Qubit Control

Driving Qubit Transitions in J-C Hamiltonian

Hamiltonian for microwave drive

$$H_d = \hbar\epsilon(t)\left(a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t}\right)$$

Unitary transform

$$\tilde{H} = U(H_{\rm JC} + H_d)U^\dagger \quad \text{with} \quad U = \exp\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)$$

$$\text{and} \quad \Delta = \omega_a - \omega_r$$

Results in dispersive approximation up to 2nd order in g

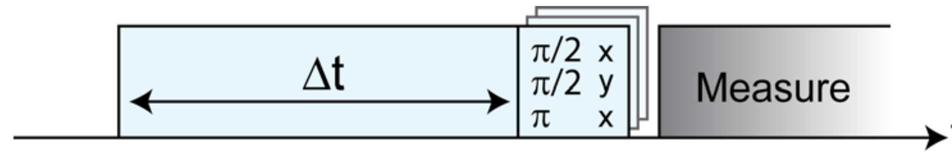
$$\tilde{H} \approx \frac{\hbar}{2}\left(\omega_q + \frac{2g^2}{\Delta}(a^\dagger a + \frac{1}{2}) - \omega_d\right)\sigma_z + \hbar\frac{g\epsilon(t)}{\Delta}\sigma_x$$
$$+\hbar(\omega_r - \omega_d)a^\dagger a + \hbar\epsilon(t)(a^\dagger + a)$$

Drive induces Rabi oscillations in qubit when in resonance with dispersively shifted qubit frequency

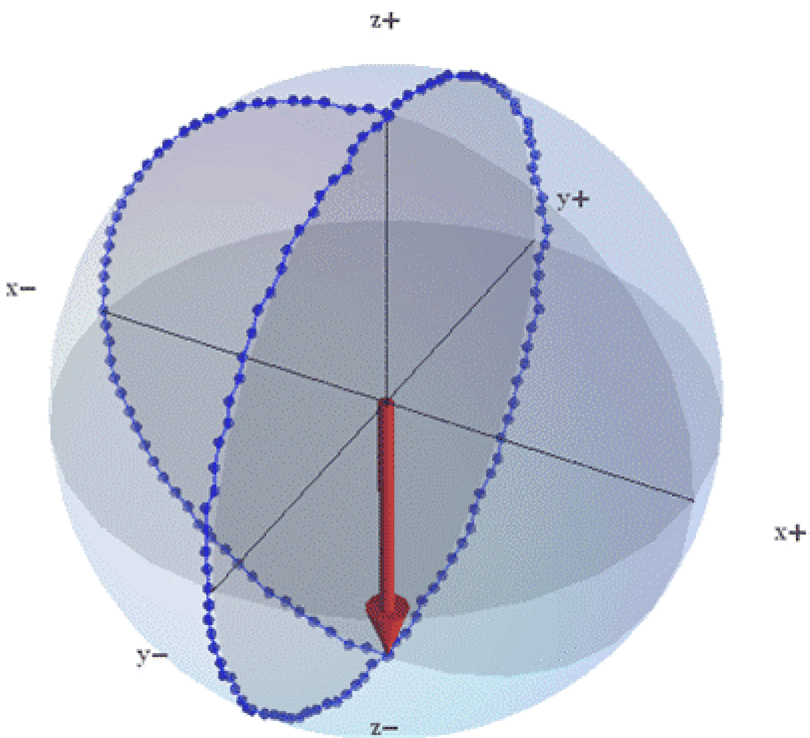
A. Blais, *et al.*, *PRA* 69, 062320 (2004)

Single Qubit Gates

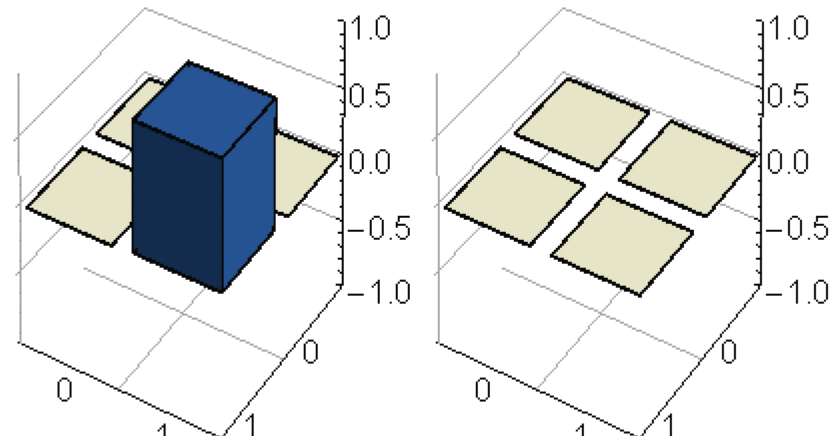
Pulse sequence for qubit rotation and readout:

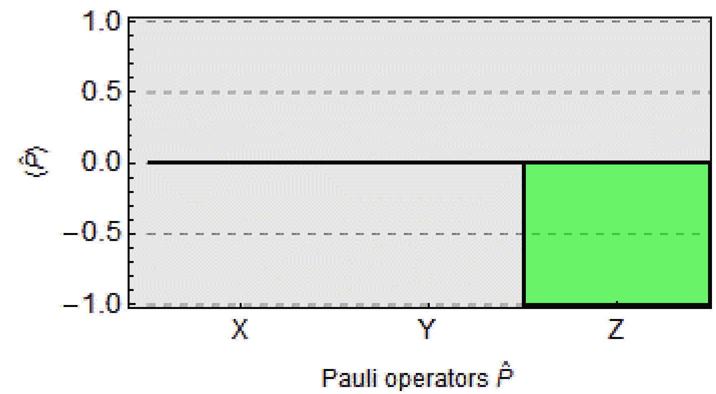


experimental Bloch vector:



experimental density matrix and Pauli set:





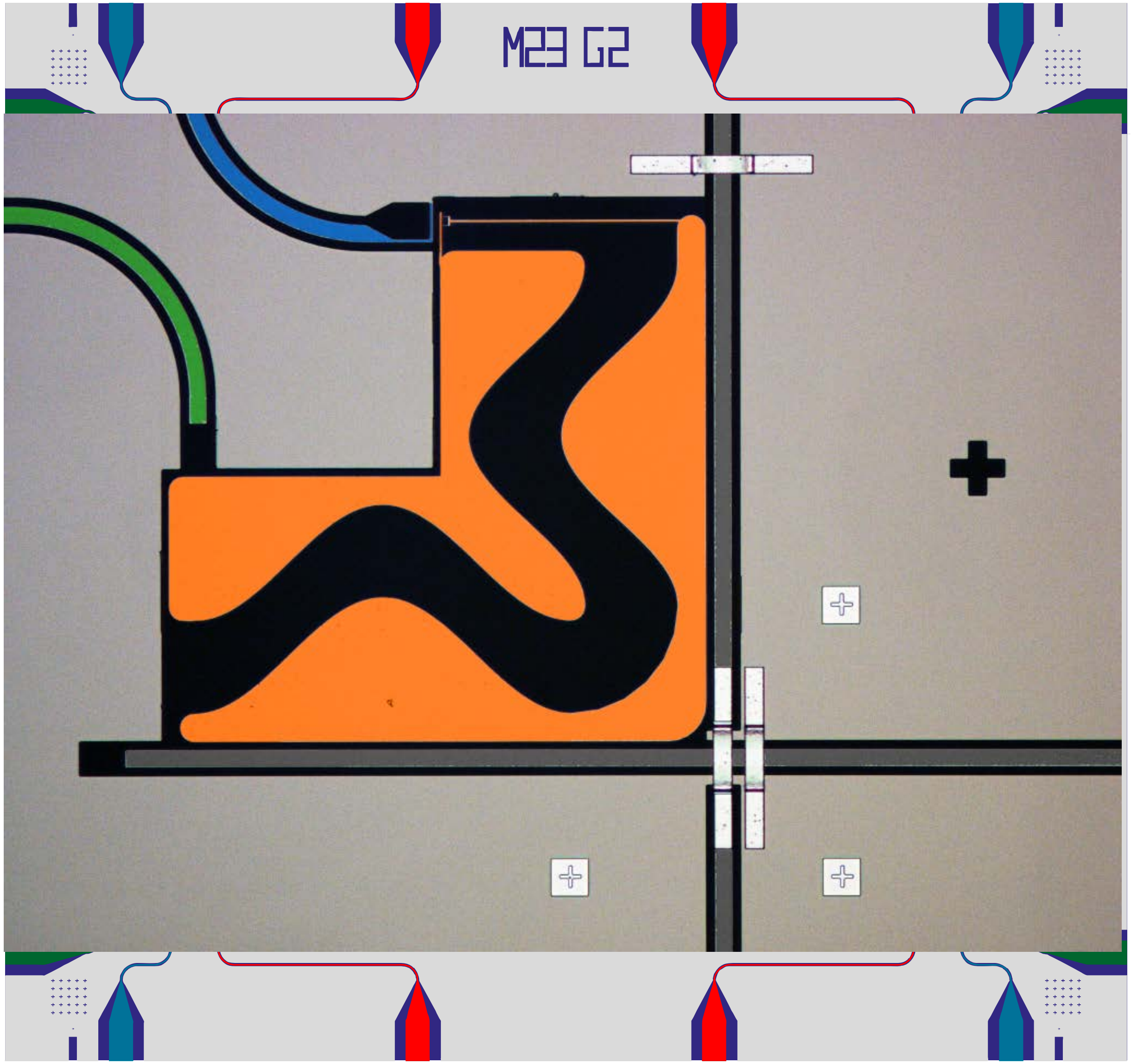
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

