

Quantum Optics with Microwave Photons

Andreas Wallraff (*ETH Zurich*)

www.qudev.ethz.ch

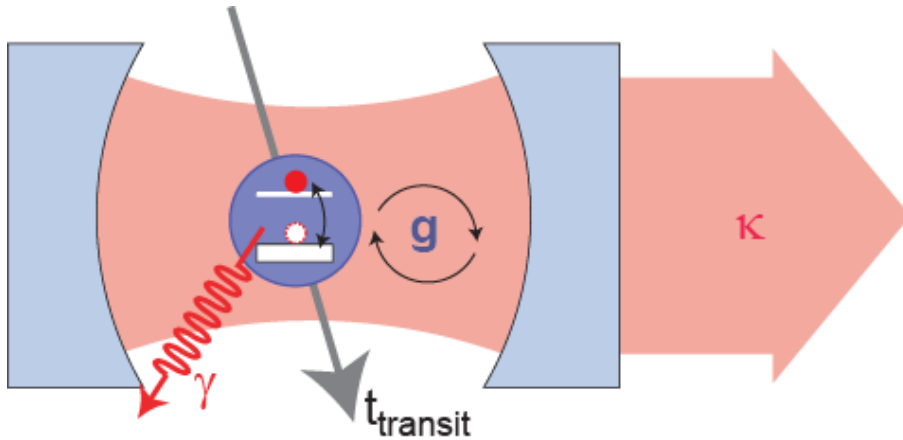
Team: A. Abdumalikov, M. Allan, J. Basset, M. Baur, S. Berger, C. Eichler, A. Fedorov, S. Filipp, T. Frey, P. Kurpiers, C. Lang, J. Mlynek, M. Mondal, M. Oppliger, M. Pechal, G. Puebla-Hellmann, Y. Salathe, M. Stammeier, L. Steffen, A. Stockklauser, T. Thiele, A. van Loo (*ETH Zurich*)

Collaborations with:

A. Blais (*Sherbrooke, Canada*), M. da Silva (*Raytheon, USA*), K. Ensslin, T. Ihn, F. Merkt, V. Wood (*ETH Zurich*)



Cavity QED with Superconducting Circuits



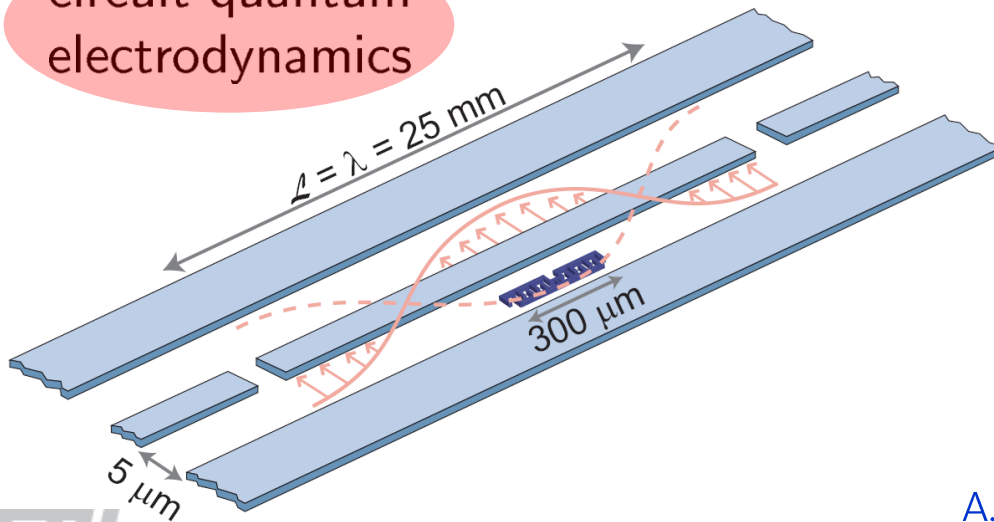
coherent interaction of photons with quantum two-level systems ...

J. M. Raimond *et al.*, *Rev. Mod. Phys.* **73**, 565 (2001)

S. Haroche & J. Raimond, *oup Oxford* (2006)

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

circuit quantum electrodynamics



Properties:

- strong coupling in solid state sys.
- 'easy' to fabricate and integrate

Research directions:

- quantum optics
- quantum information
- hybrid quantum systems

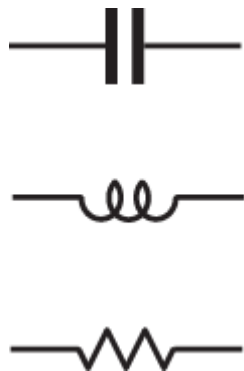
A. Blais, *et al.*, *PRA* **69**, 062320 (2004)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

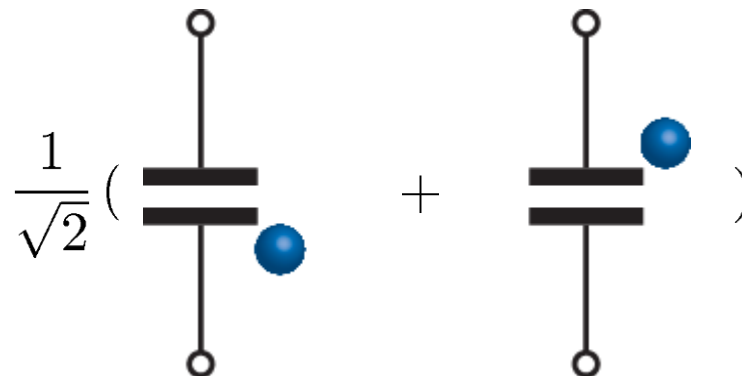
R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Classical and Quantum Electronic Circuit Elements

basic circuit elements:



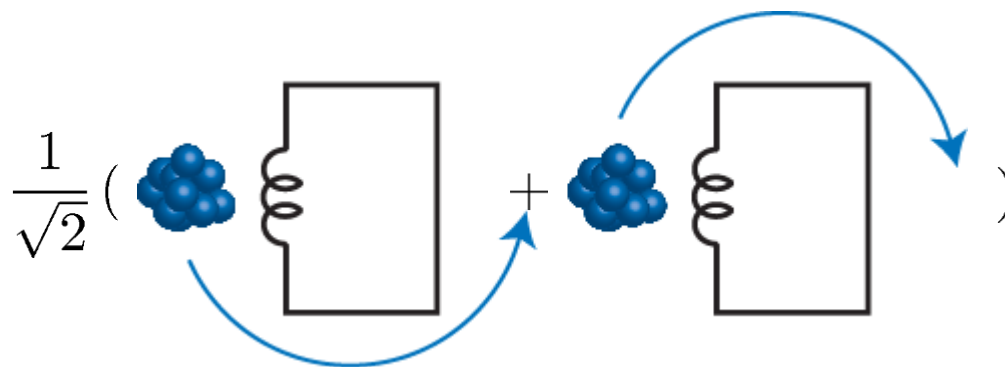
charge on a capacitor:



quantum superposition states of:

- charge q
- flux ϕ

current or magnetic flux in an inductor:

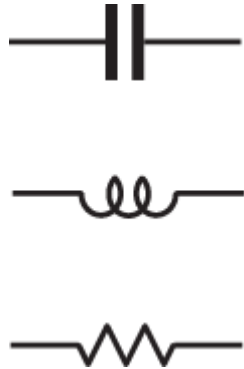


commutation relation (c.f. x, p):

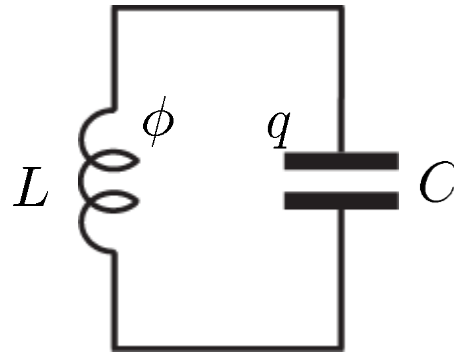
$$[\hat{\phi}, \hat{q}] = i\hbar$$

Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5 \text{ GHz}$$

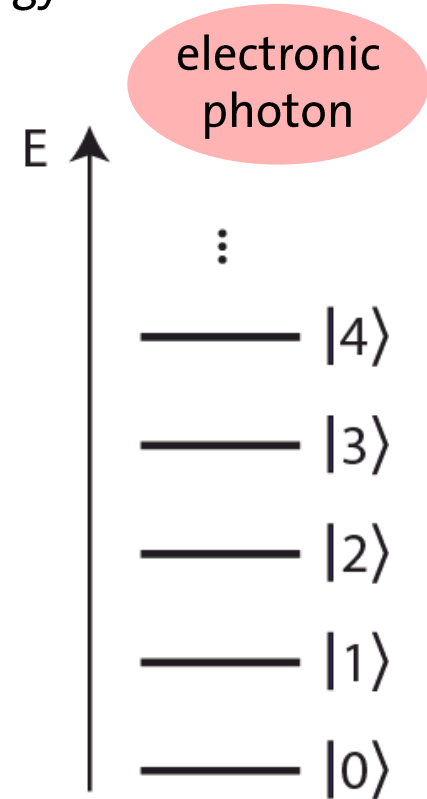
classical physics:

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

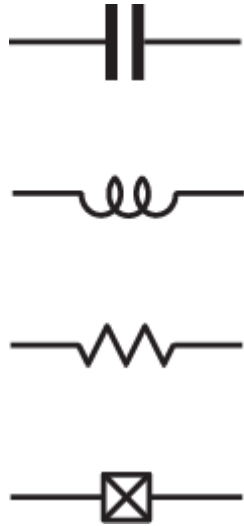
$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \quad [\hat{\phi}, \hat{q}] = i\hbar$$

energy:



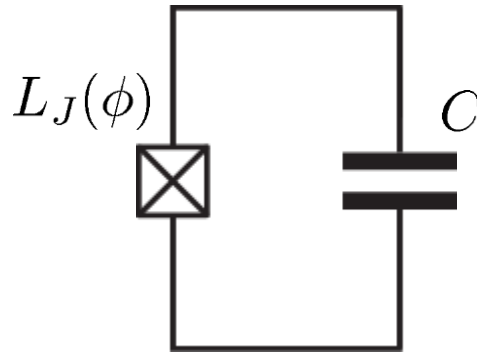
Constructing Non-Linear Quantum Electronic Circuits

circuit elements:



Josephson junction:
a non-dissipative nonlinear
element (inductor)

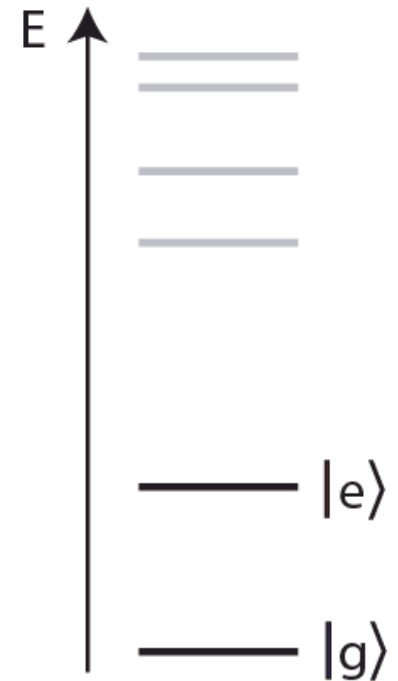
anharmonic oscillator:



$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi} \right)^{-1}$$

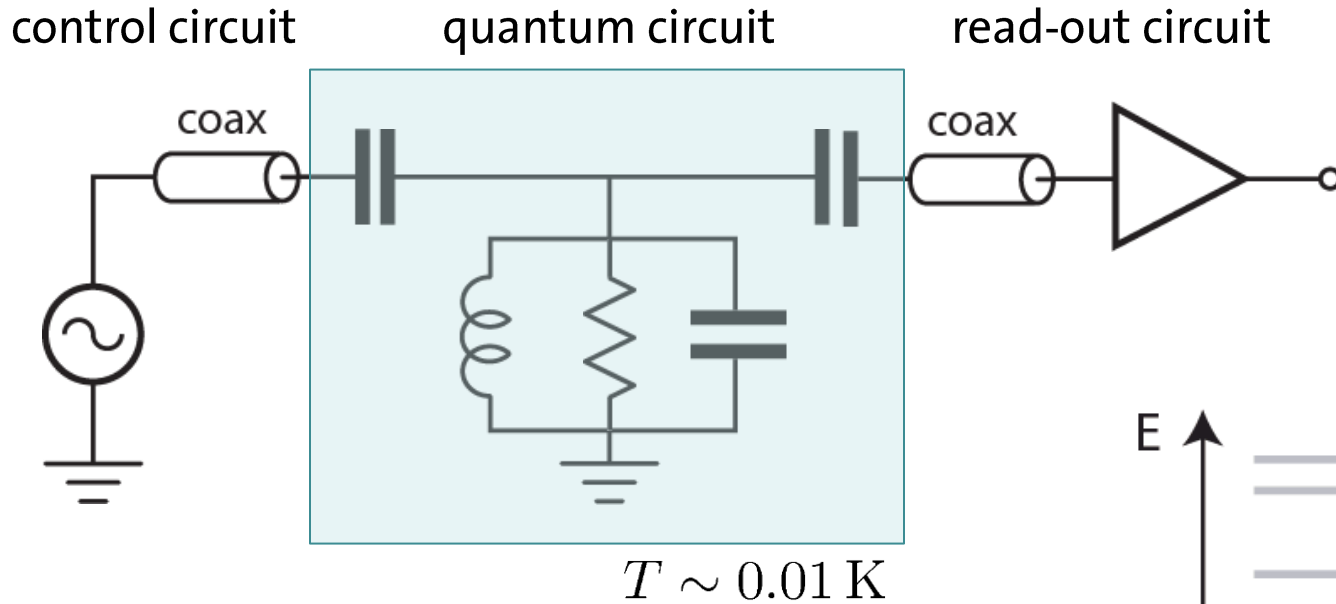
$$= \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

non-linear energy
level spectrum:



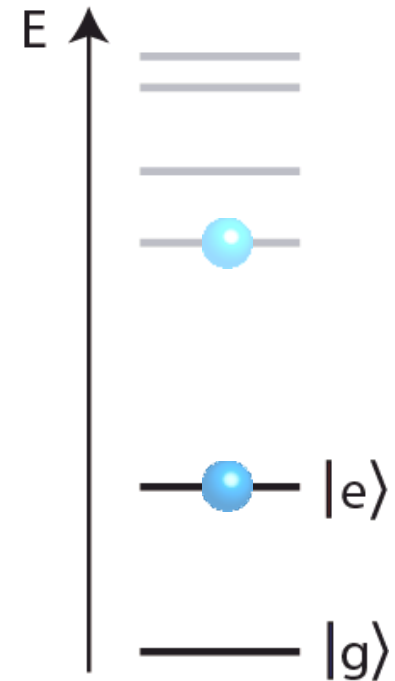
electronic
artificial atom

How to Operate Circuits in the Quantum Regime?

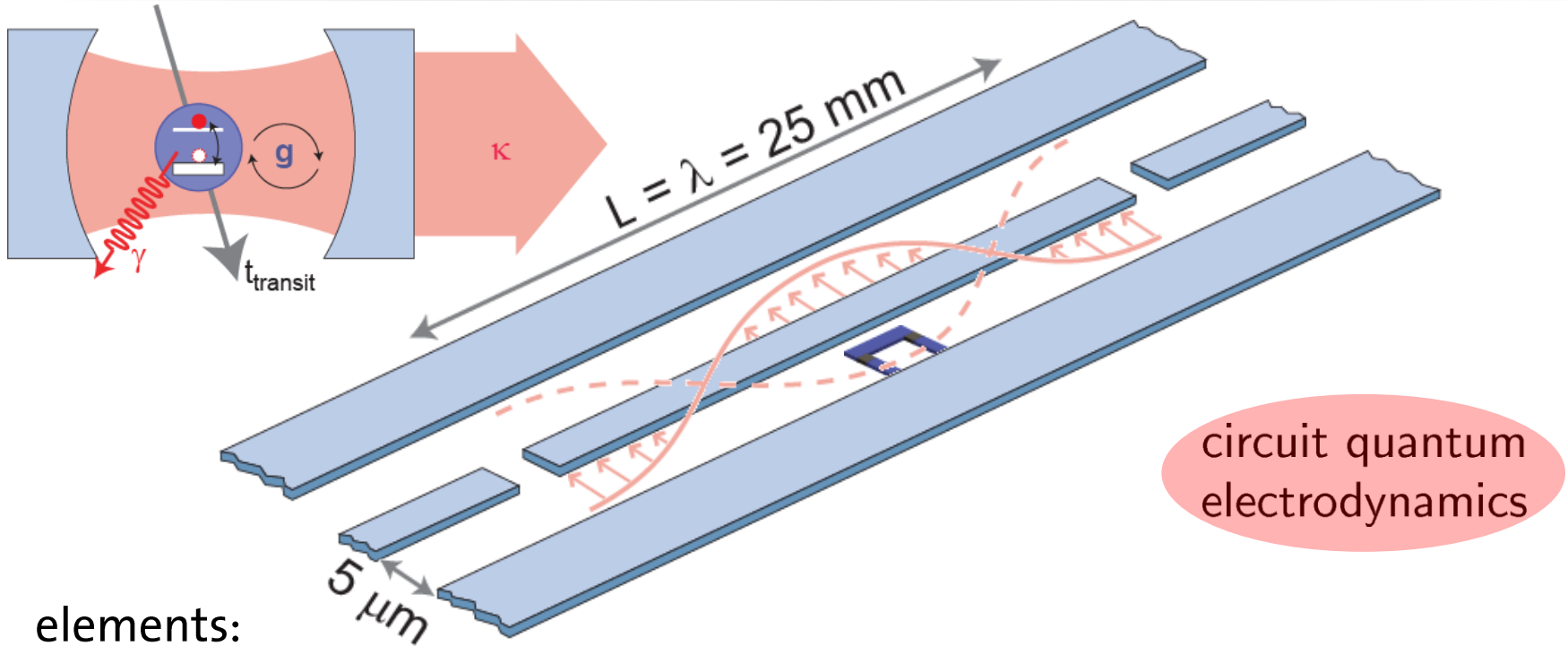


recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment



Cavity QED with Superconducting Circuits



elements:

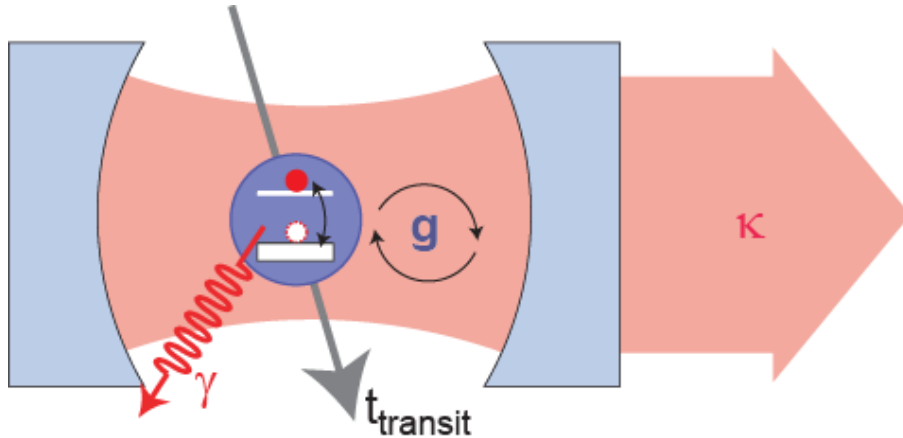
- the cavity: a superconducting 1D transmission line resonator with **large vacuum field** E_0 and **long photon life time** $1/\kappa$
- the atom: a superconducting qubit with **large dipole moment** d and **long coherence time** $1/\gamma$ and **fixed position** ...
- ... or any microscopic/macroscopic quantum element or ensemble thereof with an appreciable dipole moment

A. Blais, et al., *PRA* **69**, 062320 (2004)

A. Wallraff et al., *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

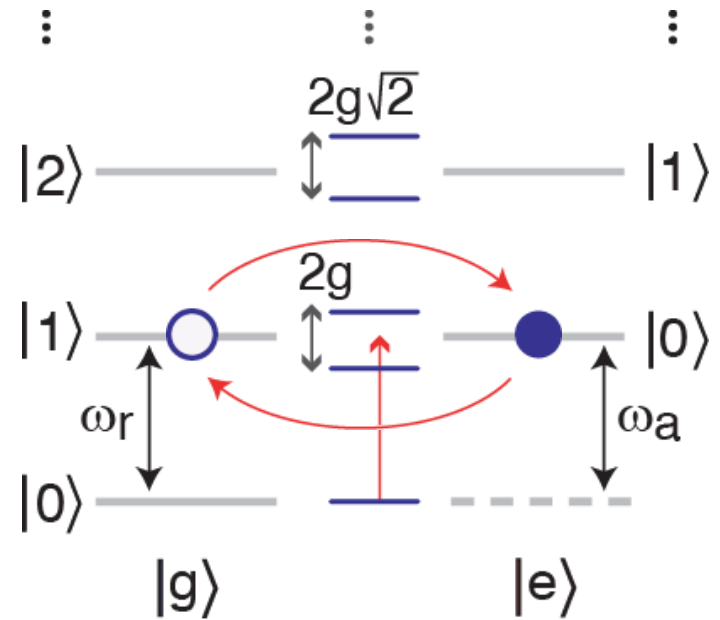
Cavity Quantum Electrodynamics



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}}$)

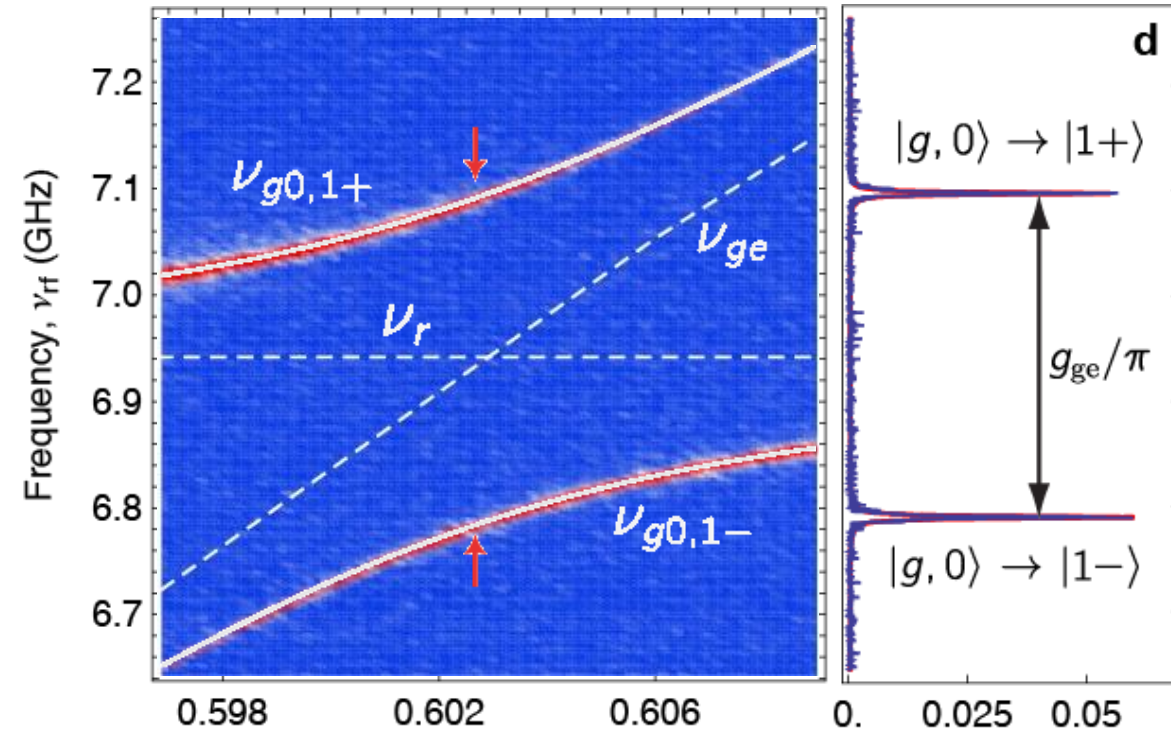


Jaynes-Cummings Ladder

Resonant Vacuum Rabi Mode Splitting ...

... with one photon ($n=1$):

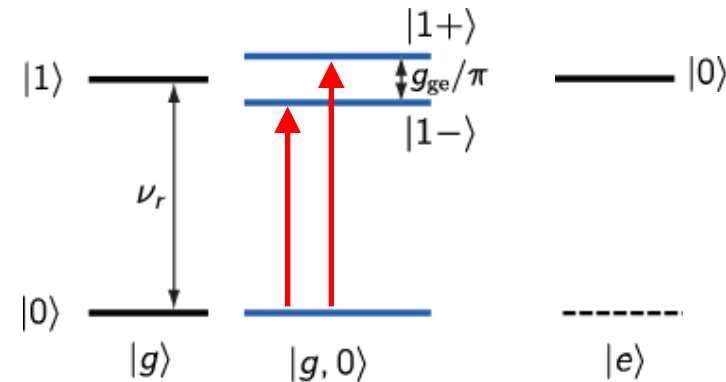
very strong coupling:



$$g_{ge}/\pi = 308 \text{ MHz}$$

$$\kappa, \gamma < 1 \text{ MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff et al., *Nature (London)* **431**, 162 (2004)

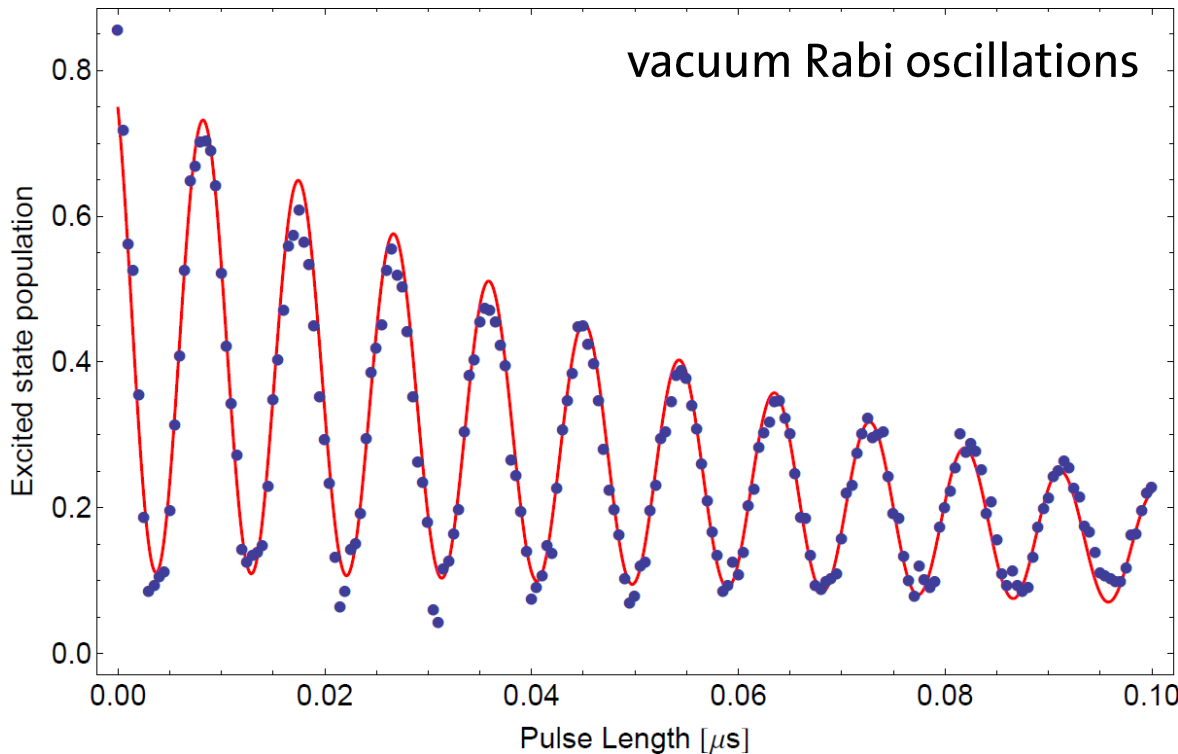
this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Resonant Vacuum Rabi Mode Splitting ...

