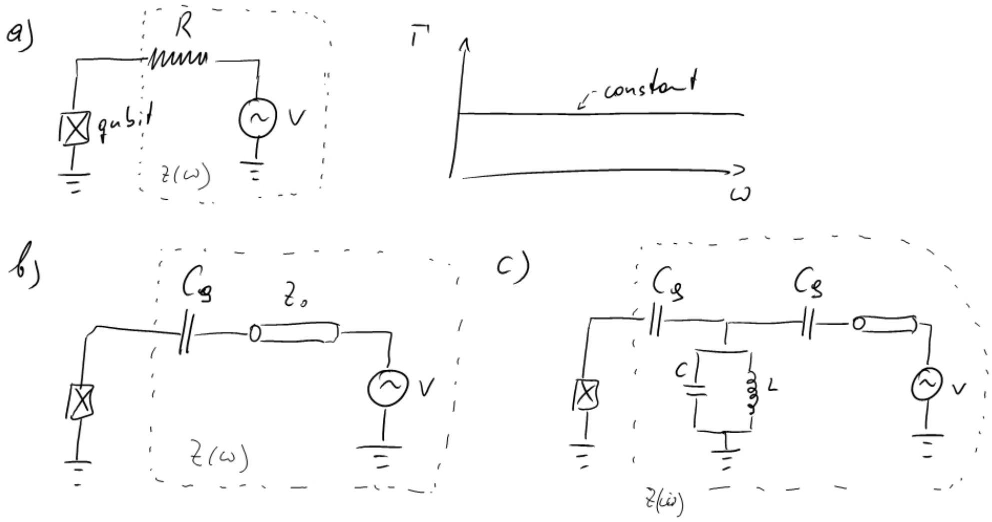
QSIT 2015 - Questions 3

2015, HIT F 13

1. **Energy relaxation of a qubit**
   In analogy to the harmonic oscillator, the energy decay time of a qubit is given by $T_1 = RC$, where C is the intrinsic capacitance of the qubit. R denotes the effective resistance $R = 1/\mathrm{Re}[Y(\omega)]$ obtained from the impedance of the environment $Z(\omega) = 1/Y(\omega)$ as seen from the position of the qubit.



   If the impedance of the environment is purely resistive, e.g. $Z(\omega) = 50\ \Omega$, the decay rate $\Gamma = 1/T_1$ is frequency independent (see Figure a).

   (a) Derive the impedance of a Cooper-pair box qubit that is capacitively coupled to a transmission line ($Z_0 = 50\ \Omega$) via a gate capacitance C_g (Figure b). Sketch the decay rate Γ as a function of frequency.

   (b) What is the spectral shape of Γ for a coupling to an LC oscillator (Figure c)?

2. **Two-level approximation for a Cooper-pair box**
The Hamiltonian for a Cooper-pair box is given by

$$H_{CPB} = \sum_n \left[E_C(\hat{n} - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right) \right].$$

   (a) Plot first four eigenvalues of this Hamiltonian as a function of the gate charge n_g.

   (b) Write down the Hamiltonian for the two-dimensional qubit subspace in terms of the Pauli matrices σ_x and σ_z by restricting the quantum states to $n = 0, 1$.

   (c) What is the transition frequency between ground and excited state?

3. **Measurement of relaxation and dephasing**
Typically all experiments starts from the qubit being in its ground state $|0\rangle$. The state of the qubit can be manipulated with the help of the microwave pulses that can induce rotations by an angle ϕ about any of the axis x, y according to $R_{x,y}(\phi) = \exp[-i(\phi/2)\sigma_{x,y}]$ (note that the manipulation are performed in the rotating frame). In order to extract the relaxation time T_1 and T_2 we can use the following schemes:

   (a) Make a π rotation (π-pulse) about x-axis, wait a time interval of Δt and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt.

   (b) Make a $\pi/2$ rotation ($\pi/2$-pulse) about x-axis, wait a time interval of Δt, make another $\pi/2$-pulse and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt.

   Using Bloch equations answer the following questions:

   (a) What is the evolution of the Bloch vector for these experiments?

   (b) How do we extract the timescales T_1, T_2?

   (c) What happens if the microwave pulses are detuned from the frequency of the qubit by $\delta\omega$ for the second experiment. (Ramsey experiment)

   (d) What happens if the one insert π pulse exactly in the middle between $\pi/2$ pulses for the second experiment (Echo experiment).