L. Steffen *et al.*, Quantum Device Lab, ETH Zurich (2008)

Coupling Superconducting Qubits and Generating Entanglement using a Controlled Phase Gate

A 4 Qubit 3 Resonator Sample

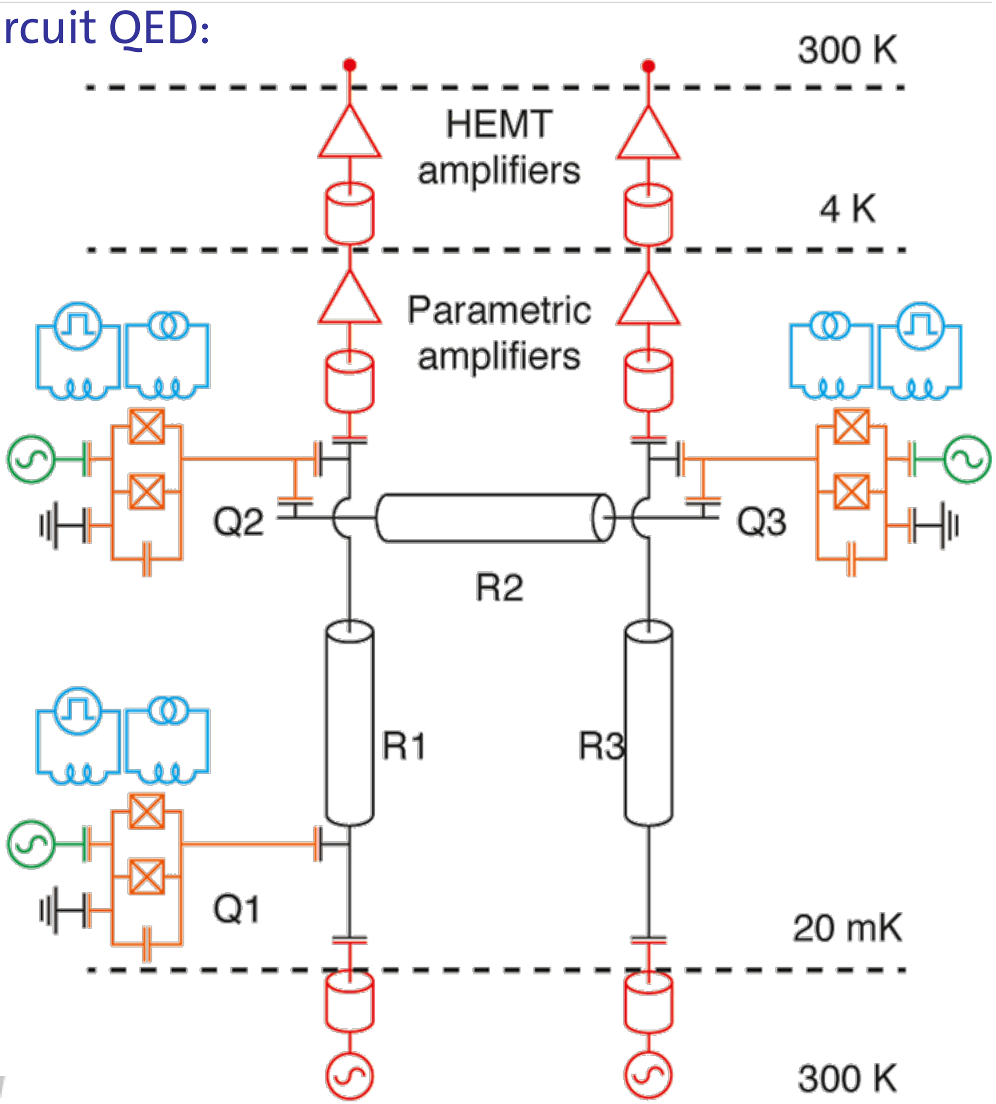
- 4 Qubits
- 3 Resonators
- single-qubit gates
- two-qubit gates (qubits in the same resonator)
- joint single-shot readout of qubits 1 & 2 and qubits 3 & 4



Steffen *et al.*, *Nature* 500, 319 (2013)

The Circuit

12-port quantum device based on circuit QED:

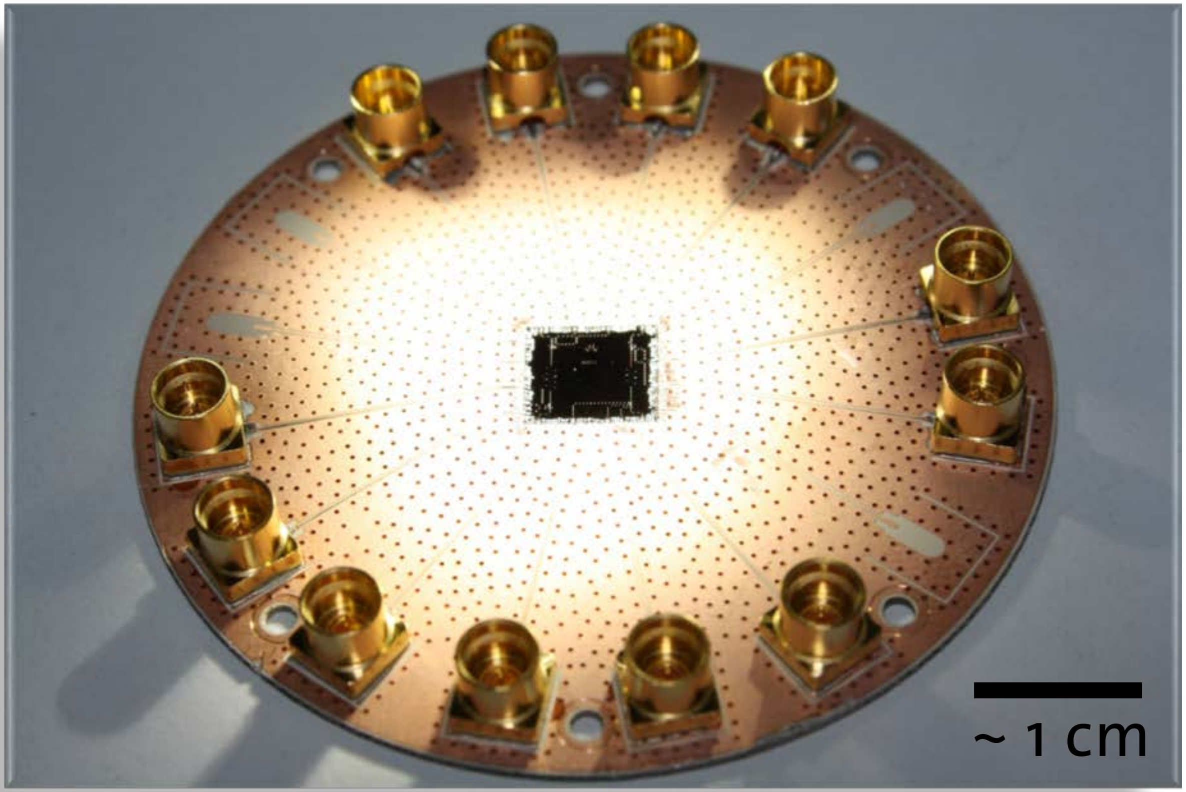


Device highlights:

- 3 high-Q resonators
- 4 transmon qubits
- individual control of all qubits
- nearest neighbor interaction via quantum bus
- individual read-out for pairs of qubits 1-2 and 3-4 through resonators
- single-shot read-out using parametric amplifiers
- qubit separation ~ 10 mm
- cross-overs for resonators

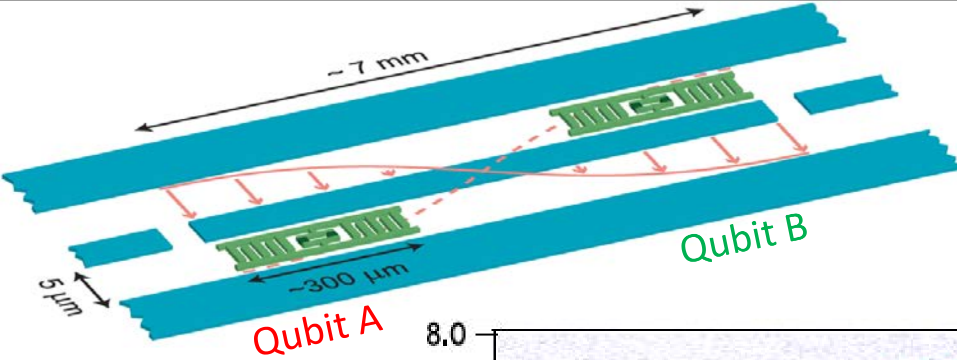
Steffen *et al., Nature* 500, 319 (2013)