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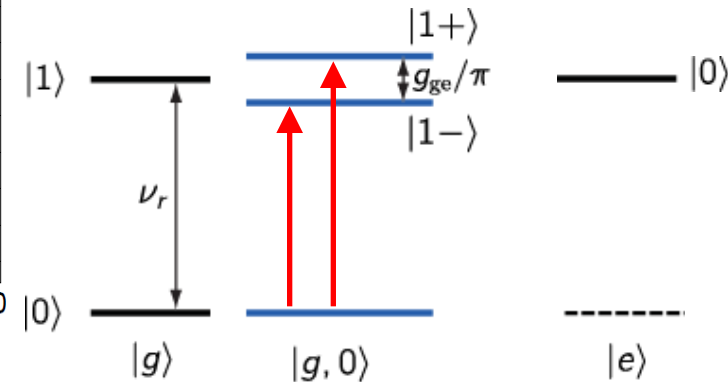
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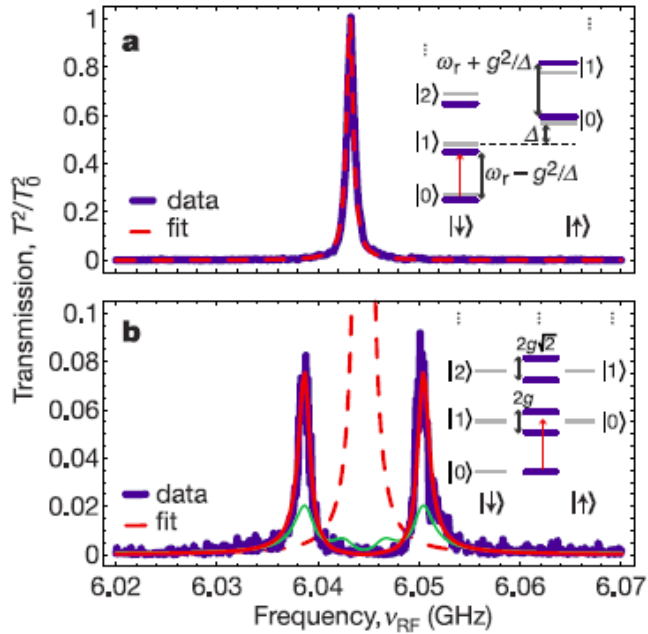
forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff et al., *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Quantum Optics with Supercond. Circuits



Strong Coherent Coupling

Chiorescu *et al.*, *Nature* **431**, 159 (2004)

Wallraff *et al.*, *Nature* **431**, 162 (2004)

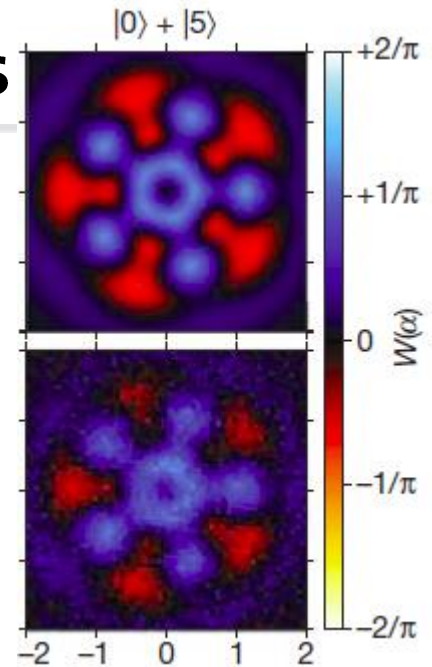
Schuster *et al.*, *Nature* **445**, 515 (2007)

Root n Nonlinearities

Fink *et al.*, *Nature* **454**, 315 (2008)

Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)

Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

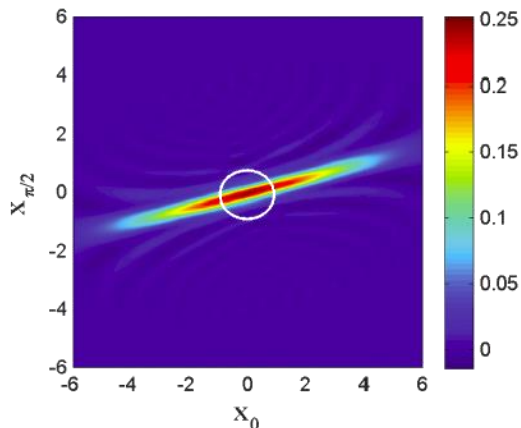


Microwave Fock and Cat States

Hofheinz *et al.*, *Nature* **454**, 310 (2008)

Hofheinz *et al.*, *Nature* **459**, 546 (2009)

Kirchmair *et al.*, *Nature* **495**, 205 (2013)



Parametric Amplification & Squeezing

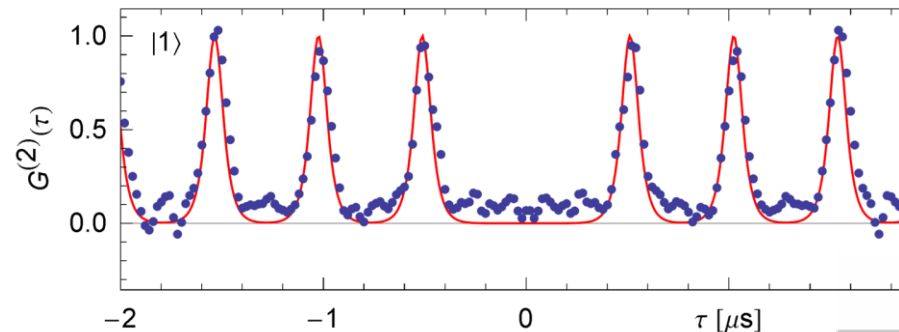
Castellanos-Beltran *et al.*,

Nat. Phys. **4**, 928 (2008)

Single Photons & Correlations

Houck *et al.*, *Nature* **449**, 328 (2007)

Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

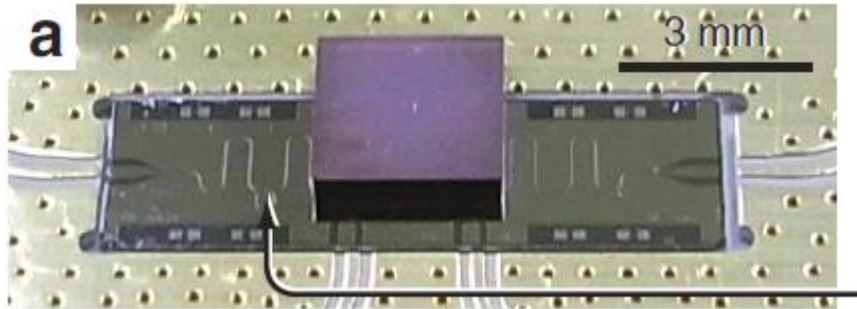


Hybrid Systems with Superconducting Circuits

Spin Ensembles: e.g. NV centers

D. Schuster *et al.*, *PRL* **105**, 140501 (2010)

Y. Kubo *et al.*, *PRL* **105**, 140502 (2010)



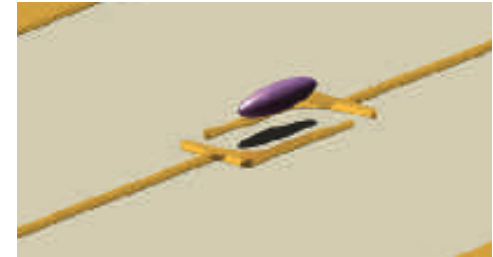
Polar Molecules, Rydberg, BEC

P. Rabl *et al.*, *PRL* **97**, 033003 (2006)

A. Andre *et al.*, *Nat. Phys.* **2**, 636 (2006)

D. Petrosyan *et al.*, *PRL* **100**, 170501 (2008)

J. Verdu *et al.*, *PRL* **103**, 043603 (2009)

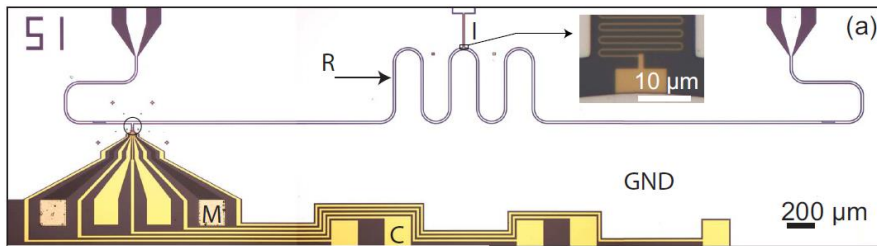


CNT, Gate Defined 2DEG, or nanowire Quantum Dots

M. Delbecq *et al.*, *PRL* **107**, 256804 (2011)

T. Frey *et al.*, *PRL* **108**, 046807 (2012)

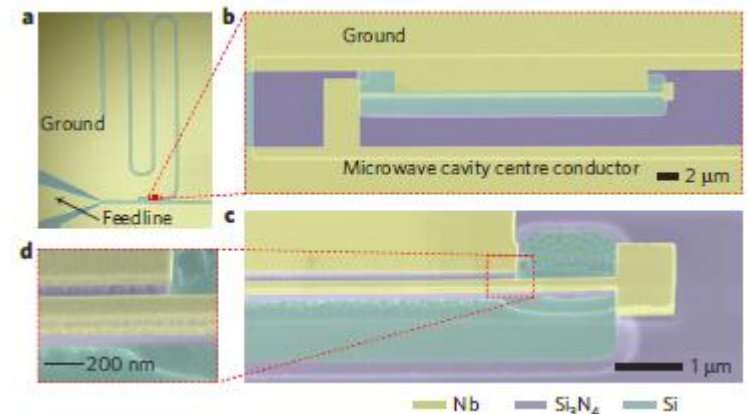
K. Petersson *et al.*, *arXiv:1205.6767* (2012)



Nano-Mechanics

J. Teufel *et al.*, *Nature* **475**, 359 (2011)

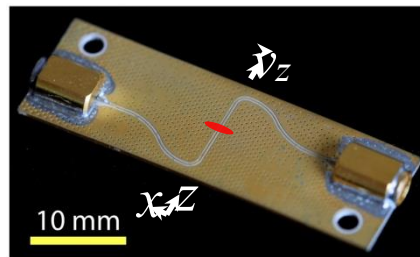
X. Zhou *et al.*, *Nat. Phys.* **9**, 179 (2013)



Rydberg Atoms

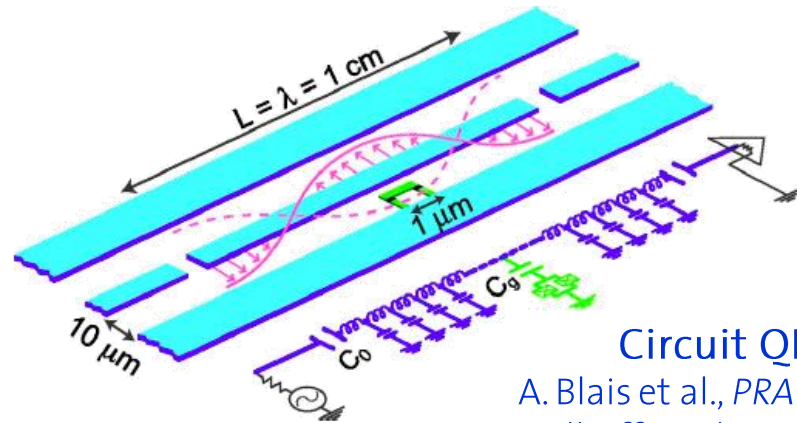
S. Hogan *et al.*, *PRL* **108**,

063004 (2012)



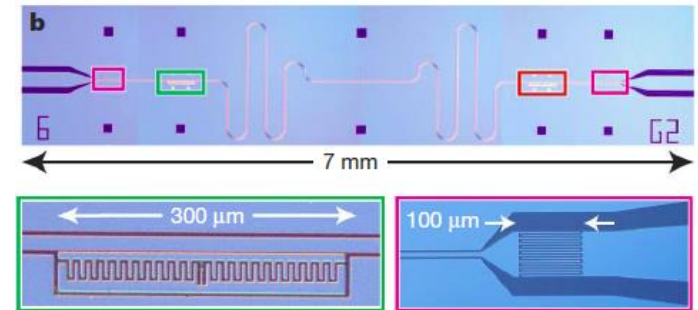
... and many more

Quantum Computing with Superconducting Circuits



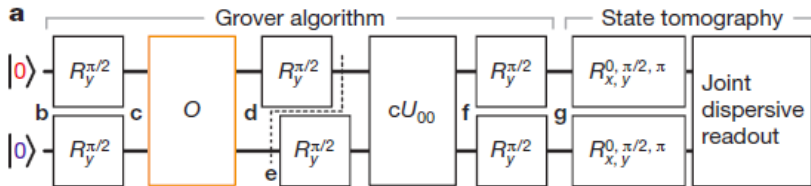
Circuit QED Architecture

A. Blais et al., *PRA* **69**, 062320 (2004)
 A. Wallraff et al., *Nature* **431**, 162 (2004)
 M. Mariani et al., *Science* **334**, 61 (2011)



Resonator as a Coupling Bus

M. Sillanpaa et al., *Nature* **449**, 438 (2007)
 H. Majer et al., *Nature* **449**, 443 (2007)

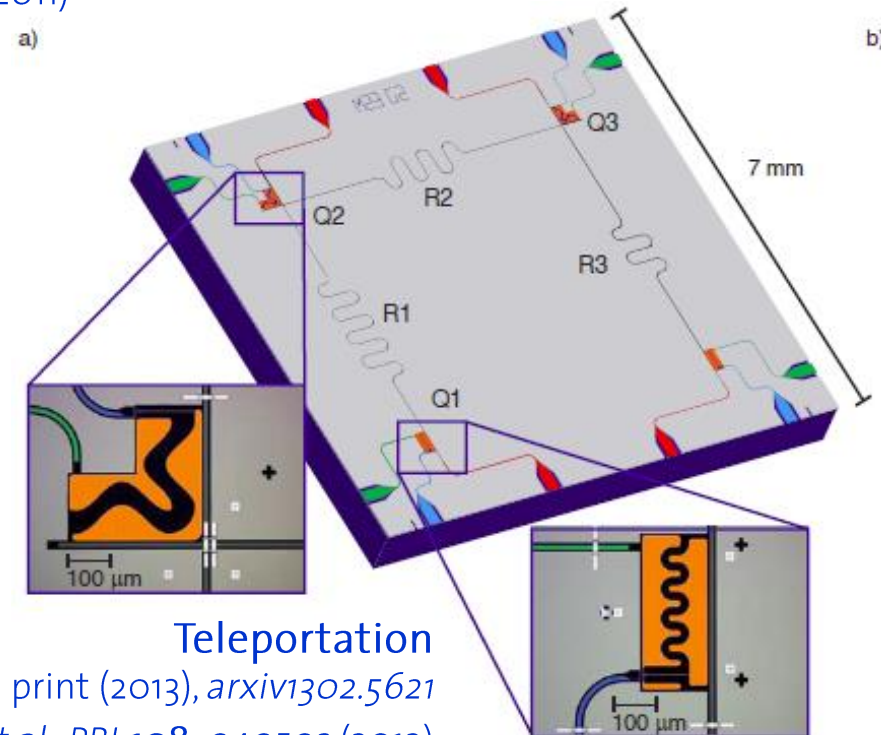


Deutsch, Grover Algorithms

L. DiCarlo et al., *Nature* **460**, 240 (2009)
 L. DiCarlo et al., *Nature* **467**, 574 (2010)

Toffoli Gates & Error Correction

A. Fedorov et al., *Nature* **481**, 170 (2012)
 M. Reed et al., *Nature* **481**, 382 (2012)



Teleportation

L. Steffen et al., *Nature*, in print (2013), *arxiv1302.5621*
 M. Baur et al., *PRL* **108**, 040502 (2012)

Exploring the Properties of Propagating Photons

quantum optics in the visible:

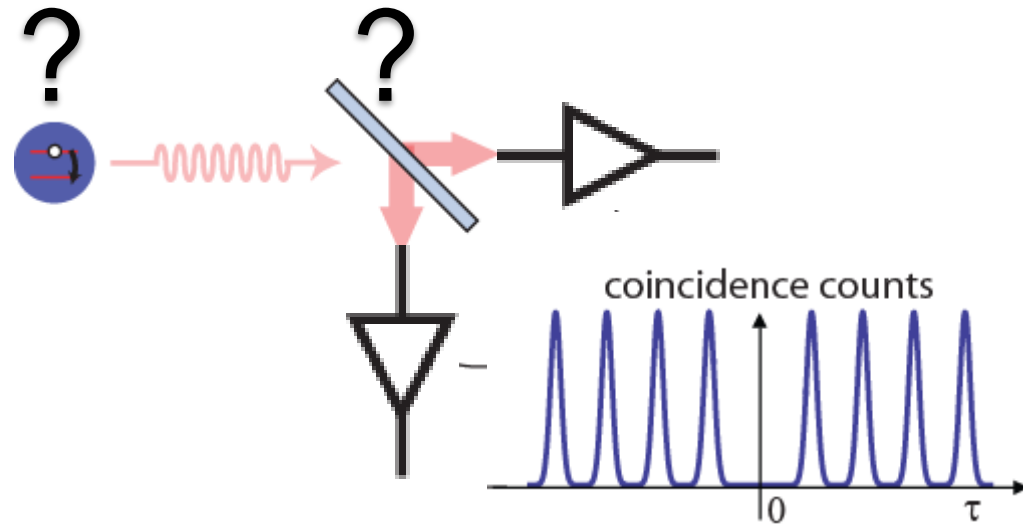
- single photon sources
- beam splitters
- photon counters

o.k. at optical frequencies

But in the microwave domain?

- smaller photon energy ...

$$\frac{\nu_{\text{opt}}}{\nu_{\mu\text{w}}} = \frac{500 \text{ THz}}{5 \text{ GHz}} = 10^5$$



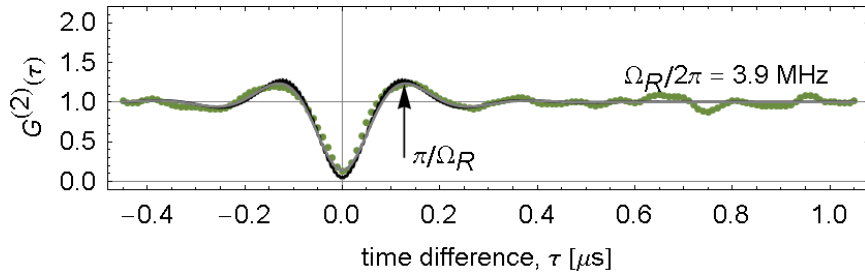
instead:

- linear amplifiers
- signal processing

- J. Gabelli et al., *Phys. Rev. Lett.* **93**, 056801 (2004)
E. P. Menzel et al., *Phys. Rev. Lett.* **105**, 100401 (2010)
M. P. da Silva et al., *Phys. Rev. A* **82**, 043804 (2010)
C. Eichler et al., *Phys. Rev. A* **86**, 032106 (2012)

Experiments with Propagating Quantum Microwaves

Single photon sources and their anti-bunching

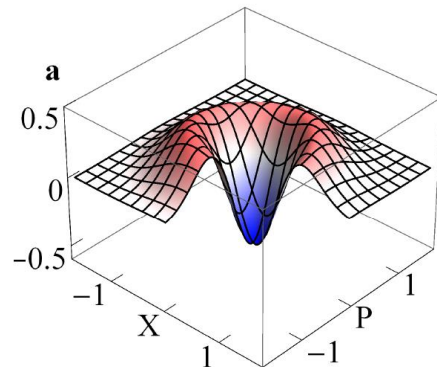


Lang et al., *PRL* 107, 073601 (2011)

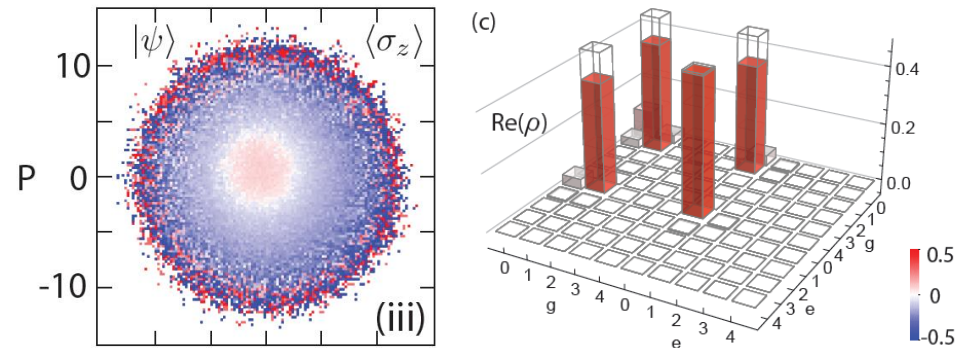
Bozyigit et al., *Nat. Phys* 7, 154 (2011)

Wigner functions and full state tomography of propagating photons:

Eichler et al., *PRL* 106, 220503 (2011)



Preparation and characterization of qubit-propagating photon entanglement

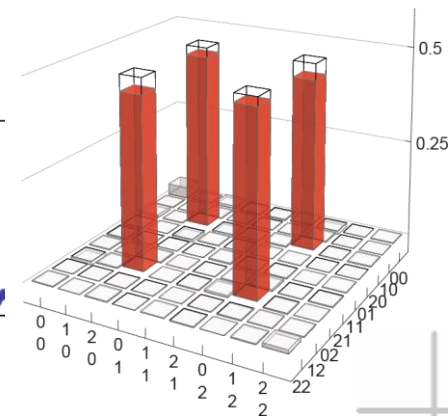
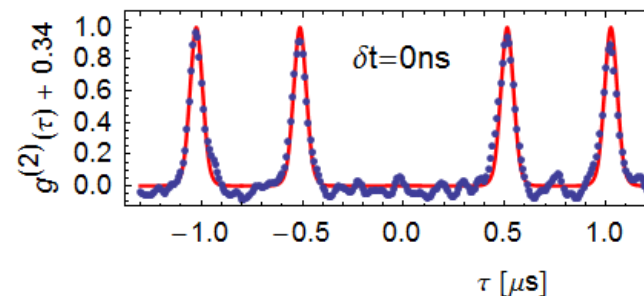


Eichler et al., *PRL* 109, 240501 (2012)

Eichler et al., *PRA* 86, 032106 (2012)

Hong-Ou-Mandel: Two-photon interference with coherences at microwave frequencies

Lang et al., *Nat. Phys.* 9, 345 (2013)



Propagating Quantum Microwaves

Correlation Function Measurements of Single Photons

Bozyigit *et al.*, *Nat. Phys* **7**, 154 (2011)

Lang *et al.*, *PRL* **107**, 073601 (2011)

Quantum State Tomography

Mallet *et al.*, *PRL* **106**, 220502 (2011)

Eichler *et al.*, *PRL* **106**, 220503 (2011)

Photon Routers

Hoi *et al.*, *PRL*, **107**, 073601 (2011)

Single Photon Detectors

Chen *et al.*, *PRL* **107**, 217401 (2011)

Positive P-Function/Dual Path Detection

Menzel *et al.* *PRL* **105**, 100401 (2010)

Eichler *et al.*, *PRA* **86**, 032106 (2012)

Photon/Qubit Entanglement

Eichler *et al.*, *PRL* **109**, 240501 (2012)

Hong-Ou-Mandel N-Photon Interference

Lang *et al.*, *Nat. Phys.* **9**, 345 (2013)

Thermal and Vacuum Noise

Mariantoni *et al.*, *PRL* **105**, 133601 (2010)

Squeezing & Two Mode Correlations

Castellanos *et al.*, *Nat. Phys.* **4**, 929 (2008)

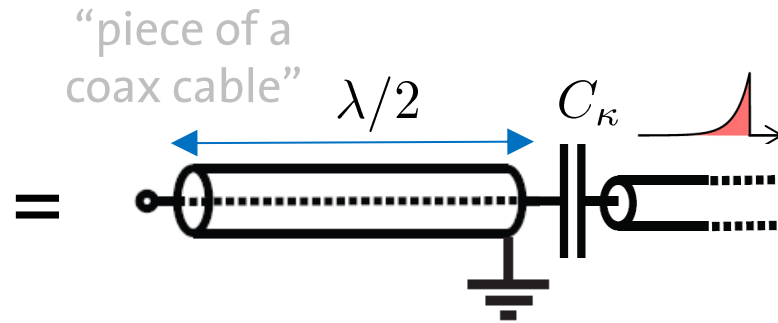
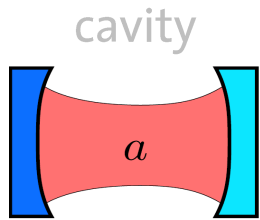
Eichler *et al.*, *PRL* **107**, 113601 (2011)

Bergeal *et al.*, *PRL* **108**, 123902 (2012)

Flurin *et al.*, *PRL* **109**, 183901 (2012)

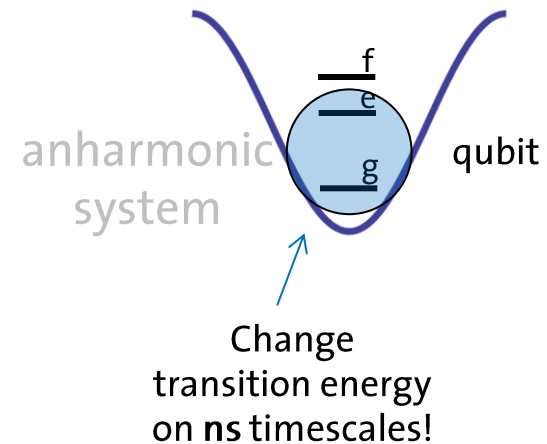
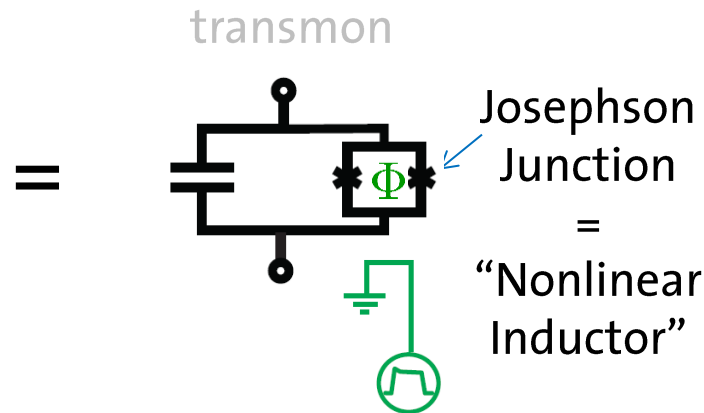
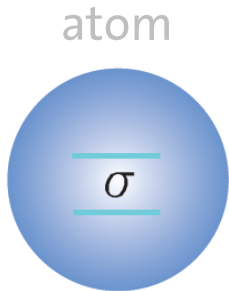
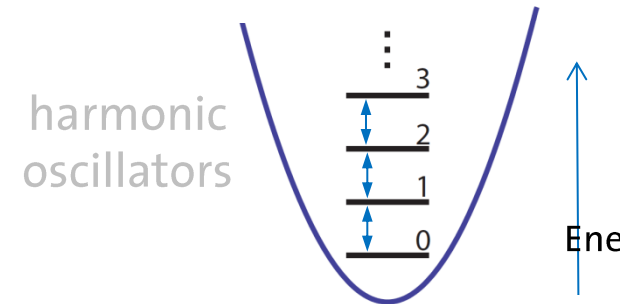
Menzel *et al.*, *PRL* **109**, 250502 (2012)

