

# Topic: Basic Elements of a Quantum Computer

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## Key Questions:

Which components (or features) does a generic quantum computer have?

Example: Memory to store information.

How are those different from a classical computer?

## Your Background:

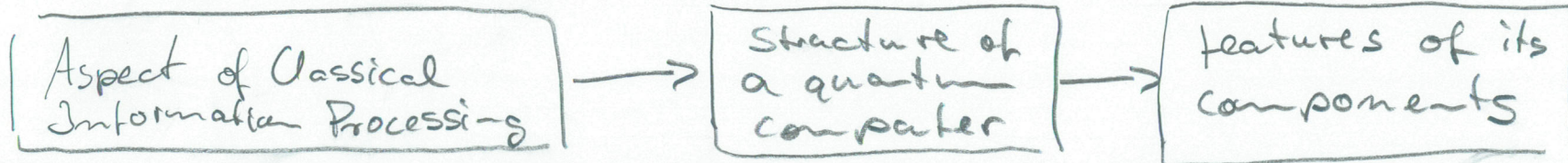
- General knowledge of how computers work
- Knowledge of quantum mechanics

COMBINE

## Goal:

- Understand basic structure and operation of a quantum computer.
- Make use of this 'hardware independent' knowledge about quantum computers to evaluate different physical realizations

## Outline:



# Classical Information Processing

- Carrier of information in binary representation

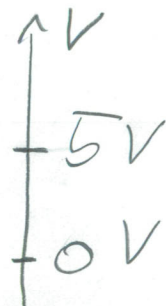
BIT

possible values

1  
0

physical representation

example:



voltage level in a circuit

- modification of information in BIT by operating with a physical process on the BIT

- any logical operation on bits can be decomposed into single and two-bit operations

- Why is this useful?
- You may want to think about why this is possible at all!
- Same is true for quantum computers.

How is information physically modified in physical realizations of bits? Give examples.

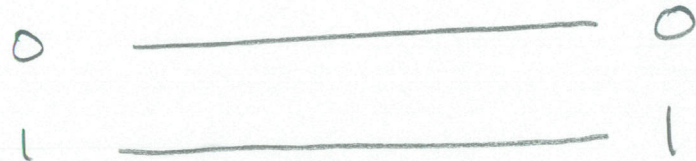
- CD
- hard disk
- RAM

2> The same question will be important in quantum computers!

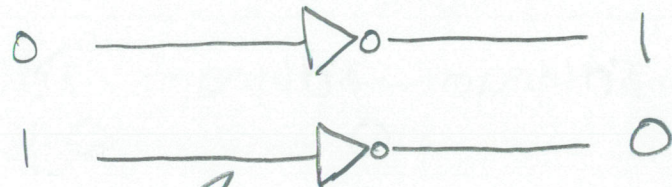
# Single Bit Operation

time →

input bit ————— output bit



IDN operation



NOT operation

Would you think single qubit operations are simple to realize in a quantum computer?

What could be potential problems?

- decoherence
- spurious interactions
- ...

- Wire represents bit
- preserves state

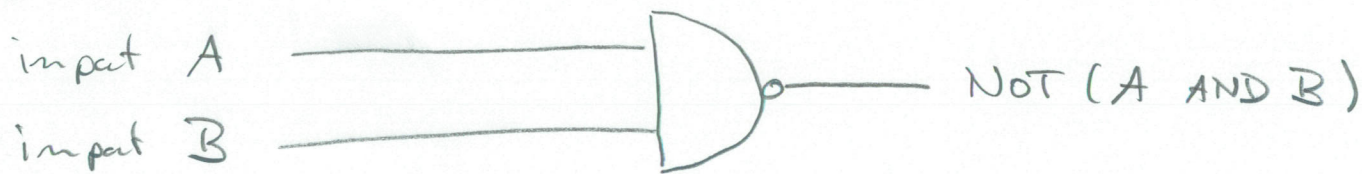
- Symbol represents operation
- changes state

Same representation of information in circuit model for quantum computation

# Two Bit Operations

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## The NAND gate



truth table	A	B	NOT (A AND B)
	0	0	1
	0	1	1
	1	0	1
	1	1	0

### Properties:

- Universal logic gate

↳ any function operating on bits can be computed using NAND gates

↳ examples: AND, OR, XOR, NOR

(ancilla bits & possibilities to make copies are required)

Why do you think universality is useful!