12-Port Device on 16-Port Sample Mount



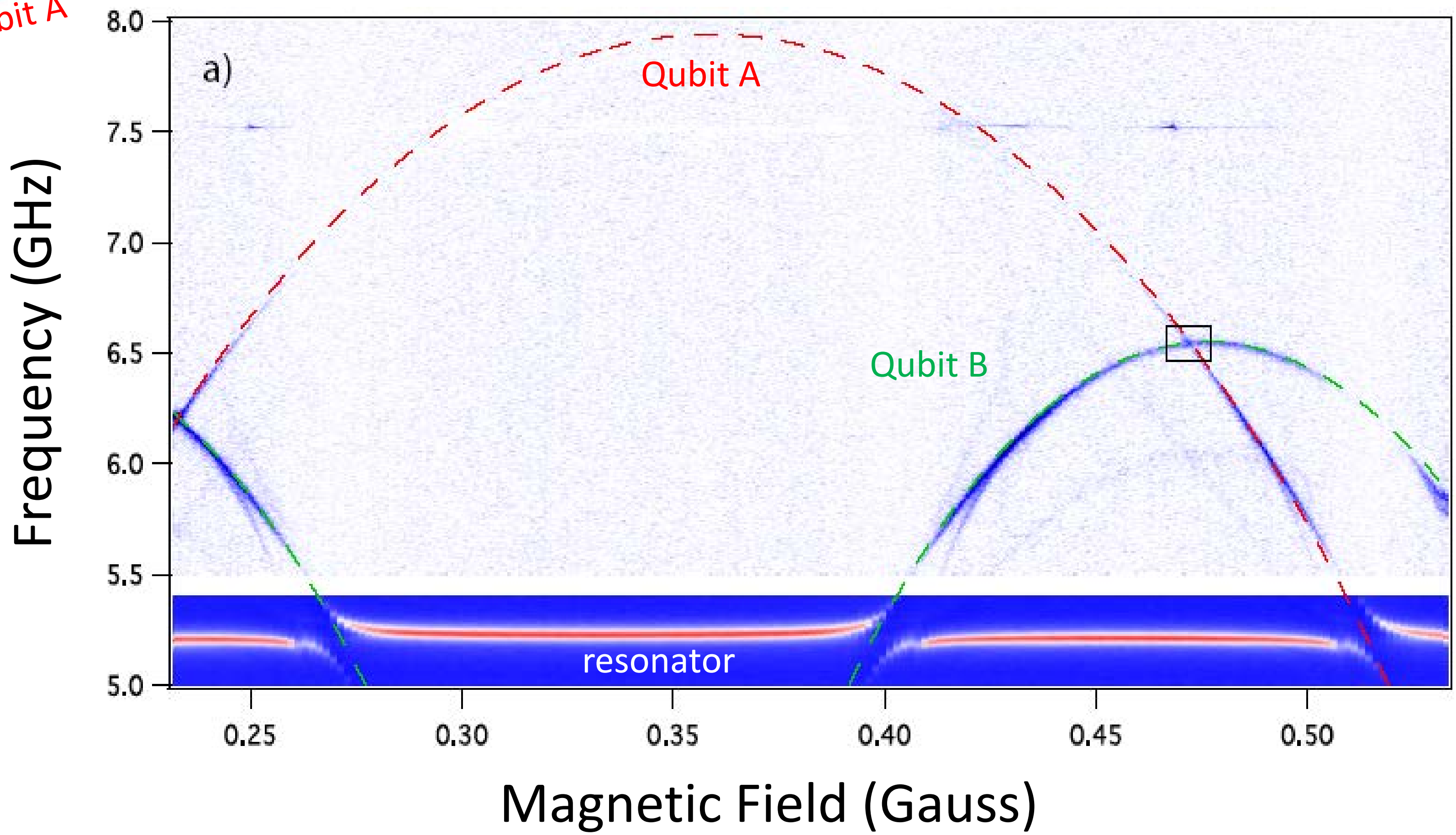
M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

Resonator as a quantum bus for qubit-qubit interactions



In the 'old days':

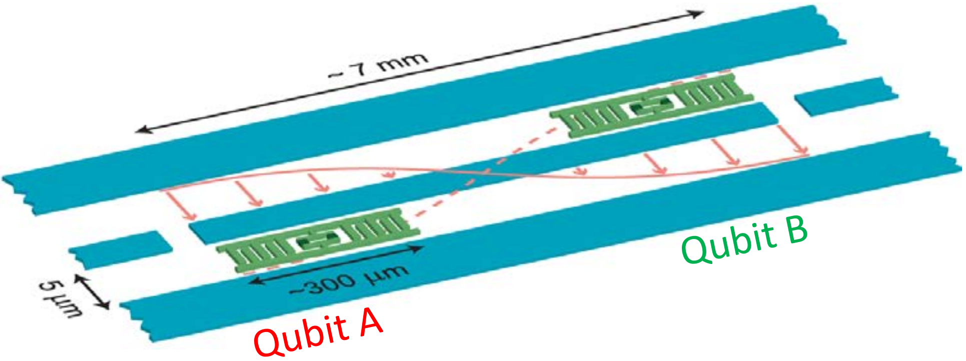
qubits tuned with static, global magnetic field



slide credit: L. DiCarlo (TUD)

J. Majer, J.M. Chow *et al.*, *Nature* **445**, 515 (2007)

Resonator as a quantum bus for qubit-qubit interactions

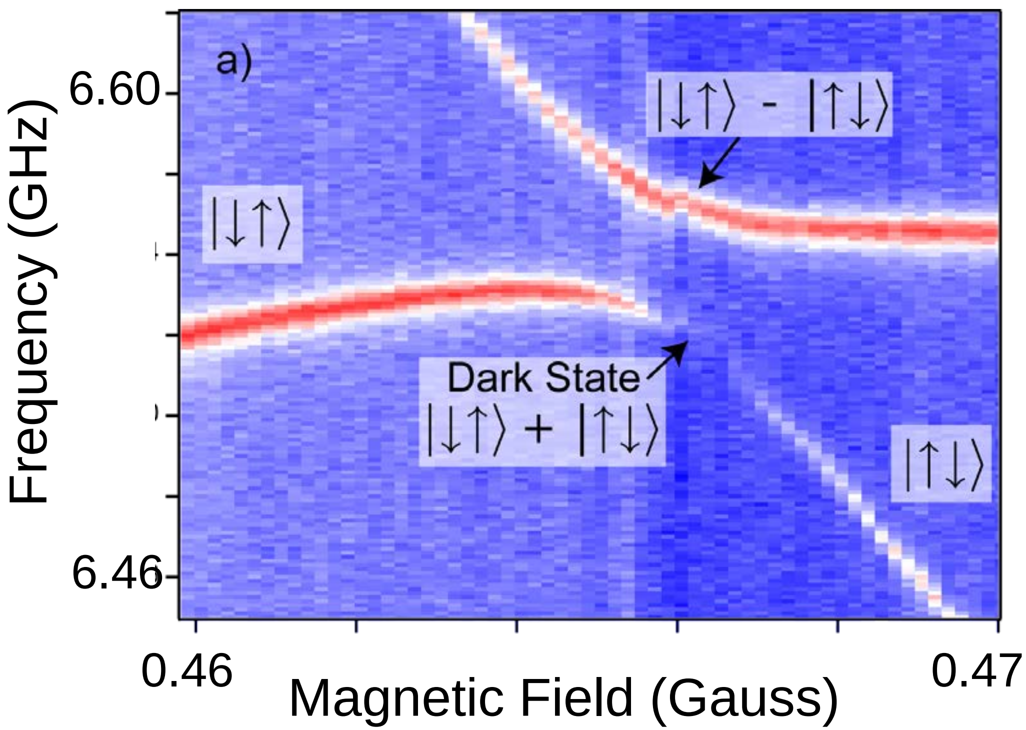


In the 'old days':

qubits tuned with static, global magnetic field

$$\tilde{H} \approx \hbar\left(\omega_r - \chi_1\sigma_1^z - \chi_2\sigma_2^z\right)a^\dagger a + \frac{\hbar\omega_1}{2}\sigma_1^z + \frac{\hbar\omega_2}{2}\sigma_2^z + \hbar J\left(\sigma_1^-\sigma_2^+ + \sigma_1^+\sigma_2^-\right)$$

