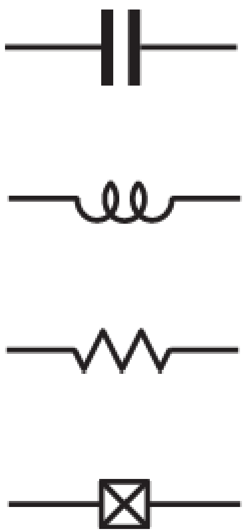
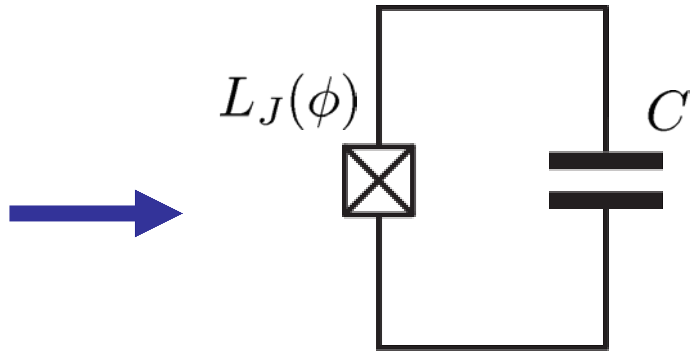
Constructing Non-Linear Quantum Electronic Circuits

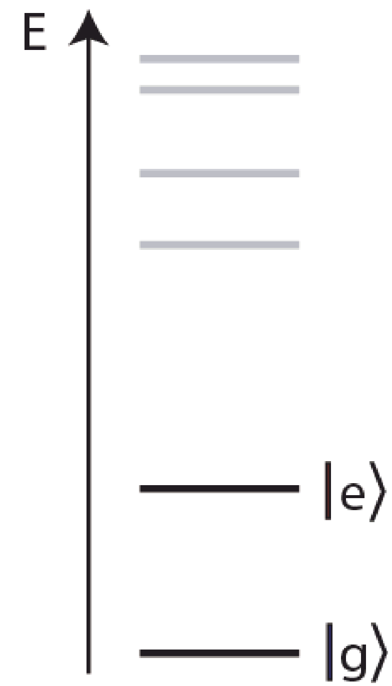
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



Josesphson junction:

a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic artificial atom

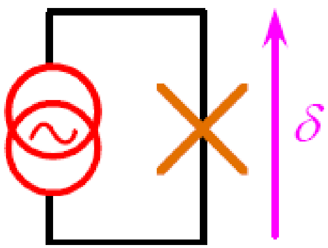
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

A Classification of Josephson Junction Based Qubits

How to make use in of Jospehson junctions in a qubit?

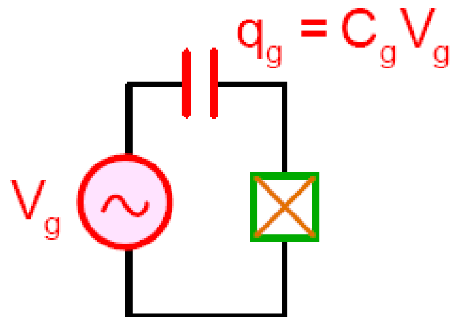
Common options of bias (control) circuits:

phase qubit



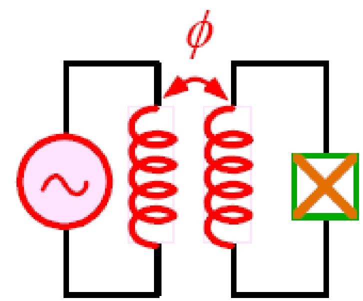
current bias

charge qubit
(Cooper Pair Box, Transmon)



charge bias

flux qubit

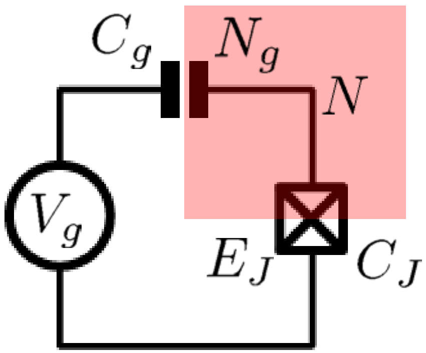


flux bias

How is the control circuit important?

The Cooper Pair Box Qubit

A Charge Qubit: The Cooper Pair Box



discrete charge on island: $$N = \frac{Q}{2e}$$

continuous gate charge: $$N_g = \frac{C_g V_g}{2e}$$

total box capacitance $$C_\Sigma = C_g + C_J$$

Hamiltonian: $$H = H_{\mathrm{el}} + H_{\mathrm{mag}}$$

electrostatic part:

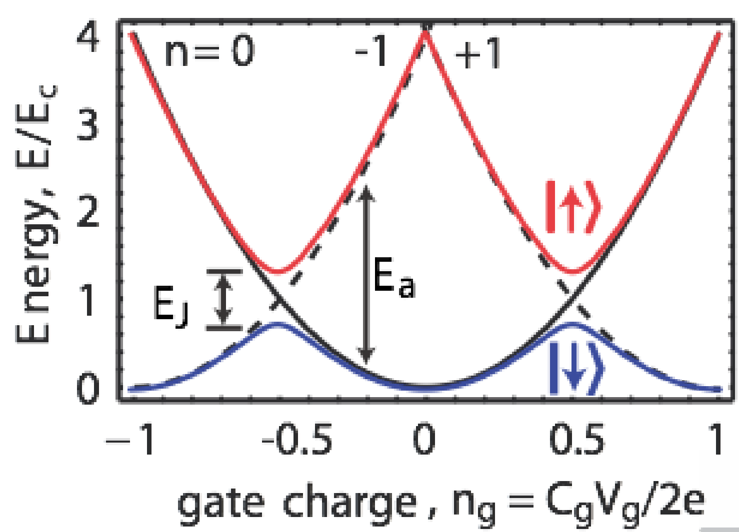
$$H_{\mathrm{el}} = \frac{Q^2}{2C} = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2$$

charging energy E_C

magnetic part:

$$H_{\mathrm{mag}} = -E_J \cos\delta \approx \frac{\phi^2}{2L_{J0}}$$

Josephson energy



Hamilton Operator of the Cooper Pair Box

Hamiltonian: $$\hat{H} = \hat{H}_{\text{el}} + \hat{H}_{\text{mag}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

$$\cos\hat{\delta} = \frac{1}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$$

commutation relation: $$[\hat{\delta}, \hat{N}] = i$$

charge number operator:

$$\hat{N}|N\rangle = N|N\rangle$$ eigenvalues, eigenfunctions

$$\sum_N |N\rangle\langle N| = 1$$ completeness

$$\langle N|M\rangle = \delta_{NM}$$ orthogonality

phase basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{iN\delta}|N\rangle$$ basis transformation

$$e^{\pm i\hat{\delta}}|N\rangle = |N \pm 1\rangle$$

Solving the Cooper Pair Box Hamiltonian

Hamilton operator in the charge basis N:

$$\hat{H} = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$

solutions in the charge basis:

$$\hat{H}|\psi_n(N)\rangle = E_n|\psi_n(N)\rangle$$

Hamilton operator in the phase basis δ:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta} = E_C(-i\frac{\partial}{\partial\delta} - N_g)^2 - E_J\cos\hat{\delta}$$

transformation of the number operator:

$$\hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi} = -i\frac{\partial}{\partial\delta}$$

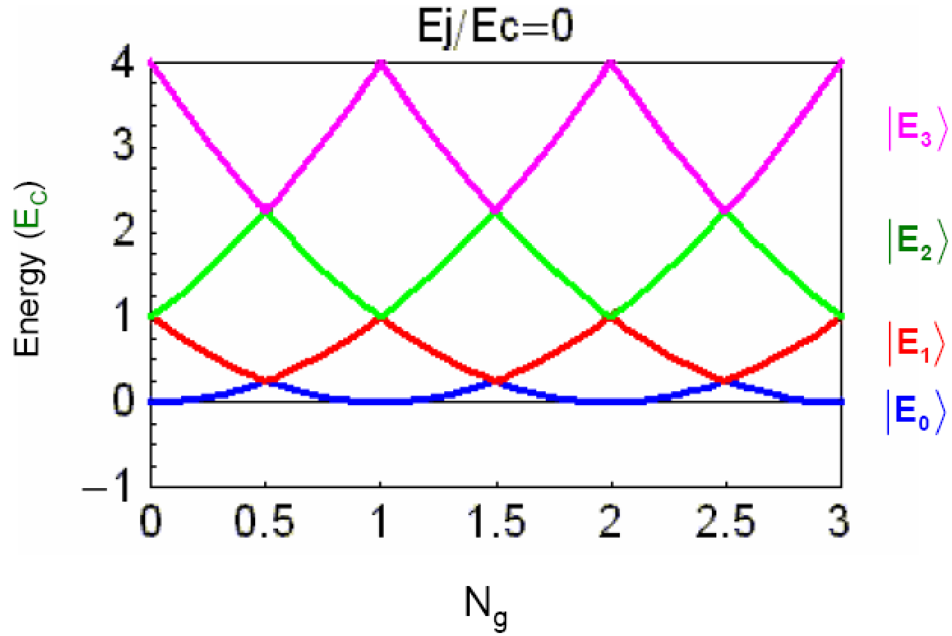
solutions in the phase basis:

$$\hat{H}|\psi_n(\delta)\rangle = E_n|\psi_n(\delta)\rangle$$

Energy Levels

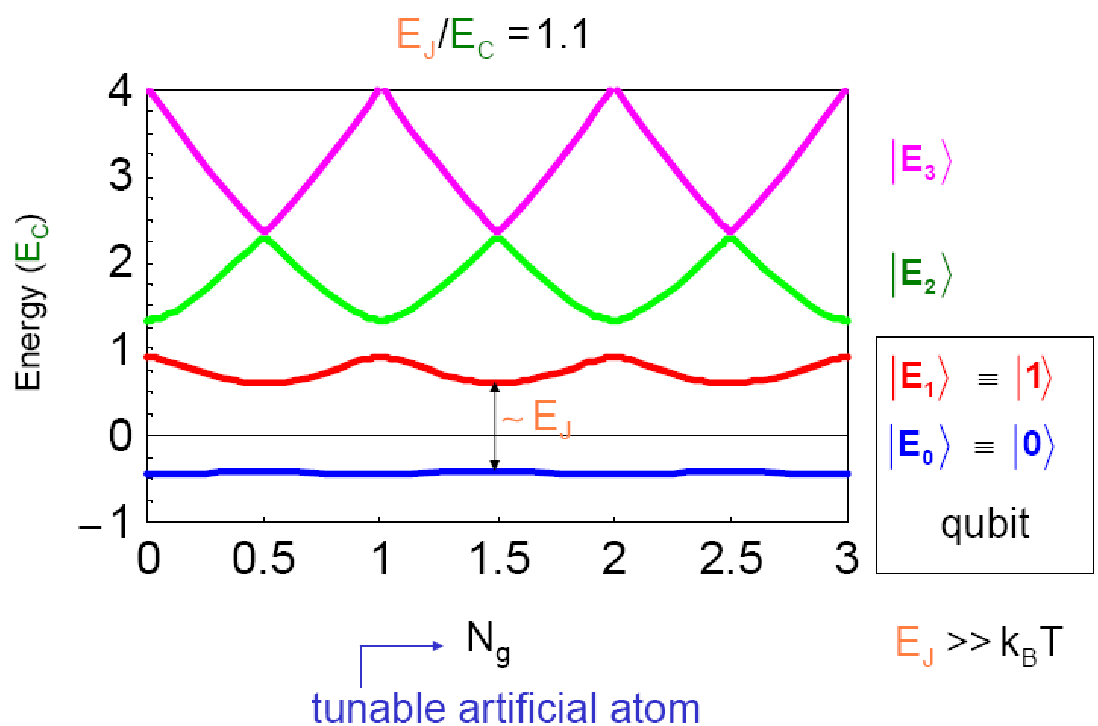
energy level diagram for E_J=0:

- energy bands are formed
- bands are periodic in N_g

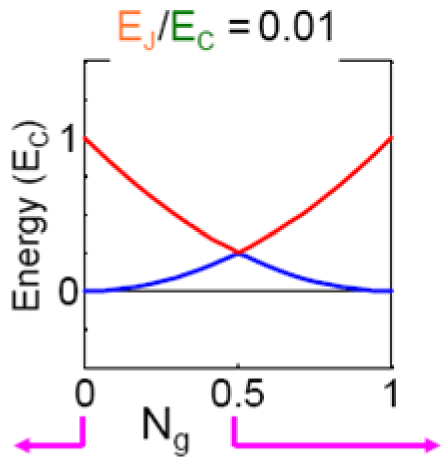


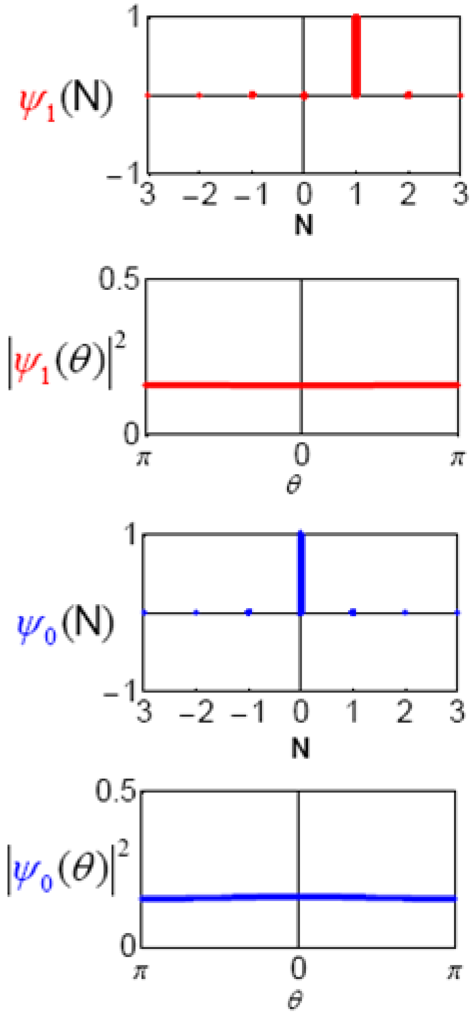
energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy



Charge and Phase Wave Functions ($E_J \ll E_C$)



