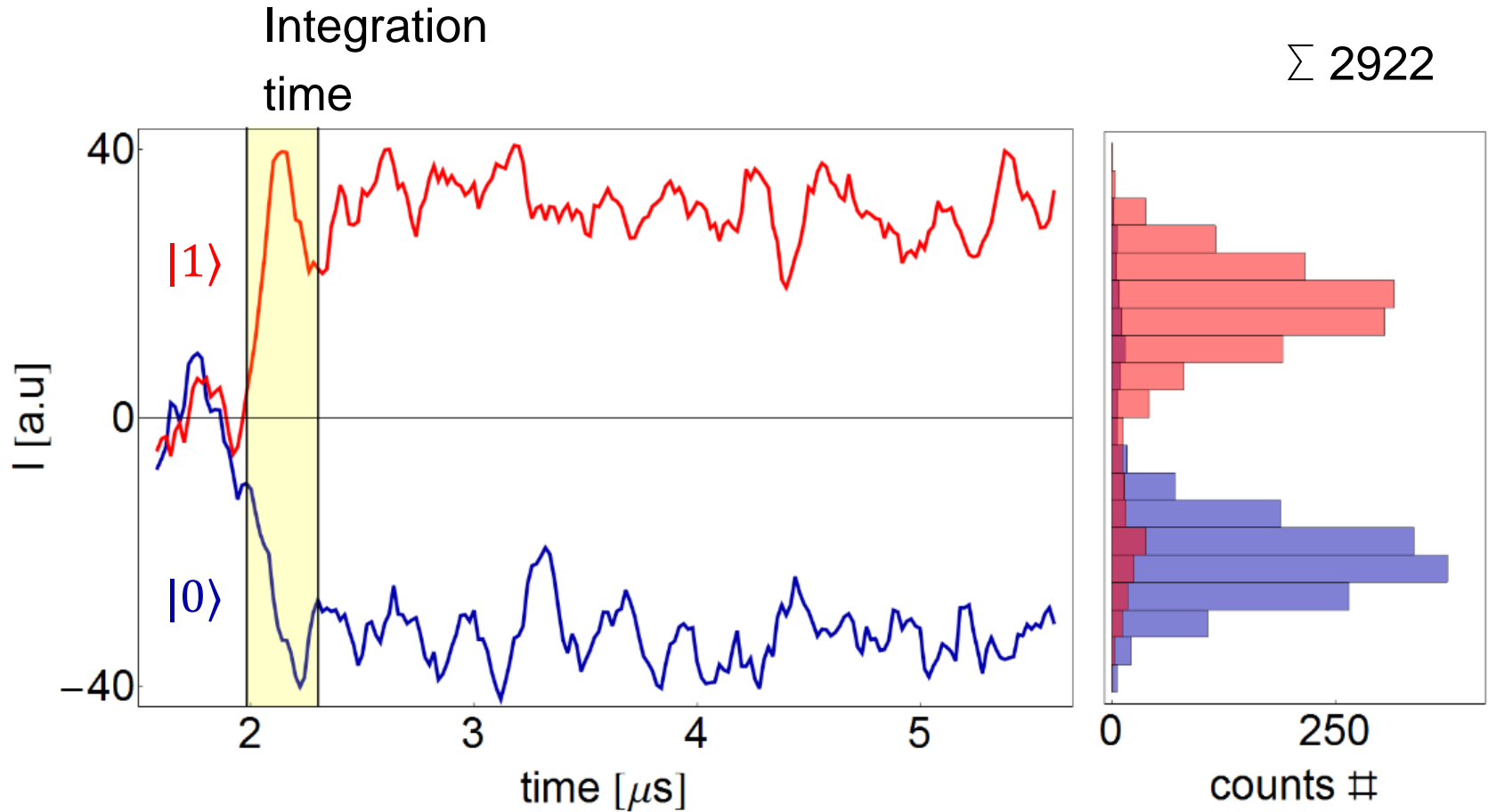


averaged measurements ($8 \cdot 10^4$):

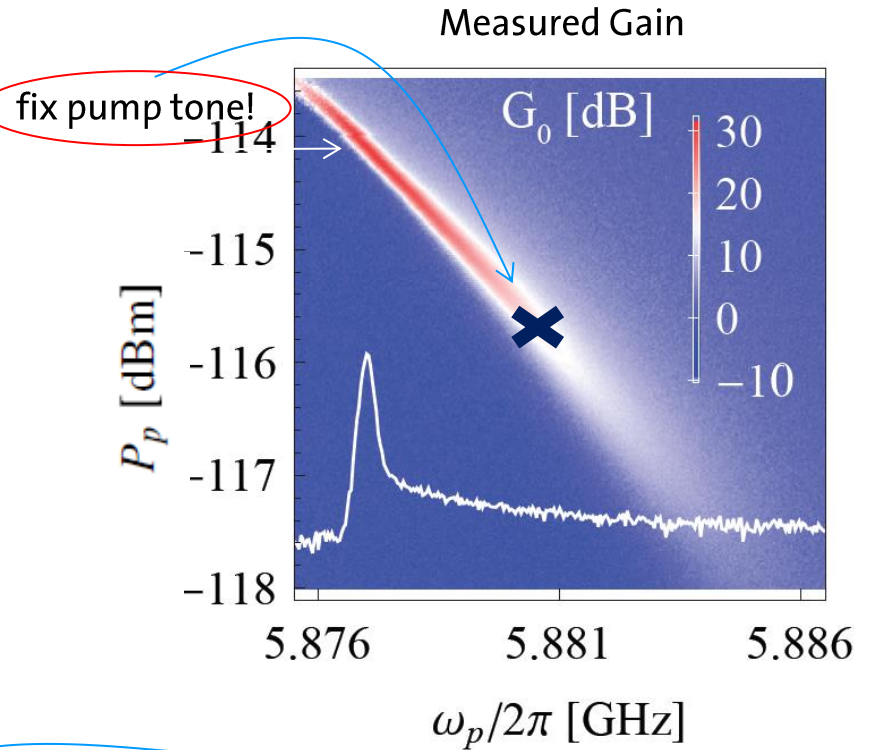
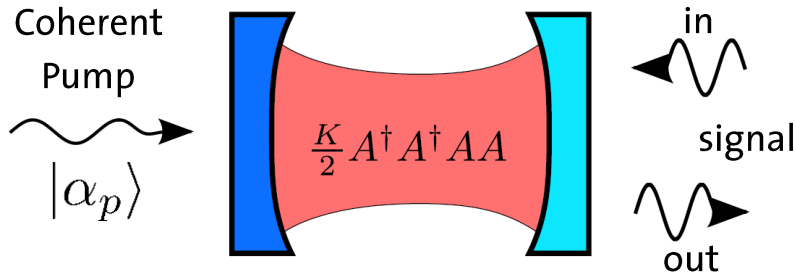