$$J = \frac{g_1 g_2}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$$

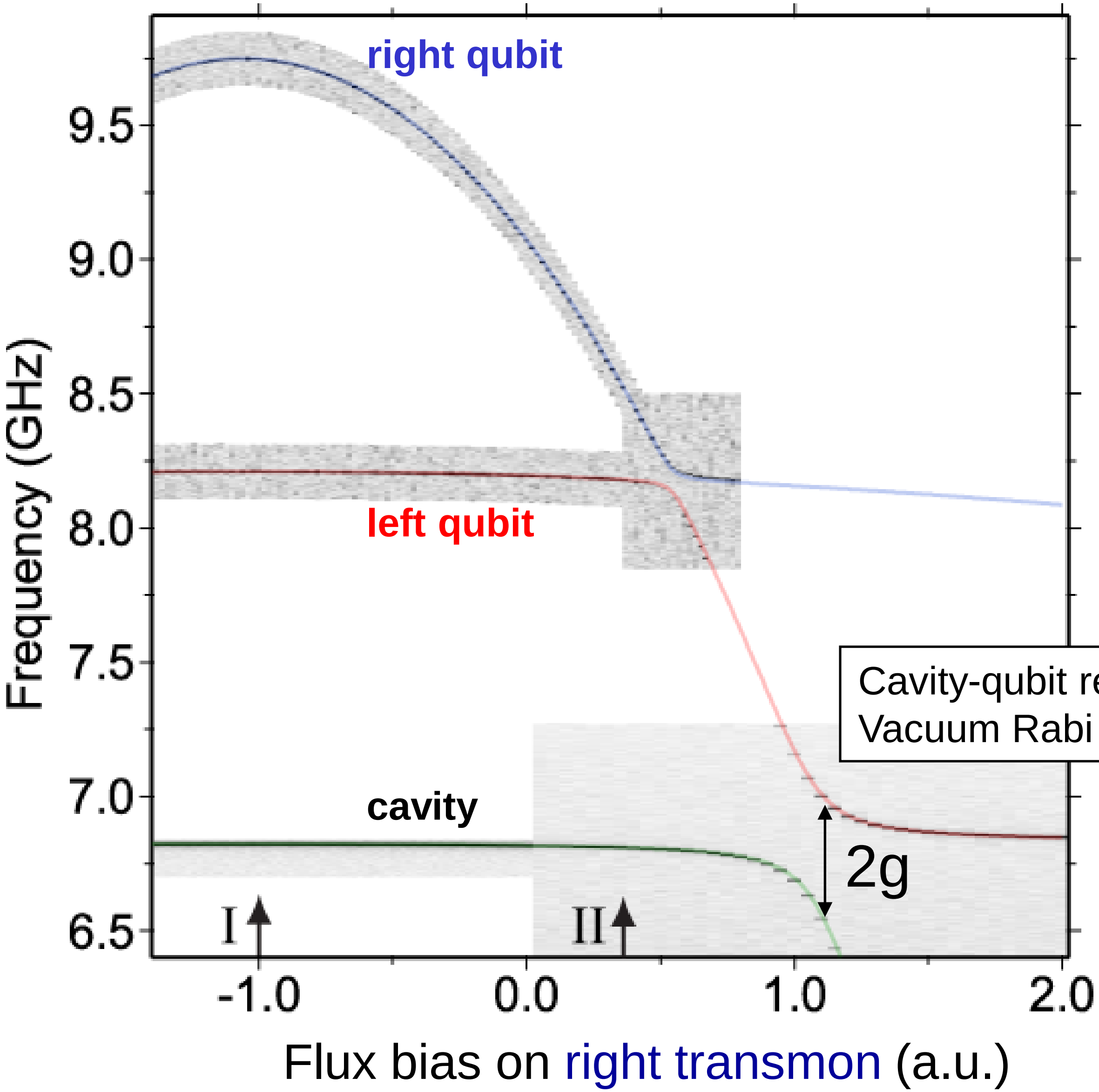


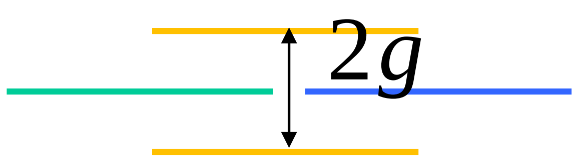
Resonator mediated qubit-qubit coupling

J. Majer, J.M. Chow *et al.*, *Nature* **449**, 443 (2007)

slide credit: L. DiCarlo (TUD)

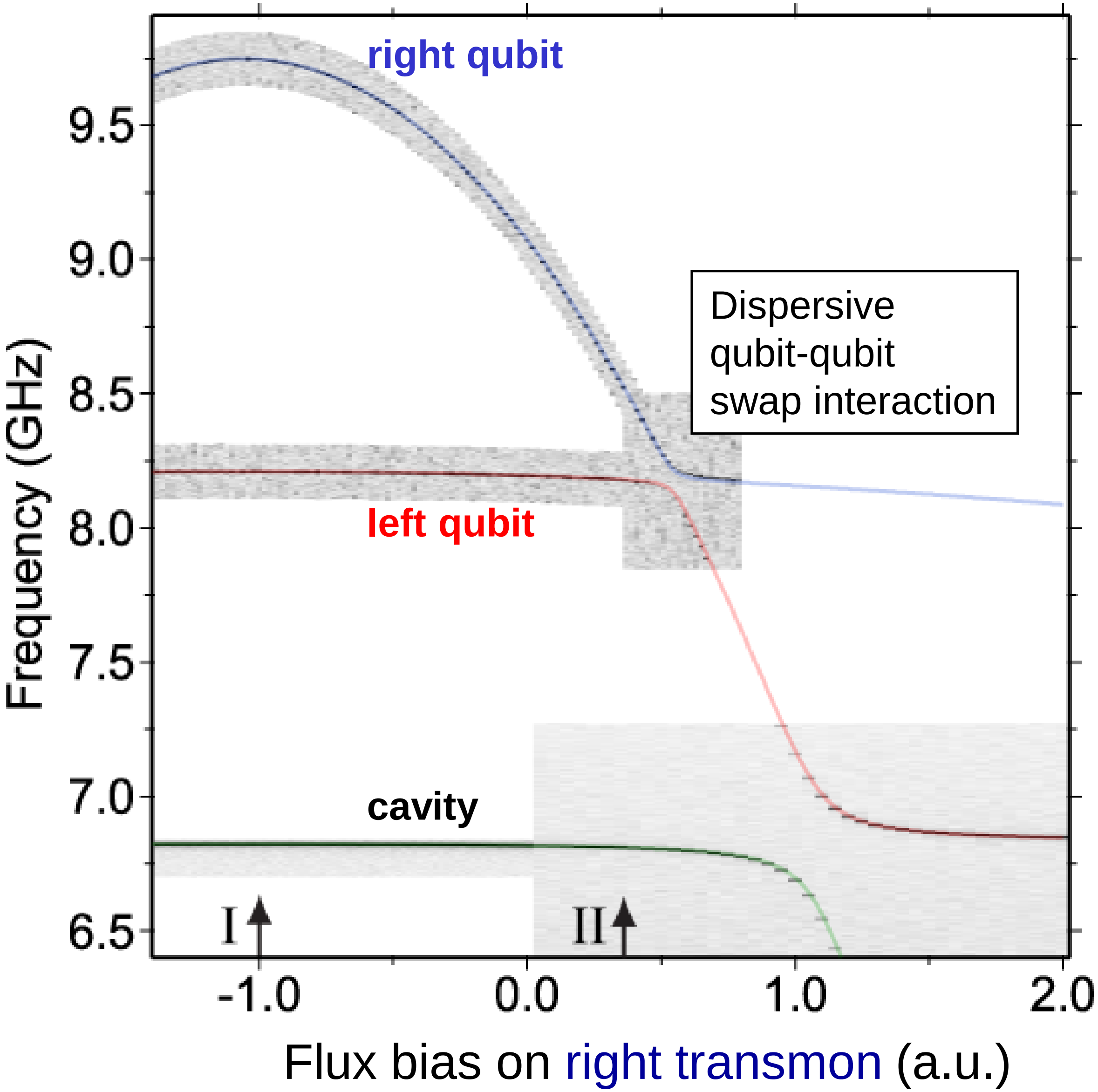
Local flux control





slide credit: L. DiCarlo (TUD)

