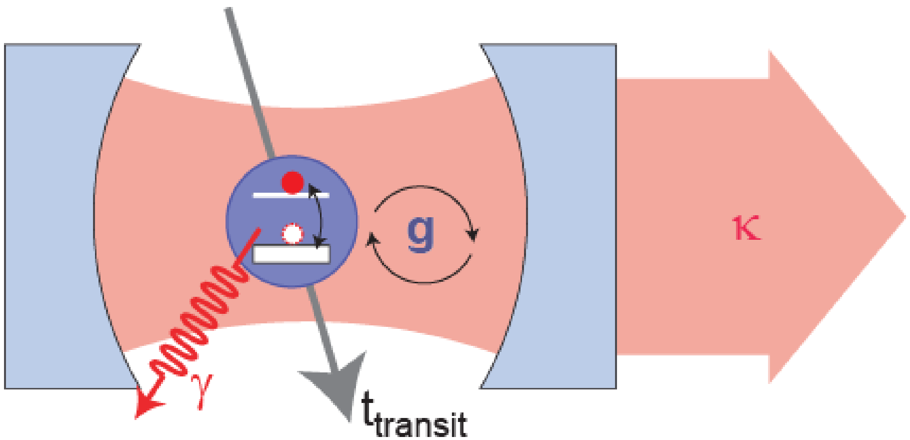
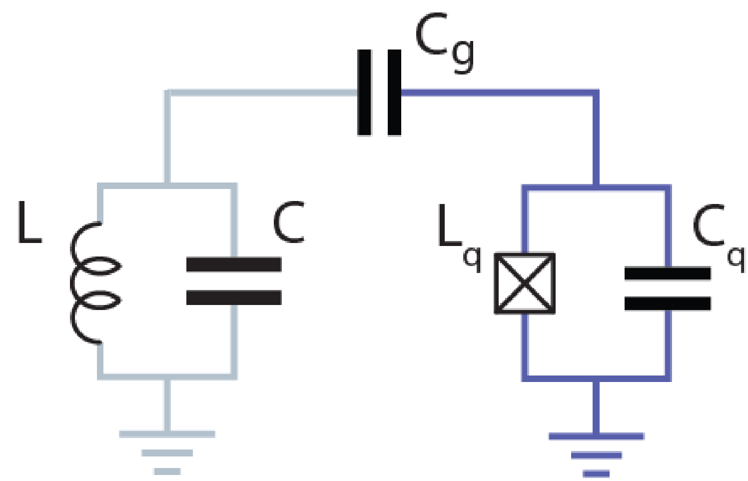
Cavity Quantum Electrodynamics (QED): Coupling a Harmonic Oscillator to a Qubit

Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

... basic approach:

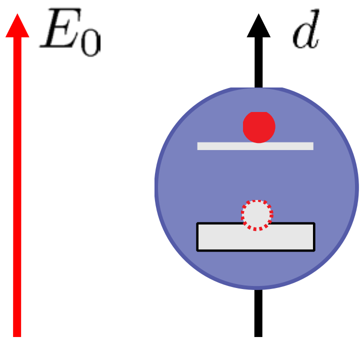


What is this good for?

- Isolating qubits from their environment
- Maintain addressability of qubits
- Reading out the state of qubits
- Coupling qubits to each other
- Converting stationary qubits to flying qubits

Controlling Light-Matter Interactions

challenging on the level of single (artificial) atoms and single photons



- mode-matching (controlling the absorption probability)
- single photon fields E_o (small in 3D)
- dipole moment d (usually small ~ ea_o)
- photon/dipole interaction $\hbar g \sim dE_0$ (usually small)

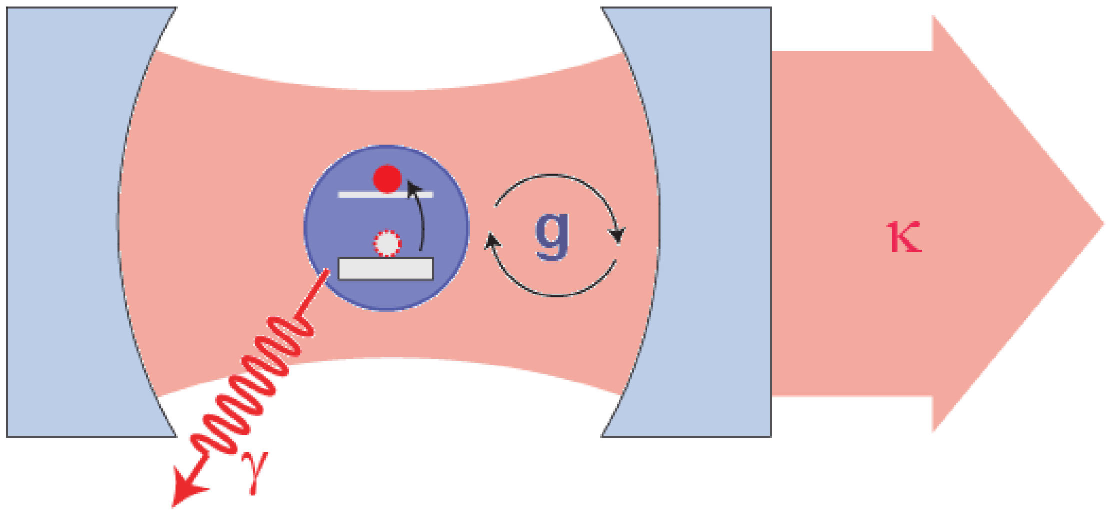
What to do?

- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit $(g = dE_0/\hbar > \gamma, \kappa\,, 1/t_{\text{transit}})$

D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

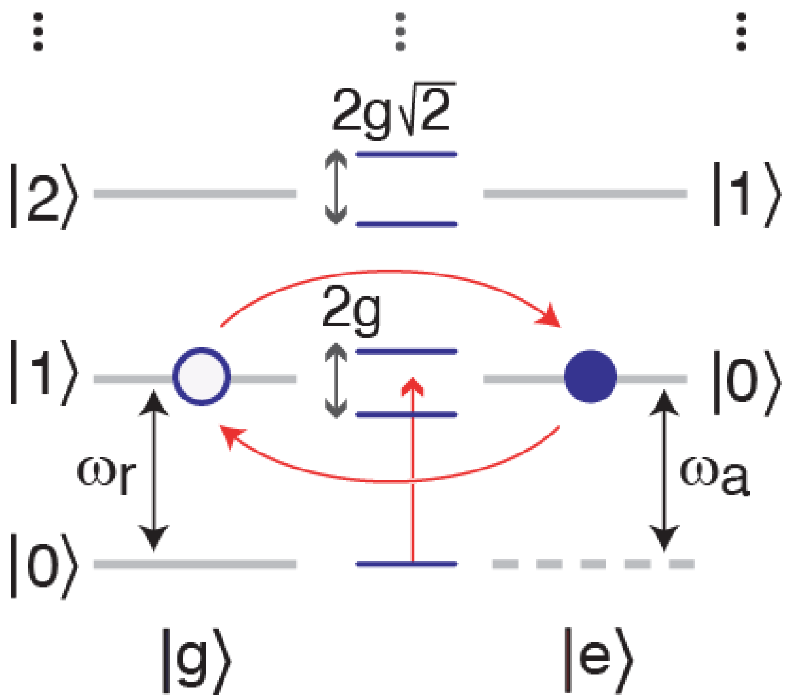
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma,\ \kappa$$



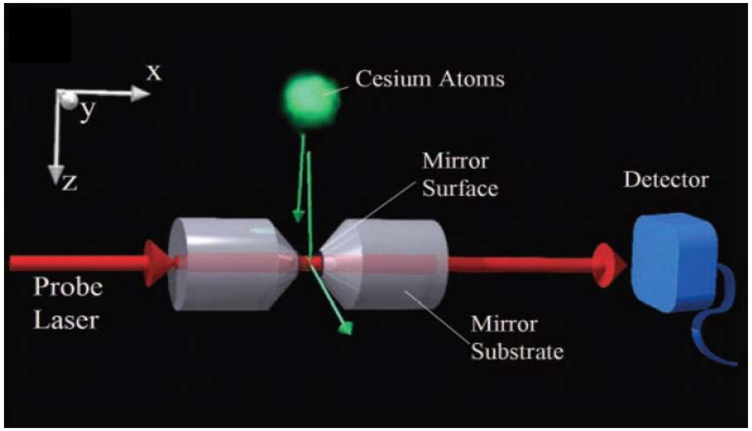
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

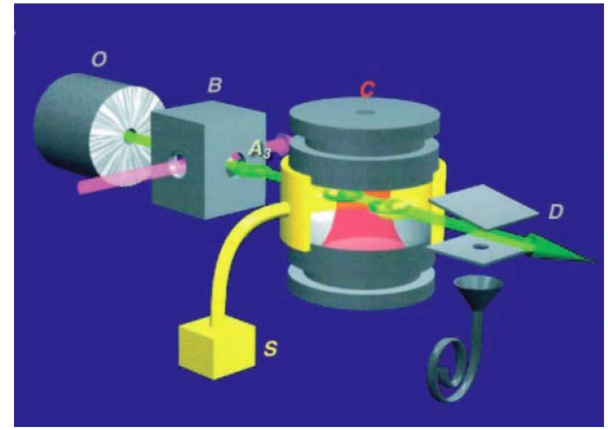
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

Cavity Quantum Electrodynamics (QED)



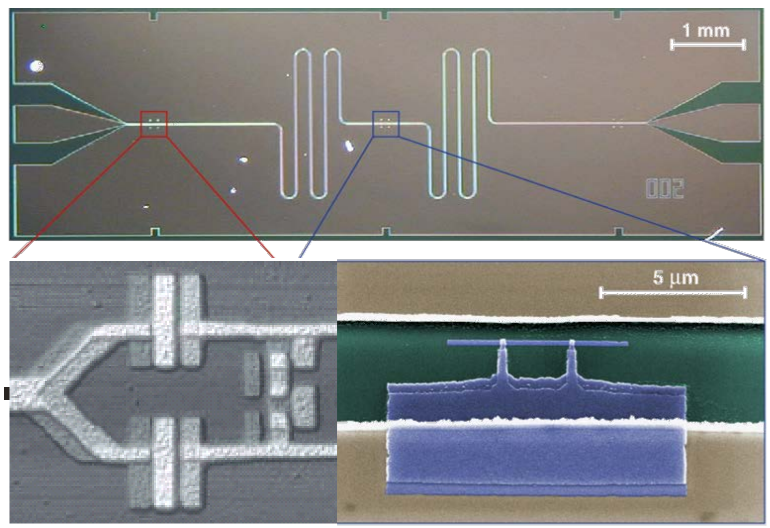
alkali atoms

MPQ, Caltech, ...



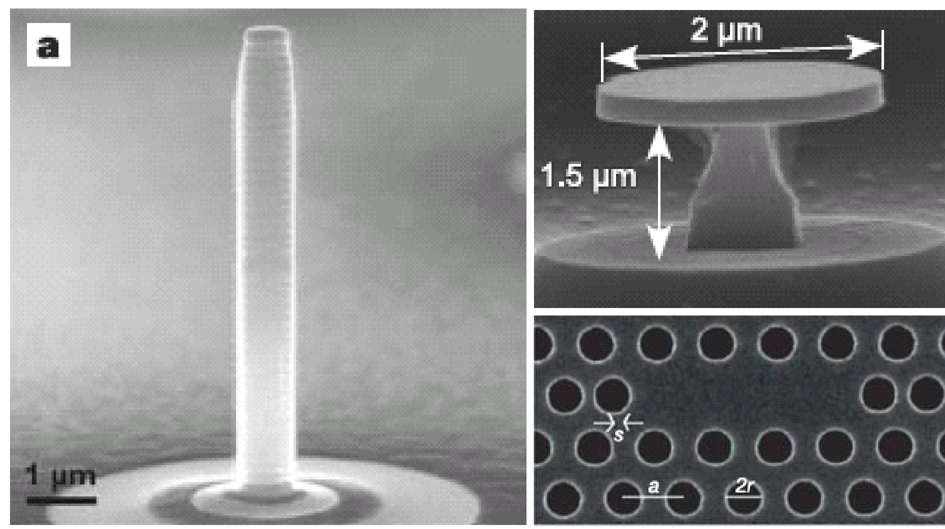
Rydberg atoms

ENS, MPQ, ...



