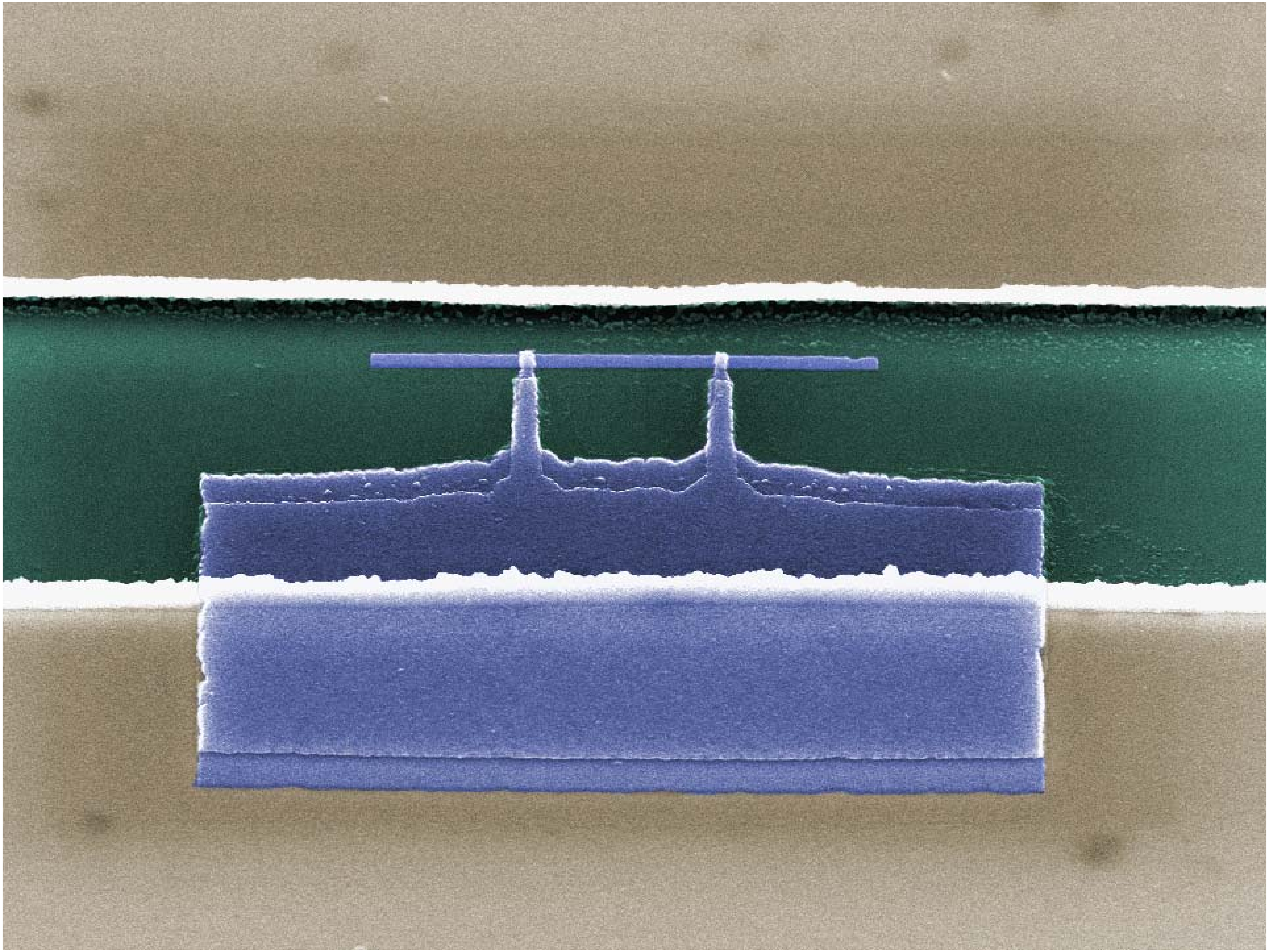
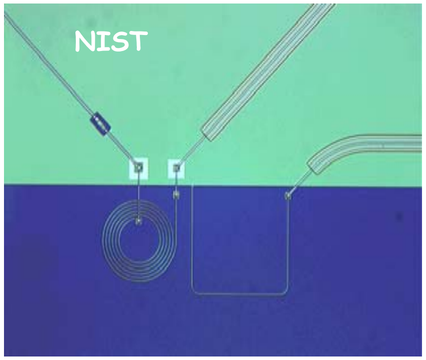
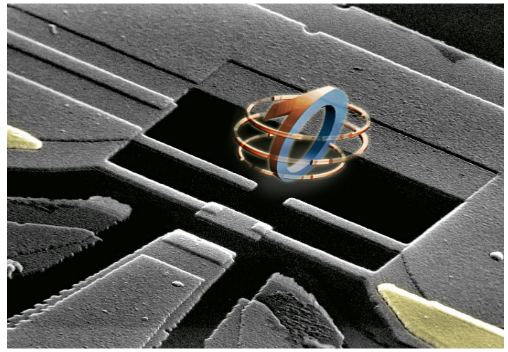
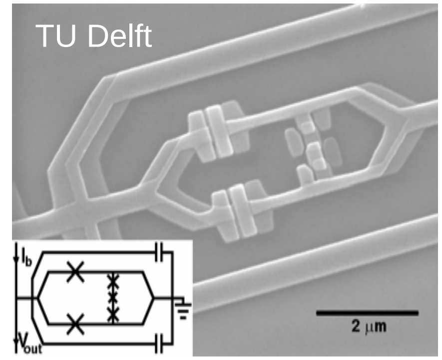




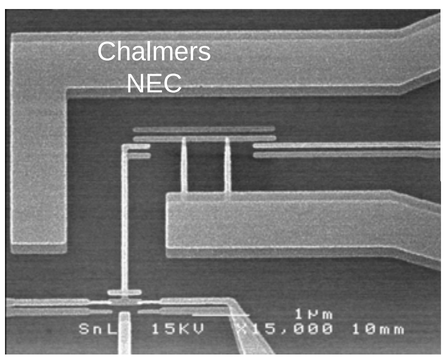


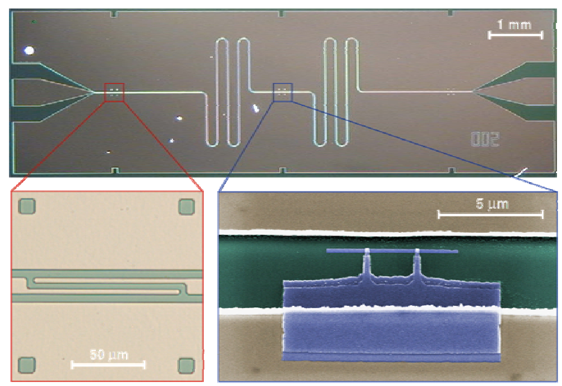


Superconducting Qubits

Andreas Wallraff

Department of Applied Physics, Yale University





with supporting material from:
M. Devoret, D. Esteve, S. Girvin, J. Mooij, R. Schoelkopf, L. Vandersypen

Motivation

long term goals:

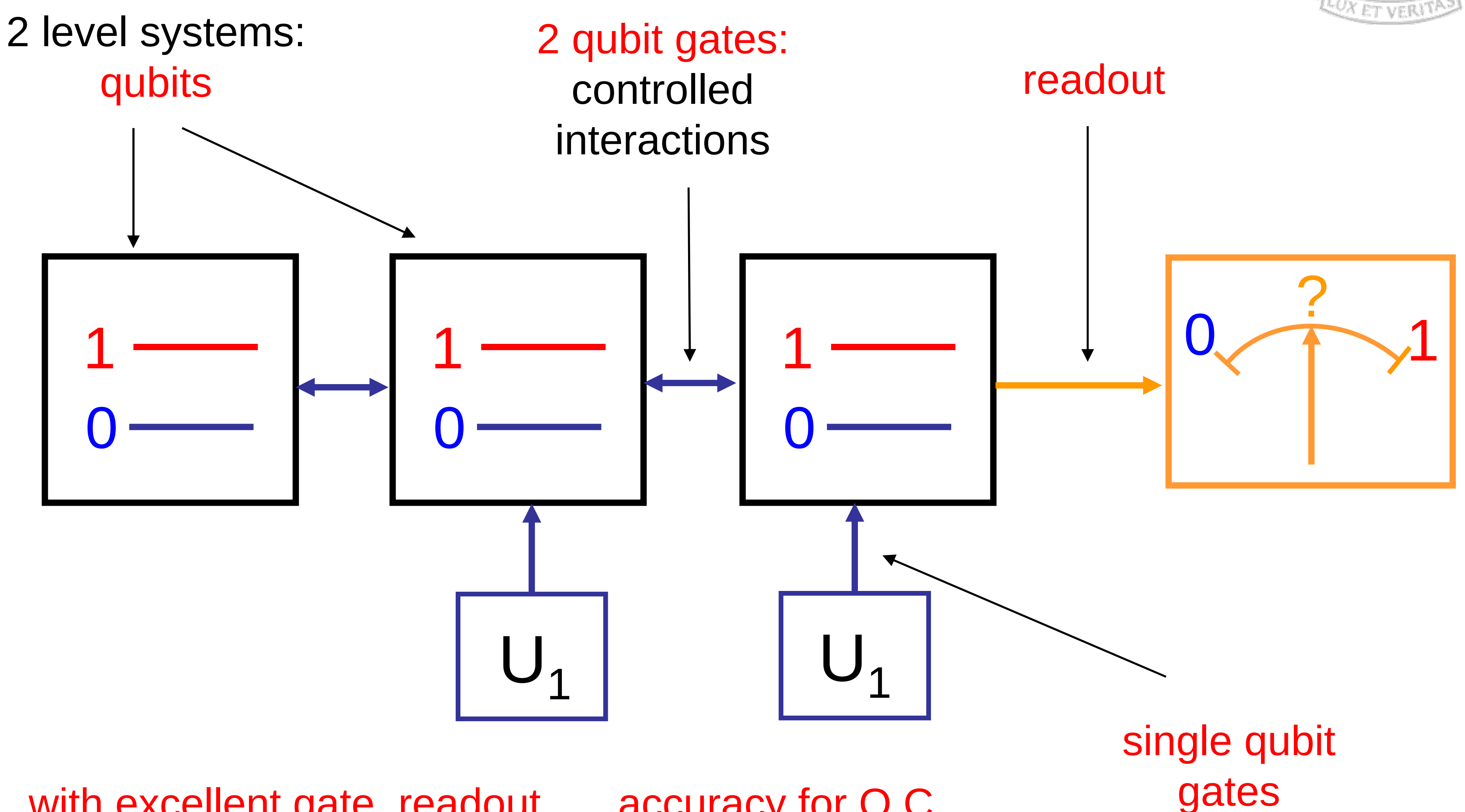
- **build a quantum computer**
- **solve computationally hard problems**

current goals for solid state implementations:

- build scalable macroscopic quantum circuits
- control open quantum systems
- investigate quantum measurement process
- learn about decoherence in solid state systems

M. Nielsen, I. Chuang, *Quantum Computation and Quantum Information* (Cambridge, 2000).

Schematic of a Generic Quantum Processor



with excellent gate, readout, … accuracy for Q.C.

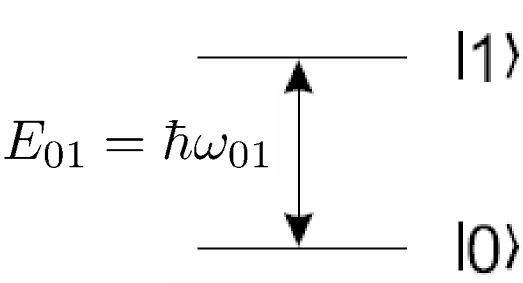
M. Nielsen, I. Chuang, Quantum Computation and Quantum Information (Cambridge, 2000).

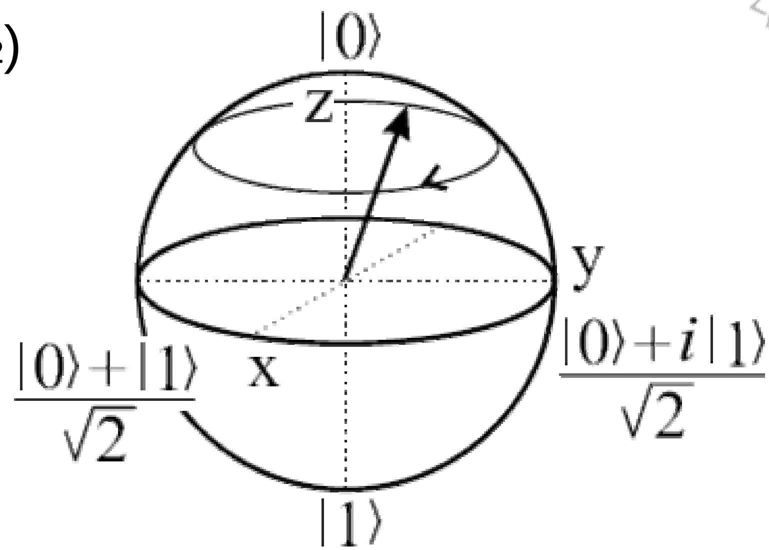
Outline

- how to make qubits from superconducting circuits
- realizations of superconducting qubits
- controlling qubits
- coherence/decoherence
- qubit readouts and measurements
- coupled qubits
- conclusions

A Generic Qubit

two-level quantum system (a spin ½)



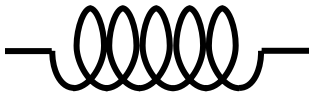


DiVincenzo criteria:

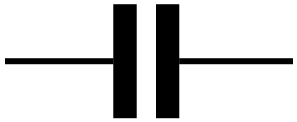
- existence of quantum two level system (a qubit)
- qubit initialization (reset)
- qubit coherence (no dissipation, no dephasing)
- qubit control (gate operations)
- qubit readout

D. P. DiVincenzo, arXiv:quant-ph/0002077 (2000)

Building Qubits with Integrated Circuits

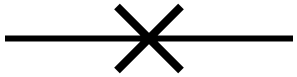


inductor

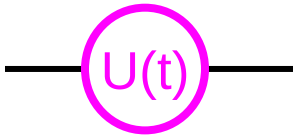


capacitor

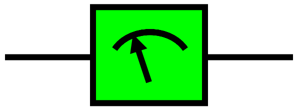
resistor



nonlinear element



voltage source



voltmeters

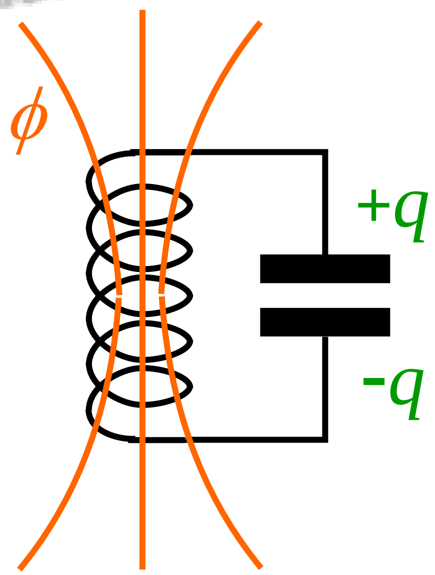
<u>requirements for quantum circuits:</u>

- low dissipation
- non-linear (non-dissipative elements)
- low (thermal) noise

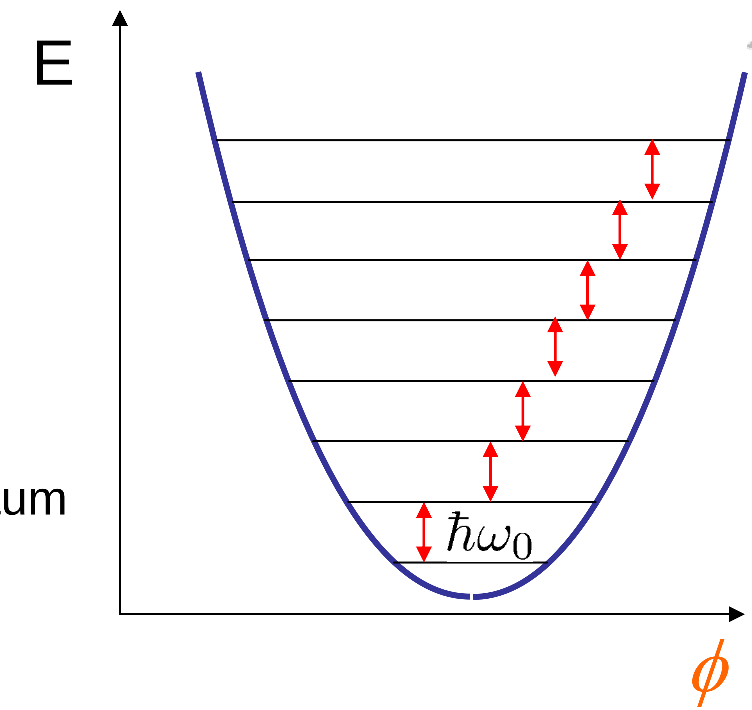
<u>a solution:</u>

- use superconductors
- use Josephson tunnel junctions
- operate at low temperatures

LC Oscillator as a Quantum Circuit



$$[\phi, q] = i\hbar$$



position momentum

hamiltonian $$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

$$\omega_0 = 1/\sqrt{LC}$$

$$H = \hbar\omega_0 \left(a^\dagger a + \frac{1}{2} \right)$$

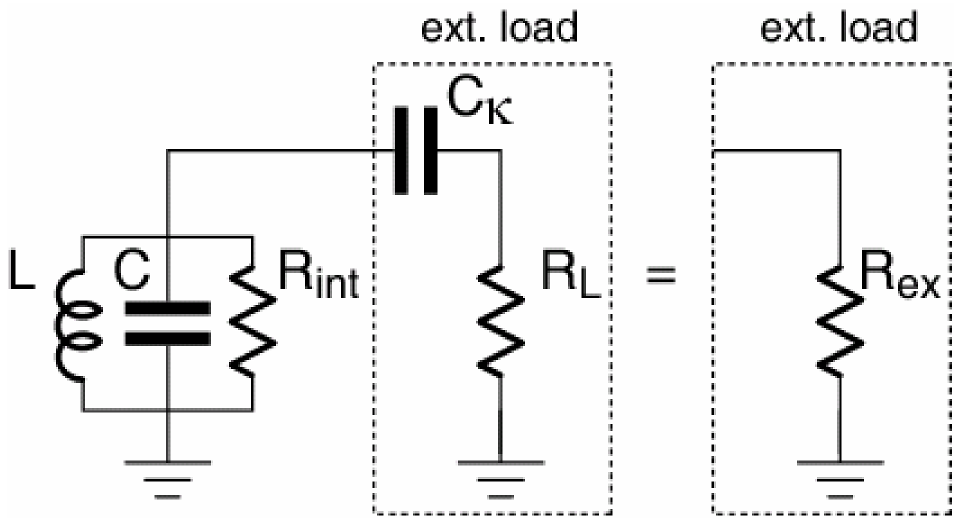
low temperature required:

$$\hbar\omega_0 \gg k_B T$$

1 GHz ~ 50 mK

problem I: equally spaced energy levels (linearity)

Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

external losses: R_{ext}
radiation, coupling

total losses $$\frac{1}{R} = \frac{1}{R_{\mathrm{int}}} + \frac{1}{R_{\mathrm{ext}}}$$

impedance $$Z = \sqrt{\frac{L}{C}}$$

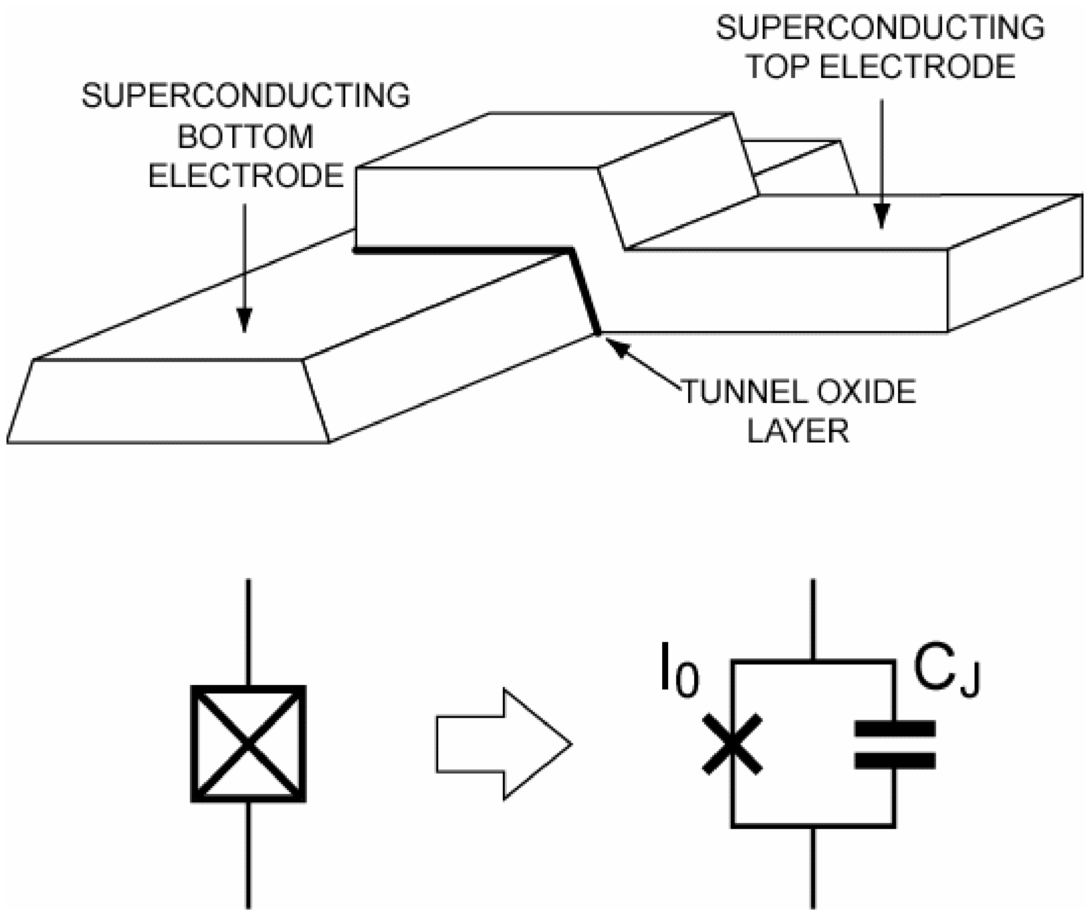
quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$

problem II: avoid internal and external dissipation

A Superconducting Nonlinear Element

Josephson junction

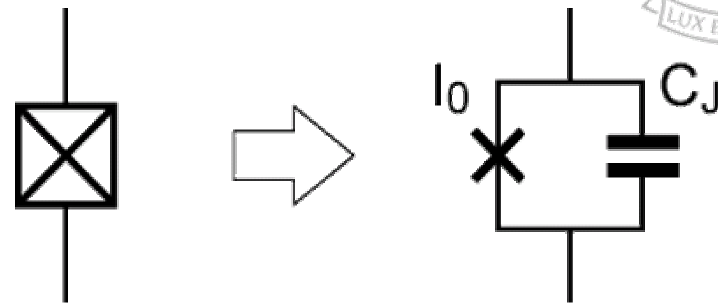


- superconductors: Nb, Al
- tunnel barrier: AlO_x
- critical current I_0
- junction capacitance C_J

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

The Josephson Junction

a nonlinear inductor without dissipation



nonlinear current flux relation: $$I = I_0 \sin\left[2\pi\Phi(t)/\Phi_0\right] = I_0 \sin\delta$$

gauge inv. phase difference: $$\delta = 2\pi\Phi(t)/\Phi_0$$

nonlinear Josephson inductance: $$L_J(\delta) = \left(\frac{\partial I}{\partial \Phi}\right)^{-1} = \underbrace{\frac{\Phi_0}{2\pi I_0}}_{L_{J0}} \frac{1}{\cos\delta}$$

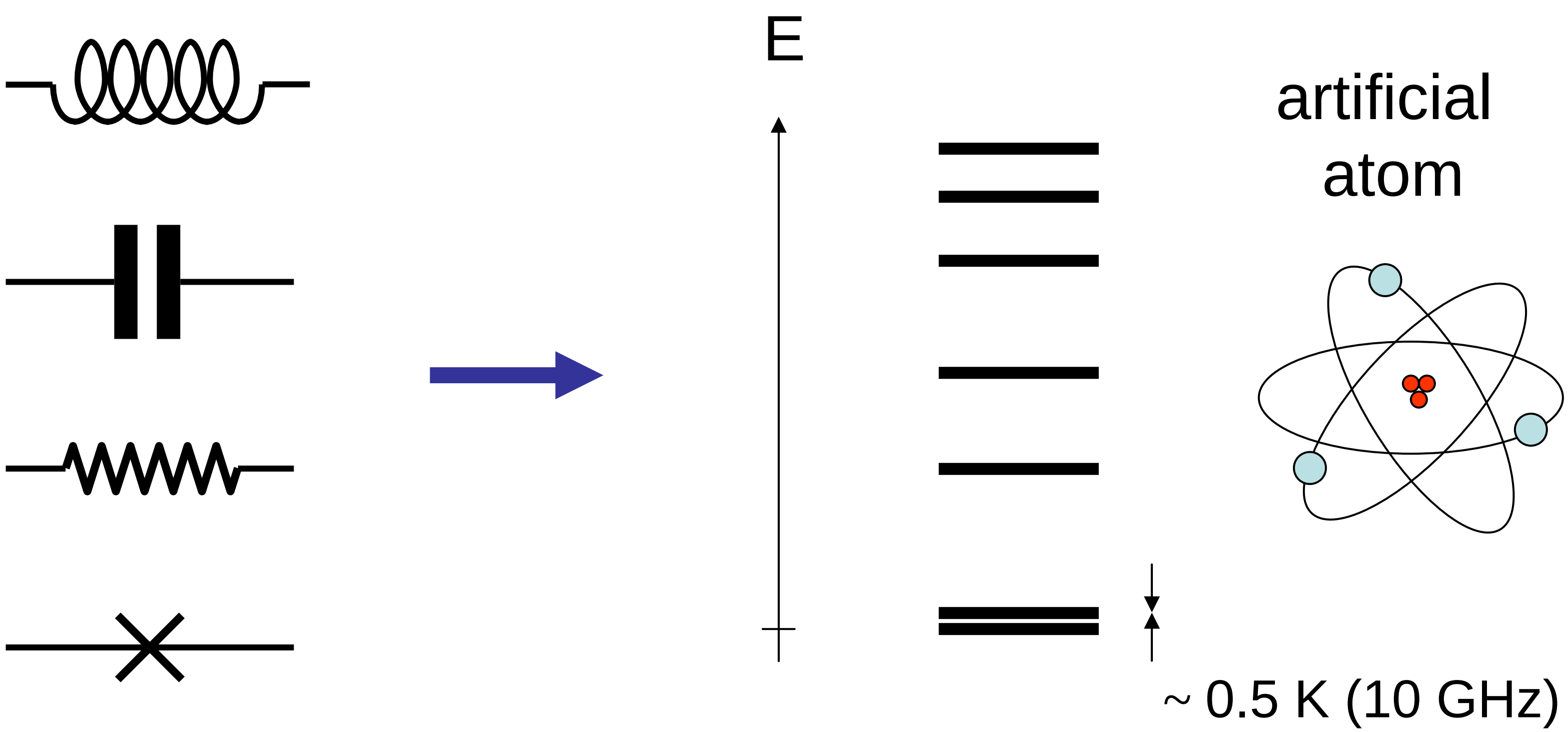
voltage: $$V = \frac{d\Phi}{dt} = \frac{\Phi_0}{2\pi}\dot{\delta}$$

Josephson energy: $$E = \underbrace{\frac{I_0\Phi_0}{2\pi}}_{E_{J0}} \cos\delta$$

Building Blocks for Qubits

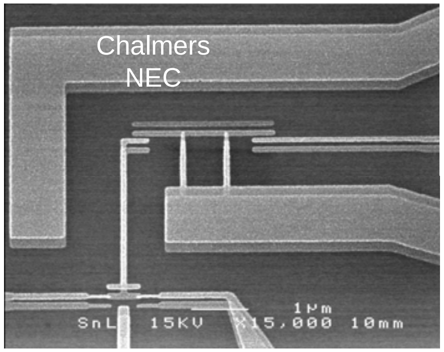
all ingredients available:

macroscopic artificial atoms:

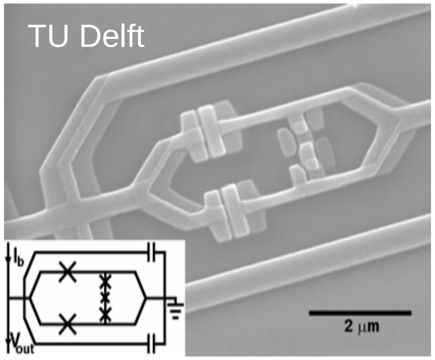


Superconducting Qubits

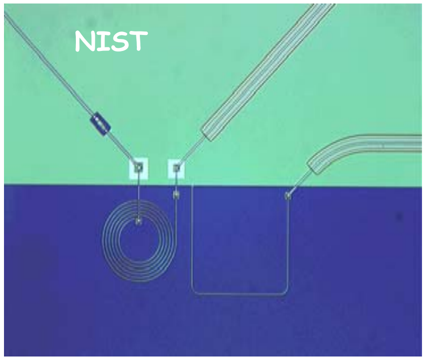
charge



flux



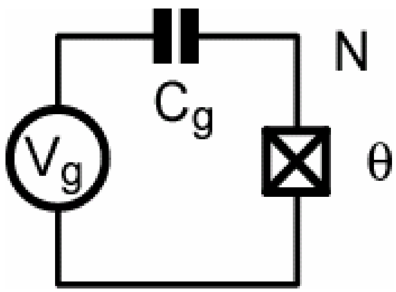
phase



classified by their control parameter

Charge Qubits

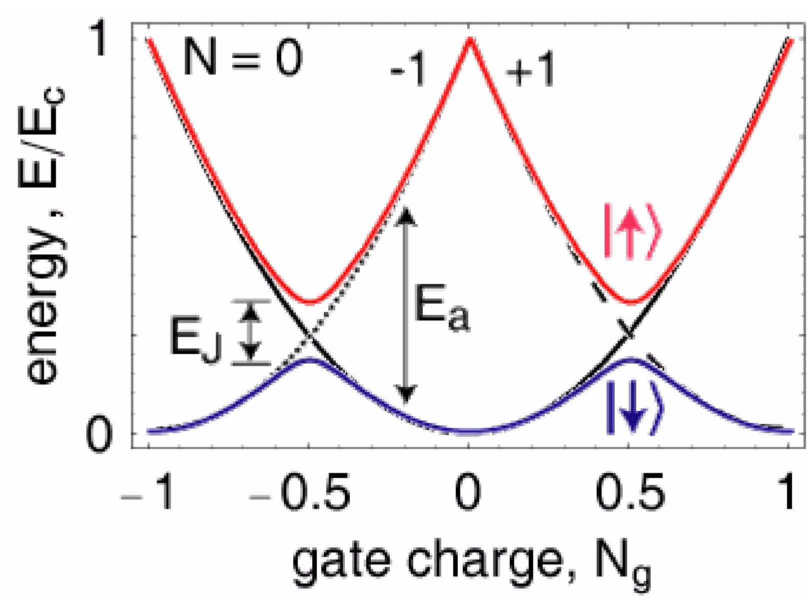
Cooper pair box



$$[\theta, N] = i$$

$$H = E_C\,(N - N_g)^2 - E_J \cos\theta$$

electrostatic energy   Josephson energy



charging energy $$E_C = \frac{(2e)^2}{2(C_J + C_g)}$$

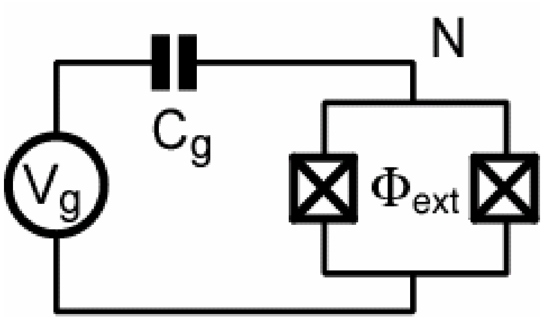
Josephson energy $$E_J = \frac{I_0 \Phi_0}{2\pi} = \frac{h\Delta}{8e^2 R_J}$$

gate charge $$N_g = \frac{C_g V_g}{2e}$$

V. Bouchiat, D. Vion, P. Joyez, D. Esteve, and M. H. Devoret, *Physica Scripta* **T76,** 165 (1998).

Tunable Charge Qubits

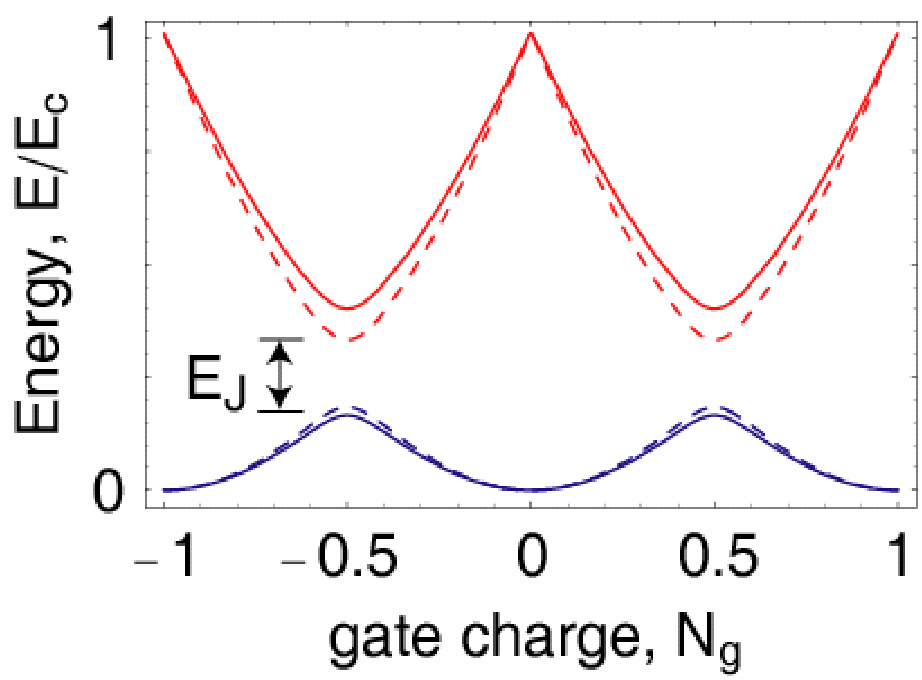
split Cooper pair box



$$H = E_C\,(N - N_g)^2 - E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\text{ext}}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\,\frac{\Phi_{\text{ext}}}{\Phi_0}\right)$$



J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

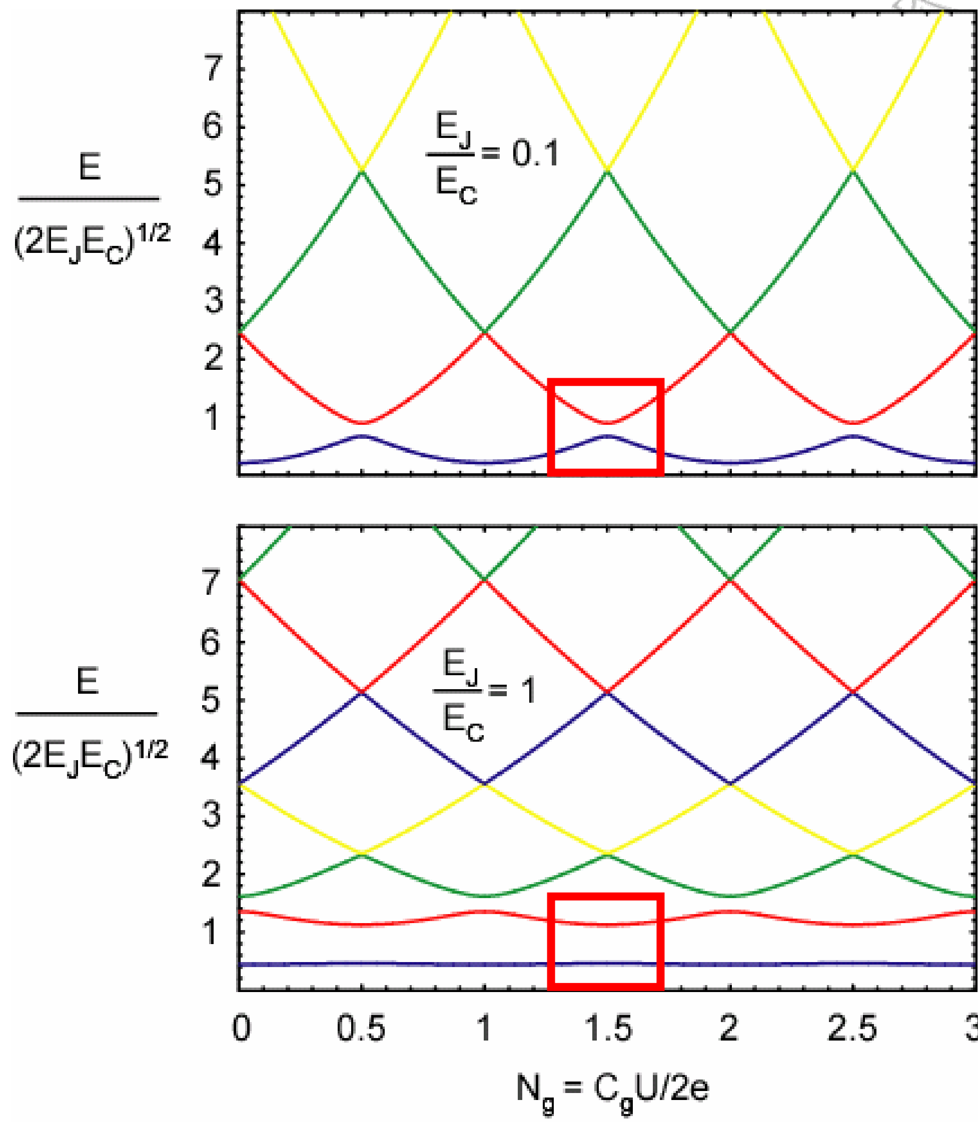
Cooper Pair Box Energy Levels

$$E_J \gg E_C$$

level separation for arbitrary charging energy and Josephson energy

$$E_J \sim E_C$$

two-state approximation close to charge degeneracy

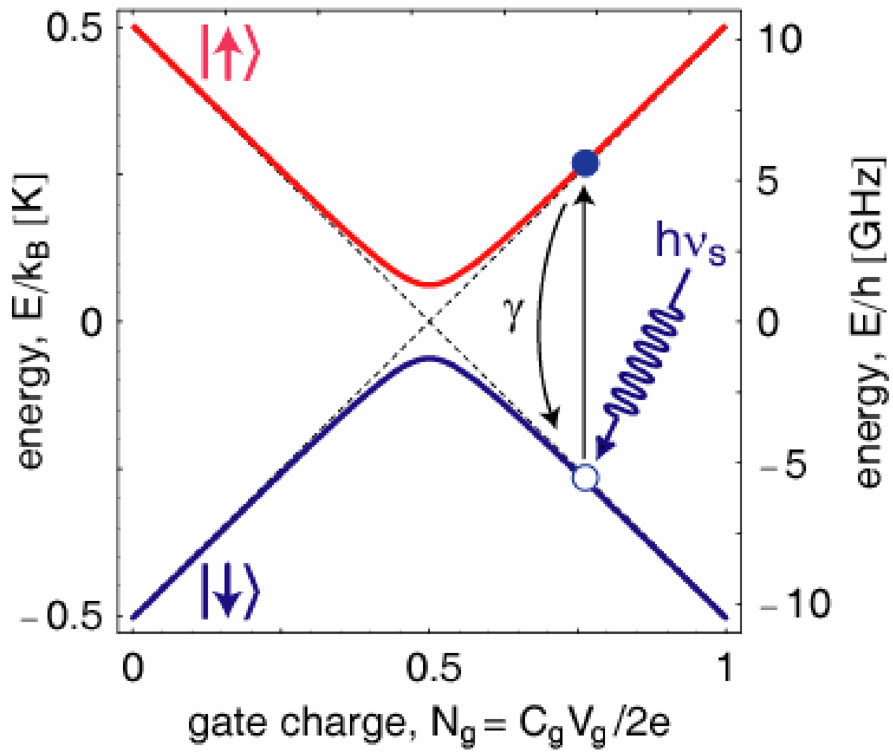


Two-State Approximation

2-state Hamiltonian and level separation:

$$H = -1/2\,(E_{\rm el}\,\sigma_x + E_{\rm J}\,\sigma_z)$$

$$E = \sqrt{E_{\rm el}^2 + E_J^2}$$



in-situ controllable parameters:

$$E_{\rm el} = E_C\,(1/2 - N_g)$$

$$E_J = E_{J,\rm max}\cos\left(\pi\Phi_{\rm ext}/\Phi_0\right)$$

E_C, $E_{J,\rm max}$ engineerable in fabrication

excited state decay rates $\Gamma_1 < 1\,\mathrm{MHz}$

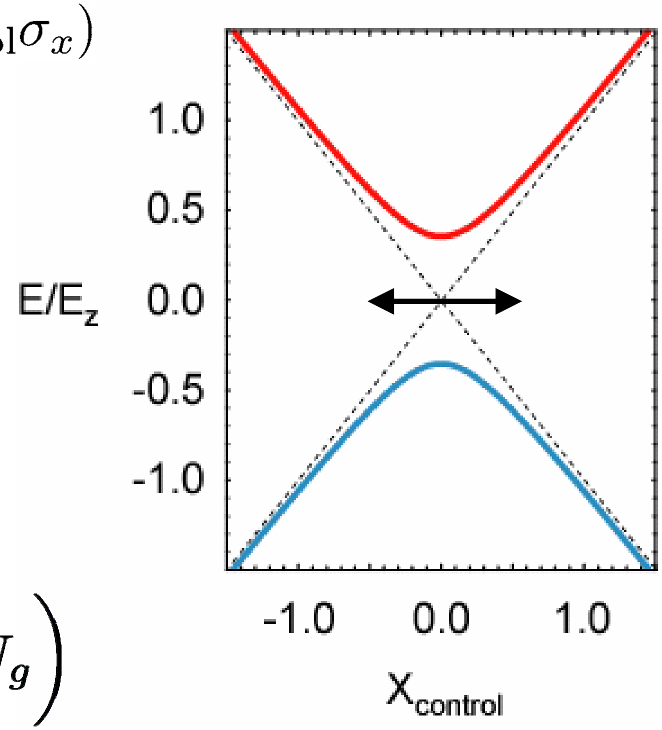
K. Lehnert *et al. PRL.* **90**, 027002 (2003).

Nakamura, Pashkin, Tsai *et al. Nature* **398**, **421**, **425** (1999,2003, 2003)

Control of Charge Qubit

effective hamiltonian

$$H_{\text{qubit}} = -E_z\left(\sigma_z + X_{\text{control}}\sigma_x\right)$$



energy splitting $$E_z = \frac{E_J}{2}$$

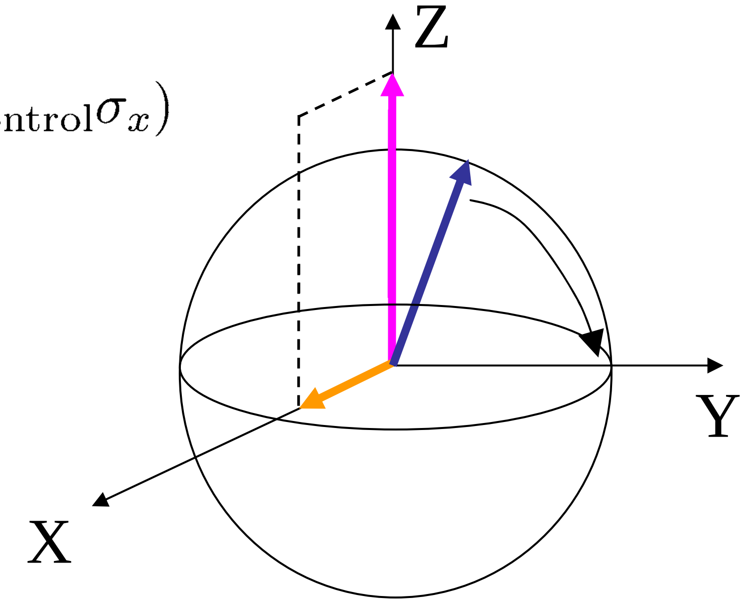
control parameter $$X_{\text{control}} = 2\frac{E_C}{E_J}\left(\frac{1}{2} - N_g\right)$$

gate charge $$N_g = \frac{C_g V_g}{2e}$$

Control of Charge Qubit

effective hamiltonian

$$H_{\text{qubit}} = -E_z \left(\sigma_z + X_{\text{control}}\sigma_x\right)$$



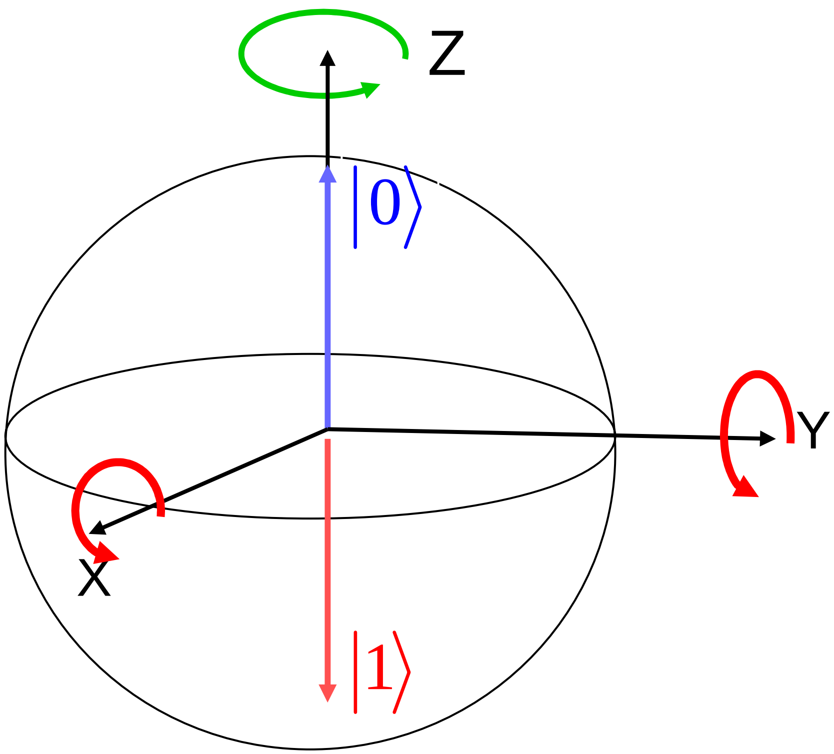
energy splitting $$E_z = \frac{E_J}{2}$$

control parameter $$X_{\text{control}} = 2\,\frac{E_C}{E_J}\left(\frac{1}{2} - N_g\right)$$

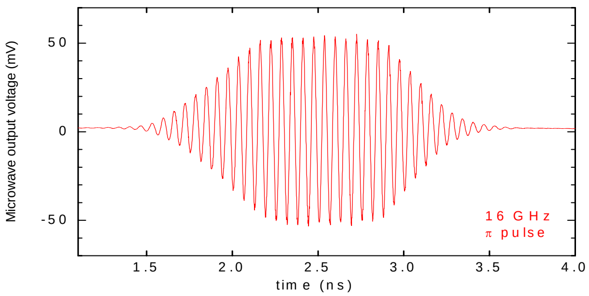
gate charge $$N_g = \frac{C_g V_g}{2e}$$

Single Qubit Control

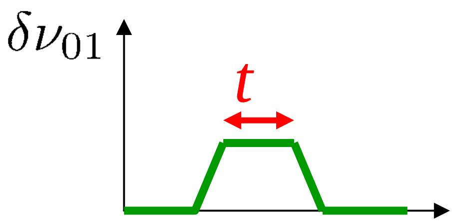
Bloch sphere representation of single qubit manipulation



x,y rotations by microwave pulses

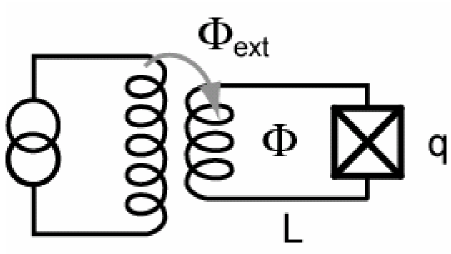


z rotations by adiabatic pulses



Flux Qubits

radio frequency superconducting quantum interference device (RF-SQUID)



$$H = \frac{q^2}{2C_J} + \frac{\Phi^2}{2L} - E_J \cos\left[2\pi \frac{\Phi - \Phi_{ext}}{\Phi_0}\right]$$

$$[\Phi, q] = i\hbar$$

kinetic energy — potential energy

charging energy $$E_C = \frac{(2e)^2}{2C_J}$$

inductive energy $$E_L = \frac{\Phi_0^2}{2L}$$

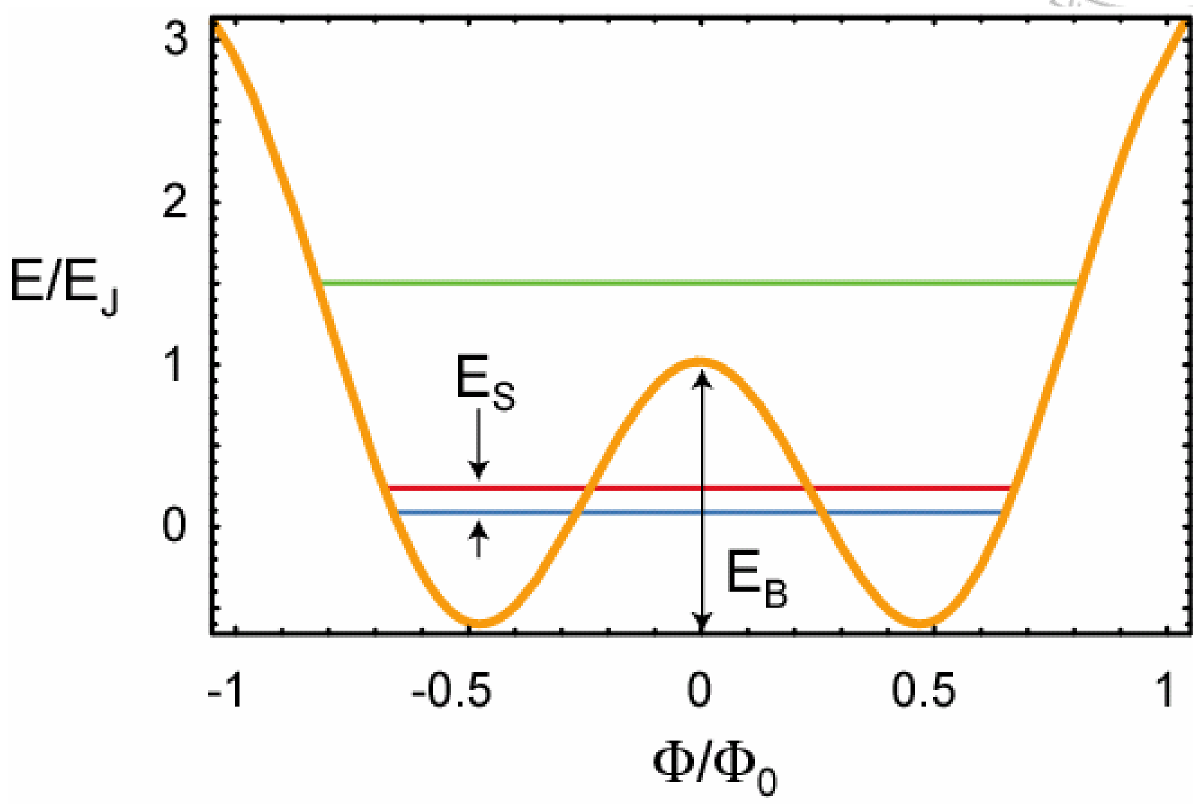
Josephson energy $$E_J = \frac{I_0 \Phi_0}{2\pi} = \frac{h\Delta}{8e^2 R_J}$$

A. Barone and G. Paterno, (Wiley, New York, 1992)

RF-SQUID Potential

parabolic potential
with cosine corrugation

- $\Phi_{\text{ext}} = \Phi_0/2$
- $E_J \gg E_C$



energy level splitting at $\Phi_{\text{ext}} = \Phi_0/2$

$$E_S \propto \eta \sqrt{E_B E_{CJ}} \exp\left(-\xi \sqrt{\frac{E_B}{E_{CJ}}}\right)$$

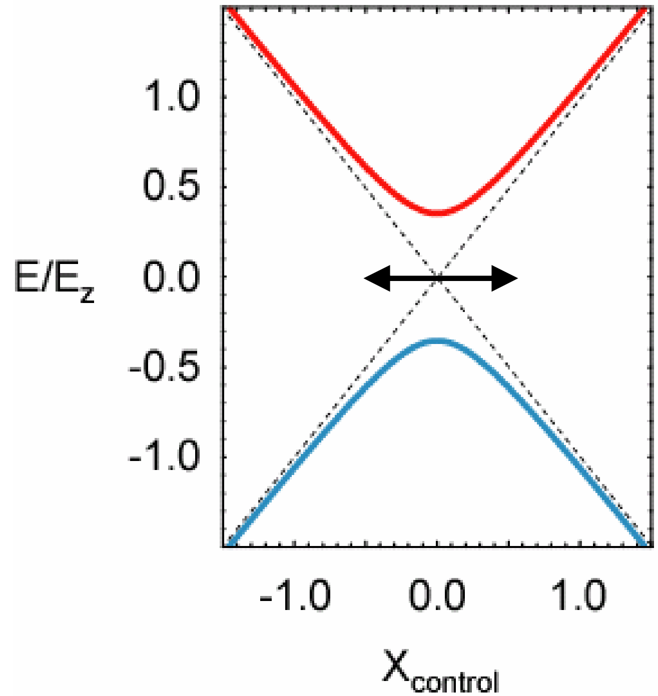
bias flux dependence

$$E_S \propto \zeta \frac{\Phi_0^2}{2L} \left(\frac{1}{2} - \frac{\Phi_{\text{ext}}}{\Phi_0}\right)$$

Control of Flux Qubits

effective hamiltonian

$$H_{\text{qubit}} = -E_z \left(\sigma_Z + X_{\text{control}} \, \sigma_X \right)$$



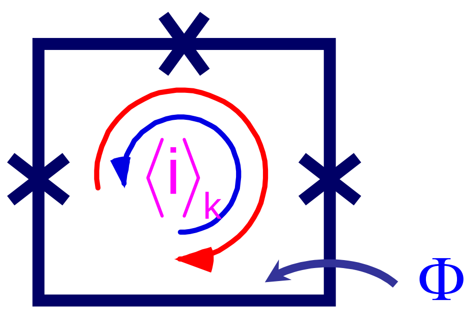
splitting energy $$E_z = \frac{E_S}{2}$$

control parameter $$X_{\text{control}} = 2 \frac{E_L}{E_S} \left(\frac{1}{2} - N_\Phi \right)$$

flux frustration $$N_\Phi = \frac{\Phi_{\text{ext}}}{\Phi_0}$$

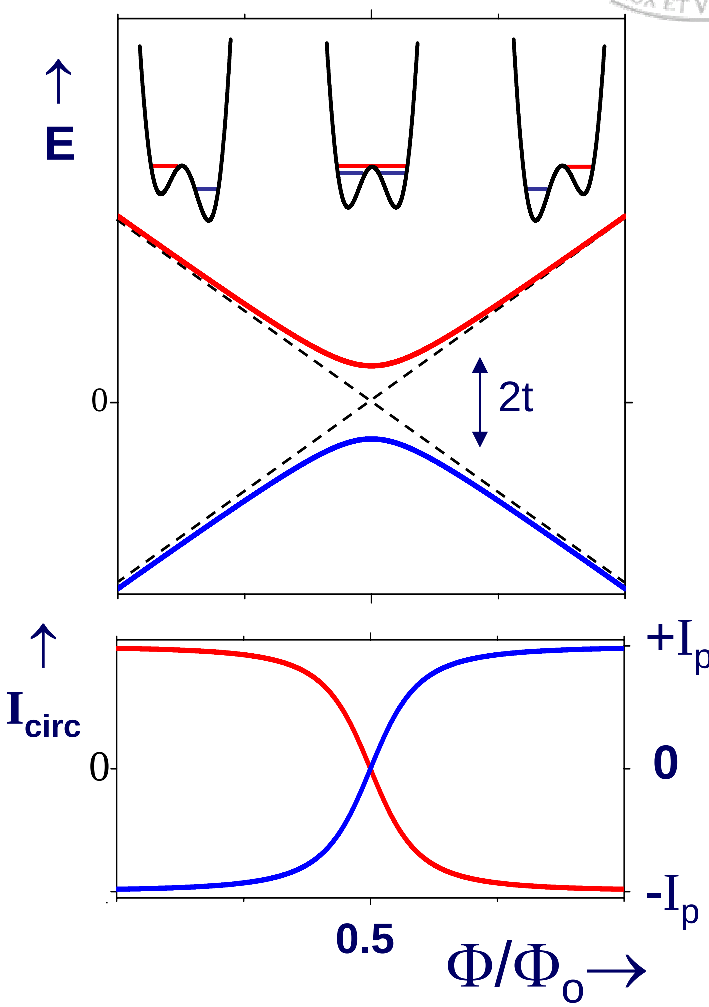
Variation of the Flux Qubit

persistent-current quantum bit:
flux qubit with three junctions,
small geometric loop inductance



$$H = h\sigma_z + t\sigma_x$$

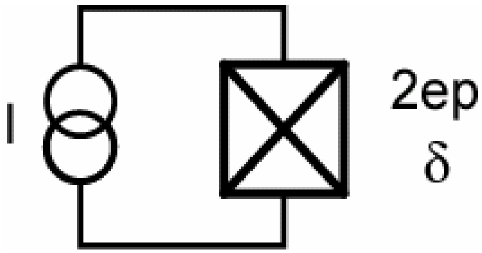
with $h=(\Phi/\Phi_o-0.5)\ \Phi_o I_p$



J. E. Mooij, T. P. Orlando, ... , C. H. van der Wal and S. Lloyd, *Science* **285**, 1036 (1999)
C. H. van der Wal, A. C. J. ter Haar, ... , S. Loyd and J. E. Mooij, *Science* **290**, 773 (2000).