Dispersive qubit-qubit interactions

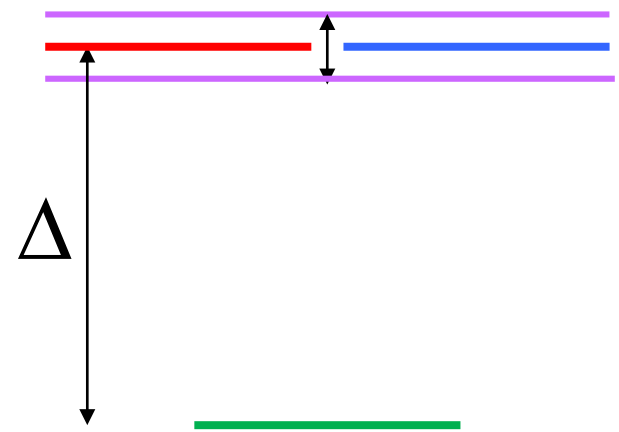


$$2g \times (g / \Delta)$$

natural speed

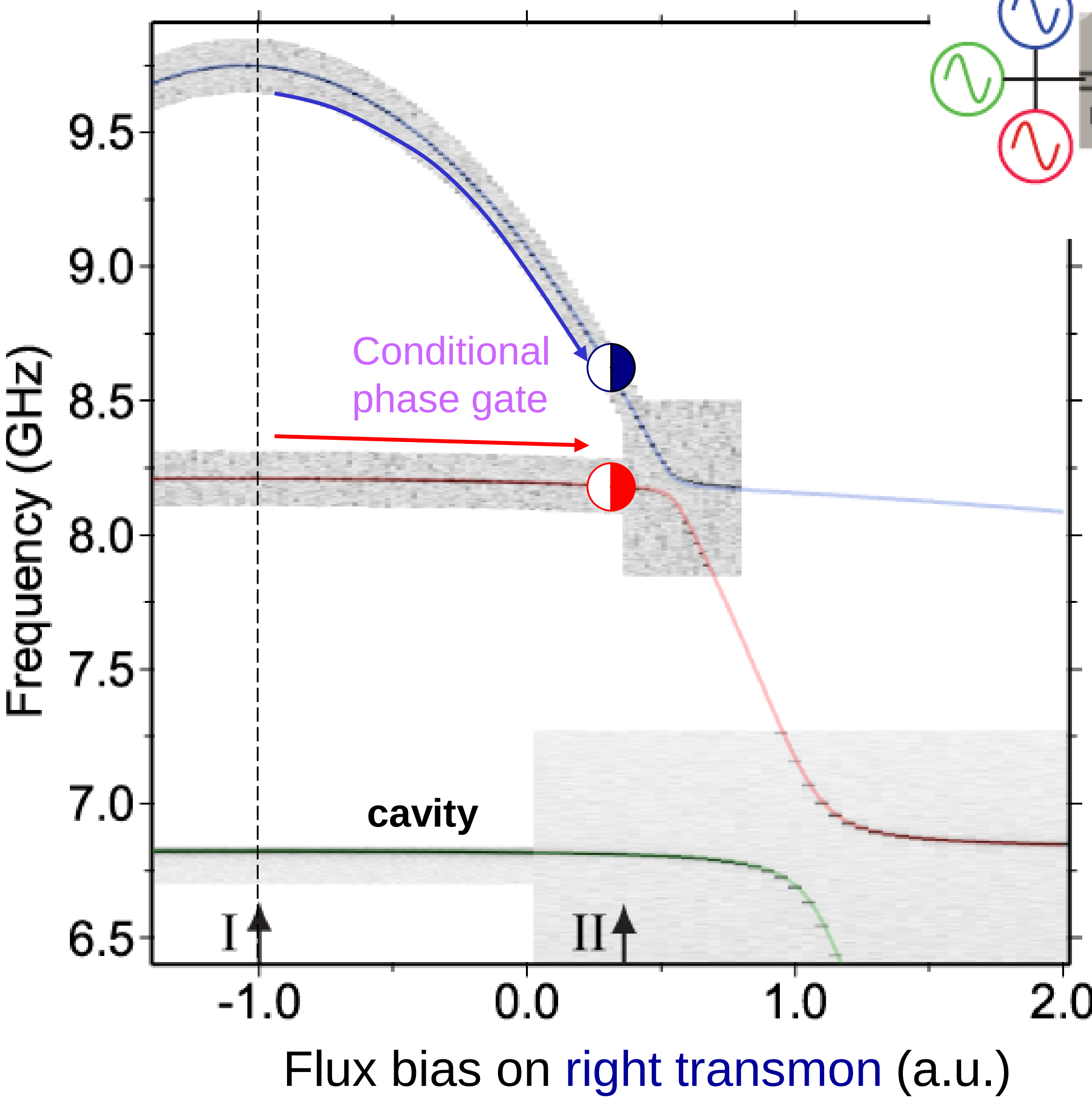
slow-down factor ~1/10

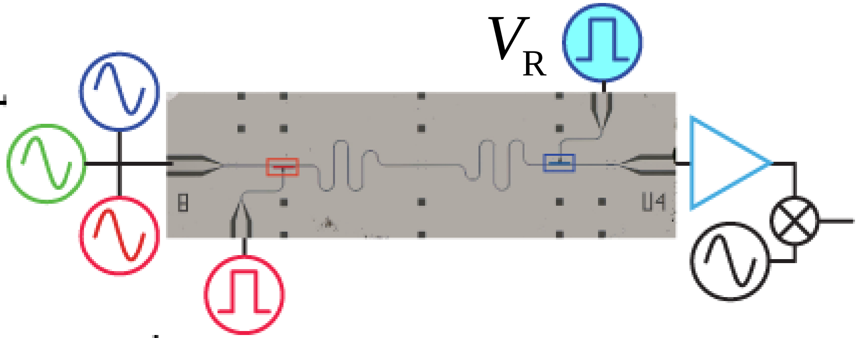
$$2g^2 / \Delta$$



slide credit: L. DiCarlo (TUD)