P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

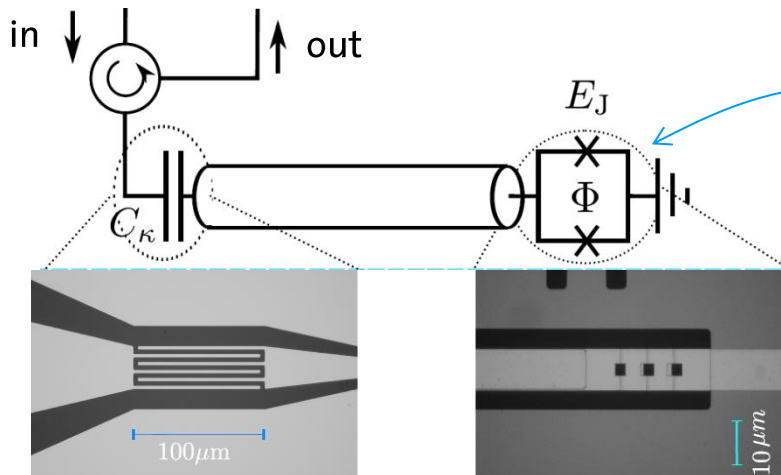
Statistics of Integrated Single-Shot Readout



Near Quantum-Limited Parametric Amplifier



Circuit QED implementation:



SQUID(-array) provides required nonlinearity

Eichler *et al.*, EPJ Quantum Technology 1, 2 (2014)
 Eichler *et al.*, Phys. Rev. Lett. 107, 113601 (2011)

Caves, Phys. Rev. D 26, 1817 (1982)

Yurke and Buks, J. Lightwave Tech. 24, 5054 (2006)

Castellanos-Beltran *et al.*, Nat. Phys. 4, 929 (2008)

The Lamb and AC-Stark Shifts

signatures of the dispersive interaction with a quantum field

for $\Delta = \omega_a - \omega_r \gg g$

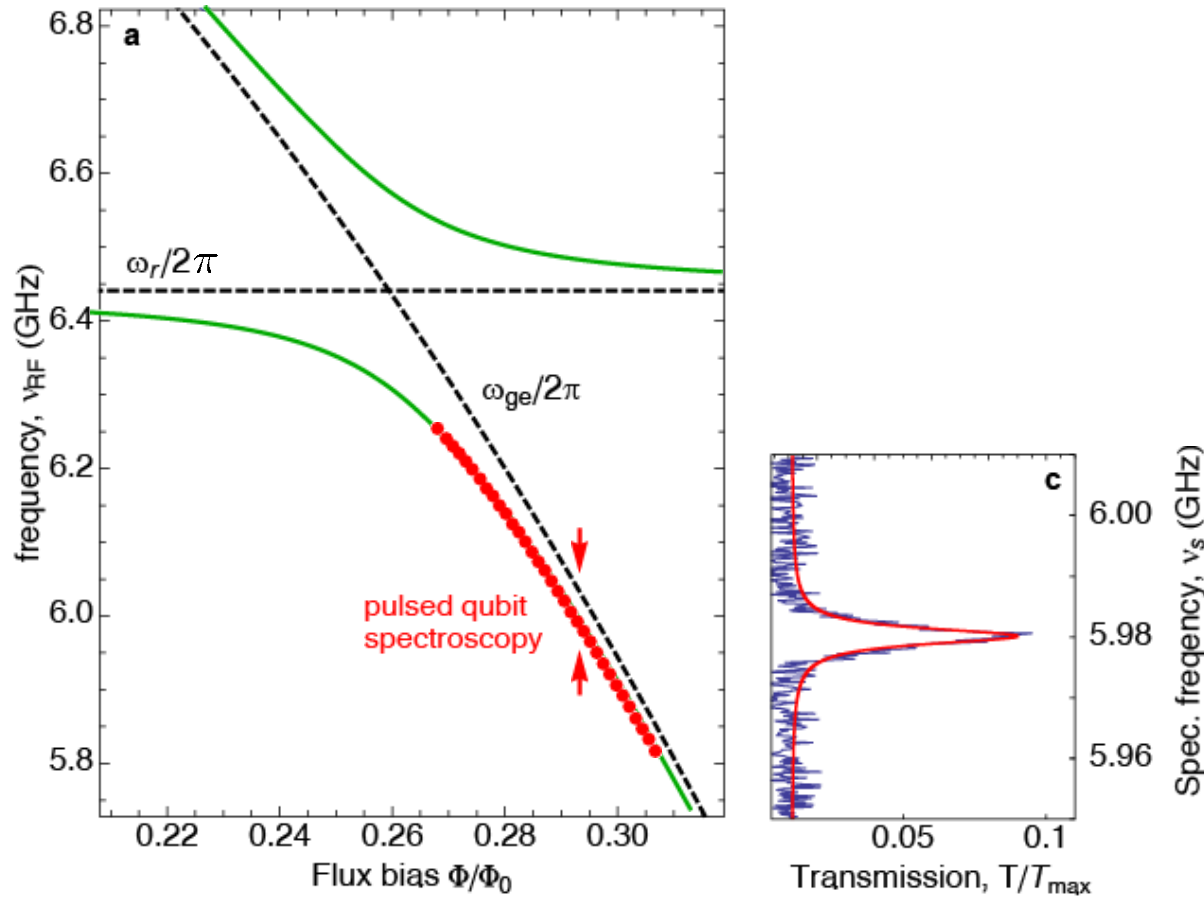
$$H \approx \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar \left(\omega_a + \overset{\text{Lamb shift}}{\parallel} \frac{g^2}{\Delta} + \overset{\text{AC Stark shift}}{\parallel} \frac{2g^2}{\Delta} a^\dagger a \right) \sigma_z$$

$$\tilde{\omega}_a \approx \omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta} n$$

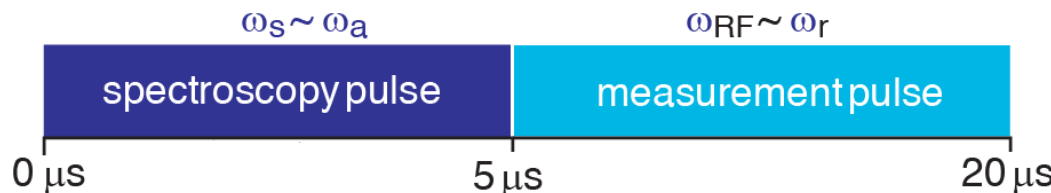
M. Brune *et al.*, *Phys. Rev. Lett.* **72**, 3339 (1994)