Phase Qubits

current biased junction



$[\delta, p] = i$

$$H = \frac{(2e)^2}{2C_J}p^2 - I\frac{\Phi_0}{2\pi}\delta - \frac{I_0\Phi_0}{2\pi}\cos\delta$$

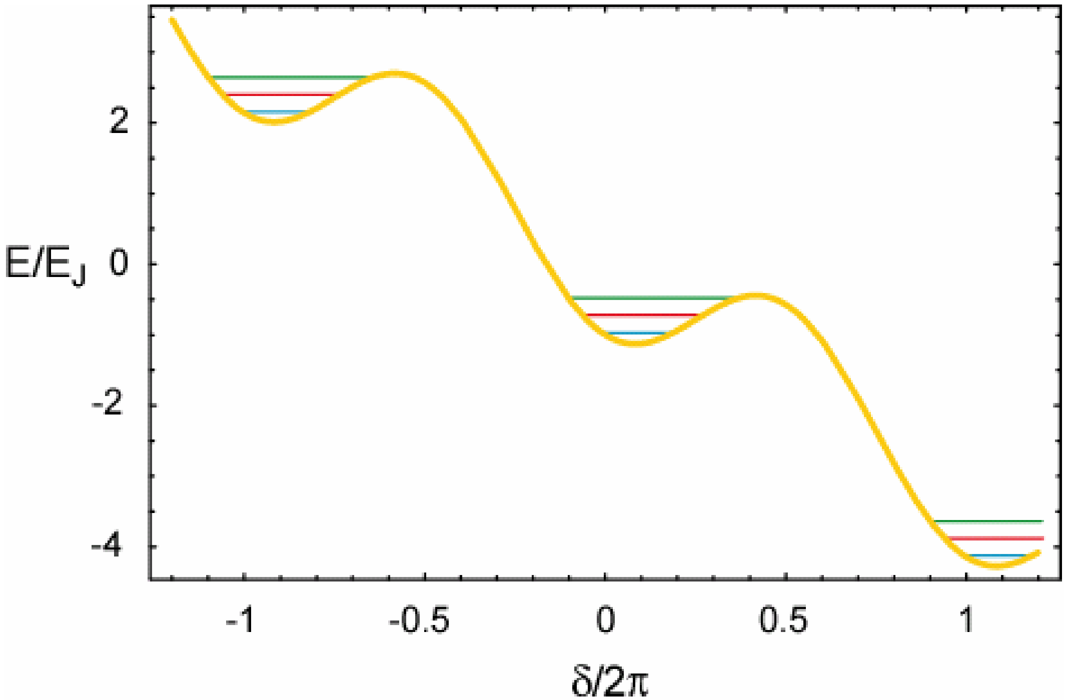
kinetic energy   potential energy

charging energy	$E_C = \frac{(2e)^2}{2C_J}$
Josephson energy	$E_J = \frac{I_0\Phi_0}{2\pi} = \frac{h\Delta}{8e^2R_J}$
bias current	I

J. M. Martinis, M. H. Devoret and J. Clarke, *Phys. Rev. B* **35**, 4682 (1987)

Potential of Current Biased Junction

particle in a washboard potential



potential

$$U(\delta) = -I\frac{\Phi_0}{2\pi}\delta - \frac{I_0\Phi_0}{2\pi}\cos\delta$$

J. M. Martinis, M. H. Devoret and J. Clarke, *Phys. Rev. B* **35**, 4682 (1987)

Energy level quantization

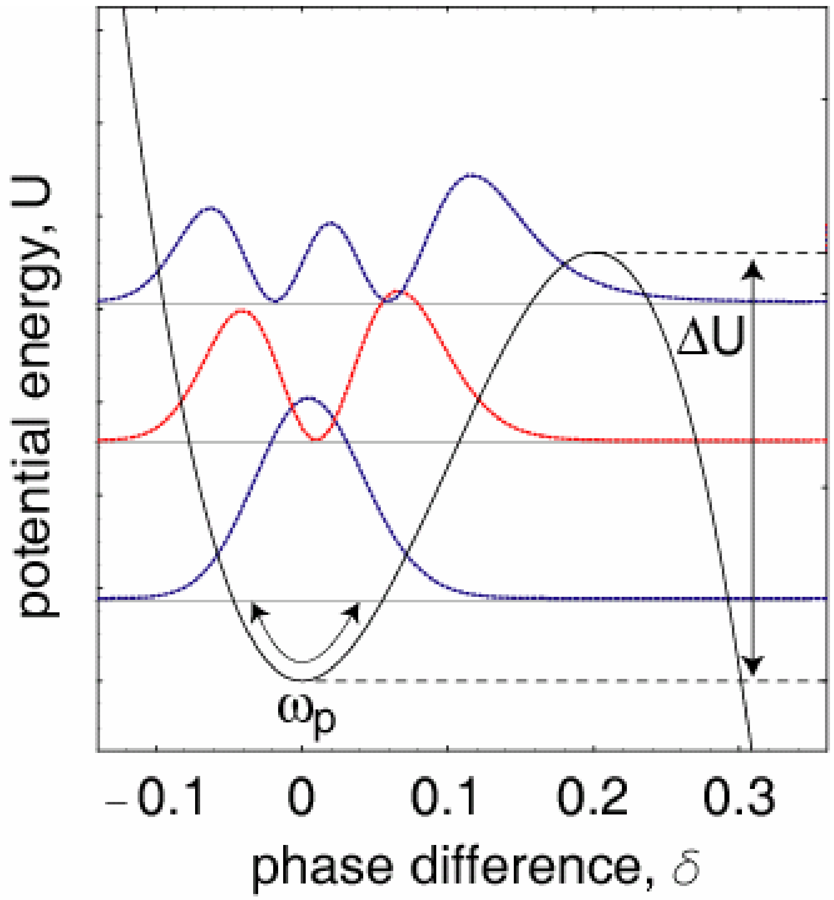
cubic potential near $I = I_0$

barrier height

$$\Delta U = \frac{2\sqrt{2}}{3} E_J \left(1 - \frac{I}{I_0}\right)^{3/2}$$

oscillation frequency

$$\omega_p = \frac{1}{\sqrt{L_{J0} C_J}} \left[1 - \left(\frac{I}{I_0}\right)^2\right]^{1/4}$$



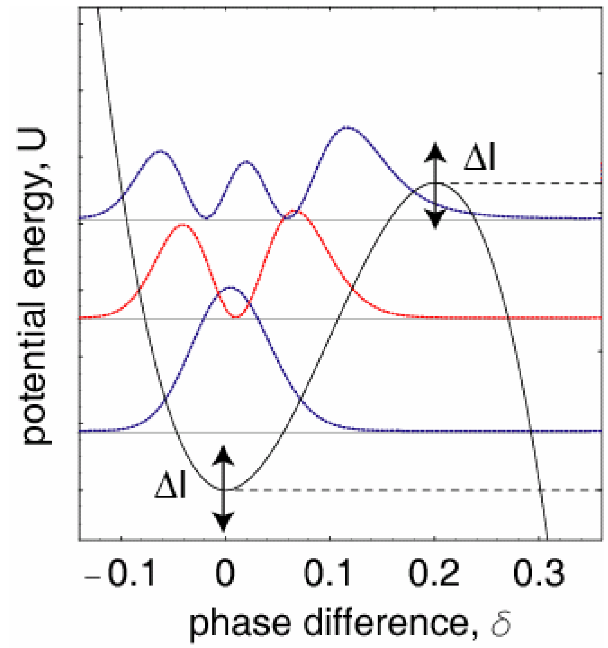
use eigenstates as basis states of qubit

J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, *Phys. Rev. Lett.* **89**, 117901 (2002)

Control of Phase Qubits

effective hamiltonian

$$H_{\text{qubit}} = \frac{1}{2}\hbar\omega_{01}\,\sigma_Z + \sqrt{\frac{\hbar}{2\omega_{01}C_J}}\Delta I(\sigma_X + \chi\sigma_Z)$$



'splitting' energy $\hbar\omega_{01}$

control parameter $\sqrt{\frac{\hbar}{2\omega_{01}C_J}}\Delta I$

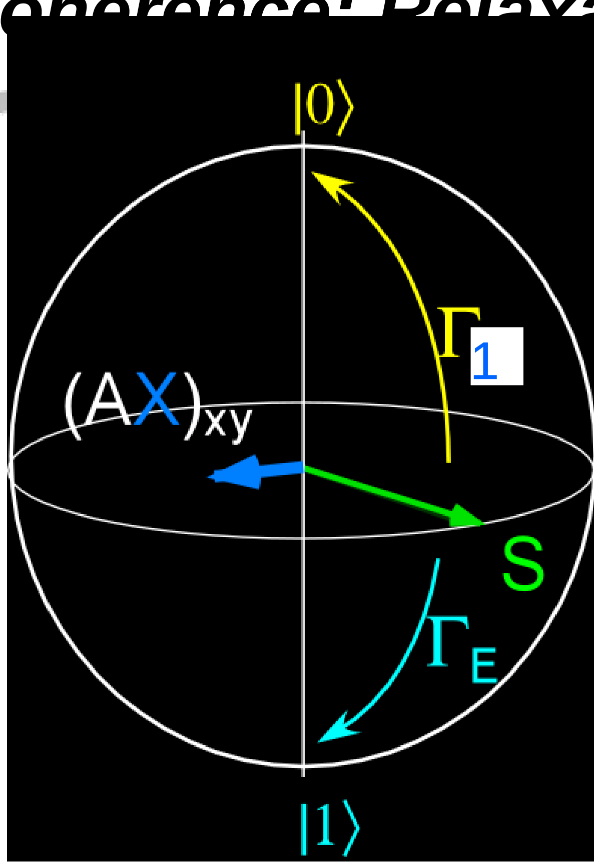
bias current $\Delta I = I - I_0$

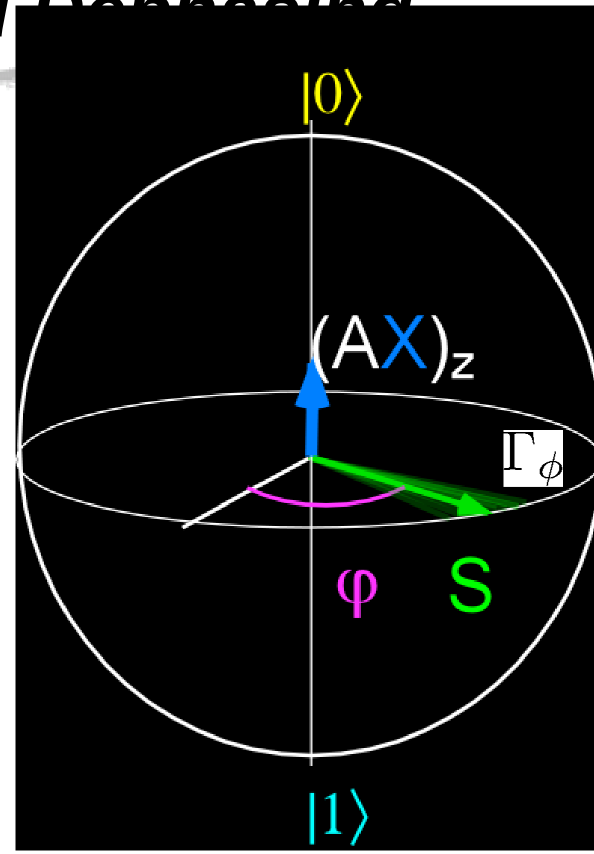
operations:

- $\Delta I \propto \sin\omega_{01}t$ performs σ_X operations
- slow variations in ΔI perform σ_Z operations

M. Steffen, J. Martinis and I. L. Chuang, *Phys. Rev. B* **68**, 224518 (2003).

Decoherence: Relaxation and Dephasing





relaxation: transverse fluctuations at qubit transition frequency

dephasing: parallel fluctuations (in qubit level sep.) at low frequencies

life time

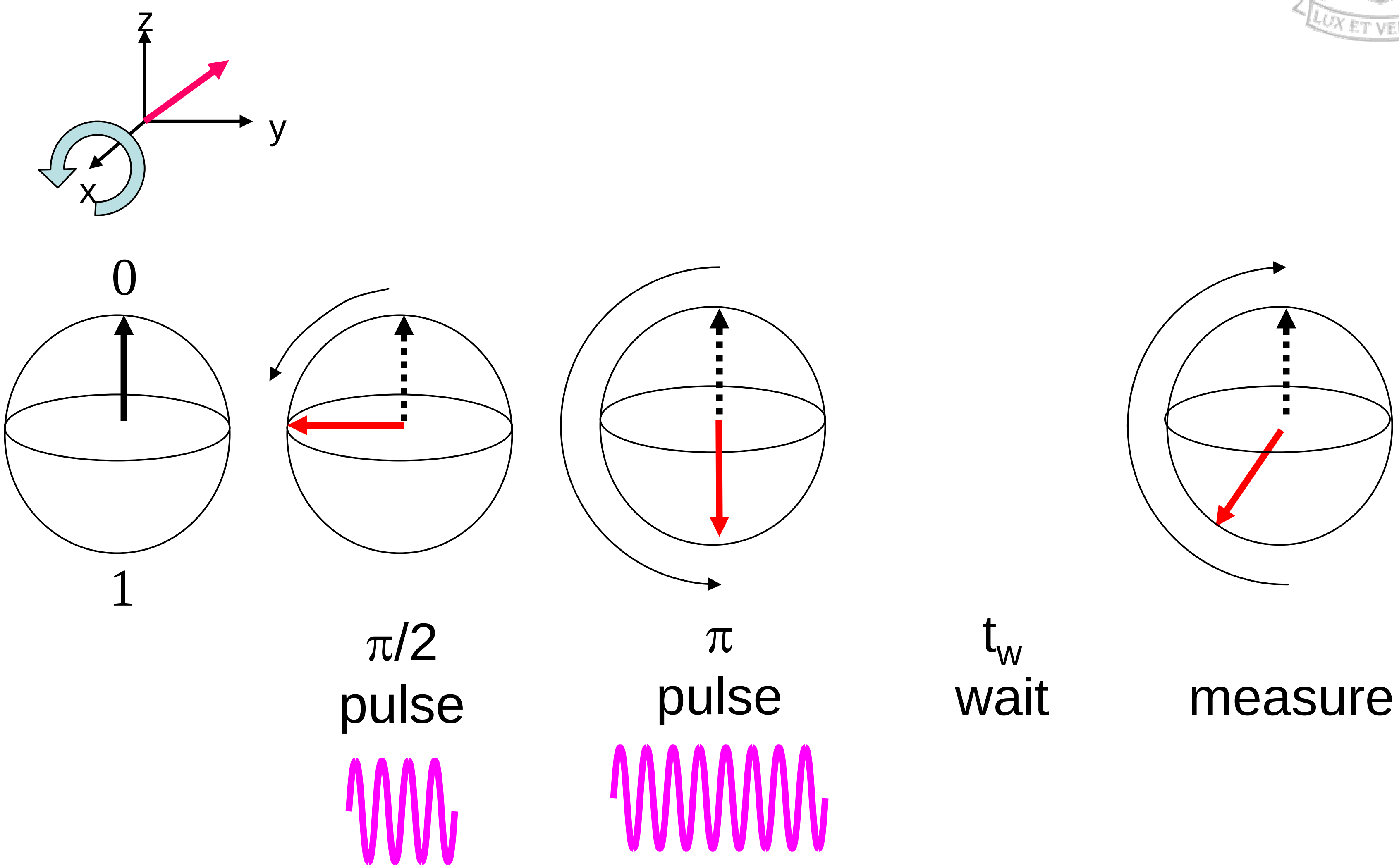
$$T_1 = \Gamma_1^{-1}$$

coherence time

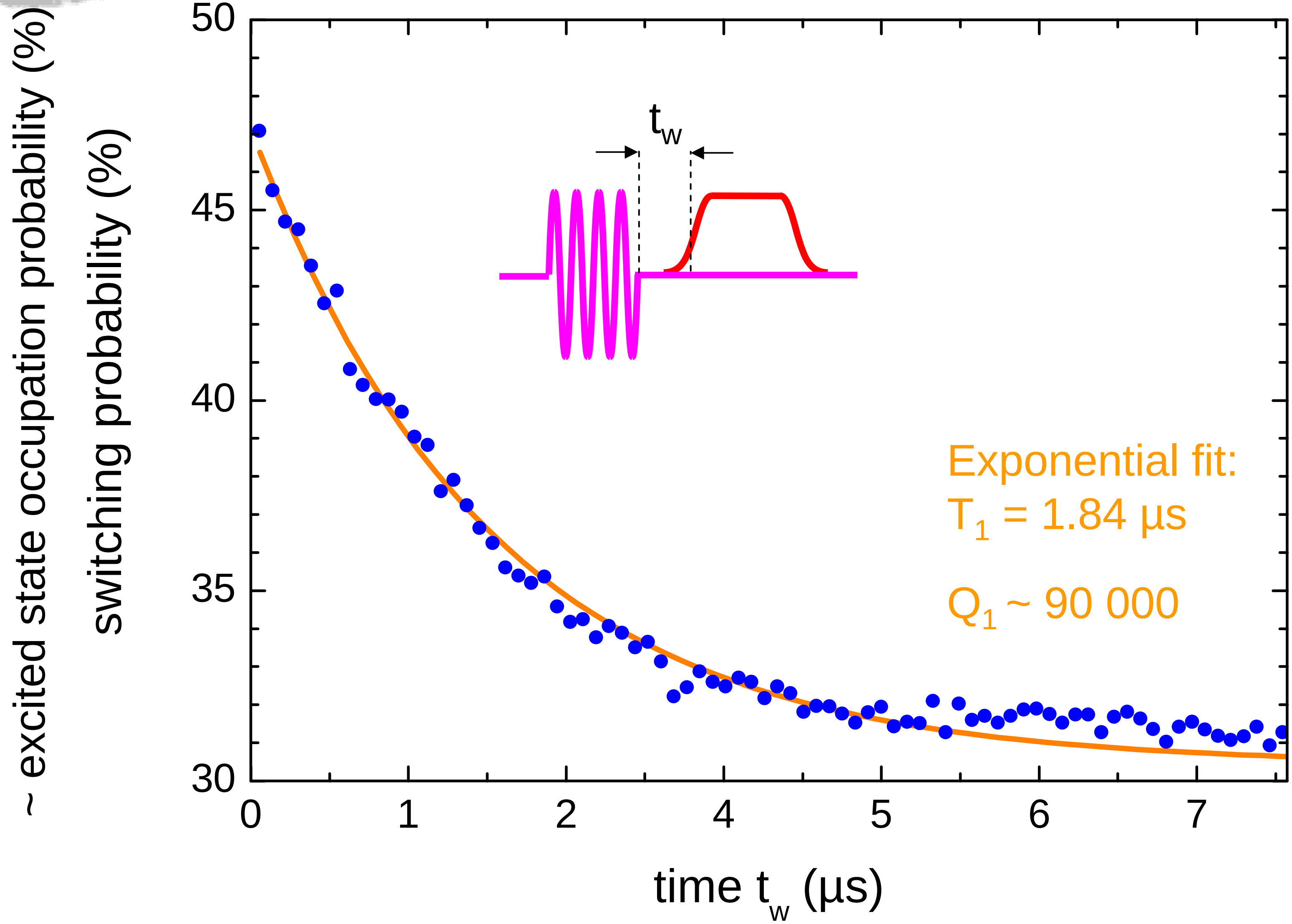
$$T_2 = (\Gamma_\phi + \Gamma_1/2)^{-1}$$