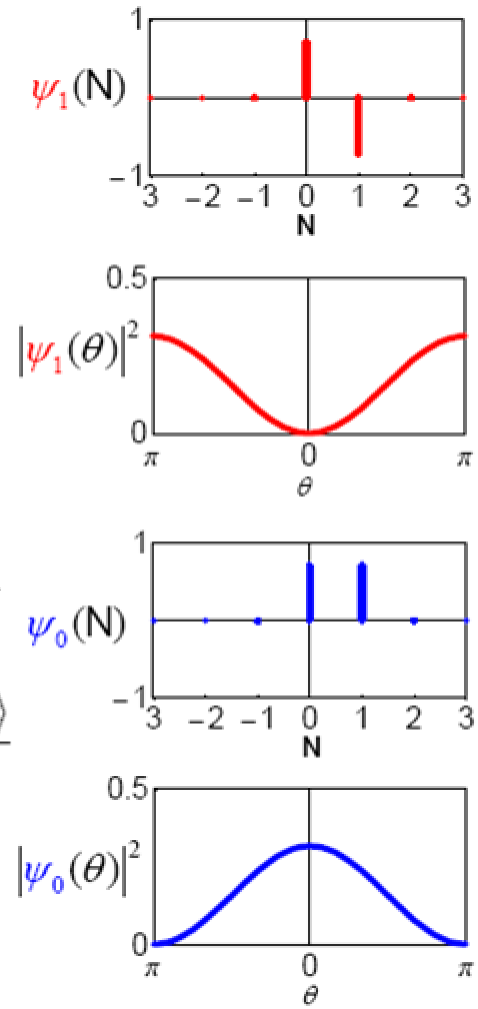


$$|\psi_1\rangle \approx |N=1\rangle$$

$$|\psi_0\rangle \approx |N=0\rangle$$

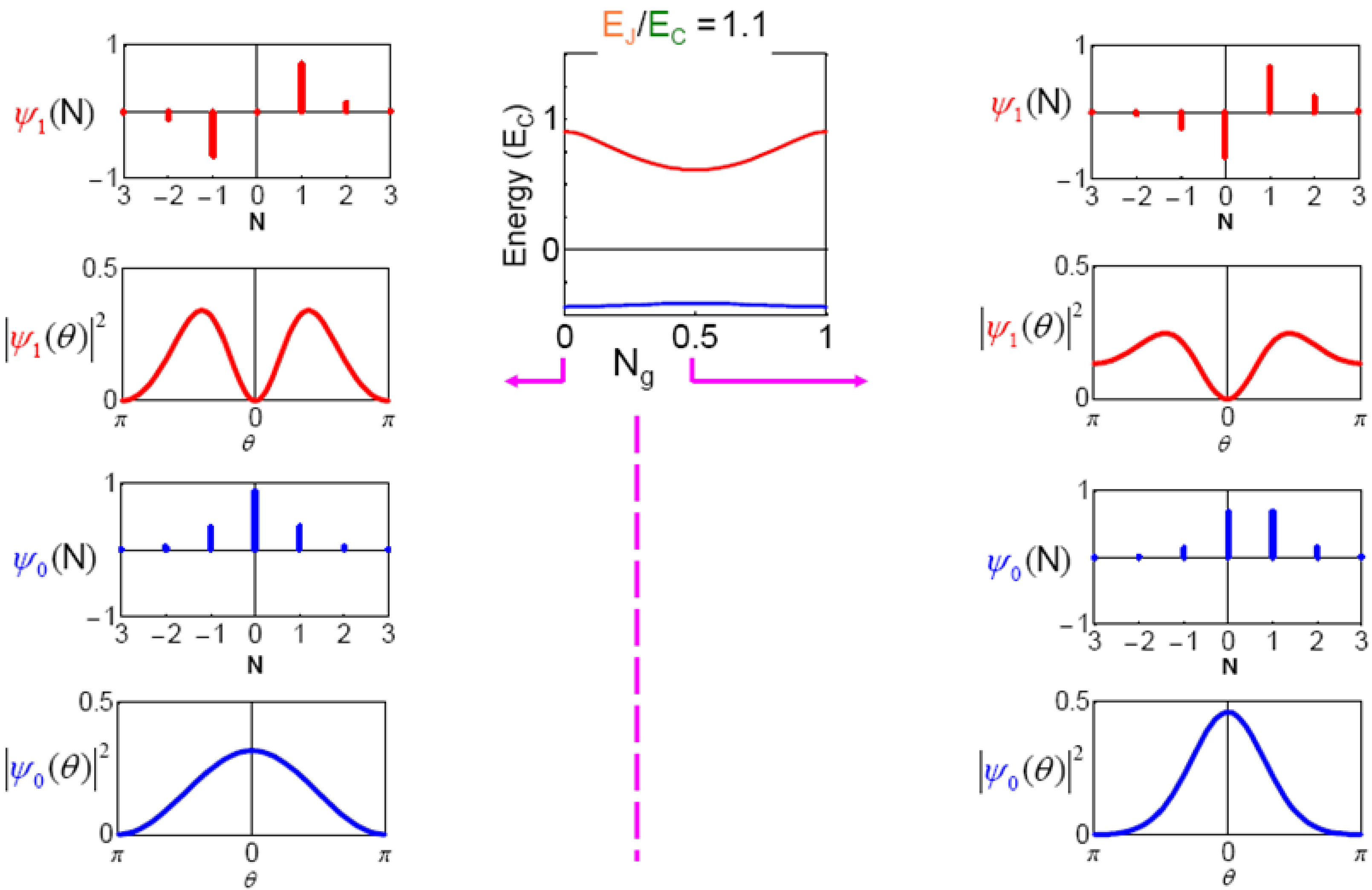
$$|\psi_1\rangle \approx \frac{|N=0\rangle - |N=1\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle \approx \frac{|N=0\rangle + |N=1\rangle}{\sqrt{2}}$$



courtesy CEA Saclay

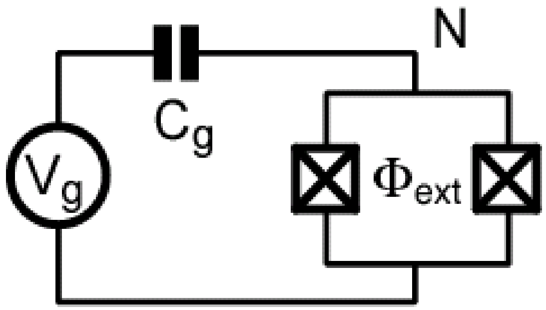
Charge and Phase Wave Functions ($E_J \sim E_C$)



courtesy CEA Saclay

Tuning the Josephson Energy

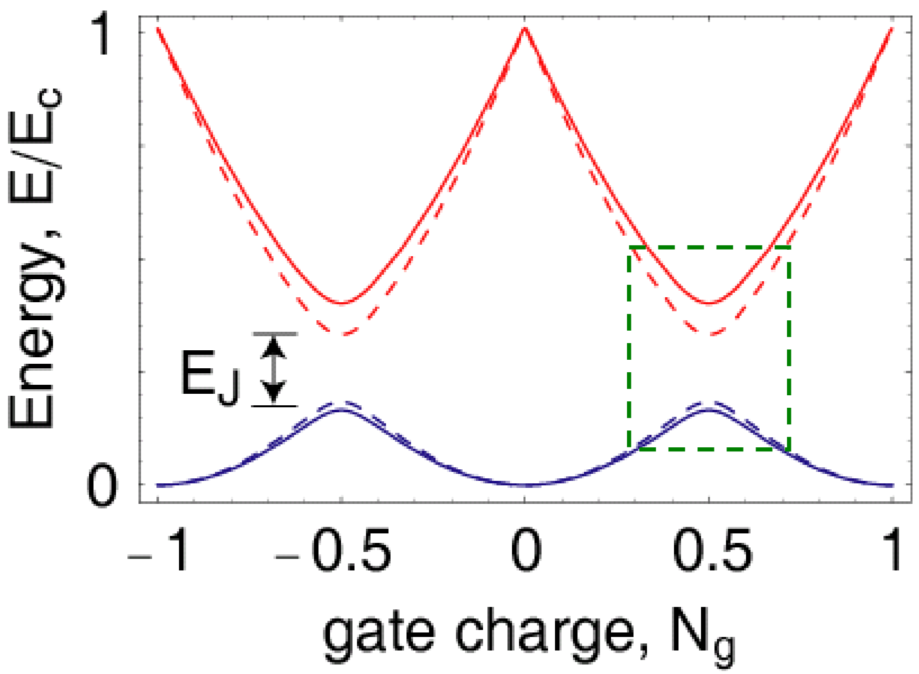
split Cooper pair box in perpendicular field



$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_{J,max}\cos\left(\pi\frac{\phi_{ext}}{\phi_0}\right)\cos(\hat{\delta})$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\phi_{\rm ext}}{\phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* 77, 1208 (1989)

Two-State Approximation

$$\hat{H}_{\mathrm{CPB}} = \hat{H}_{\mathrm{el}} + \hat{H}_{\mathrm{J}} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$$

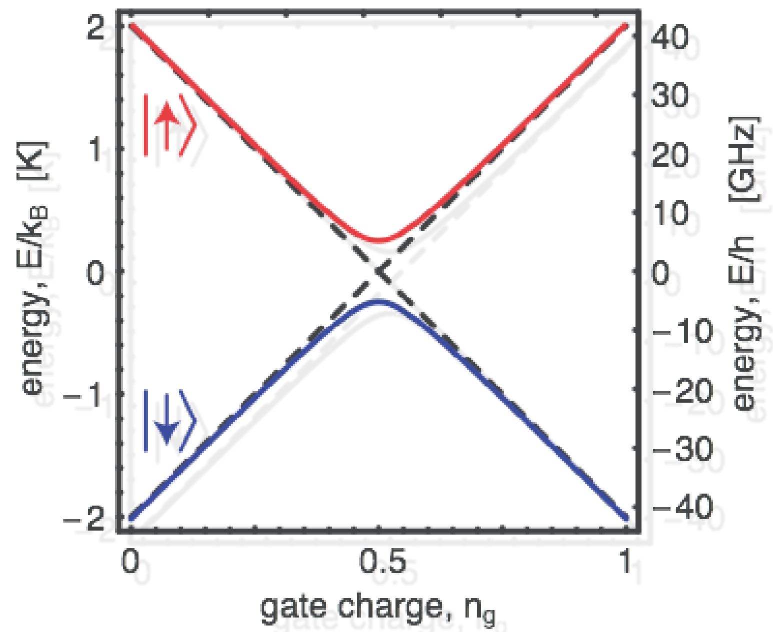
$$\hat{H}_{\mathrm{CPB}} = \sum_N \left[E_C(N - N_g)^2 \left|N\right\rangle \left\langle N\right| - \frac{E_{\mathrm{J}}}{2} \left(\left|N\right\rangle \left\langle N+1\right| + \left|N+1\right\rangle \left\langle N\right| \right) \right]$$

Restricting to a two-charge Hilbert space:

$$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - \hat{\sigma}_z}{2}$$

$$\cos\hat{\delta} = \frac{\hat{\sigma}_x}{2}$$

$$\begin{aligned} \hat{H} &= -\frac{E_C}{2}(1 - 2N_g)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x \\ &= -\frac{1}{2}(E_{\mathrm{el}}\hat{\sigma}_z + E_J\hat{\sigma}_x) \end{aligned}$$