Circuit QED components



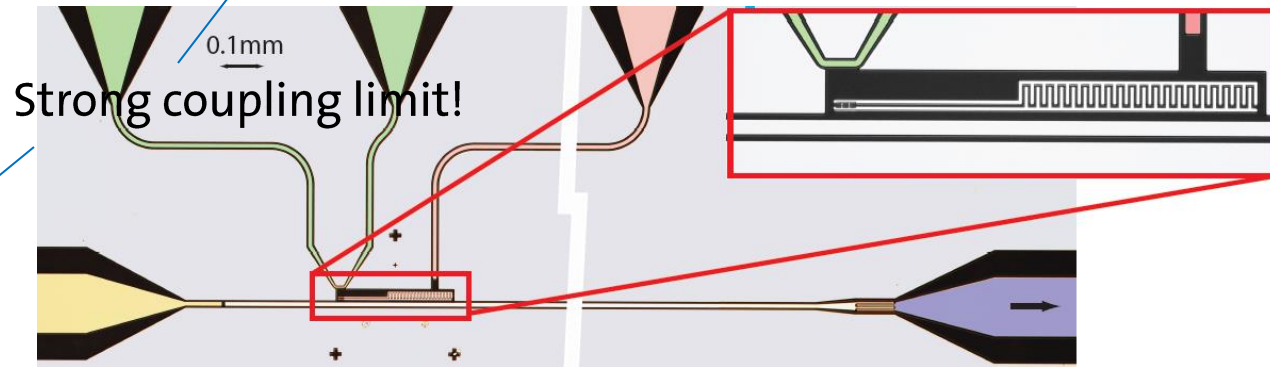
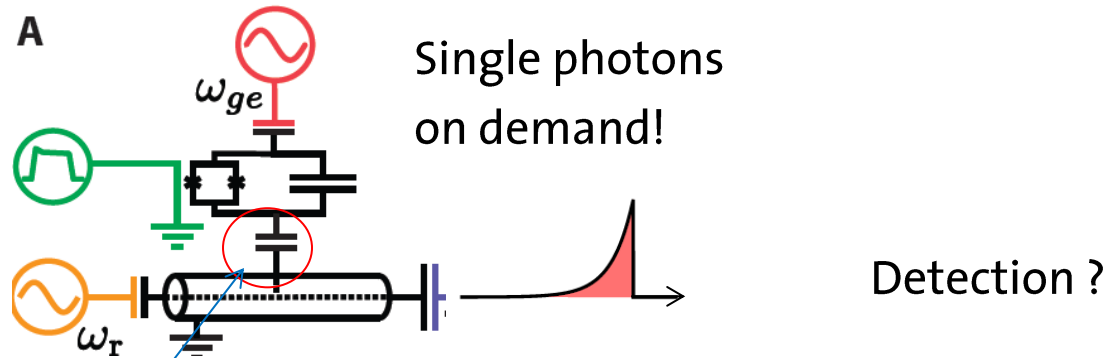
Radiation field stored inside:

$$H = \hbar\omega a^\dagger a$$

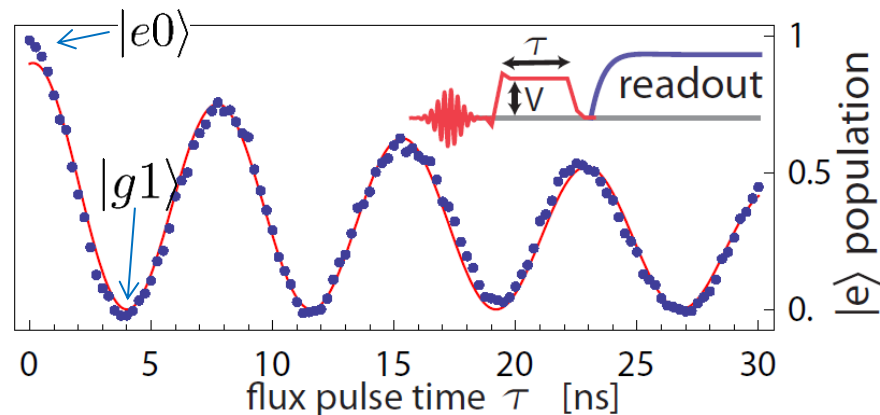


Strong coupling regime

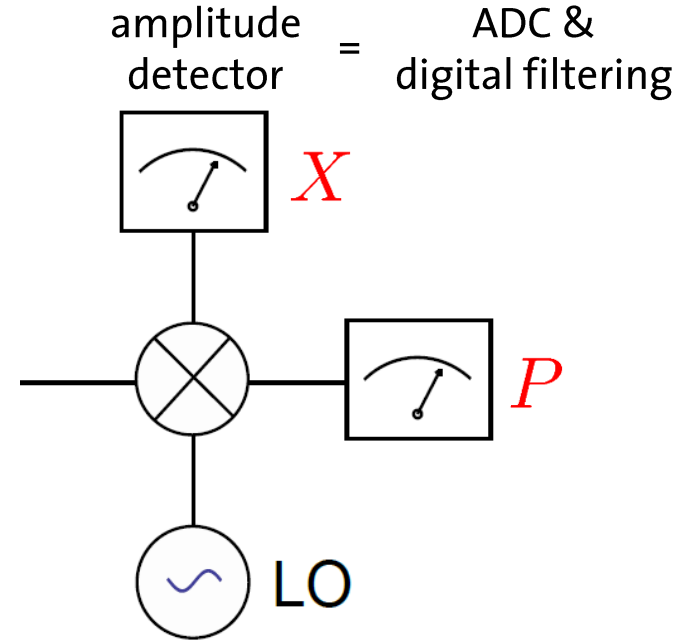
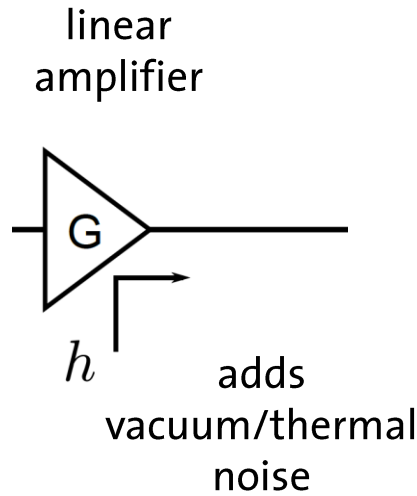
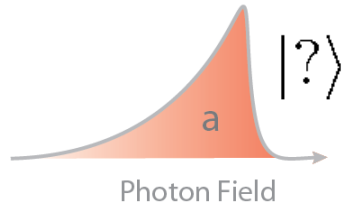
- Transmon qubit
 - $T_1 = 1.1 \mu s$
 - $T_2 = 550 ns$
 - $T_2^* = 220 ns$
- Single sided resonator
 - $1/\kappa = 25 ns$
- Coupling strength
 - $\pi/g = 7.7 ns$



Vacuum Rabi oscillations



Microwave Photon Field Detection



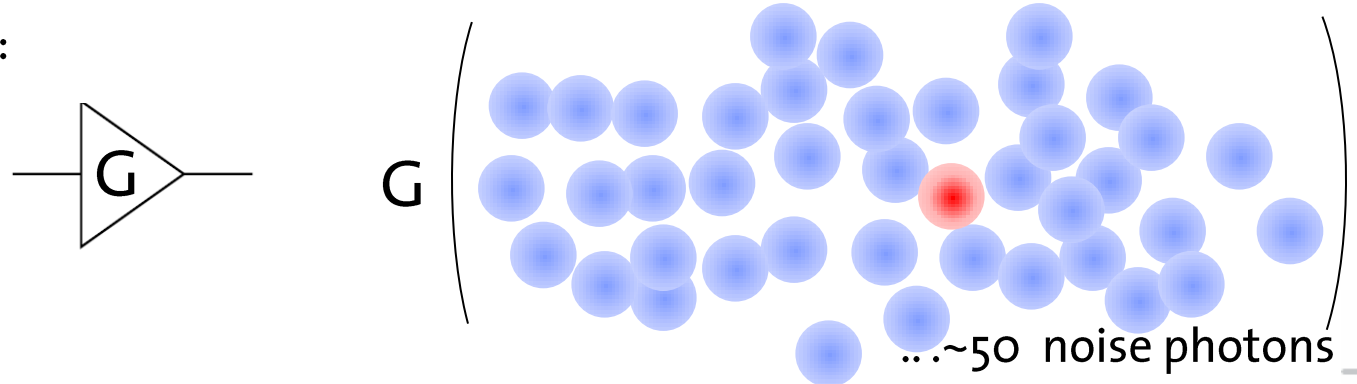
complex amplitude:

$$S = X + iP = a + h^\dagger$$

“signal”
“noise”

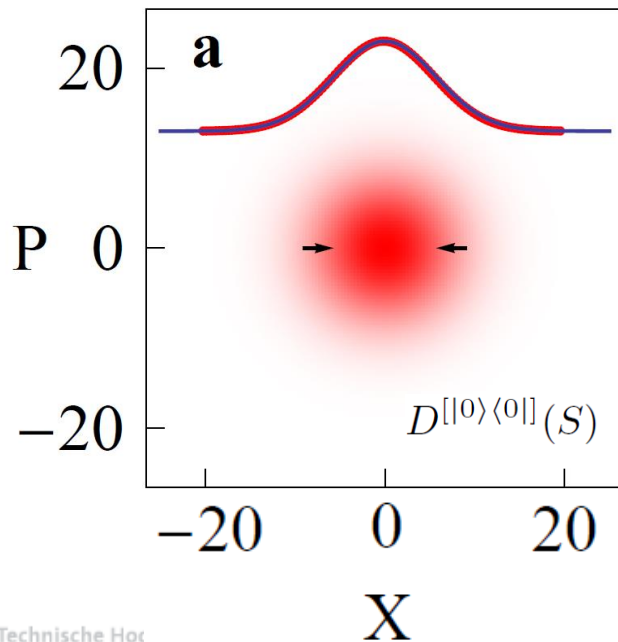
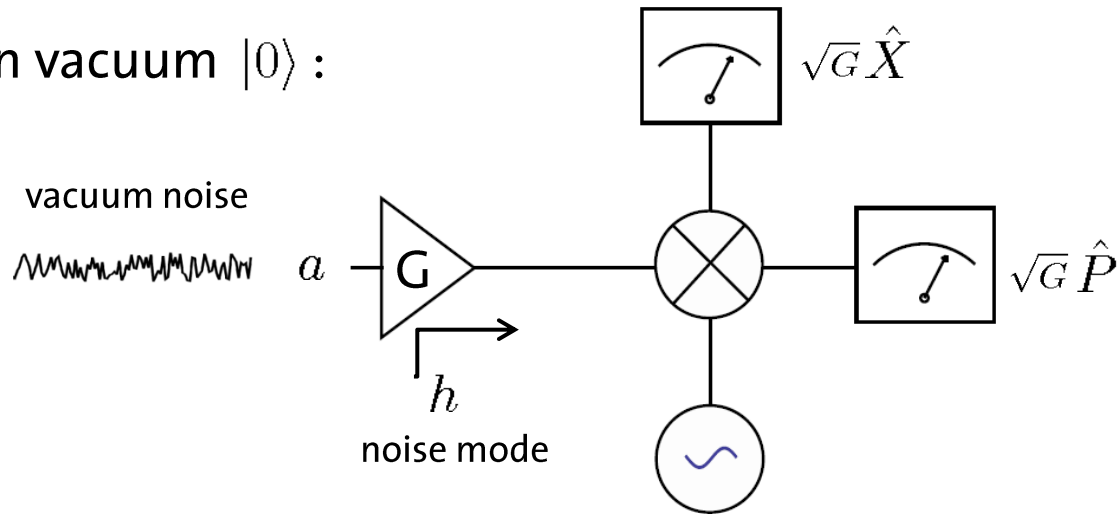
Eichler et al., *PRA* 86, 032106 (2012)
 M. P. da Silva et al., *PRA* 82, 043804 (2010).
 C. M. Caves, *PRD* 26, 1817 (1982).

typical added noise:



Full Tomography of a Single Propagating Mode

1) prepare a in vacuum $|0\rangle$:



← record histogram $D^{[|0\rangle\langle 0|]}(S)$
of measurement results $S/\sqrt{G} = X + iP$

→ normal distribution with variance

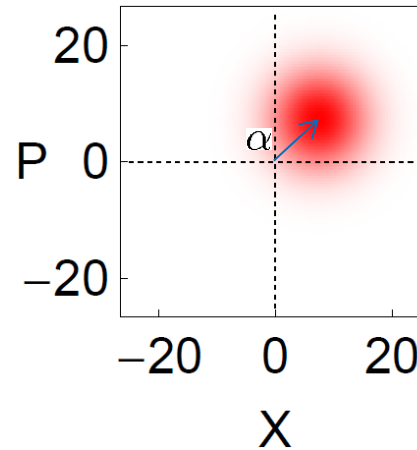
$$2\sigma^2 = \langle \hat{S}^\dagger \hat{S} \rangle / G = \frac{1}{G} \int d^2 S D^{[|0\rangle\langle 0|]}(S) S^* S = 67$$

h introduces thermal noise
with mean photon number N_{noise}

Coherent State Histograms

2) prepare a in coherent state $|\alpha\rangle$:

MW generator



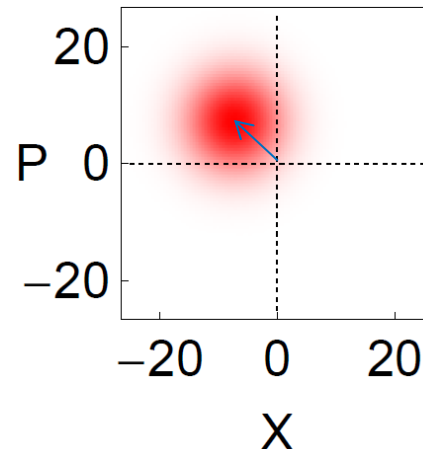
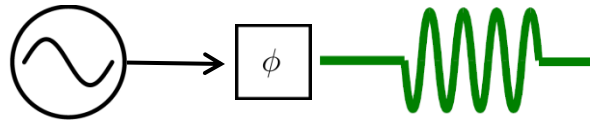
$$|\alpha| \approx 6.3$$

$$\Leftrightarrow$$

$$\langle a^\dagger a \rangle \approx 41 \sim N_{\text{noise}}$$

3) rotate phase $|e^{i\phi}\alpha\rangle$:

MW generator



Question: What can we learn about state when $\langle a^\dagger a \rangle \leq 1$?

Single Photon Source Histogram

store 2D histogram $D^{[\rho]}(S)$ from $S/\sqrt{G} = X + iP$ measurement results:

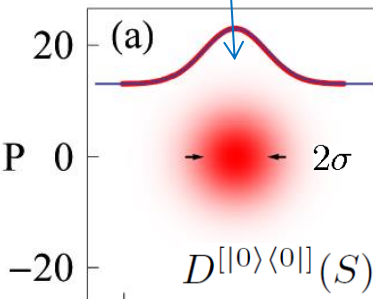
corresponding phase space distribution

signal mode a
in vacuum

Q - function
of noise mode :

$$Q_h$$

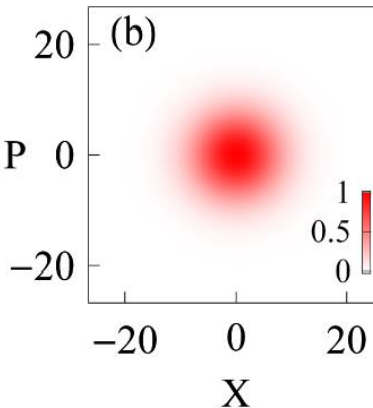
← P



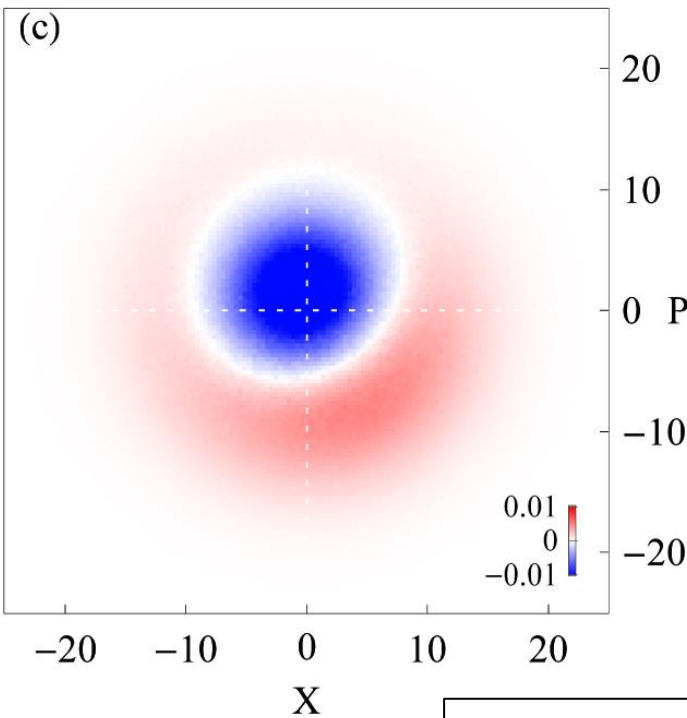
convolution
with P - function
of signal

$$Q_h * P_a$$

← P



signal mode a
in single photon
Fock state



← subtracted
histograms
to visualize
difference

separate noise h from
signal a systematically!

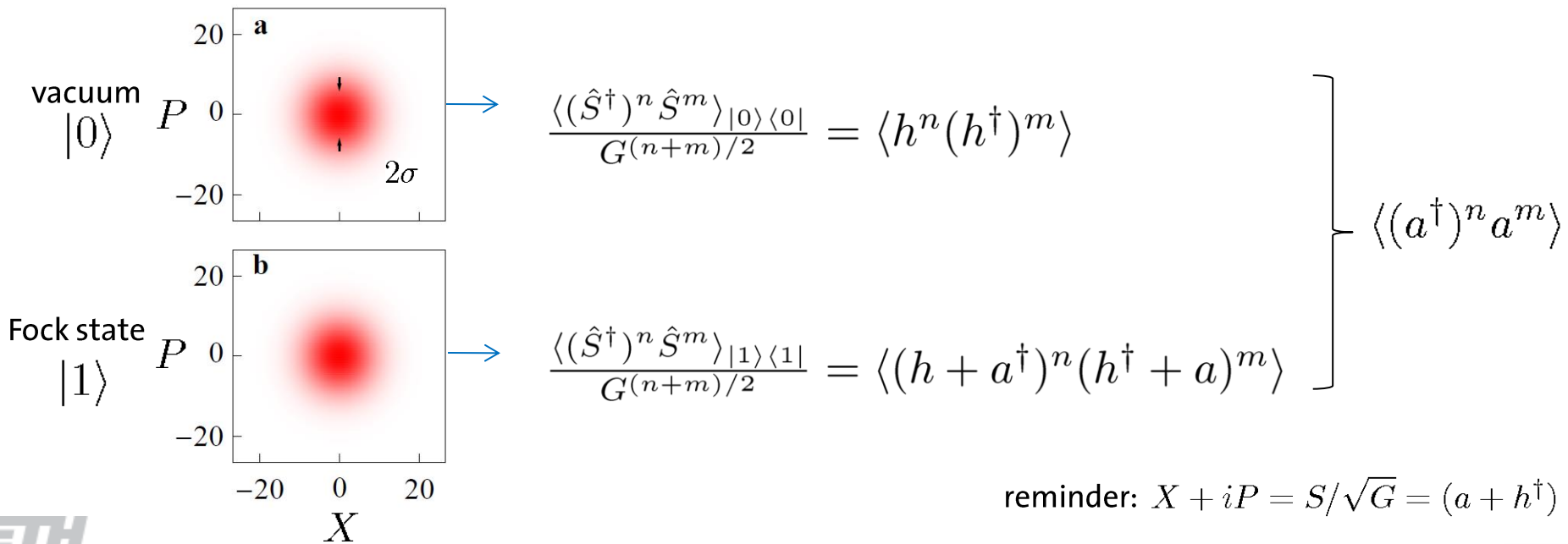
Statistical Analysis of Histograms

systematic mode separation:

histogram moments: $\langle (\hat{S}^\dagger)^n \hat{S}^m \rangle_\rho = \int d^2 S (S^*)^n S^m D^{[\rho]}(S)$

1. calculate histogram moments

2. algebraic inversion

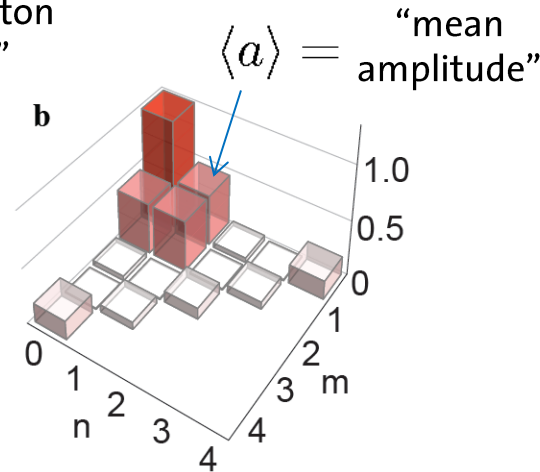
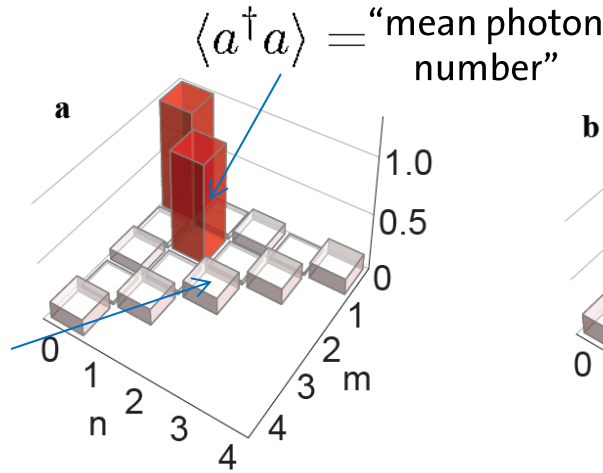


State Dependent Moments of Probability Distribution

moments $|\langle (a^\dagger)^n a^m \rangle|$ for different prepared states:

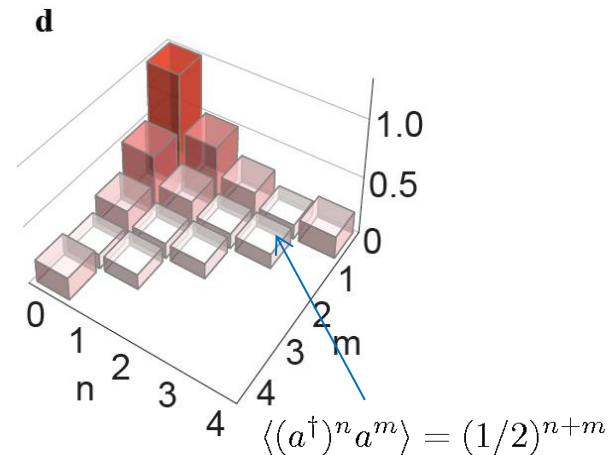
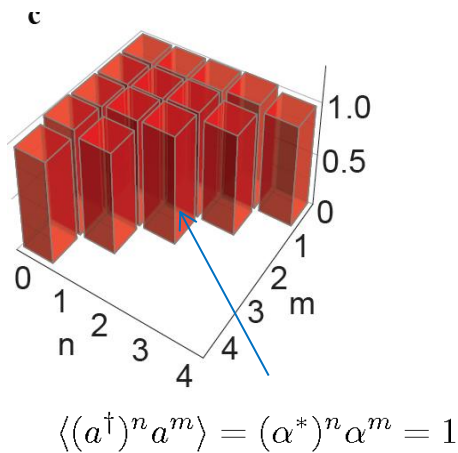
Fock state
 $|1\rangle$

$\langle (a^\dagger)^2 a^2 \rangle \approx 0$
“anti bunching”



superposition
 $\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

coherent state
 $|\alpha = 1\rangle$

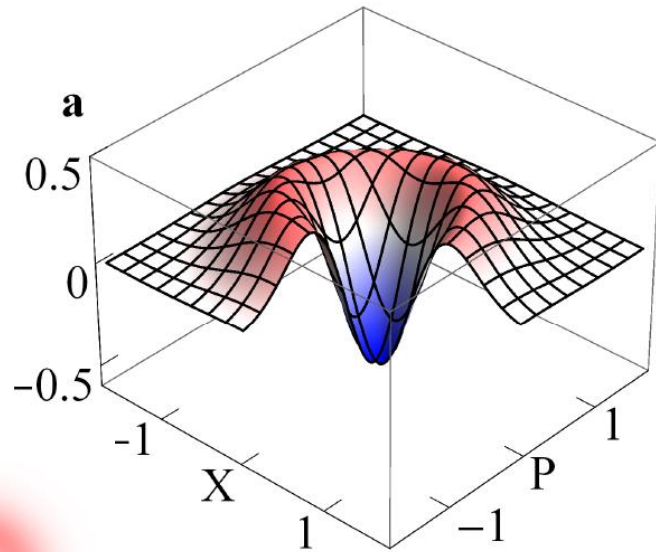


coherent state
 $|\alpha = 0.5\rangle$

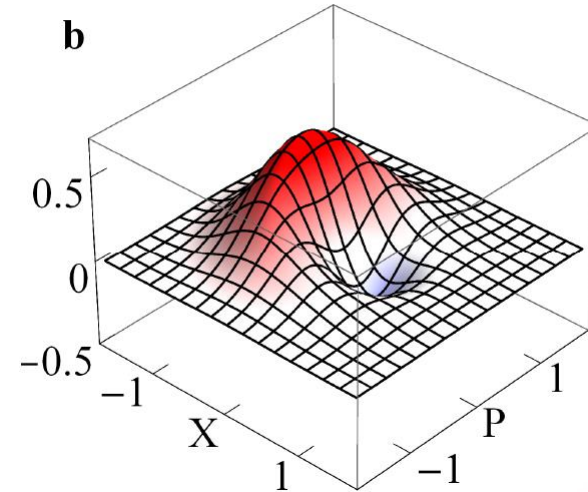
Reconstructed Wigner Function of Itinerant Photon

Wigner function reconstructed from measured moments:

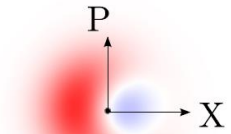
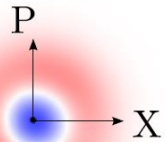
$$W(\alpha) = \sum_{n,m} \int d^2\lambda \frac{\langle (a^\dagger)^n a^m \rangle (-\lambda^*)^m \lambda^n}{\pi^2 n! m!} e^{(-1/2)|\lambda|^2 + \alpha\lambda^* - \alpha^*\lambda} \quad \text{with} \quad n + m < 4$$



Fock state
 $|1\rangle$

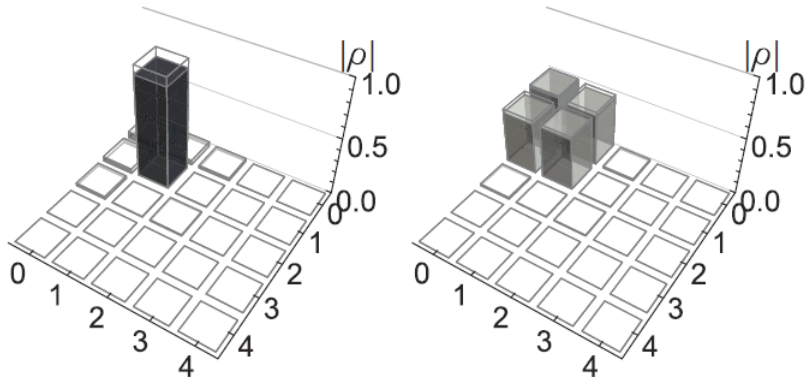


superposition
 $\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

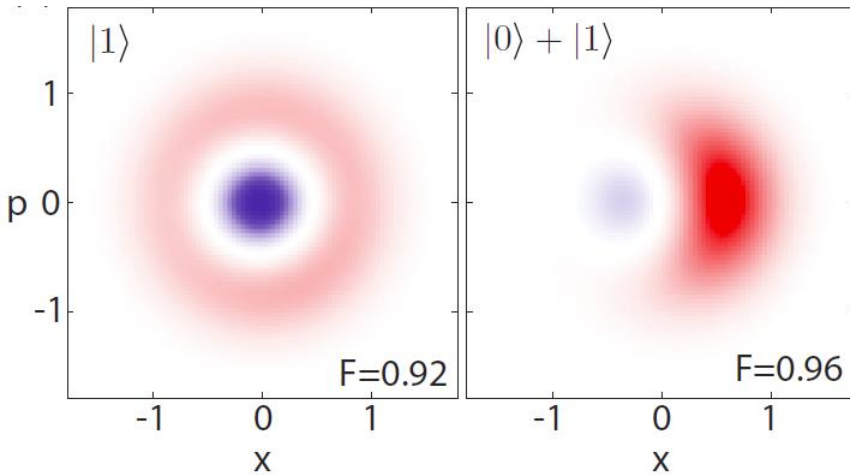
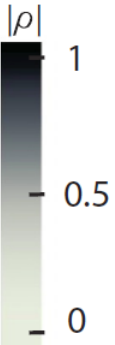


Reconstruct Density Matrices and Wigner functions...