A. Abragam, *Principles of Nuclear Magnetic Resonance* (Oxford University Press, Oxford, 1985)

Measuring Relaxation



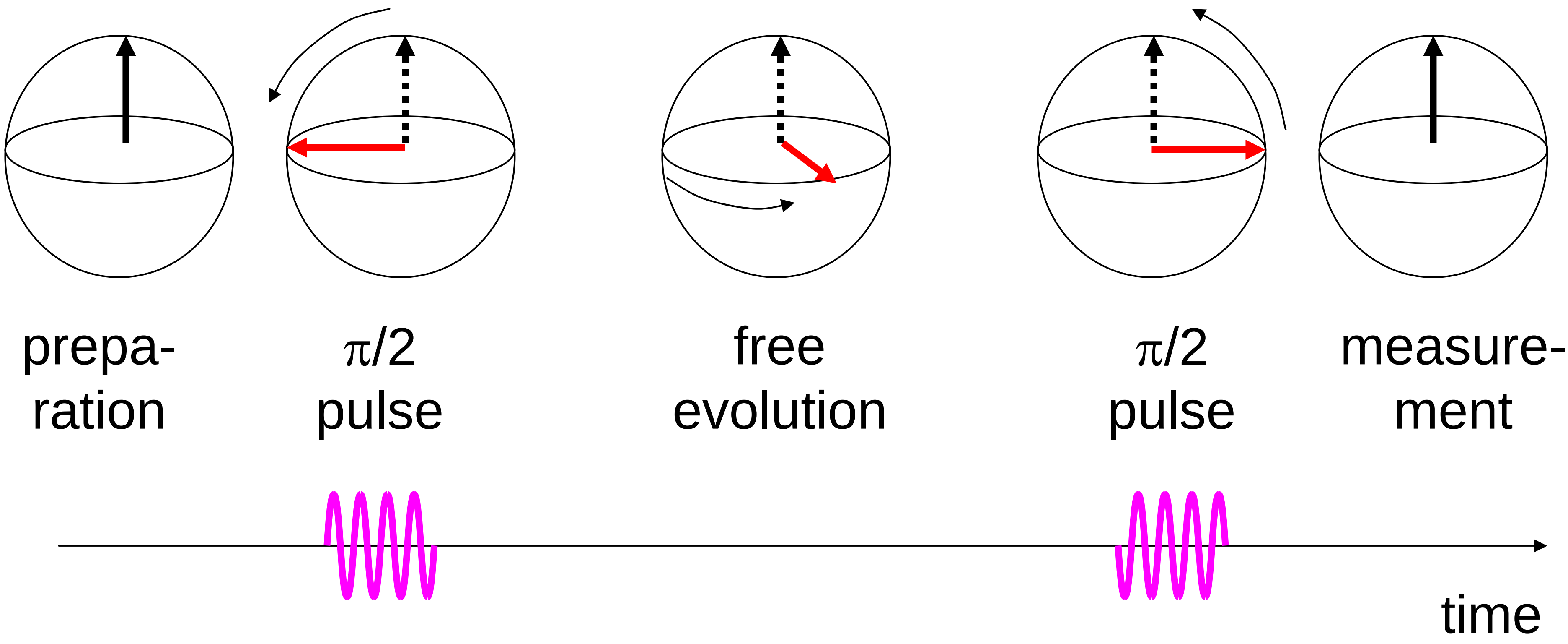
Relaxation Measurement



D. Vion, A. Aassime, A. Cottet, ... , D. Esteve, and M.H. Devoret, *Science* **296**, 286 (2002).

Measuring Quantum Coherence (I)

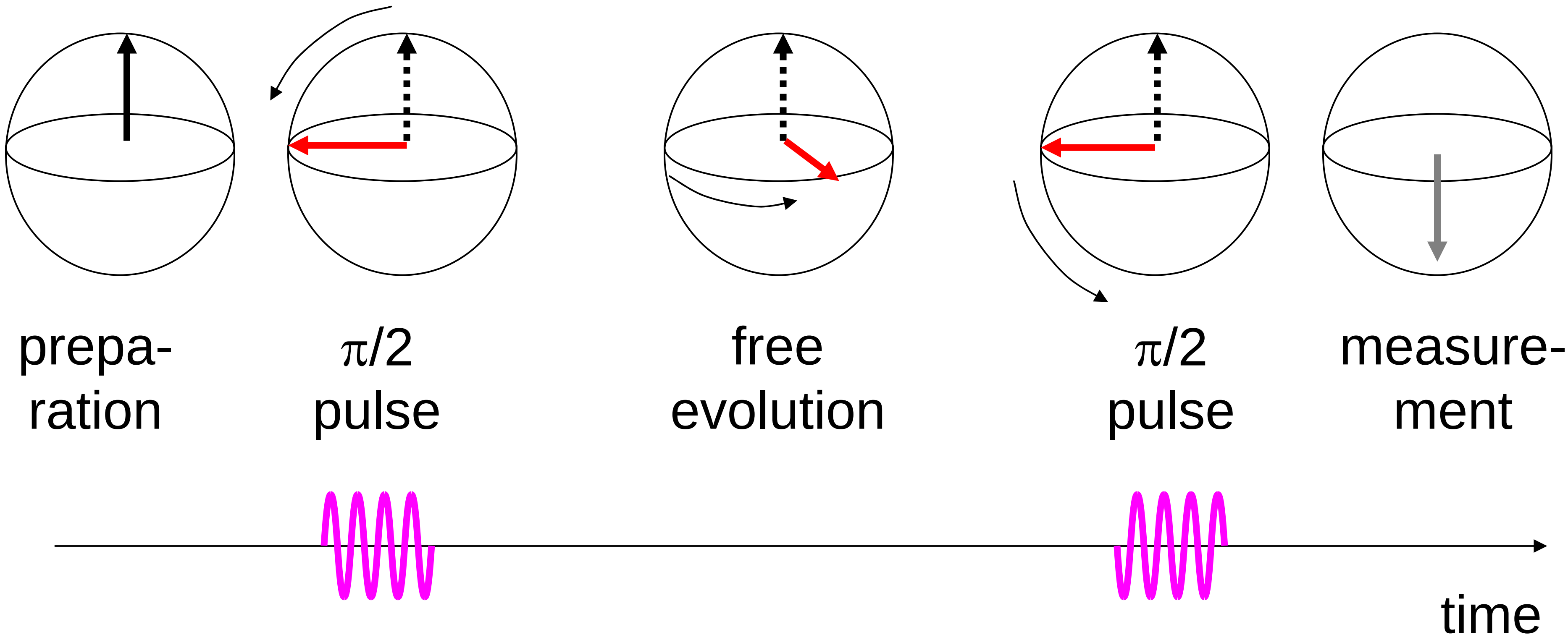
Ramsey fringe experiment



determine coherence time T_2

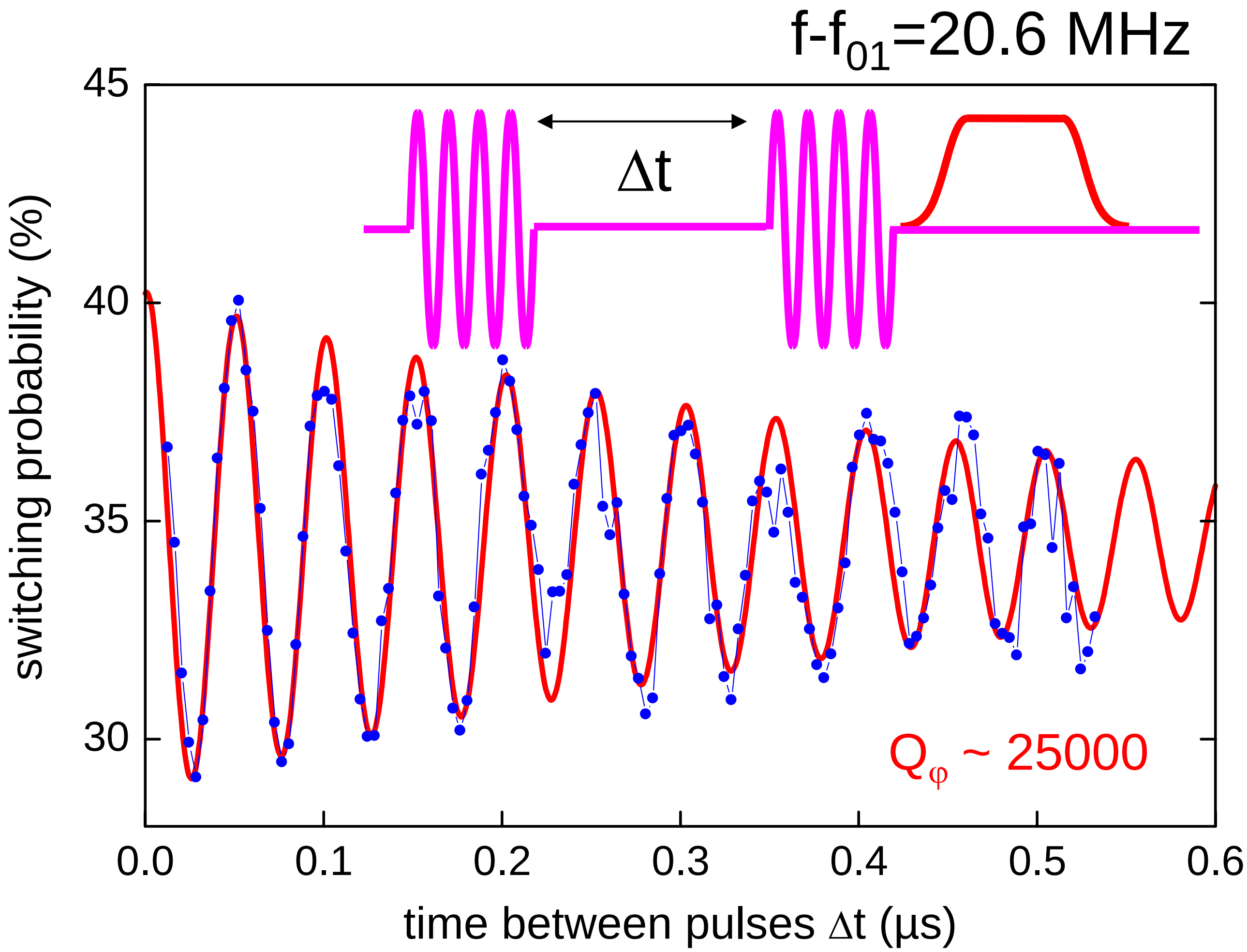
Measuring Quantum Coherence (II)

Ramsey fringe experiment



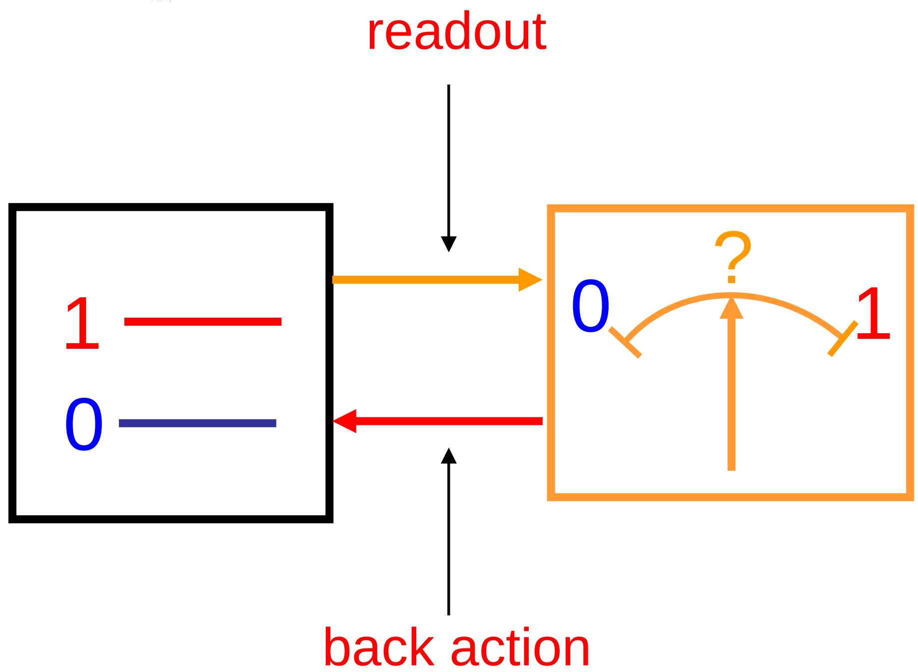
determine coherence time T_2

Measurement of Ramsey Fringes



D. Vion, A. Aassime, A. Cottet, ... , D. Esteve, and M.H. Devoret, *Science* **296**, 286 (2002).

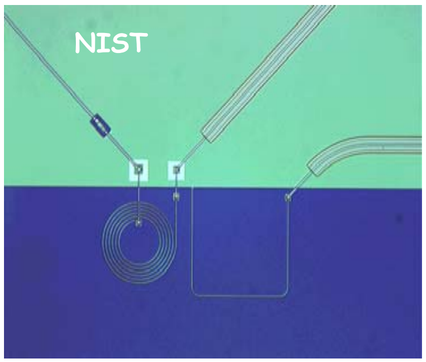
Qubit Readouts



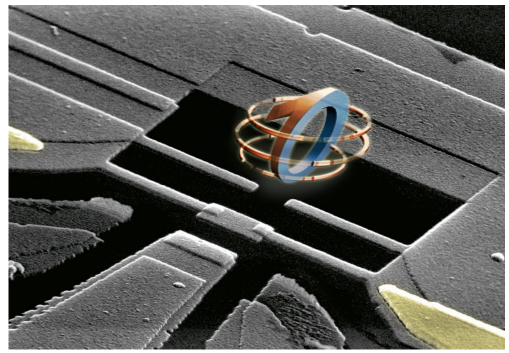
- negligible coupling between readout and qubit in OFF state
  - no dephasing, no relaxation
- strong coupling in ON state
  - minimal relaxation (QND)
  - high fidelity

Readouts for Superconducting Qubits

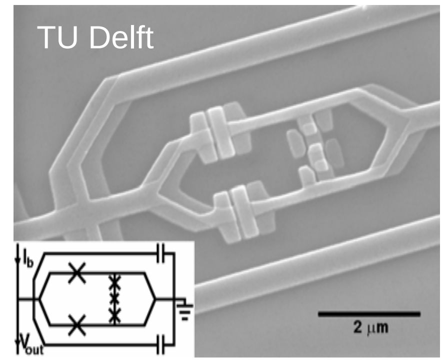
phase



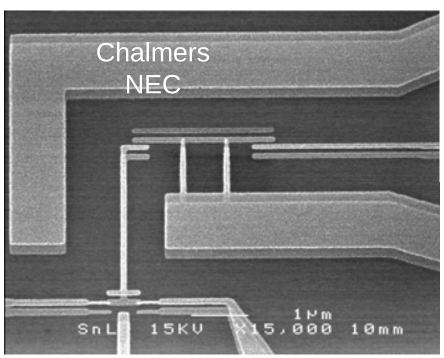
charge-phase



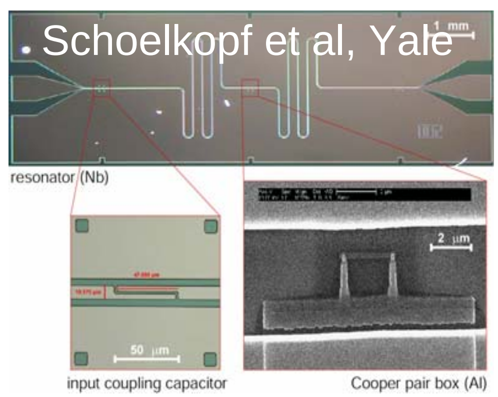
flux



charge



dispersive charge



Phase Qubit Direct Tunneling Readout

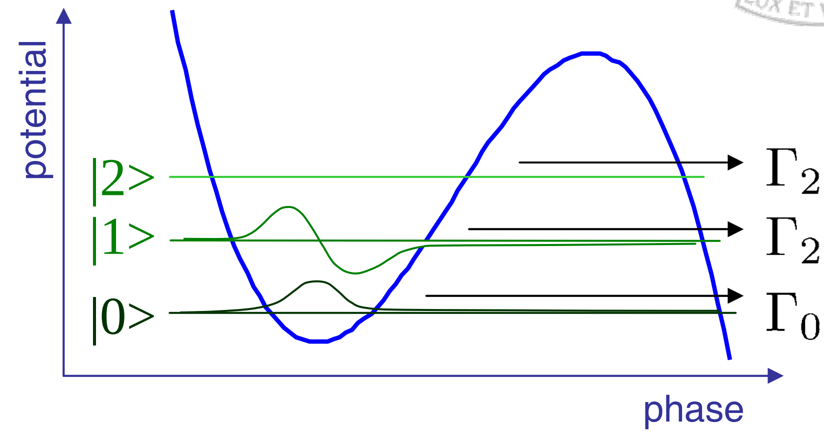
tunneling rates

$$\Gamma_2 \gg \Gamma_1 \gg \Gamma_0$$

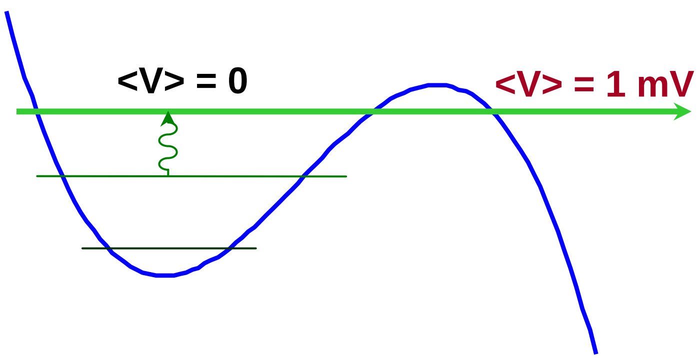
state measurement :

|0> : zero voltage

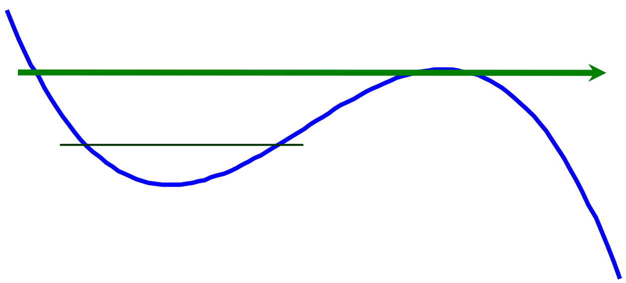
|1> : voltage



pump&probe: ω_{21} microwave pulse



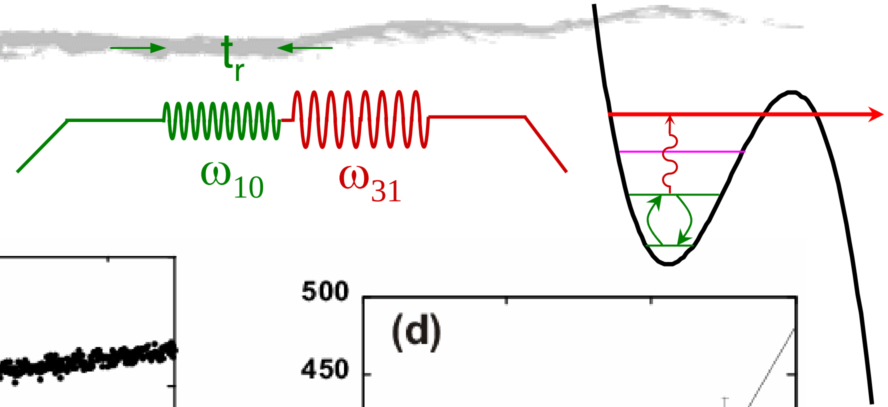
current pulse (lower barrier)