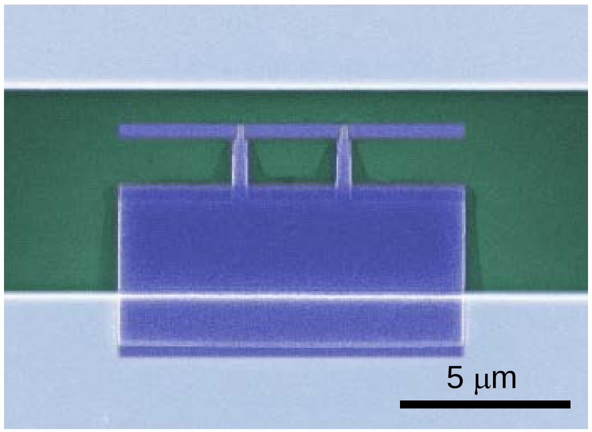


Shnirman *et al., Phys. Rev. Lett.* 79, 2371 (1997)

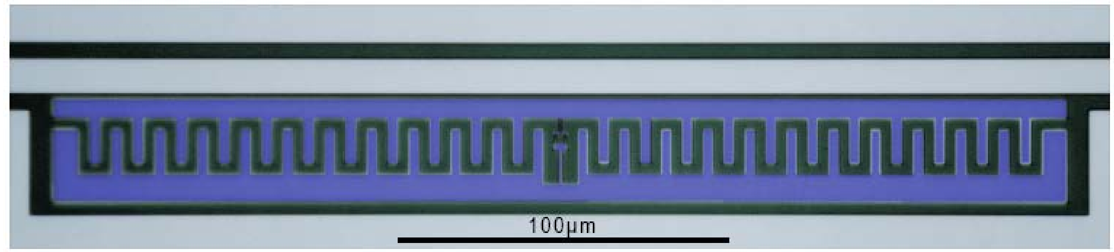
A Variant of the Cooper Pair Box

a Cooper pair box with a small charging energy

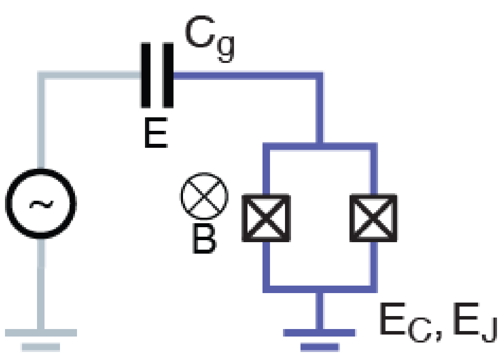
standard CPB:

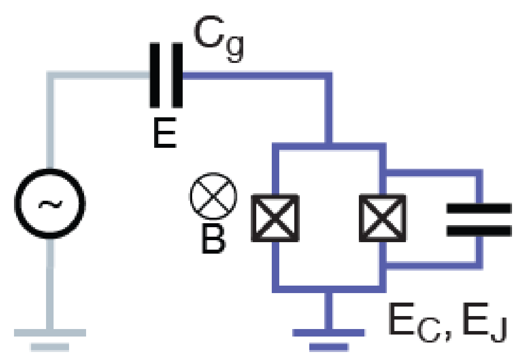


Transmon qubit:



circuit diagram:

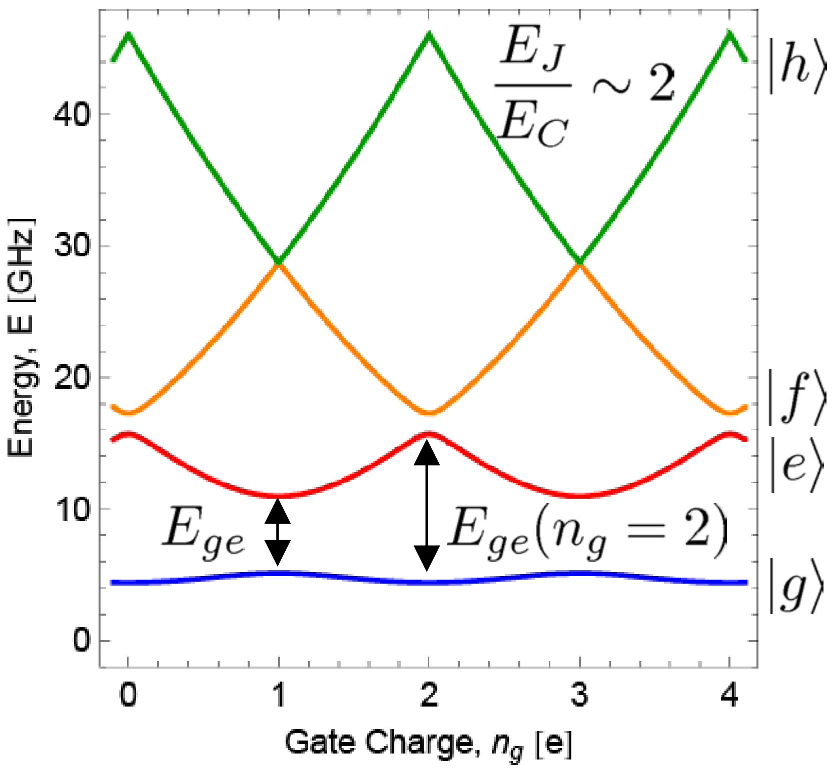




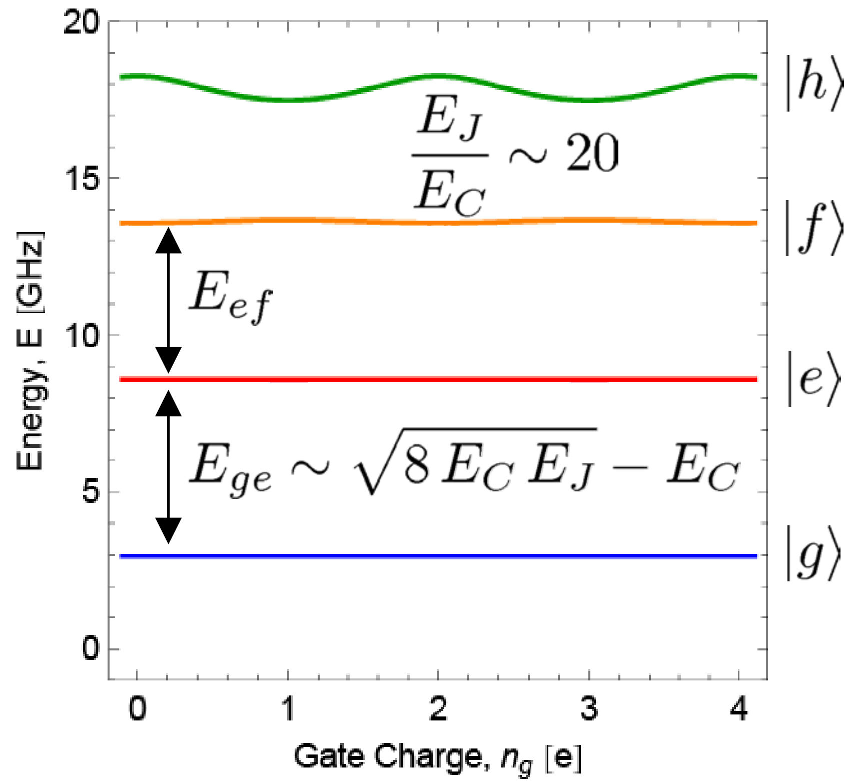
J. Koch *et al.*, Phys. Rev. A 76, 042319 (2007)
J. Schreier *et al.*, Phys. Rev. B 77, 180502 (2008)

The Transmon: A Charge Noise Insensitive Qubit

Cooper pair box energy levels:



Transmon energy levels:



dispersion:

$$\epsilon = E_{ge}(n_g = 1) - E_{ge}(n_g = 2)$$

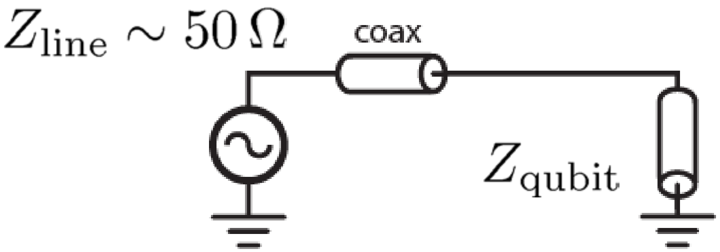
relative anharmonicity:

$$\alpha_r = \frac{E_{ef} - E_{ge}}{E_{ge}}$$

J. Koch *et al.*, Phys. Rev. A 76, 042319 (2007)

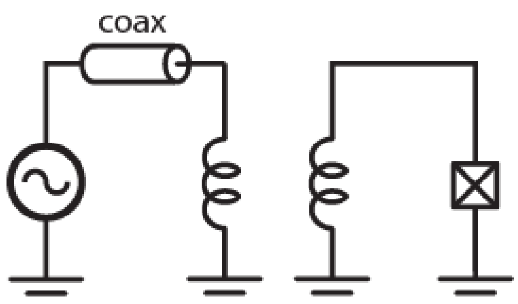
Control of Coupling to Electromagnetic Environment