A. Blais *et al.*, *PRA* **69**, 062320 (2004)

Measurements of the Lamb and Quantized Stark Shifts

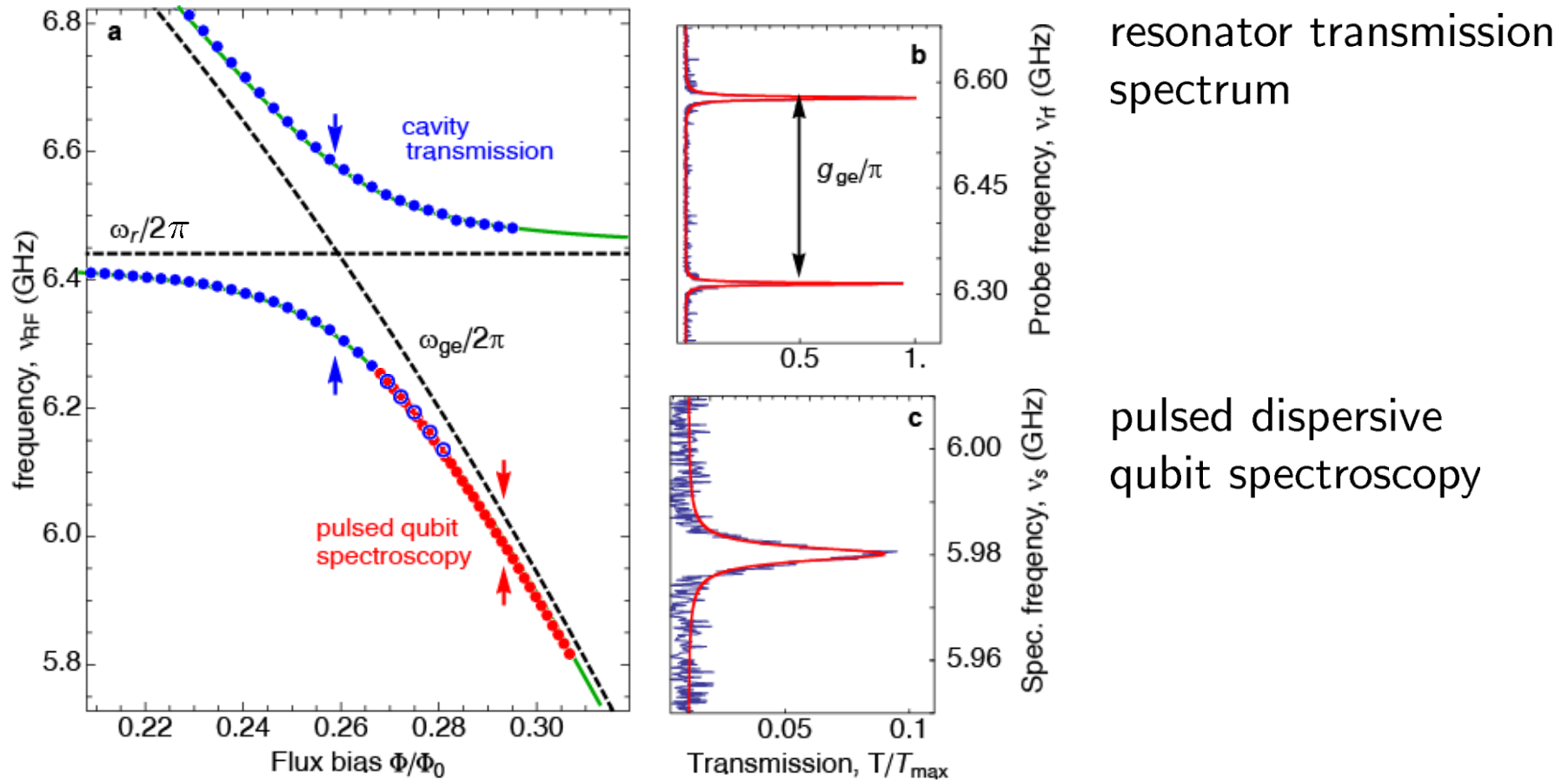


pulsed dispersive
qubit spectroscopy



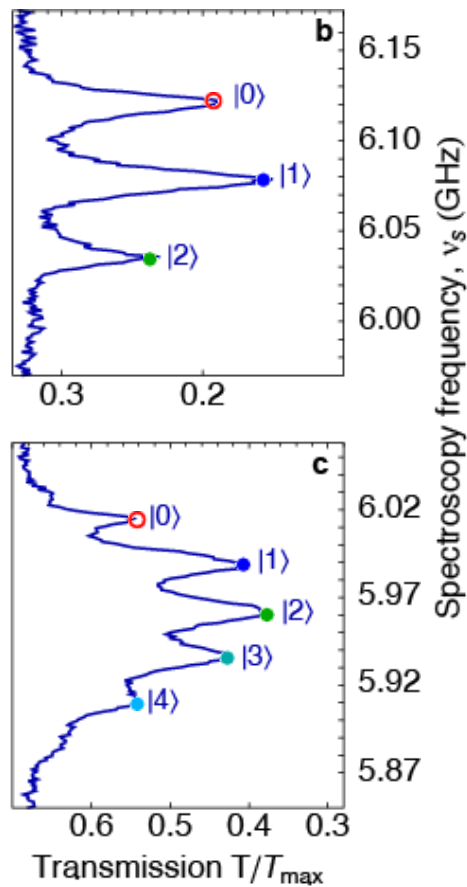
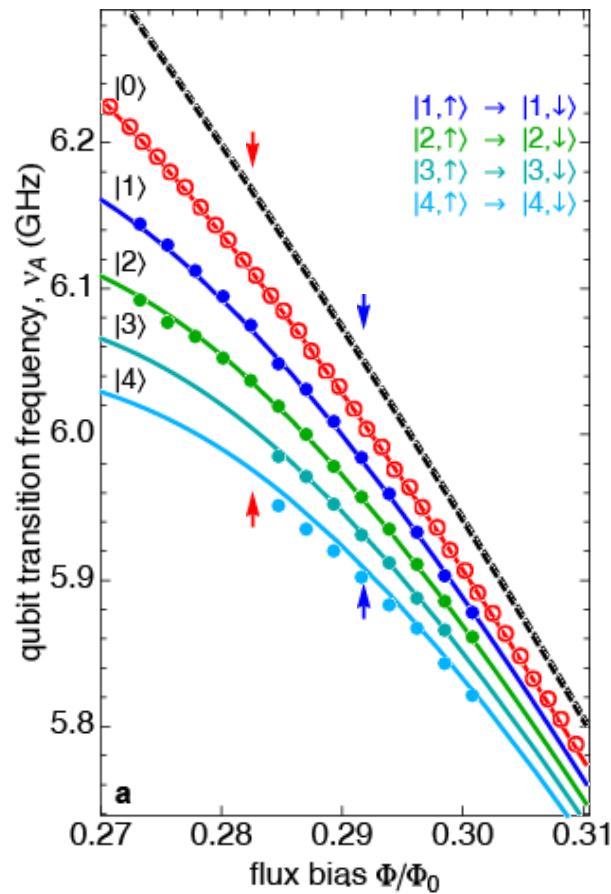
pulsed spectroscopy scheme

Measurement of the Lamb Shift



- qubit and photon component of joint state are measured
- accurate knowledge of qubit parameters possible