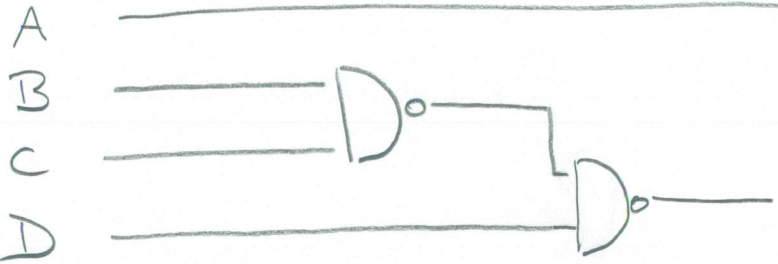
### NOTE:

Universal logic gates also do exist for quantum computers!

# Circuit Representation

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INPUT



OUTPUT

$$E = A$$

$$F = (B \text{ NAND } C) \text{ NAND } D$$

Any computable function can be represented as a circuit composed of logic universal gates acting on a set of input bits generating a set of output bits

Circuit properties :

- bits can be copied (FAN OUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than number of input bits
- loops are not allowed

In your opinion, which of these properties might be less / hard to realize in a quantum computer?

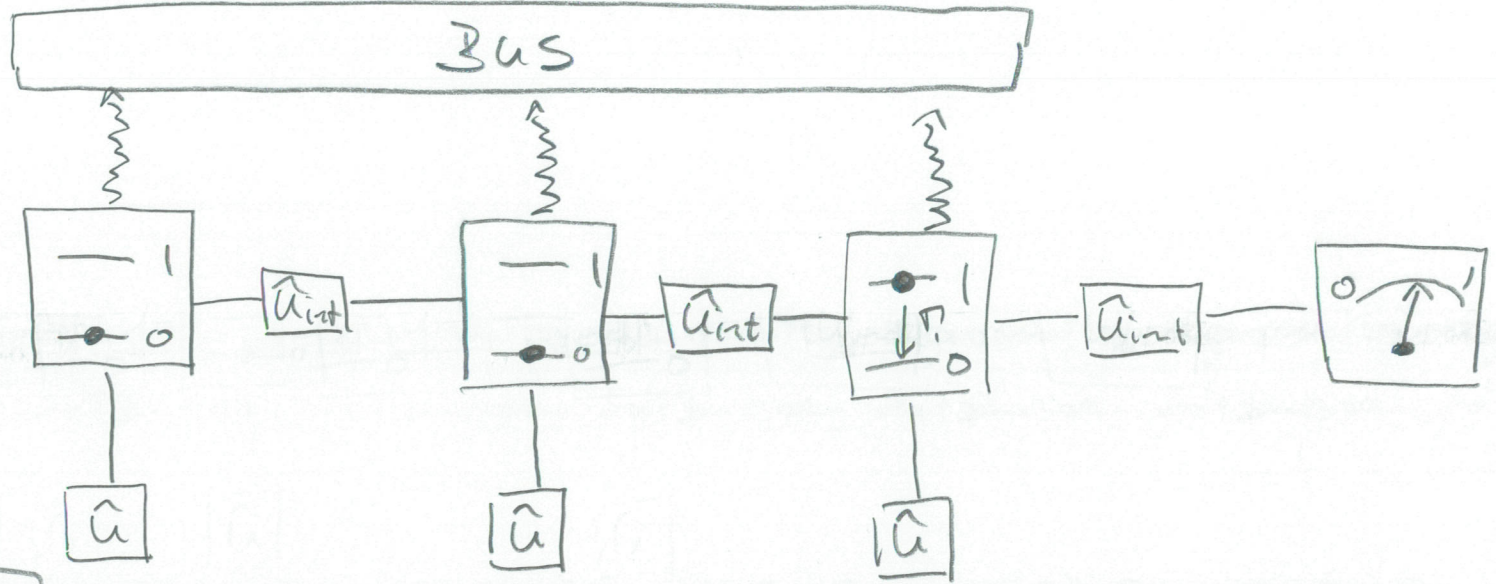
- reversibility?
- preservation of state?
- copying of information
- universality?