... for propagating multi-photon Fock states and their superpositions:



Density matrices



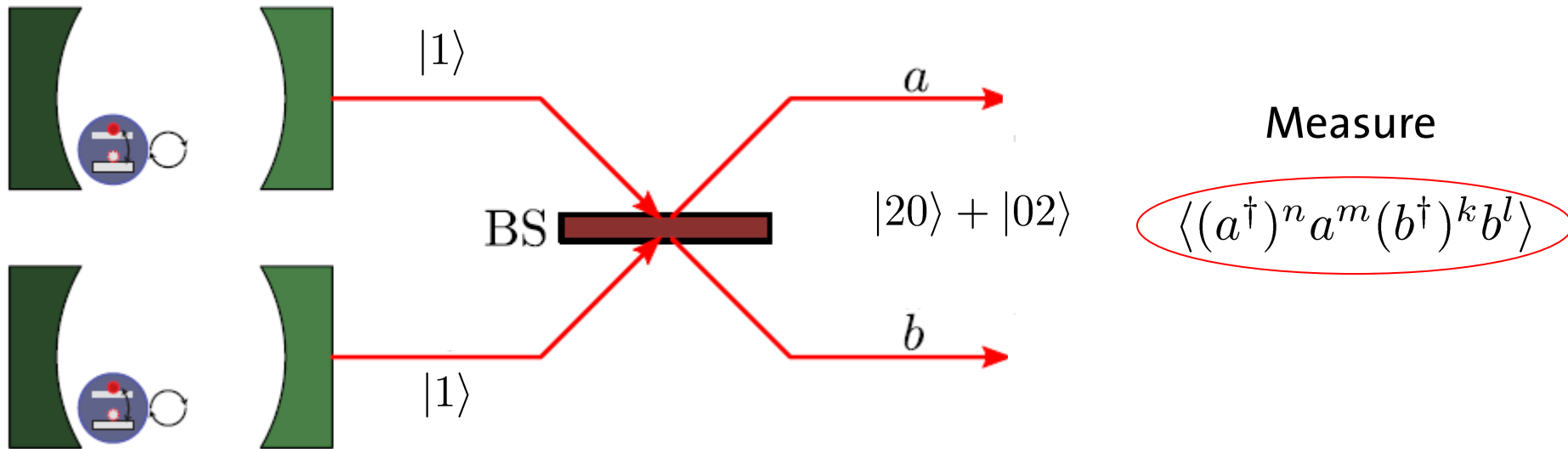
Wigner functions



measured using near-quantum-limited parametric amplifier

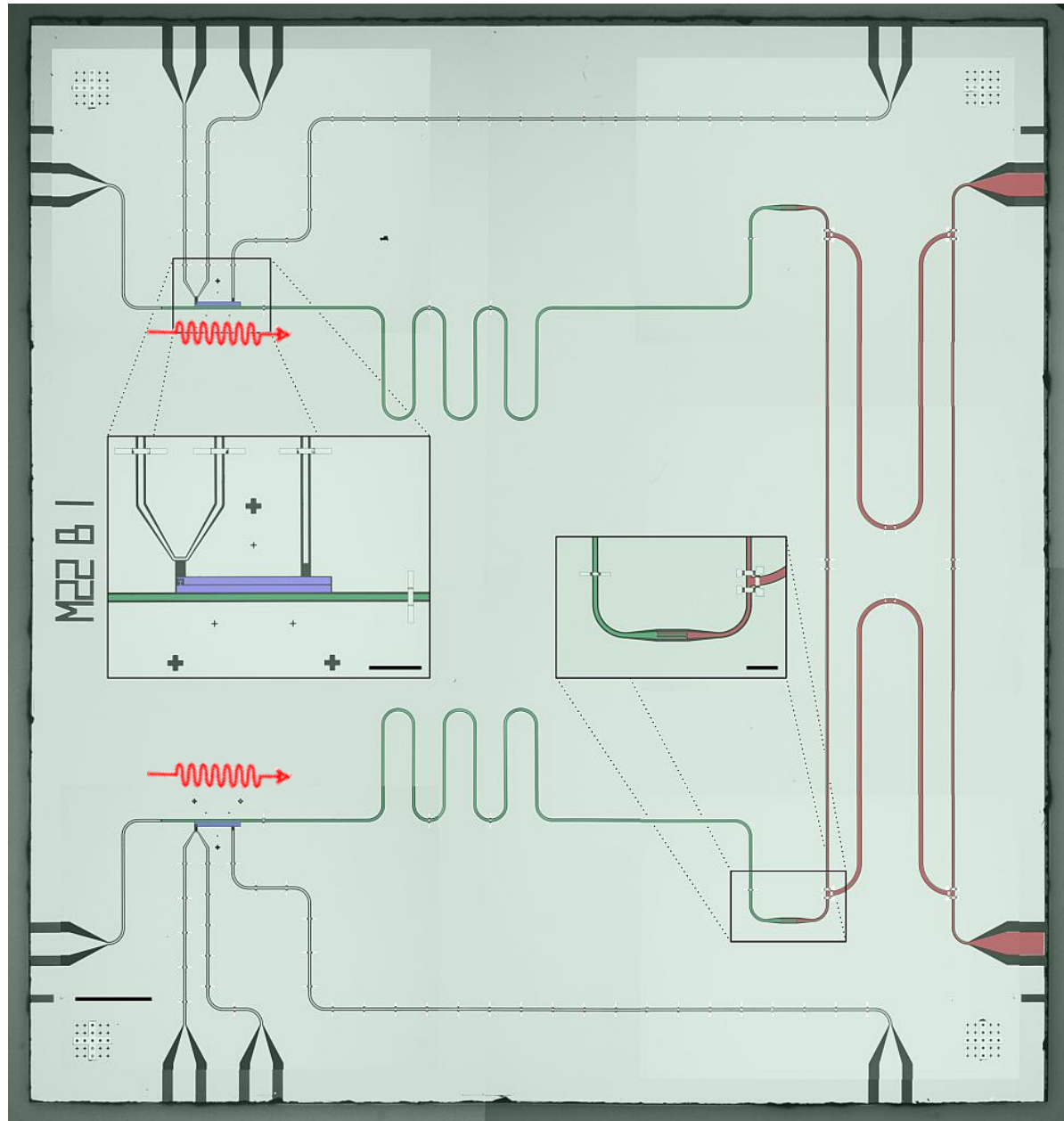
Hong-Ou-Mandel Experiments with Microwaves

Measure field – field correlations in two spatial modes:

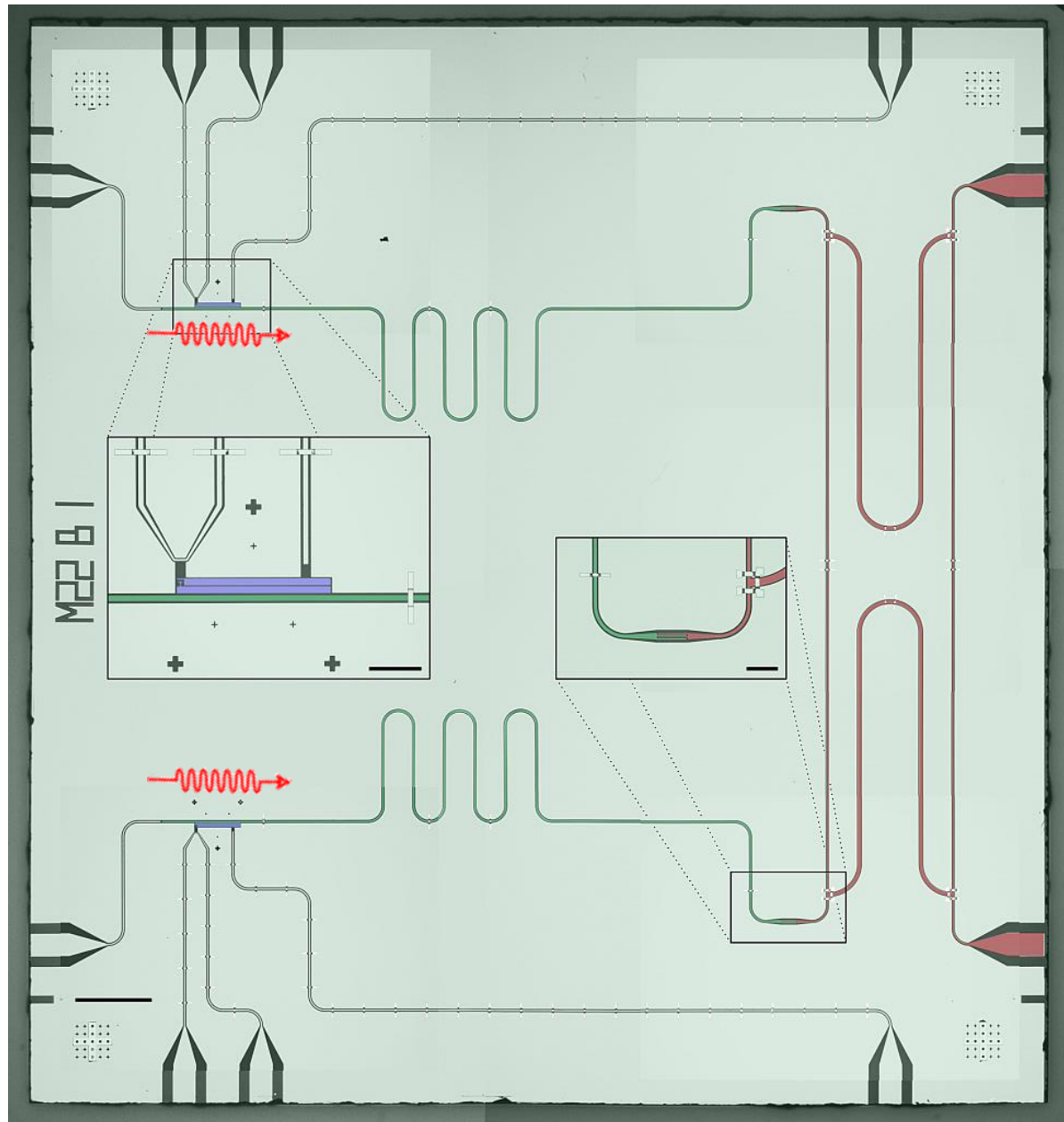


... from recorded 4D histograms: $D(X_a, P_a, X_b, P_b)$

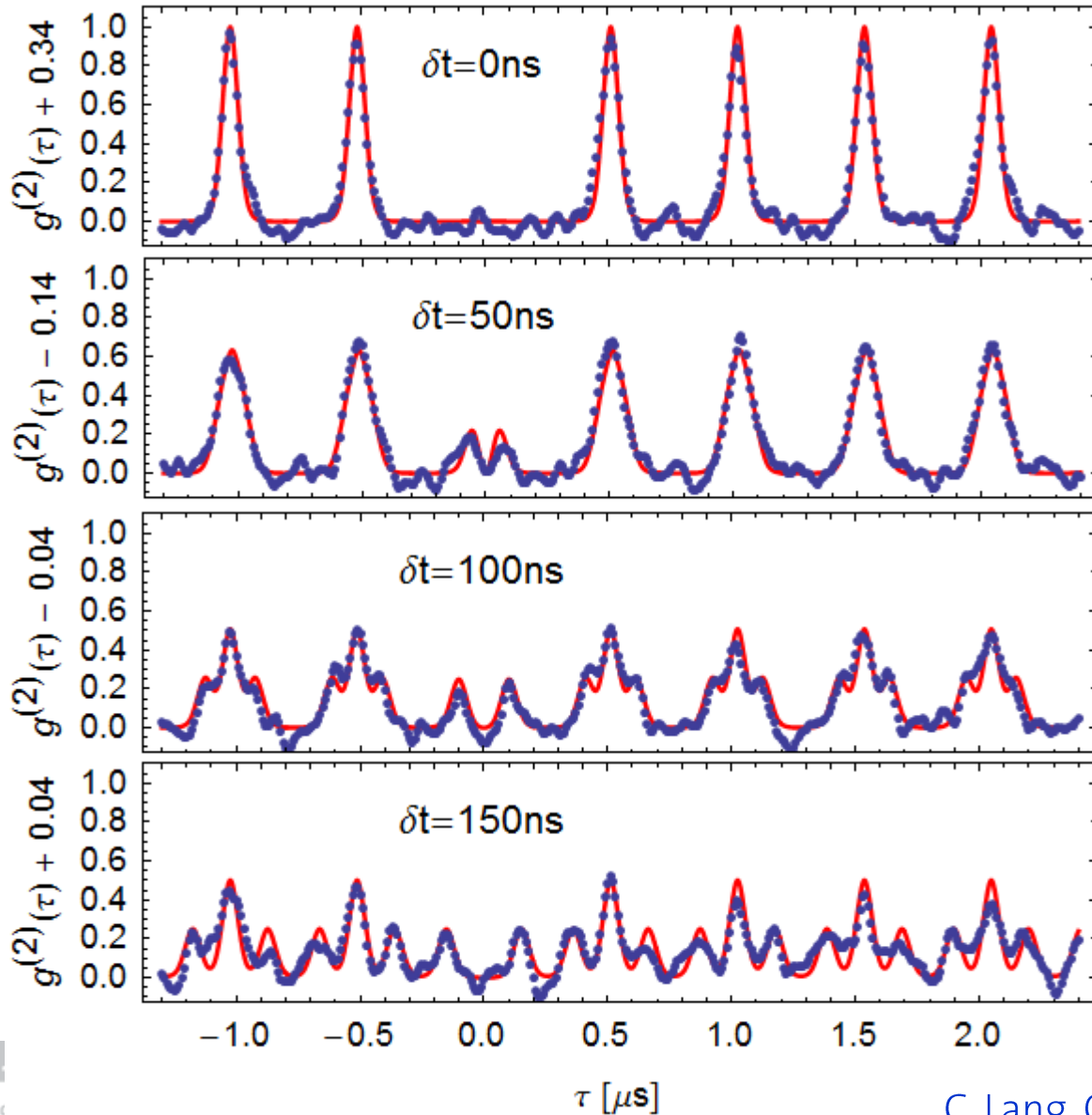
Two Single Photon Sources and Beam Splitter



Two Single Photon Sources and Beam Splitter



Hong-Ou-Mandel $g^{(2)}(\tau)$ for Microwave Photons



Observations:

- Photon-Pair anti-bunching

For $\tau > 0$:

- Broadening of satellite peaks
- Triple-peak structure of satellite peaks
- Full recovery of double-peak at $\tau \approx 0$



Learning More: Two-Channel Tomography

Idea: Measure 4D histogram and evaluate relevant photon statistics

$$D_{\text{ON}}(X_a, P_a, X_b, P_b)$$

$$D_{\text{OFF}}(X_a, P_a, X_b, P_b)$$

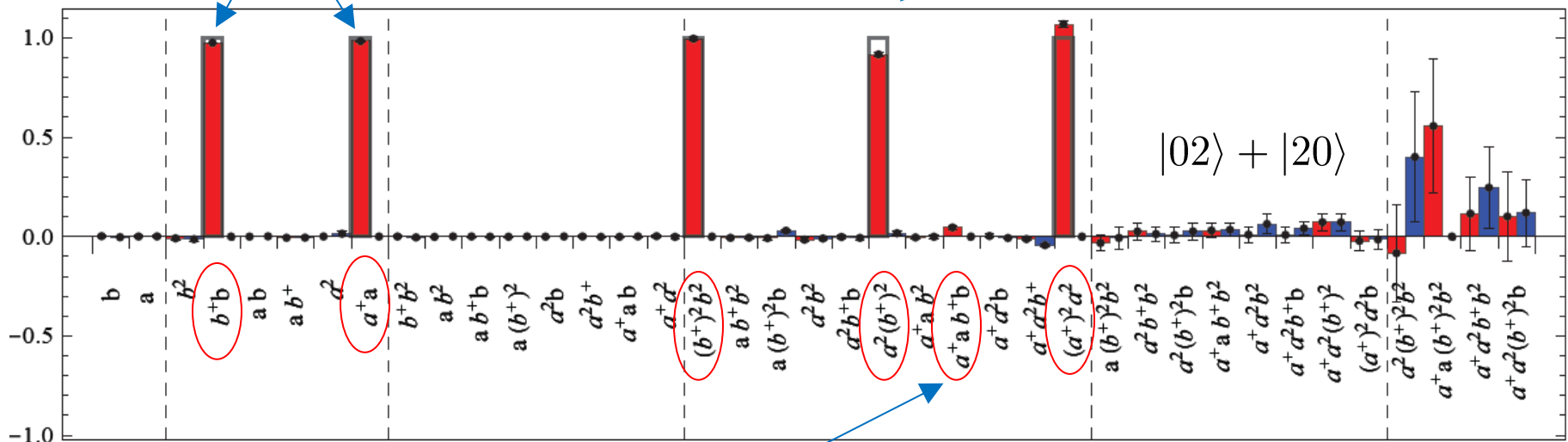
analogous to
1-channel case

$$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$$

1 average photon
in each channel

2-photon
bunching

Quantum
superposition!



never simultaneous
"click" in both channels

Density Matrix Displaying Two-Mode Entanglement

Density matrix reconstruction:

$$\langle (a^\dagger)^n a^m (b^\dagger)^k b^l \rangle$$

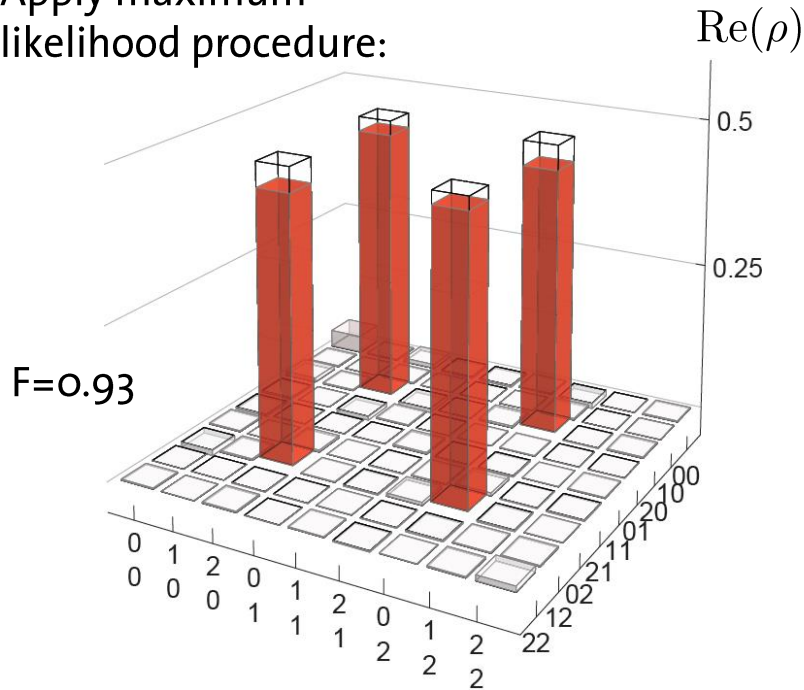
moments

linear map
→

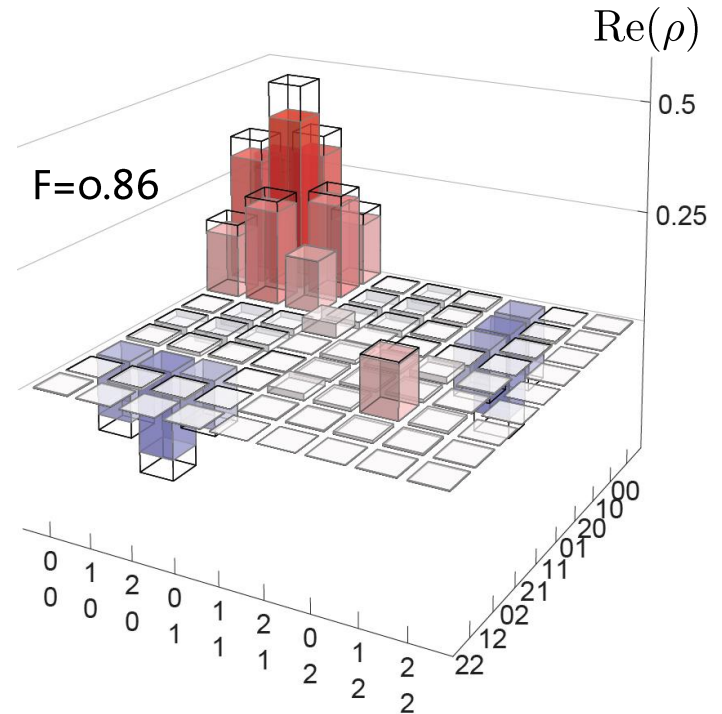
$$\langle nm | \rho | kl \rangle$$

Fock space
density matrix

Apply maximum
likelihood procedure:



$$|02\rangle + |20\rangle$$



$$|00\rangle + \sqrt{2}|10\rangle + (|20\rangle - |02\rangle)\sqrt{2}$$

C. Lang, C Eichler *et al.*, *Nat. Phys.* 9, 345 (2013)

Photon/Qubit Entanglement



Atom-Photon Entanglement

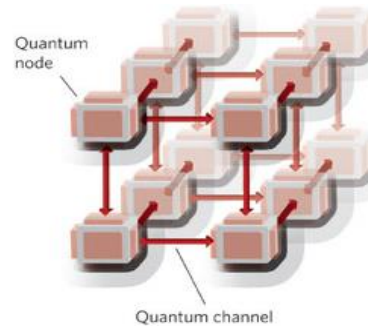
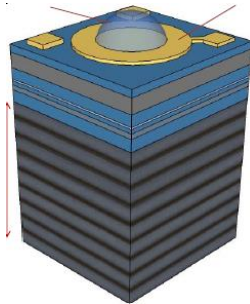
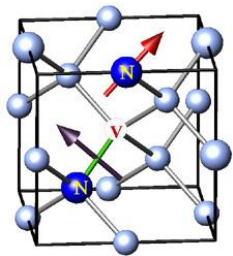
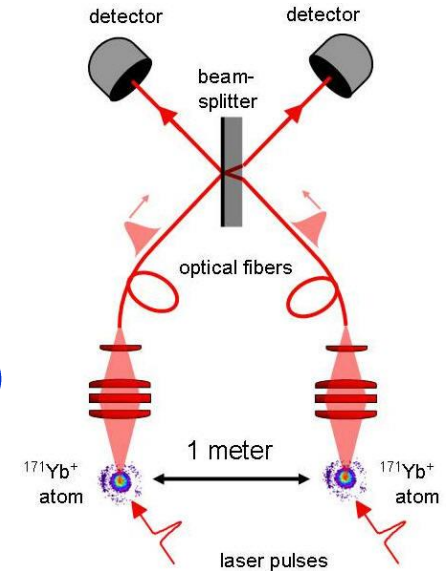
Blinov *et al.*, *Nature* 428, 153 (2004)

Volz *et al.*, *PRL* 96, 030404 (2006)

Atom-Atom Entanglement

Moehring *et al.*, *Nature* 449, 68 (2007)

Ritter *et al.*, *Nature* 484, 195 (2012)



Spin-Photon Entanglement

Togan *et al.*, *Nature* 466, 730 (2010)

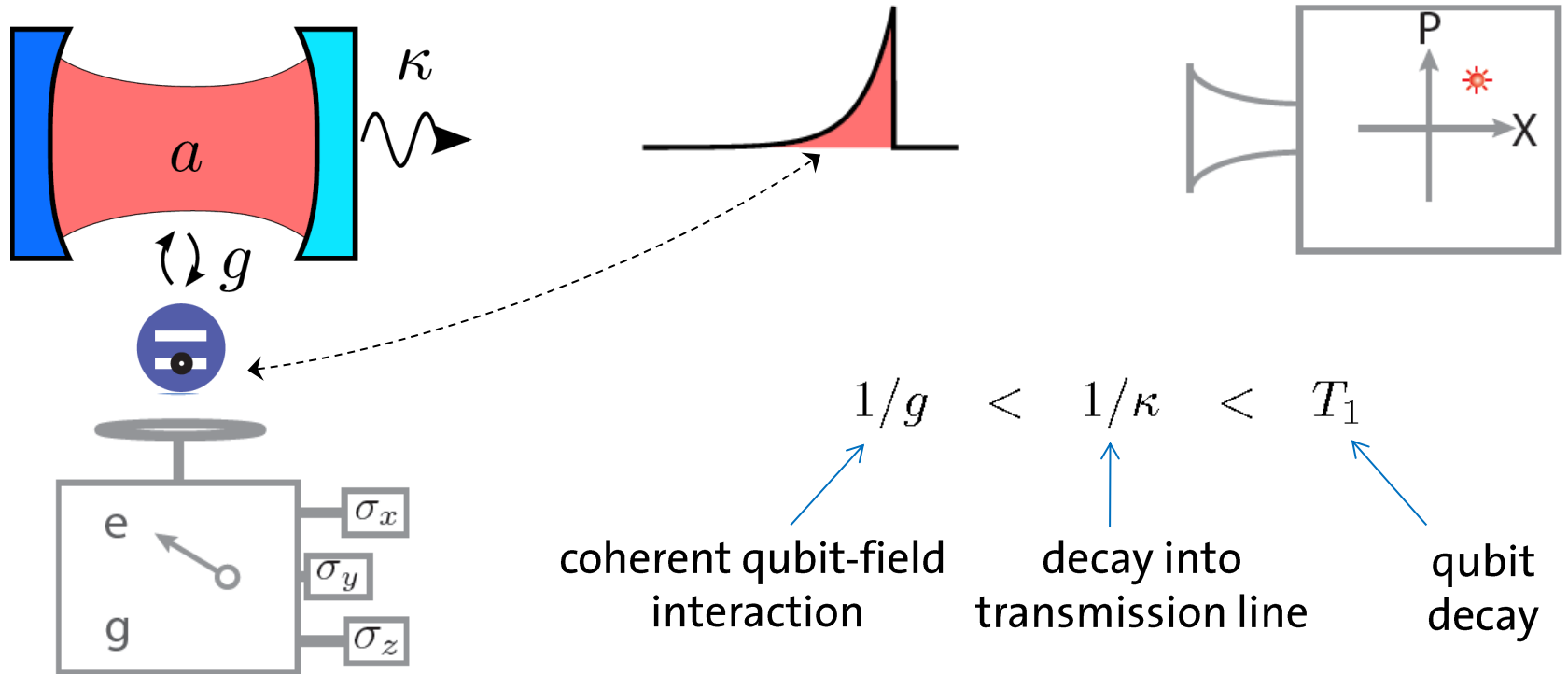
Gao *et al.*, *Nature* 491, 426 (2012)

Quantum networks

Kimble, *Nature* 453, 1023 (2008)

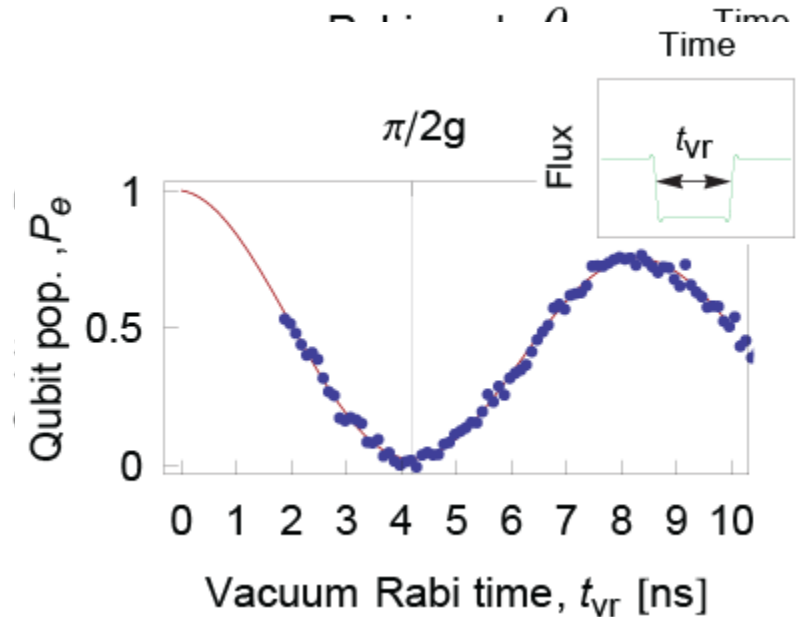
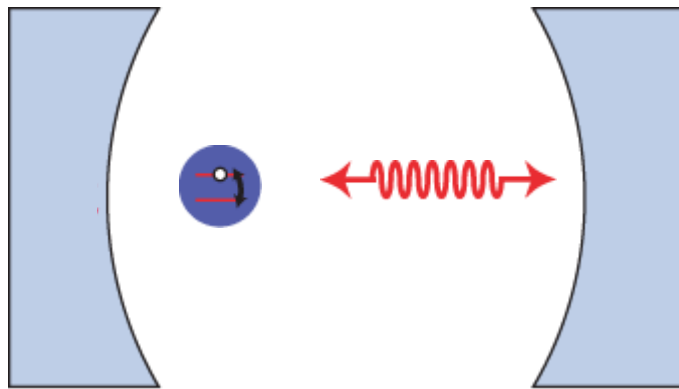
What about superconducting circuits?

