Algorithms in Superconducting Circuits

Axel Dahlberg, Marco Roth

April 24, 2015

Motivation

1. Implementing Grover's algorithm.

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2. Superconducting circuits.

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2. Superconducting circuits.
3. Control over parameters.

Outline

Grover's Algorithm

1. A quantum algorithm for finding an element in an unstructured set of $N = 2^n$ elements.

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2. **Assumption**: Given a function $f(x) : \{0, 1\}^n \to \{0, 1\}$ such that

$$f(x) = \begin{cases} 1 & x = x_k \\ 0 & else \end{cases}, \tag{1}$$

for some k.

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3. **Task**: Find x_k with the least amount of calls to the function f.
4. To evaluate the function f a gate called the oracle O_f is implemented. The gate does the following

$$|x\rangle \to (-1)^{f(x)} |x\rangle . \tag{2}$$

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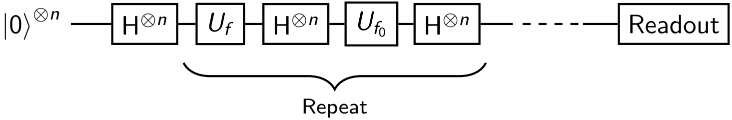
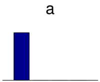


The function f_0 outputs a 1 if the input is $0^{\otimes n}$ and otherwise 0.

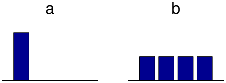
Grover's Algorithm



$$|\varphi_a\rangle = |0,0\rangle$$

One iteration of Grover's algorithm on two qubits.

Grover's Algorithm



$$|\varphi_{\mathsf{a}}\rangle = |0,0\rangle$$

$$\downarrow \quad \mathsf{H}^{\otimes 2}$$

$$|\varphi_{\mathsf{b}}\rangle = (|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)/2$$

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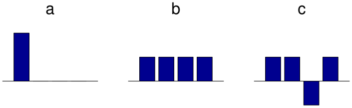

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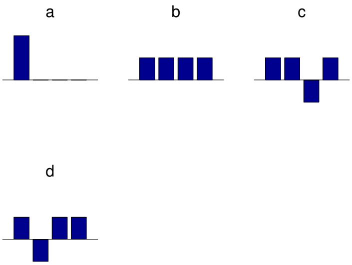

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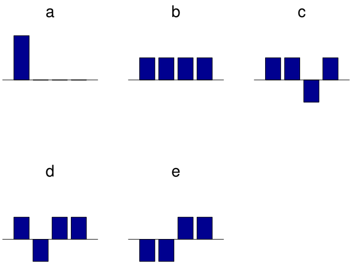

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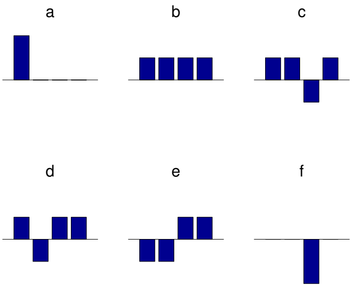

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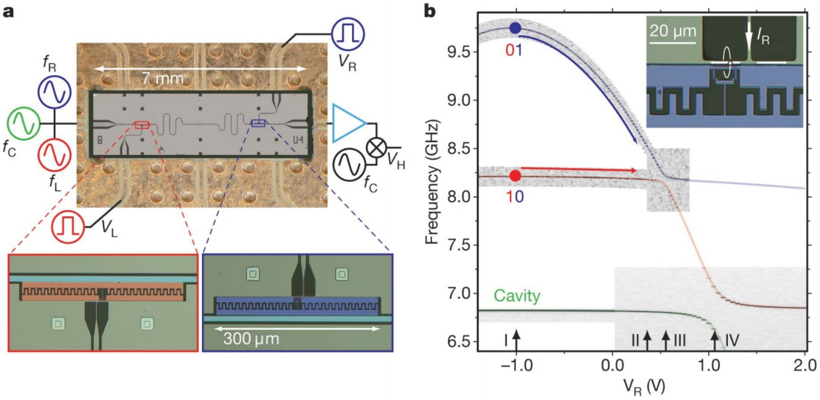
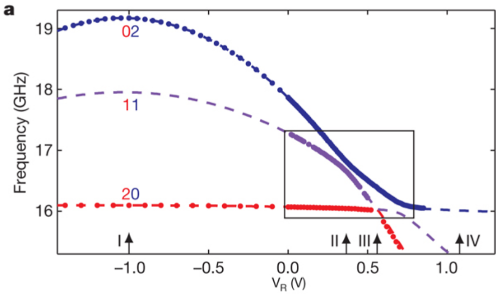
One iteration of Grover's algorithm on two qubits.