superconductor circuits

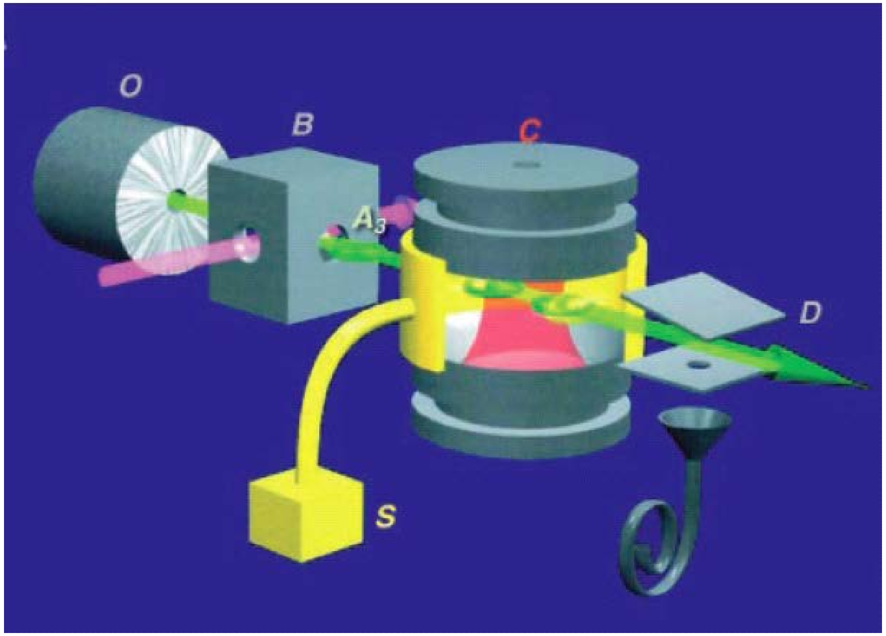
Yale, Delft, NTT, ETHZ, NIST, …

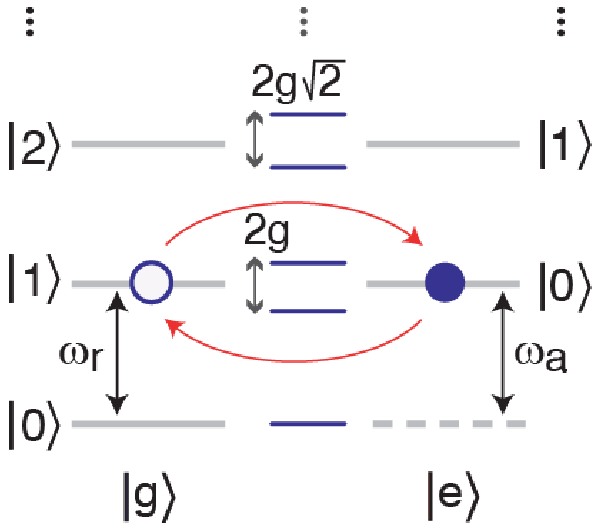


semiconductor quantum dots

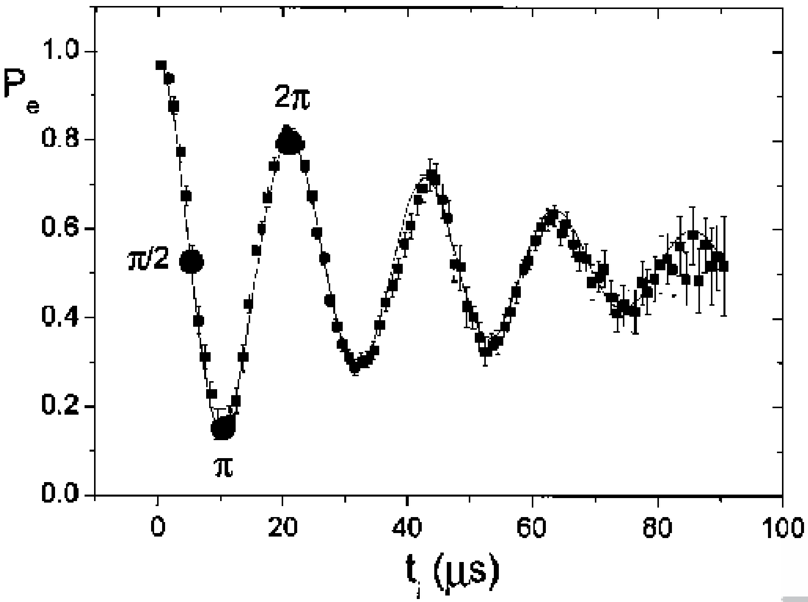
Wurzburg, ETHZ, Stanford …

Vacuum Rabi Oscillations with Rydberg Atoms

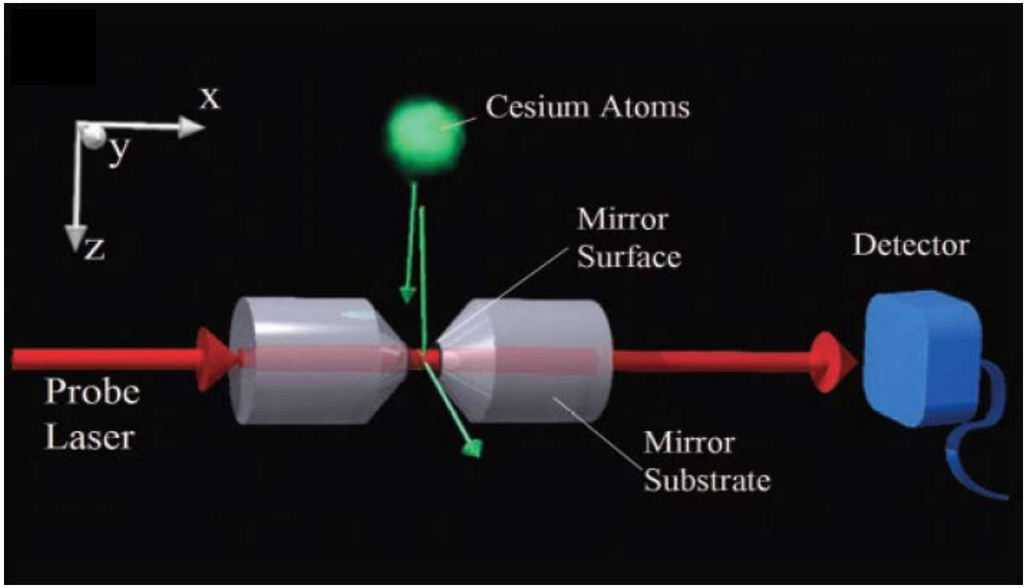


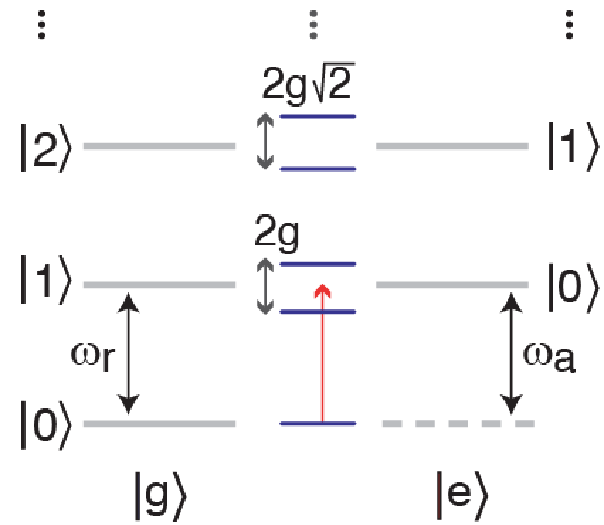


Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)

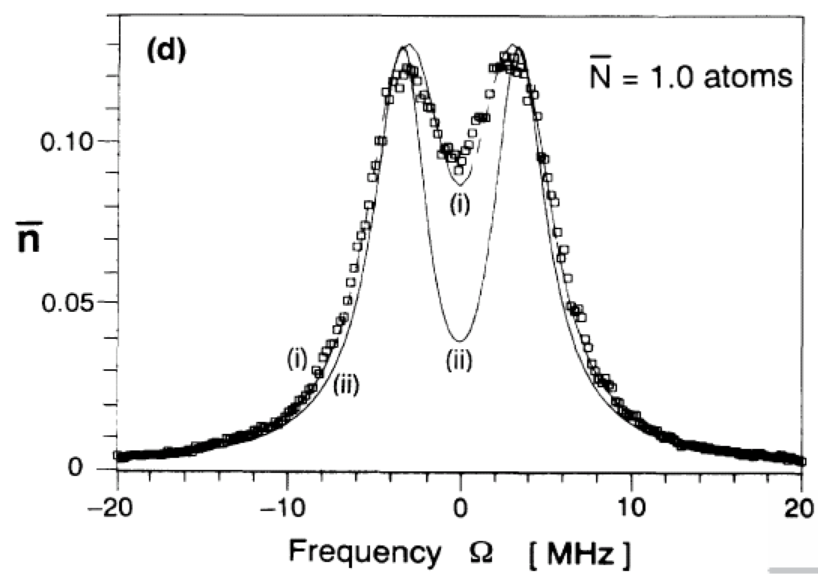


Vacuum Rabi Mode Splitting with Alkali Atoms

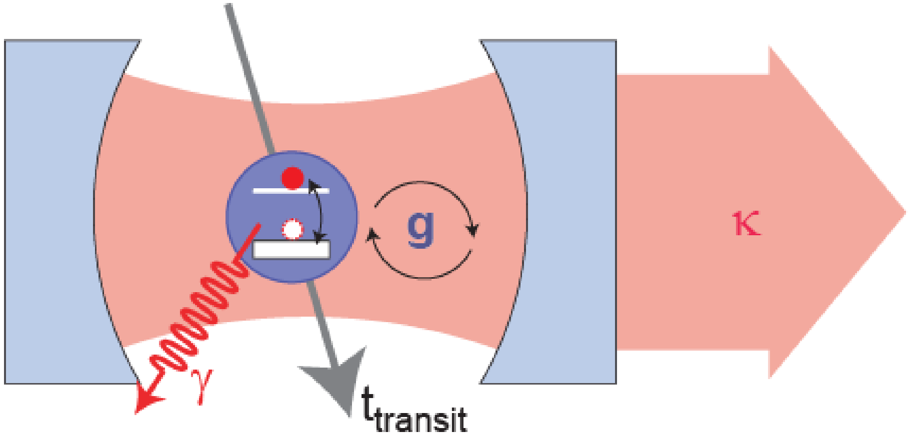




R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble,
Phys. Rev. Lett. **93**, 233603 (2004)

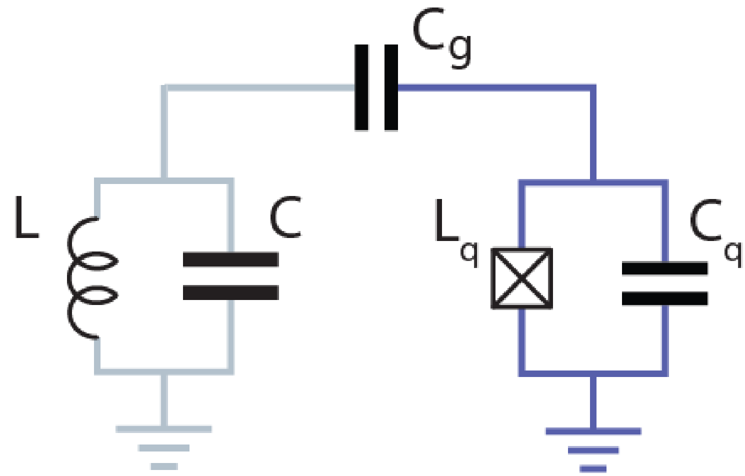


Cavity QED with Superconducting Circuits

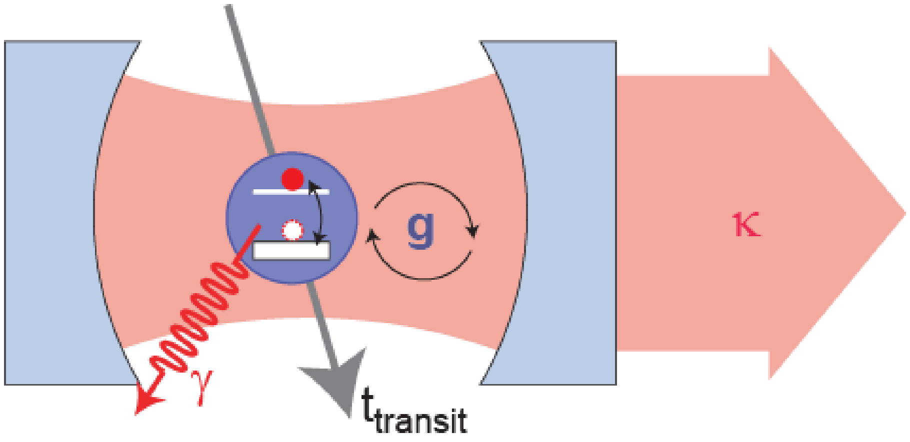


coherent quantum mechanics
with individual photons and qubits ...

... basic approach:



Proposals for Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

a number of approaches suggested at the time:

discrete LC circuits:

- Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001).
- O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).

large Josephson junctions:

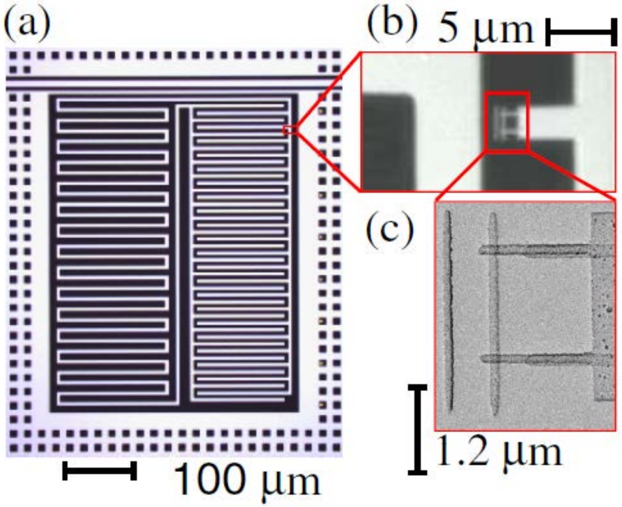
- F. Marquardt and C. Bruder, Phys. Rev. B 63, 054514 (2001).
- F. Plastina and G. Falci, Phys. Rev. B 67, 224514 (2003).
- A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. 90, 127901 (2003).

3D cavities:

- W. Al-Saidi and D. Stroud, Phys. Rev. B 65, 014512 (2001).
- C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A 67, 042311 (2003).
- J. Q. You and F. Nori, Phys. Rev. B 68, 064509 (2003).