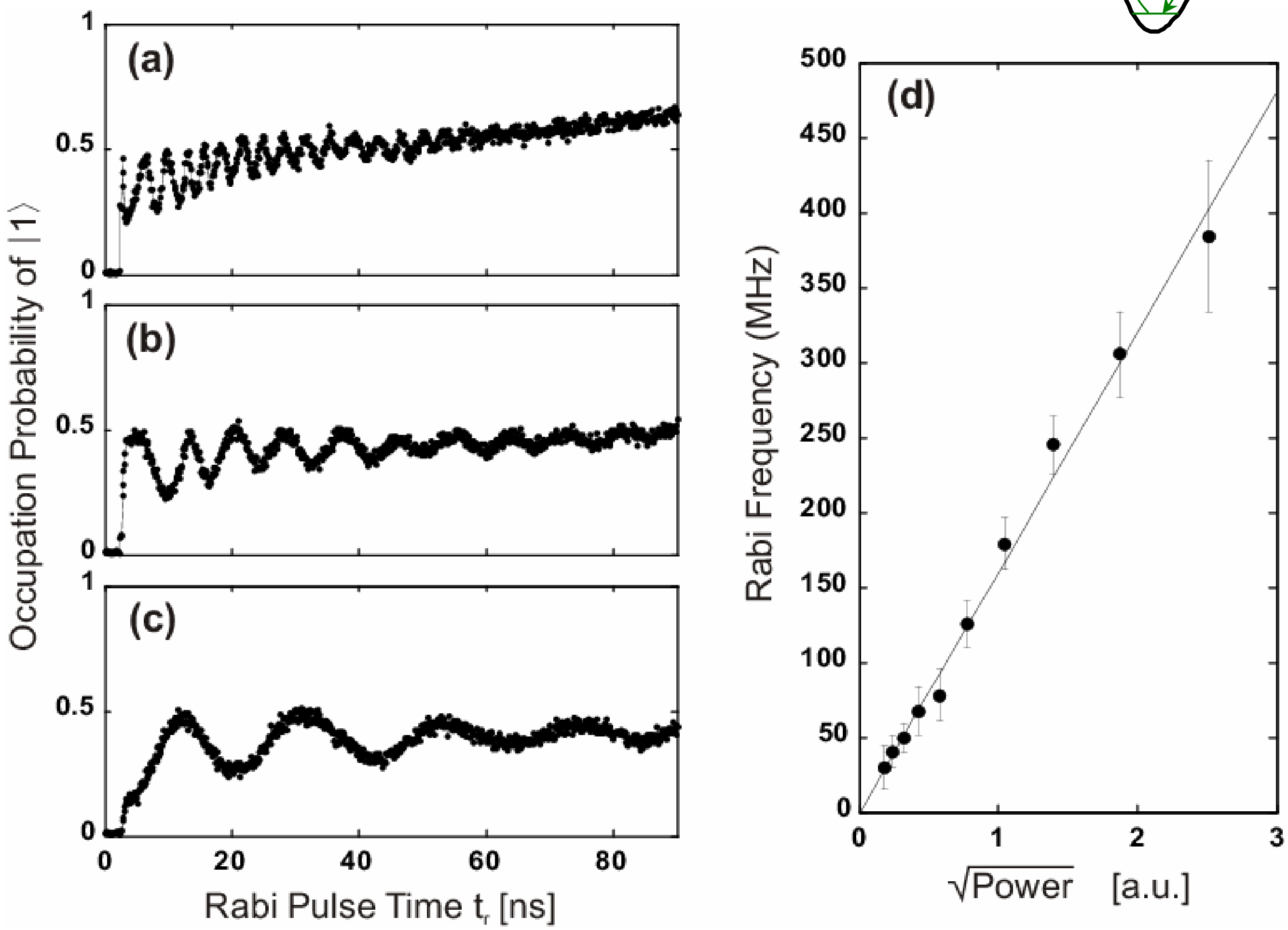


advantages:
on-chip built-in amplification

disadvantages:
- on-chip dissipation
- quasi particle generation
- decoherence

Phase Qubit: Rabi Oscillations

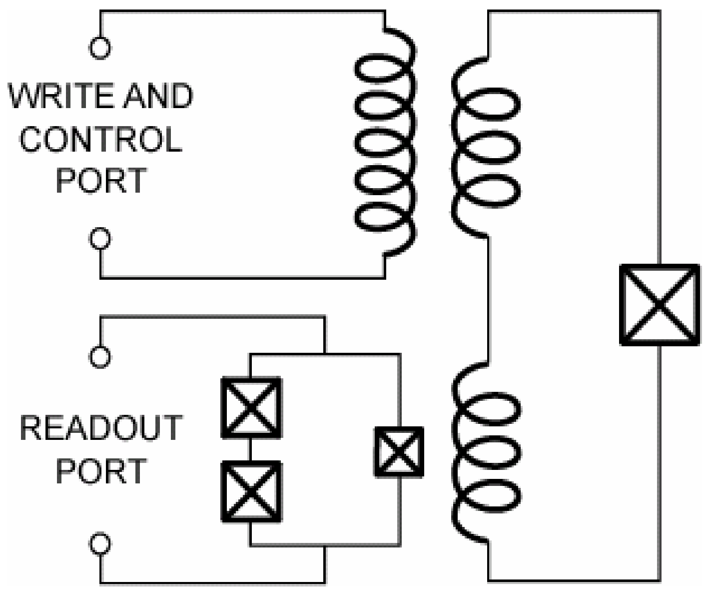


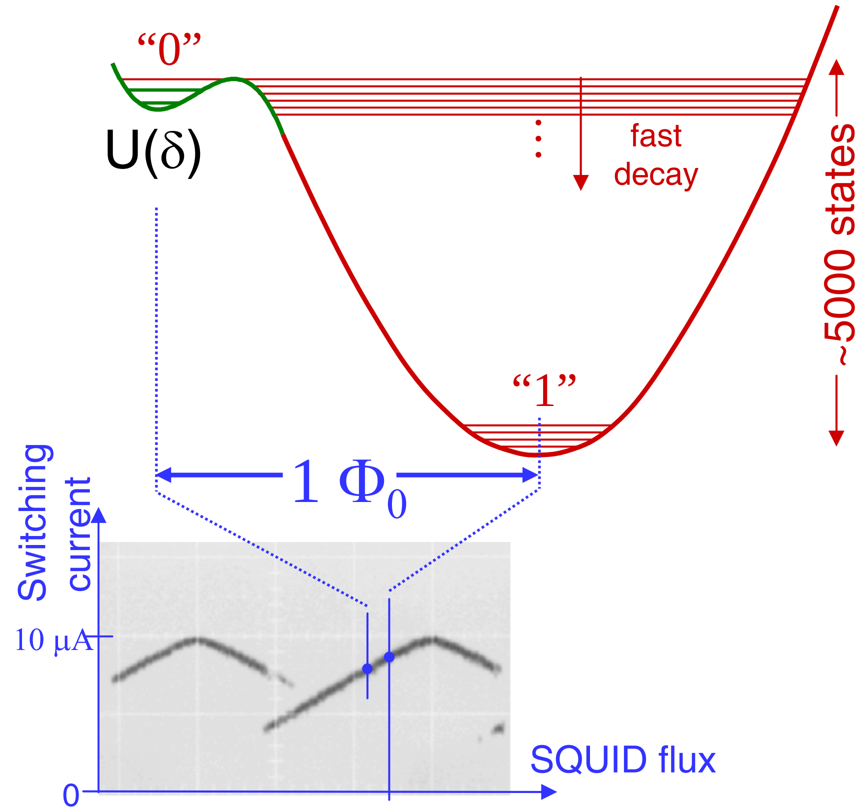


J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, *Phys. Rev. Lett.* **89**, 117901 (2002)

Phase Qubit SQUID Readout

tunneling readout with on-chip DC-SQUID amplifier

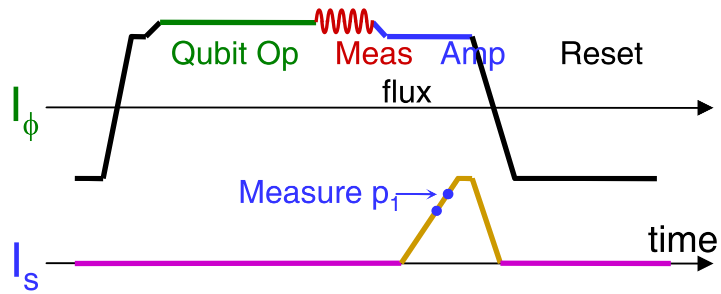




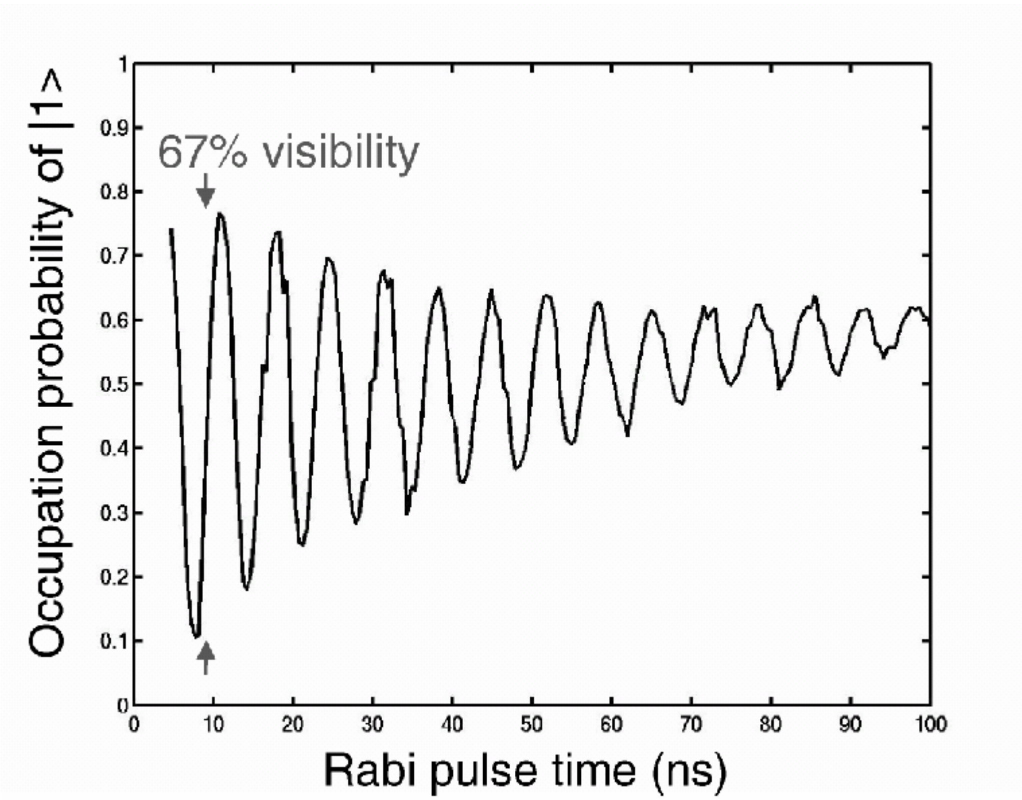
- sample and hold readout
- no quasi particles

R. W. Simmonds, K. M. Lang, ... and J. M. Martinis, *Phys. Rev. Lett.* **93**, 077003 (2004)

Phase Qubit: Rabi oscillations



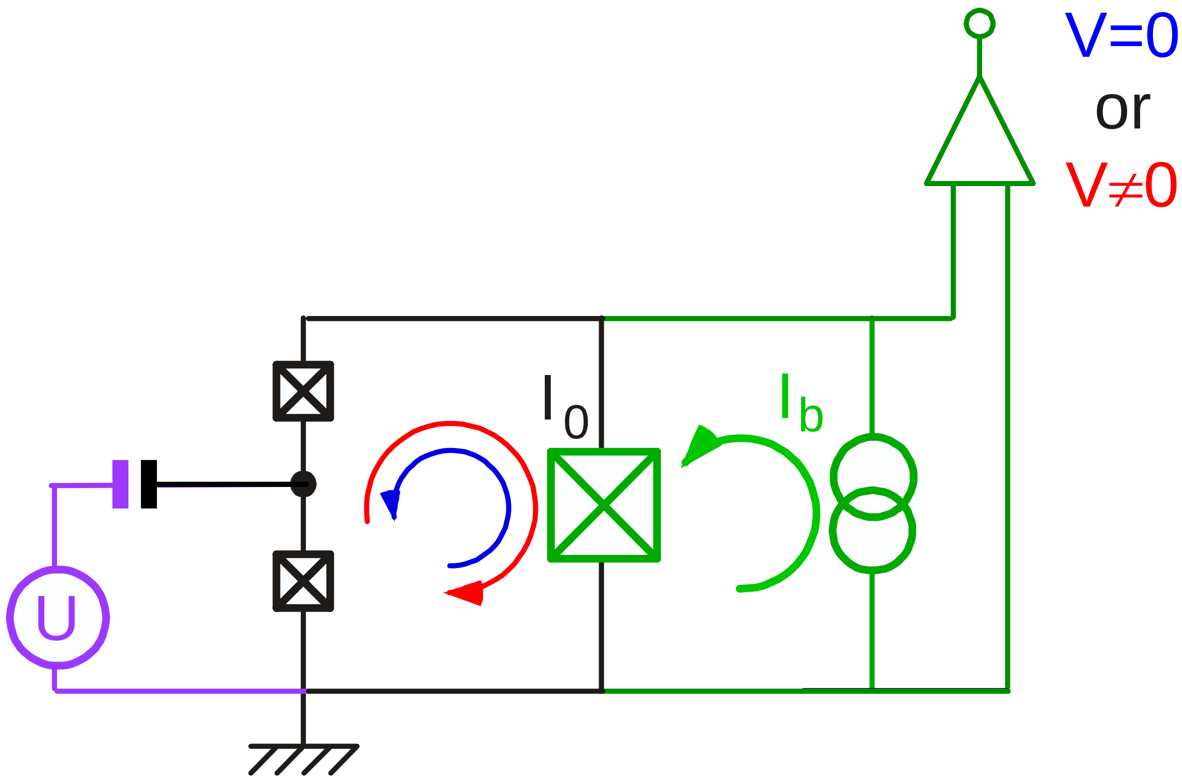
qubit cycle



large visibility

R. W. Simmonds, K. M. Lang, ... and J. M. Martinis, *Phys. Rev. Lett.* **93**, 077003 (2004)

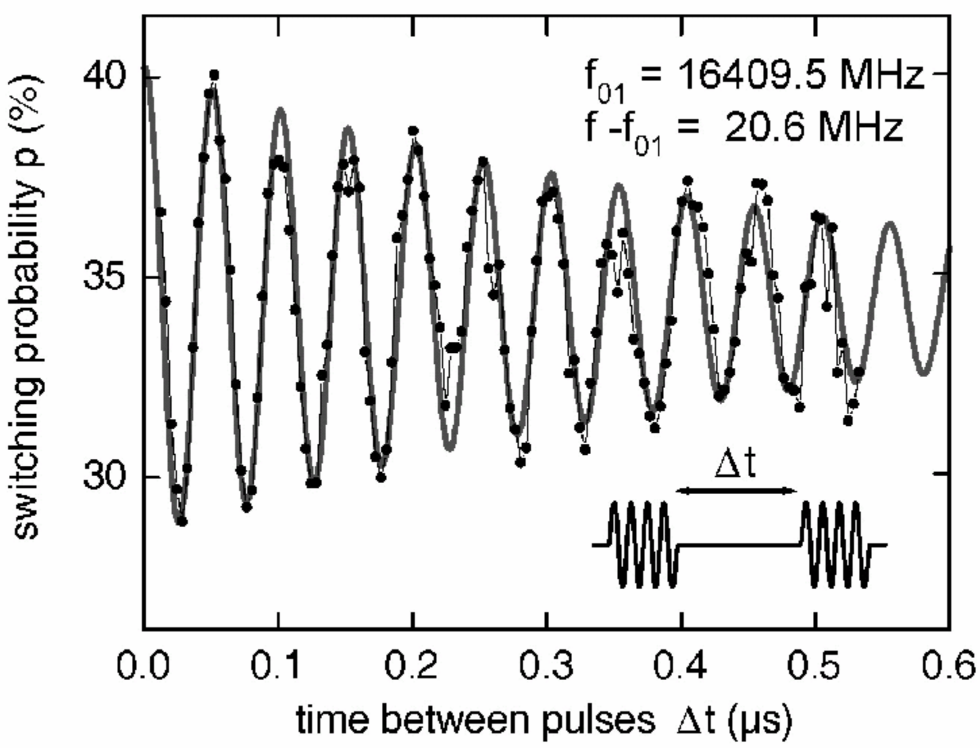
Cooper Pair Box Readout: Quantronium



- high impedance capacitively coupled write and control port
- low impedance inductive readout