Superconducting circuit with two qubits used for realising Grover's algorithm.



A four-port device with a coplanar waveguide cavity bus coupling two transmon qubits (Fig. taken from [1]).

Implementing the C-Phase gate



Flux dependence of transition frequencies (Fig. taken from [1]).

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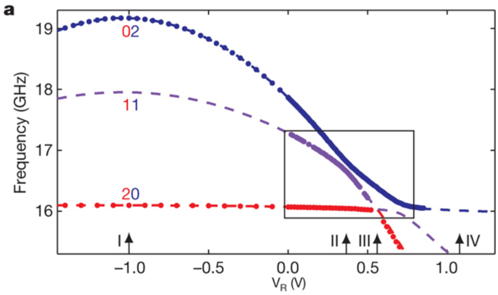

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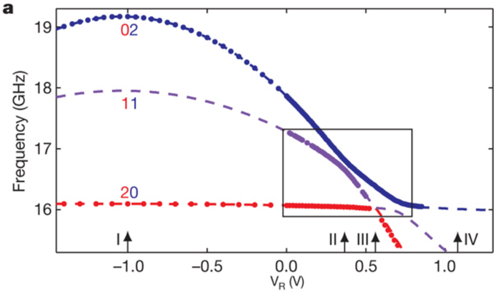

Flux dependence of transition frequencies (Fig. taken from [1]).

1. Cavity-qubit interactions produce a frequency shift between the $|1,1\rangle$ and the $|0,2\rangle$ state
2. This shift can be exploited to create a C-Phase gate:

Implementing the C-Phase gate

1. The interaction between the $|1,1\rangle$ state and the non-computational $|0,2\rangle$ state introduces a phase difference:

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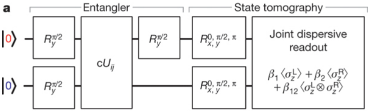
2. Which effectively produces the gate:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (3)$$

Creating entanglement



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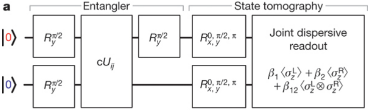

$$|\varphi_a\rangle = |0,0\rangle$$

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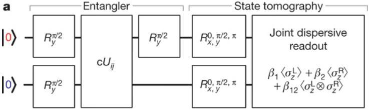

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$$\downarrow \quad R_y^{\pi/2} \otimes R_y^{\pi/2}$$

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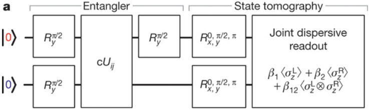

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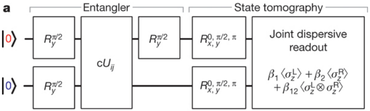
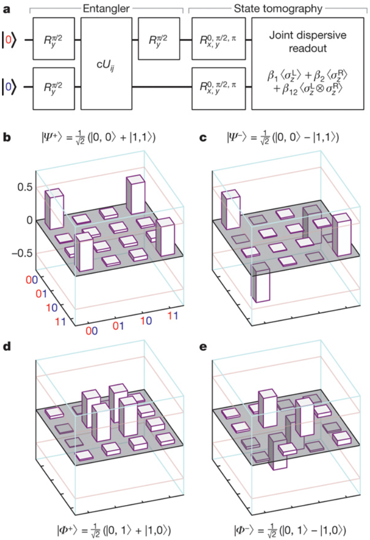


$$
\begin{aligned}
|\varphi_{\mathsf{a}}\rangle &= |0,0\rangle \\
&\downarrow \quad \mathrm{R}_y^{\pi/2} \otimes \mathrm{R}_y^{\pi/2} \\
|\varphi_{\mathsf{b}}\rangle &= \frac{1}{2}(|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle) \\
&\downarrow \quad U_{10} \\
|\varphi_{\mathsf{c}}\rangle &= \frac{1}{2}(|0,0\rangle + |0,1\rangle - |1,0\rangle + |1,1\rangle) \\
&\downarrow \quad \mathrm{R}_y^{\pi/2} \\
|\varphi_{\mathsf{d}}\rangle &= \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle) = \left|\Psi^+\right\rangle
\end{aligned}
$$

Creating entanglement



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$$\downarrow \quad U_{10}$$

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$$\downarrow \quad R_y^{\pi/2}$$

$$|\varphi_d\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle+|1,1\rangle) = |\Psi^+\rangle$$

The C-Phase gate can be used to create entanglement (Fig. taken from [1]).