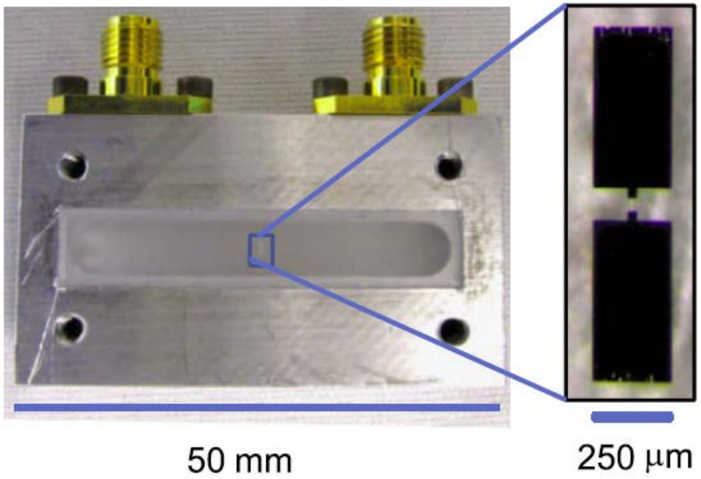
Circuit QED and its Different Realizations

lumped element resonator:



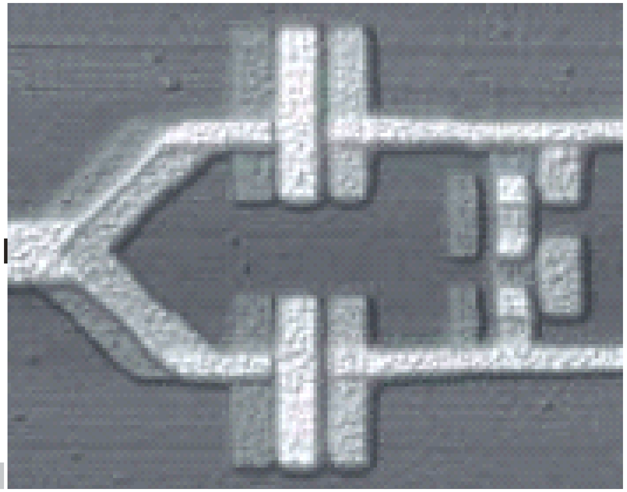
Z. Kim *et al.*, *PRL* 106, 120501 (2011)

3D cavity:



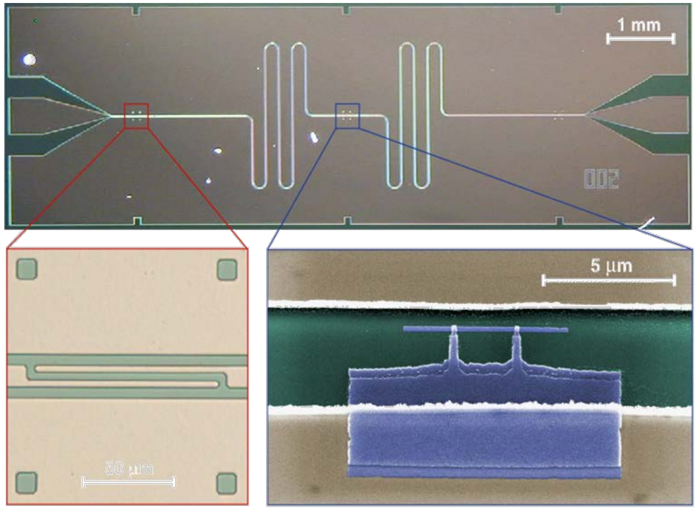
H. Paik *et al.*, *PRL* 107, 240501 (2011)

weakly nonlinear junction:



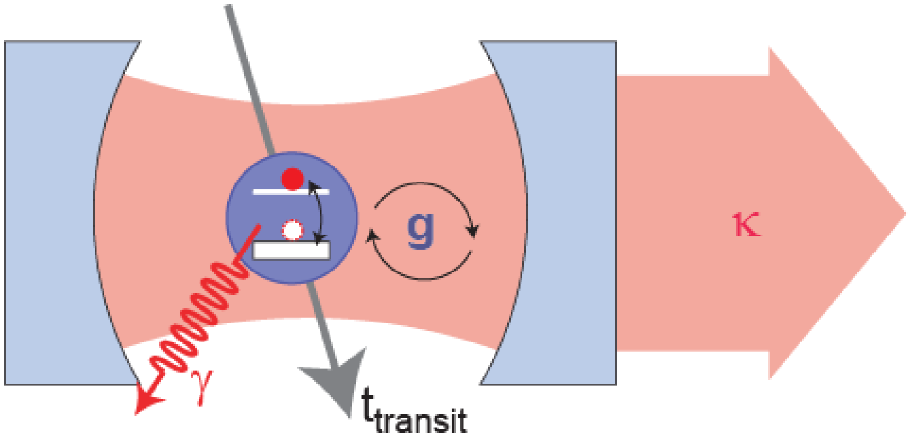
I. Chiorescu *et al.*, *Nature* 431, 159 (2004)

planar transmission line resonator:



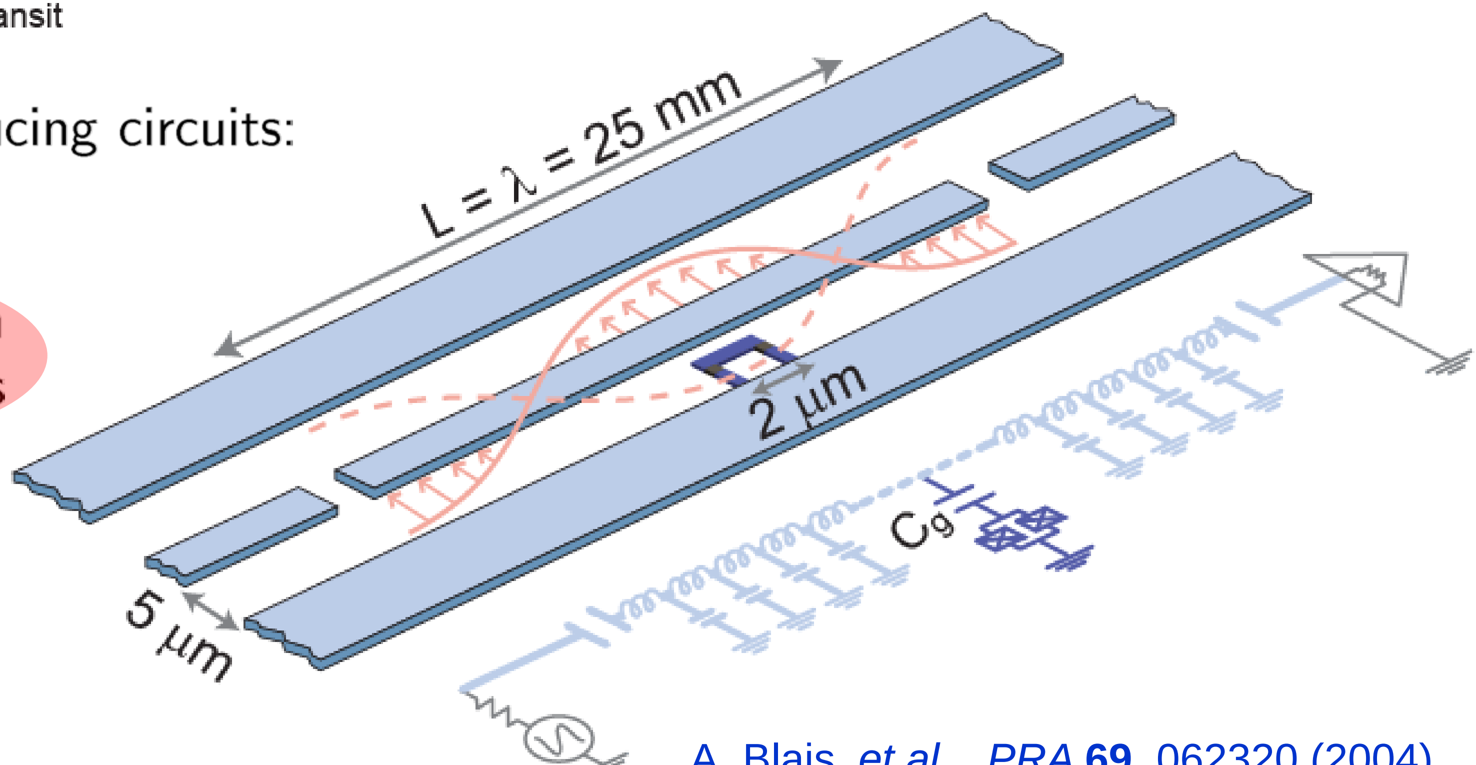
A. Wallraff *et al.*, *Nature* 431, 162 (2004)

Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

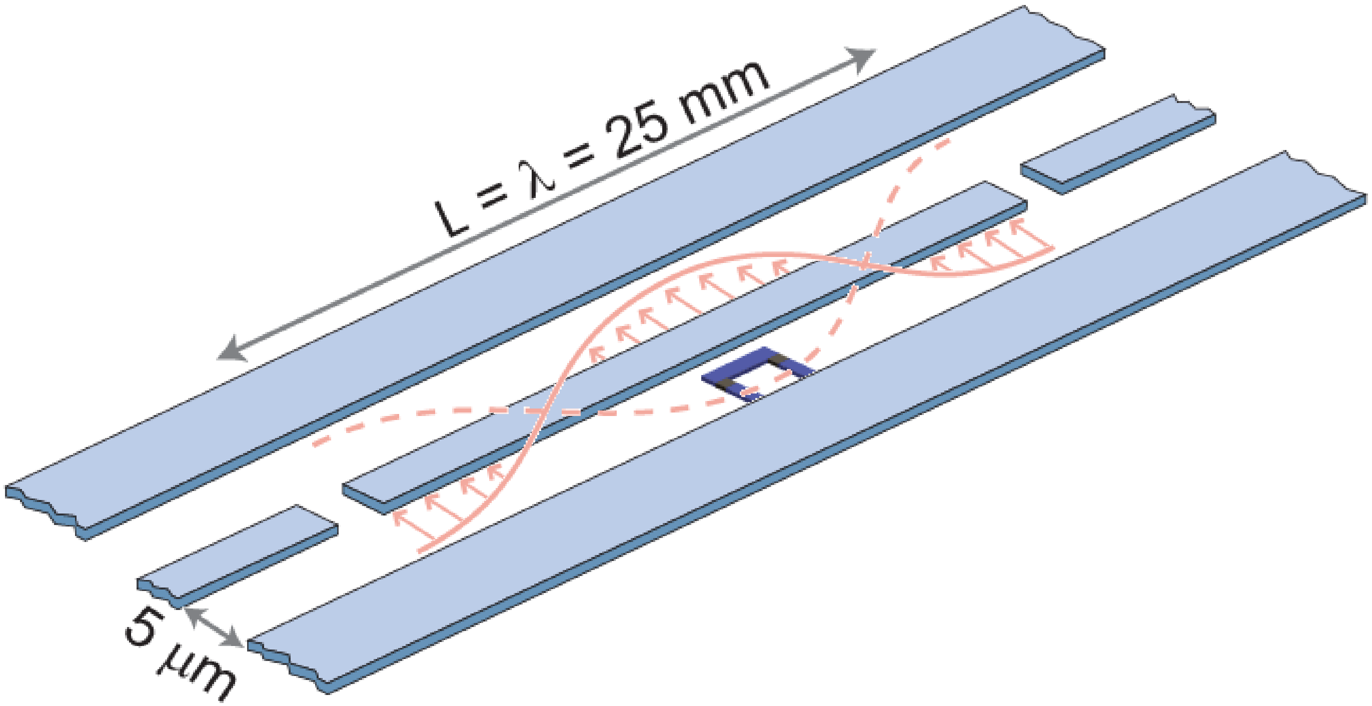
... in superconducing circuits:



circuit quantum electrodynamics

A. Blais, *et al.* , *PRA* **69,** 062320 (2004)
A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Circuit Quantum Electrodynamics



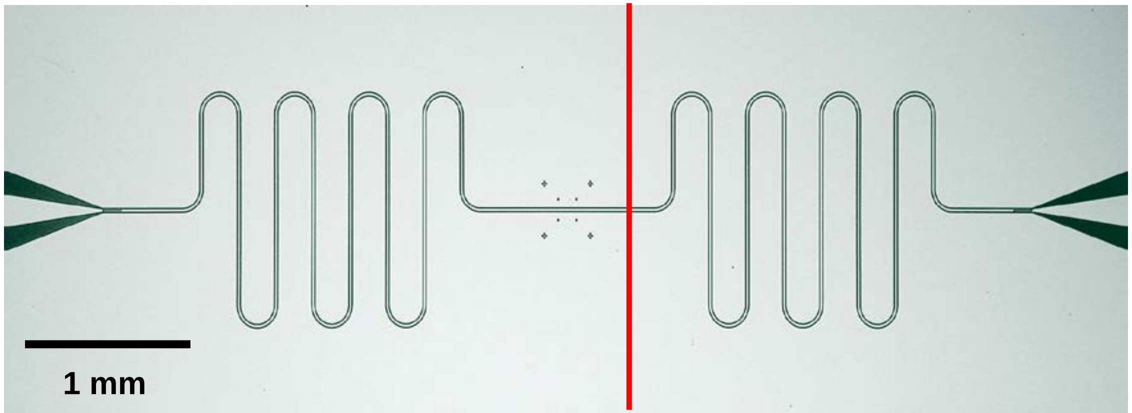
elements

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_o and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment *d* and long coherence time $1/\gamma$

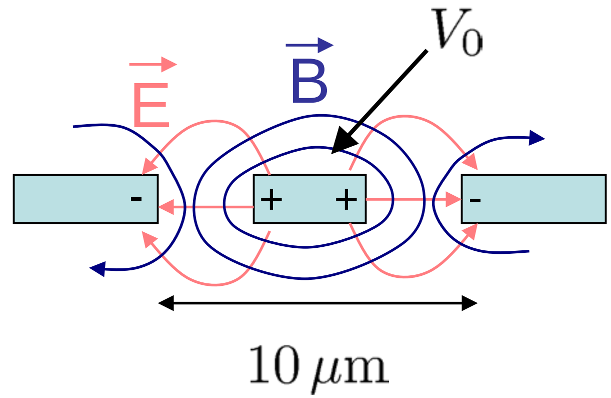
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state $(n = 0)$

$$V_{0,\mathrm{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1\,\mu\mathrm{V}$$

harmonic oscillator

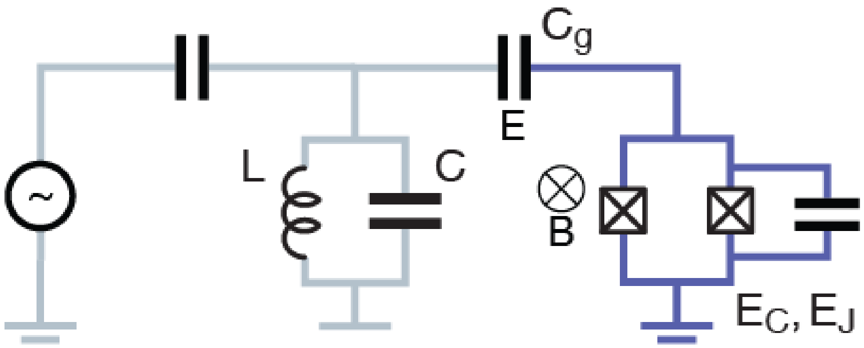
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right)$$

$$E_0 = \frac{V_{0,\mathrm{rms}}}{b} \approx 0.2\,\mathrm{V/m}$$

$\times 10^6$ larger than E_0 in 3D microwave cavity

for $\omega_r/2\pi \approx 6\,\mathrm{GHz}$ $(C \sim 1\,\mathrm{pF})$, $b \approx 5\,\mu\mathrm{m}$

Qubit/Photon Coupling



Hamilton operator of qubit (2-level approx.) coupled to resonator:

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L} + \frac{E_C}{2}(1 - 2(N_g + \hat{N}_g))\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

quantum part of gate voltage due to resonator

$$\hat{N}_g = \frac{C_g}{2e}\hat{V}_g = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})$$

Jaynes-Cummings Hamiltonian

Consider bias at charge degeneracy N_g = 1/2 and change of qubit basis (*z* to *x*, *x* to *-z*)

$$\hat{H} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2) + \frac{E_J}{2}\hat{\sigma}_z + \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x$$

Use qubit raising and lowering operators $\hat{\sigma}_x = \hat{\sigma}^+ + \hat{\sigma}^-$

Coupling term in the rotating wave approximation (RWA)

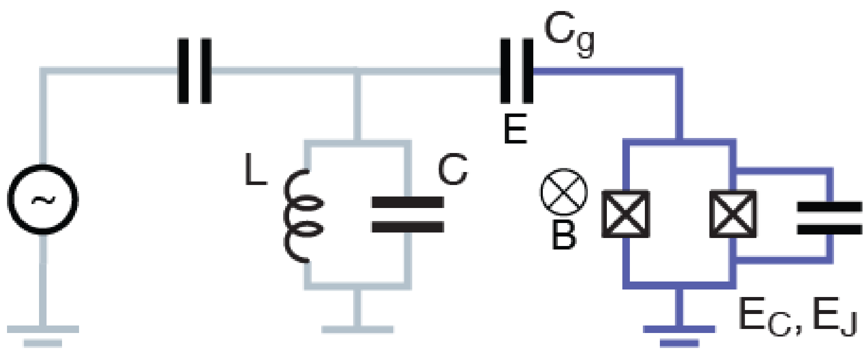
$$\hat{H}_g = \frac{E_C}{2}\frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger\hat{\sigma}^- + \cancel{\hat{a}\hat{\sigma}^-} + \cancel{\hat{a}^\dagger\hat{\sigma}^+} + \hat{a}\hat{\sigma}^+) \approx \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+)$$

Coupling strength of the Jaynes Cummings Hamiltonian $\hbar g = \frac{C_g}{C_\Sigma}2e\sqrt{\frac{\hbar\omega_r}{2C}}$

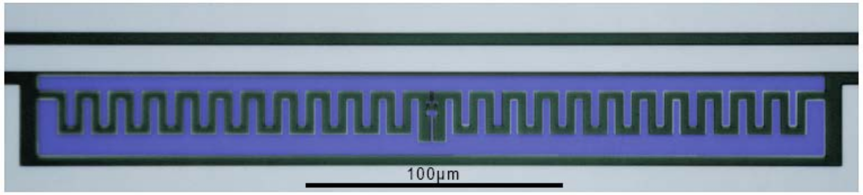
Vacuum-Rabi frequency $\nu_R = \frac{2g}{2\pi} \approx 1\ldots 300\,\mathrm{MHz}$

$g \gg [\kappa, \gamma]$ possible!