# A Generic Quantum Processor

(5)

features:

- (1) quantum bits
- (2) initialization
- (3) coherence
- (4) universal gates
- (5) read-out



## Di Vincenzo Criteria

- (6) conversion
- (7) transfer

In your opinion, what kind of components are absolutely essential to realize any quantum computer?

# The Quantum Bit

- Quantum mechanical system with two distinct states



Give examples of systems with this energy level spectrum. Under which conditions would you be allowed to approximate a given quantum system as a two-level system? (6)

- Representation of qubit basis states  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

vectors in 2D Hilbert space  
(1st QM postulate)

- general qubit state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \text{with } \alpha, \beta \in \mathbb{C}$$

properties: - qubit can be in superposition of states  $|0\rangle$  and  $|1\rangle$

- probability of states  $P_0 = |\alpha|^2$

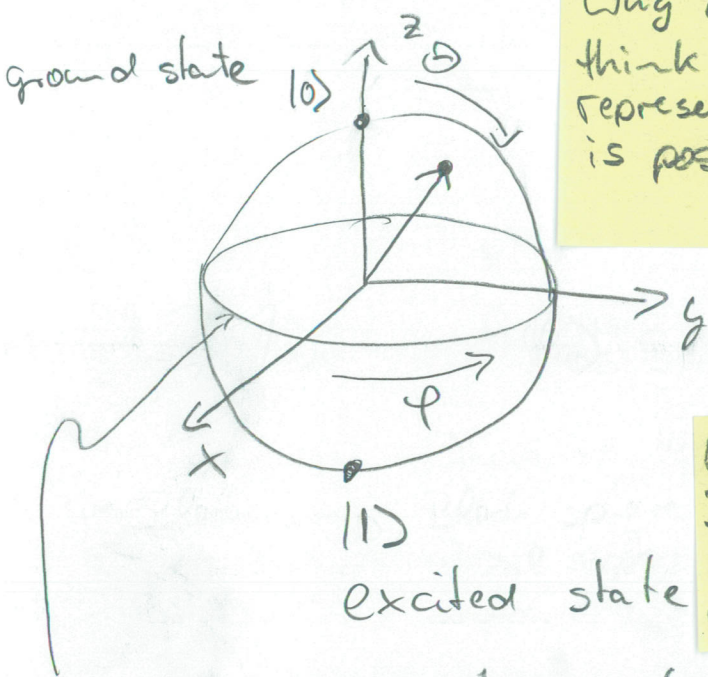
$$P_1 = |\beta|^2$$

$$\text{with } P_1 + P_2 = 1$$

What is the main difference in comp. to a classical bit?

# The Bloch Sphere

Representation of pure single qubit state as vector to the surface of a sphere



Why do you think this representation is possible?

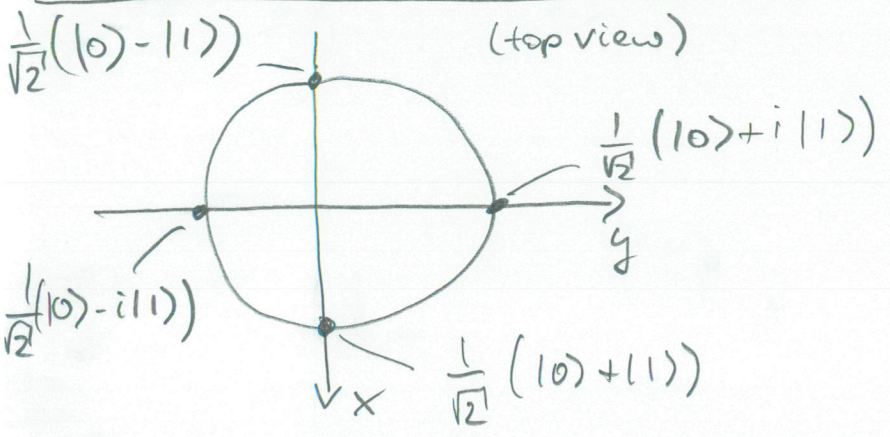
Where on the Bloch sphere are equal superposition states?