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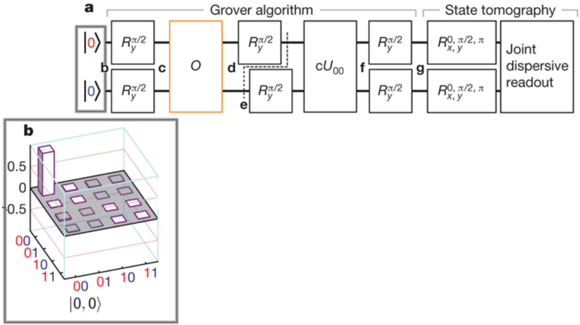

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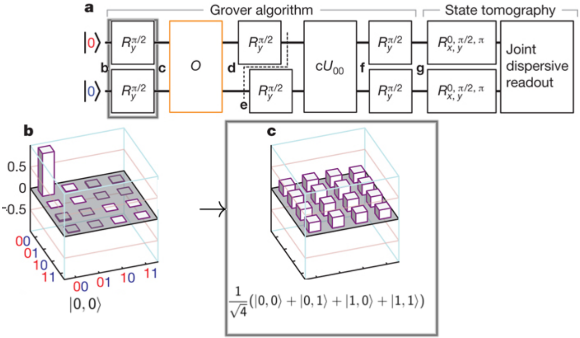

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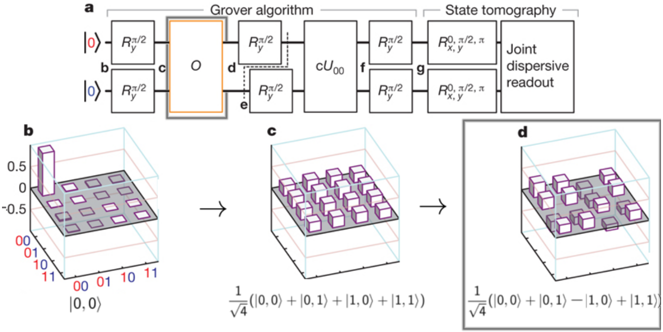

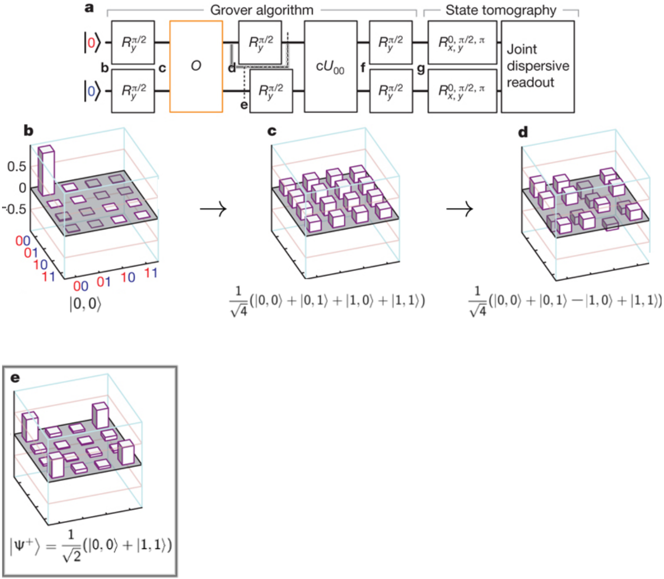
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Grover algorithm
State tomography
|0⟩
|0⟩
$R_y^{\pi/2}$
O
cU00
$R_{x,y}^{0,\pi/2,\pi}$
Joint dispersive readout
b
c
d
e
f
g
0.5
0
-0.5
00
01
10
11
$|0,0\rangle$
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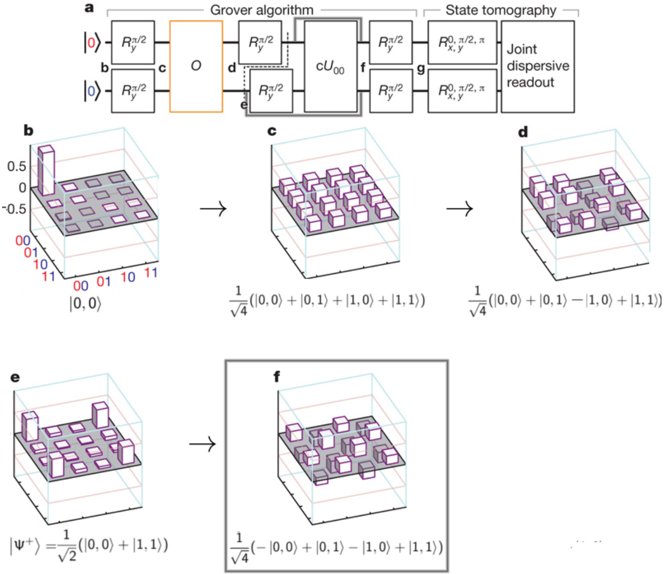