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g \quad = \quad e V_{0,\mathrm{rms}} \frac{C_g}{C_\Sigma}$$

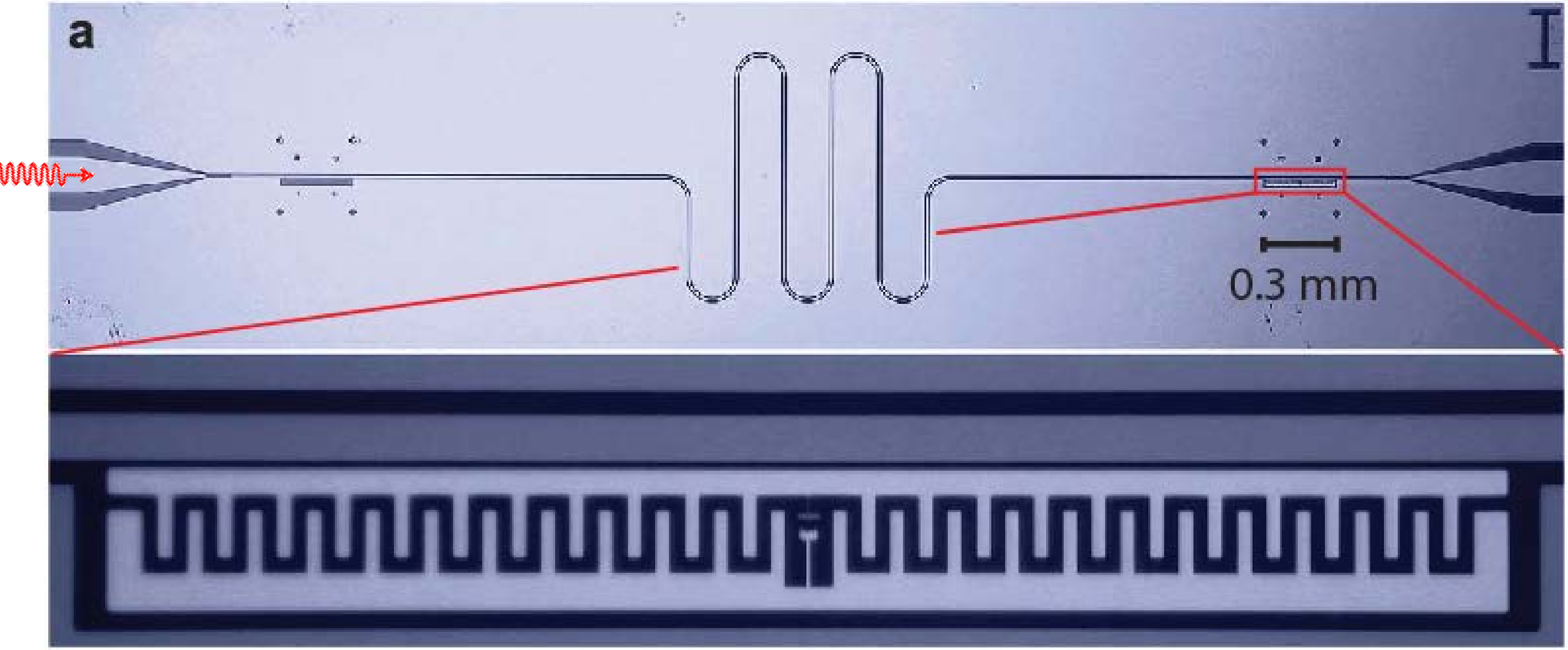
$$\Longrightarrow \nu_{\mathrm{vac}} \quad = \quad \frac{g}{\pi} \approx 1 \ldots 300\,\mathrm{MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

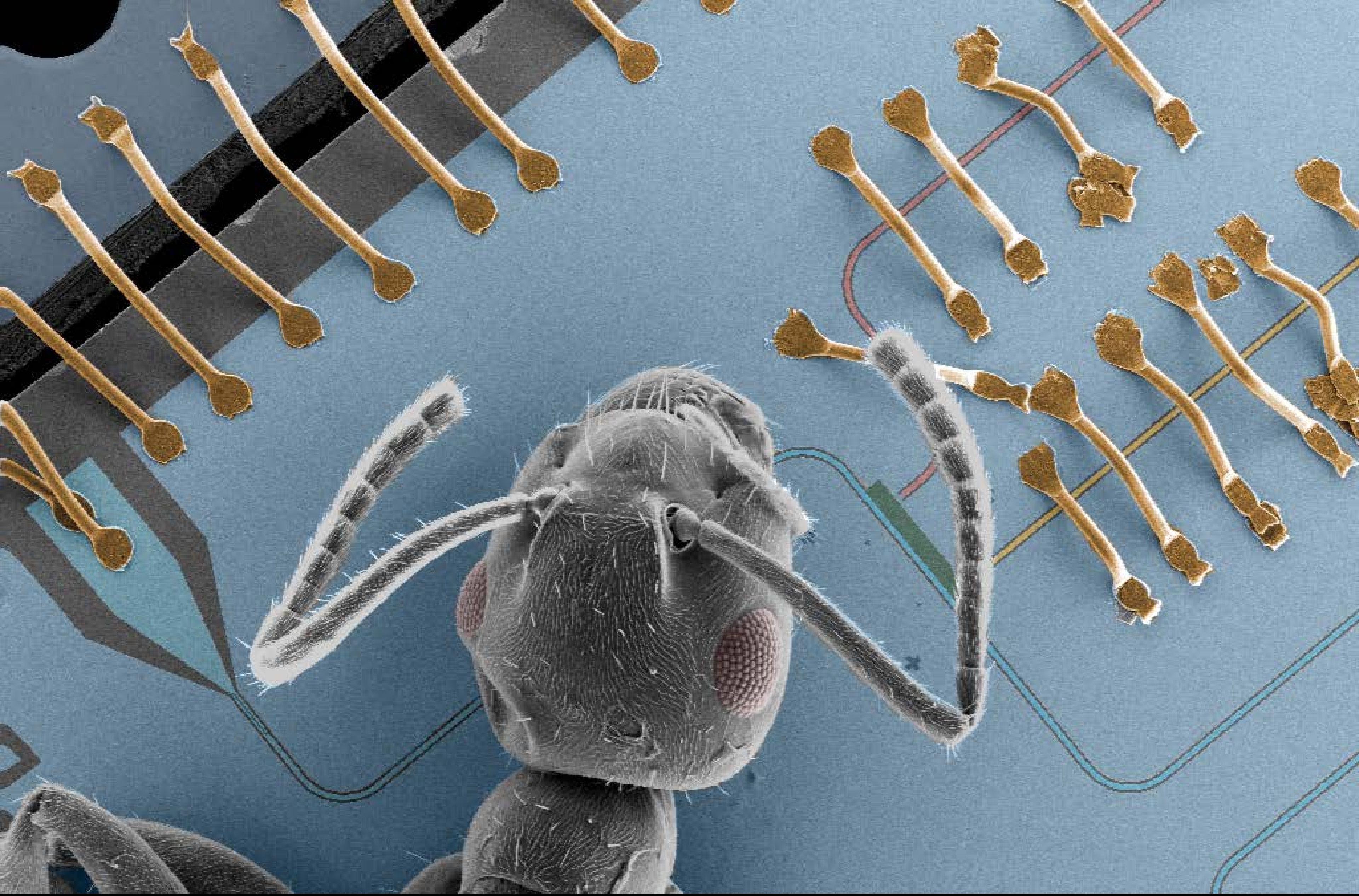
$$d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4\, e a_0$$