Concept of Photon/Qubit Entanglement Experiment



Conditions for generation and detection of qubit/photon entanglement

Create Qubit/Photon Entanglement



$\alpha |g\rangle + \beta |e\rangle$
 $\left\{ \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right\}$

Step 1:

Prepare qubit state by Rabi oscillation

D. Bozyigit et al., *Nat. Phys.* 7, 154 (2011)

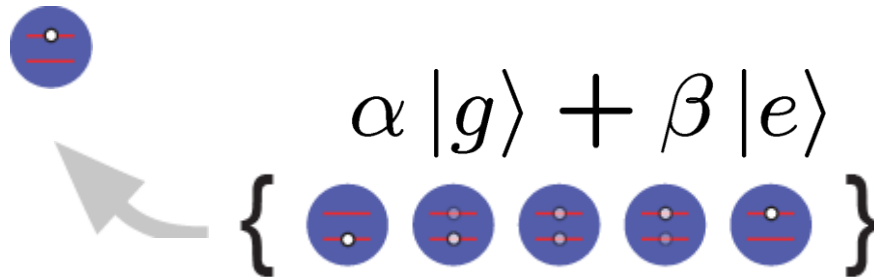
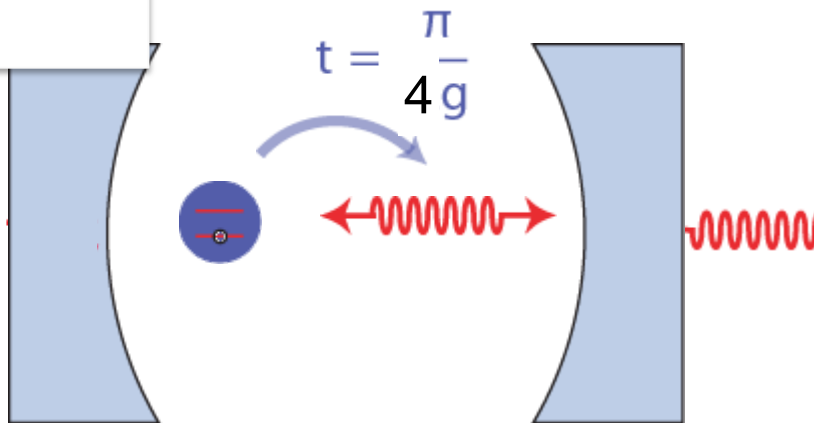
S. Deleglise et al. *Nature* 455, 510514 (2008); M. Hofheinz et al., *Nature* 454, 310 (2008)

Create Qubit/Photon Entanglement

Step 2:

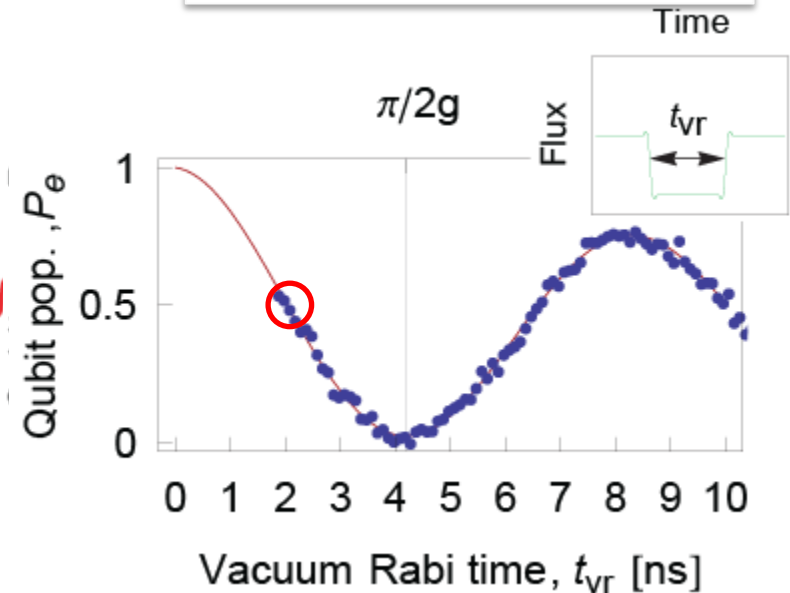
Entangle qubit with resonator by 1/4 vacuum Rabi oscillation

$$\frac{1}{\sqrt{2}} (|0e\rangle + |1g\rangle)$$



Step 3:

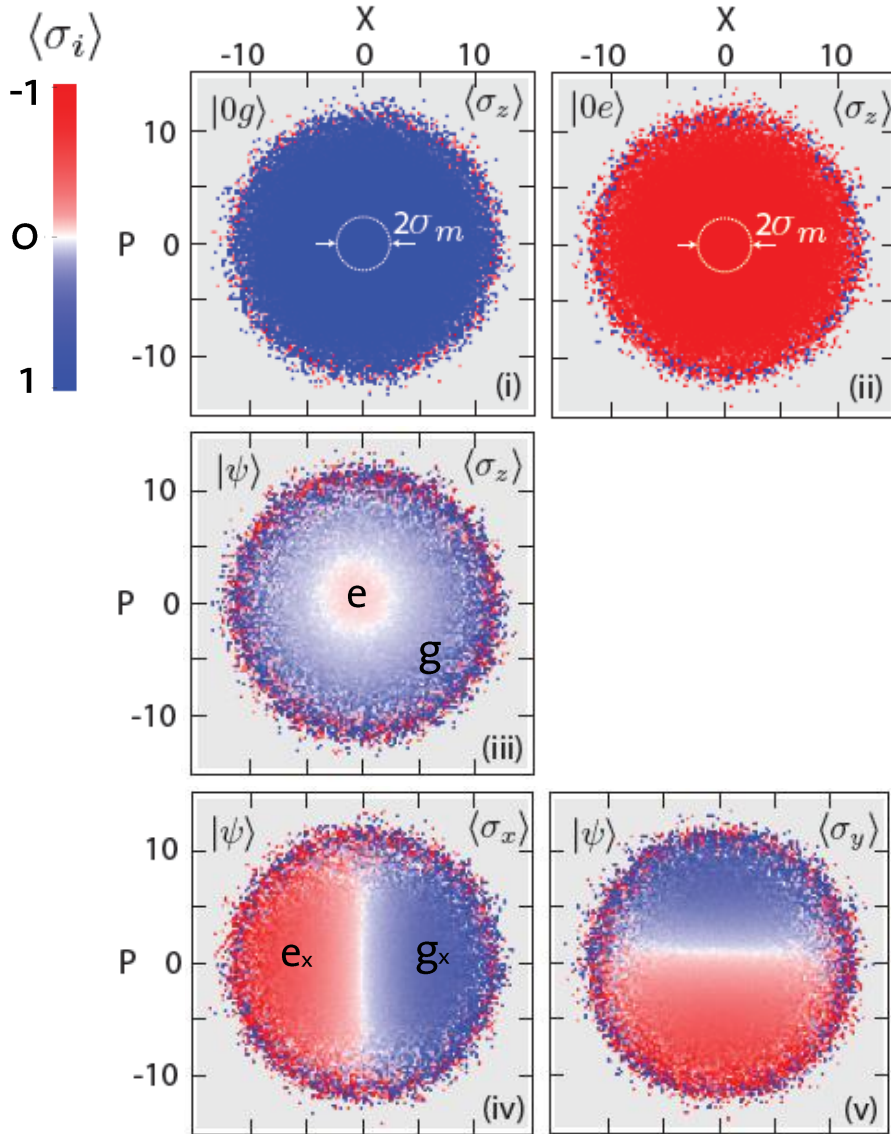
Measure qubit and photon state.



Step 1:

Prepare qubit state by Rabi oscillation

Measurement Results



und state
e
ation vs.
 $+ iP$



as expected $\langle \sigma_z \rangle_\alpha$
independent
of α

Analyzing the Bell state

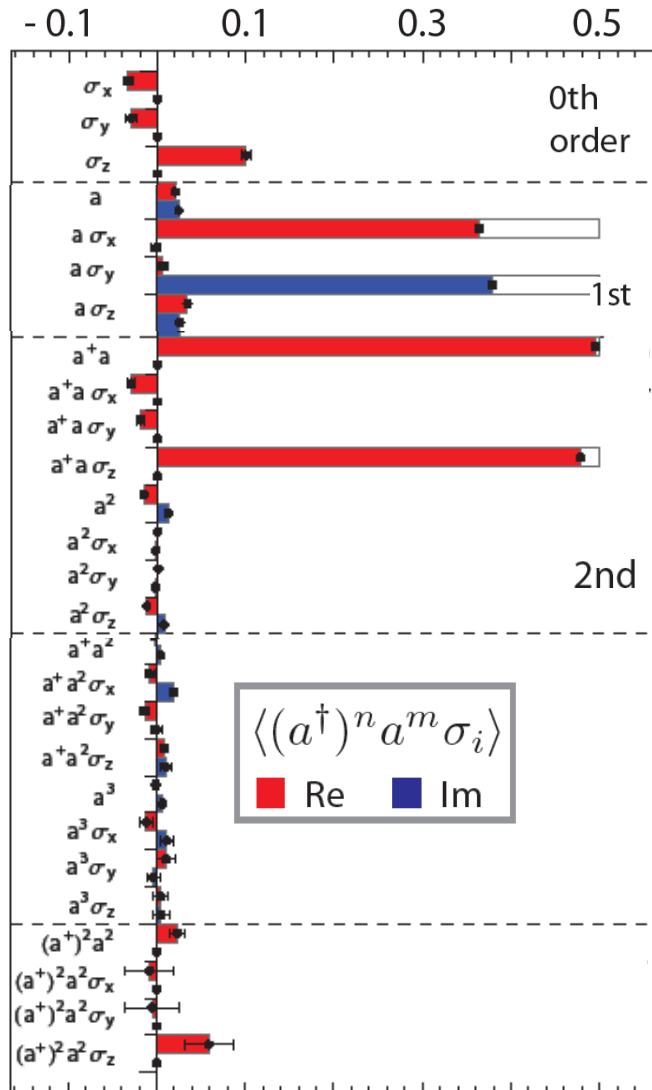
$$|\psi\rangle = |0e\rangle + |1g\rangle$$

Probing coherences:

$$= |e_x\rangle \underbrace{(|1\rangle - |0\rangle)}_{\langle \hat{X} \rangle < 0} + |g_x\rangle \underbrace{(|1\rangle + |0\rangle)}_{\langle \hat{X} \rangle > 0}$$

exp: C. Eichler *et al.*, *PRL* 109, 240501 (2012)
theo: C. Eichler *et al.*, *PRA* 86, 032106 (2012)

Measured Moments for Bell state



0th : qubit state with photon discarded/traced out

1st : phase correlations between qubit and photon field

2nd : number correlations!

e <-> no photon

g <-> one photon

3rd, 4th : no higher photon number states!

exp: C. Eichler *et al.*, *PRL* 109, 240501 (2012)
 theo: C. Eichler *et al.*, *PRA* 86, 032106 (2012)

Photon/Qubit Joint State Density Matrix

Reconstruction from measured moments

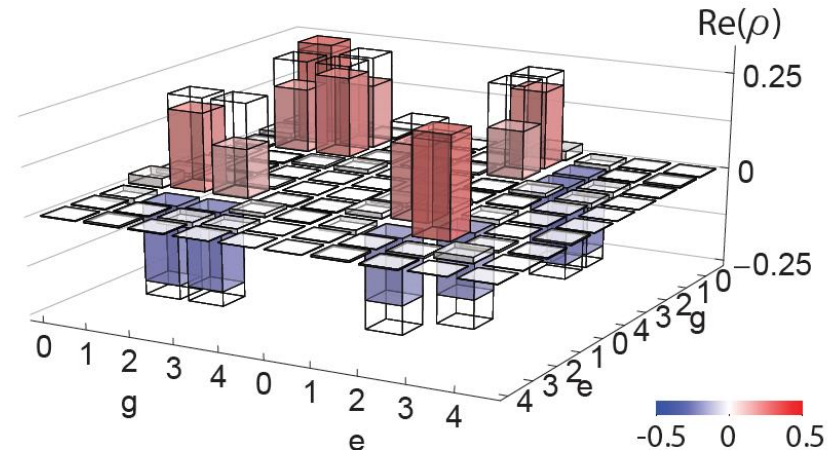
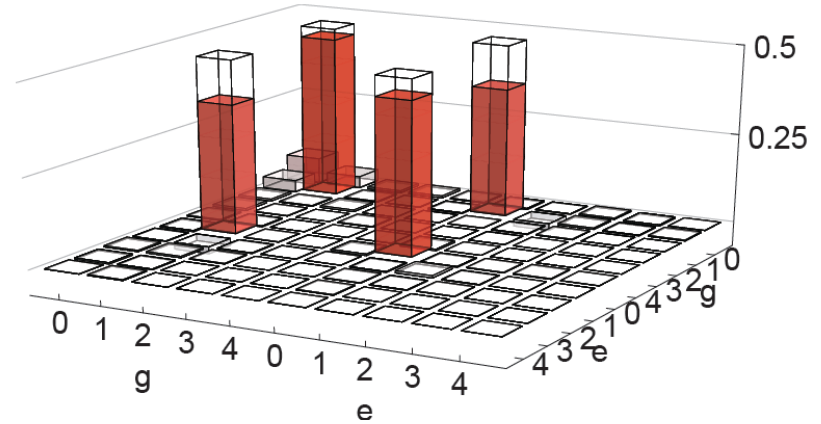
Fidelity: $F = \langle \psi | \rho | \psi \rangle = 0.83$

Limited by qubit decay during time required for photon detection in same mode.

Extension to states with more than a single photon:

$$\frac{1}{2} [|g\rangle(|1\rangle + |2\rangle) + |e\rangle(|1\rangle - |2\rangle)]$$

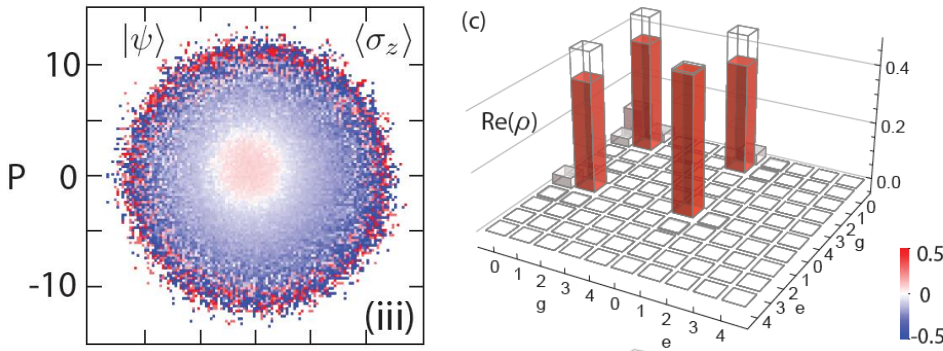
$F = 0.80$



exp: C. Eichler *et al.*, *PRL* 109, 240501 (2012)
theo: C. Eichler *et al.*, *PRA* 86, 032106 (2012)

Experiments with Propagating Quantum Microwaves

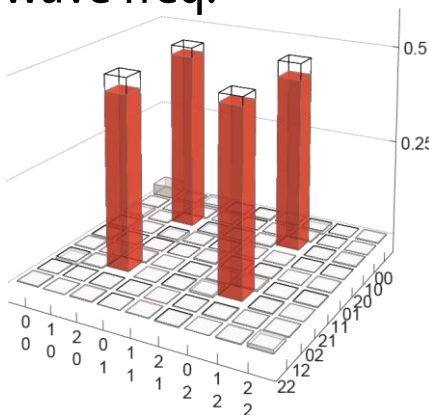
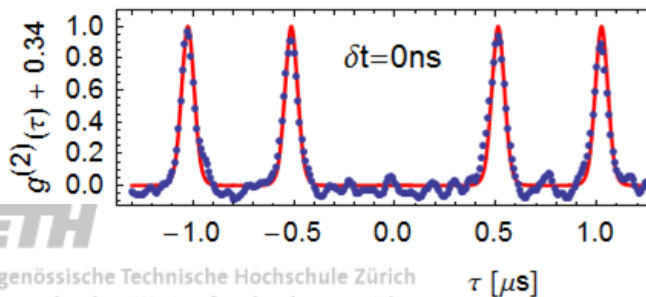
Preparation and characterization of qubit-propagating photon entanglement



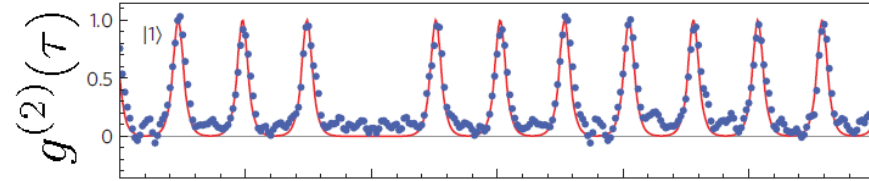
Eichler *et al.*, *PRL* 109, 240501 (2012)
 Eichler *et al.*, *PRA* 86, 032106 (2012)

Hong-Ou-Mandel: Two-photon interference incl. msrmt of coherences at microwave freq.

Lang *et al.*, *Nat. Phys.* 9, 345 (2013)



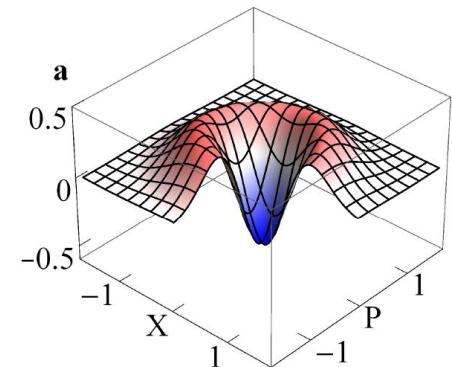
Single photon sources and their anti-bunching



Bozyigit *et al.*, *Nat. Phys* 7, 154 (2011)
 Lang *et al.*, *PRL* 107, 073601 (2011)

Full state tomography and Wigner functions of propagating photons

Eichler *et al.*, *PRL* 106, 220503 (2011)



Teleportation

Task:

- transfer unknown state of qubit (A) from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)
- a small quantum computer
- classical communication

Alice



classical communication

Bob



Features:

- exploits non-local quantum correlations
- uses many essential ingredients required for realizing a quantum computer: important benchmark to pass!

Applications:

- quantum repeaters
- simplification of quantum circuits
- universal computation

Has only been demonstrated for photons and ions. But work on solid state realizations is progressing!

Teleportation Protocol

Task:

- transfer unknown quantum state from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)

Alice



Bell measurement

Qubit: Q1 Q2



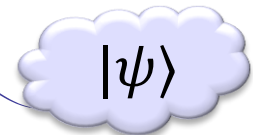
arbitrary unknown qubit state

entangled qubit state

Bob



Q3



Teleportation Protocol

Task:

- transfer unknown quantum state from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)
- classical communication

Alice



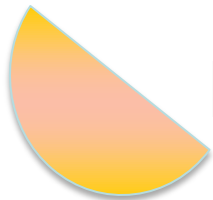
Q1



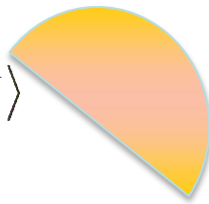
Bob



Qubit: Q1

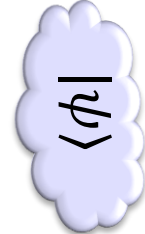


Q2



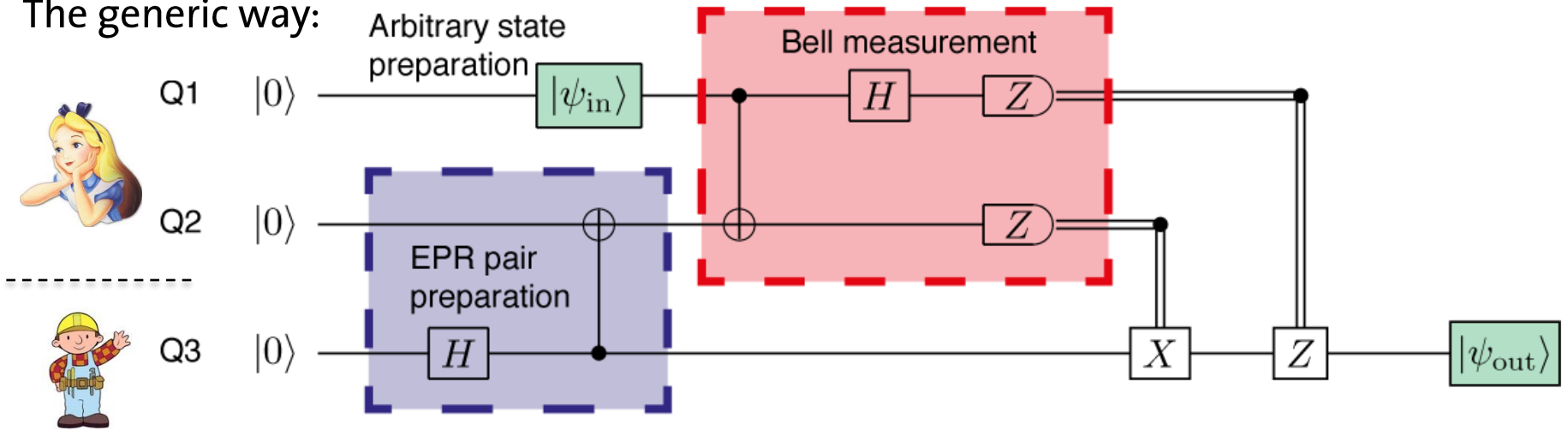
$|\Psi^+\rangle$

Q3



Implementation of the Teleportation Protocol

The generic way:



Hadamard

Rotation around Y-axis



Controlled NOT

Controlled phase gate



Measurement along Z-axis

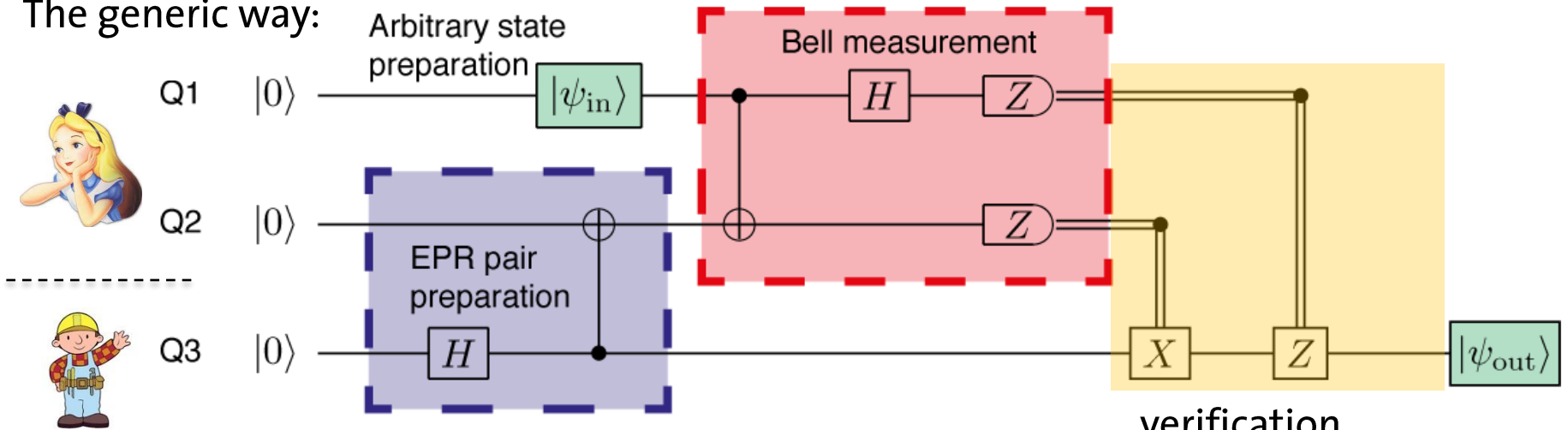
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).

implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Implementation of the Teleportation Protocol

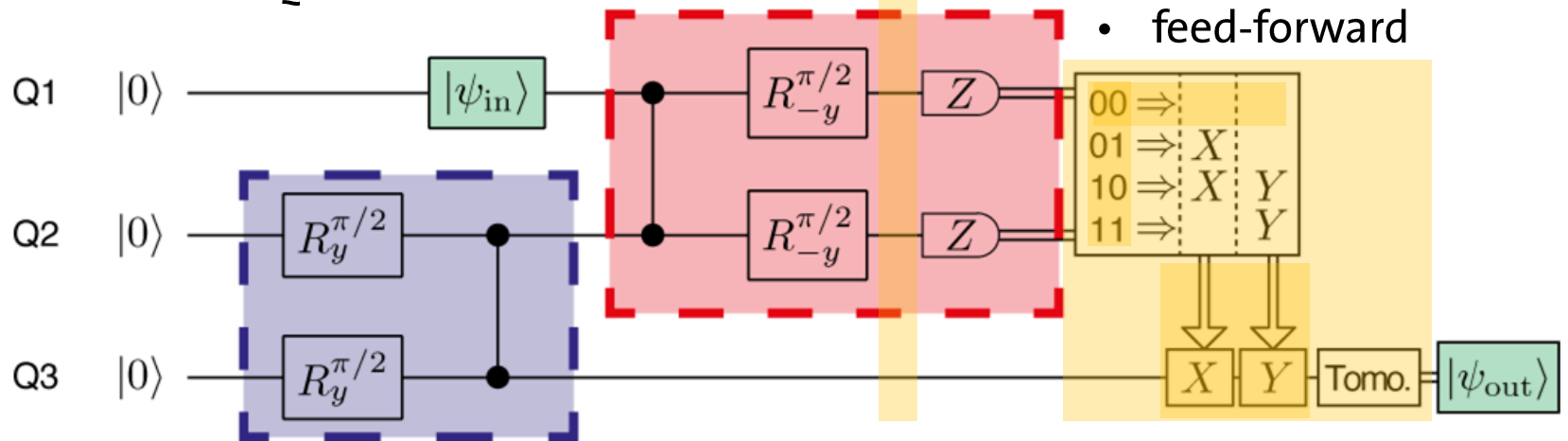
The generic way:



verification

- 3-qubit tomography
- post selection (1 state)
- deterministic (4 states)
- feed-forward

Realization in circuit QED:



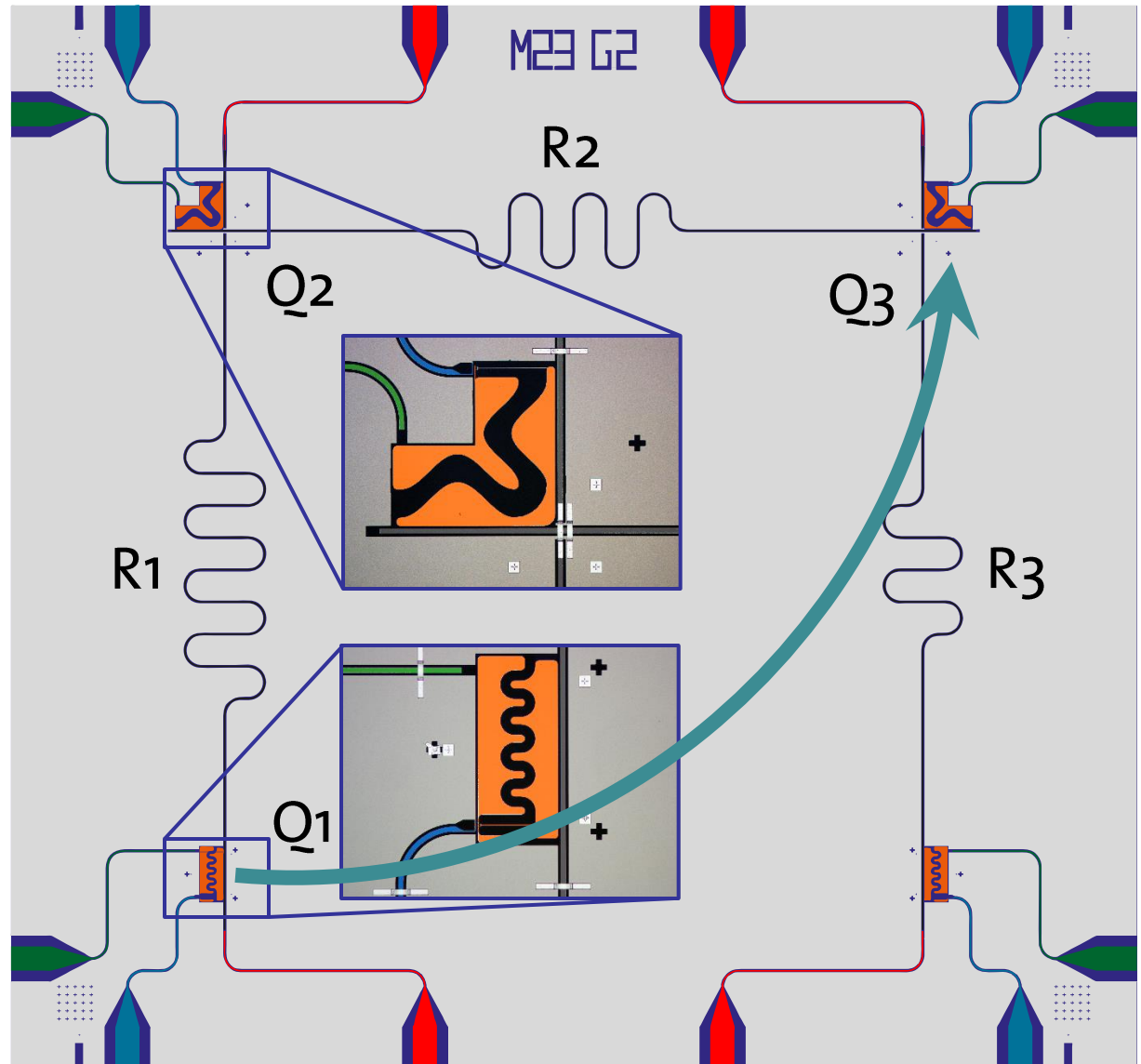
The Sample

- 3 Resonators
- 3 Qubits
- single-qubit gates
- two-qubit gates (qubits in the same resonator)
- joint single-shot readout of qubits 1 & 2
- single-shot readout of qubit 3
- with two parametric amplifiers

Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006).