- general state  $|ψ\rangle = α|0\rangle + β|1\rangle$
- 4 parameters  $Re\ α, Im\ α, Re\ β, Im\ β$
- + 1 normalization constraint  $|α|^2 + |β|^2 = 1$

•  $|ψ\rangle = e^{iγ} \left[ \cos\frac{\theta}{2} |0\rangle + e^{iφ} \sin\frac{\theta}{2} |1\rangle \right]$

with  $γ$ : global phase factor  
 $\theta$ : polar angle  
 $φ$ : azimuthal angle

equal superposition states:



equivalent representation

- $|ψ\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{iφ} |1\rangle)$
- $\Rightarrow \theta = \frac{\pi}{2}$ : defines equator
- $φ = 0, \pi/2, \pi, 3/2\pi$  defines phase angle



# Single Qubit Gates

circuit representation



•  $\hat{I} = \hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  Identity

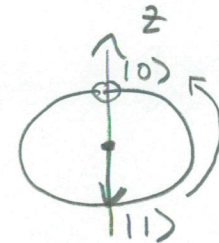
•  $\hat{X} = \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  Bit flip

•  $\hat{Y} = \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$  Conjugate bit flip

•  $\hat{Z} = \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  Phase flip

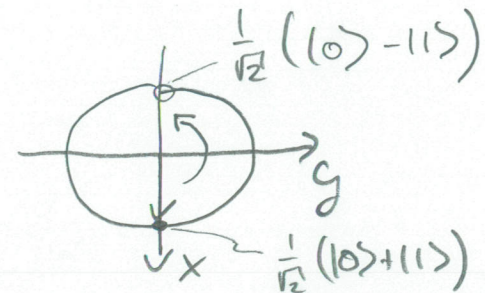
↑  
Pauli matrices

Work out the effect of the single qubit operations on some simple state on the Bloch sphere!



$$|10\rangle \rightarrow -i |11\rangle$$

$$|11\rangle \rightarrow i |10\rangle$$



# The Hadamard Gate

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generation of Superposition from basis states

$$|0\rangle \longrightarrow \boxed{\hat{H}} \longrightarrow \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$|1\rangle \longrightarrow \boxed{\hat{H}} \longrightarrow \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

• matrix representation

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{X} + \hat{Z} \\ \hat{X} - \hat{Z} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Note: - this gate is used in many quantum algorithms to prepare superposition states from basis states

Can you think of alternative ways to generate a superposition state?

# Single Qubit Dynamics

spin 1/2 particle in external field

- Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B}$$

- corresponding Operator

$$\hat{H} = -\frac{g\mu_B B_z}{2} \hat{z}$$

- time independent Schrödinger equation

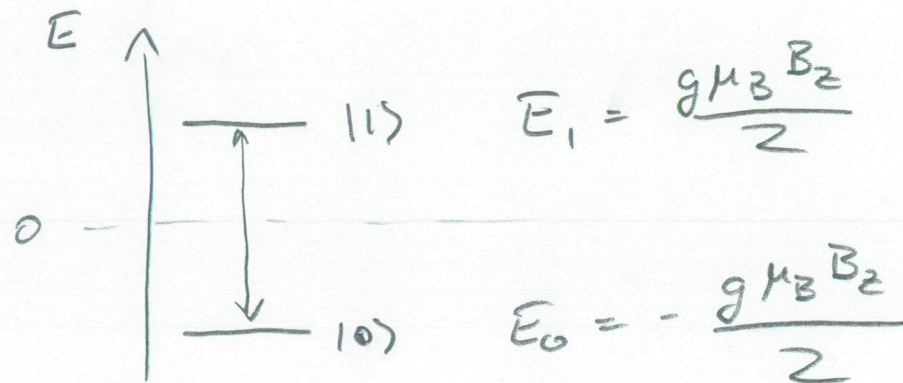
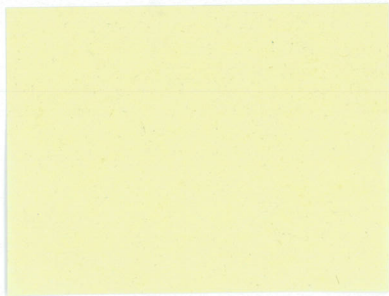
$$\hat{H} |\psi_i\rangle = E_i |\psi_i\rangle$$