Circuit QED with One Photon



superconducting cavity QED circuit

A. Wallraff, ..., R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

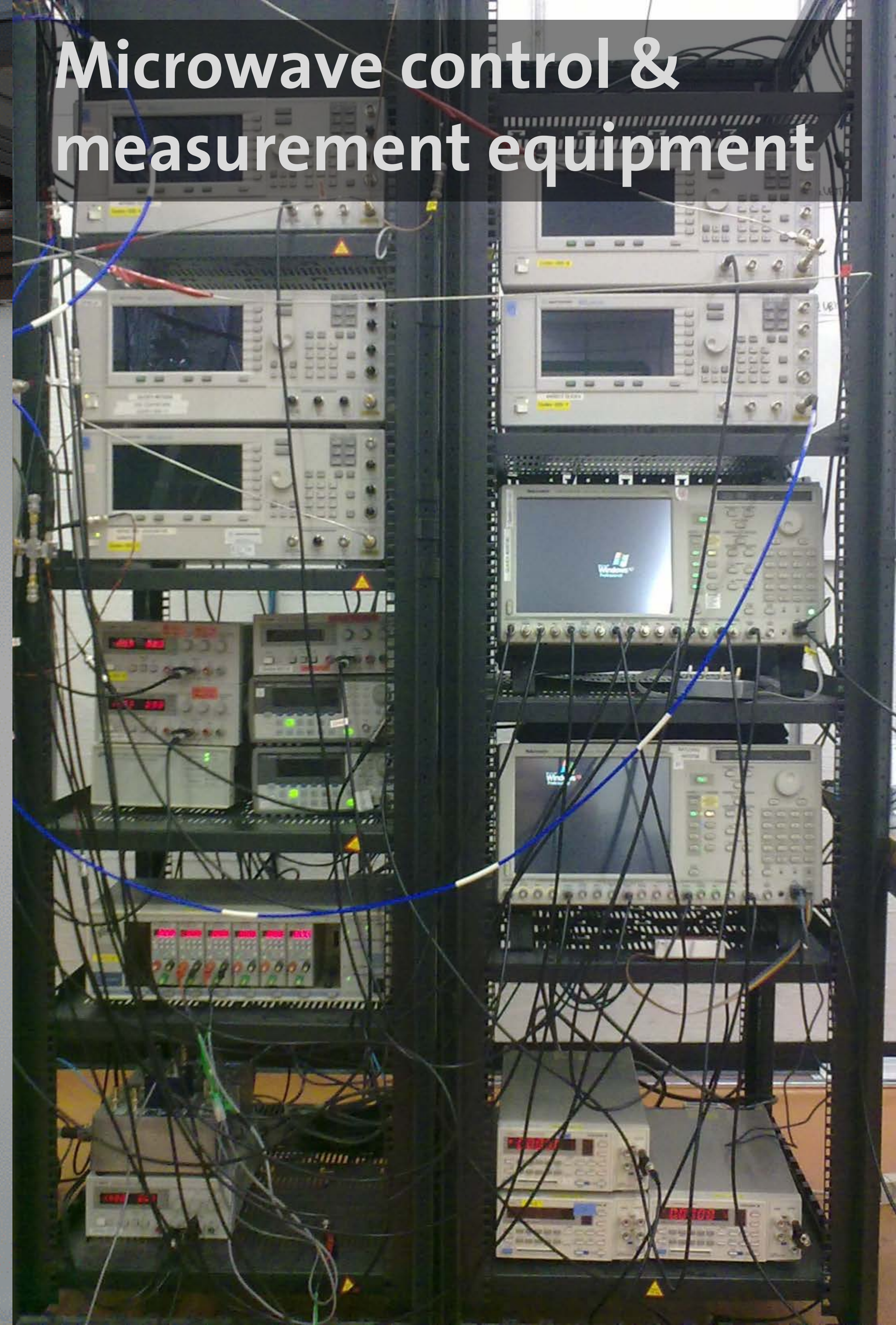


J. Mlynek *et al.,* Quantum Device Lab, ETH Zurich (2012)

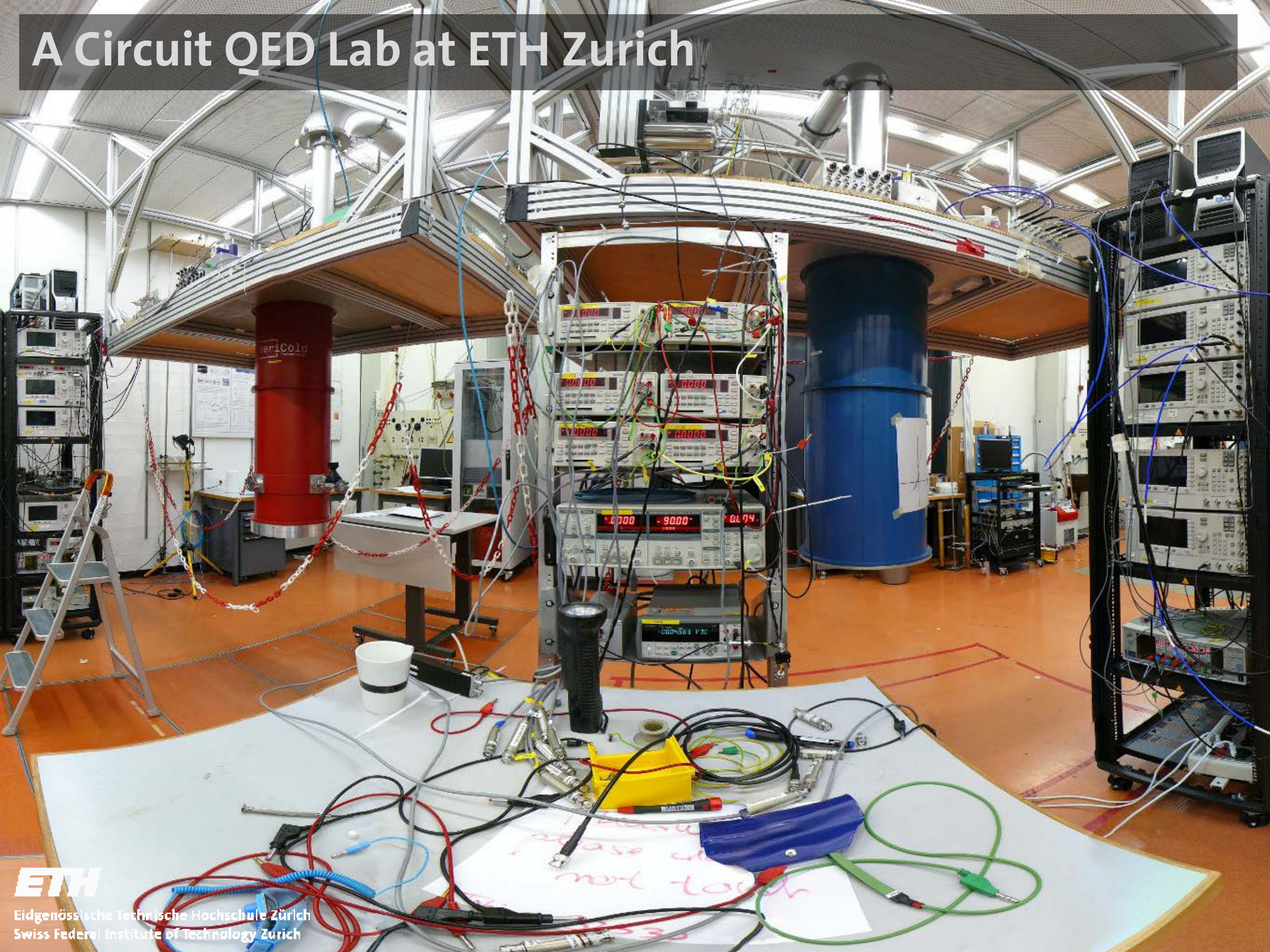
Sample Mount

M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)


Cryostate for temperatures down to 0.02 K
VeriCold
Technologies
~ 20 cm
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich


Microwave control & measurement equipment

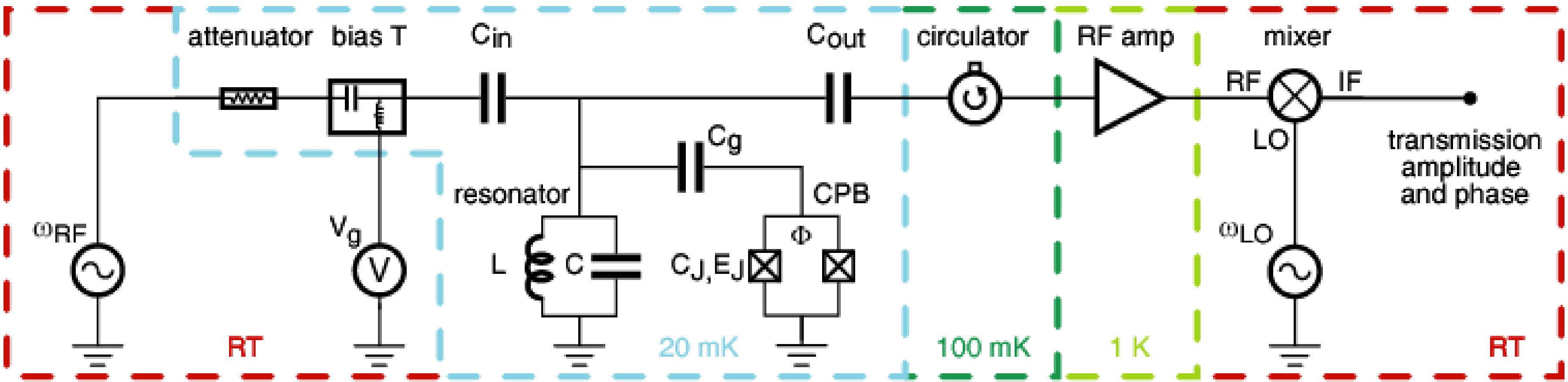
A Circuit QED Lab at ETH Zurich



How to Measure Single Microwave Photons

- average power to be detected

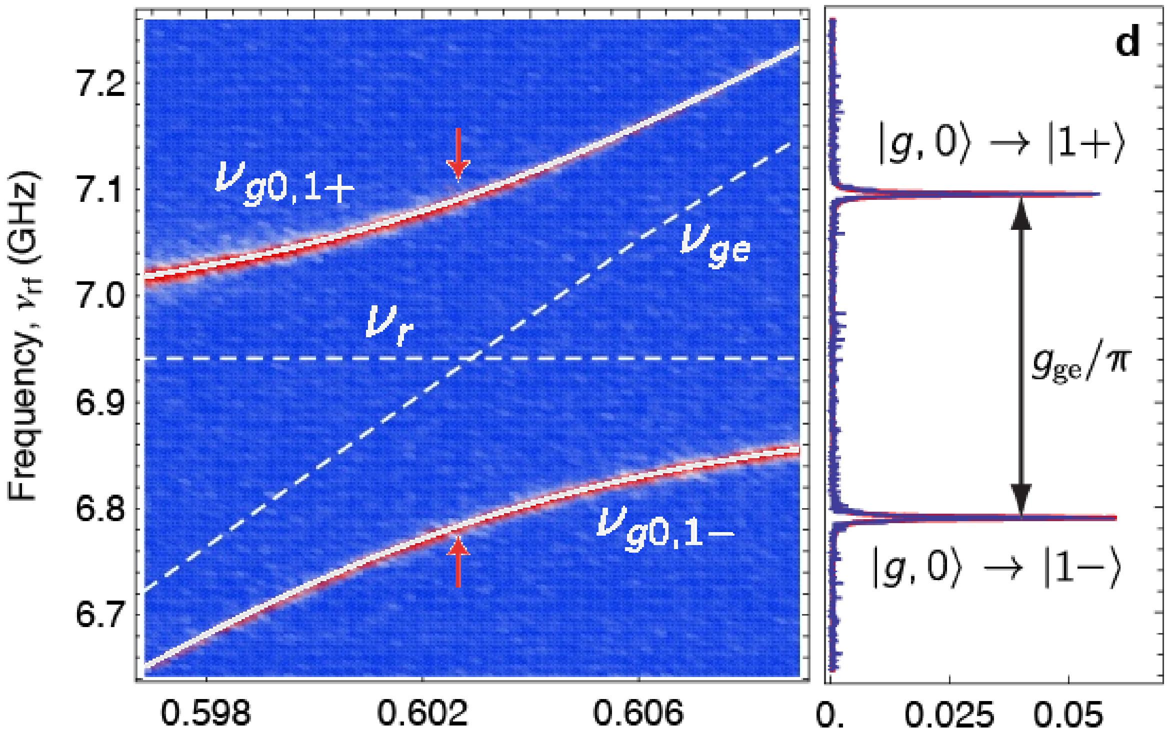
$$\rightarrow \langle n=1\rangle \hbar\omega_r\kappa/2 \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6\,\mathrm{K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)

Resonant Vacuum Rabi Mode Splitting ...

... with one photon (*n=1*):

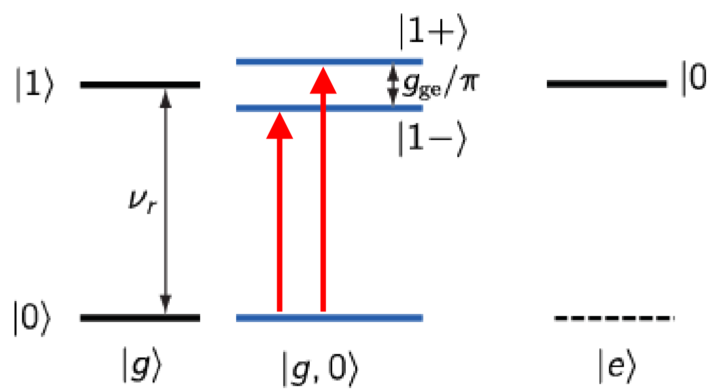


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff et *al., Nature (London)* 431, 162 (2004)

this data: J. Fink et al., *Nature (London)* 454, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* 451, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

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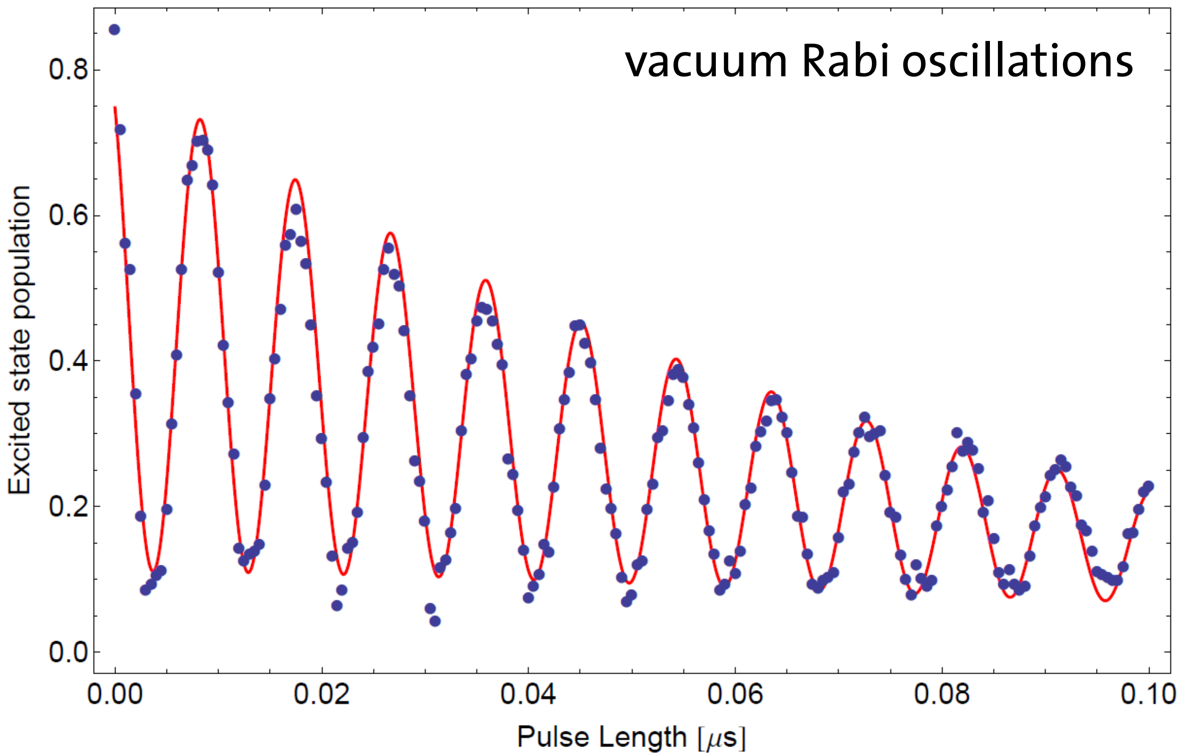

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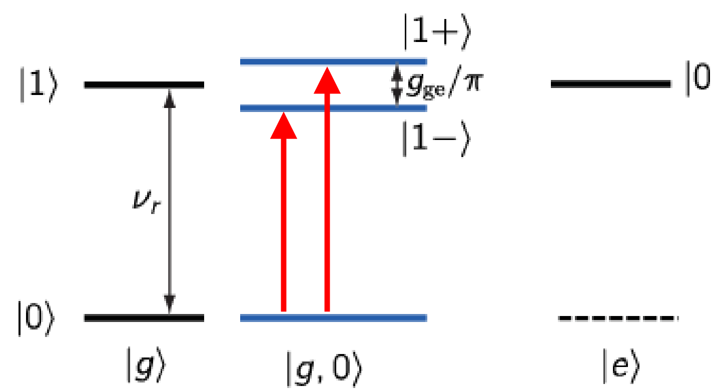