Quantum AC-Stark Shift and Lamb Shift



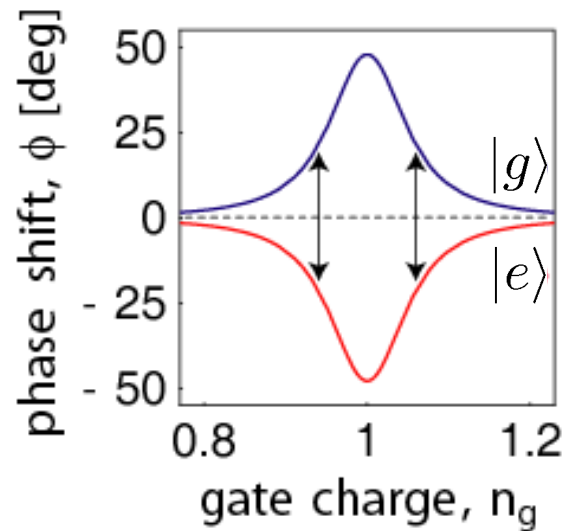
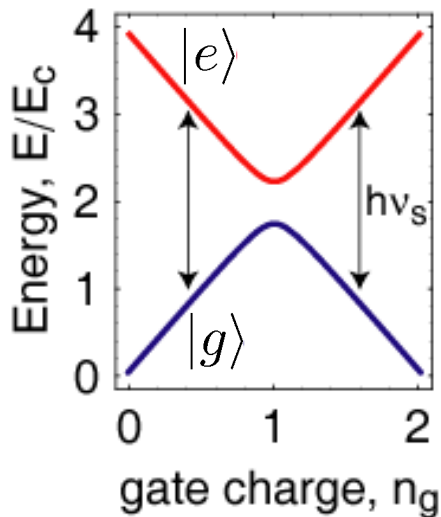
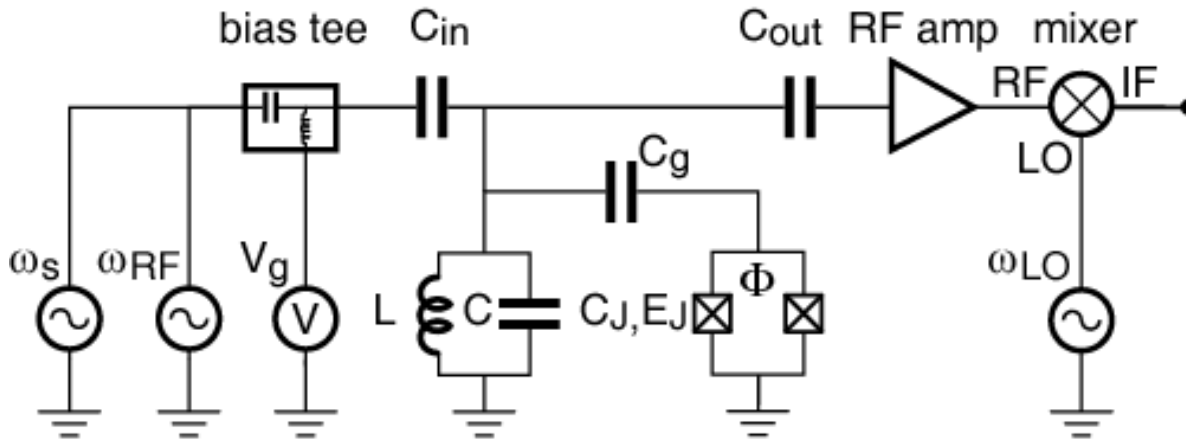
- populate resonator with small coherent field (Poisson distribution of photon number)
- spectroscopic measurement of qubit line shape
- qubit frequencies ac-Stark shifted by quantized cavity field



D. Schuster *et al.*, *Nature* 445, 515 (2007)
A. Fragner *et al.*, *Science* 322, 1357 (2008)

Qubit Control

Qubit Spectroscopy with Dispersive Read-Out



Driving Qubit Transitions in J-C Hamiltonian

Hamiltonian for microwave drive

$$H_d = \hbar\epsilon(t) (a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t})$$

Unitary transform

$$\begin{aligned} \tilde{H} &= U(H_{\text{JC}} + H_d)U^\dagger & \text{with } U &= \exp \frac{g}{\Delta} (a\sigma^+ - a^\dagger\sigma^-) \\ & & \text{and } \Delta &= \omega_a - \omega_r \end{aligned}$$

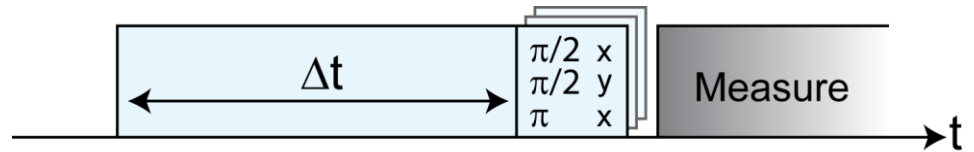
Results in dispersive approximation up to 2nd order in g

$$\begin{aligned} \tilde{H} \approx & \frac{\hbar}{2} \left(\omega_q + \frac{2g^2}{\Delta} (a^\dagger a + \frac{1}{2}) - \omega_d \right) \sigma_z + \hbar \frac{g\epsilon(t)}{\Delta} \sigma_x \\ & + \hbar(\omega_r - \omega_d) a^\dagger a + \hbar\epsilon(t)(a^\dagger + a) \end{aligned}$$

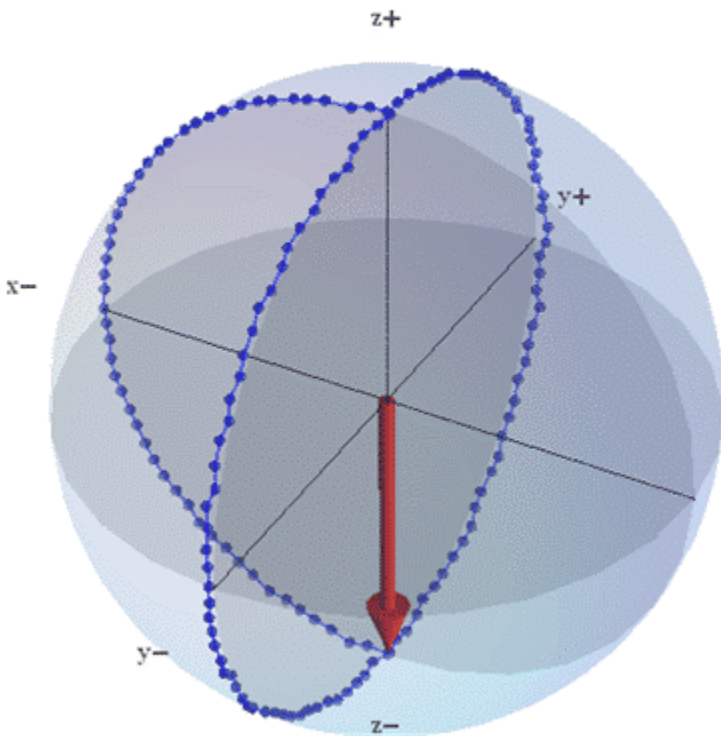
Drive induces Rabi oscillations in qubit when in resonance with dispersively shifted qubit frequency