Two-qubit gate: turn on interactions

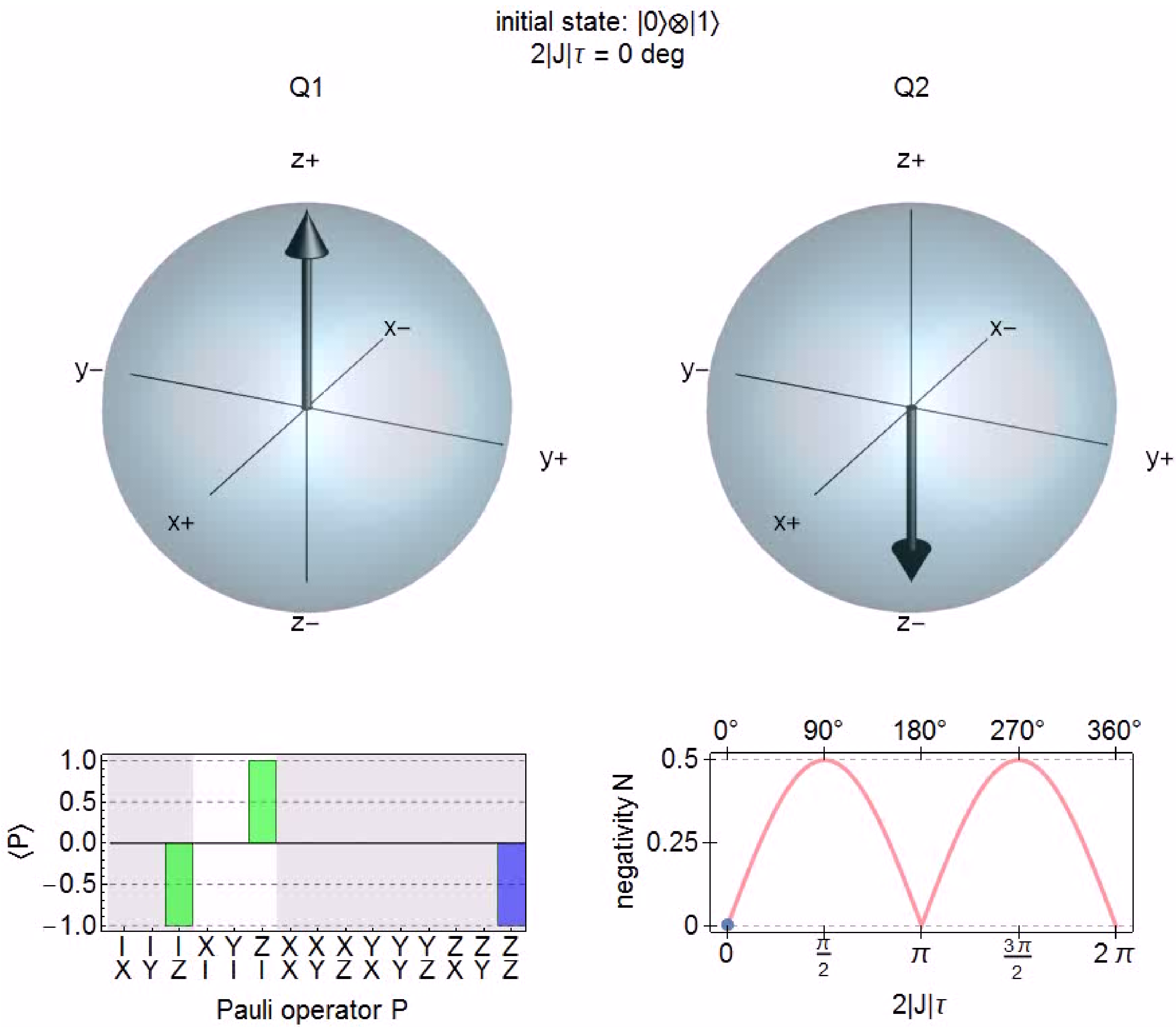




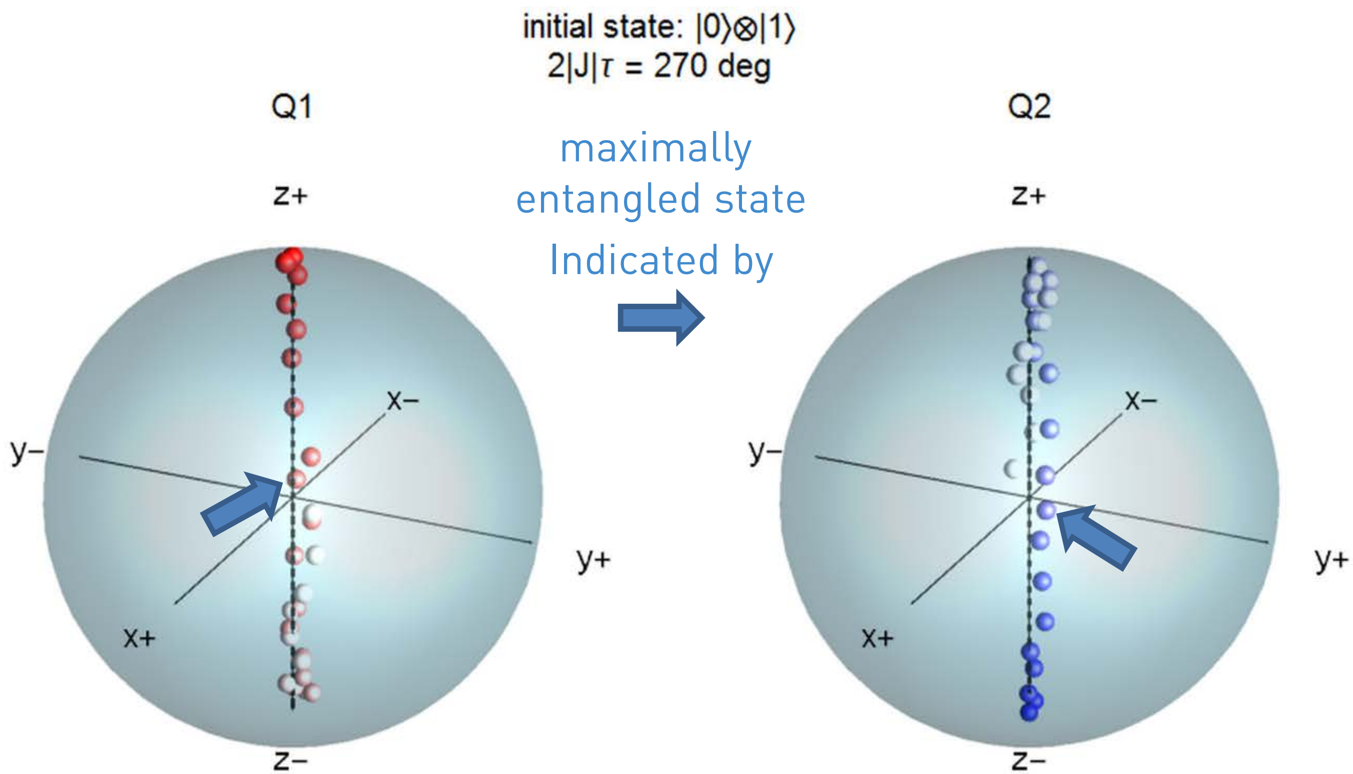
Use control lines to push qubits near a cavity-mediated resonance

slide credit: L. DiCarlo (TUD)

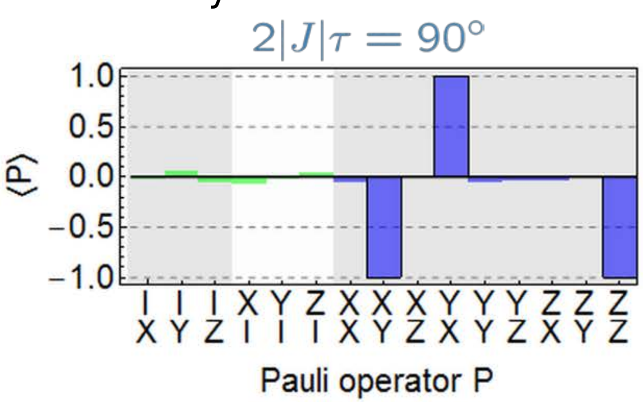
Exchange (XY) Interaction, J-Coupling: Calculation

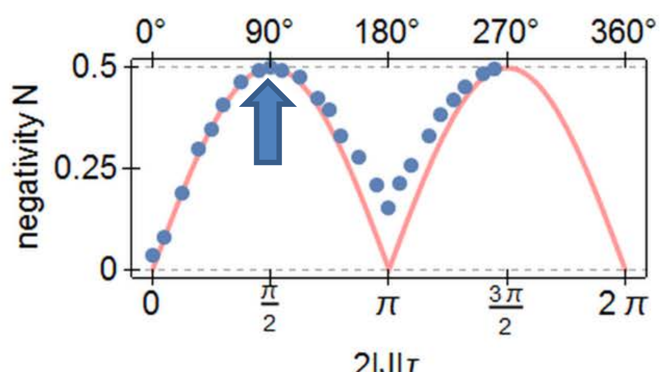


Exchange (XY) Interaction, J-Coupling: Experiment



state fidelity: 99.7 %



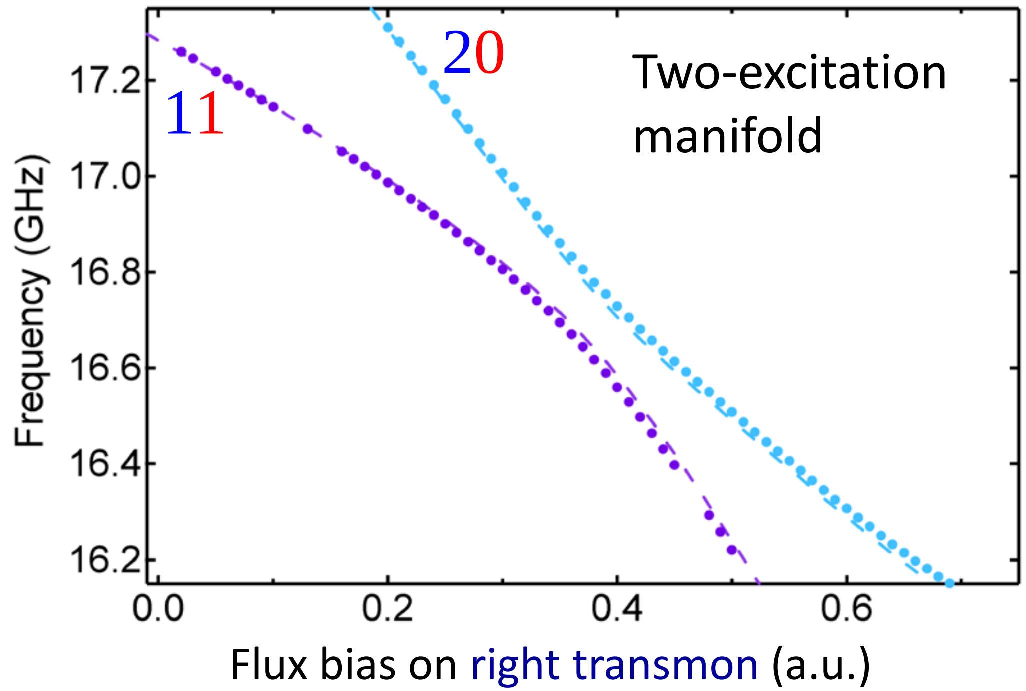


Two-excitation manifold of system

- Transmon "qubits" have multiple levels…

- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |20\rangle$$

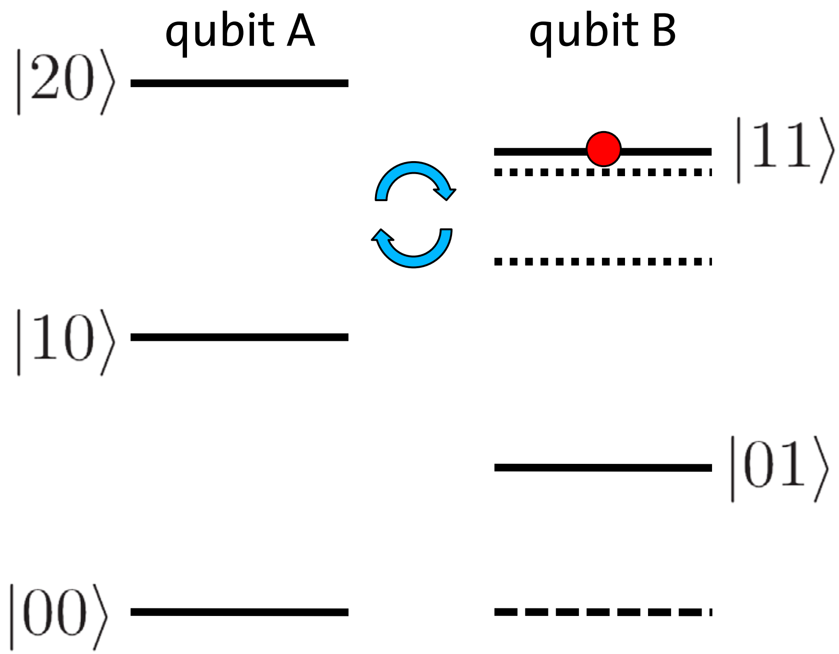


Strauch *et al*. PRL (2003):
proposed using interactions with higher levels for computation in phase qubits

slide credit: L. DiCarlo (TUD)

Universal Two-Qubit Controlled Phase Gate

Tune levels into resonance using magnetic field



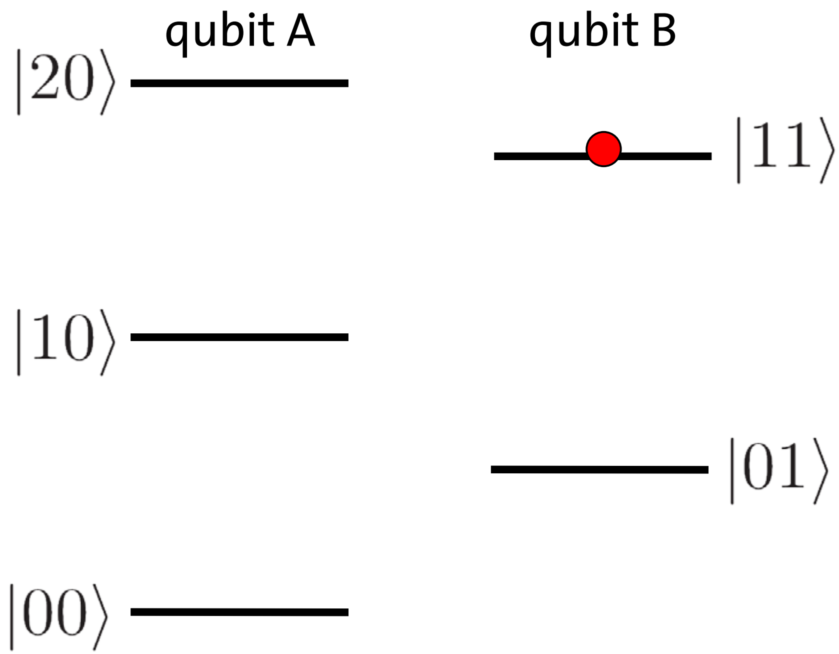
Interaction mediated by virtual photon exchange through resonator

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* 91, 167005 (2003).

first implementation: L. DiCarlo, *Nature* 460, 240 (2010).