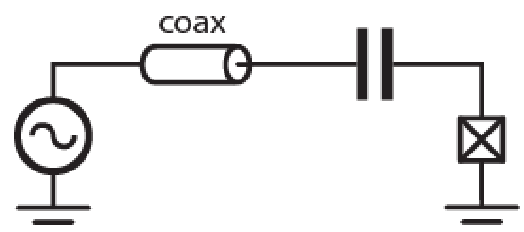
coupling to environment (bias wires):



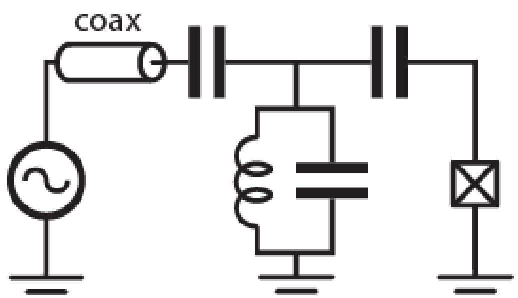
decoherence due to energy relaxation stimulated by the vacuum fluctuations of the environment (spontaneous emission)

Decoupling schemes using non-resonant impedance transformers ...



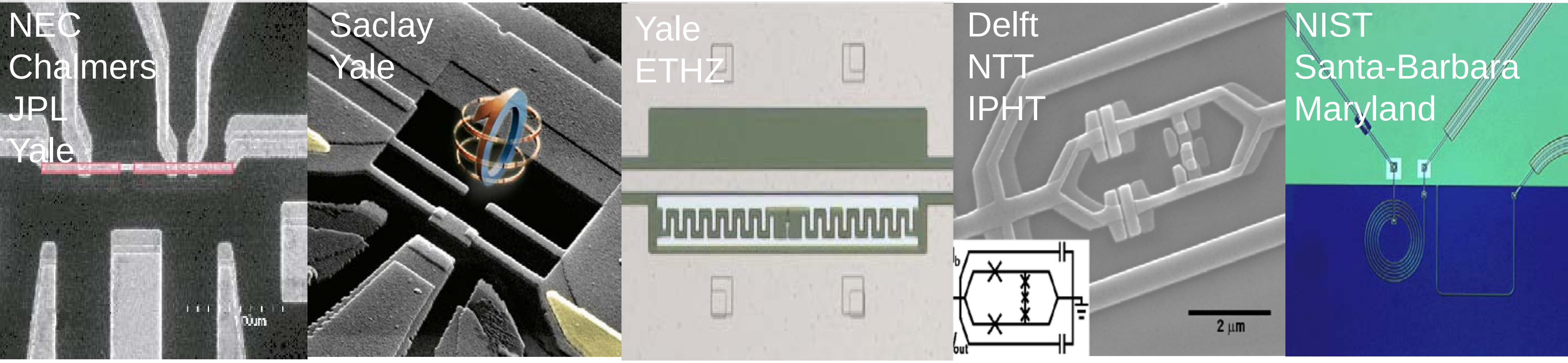


... or resonant impedance transformers



control spontaneous emission by circuit design

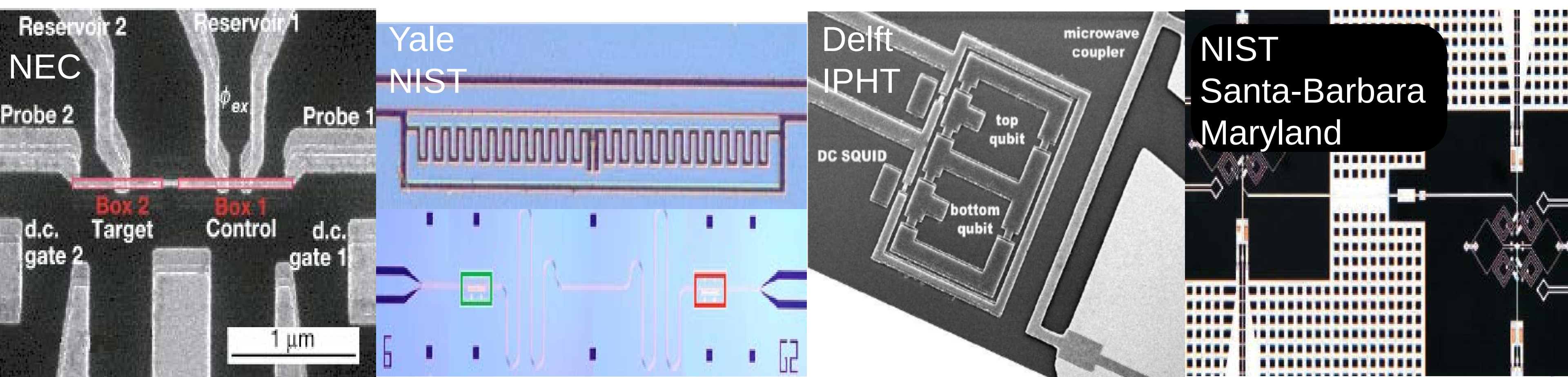
Realizations of Superconducting Artificial Atoms



'artificial atoms' -- single superconducting qubits

review:
J. Clarke and F. Wilhelm
Nature 453, 1031 (2008)

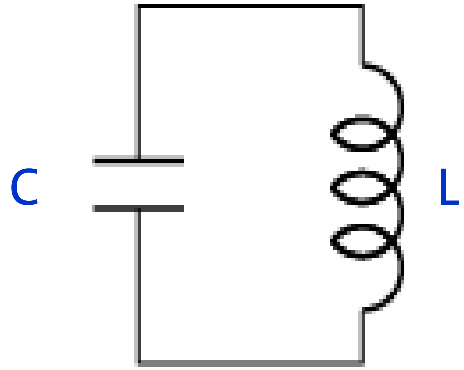
'artificial molecules' -- coupled superconducting qubits



Realizations of Harmonic Oscillators

Superconducting Harmonic Oscillators

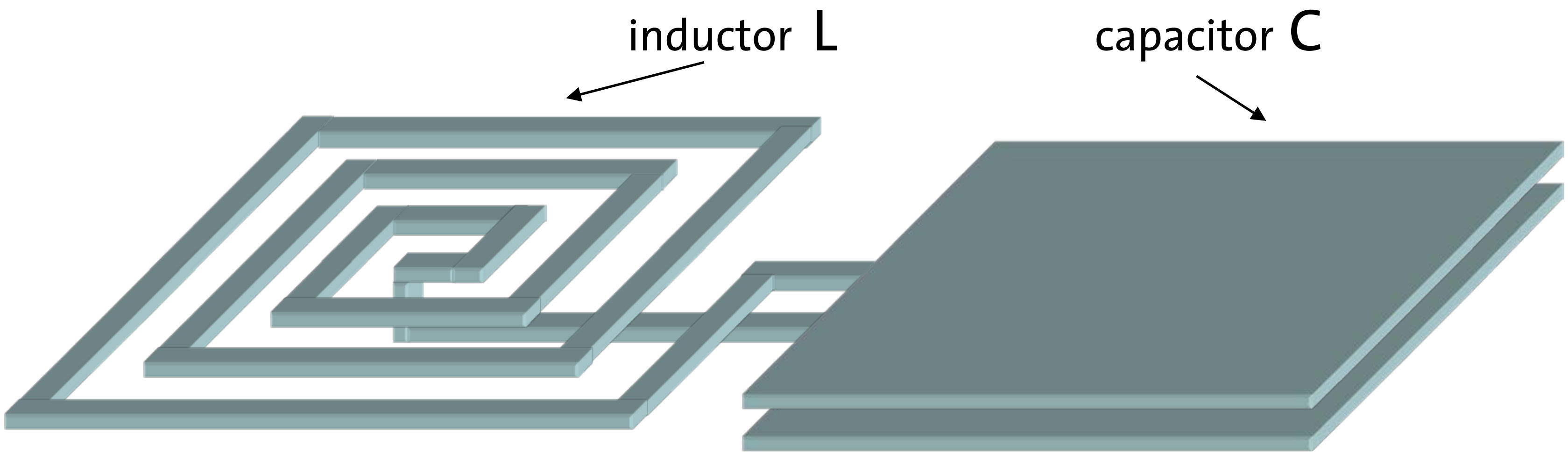
a simple electronic circuit:



- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm
- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF
- resonance frequency