D. Vion, A. Aassime, A. Cottet, ... , D. Esteve, and M.H. Devoret, *Science* **296**, 286 (2002).

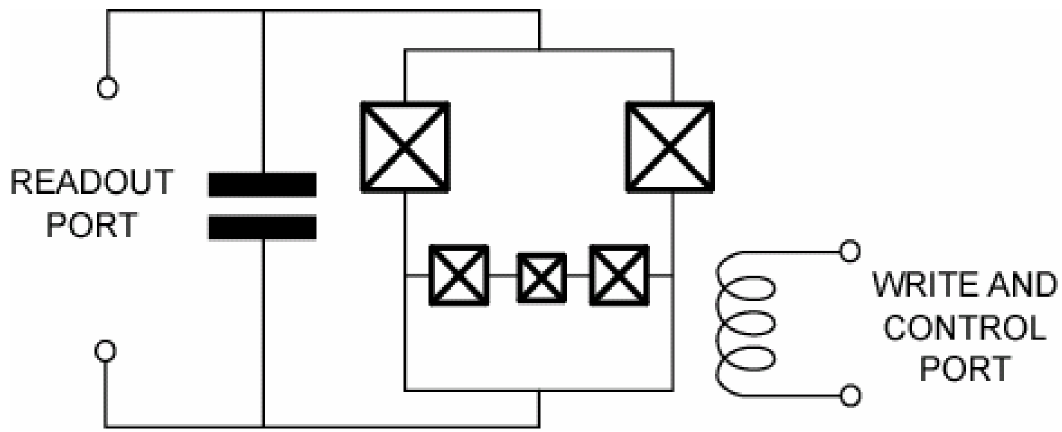
Ramsey oscillations in the Quantronium



- operation at optimal point
- long coherence time

D. Vion, A. Aassime, A. Cottet, ... , D. Esteve, and M.H. Devoret, *Science* **296**, 286 (2002).

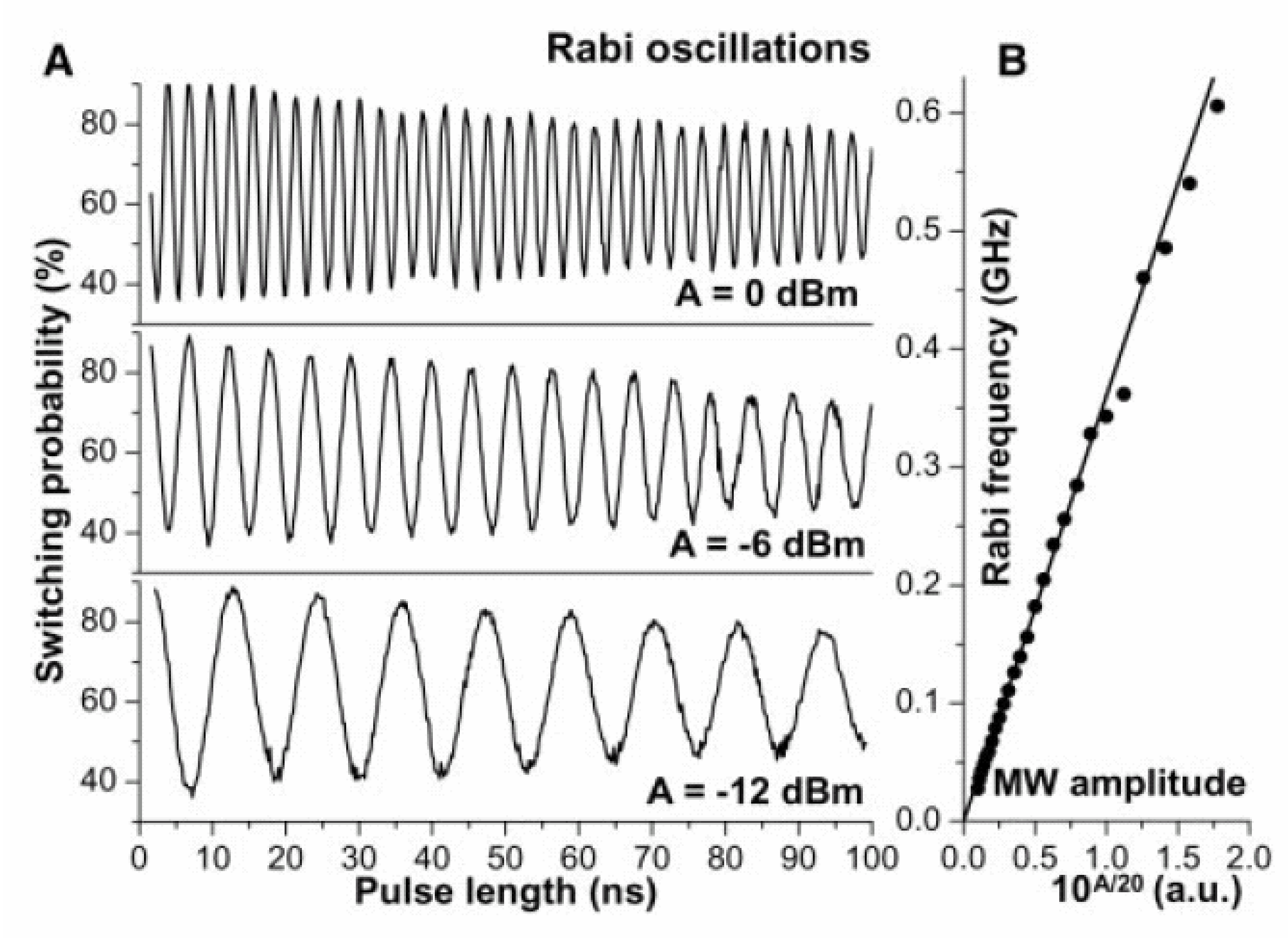
Flux Qubit with Bulit-In Readout



- inductively coupled hysteretic DC-SQUID for readout
- high impedance inductive write and control port

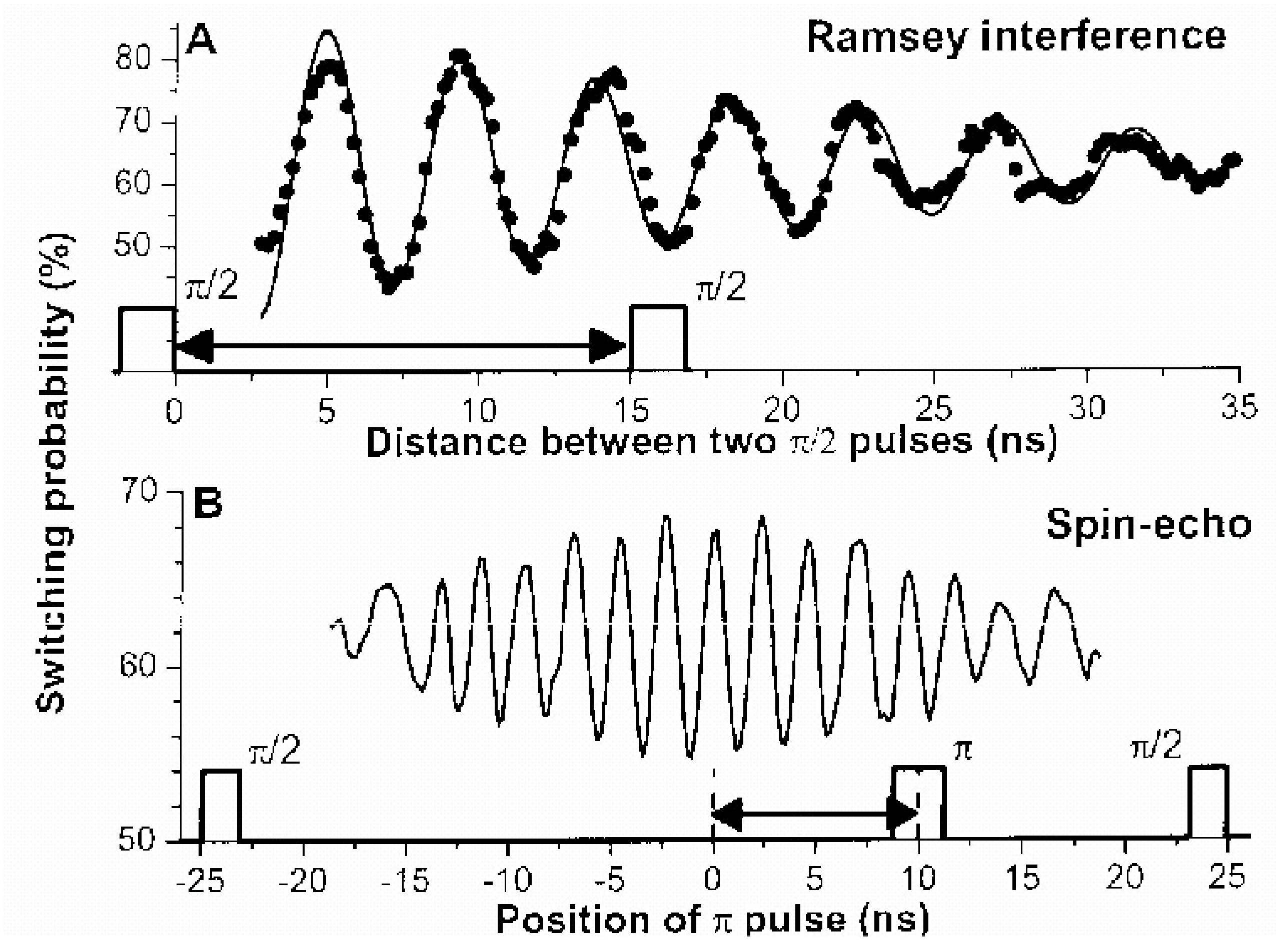
I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, *Science* **299**, 1869 (2003).

Rabi Oscillations with Flux Qubit



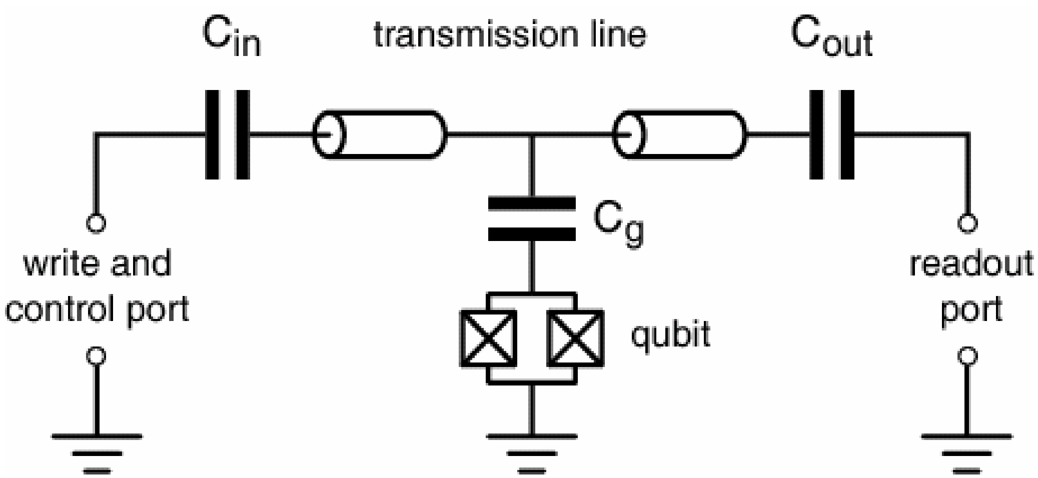
I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, *Science* **299**, 1869 (2003).

Ramsey Fringes with Flux Qubit



I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, *Science* **299**, 1869 (2003).

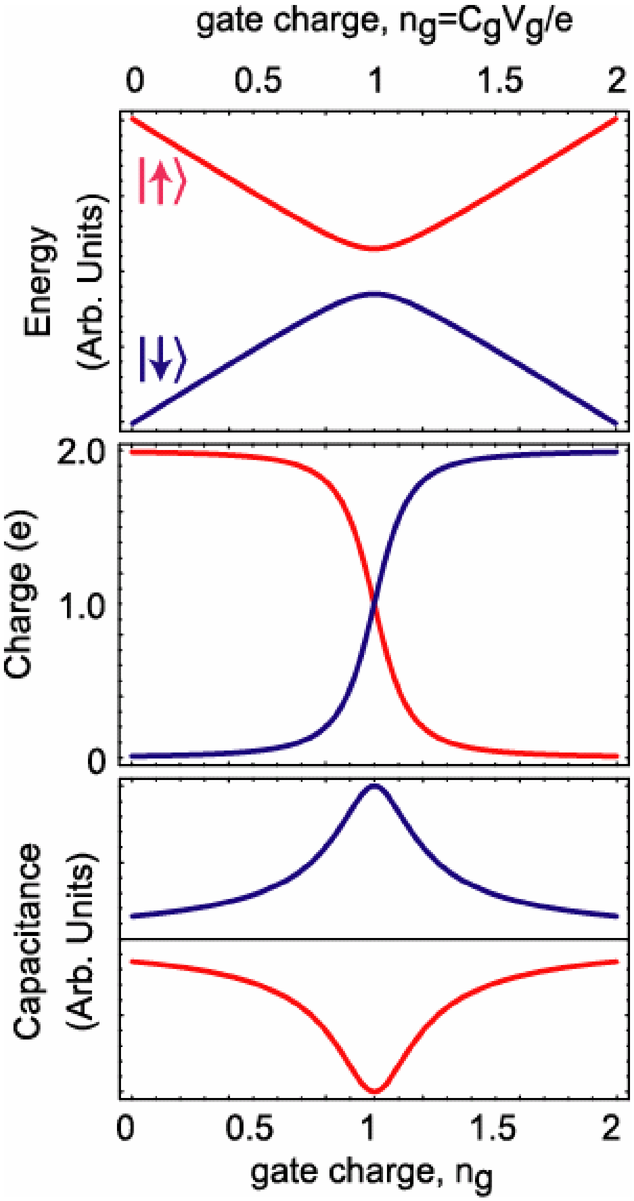
Dispersive Read-Out of Charge Qubit



- dispersive measurement of qubit susceptibility
- no on-chip dissipation
- quantum non-demolition measurement (QND)
- measurement back-action understood

A. Wallraff, D. Schuster, A. Blais, ..., S. M. Girvin and R. J. Schoelkopf, *Nature* **431**, 162 (2004).

The CPB: State Dependent Capacitance



$$E = \frac{1}{2}CV^2$$

$$\frac{dE}{dV} = CV = Q$$

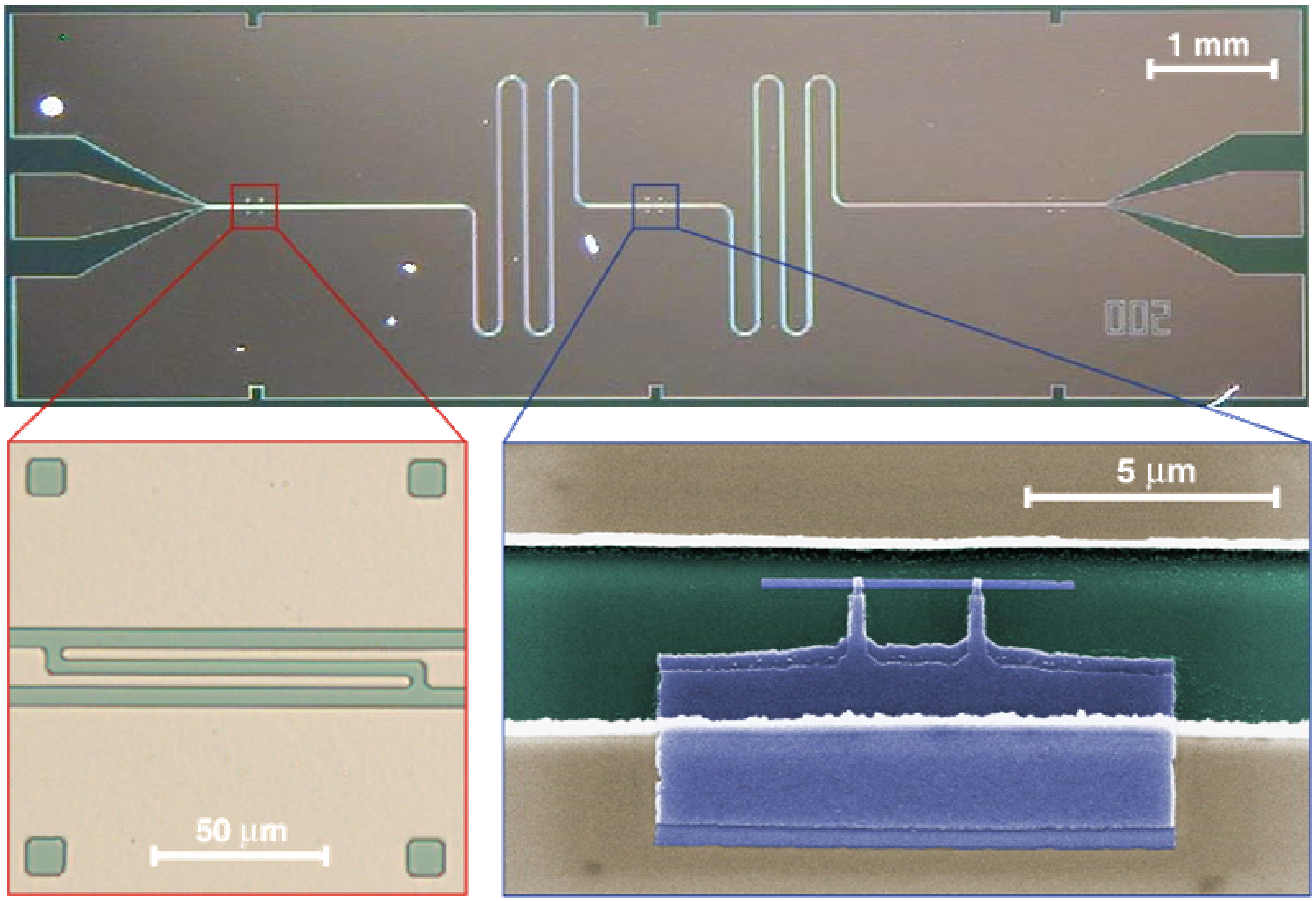
$$\frac{d^2E}{dV^2} = C$$

- n_g-dependent capacitance
- induces shift in resonator ν_r

at magic point $(n_g = 1)$

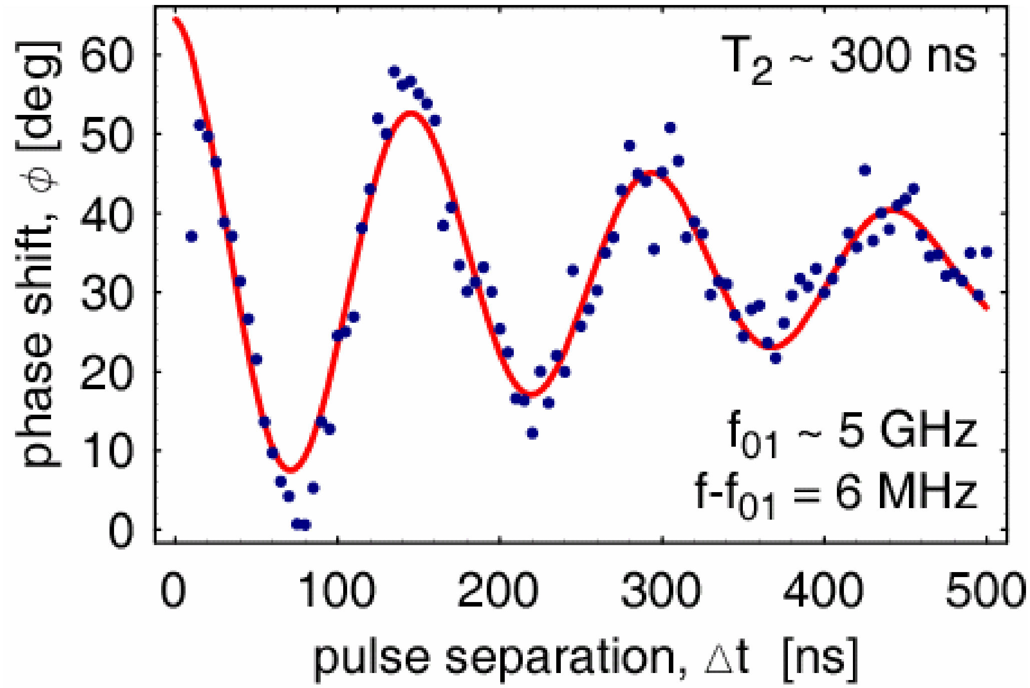
- mininal dephasing
- no charge signal
- BUT maximum phase shift