Single Qubit Gates

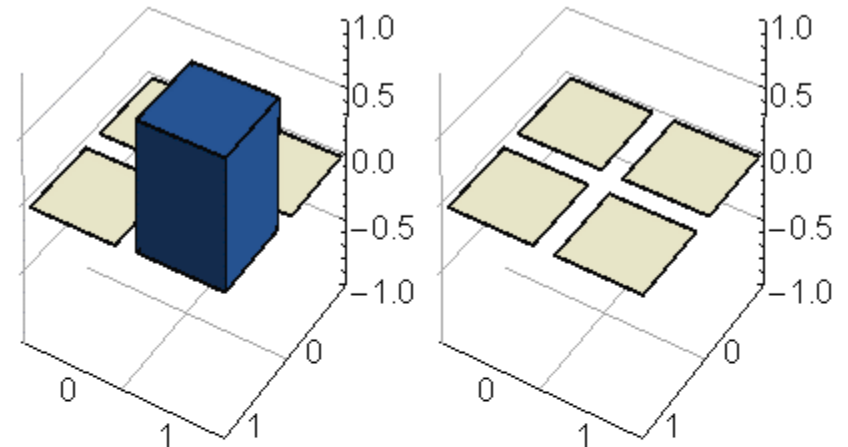
Pulse sequence for qubit rotation and readout:



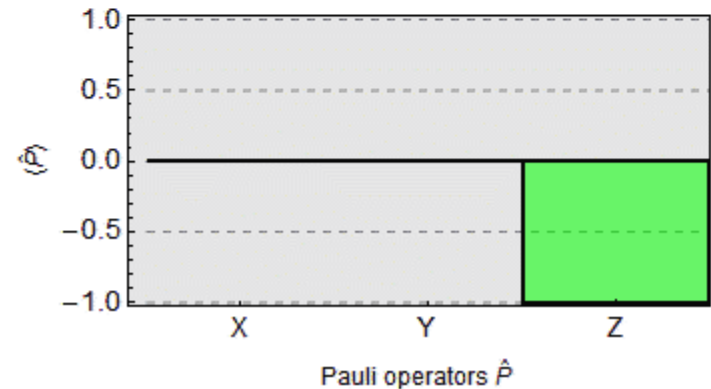
experimental Bloch vector:



experimental density matrix and Pauli set:

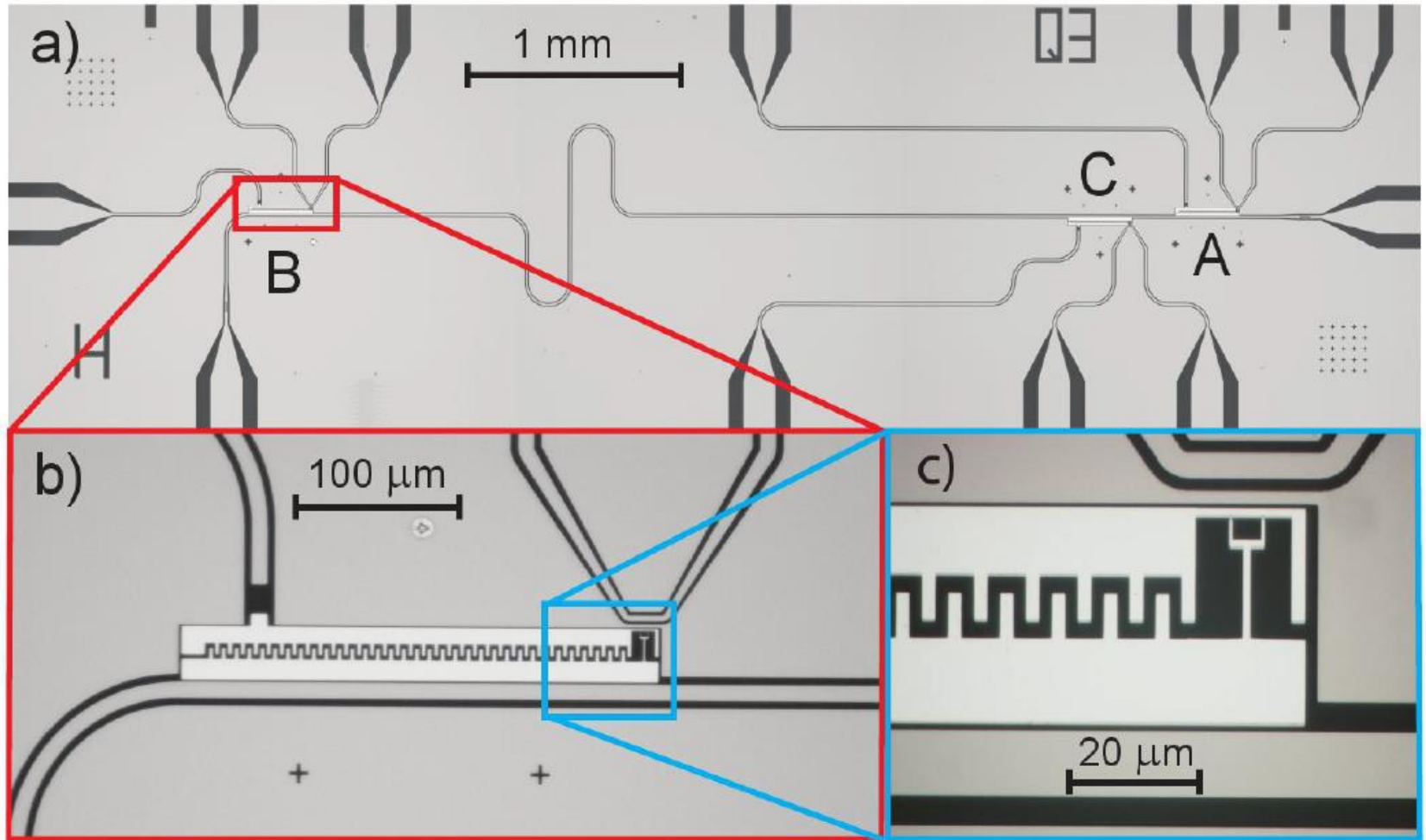


x+



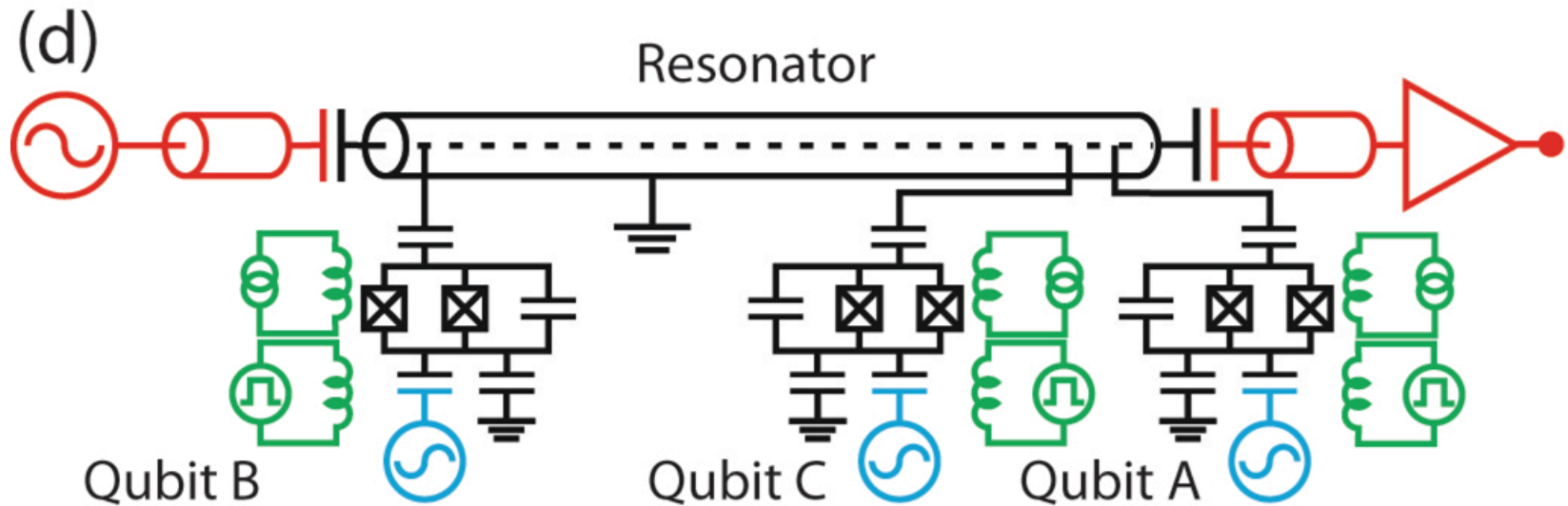
Coupling Superconducting Qubits and Generating Entanglement using a Controlled Phase Gate

Quantum Processor with 3 Qubits: The Chip



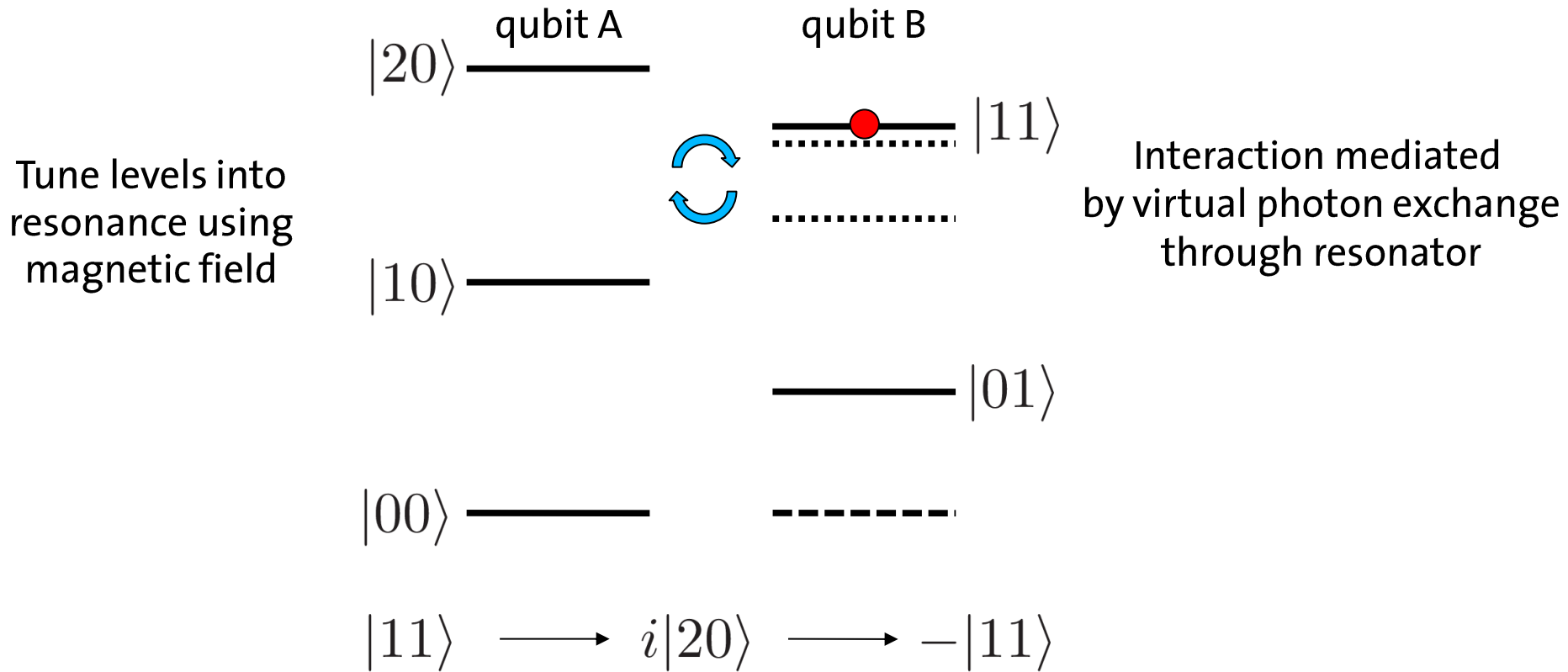
- three transmon qubits: $T_1 \sim 1.0 \mu\text{s}$, $T_2 \sim 0.6 \mu\text{s}$, individual local control
- one resonator: $f_0 \sim 8.625 \text{ GHz}$, coupling to qubits $g/2\pi \sim 300 \text{ MHz}$

Quantum Processor with 3 Qubits: Circuit Diagram

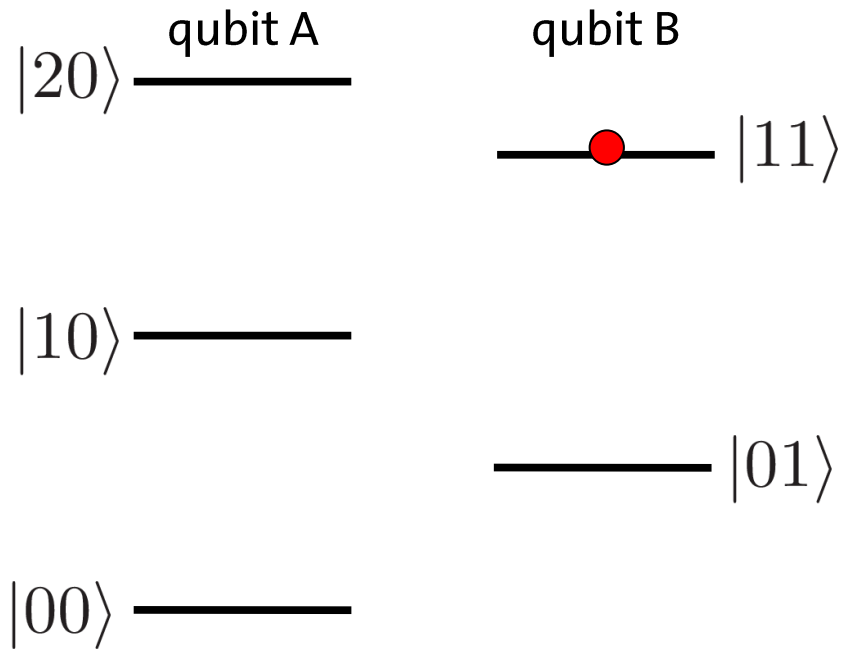


- **qubit state measurement** through resonator
- individual qubit control through local **microwave gates**
- two-qubit interactions by tuning qubits into resonance using local **flux gates**