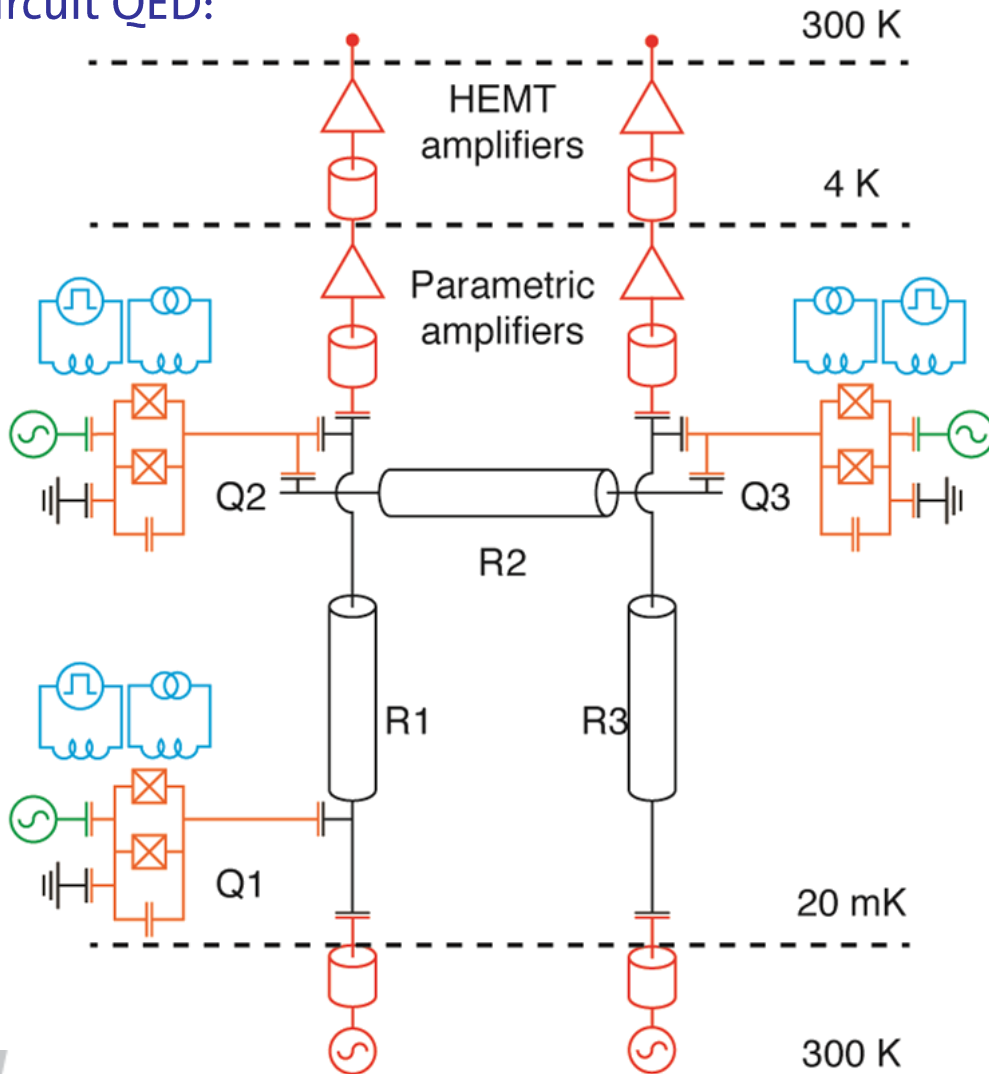
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008).

Eichler et al., *PRL* **107**, 113601 (2011).



The Circuit

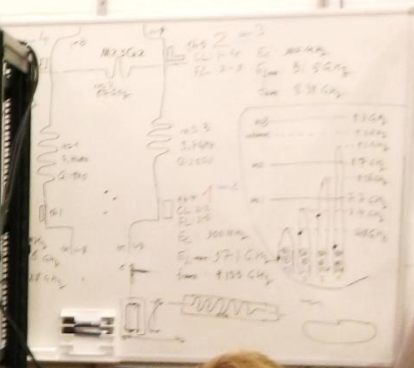
12-port quantum device based
on circuit QED:



Device highlights:

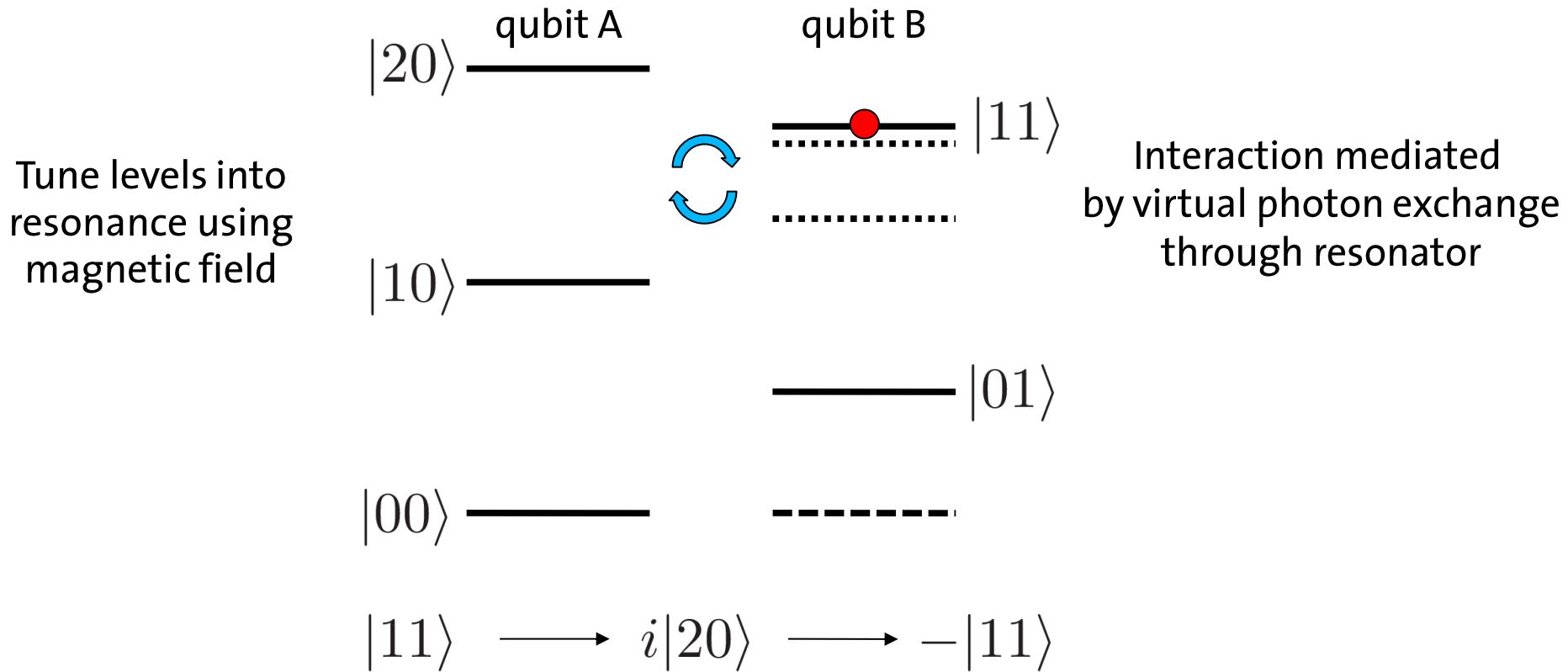
- 3 high-Q resonators
- 4 transmon **qubits**
- individual control of all qubits
- nearest neighbor interaction via quantum bus
- individual read-out for pairs of **qubits 1-2** and **3-4** through resonators
- single-shot read-out using parametric amplifiers
- qubit separation ~ 10 mm
- cross-overs for resonators

Quantum Teleportation via Superconducting Circuits



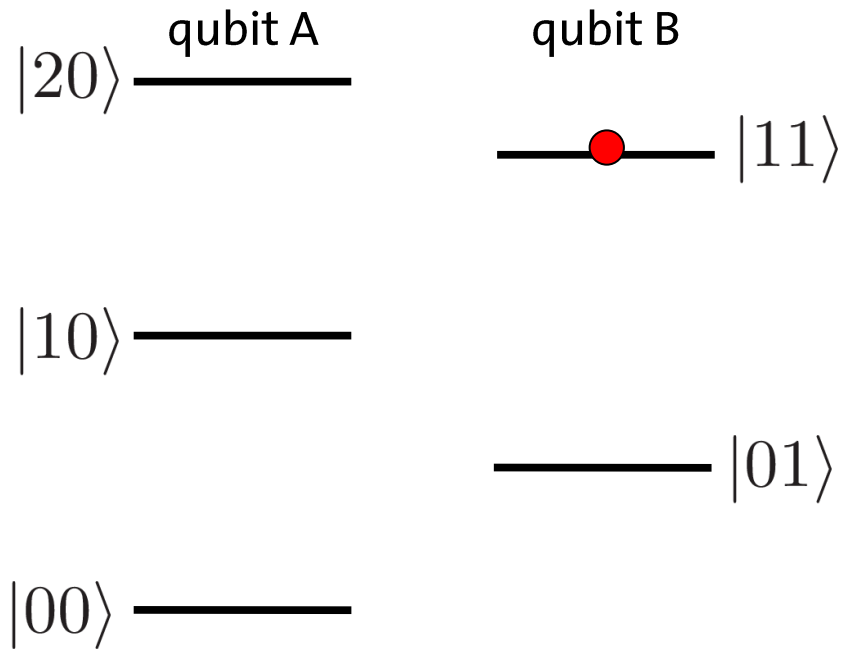
1 μm V_{cos} 2.4 GHz
10 μm V_{cos} 2.4 GHz
100 μm V_{cos} 2.4 GHz
1 mV \sin 2.4 GHz
10 mV \cos 2.4 GHz

Universal Two-Qubit Phase Gate



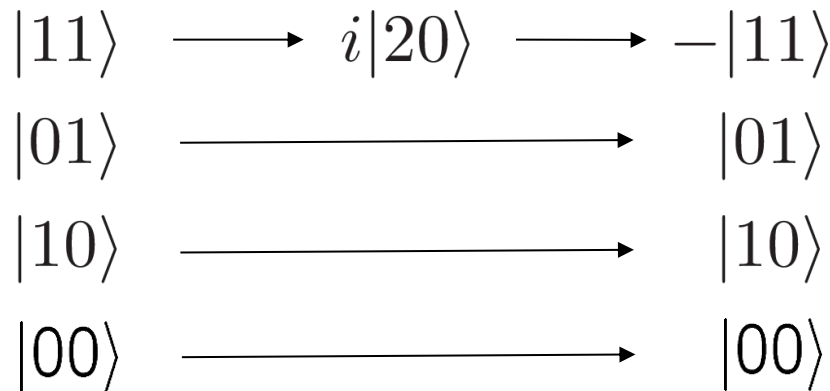
proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
 first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Controlled Phase Gate



How to verify the operation of this gate?

Universal two-qubit gate. Used together with single-qubit gates to create any quantum operation.



C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).
 first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

Process Tomography: C-Phase Gate

arbitrary quantum process

$$\rho' = \mathcal{E}(\rho)$$

decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

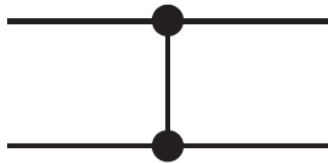
$$\{\tilde{E}_k\}$$

$$\chi$$

is an operator basis

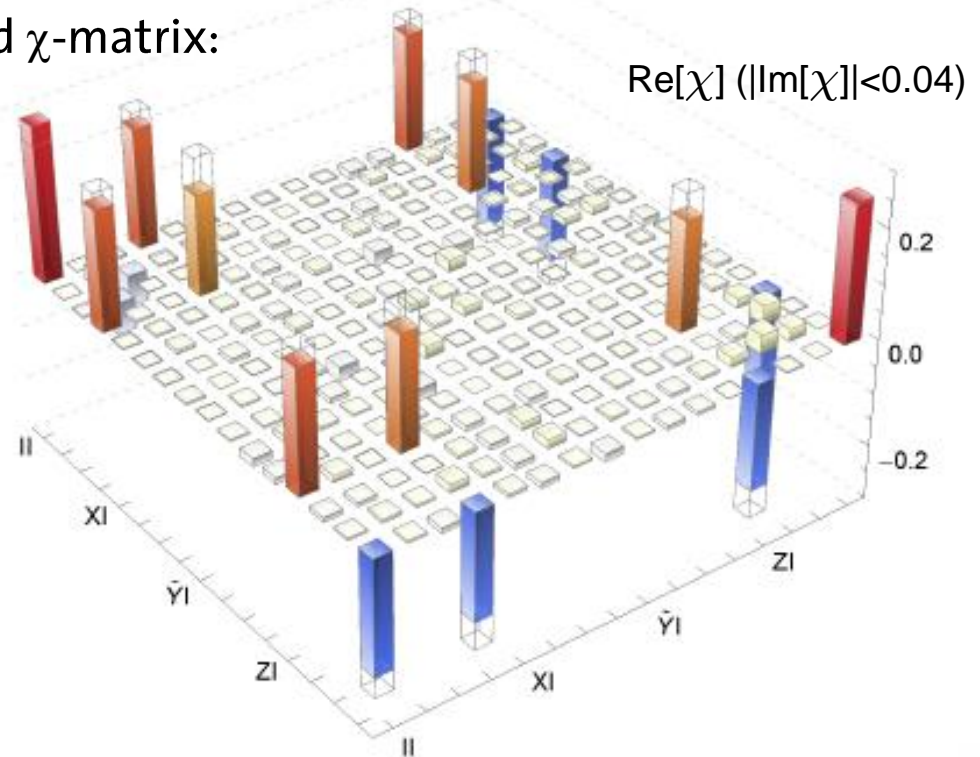
is a positive semi definite Hermitian matrix characteristic for the process

Controlled phase gate



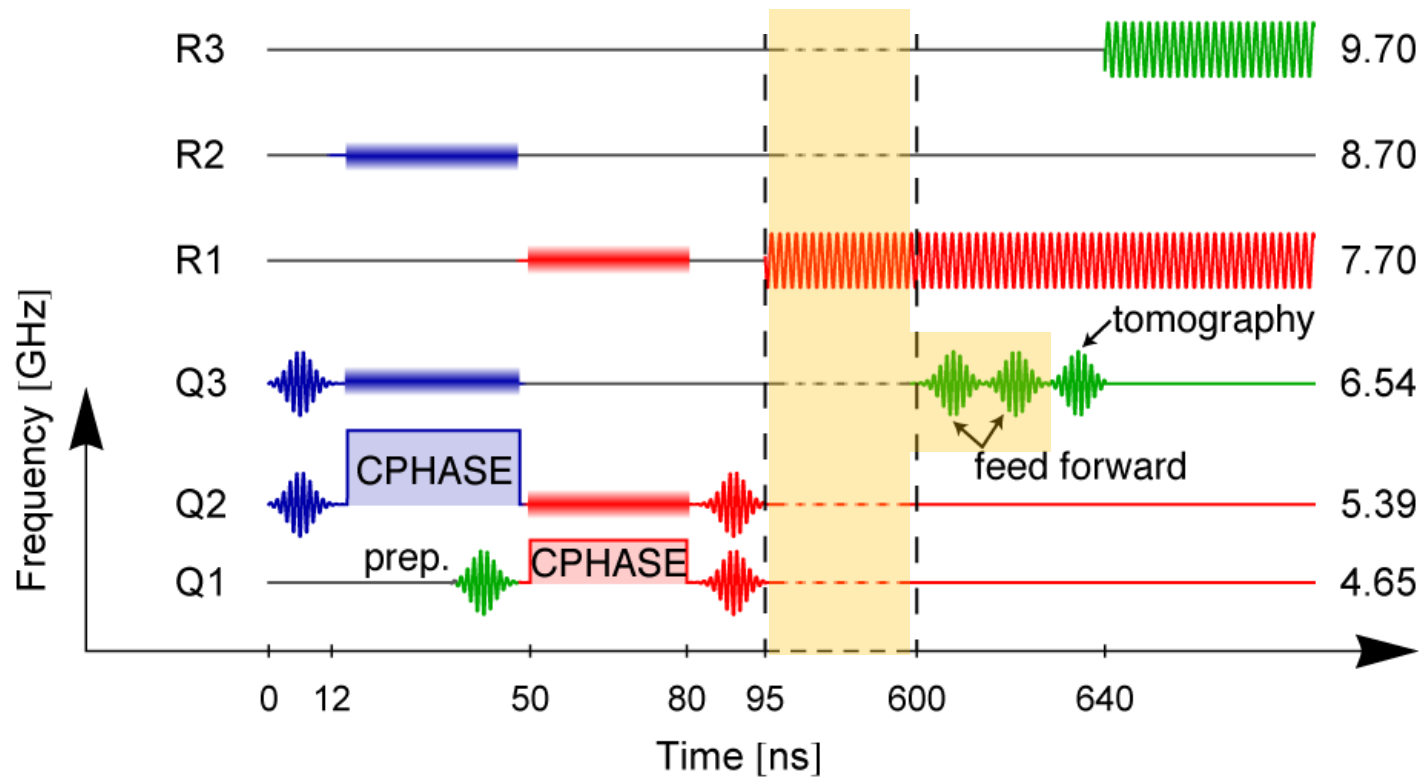
$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured χ -matrix:

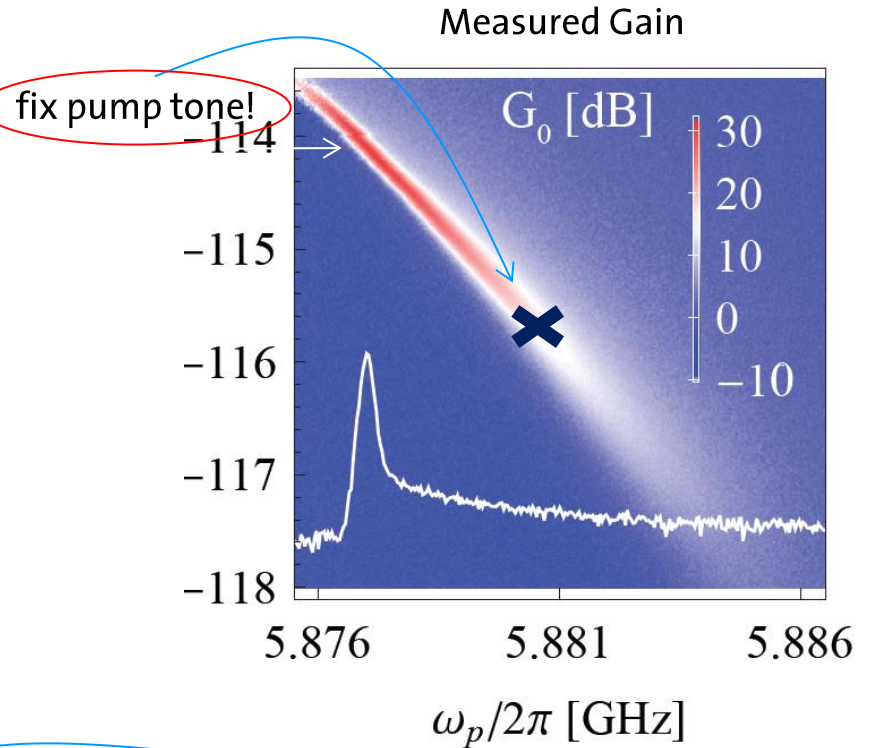
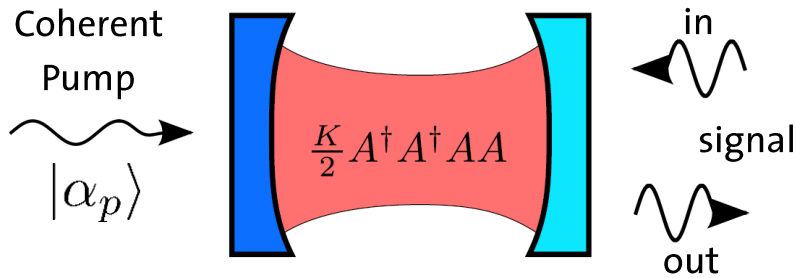


$$F = \text{Tr}[\chi_{\text{meas}} \chi_{\text{ideal}}] = 0.86$$

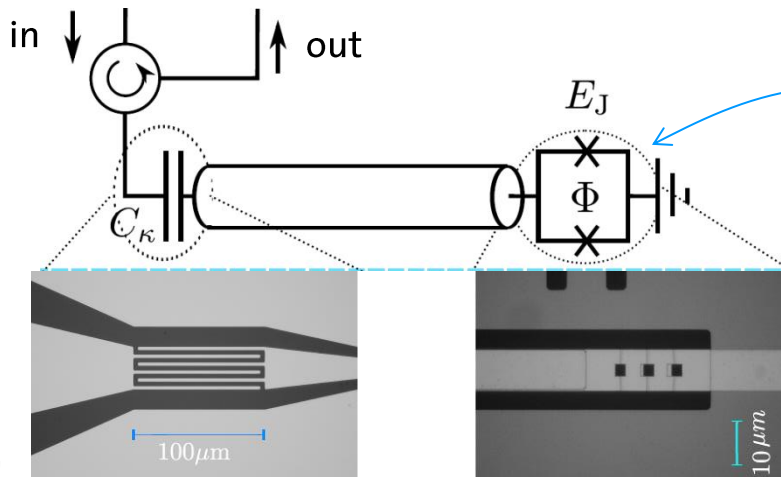
Pulse scheme



Near Quantum-Limited Parametric Amplifier



Circuit QED implementation:



SQUID provides nonlinearity!

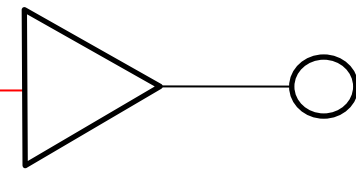
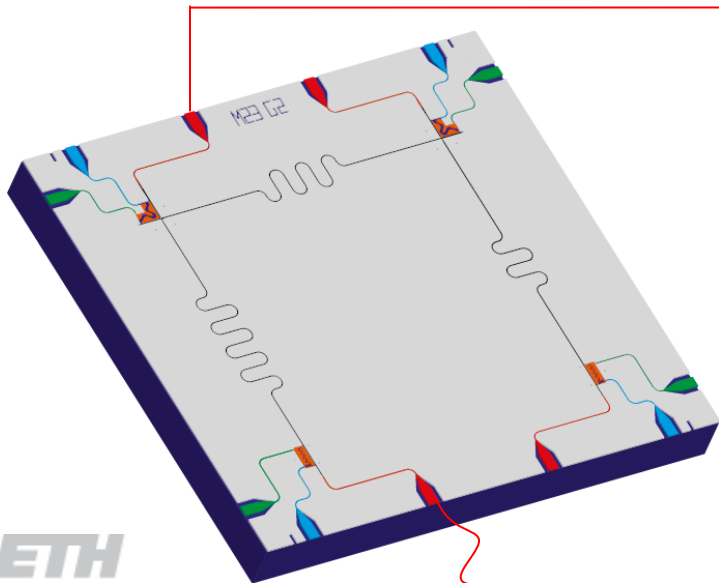
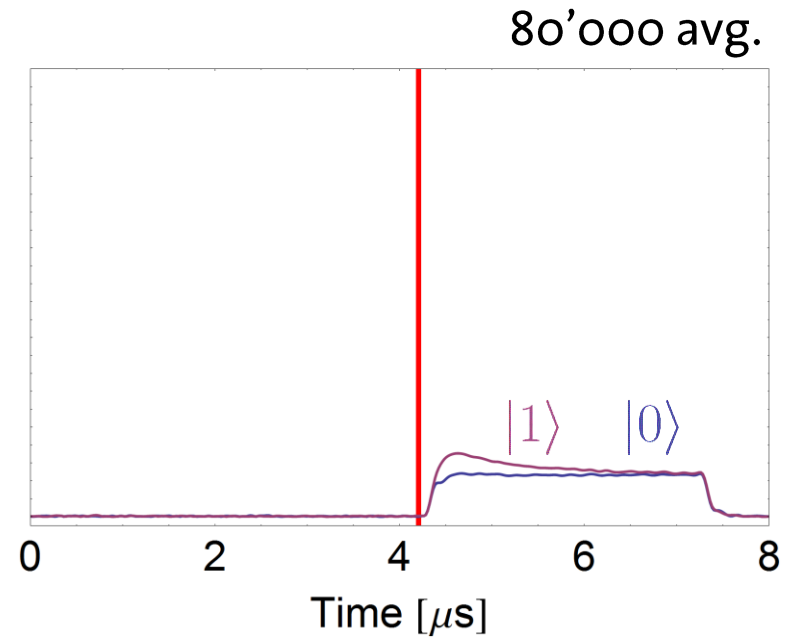
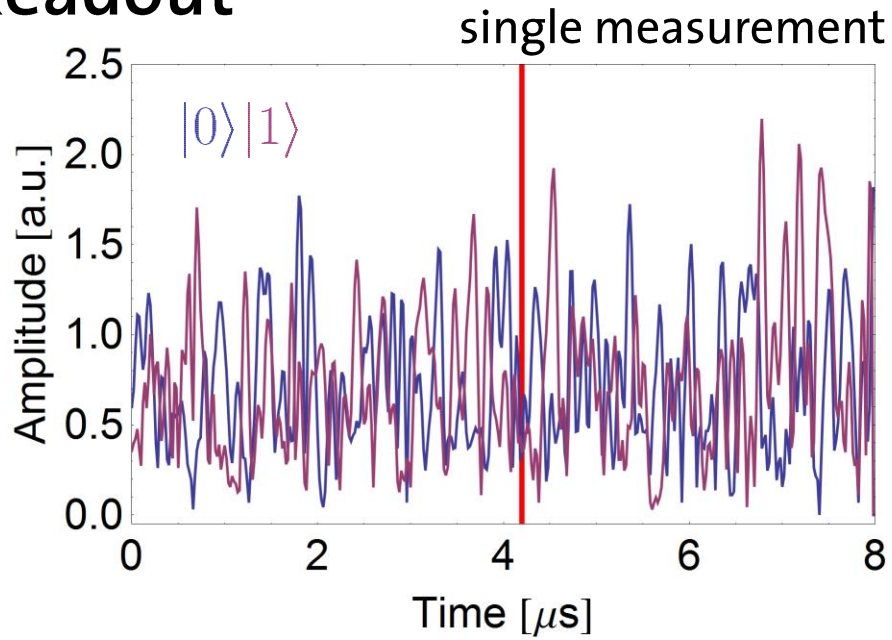
Caves, *Phys. Rev. D* 26, 1817 (1982)

Yurke and Buks, *J. Lightwave Tech.* 24, 5054 (2006)

Castellanos-Beltran et al., *Nat. Phys.* 4, 929 (2008)