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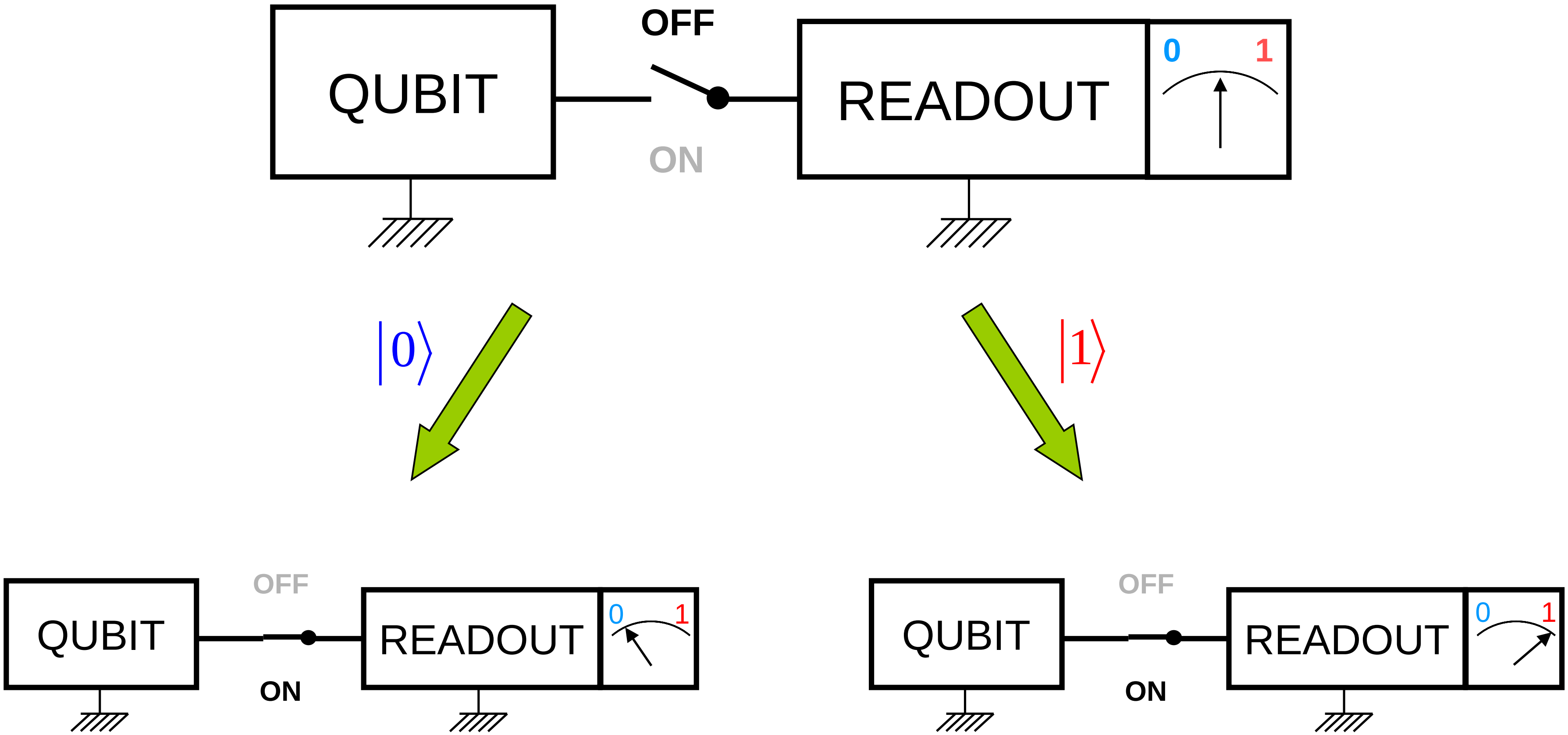
Read-Out …

… of superconducting qubits

Qubit Read Out



desired: good on/off ratio
no relaxation in on state (QND)

Dispersive Approximation of the J-C Hamiltonian

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Unitary transformation

$$\tilde{H} = UHU^\dagger \quad \text{with} \quad U = \exp\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)$$

$$\text{and} \quad \Delta = \omega_a - \omega_r$$

Results in dispersive approximation up to 2nd order in g

$$\tilde{H} \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

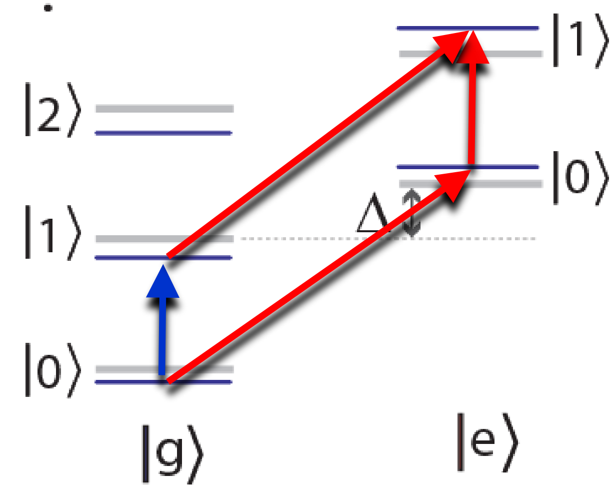
A. Blais, *et al.*, *PRA* 69, 062320 (2004)

Non-Resonant (Dispersive) Interaction

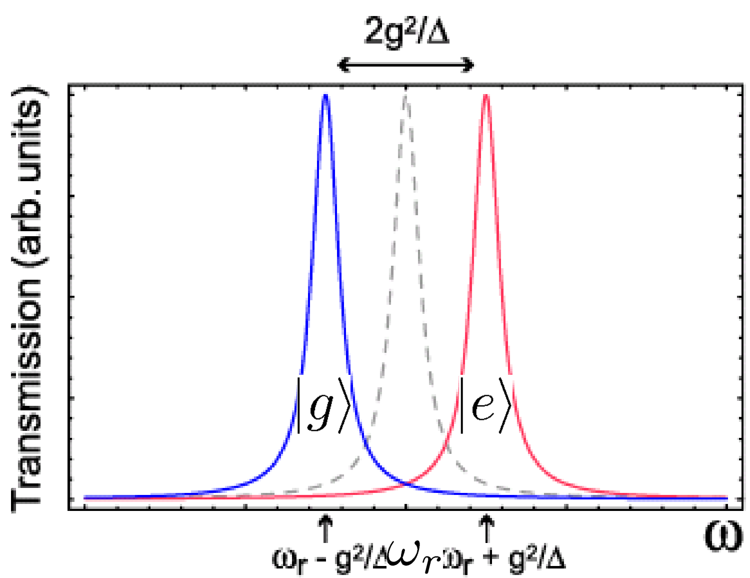
approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$:

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

cavity frequency shift



qubit detuned by Δ
from resontaor



A. Blais *et al., PRA* 69, 062320 (2004)
A. Wallraff *et al., Nature (London)* 431, 162 (2004)
D. I. Schuster *et al., Phys. Rev. Lett.* 94, 123062 (2005)
A. Fragner *et al., Science* 322, 1357 (2008)

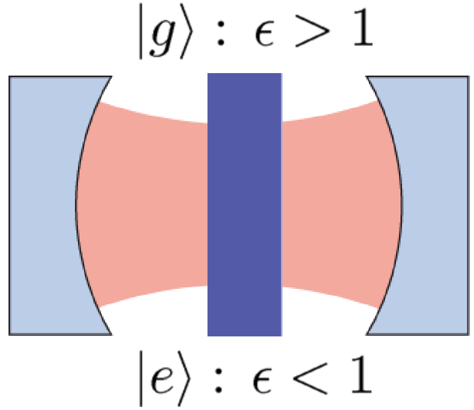
Dispersive Read-Out

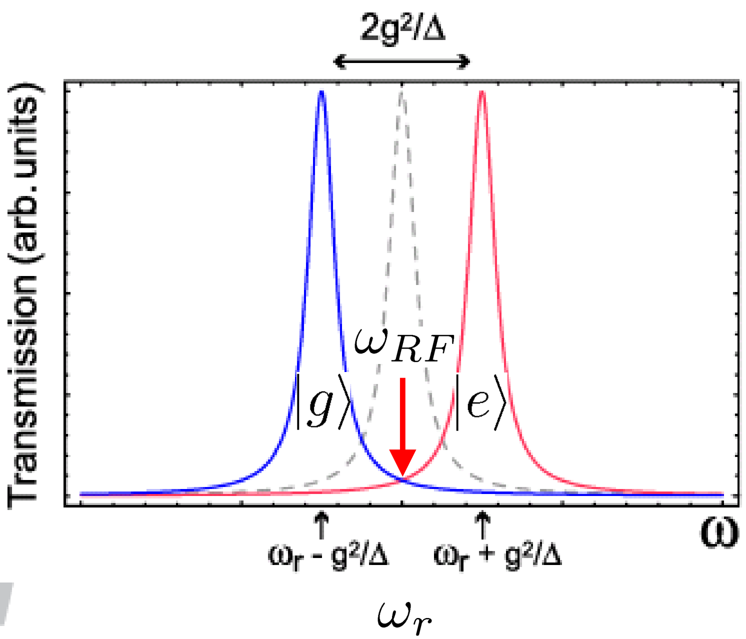
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

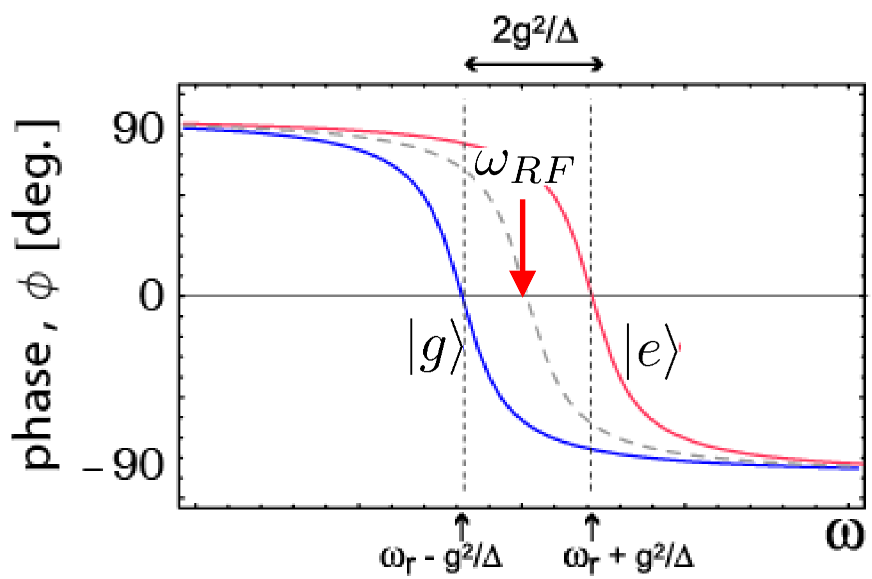
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//

cavity frequency shift



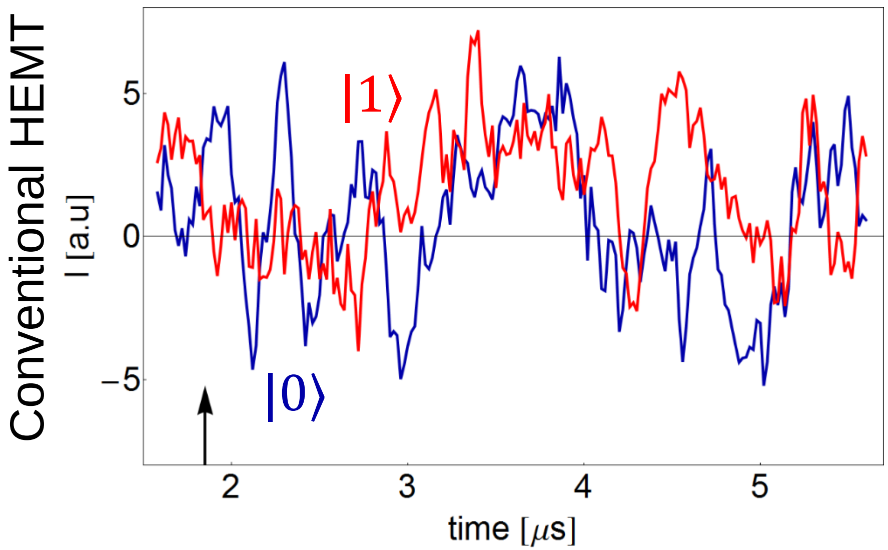




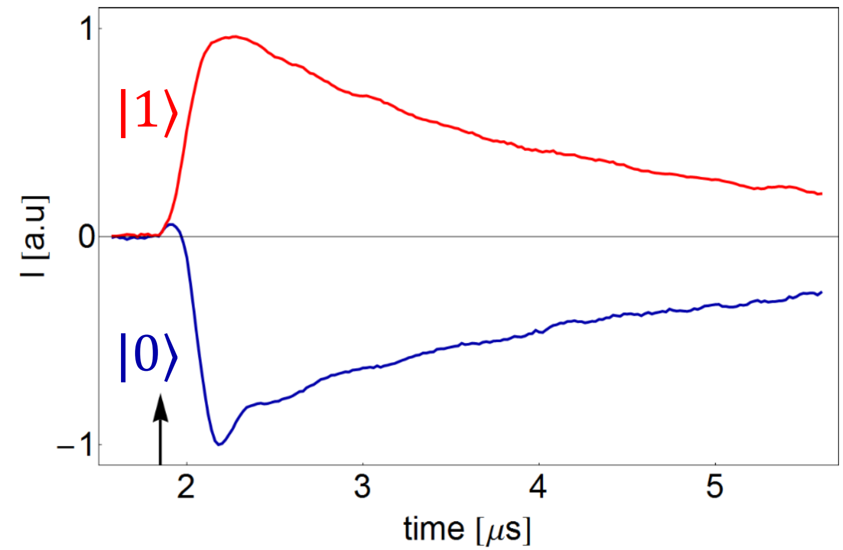
A. Blais *et al.*, *PRA* **69,** 062320 (2004)

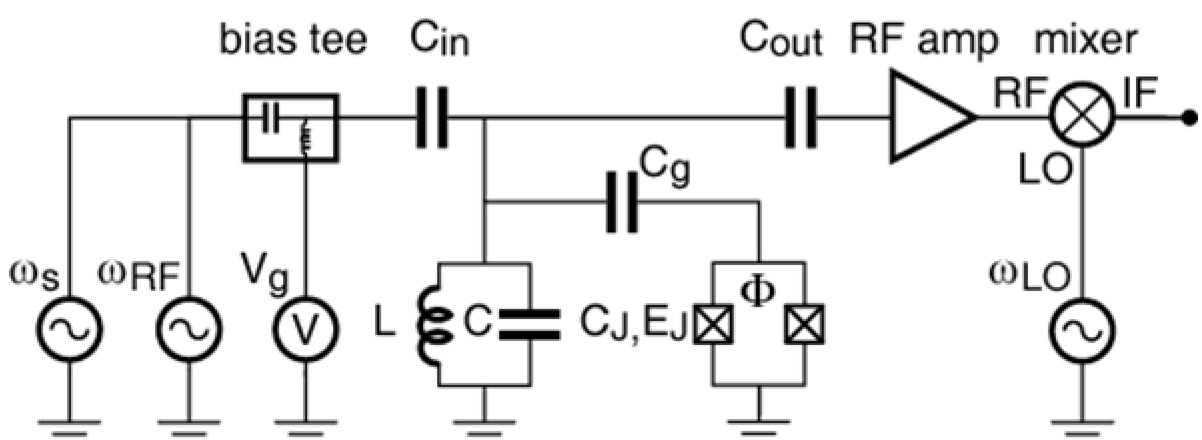
Qubit-Readout (Averaged)

single-shot measurements:



averaged measurements (8 10⁴):