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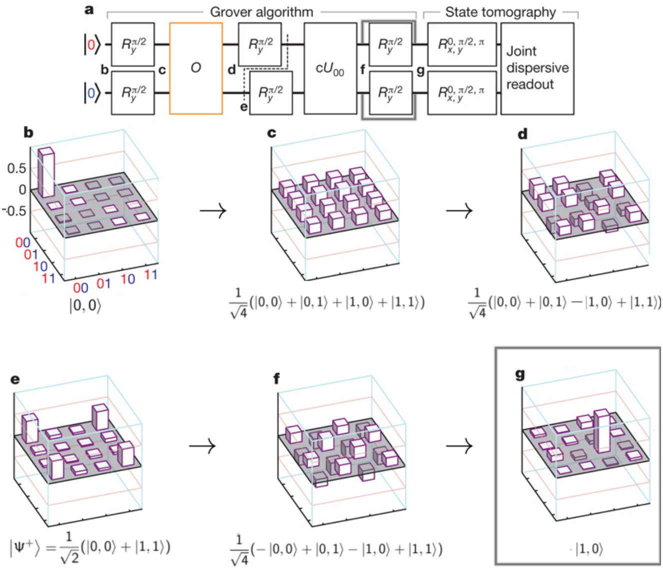

Fidelity of Results

Table 1 | Summary of algorithmic performance

Element		Grover search oracle*			
		f_{00}	f_{01}	f_{10}	f_{11}
$\langle 0,0 \vert \rho \vert 0,0 \rangle$	Ideal	1	0	0	0
	Measured	0.81(1)	0.08(1)	0.07(2)	0.065(7)
$\langle 0,1 \vert \rho \vert 0,1 \rangle$	Ideal	0	1	0	0
	Measured	0.066(7)	0.802(9)	0.05(1)	0.054(8)
$\langle 1,0 \vert \rho \vert 1,0 \rangle$	Ideal	0	0	1	0
	Measured	0.08(1)	0.05(1)	0.82(2)	0.07(1)
$\langle 1,1 \vert \rho \vert 1,1 \rangle$	Ideal	0	0	0	1
	Measured	0.05(2)	0.07(1)	0.06(1)	0.81(1)

Fidelity $F(\rho, \psi) = \langle\psi| \rho |\psi\rangle$ of final states of Grover's algorithm (Figure taken from [1]).

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2. Basic algorithms have been implemented in superconducting circuit systems.

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2. Basic algorithms have been implemented in superconducting circuit systems.
3. The present architecture can easily be expanded to several qubits.

Sources

1 L. DiCarlo, J. M. Chow, J. M. Gambetta, Lev S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin & R. J. Schoelkopf
Demonstration of two-qubit algorithms with a superconducting quantum processor
Nature 460, 7252 (2009)

2 Clarke, J. & Wilhelm, F.K.
Superconducting quantum bits
Nature 453, 1031 (2008)

3 Schoelkopf, R.J. & Girvin, S.M.
Wiring up quantum systems
Nature 451, 664 (2008)

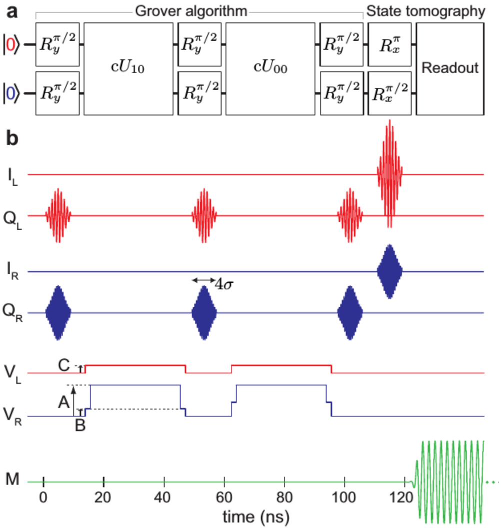
4 Devoret, M.H. & Martinis, J.M. *Implementing Qubits with Superconducting Integrated Circuits*
Quant. Inf. Proc, 3 163 (2004)

Creating Phase Gates

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\Phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\Phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\Phi_{11}} \end{pmatrix} \tag{4}$$

- Φ_{01} is adjusted by tuning the rising or falling edge of the pulse
- Φ_{10} is adjusted by varying the amplitude of a simultaneous V_L pulse

Creating Phase Gates



Creating U_{01}