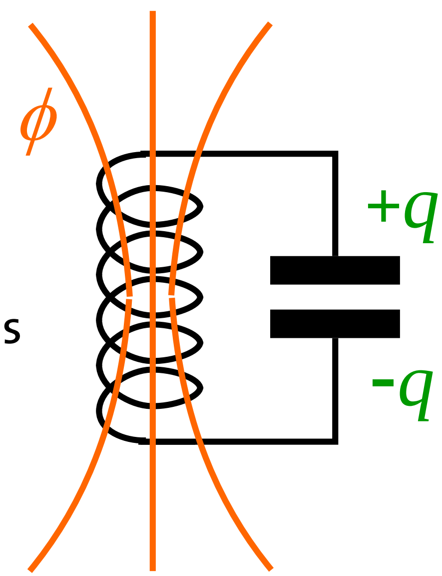
$$\frac{1}{2\pi\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

Realization of H.O.: Lumped Element Resonator



a harmonic oscillator

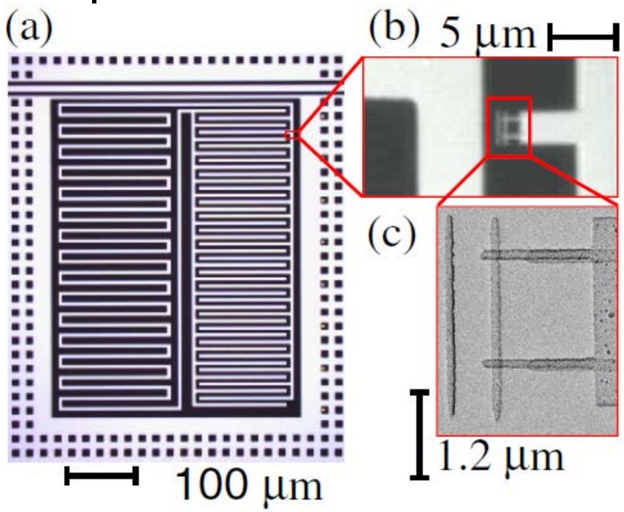
currents and magnetic fields



charges and electric fields

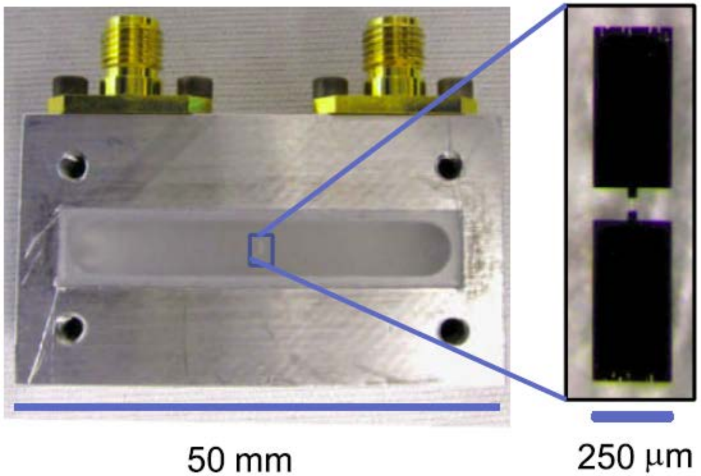
Types of Superconducting Harmonic Oscillators

lumped element resonator:



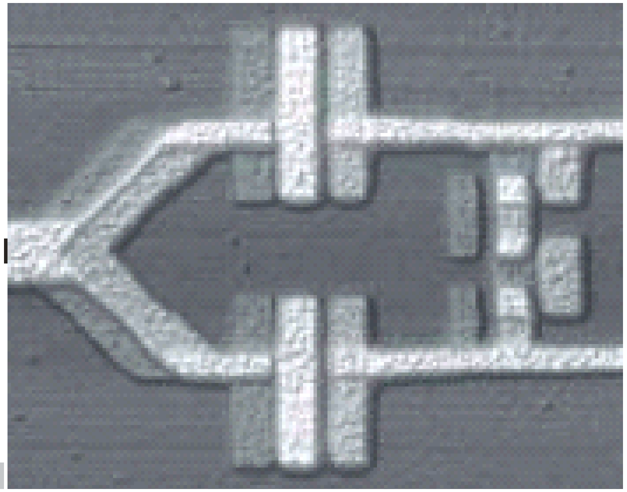
Z. Kim *et al.*, *PRL* 106, 120501 (2011)

3D cavity:



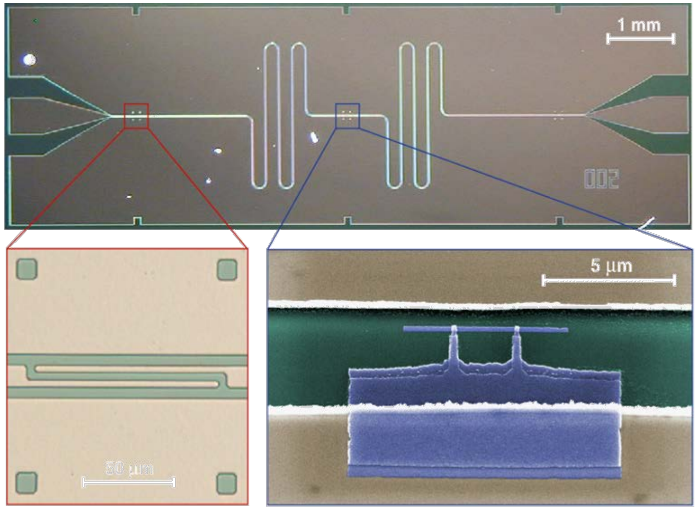
H. Paik *et al.*, *PRL* 107, 240501 (2011)

weakly nonlinear junction:



I. Chiorescu *et al.*, *Nature* 431, 159 (2004)

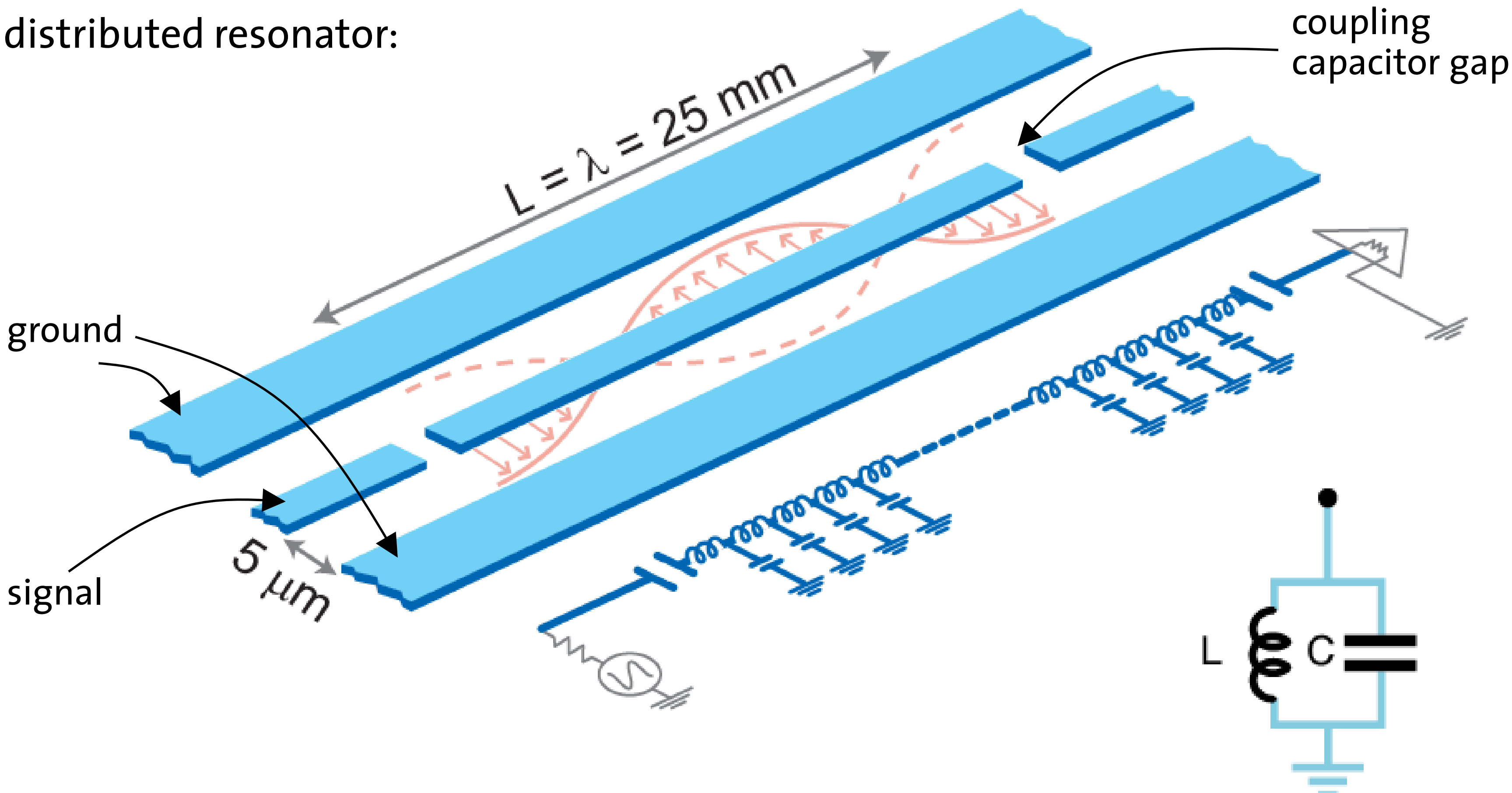
planar transmission line resonator:



A. Wallraff *et al.*, *Nature* 431, 162 (2004)

Realization of H.O.: Transmission Line Resonator

distributed resonator:

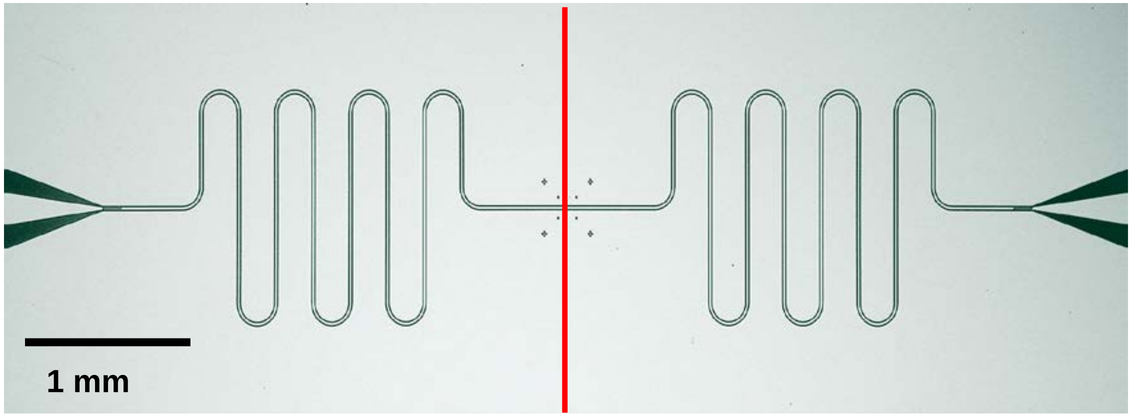


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

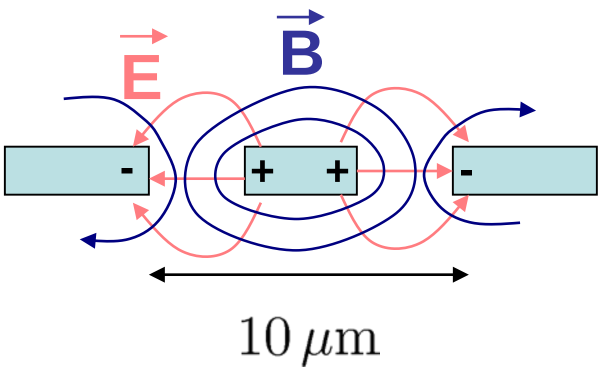
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* 104, 113904 (2008)

Realization of Transmission Line Resonator

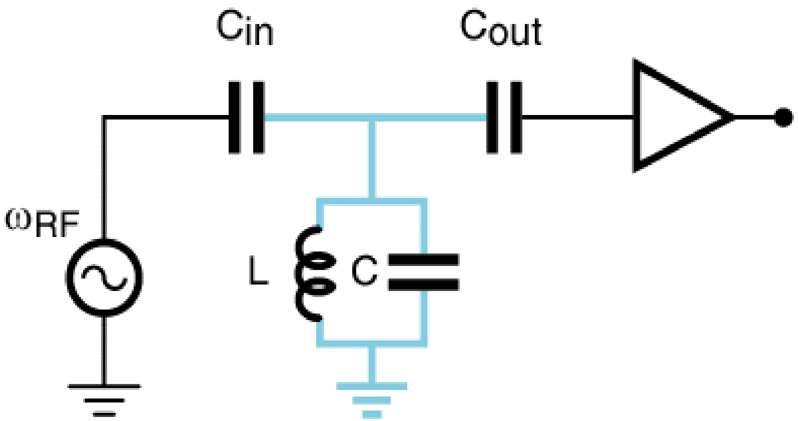
coplanar waveguide:



cross-section of transm. line (TEM mode):



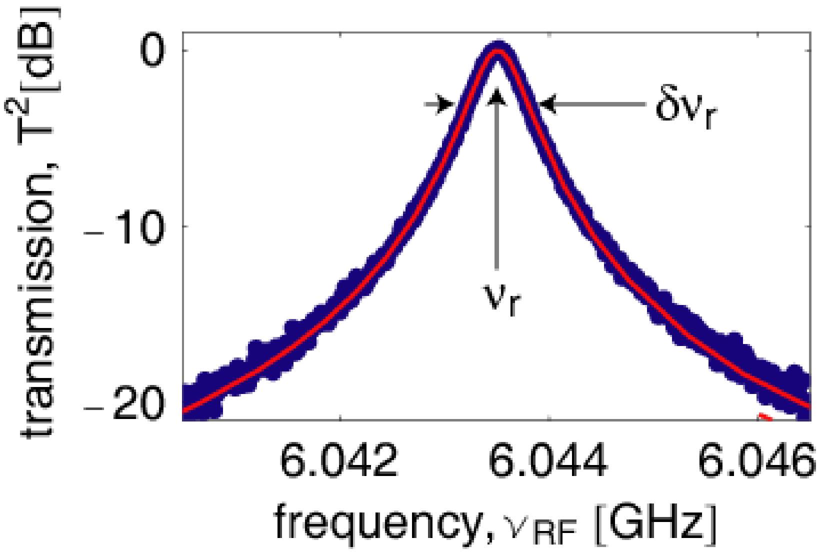
measuring the resonator:

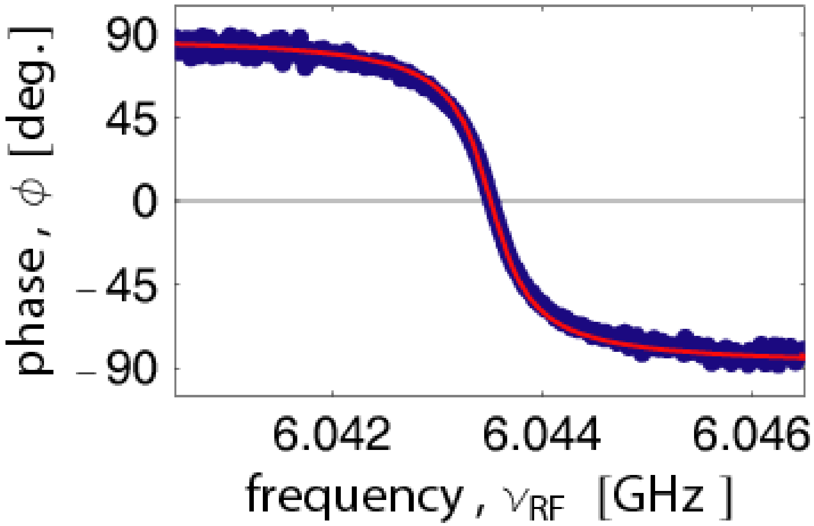


photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

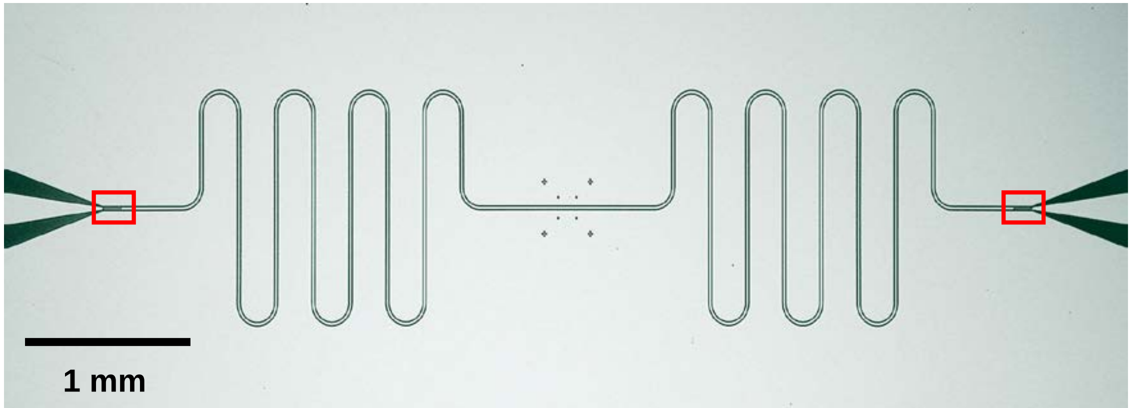
photon decay rate:

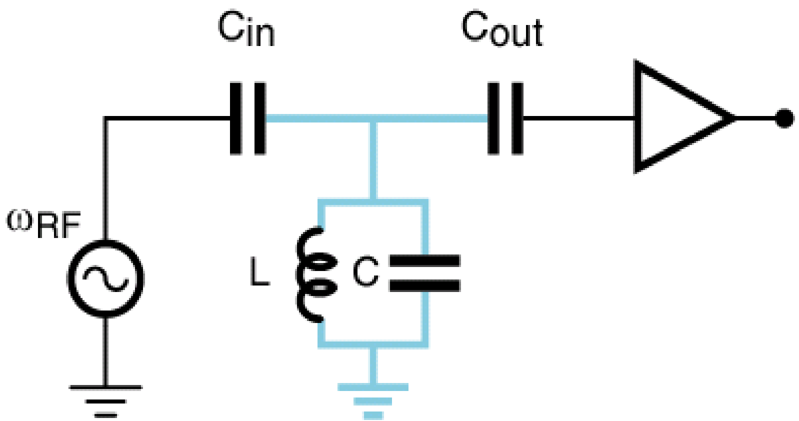
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

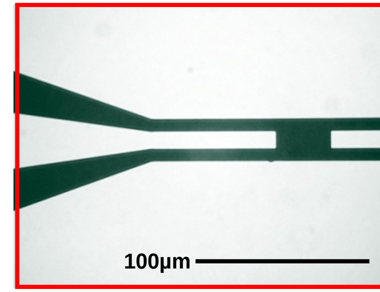
photon lifetime:

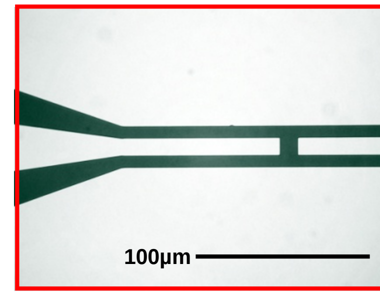
$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

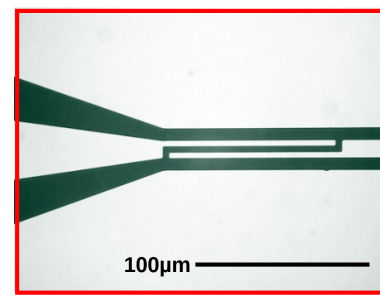
Controlling the Photon Life Time

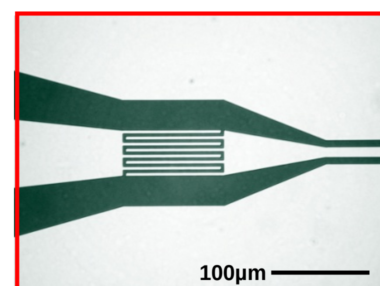






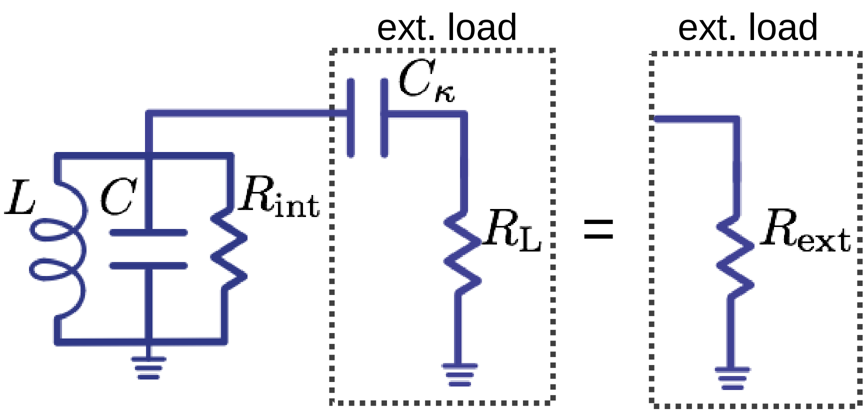


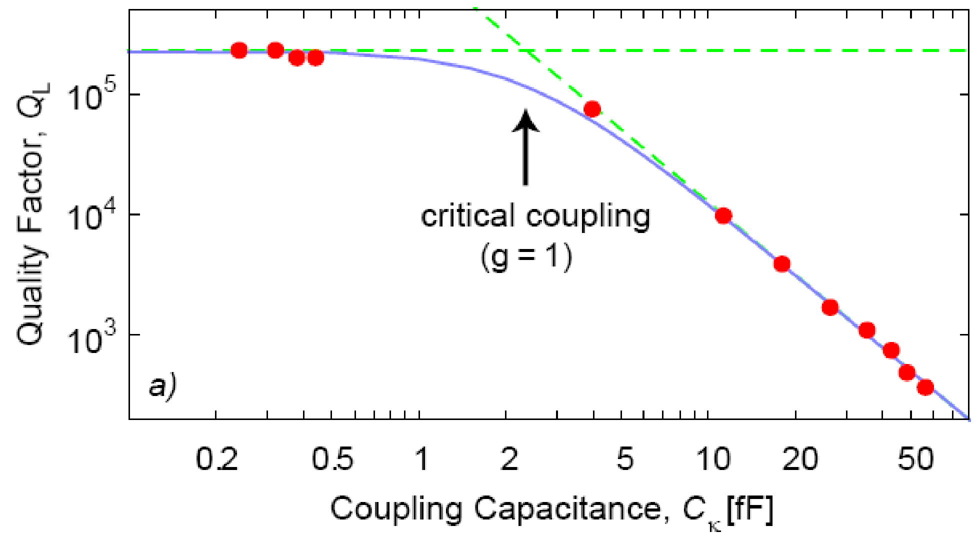


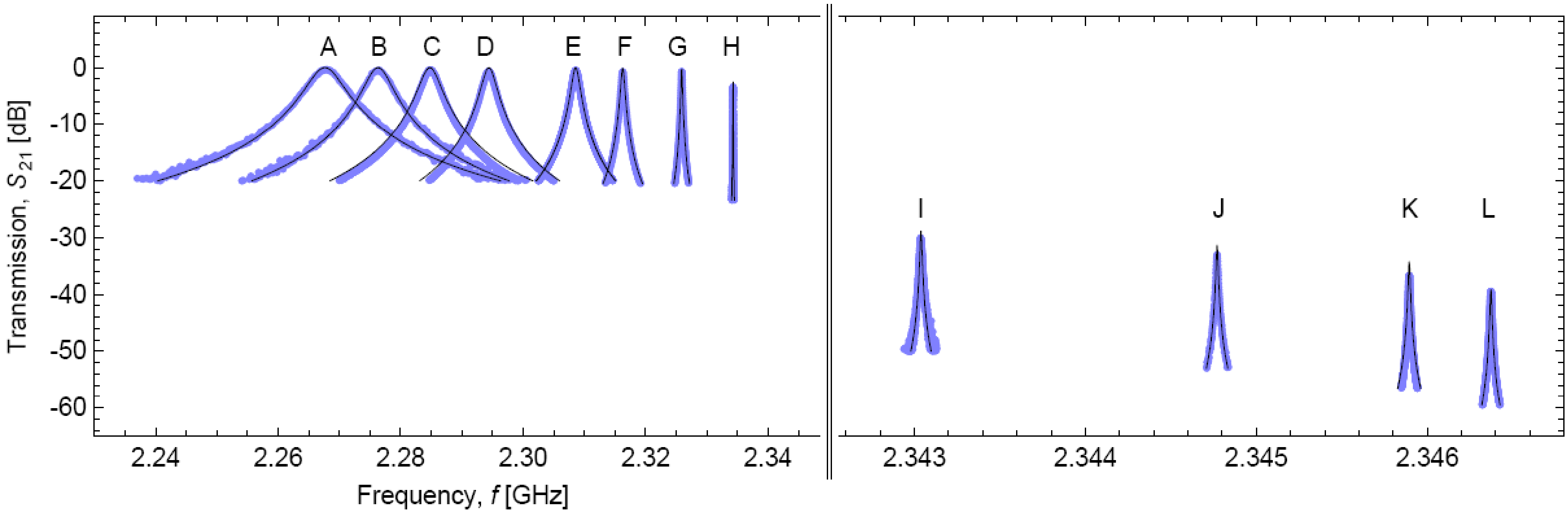


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Quality Factor Measurement







M. Goeppl *et al.*, *J. Appl. Phys.* 104, 113904 (2008)