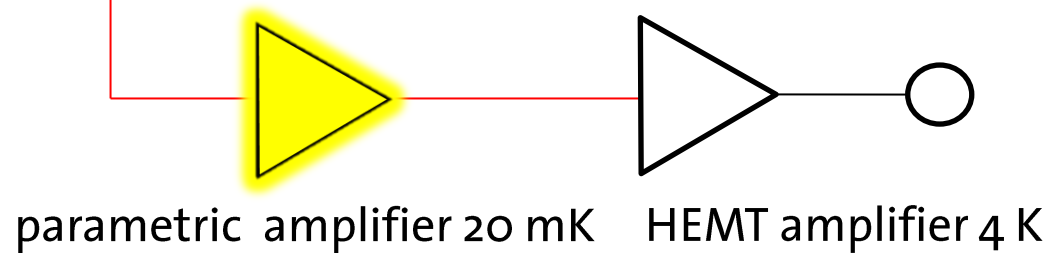
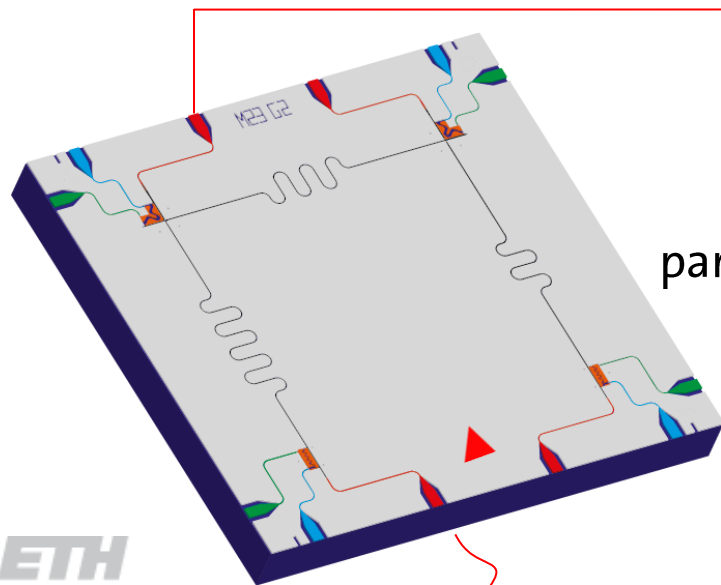
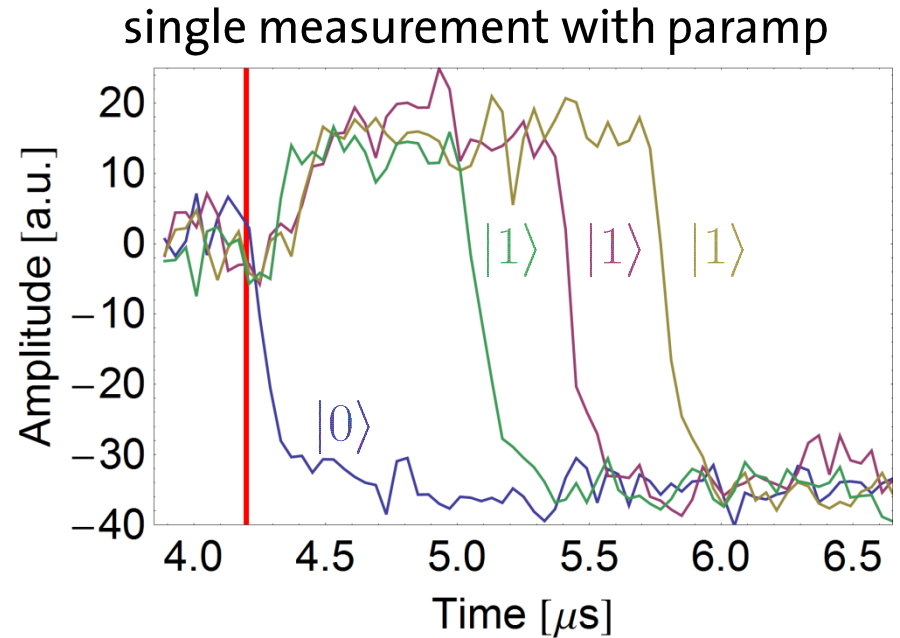
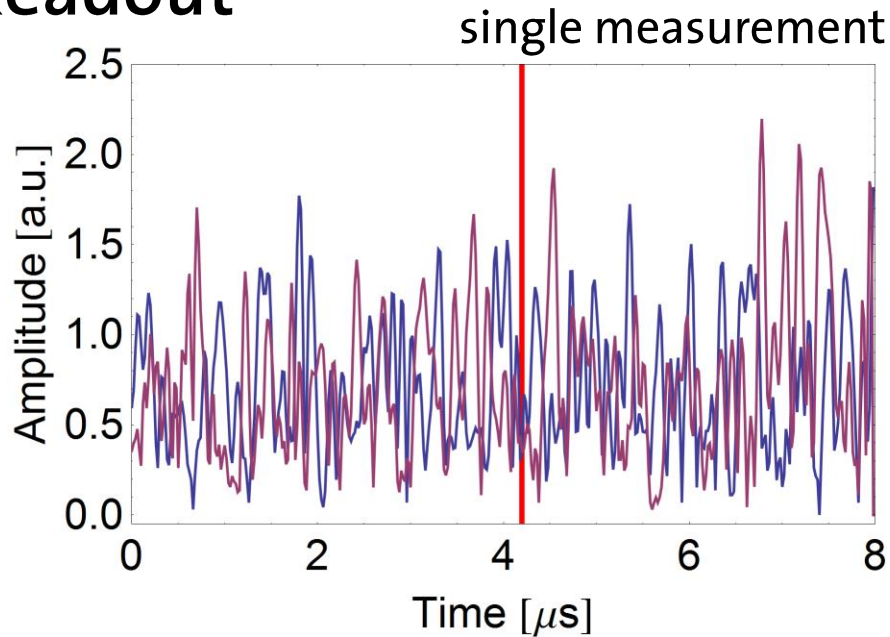
Eichler et al., *Phys. Rev. Lett.* 107, 113601 (2011)

Readout



HEMT amplifier 4 K

Readout



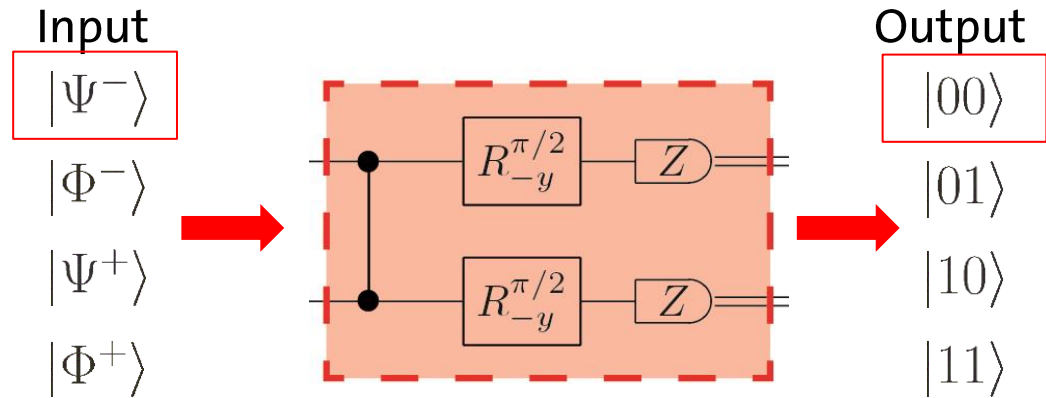
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)

Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)

Eichler et al., *PRL* **107**, 113601 (2011)

R. Vijay et al, *PRL* **106**, 110502 (2011)

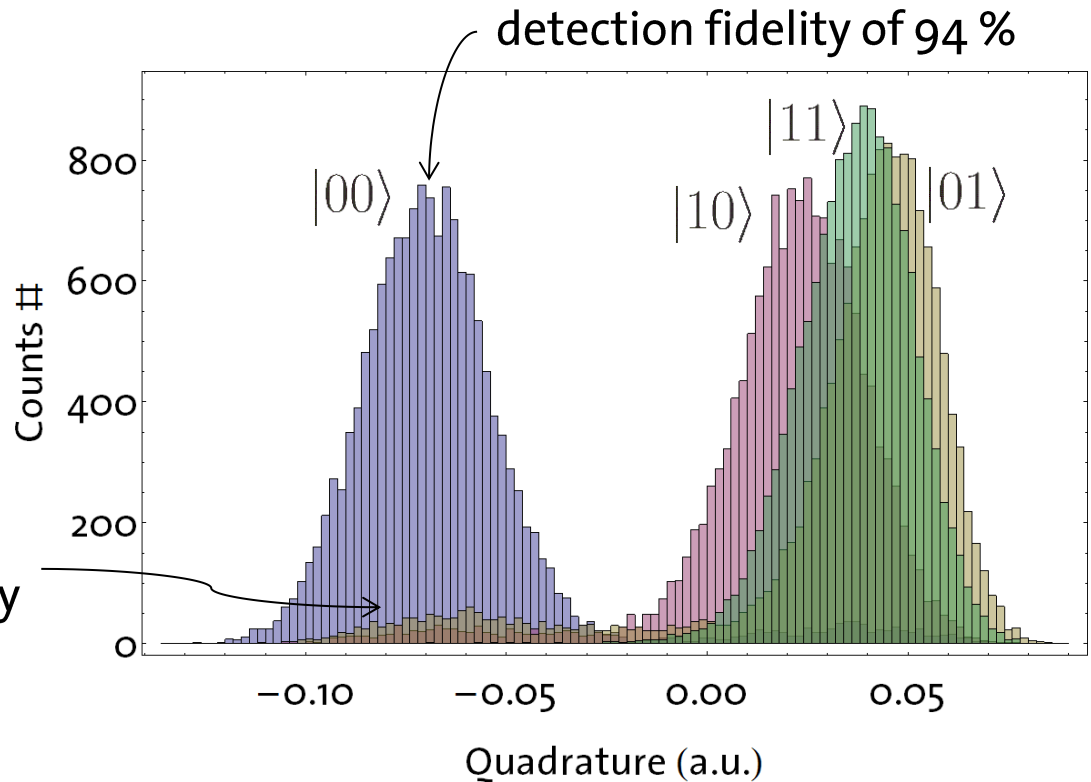
Post-Selected Teleportation: Bell Measurement



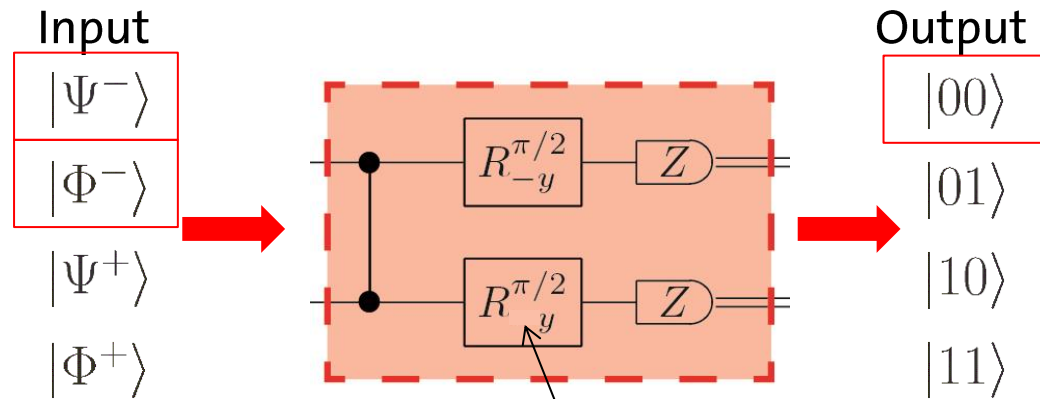
Operate parametric amplifier in phase sensitive mode

Maximize contrast of $|00\rangle$ to other states

Limited by decay



Post-Selecting Every State Individually



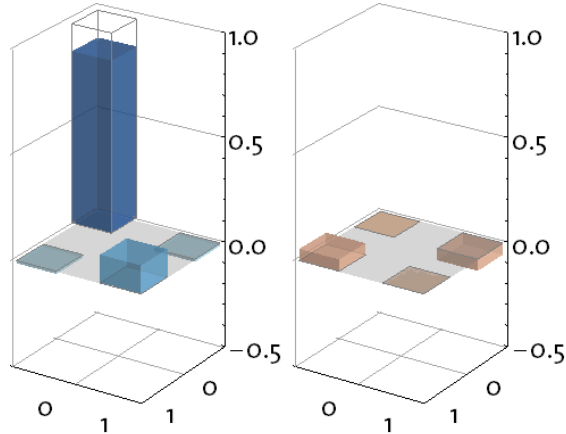
How to post-select on the other Bell states:

- Change the phases of the $\pi/2$ pulses
- Another Bell state is transformed to the $|00\rangle$ state
- Possibility to post-select on all four Bell states

Tomography of Teleported States with Post-Selection

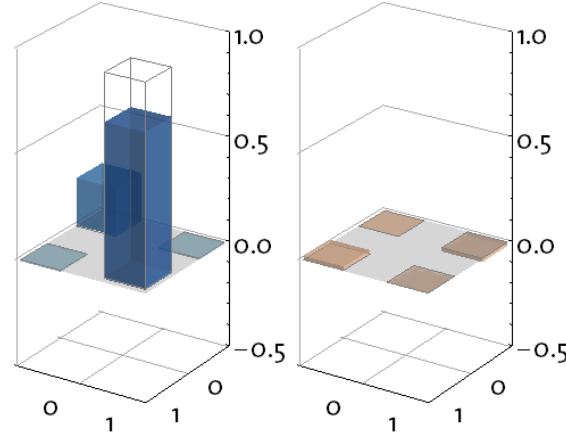
$$\psi_{in} = |0\rangle$$

82.2 %



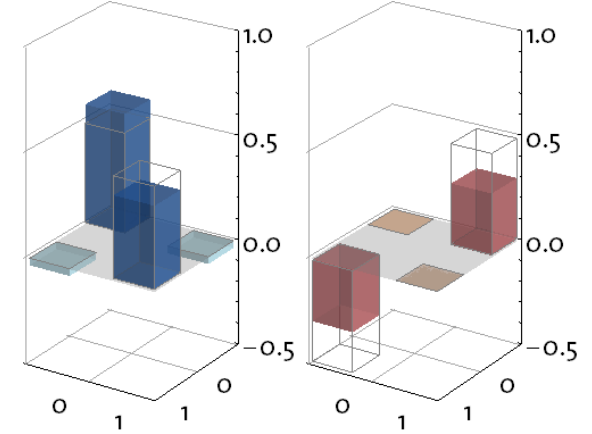
$$\psi_{in} = |1\rangle$$

80.5 %



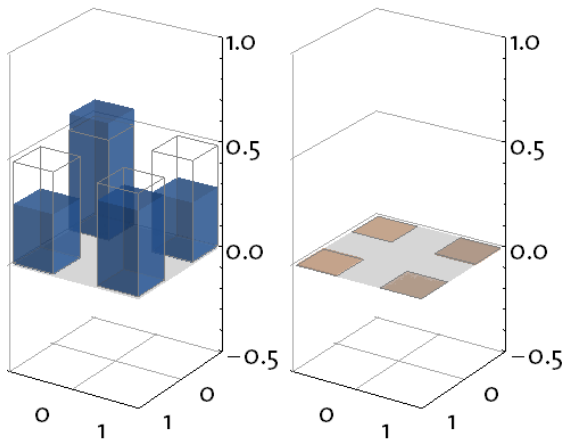
$$\psi_{in} = |0\rangle - i|1\rangle$$

79.4 %



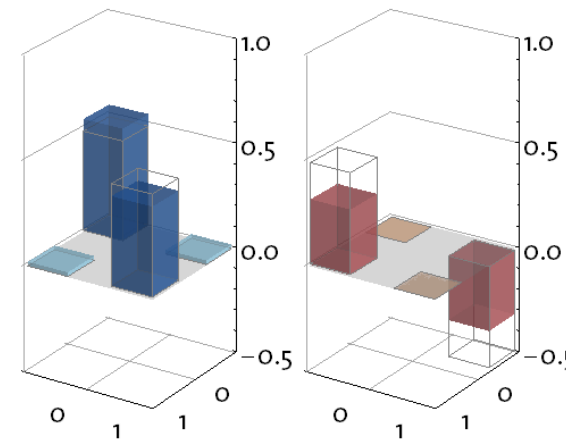
$$\psi_{in} = |0\rangle + |1\rangle$$

84.2 %



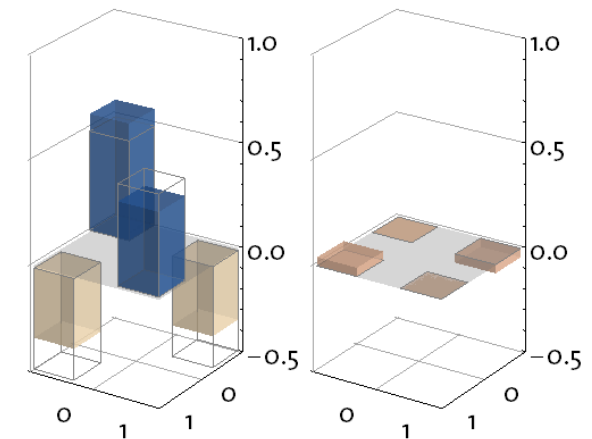
$$\psi_{in} = |0\rangle + i|1\rangle$$

79.5 %



$$\psi_{in} = |0\rangle - |1\rangle$$

83.6 %



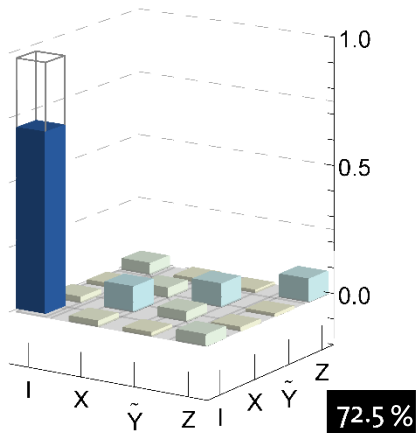
Average state fidelity **78.1 ± 0.9 %**

Steffen *et al.*, *Nature* in print (2013), [arxiv1302.5621](https://arxiv.org/abs/1302.5621)

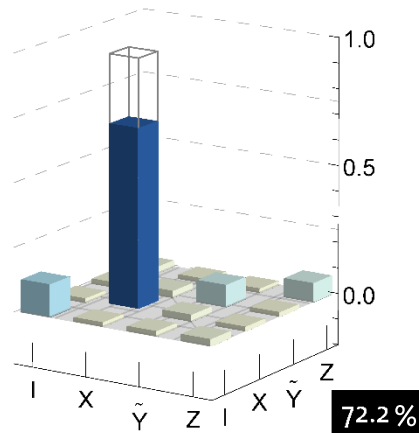
Process Tomography: Teleportation with Post-Selection

absolute value of process matrices $|\chi|$ for state transfer from qubit 1 to qubit 3:

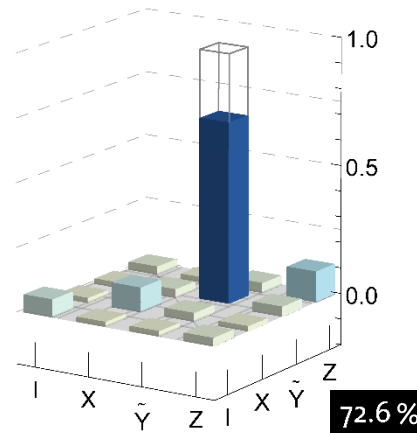
$|00\rangle \hat{=} |\Phi^-\rangle$



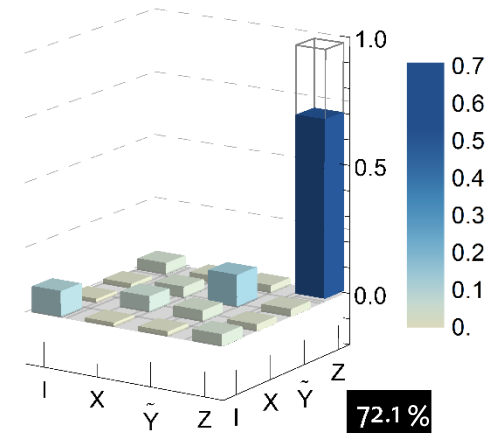
$|01\rangle \hat{=} |\Psi^-\rangle$



$|11\rangle \hat{=} |\Psi^+\rangle$



$|10\rangle \hat{=} |\Phi^+\rangle$



$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = X |\psi_{\text{in}}\rangle$$

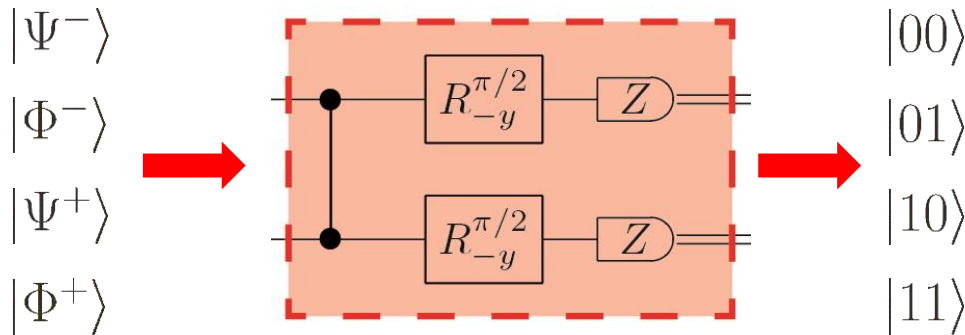
$$|\psi_{\text{out}}\rangle = \tilde{Y} |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = Z |\psi_{\text{in}}\rangle$$

$$X = \hat{\sigma}_x, \tilde{Y} = i\hat{\sigma}_y, Z = \hat{\sigma}_z$$

Average process fidelity **72.3 ± 0.7 %**

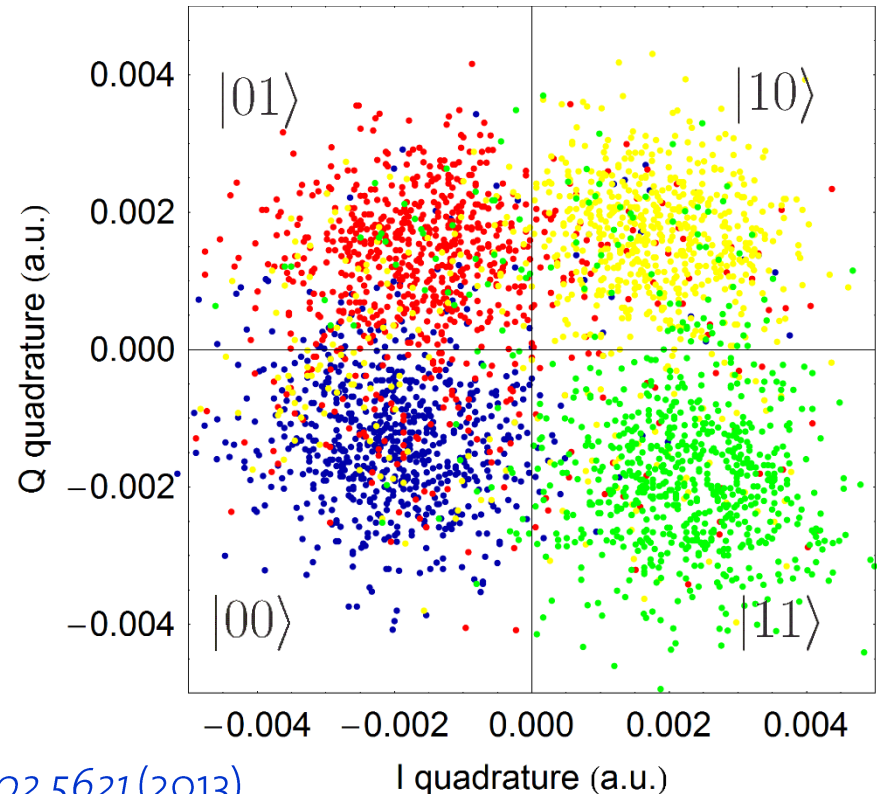
Deterministic Measurement of all 4 Bell States



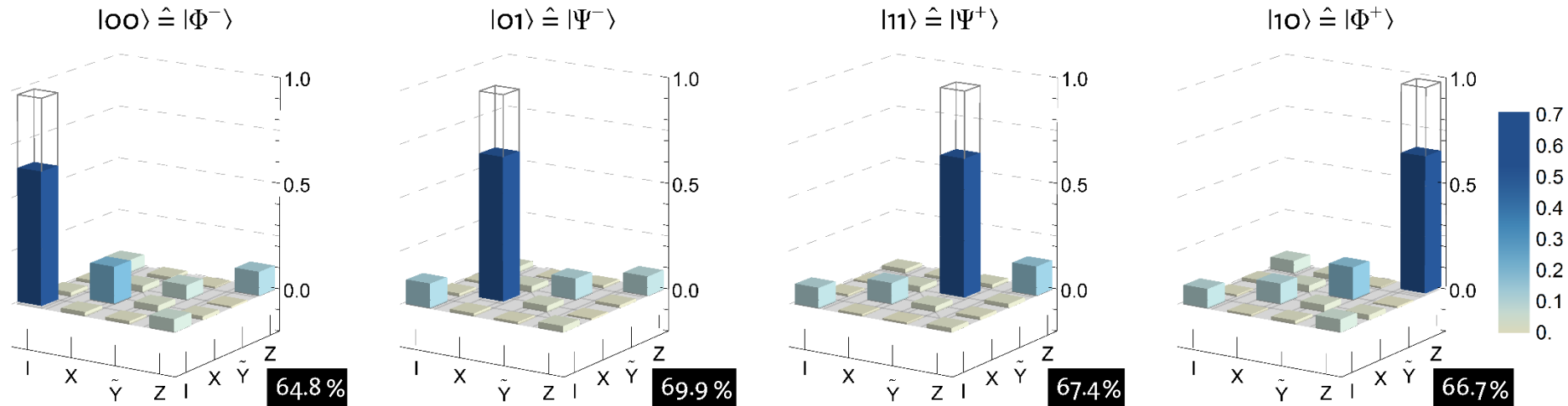
- operate paramp in the phase preserving mode
- both quadratures are amplified equally

States are identified correctly with ~80% probability

	upper left	upper right	lower left	lower right
identified as	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$
$ 00\rangle$	0.86	0.09	0.02	0.02
$ 01\rangle$	0.14	0.73	0.04	0.09
$ 10\rangle$	0.03	0.05	0.84	0.09
$ 11\rangle$	0.08	0.10	0.09	0.73



Teleportation with Deterministic Bell Measurement



$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = X |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = \tilde{Y} |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = Z |\psi_{\text{in}}\rangle$$