- eigenstates of  $\hat{H}$  are  $|0\rangle$  and  $|1\rangle$

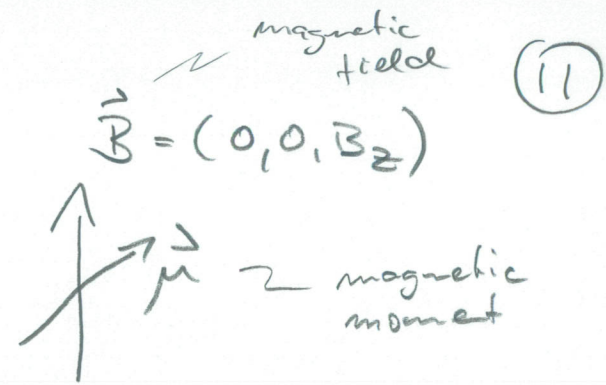
$$\hat{H} |0\rangle = E_0 |0\rangle$$

$$\hat{H} |1\rangle = E_1 |1\rangle$$

- energy level diagram



$$\Delta E = g\mu_B B_z = \hbar \Omega_z = E_1 - E_0$$



$g$ : gyromagnetic ratio  
 $\mu_B$ : Bohr magneton

- time dependent Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$

- general solution for time independent  $\hat{H}$

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \hat{H} t\right) |\psi(0)\rangle$$

with

$$\exp(i\theta \hat{O}) = \cos \theta \hat{I} + i \sin \theta \hat{O}$$

for operators with  $\hat{O}^2 = \hat{I}$  and  $\theta \in \mathbb{R}$

e.g. for all Pauli matrices

- for spin  $1/2$  example

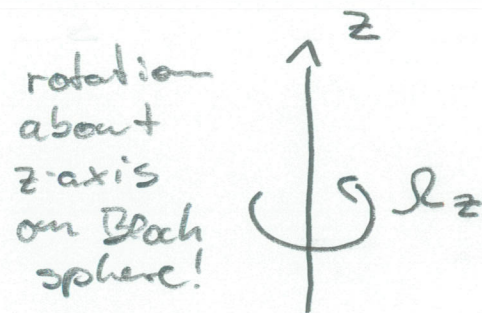
$$\hat{H} = -\frac{\hbar \mathcal{L}_z}{2} \hat{Z}$$

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \hat{H} t\right) |\psi(0)\rangle$$

$$= \left(\cos \frac{\theta_z}{2} \hat{I} + i \sin \frac{\theta_z}{2} \hat{Z}\right) |\psi(0)\rangle = R_z(\theta_z) |\psi(0)\rangle$$

with  $\theta_z = \mathcal{L}_z t$

How would you determine the dynamics of a system described by the operator  $\hat{H}$ ?



# Dynamics of Superposition State

- initial state
- Hamilton operator
- final state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$\hat{H} = -\frac{\hbar J_z}{2} (|0\rangle\langle 0| - |1\rangle\langle 1|)$$

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \hat{H} t\right) |\psi(0)\rangle$$

$$= \frac{1}{\sqrt{2}} \left( e^{i \frac{J_z t}{2}} |0\rangle + e^{-i \frac{J_z t}{2}} |1\rangle \right)$$

upto global phase

$$= \frac{1}{\sqrt{2}} \left( |0\rangle + e^{-i J_z t} |1\rangle \right)$$

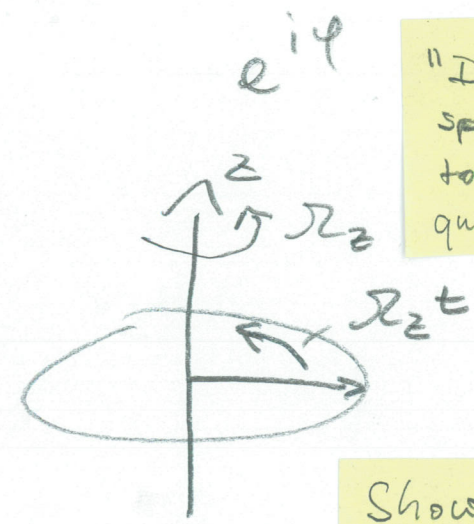
- on Bloch sphere

Can you work out what dynamics the Hamiltonian  $\hat{H}_x = -\frac{\hbar J_x}{2} \hat{X}$  induces?

$$\Theta = \frac{\pi}{2}$$

$$\varphi = -J_z t$$

How is this useful for controlling the qubit state?



"Do the Bloch sphere dance to illustrate qubit dynamics!"

Show slides with other rotation operators.