Universal Two-Qubit Controlled Phase Gate



How to verify the operation of this gate?

$$|11\rangle \longrightarrow i|20\rangle \longrightarrow -|11\rangle$$
$$|01\rangle \longrightarrow |01\rangle$$
$$|10\rangle \longrightarrow |10\rangle$$
$$|00\rangle \longrightarrow |00\rangle$$

Universal two-qubit gate. Used together with single-qubit gates to create any quantum operation.

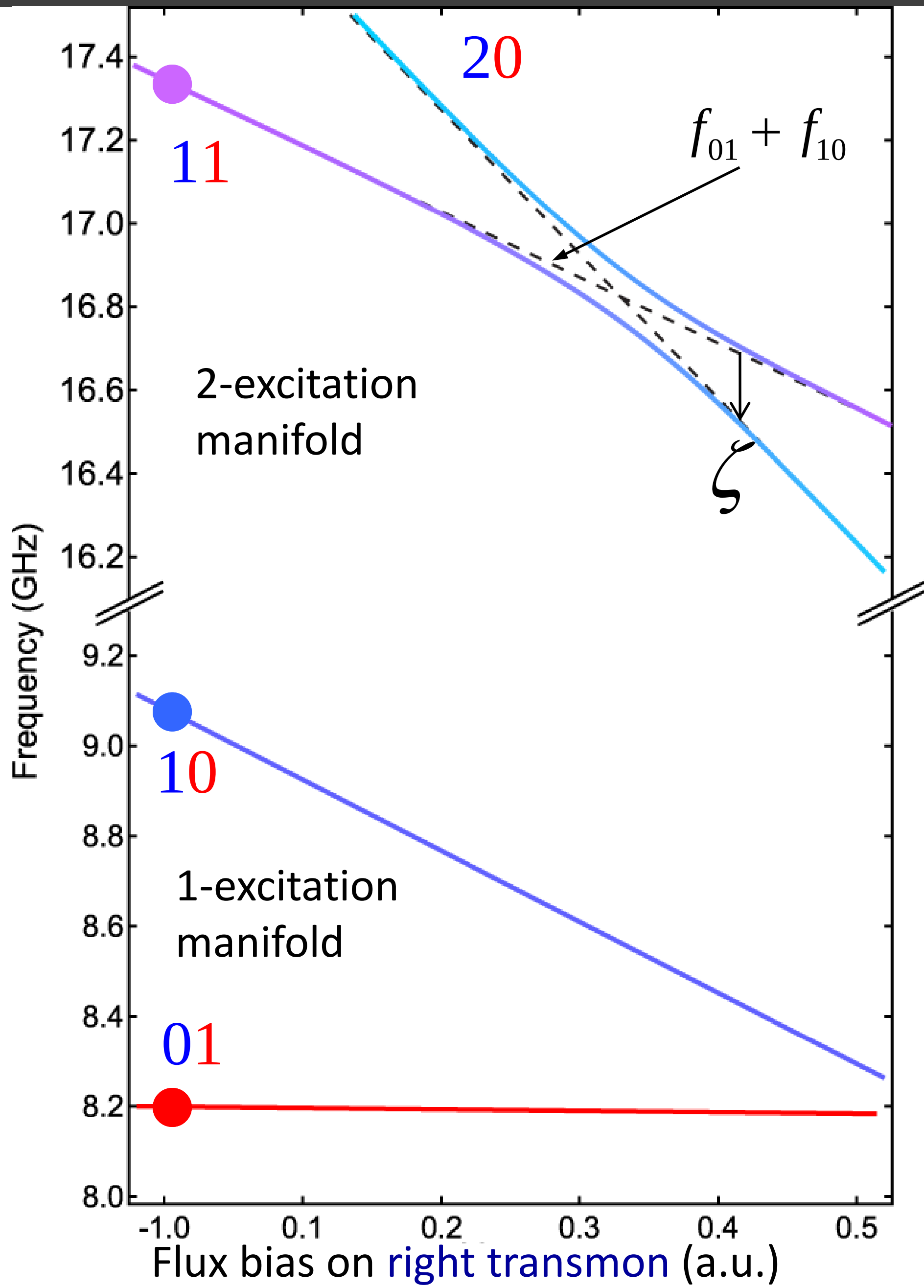
C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* 91, 167005 (2003).

first implementation: L. DiCarlo, *Nature* 460, 240 (2010).

Adiabatic conditional-phase gate



$$\varphi_a = -2\pi \int_{t_0}^{t_f} \delta f_a(t)\,dt$$

$$|11\rangle \rightarrow e^{i\varphi_{11}} |11\rangle$$

$$\varphi_{11} = \varphi_{10} + \varphi_{01} - 2\pi \int_{t_0}^{t_f} \zeta(t)\,dt$$

$$|10\rangle \rightarrow e^{i\varphi_{01}} |10\rangle$$

$$|01\rangle \rightarrow e^{i\varphi_{10}} |01\rangle$$

slide credit: L. DiCarlo (TUD)

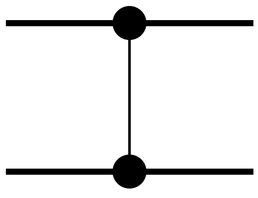
Implementing C-Phase with 1 pulse

$$\begin{array}{c} \\ |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\varphi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\varphi_{11}} \end{pmatrix} \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array}$$

Adjust timing of flux pulse so that only quantum amplitude of $|11\rangle$ acquires a minus sign:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



slide credit: L. DiCarlo (TUD)

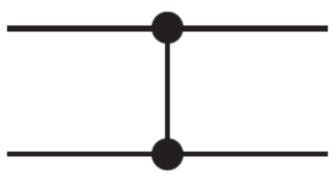
Process Tomography: C-Phase Gate

arbitrary quantum process $\rho' = \mathcal{E}(\rho)$ decomposed into $\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$

$\{\tilde{E}_k\}$ is an operator basis

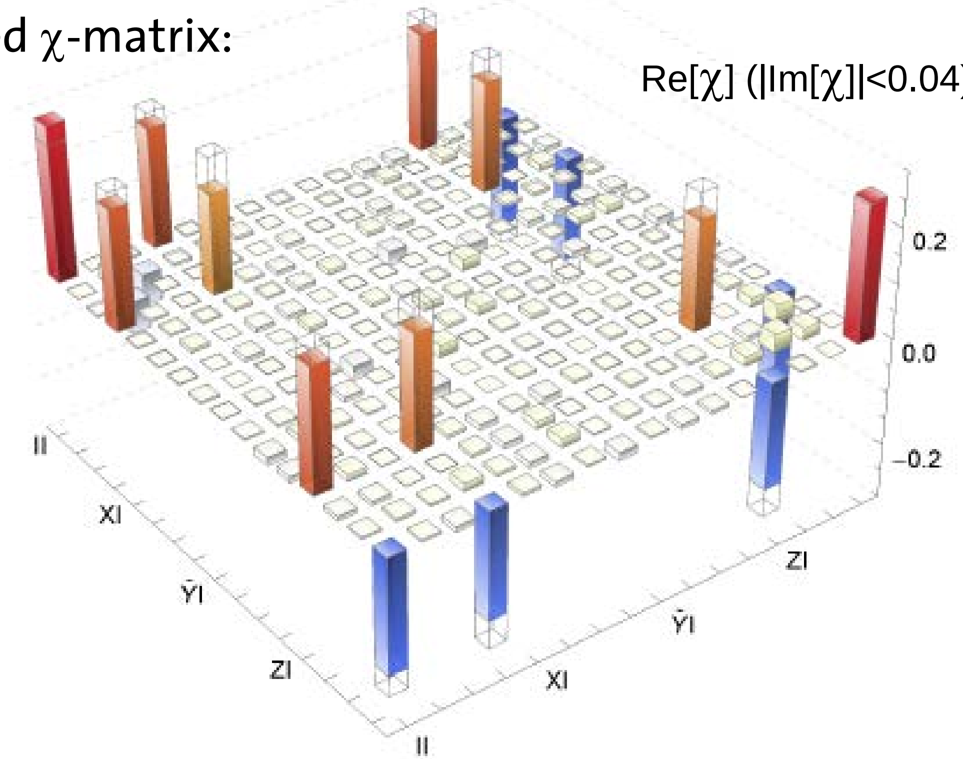
χ is a positive semi definite Hermitian matrix characteristic for the process