Average process fidelity **67.1 ± 0.5 %**

Average state fidelity **78.1 ± 0.9 %**

$$\mathcal{F}_p = (\mathcal{F}_s(d + 1) - 1)/d$$

Feed-Forward Characterization

process tomography for qubit 3 assuming input = $|\psi\rangle$

preparation:

$$|00\rangle \otimes |\psi\rangle$$

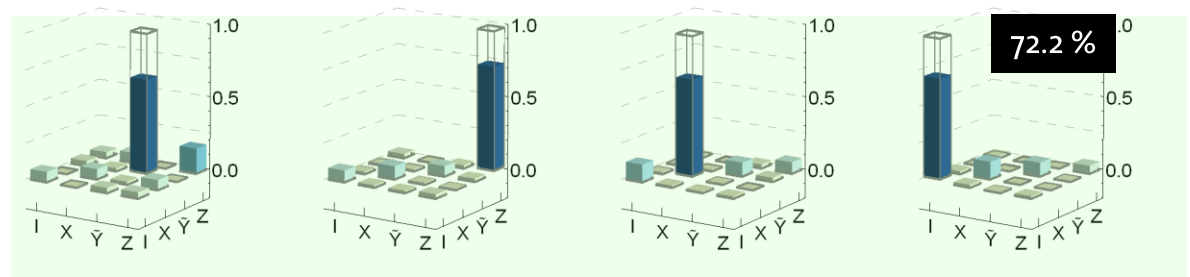
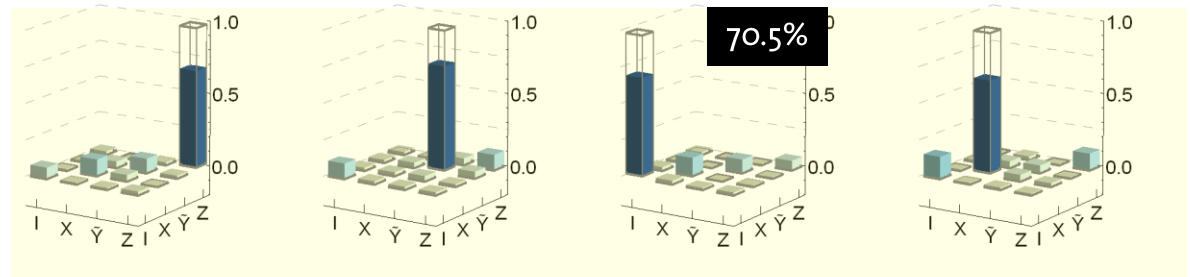
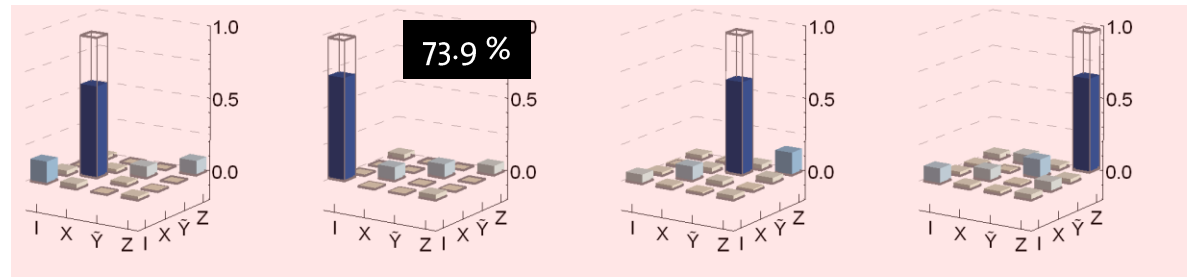
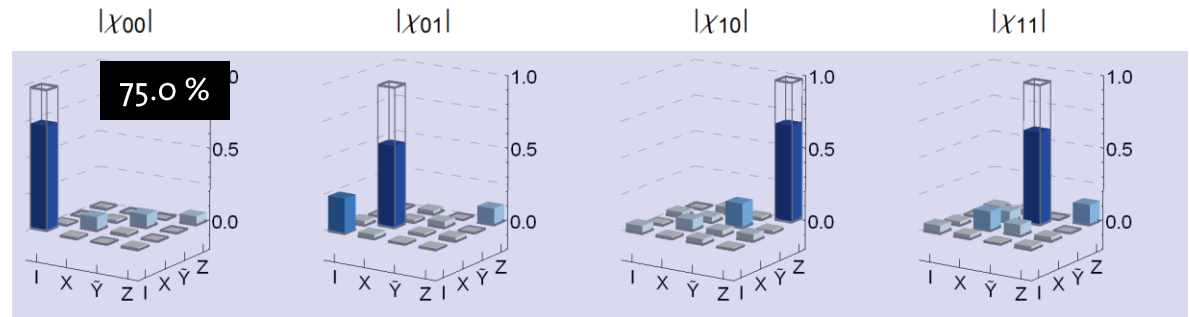
$$|01\rangle \otimes \sigma_x |\psi\rangle$$

$$|10\rangle \otimes \sigma_z |\psi\rangle$$

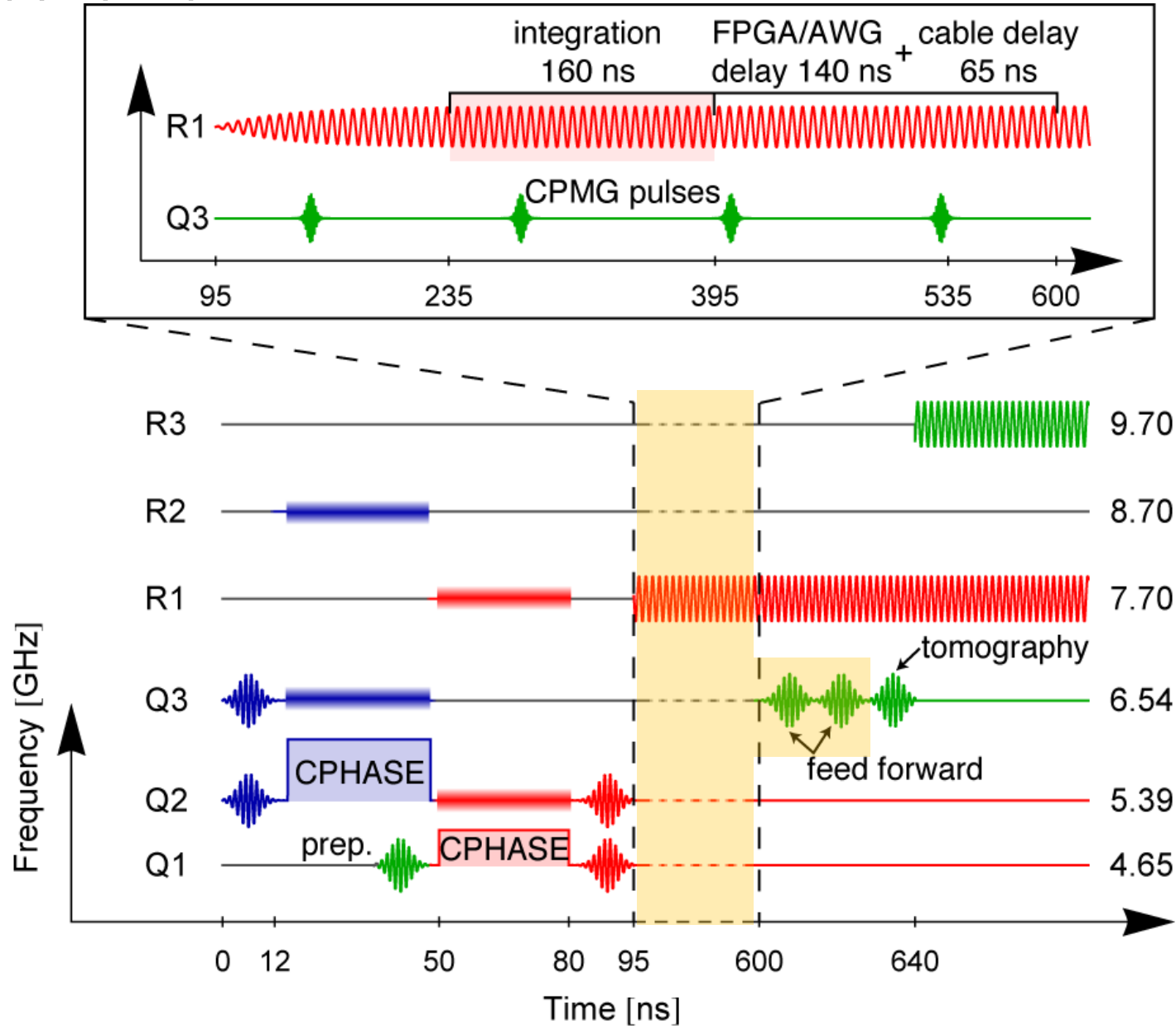
$$|11\rangle \otimes i\sigma_y |\psi\rangle$$

readout of qubit 1 and 2

	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$
$ 00\rangle$	0.91	0.05	0.02	0.02
$ 01\rangle$	0.1	0.81	0.03	0.05
$ 10\rangle$	0.04	0.04	0.8	0.12
$ 11\rangle$	0.06	0.03	0.11	0.8

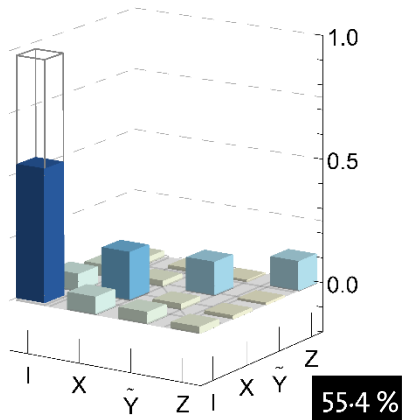


Pulse scheme

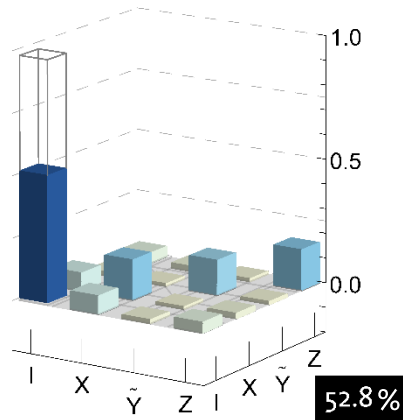


Teleportation Process with Feed-Forward

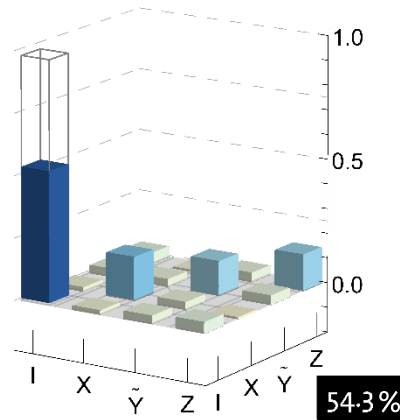
$$|00\rangle \hat{=} |\Phi^-\rangle$$



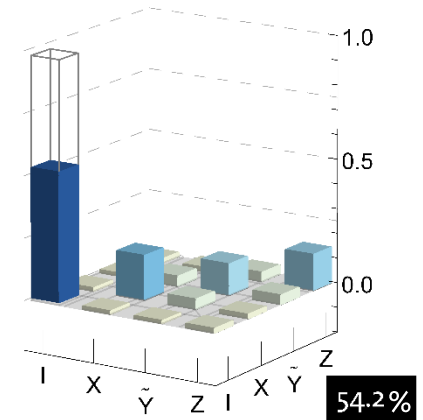
$$|01\rangle \hat{=} |\Psi^-\rangle$$



$$|11\rangle \hat{=} |\Psi^+\rangle$$



$$|10\rangle \hat{=} |\Phi^+\rangle$$



$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

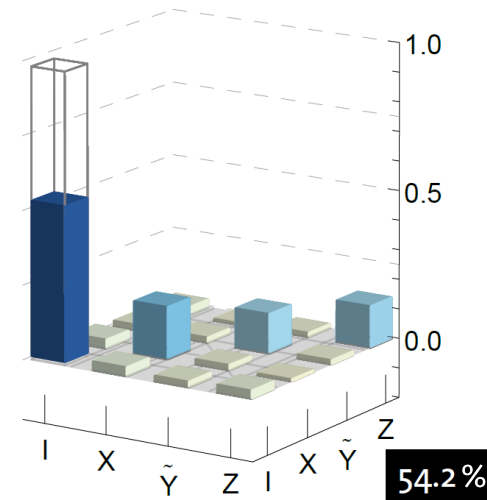
$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

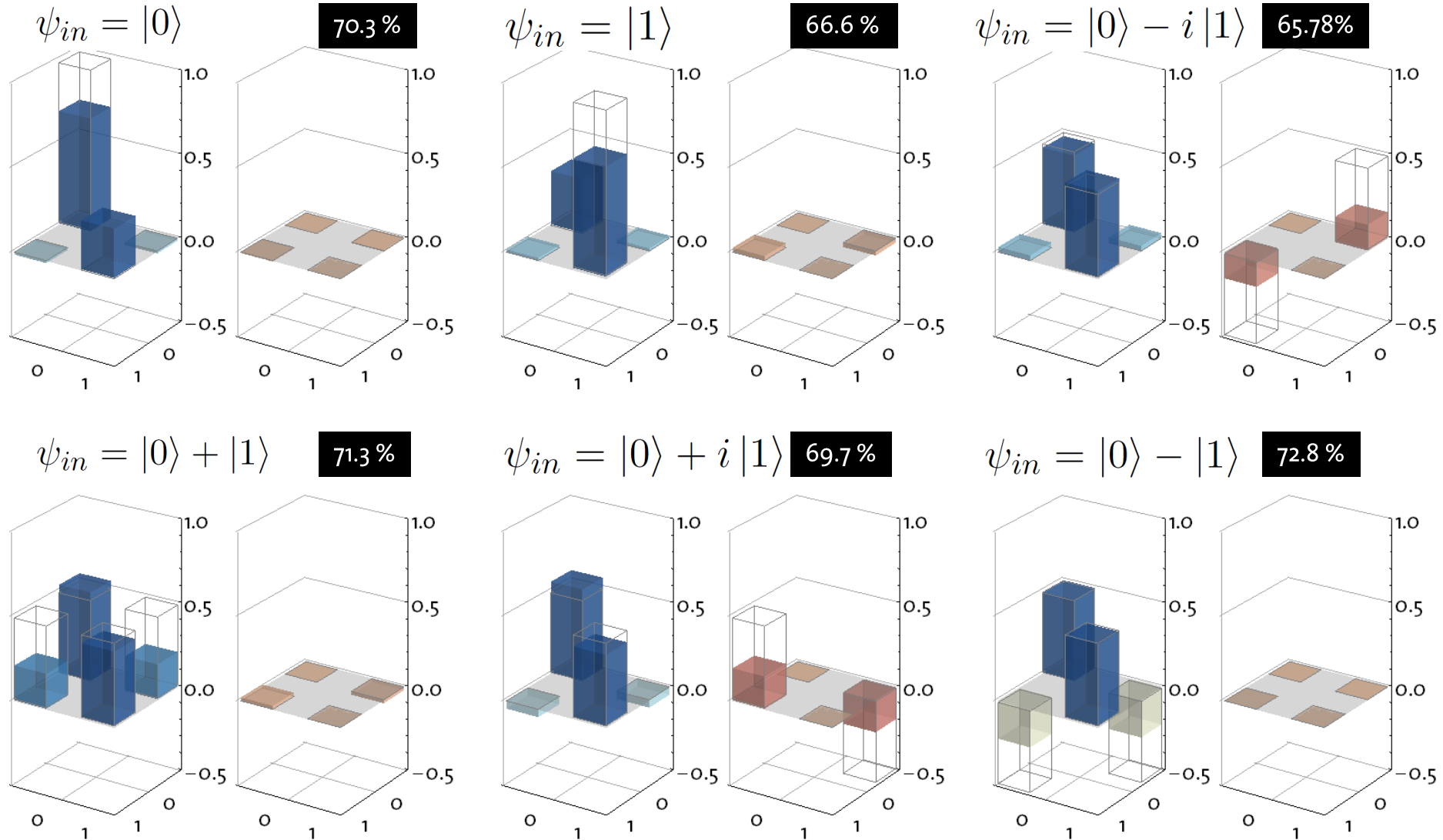
$$|\psi_{\text{out}}\rangle = |\psi_{\text{in}}\rangle$$

Average process fidelity **54.2 ± 0.1 %**

$$|\chi\rangle$$



Tomography of Teleported States with Feed-Forward



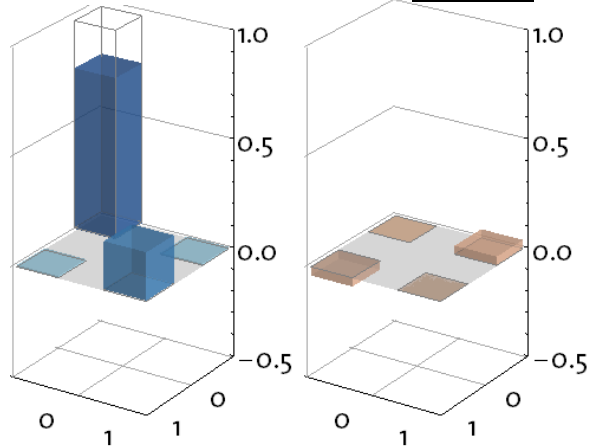
Average state fidelity of **69.5±0.1 %**

Tomography of Teleported States with Feed-Forward

averaged readout of qubit 3

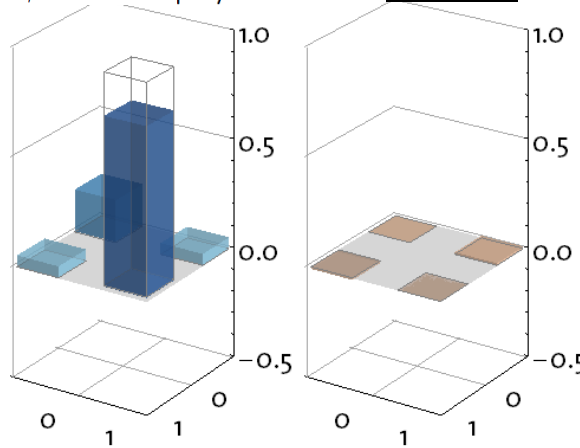
$$\psi_{in} = |0\rangle$$

77.5 %



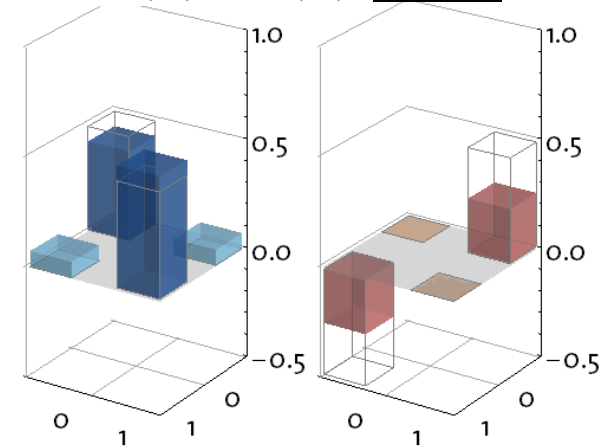
$$\psi_{in} = |1\rangle$$

79.9 %



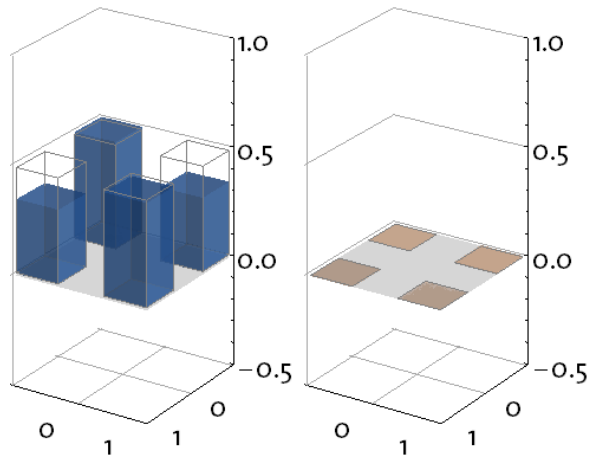
$$\psi_{in} = |0\rangle - i|1\rangle$$

76.2 %



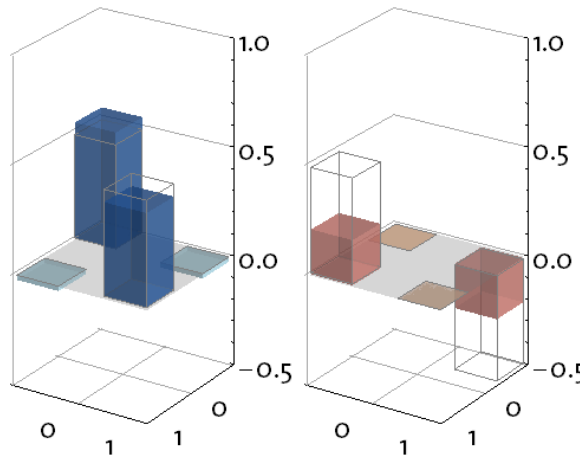
$$\psi_{in} = |0\rangle + |1\rangle$$

85.3 %



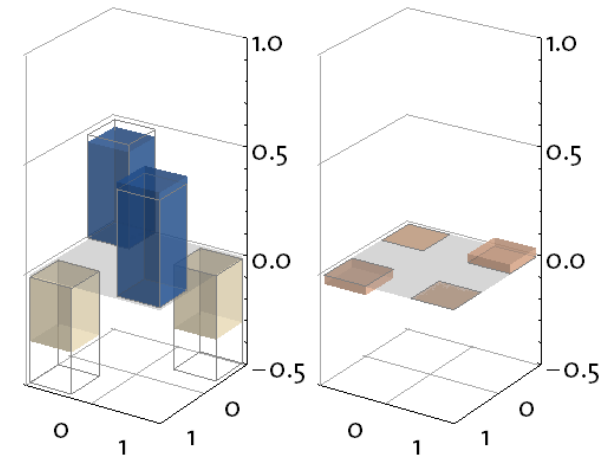
$$\psi_{in} = |0\rangle + i|1\rangle$$

71.2 %



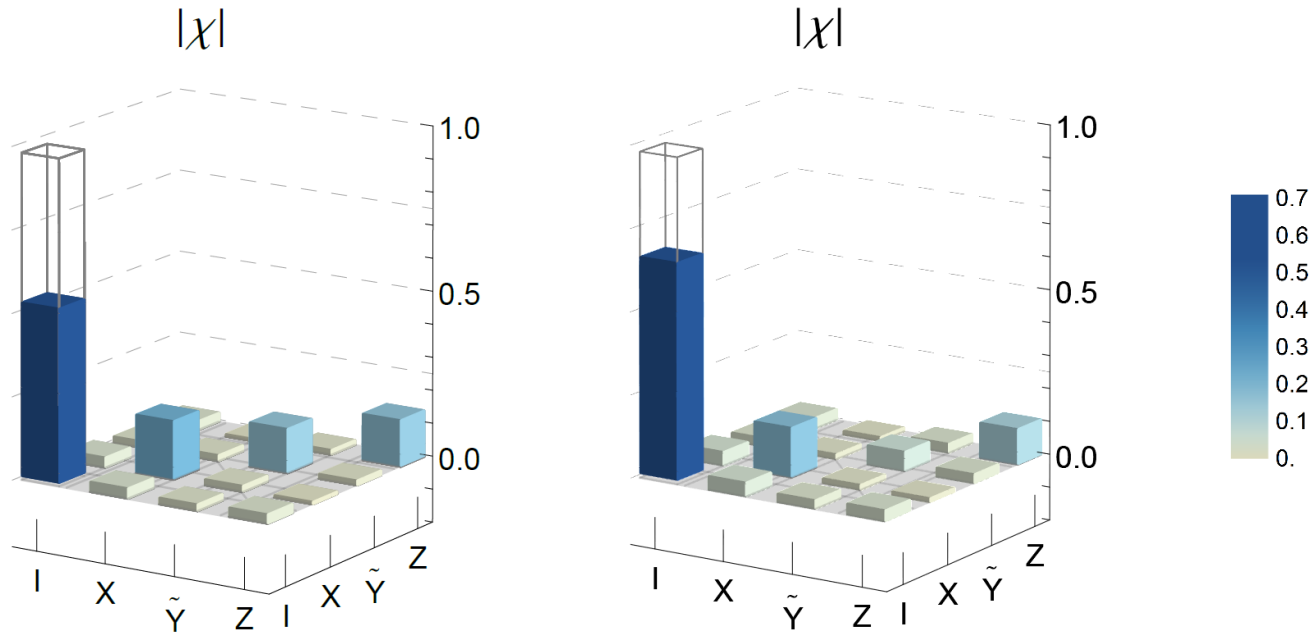
$$\psi_{in} = |0\rangle - |1\rangle$$

80.7 %



Average state fidelity of **78.5 ± 0.9%**

Teleportation Process with Feed-Forward

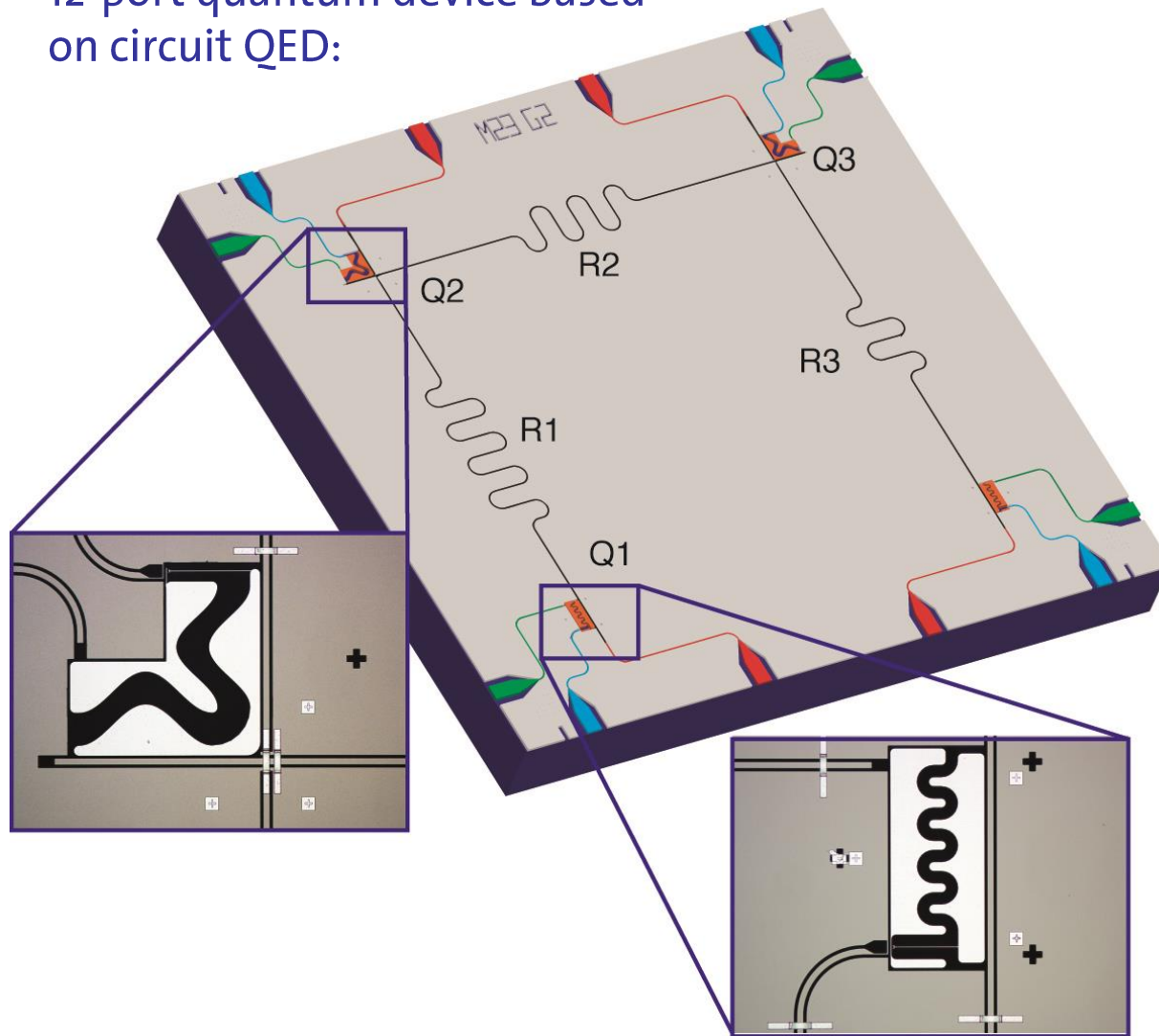


Average process fidelity with single shot readout: **$54.2 \pm 0.1 \%$**

Average process fidelity with averaged readout: **$67.7 \pm 1.1 \%$**

Teleportation

12-port quantum device based on circuit QED:



Experimental highlights:

- teleportation in a (macroscopic) solid state system
- post-selection on either of 4 Bell states individually
- simultaneous and det. measurement of all 4 Bell states
- implementation of feed-forward
- fidelities $>$ classical threshold
- $O(1)$ success probability
- teleportation rate $>$ 10 kHz
- distance \sim 10 mm

Next steps:

- improve fidelities
- increase distances
- apply teleportation

The ETH Zurich Quantum Device Lab

Postdoc and PhD positions available



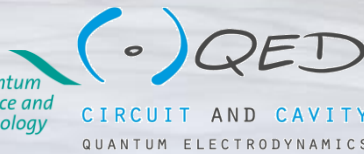
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Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



National Centre of Competence in Research



Selected Circuit QED Publications

Circuit QED Proposal:

- Blais et al., *PRA* **69**, 062320 (2004)

Strong Coupling & Vacuum Rabi Mode Splitting:

- Wallraff et al., *Nature* **431**, 162 (2004)
- Fink et al., *Nature* **454**, 315 (2008)
- Fink et al., *PRL* **105**, 163601 (2010)

Tavis-Cummings Multi-Atom QED:

- Fink et al., *PRL* **103**, 083601 (2009)

AC-Stark & Lamb Shift, Autler-Townes and Mollow Transitions

- Schuster et al., *PRL* **94**, 123062 (2005)
- Gambetta et al., *PRA* **74**, 042318 (2006)
- Schuster et al., *Nature* **445**, 515 (2007)
- Fragner et al., *Science* **322**, 1357 (2008)
- Baur et al., *PRL* **102**, 243602 (2009)

Device Fabrication:

- Frunzio et al., *IEEE Trans. Appl. Sup.* **15**, 860 (2005)
- Goeppl et al., *J. Appl. Phys.* **104**, 113904 (2008)

Geometric Phases:

- Leek et al., *Science* **318**, 1889 (2007)
- Pechal et al., *PRL* **108**, 170401 (2012)
- Abdumalikov et al., *Nature* **496**, 482 (2013)

One-, Two-, Three-Qubit Gates, Algorithms and Teleportation:

- Wallraff et al., *PRL* **95**, 060501 (2005)
- Blais et al., *PRA* **75**, 032329 (2007)
- Wallraff et al., *PRL* **99**, 050501 (2007)
- Majer et al., *Nature* **449**, 443 (2007)
- Leek et al., *PRB* **79**, 180511(R) (2009)
- Filipp et al., *PRL* **102**, 200402 (2009)
- Leek et al., *PRL* **104**, 100504 (2010)
- Bianchetti et al., *PRL* **105**, 223601 (2010)
- Fedorov et al., *Nature* **481**, 170 (2012)
- Baur et al., *PRL* **108**, 040502 (2012)
- Steffen et al., *PRL* **108**, 260506 (2012)
- Steffen et al., *Nature* in print (2013), *arxiv*1302.5621

Review (gr.):

- Wallraff, *Physik Journal* **7** (12), 39 (Dez. 2008)

Additional Information: www.qudev.ethz.ch

Selected Circuit QED Publications (cont'd)

Itinerant Photons, Tomography, Photon Blockade, Correlation Functions, Qubit-Photon Entanglement, Hong-Ou-Mandel Effect:

- da Silva et al., *PRA* **82**, 043804 (2010)
- Bozyigit et al., *Nat. Phys.* **7**, 154 (2011)
- Eichler et al., *PRL* **106**, 220503 (2011)
- Lang et al., *PRL* **106**, 243601 (2011)
- Eichler et al., *PRL* **107**, 113601 (2011)
- Eichler et al., *PRA* **86**, 032106 (2012)
- Eichler et al., *PRL* **109**, 240501 (2012)
- Lang et al., *Nat. Phys.* **9**, 345 (2013)

Hybrid Systems: Quantum Dots

- Frey et al., *PRL* **108**, 046807 (2012)
- Frey et al., *PRB* **86**, 115303 (2012)

Hybrid Systems: Rydberg Atoms

- Hogan et al., *PRL* **108**, 063004 (2012)