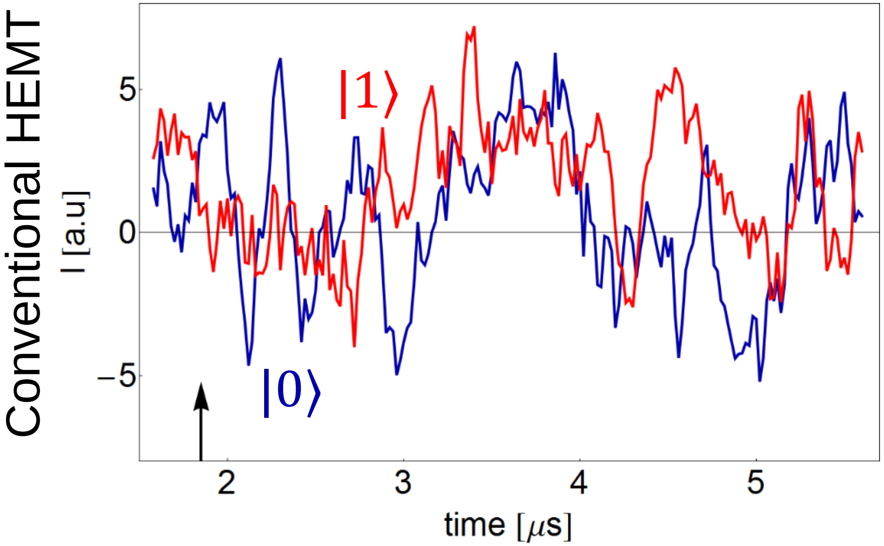




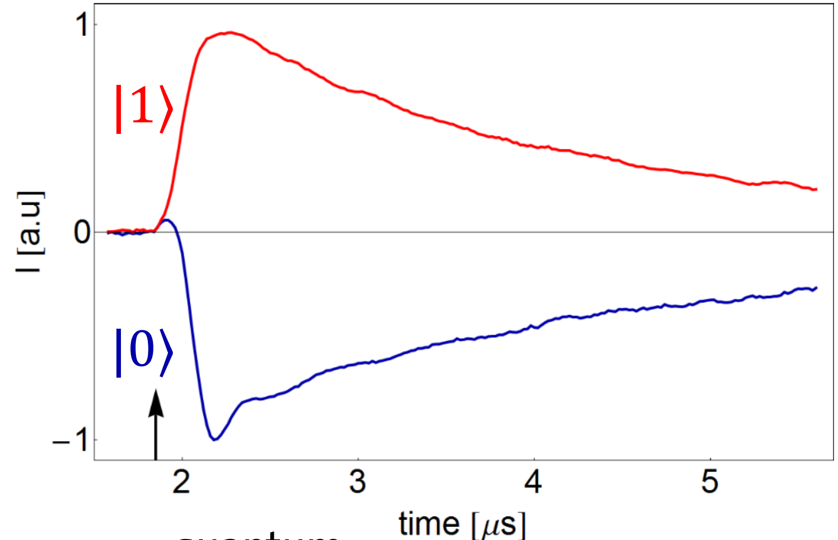
P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al., PRL* 106, 110502 (2011)

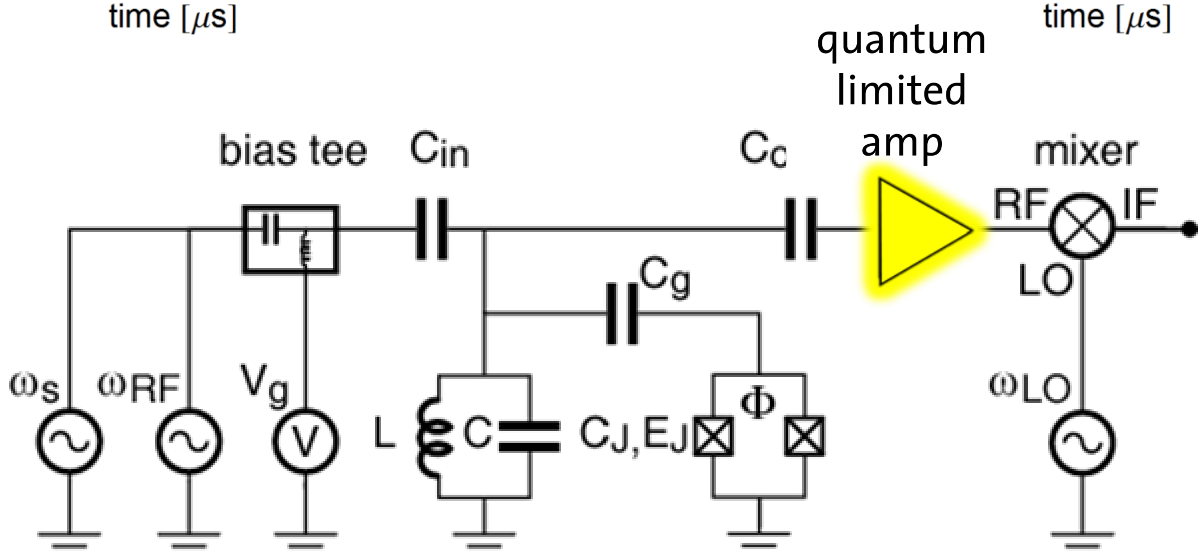
Improved using a Quantum Limited Amplifier

single-shot measurements:



averaged measurements (8 10^4):

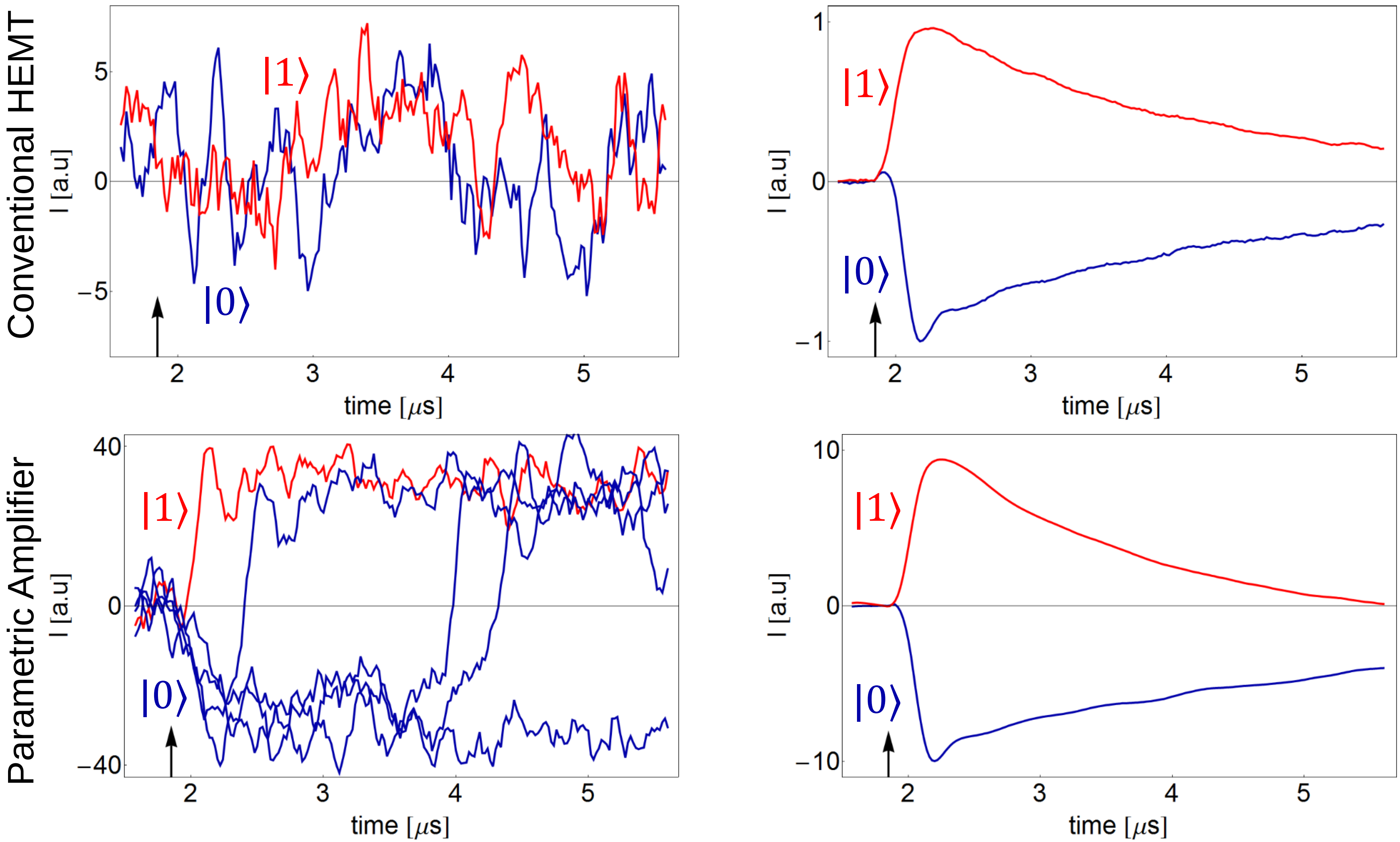




P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

Single-Shot Single-Qubit Readout

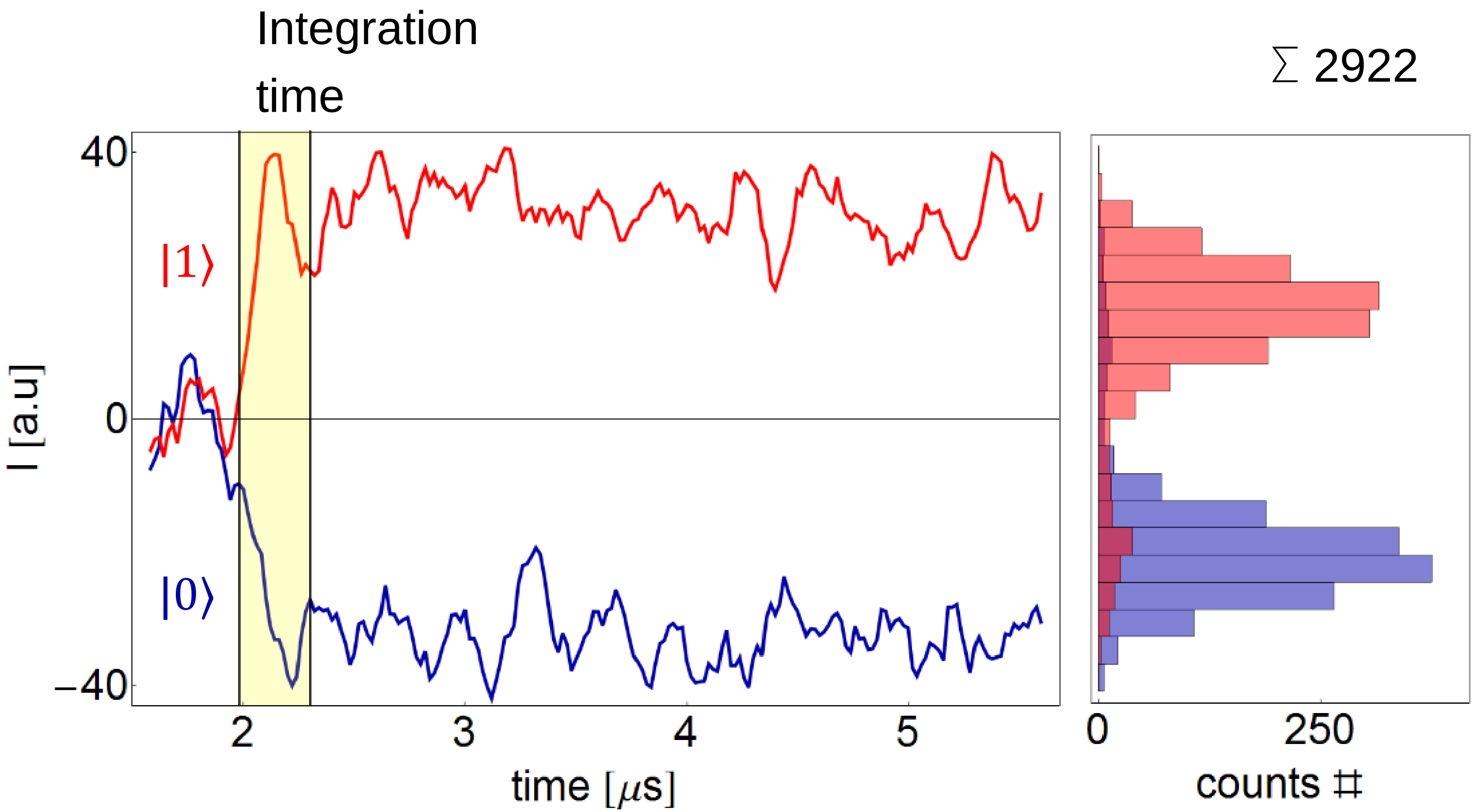
single-shot measurements: averaged measurements (8 10⁴):



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

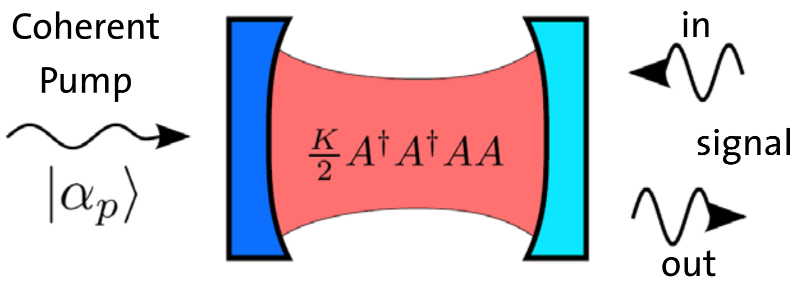
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Statistics of Integrated Single-Shot Readout

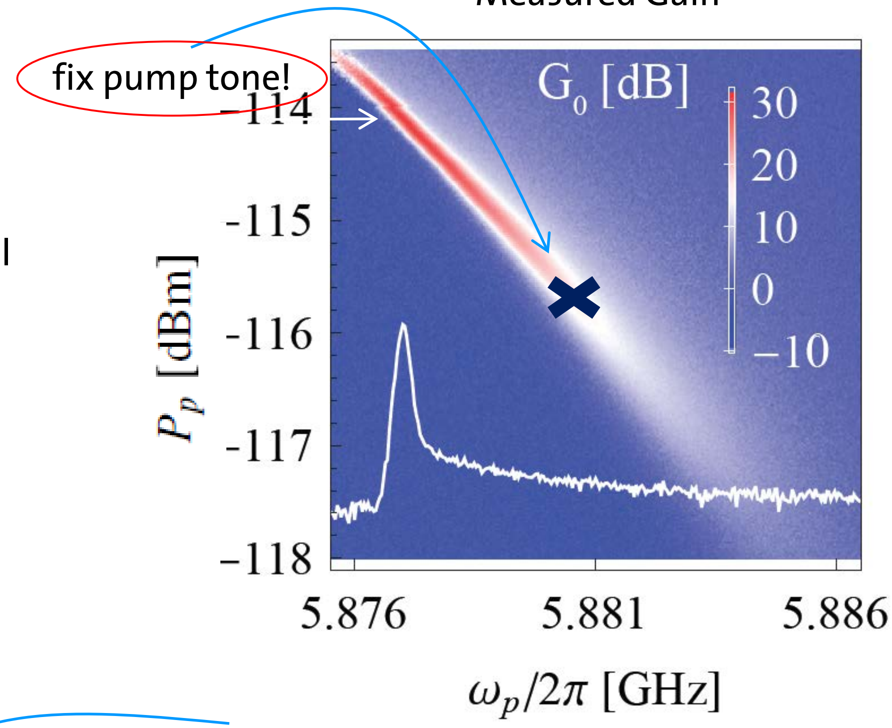


P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)