Universal Two-Qubit Controlled Phase Gate

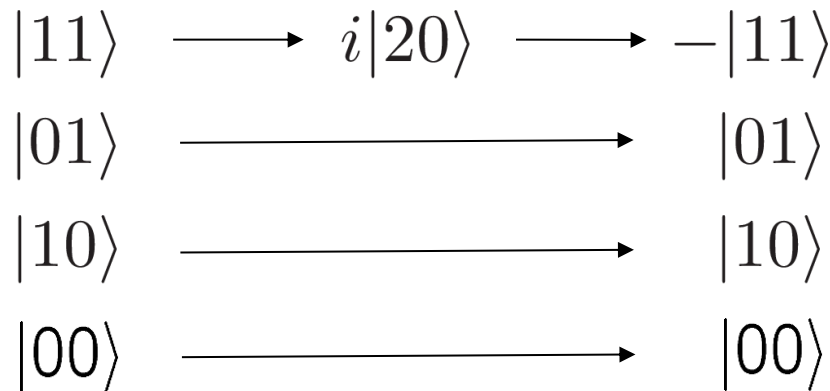


Universal Two-Qubit Controlled Phase Gate



How to verify the operation of this gate?

Universal two-qubit gate. Used together with single-qubit gates to create any quantum operation.



C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
 first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Process Tomography: C-Phase Gate

arbitrary quantum process

$$\rho' = \mathcal{E}(\rho)$$

decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

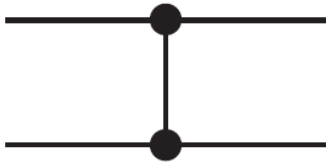
$$\{\tilde{E}_k\}$$

$$\chi$$

is an operator basis

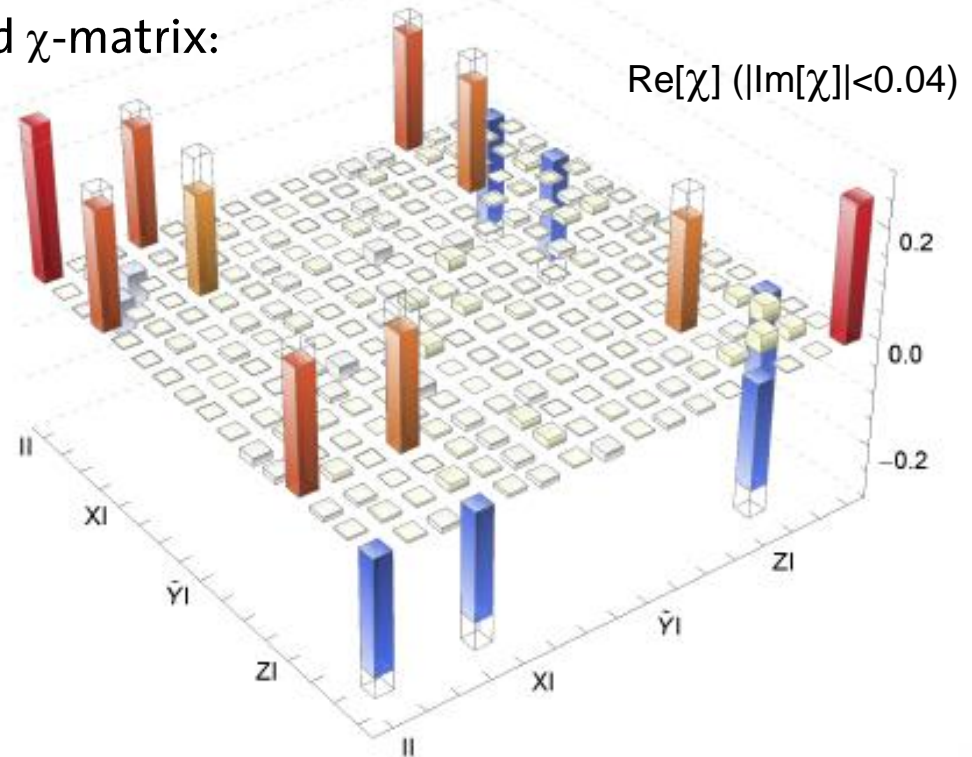
is a positive semi definite Hermitian matrix characteristic for the process

Controlled phase gate



$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured χ -matrix:



$$F = \text{Tr}[\chi_{\text{meas}} \chi_{\text{ideal}}] = 0.86$$

Process Tomography: C-NOT Gate

arbitrary quantum process

$$\rho' = \mathcal{E}(\rho)$$

decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

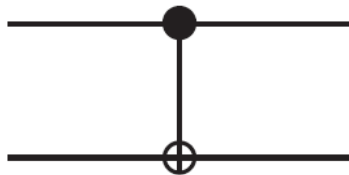
$$\{\tilde{E}_k\}$$

$$\chi$$

is an operator basis

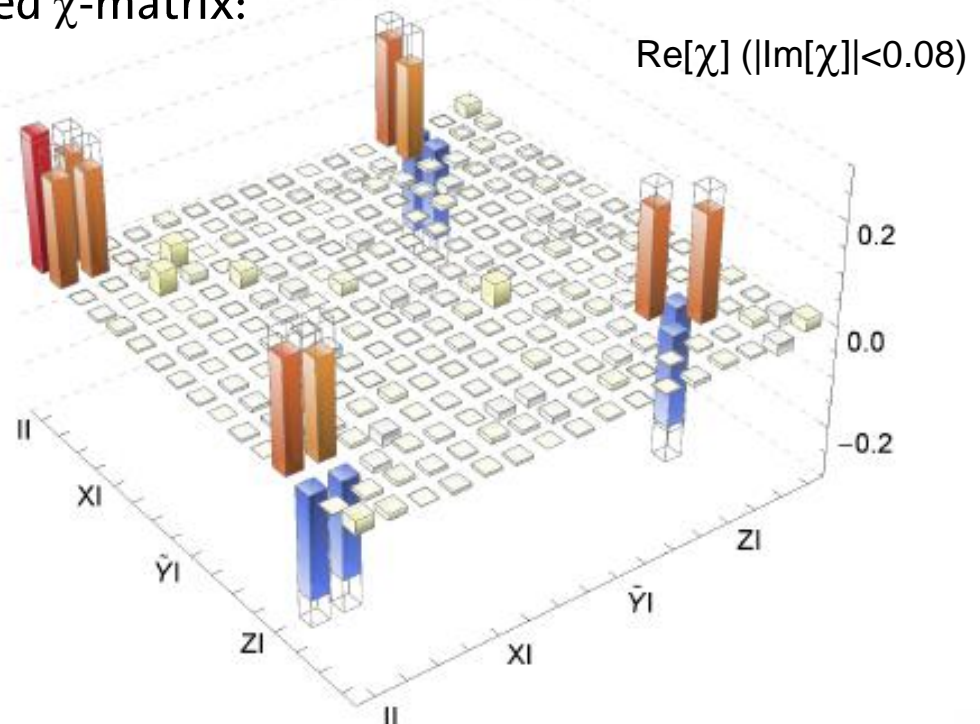
is a positive semi definite Hermitian matrix characteristic for the process