Controlled phase gate



$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured χ-matrix:



$$F = \mathrm{Tr}[\chi_{\mathrm{meas}}\chi_{\mathrm{ideal}}] = 0.86$$

Process Tomography: C-NOT Gate

arbitrary quantum process

$$\rho' = \mathcal{E}(\rho)$$

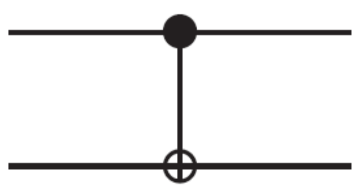
decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

$\{\tilde{E}_k\}$ is an operator basis

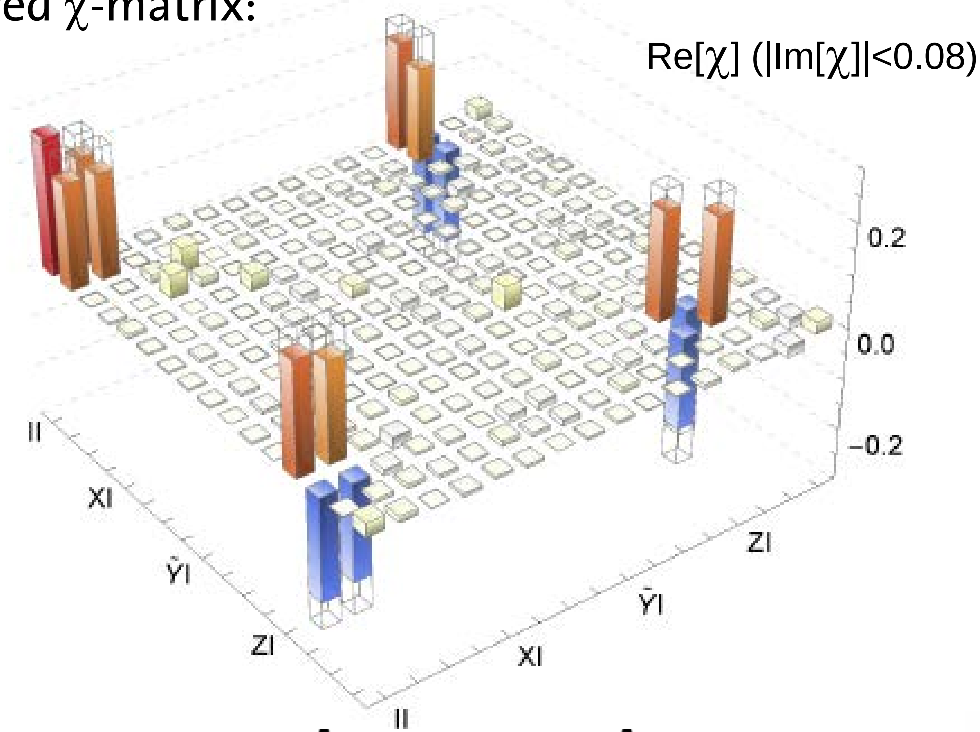
χ is a positive semi definite Hermitian matrix characteristic for the process

Controlled-NOT gate



$$\mathrm{C-NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Measured χ-matrix:

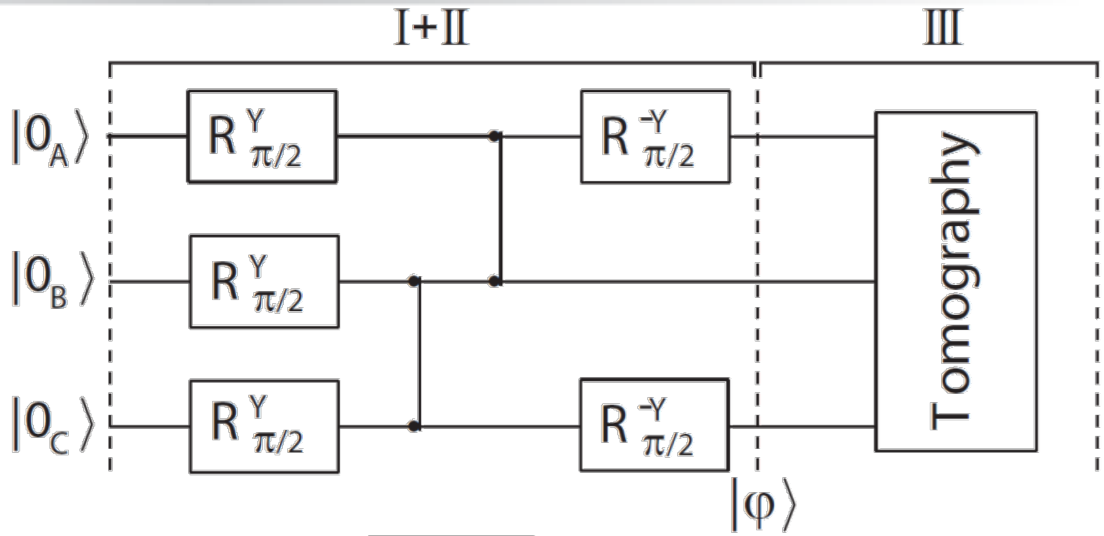


$$F = \mathrm{Tr}[\chi_{\mathrm{meas}} \chi_{\mathrm{ideal}}] = 0.81$$

Maximally Entangled Three Qubit States

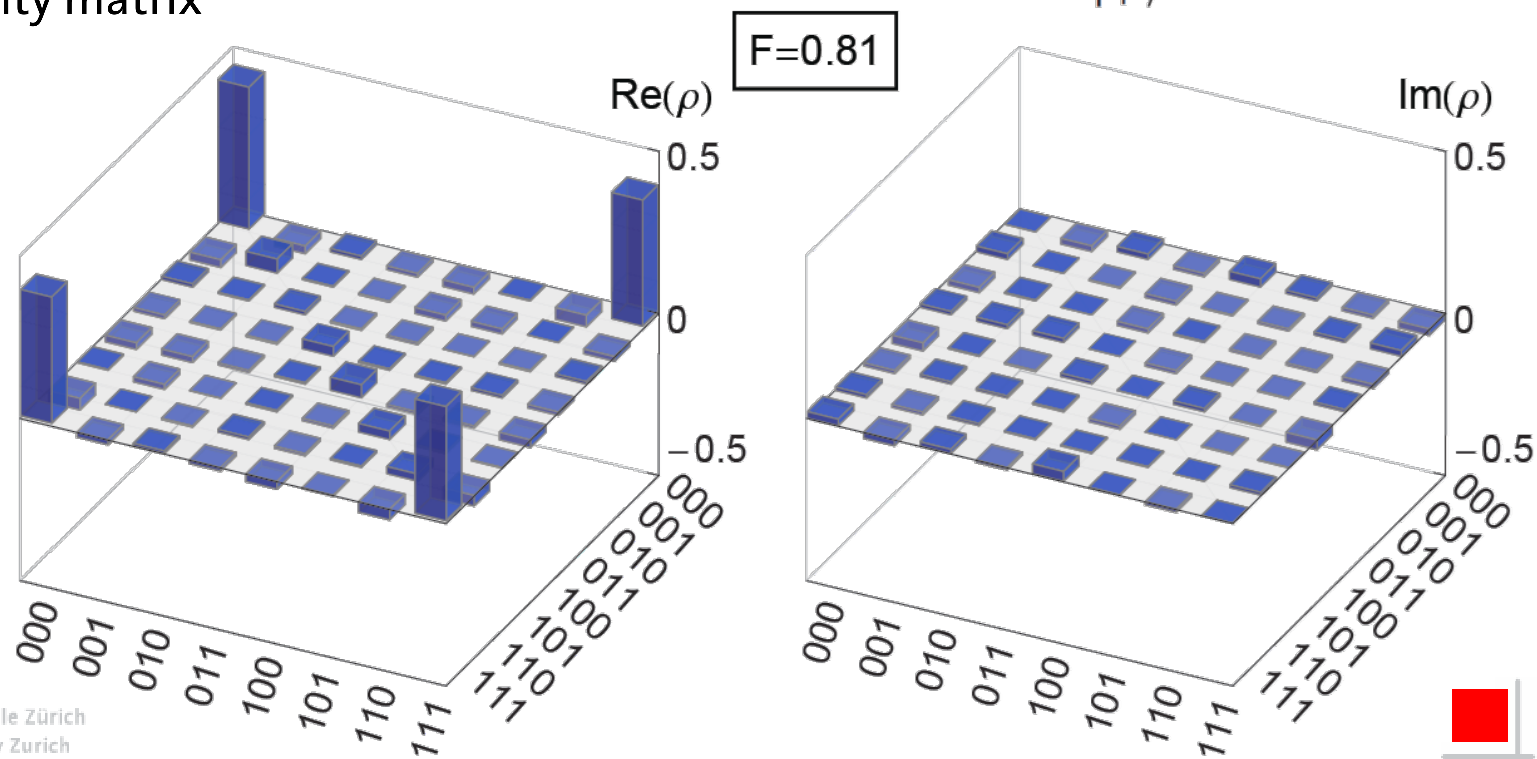
Generation of GHZ class, e.g. |000>+|111>, states:

- single qubit gates
- C-PHASE gates



Measured density matrix

- high fidelity



DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A scalable physical system with well-characterized qubits. ✓
#2. The ability to initialize the state of the qubits. ✓
#3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
#4. A universal set of quantum gates. ✓
#5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
#7. The ability to faithfully transmit flying qubits between specified locations. ✓