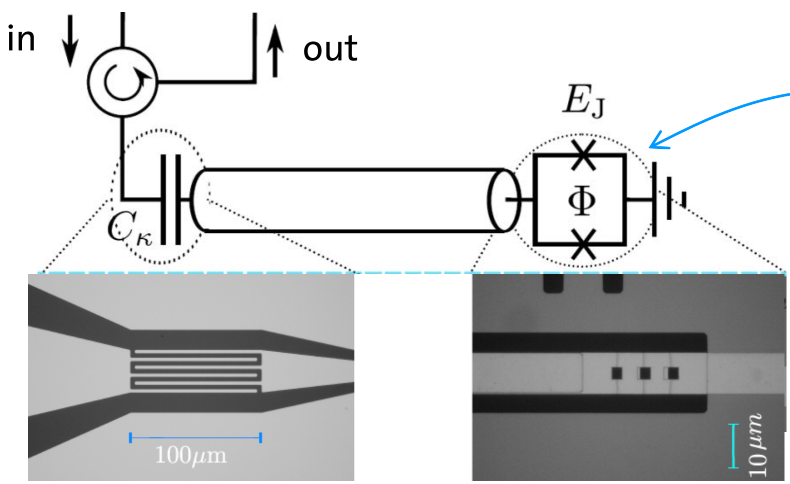
Near Quantum-Limited Parametric Amplifier



Measured Gain



Circuit QED implementation:



SQUID(-array) provides
required nonlinearity

Eichler *et al.,* EPJ Quantum Technology 1, 2 (2014)
Eichler *et al., Phys. Rev. Lett.* 107, 113601 (2011)

Caves, *Phys. Rev. D* 26, 1817 (1982)
Yurke and Buks, *J. Lightwave Tech.* 24, 5054 (2006)
Castellanos-Beltran *et al., Nat. Phys.* 4, 929 (2008)

The Lamb and AC-Stark Shifts

signatures of the dispersive interaction with a quantum field

for $\Delta = \omega_a - \omega_r \gg g$

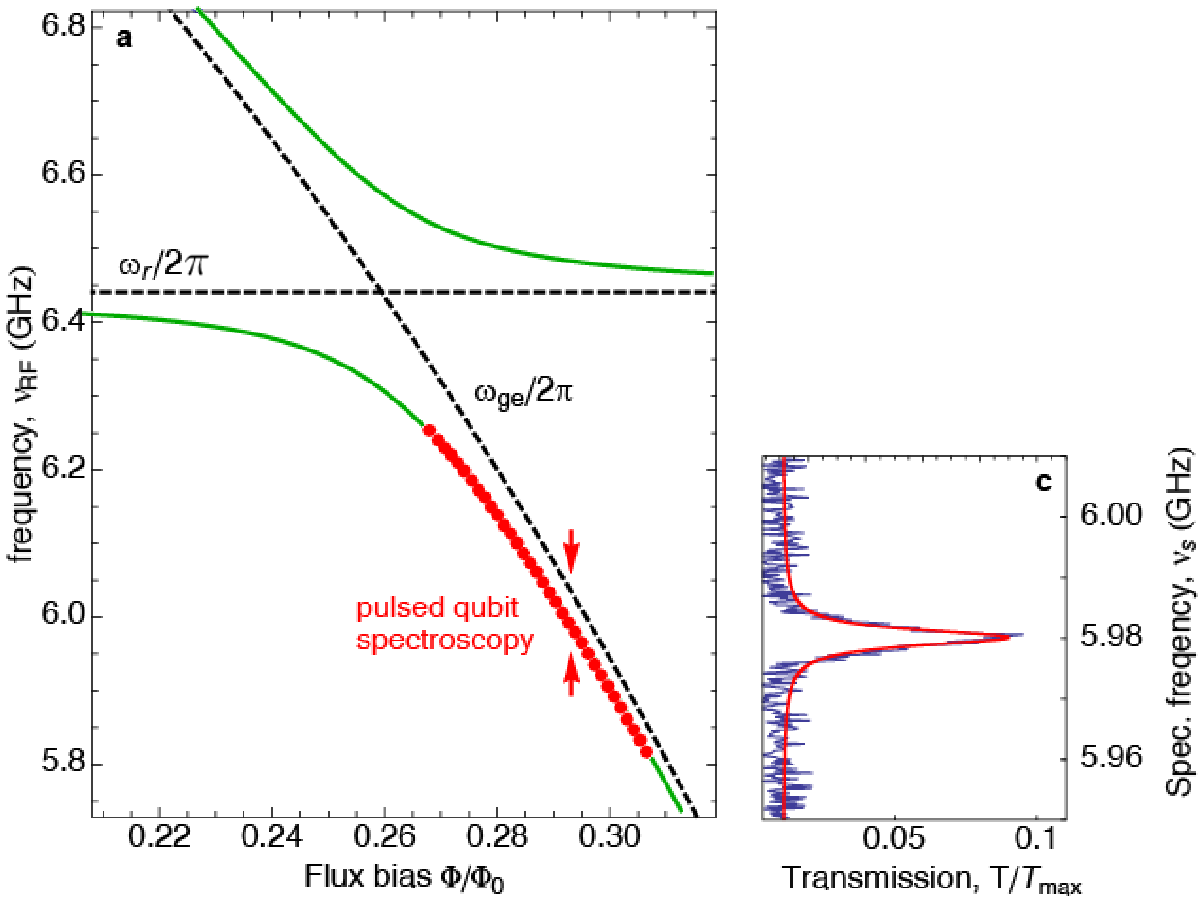
Lamb shift // AC Stark shift //

$$H \approx \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta}a^\dagger a\right)\sigma_z$$

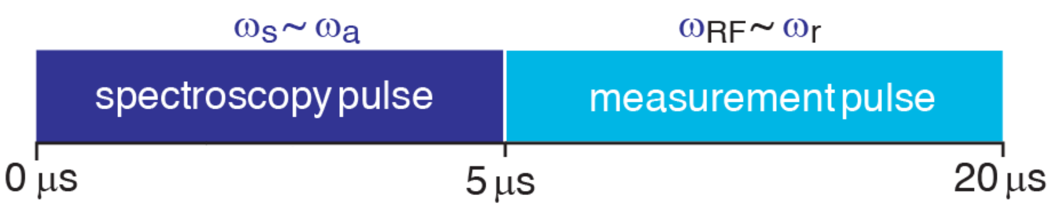
$$\tilde{\omega}_a \approx \omega_a + \frac{g^2}{\Delta} + \frac{2g^2}{\Delta}n$$

M. Brune *et al.*, *Phys. Rev. Lett.* **72**, 3339 (1994)
A. Blais *et al.*, *PRA* **69,** 062320 (2004)

Measurements of the Lamb and Quantized Stark Shifts



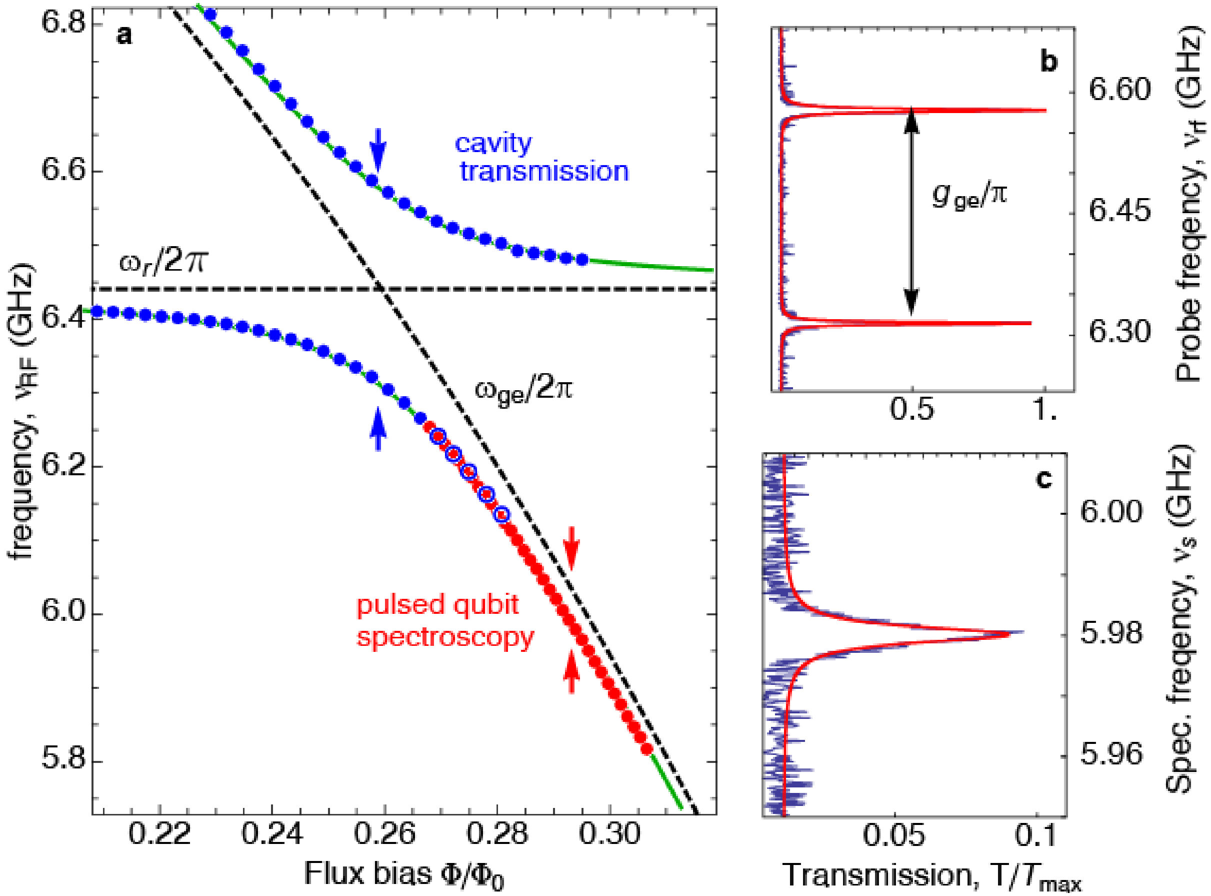
pulsed dispersive qubit spectroscopy



pulsed spectroscopy scheme

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

Measurement of the Lamb Shift



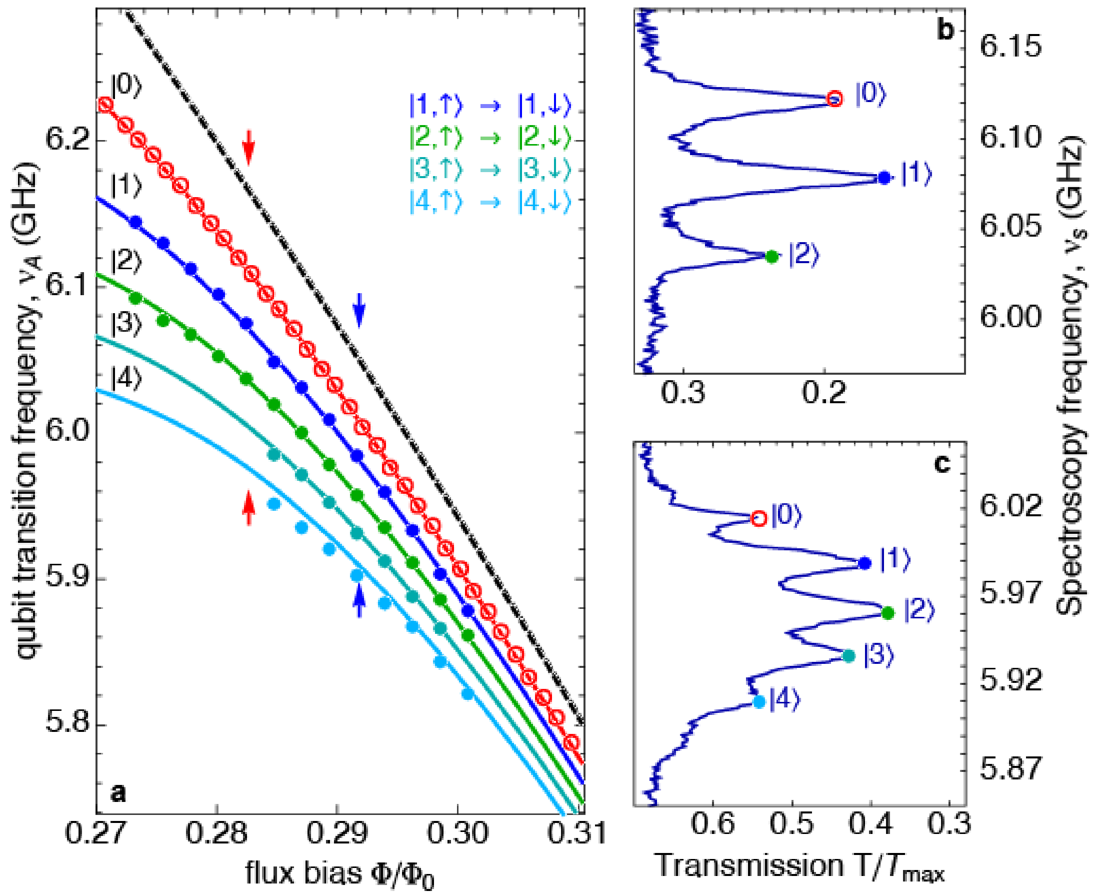
resonator transmission spectrum

pulsed dispersive qubit spectroscopy

- qubit and photon component of joint state are measured
- accurate knowledge of qubit parameters possible

A. Fragner *et al., Science* 322, 1357 (2008)

Quantum AC-Stark Shift and Lamb Shift



- populate resonator with small coherent field (Poisson distribution of photon number)
- spectroscopic measurement of qubit line shape
- qubit frequencies ac-Stark shifted by quantized cavity field

D. Schuster *et al., Nature* 445, 515 (2007)
A. Fragner *et al., Science* 322, 1357 (2008)