Controlled-NOT gate



$$\text{C-NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Measured χ -matrix:



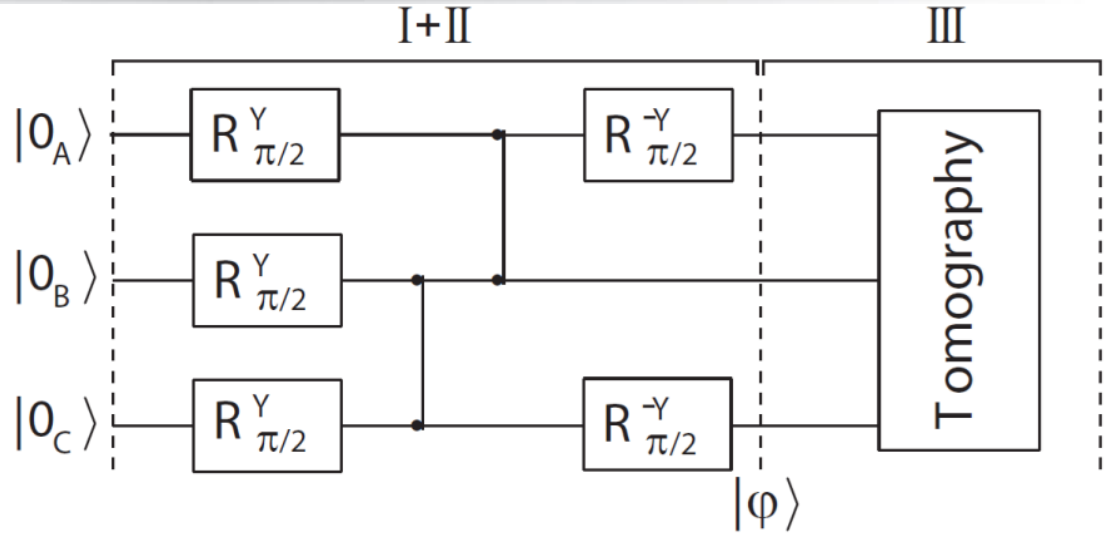
$$F = \text{Tr}[\chi_{\text{meas}} \chi_{\text{ideal}}] = 0.81$$

Maximally Entangled Three Qubit States

Generation of GHZ class, e.g.

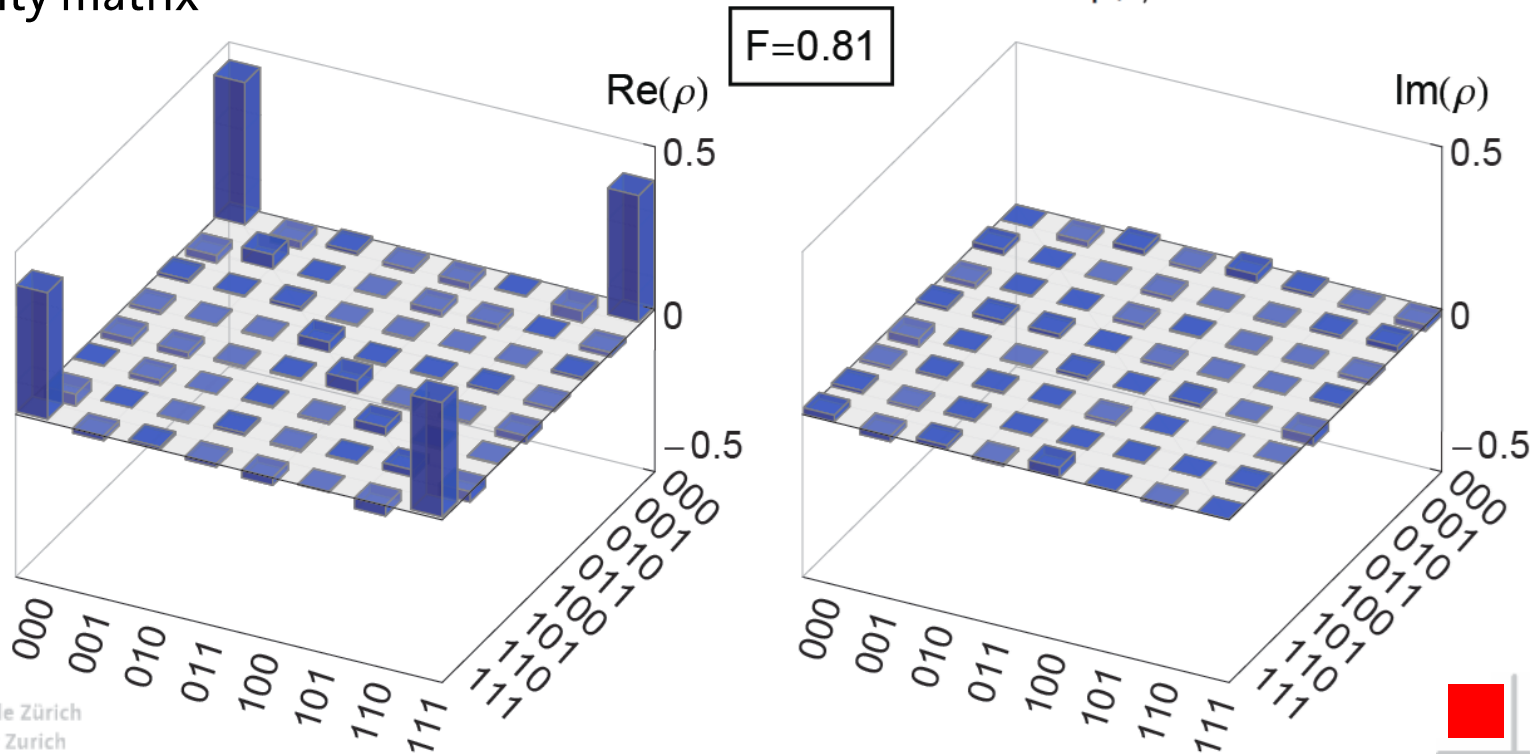
$|000\rangle + |111\rangle$, states:

- single qubit gates
- C-PHASE gates



Measured density matrix

- high fidelity



DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

- #1. A scalable physical system with well-characterized qubits. ✓
- #2. The ability to initialize the state of the qubits. ✓
- #3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
- #4. A universal set of quantum gates. ✓
- #5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

- #6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
- #7. The ability to faithfully transmit flying qubits between specified locations. ✓