A Cooper Pair Box in a Cavity



realization of superconducting cavity QED circuit

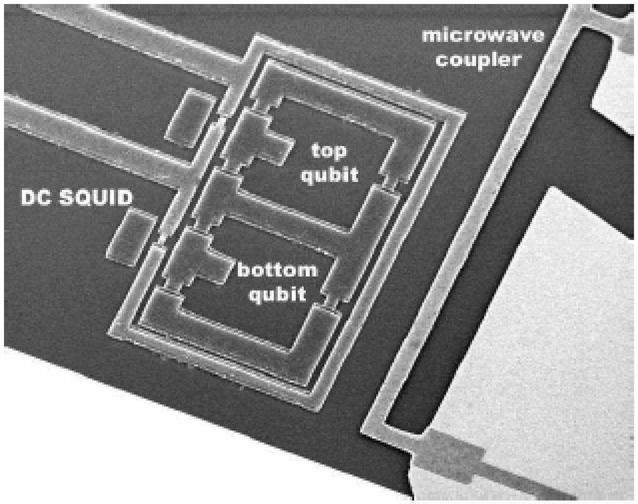
Ramsey Fringes with Circuit QED Readout

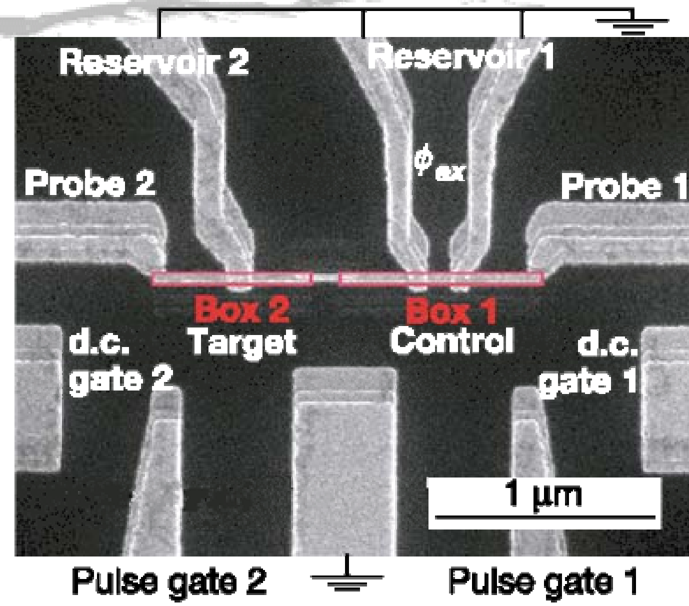


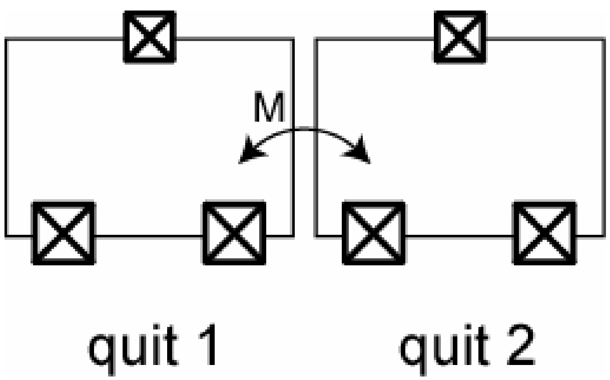
- long life time T_1 ~ 5 μs
- long coherence time

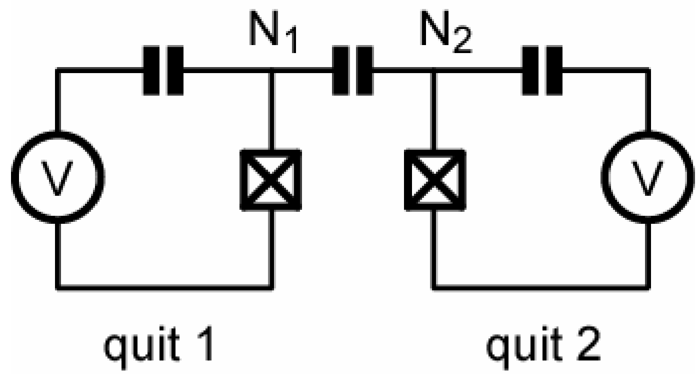
A. Wallraff, D. Schuster, A. Blais, ..., S. M. Girvin and R. J. Schoelkopf, *unpublished* (2004).

Realizations of Coupled Qubits









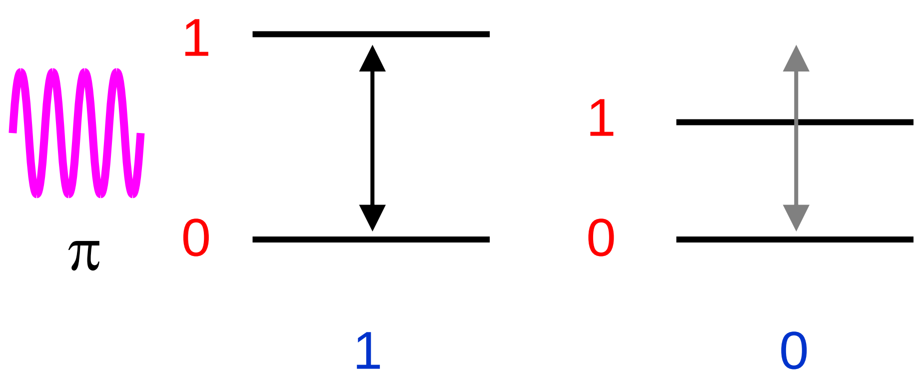
J. B. Majer, F. G. Paauw, A. C. J. ter Haar, C. P. J. Harmans, J. E. Mooij, cond-mat/0308192.
Pashkin Yu. A., … , Nakamura Y., Averin D. V., and Tsai J. S., *Nature* **421**, 823-826 (2003).

Coupled Qubits

$$H = \frac{E_{z1}}{2}\sigma_{z1} + \frac{E_{z2}}{2}\sigma_{z2} + J\sigma_{z1}\sigma_{z2}$$

$$= \frac{1}{2}\left(E_{z1} + 2J\sigma_{z2}\right)\sigma_{z1} + \frac{E_{z2}}{2}\sigma_{z2}$$

Ising coupling



target bit (1)   control bit (2)

CNOT

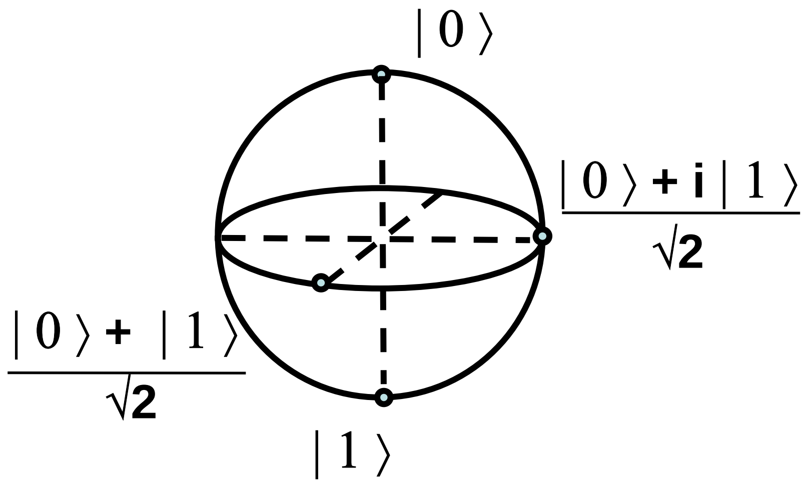
00 → 00

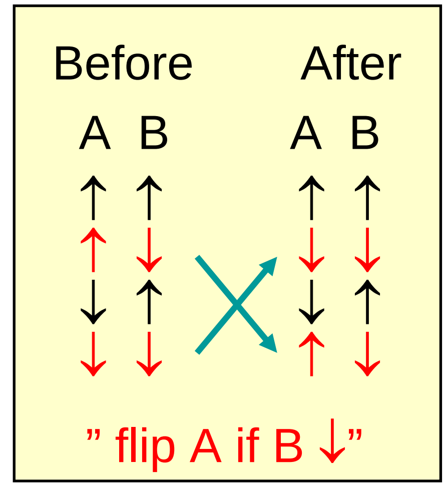
01 → 11

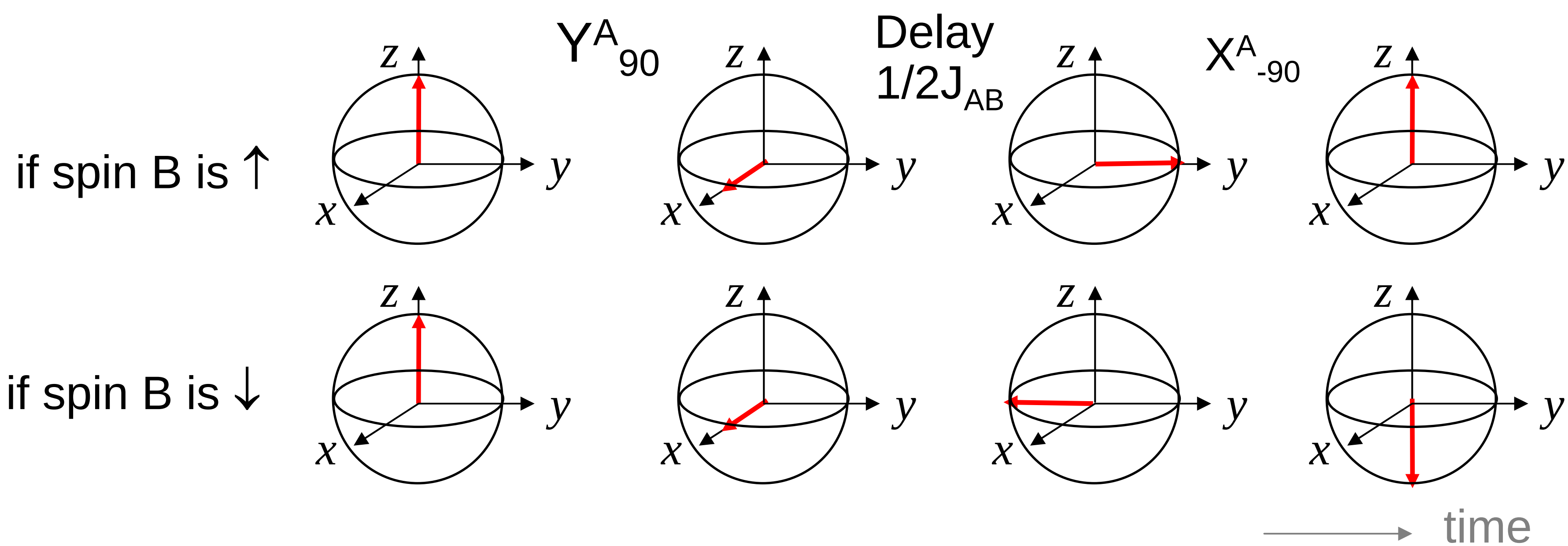
10 → 10

11 → 01

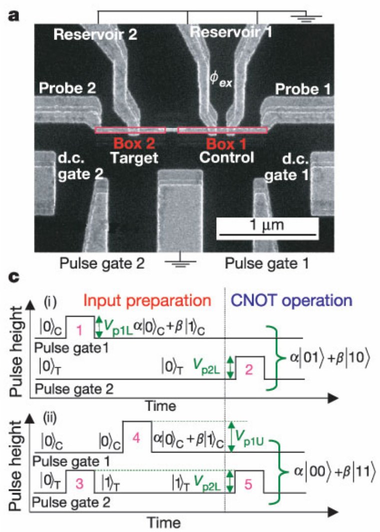
2 Qubit Gates: Controlled-NOT

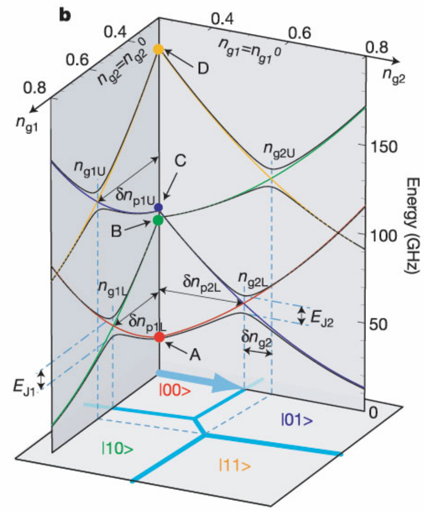


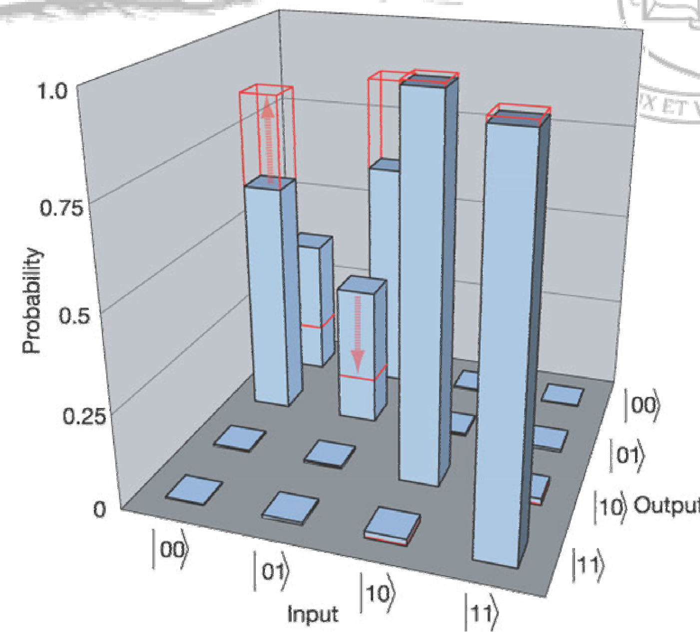


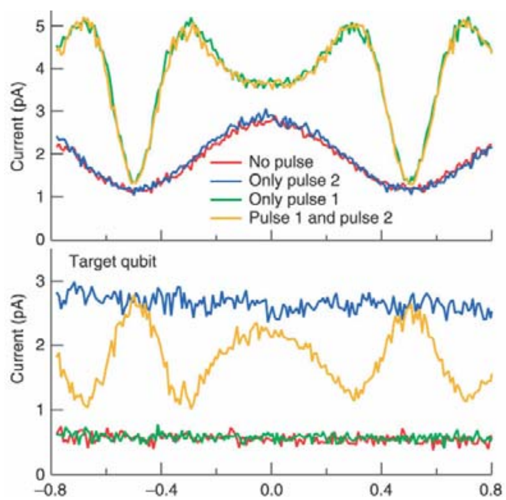


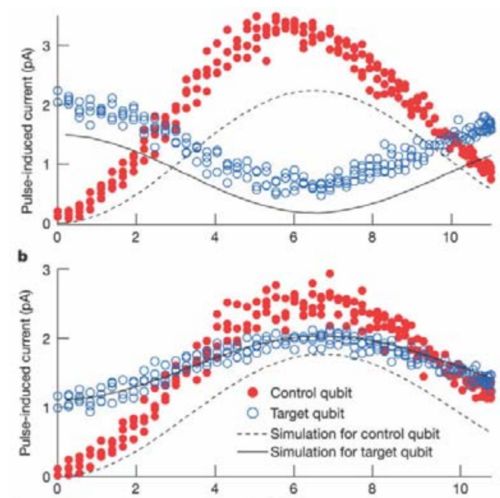
Realization of Controlled-NOT











T. Yamamoto, Yu. A. Pashkin, O. Astaflev, Y. Nakamura, J. S. Tsai *Nature* **425**, 941 (2003)

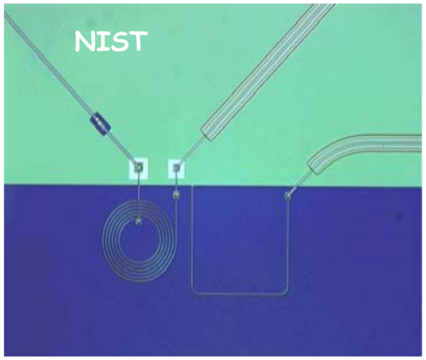
Conclusions

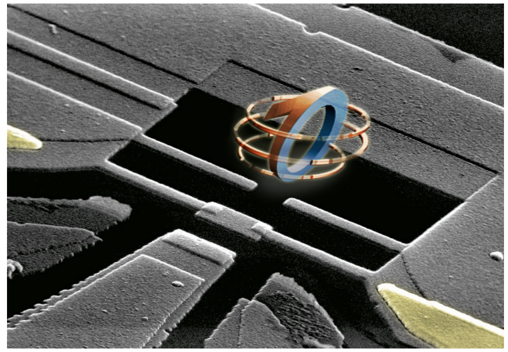
achievements:

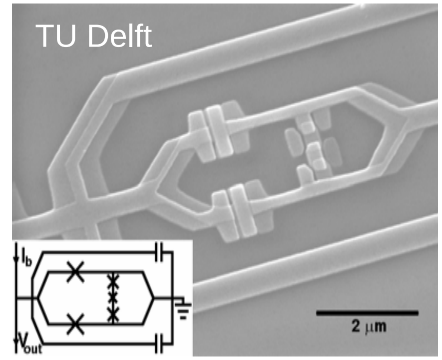
- superconducting qubit architectures have been realized
- different readout strategies have been tested
- qubit initialization, single qubit control has been demonstrated
- first two-qubit gates have been implemented

challenges:

- realize high fidelity, single-shot qubit readout
- control decoherence (increase T_1 and T_2)
- understand limitations imposed by circuit materials and fabrication
- integrate multi-qubit circuits







many thanks to:

M. Devoret, D. Esteve, S. Girvin, J. Mooij, R. Schoelkopf, L. Vandersypen